Supplementary Materials

Part A. Mathematical Proofs

A. Proof of Corollary 1

Differentiating Equation (24) with respect to parameters θ and g

$$\frac{\partial \alpha_1}{\partial \theta} = -\frac{2\theta\rho\sigma_d^2\left[p + g^2\gamma\left(1-\gamma\right)\right]}{p\left[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)\right]^2} < 0 \;\; \forall \theta > 0,\; \rho > 0,\; g > 0,\; \sigma_d > 0,\; \sigma_u > 0,\; p > 0,\; 0 < \gamma < 1$$

$$\frac{\partial \alpha_1}{\partial g} = \frac{2g\gamma(1-\gamma)}{p\left[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)\right]} > 0 \;\; \forall \theta > 0,\; \rho > 0,\; g > 0,\; \sigma_d > 0,\; \sigma_u > 0,\; p > 0,\; 0 < \gamma < 1,$$

which completes the proof. □

B. Proof of Corollary 2

Differentiating each component of the profit function expressed in Equation (31) with respect to parameters θ and g

$$\frac{\partial \Lambda_1}{\partial \theta} = \frac{g(1+p)\left(1-\gamma\right)\left[p+g^2\gamma\left(1-\gamma\right)\right]}{2\left[1-g\theta\left(1-\gamma\right)\right]^2} > 0 \;\; \forall \theta > 0,\; \rho > 0,\; g > 0,\; p > 0,\; 0 < \gamma < 1$$

$$\frac{\partial \Lambda_1}{\partial g} = \frac{(1+p)(1-\gamma)\left\{p\theta + g\gamma\left[2-g\theta(1-\gamma)\right]\right\}}{2\left[1-g\theta\left(1-\gamma\right)\right]^2} > 0 \;\; \forall \theta > 0,\; \rho > 0,\; g > 0,\; p > 0,\; 0 < \gamma < 1$$

$$\frac{\partial \Lambda_2}{\partial \theta} = \frac{\left[p+g^2\gamma\left(1-\gamma\right)\right]^2}{\left[1-g\theta\left(1-\gamma\right)\right]^3\left[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)\right]^2}\left\{-\rho\theta\sigma_d^2\left[1-2g\theta(1-\gamma)\right] + g\left(1-\gamma\right)\left(1+\rho\sigma_u^2\right)\right\}.$$

Then

$$\frac{\partial \Lambda_2}{\partial \theta} > (\leq) 0 \Leftrightarrow g\left(1-\gamma\right)\left(1+\rho\sigma_u^2\right) > (\leq)\rho\theta\sigma_d^2\left[1-2g\theta(1-\gamma)\right]$$

as long as

$$g\theta\left(1-\gamma\right) < 1 \Leftrightarrow g < \frac{1}{\theta\left(1-\gamma\right)} \;\; \forall \theta > 0,\; \rho > 0,\; \sigma_d > 0,\; \sigma_u > 0,\; p > 0,\; 0 < \gamma < 1$$

holds true, which consists of a regularity condition that is imposed to ensure that the manager effectively incurs in a positive level of effort at this stage, as observed in Equation (26). Economically speaking, this restriction means that the indirect network externality promoted by viewers on online gamers cannot be excessively strong. Otherwise, online gamers would have an extremely high incentive on attracting viewers that the manager would face the risk of not obtaining any kind of compensation. Moreover

$$\frac{\partial \Lambda_2}{\partial g} = \frac{\left(1-\gamma\right)\left[p+g^2\gamma\left(1-\gamma\right)\right]}{\left[1-g\theta\left(1-\gamma\right)\right]^3\left[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)\right]}\left\{p\theta + g\gamma\left[2-g\theta(1-\gamma)\right]\right\} > 0$$

for

$$g\theta\left(1-\gamma\right)<1\ \ \forall\theta>0,\rho>0,\sigma_d>0,\sigma_u>0,p>0,0<\gamma<1.$$

If inequality $g\theta\left(1-\gamma\right)<1$ holds, then inequality $g\theta\left(1-\gamma\right)<2$ is necessarily satisfied. Part (II) is a direct consequence from solving Part (I) because the profit function of the platform corresponds to a linear combination of both components: $\Pi=\Lambda_1+\Lambda_2$. Therefore, even without additional computation one can immediately confirm that Corollary 2 is satisfied. □

C. Proof of Proposition 1

Since equilibrium outcomes result from the direct substitution of Equation (32) into Equations (26)−(31), this step is omitted for the sake of brevity. However, one needs to justify restrictions imposed on parameters ρ, σ_d and g. Focusing on the second order condition (SOC) of the price stage follows that

$$\frac{\partial^2\Pi}{\partial p^2}=\frac{-2[1-g\theta\left(1-\gamma\right)]+\dfrac{1}{1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)}}{[1-g\theta\left(1-\gamma\right)]^2}\Leftrightarrow$$

$$\frac{\partial^2\Pi}{\partial p^2}=-\frac{2}{1-g\theta\left(1-\gamma\right)}+\frac{1}{[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)][1-g\theta\left(1-\gamma\right)]^2}\Leftrightarrow$$

$$\frac{\partial^2\Pi}{\partial p^2}=\frac{1-2[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)][1-g\theta\left(1-\gamma\right)]}{[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)][1-g\theta\left(1-\gamma\right)]^2}.$$

Profit maximization requires

$$\frac{\partial^2\Pi}{\partial p^2}<0\Leftrightarrow 1<2[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)][1-g\theta\left(1-\gamma\right)]\Leftrightarrow\frac{1}{2[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}<1-g\theta\left(1-\gamma\right)\Leftrightarrow$$

$$g\theta\left(1-\gamma\right)<1-\frac{1}{2[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}\Leftrightarrow g<\frac{2[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]-1}{2\theta\left(1-\gamma\right)[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}.$$

Therefore

$$g<\overline{g}:=\frac{1+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}{2\theta\left(1-\gamma\right)[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}$$

as claimed in Proposition 1. Nevertheless, one still needs to confirm that this new inequality is more restrictive than the one imposed in Corollary 2. Define $\widetilde{g}:=1/\theta\left(1-\gamma\right)$ and suppose by contradiction that $\widetilde{g}$ is more restrictive than $\overline{g}$ such that

$$\widetilde{g}<\overline{g}\Leftrightarrow\frac{1}{\theta\left(1-\gamma\right)}<\frac{1+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}{2\theta\left(1-\gamma\right)[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}\Leftrightarrow 2[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]<1+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]\Leftrightarrow$$

$$2+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)<1+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]\Leftrightarrow 2<1.$$

By definition, this inequality is impossible. Focusing on the equilibrium price charged to viewers given by Equation (32), one must ensure that it is non-negative since one assumes that viewers are not subsidized to join the platform.

Then

$$p^* \geq 0 \Leftrightarrow \frac{1 - g^2\gamma\,(1-\gamma)}{2} + \frac{1}{2}\left(\frac{1 + g^2\gamma\,(1-\gamma)}{1 - 2g\theta(1-\gamma) + 2\rho\left(\theta^2\sigma_d^2 + \sigma_u^2\right)\left[1 - g\theta\,(1-\gamma)\right]}\right) \geq 0 \Leftrightarrow$$

$$\frac{1 - g^2\gamma\,(1-\gamma)}{1 + g^2\gamma\,(1-\gamma)} \geq -\frac{1}{1 - 2g\theta(1-\gamma) + 2\rho\left(\theta^2\sigma_d^2 + \sigma_u^2\right)\left[1 - g\theta\,(1-\gamma)\right]} \Leftrightarrow$$

$$1 - 2g\theta(1-\gamma) + 2\rho\left(\theta^2\sigma_d^2 + \sigma_u^2\right)\left[1 - g\theta\,(1-\gamma)\right] \geq -\frac{1 + g^2\gamma\,(1-\gamma)}{\left[1 - g^2\gamma\,(1-\gamma)\right]} \Leftrightarrow$$

$$2\rho\left(\theta^2\sigma_d^2 + \sigma_u^2\right)\left[1 - g\theta\,(1-\gamma)\right] \geq -\frac{1 + g^2\gamma\,(1-\gamma)}{\left[1 - g^2\gamma\,(1-\gamma)\right]} - 1 + 2g\theta(1-\gamma)$$

Rearranging the inequality, one finds that

$$\rho \leq -\frac{1 - g\theta(1-\gamma)[1 - g^2\gamma\,(1-\gamma)]}{[1 - g^2\gamma\,(1-\gamma)][1 - g\theta\,(1-\gamma)]\left(\theta^2\sigma_d^2 + \sigma_u^2\right)} \Leftrightarrow$$

$$\rho \leq \overline{\rho} := \frac{g\theta(1-\gamma)[1 - g^2\gamma\,(1-\gamma)] - 1}{[1 - g^2\gamma\,(1-\gamma)][1 - g\theta\,(1-\gamma)]\left(\theta^2\sigma_d^2 + \sigma_u^2\right)}$$

as claimed in Proposition 1. The reader may observe that the lowest possible price corresponds to

$$\lim_{\rho \to \overline{\rho}} p^* = 0$$

and it corresponds to a free access regime applied to viewers when the risk aversion faced by this side of the market reaches the highest possible level. Lastly, one must ensure that the surplus enjoyed by viewers is strictly positive, but not excessive. Knowing that the surplus enjoyed by viewers is generically given by

$$CS_u = \frac{1}{2} - \frac{1}{2}v^2$$

and knowing that, in equilibrium, it corresponds to

$$CS_u^* = \frac{1}{2} - \frac{1}{2}\left(\frac{\rho\left(\theta^2\sigma_d^2 + \sigma_u^2\right)\left[1 - g(1-\gamma)(g\gamma + 2\theta)\right] - g(1-\gamma)(g\gamma + 2\theta)}{1 - 2g\theta(1-\gamma) + 2\rho\left(\theta^2\sigma_d^2 + \sigma_u^2\right)\left[1 - g\theta(1-\gamma)\right]}\right)^2$$

then, it is clear the impossibility to sustain any

$$CS_u^* > \frac{1}{2}$$

because $v^* < 0$ is, by definition, impossible. This means that one must ensure that the previous inequality never holds in equilibrium, which is equivalent to say that one must solve the equality $v^* = 0$ and identify the region of parameters where $v^* \geq 0$ is unambiguously verified, while disregarding the region of parameters associated with $v^* < 0$. Two zeros are found

$$v^* = 0 \Leftrightarrow \sigma_u = \pm\sqrt{\frac{g(1-\gamma)(g\gamma+2\theta) - \rho\theta^2\sigma_d^2[1-g(1-\gamma)(g\gamma+2\theta)]}{\rho\,[1-g(1-\gamma)(g\gamma+2\theta)]}}$$

Knowing that the lowest zero is strictly negative and after observing that the coefficient associated with the quadratic term of the respective polynomial function has a negative value

$$-\rho[1-g(1-\gamma)(g\gamma+2\theta)]\sigma_u^2$$

for any $g < \overline{g}$ and remaining parameter values, the restriction that must be satisfied requires to choose the strictly positive zero and impose

$$0 < \sigma_u \leq \overline{\sigma}_u := \sqrt{\frac{g(1-\gamma)(g\gamma+2\theta) - \rho\theta^2\sigma_d^2[1-g(1-\gamma)(g\gamma+2\theta)]}{\rho\,[1-g(1-\gamma)(g\gamma+2\theta)]}}$$

as claimed in Proposition 1. The reader may observe that the highest possible surplus enjoyed by viewers is given by

$$\lim_{\sigma_u \to \overline{\sigma}_u} CS_u^* = \frac{1}{2}$$

and it corresponds to the case where the uncertainty on the adherence of members from this side of the market to the platform reaches the highest possible level. Finally, $\overline{\sigma}_u$ can be alternatively rewritten as follows

$$\overline{\sigma}_u := \sqrt{\frac{g(1-\gamma)(g\gamma+2\theta)}{\rho\,[1-g(1-\gamma)(g\gamma+2\theta)]} - \theta^2\sigma_d^2}$$

which completes the proof. □

D. Proof of Lemma 1

Differentiating the equilibrium price charged to viewers with respect to θ

$$\frac{\partial p^*}{\partial\theta} = -\frac{[1+g^2\gamma(1-\gamma)]}{\{1+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right) - 2g\theta(1-\gamma)[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]\}^2}\{2\rho\theta\sigma_d^2 - g(1-\gamma)[1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)]\}.$$

Then

$$\frac{\partial p^*}{\partial\theta} > (\leq) 0 \Leftrightarrow -2\rho\theta\sigma_d^2 + g(1-\gamma)[1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)] > (\leq) 0 \Leftrightarrow$$

$$-g[1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)]\gamma + g[1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)] - 2\rho\theta\sigma_d^2 > (\leq) 0 \Leftrightarrow$$

$$\gamma > (\leq) -1 + \frac{2\rho\theta\sigma_d^2}{g[1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)]} \Leftrightarrow$$

$$\gamma < (\geq) \widetilde{\gamma} := 1 - \frac{2\rho\theta\sigma_d^2}{g[1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)]}$$

as clarified in Lemma 1. By definition, $\gamma \in (0,1)$ must hold in equilibrium. It is straightforward to check that

$$\widetilde{\gamma} < 1 \;\; \forall \theta > 0,\; 0 < \rho \leq \overline{\rho},\; \sigma_d > 0,\; 0 < \sigma_u \leq \overline{\sigma}_u,\; 0 < g < \overline{g}$$

Evaluating at the floor follows

$$\widetilde{\gamma} > 0 \Leftrightarrow g > \widetilde{g} := \frac{2\rho\theta\sigma_d^2}{1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)}$$

as clarified in Lemma 1. Since this critical value is strictly positive, we only need to confirm that the inequality $\widetilde{g} < \overline{g}$ constitutes a non-empty space (i.e. that it holds in equilibrium). Suppose, by contradiction, that the opposite is true. Then

$$\widetilde{g} > \overline{g} \Leftrightarrow \frac{2\rho\theta\sigma_d^2}{1+\rho(3\theta^2\sigma_d^2+\sigma_u^2)} - \frac{1+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)]}{2\theta\left(1-\gamma\right)\left[1+\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)\right]} > 0$$

but this inequality is never satisfied, $\forall \theta > 0$, $0 < \rho \leq \overline{\rho}$, $\sigma_d > 0$, $0 < \sigma_u \leq \overline{\sigma}_u$.

In turn, differentiating the equilibrium price charged to viewers with respect to g

$$\frac{\partial p^*}{\partial g} = -g\gamma(1-\gamma) + \frac{g\gamma(1-\gamma)}{1-2g\theta(1-\gamma)+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)[1-g\theta(1-\gamma)]} + \frac{\theta(1-\gamma)[1+g^2(1-\gamma)][1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]}{\{1-2g\theta(1-\gamma)+2\rho\left(\theta^2\sigma_d^2+\sigma_u^2\right)[1-g\theta(1-\gamma)]\}^2}$$

Solving the previous equation one obtains

$$\frac{\partial p^*}{\partial g} = 0 \Leftrightarrow \rho = \pm\frac{\sqrt{4g^2\gamma^2+4g\gamma\theta[3+g^2\gamma(1-\gamma)]+\theta^2\{1-g^2\gamma(1-\gamma)[10+7g^2\gamma(1-\gamma)]\}}}{8g\gamma[1+g\theta(1-\gamma)]^2\left(\theta^2\sigma_d^2+\sigma_u^2\right)^2} \pm \frac{\theta-g\gamma\left\{2-g\theta(1-\gamma)[11-8g\theta(1-\gamma)]\right\}}{8g\gamma[1+g\theta(1-\gamma)]^2\left(\theta^2\sigma_d^2+\sigma_u^2\right)^2}$$

Knowing that the lowest (highest) zero is strictly negative (positive) and given the negative value of the coefficient associated with the quadratic term of the respective polynomial function, it follows that Part (II) of Lemma 1 becomes straightforward, respectively. Naturally, the critical threshold

$$\widetilde{\rho} = \frac{\sqrt{4g^2\gamma^2+4g\gamma\theta[3+g^2\gamma(1-\gamma)]+\theta^2\{1-g^2\gamma(1-\gamma)[10+7g^2\gamma(1-\gamma)]\}}}{8g\gamma[1+g\theta(1-\gamma)]^2\left(\theta^2\sigma_d^2+\sigma_u^2\right)^2} + \frac{\theta-g\gamma\left\{2-g\theta(1-\gamma)[11-8g\theta(1-\gamma)]\right\}}{8g\gamma[1+g\theta(1-\gamma)]^2\left(\theta^2\sigma_d^2+\sigma_u^2\right)^2}$$

indicated in Lemma 1 is immediately identified. □

E. Proof of Lemma 2

Substituting $\theta := g(1-d)$ in the equilibrium variable incentive exposed in Proposition 1 one obtains

$$\alpha_1^* = \frac{[1-g^2(1-\gamma)(1-d)][1+g^2\gamma(1-\gamma)]}{1-g(1-\gamma)[(1-d)g-(1-d)g^3\gamma(1-\gamma)]+\rho[(1-d)^2g^2\sigma_d^2+\sigma_u^2][1-(1-d)g^2(1-\gamma)][1-g^2\gamma(1-\gamma)]}$$

Differentiating this equation with respect to d

$$\frac{\partial\alpha_1^*}{\partial d} = \frac{g[1-g^2\gamma(1-\gamma)]\{g^3\gamma(1-\gamma)^2+2\rho g t(1-d)[1-g^2(1-d)(1-\gamma)]^2[1-g^2\gamma(1-\gamma)]\}}{\{1-(1-d)g^2(1-\gamma)[1-g^2\gamma(1-\gamma)]+\rho[(1-d)^2g^2\sigma_d^2+\sigma_u^2][1-g^2\gamma(1-\gamma)][1-g^2(1-d)(1-\gamma)]\}^2}$$

Evaluating this derivative at $(\sigma_d, \sigma_u) = (0, 0)$ follows

$$\left.\frac{\partial\alpha_1^*}{\partial d}\right|_{(\sigma_d,\sigma_u)=(0,0)} = \frac{g^4(1-\gamma)^2\gamma[1+g^2\gamma(1-\gamma)]}{\{1-g^2(1-d)(1-\gamma)[1-g^2\gamma(1-\gamma)]\}^2}$$

such that

$$\left.\frac{\partial\alpha_1^*}{\partial d}\right|_{(\sigma_d,\sigma_u)=(0,0)} > 0 \quad \forall\theta > 0,\ 0<\rho\le\overline{\rho},\ \sigma_d>0,\ 0<\sigma_u\le\overline{\sigma}_u,\ 0<g<\overline{g},\ 0<\gamma<1.$$

Considering only the presence of uncertainty on the side of viewers one obtains

$$\left.\frac{\partial\alpha_1^*}{\partial d}\right|_{\sigma_d\to 0} = \frac{g^4(1-\gamma)^2\gamma[1+g^2\gamma(1-\gamma)]}{\{1+\rho\sigma_u^2+(1-d)g^4(1-\gamma)^2\gamma(1+\rho\sigma_u^2)-g^2(1-d)(1-\gamma)[1-g^2\gamma(1-\gamma)]\}^2}.$$

Differentiating it with respect to the parameter $s := \sigma_u^2$, which can considered the parameter under evaluation rather than σ_u for the sake of simplicity, yields

$$\left.\frac{\partial^2\alpha_1^*}{\partial s\partial d}\right|_{\sigma_d\to 0} = \frac{2g^4(1-\gamma)^2\gamma[1+g^2\gamma(1-\gamma)][g^2\rho(1-d+\gamma)(1-\gamma)-g^2\gamma\rho(1-\gamma)^2(1-d)-\rho]}{\{1+\rho\sigma_u^2+(1-d)g^4(1-\gamma)^2\gamma(1+\rho\sigma_u^2)-g^2(1-d)(1-\gamma)[1-g^2\gamma(1-\gamma)]\}^3}$$

such that

$$\left.\frac{\partial^2\alpha_1^*}{\partial s\partial d}\right|_{\sigma_d\to 0} > 0 \quad \forall\theta > 0, 0<\rho\le\overline{\rho}, \sigma_d>0, 0<\sigma_u\le\overline{\sigma}_u, 0<g<\overline{g}, 0<\gamma<1.$$

Considering only the presence of uncertainty on the side of online gamers follows that

$$\left.\frac{\partial\alpha_1^*}{\partial d}\right|_{\sigma_u\to 0} = g[1+g^2\gamma(1-\gamma)] \times \frac{1}{g(.)} \times f(.)$$

where

$$g(.) := \left\langle 1-(1-d)g^2[1-g^2\gamma(1-\gamma)]\{1-\gamma+\sigma_d^2\rho(1-d)[1-(1-d)g^2(1-\gamma)]\}\right\rangle^2$$

and

$$f(.) := g^3(1-\gamma)^2\gamma + 2\rho g\sigma_d^2(1-d)[1-g^2(1-d)(1-\gamma)]^2[1-g^2\gamma(1-\gamma)]^2.$$

The sign of $\partial\alpha_1^*/\partial d|_{\sigma_u\to 0}$ is dependent on the sign of the derivative of $f(.)$ and $g(.)$ with respect to the transformed parameter $t := \sigma_d^2$, which can be considered the parameter under evaluation rather than σ_d just for the sake of simplicity. The difference relies on the magnitude of effect, which is leveraged by the factor $g[1+g^2\gamma(1-\gamma)]$ in the case of $\partial\alpha_1^*/\partial d|_{\sigma_u\to 0}$. Both $\partial f(.)/\partial t < 0$ and $\partial g(.)/\partial t < 0$ hold for the relevant region of parameters, thus, one necessarily has

$$\left.\frac{\partial\alpha_1^*}{\partial t\partial d}\right|_{\sigma_u\to 0} > 0 \;\; \forall\theta > 0,\; 0<\rho\le\overline{\rho},\; \sigma_d > 0,\; 0<\sigma_u\le\overline{\sigma}_u,\; 0<g<\overline{g},\; 0<\gamma<1$$

because $g[1+g^2\gamma(1-\gamma)]$ is strictly positive for the relevant region of parameters. With a simple linear transformation by applying the rule on the derivative of the composite function on t and s follows that

$$\begin{cases} \left.\dfrac{\partial^2\alpha_1^*}{\partial\sigma_u\partial d}\right|_{\sigma_d\to 0} > 0 \\ \left.\dfrac{\partial\alpha_1^*}{\partial\sigma_d\partial d}\right|_{\sigma_u\to 0} > 0,\; \forall\theta>0, 0<\rho\le\overline{\rho}, \sigma_d>0, 0<\sigma_u\le\overline{\sigma}_u, 0<g<\overline{g}, 0<\gamma<1 \end{cases}$$

are also satisfied, which completes the proof. □

F. Proof of Lemma 3

Substituting $\theta := g(1-d)$ in the equilibrium variable incentive exposed in Proposition 1 follows

$$e^* = \frac{1+g^2\gamma(1-\gamma)}{1+2g^2(1-d)(1-\gamma)+2\rho[(1-d)^2g^2\sigma_d^2+\sigma_u^2]\{1-g^2[1-d-\gamma(1-d)]\}}$$

Differentiating the equilibrium effort level with respect to d

$$\frac{\partial e^*}{\partial d} = \frac{2g^2[1+g^2\gamma(1-\gamma)]\left\langle -1+\gamma-\rho\{\sigma_u^2-\sigma_d^2(1-d)[2-3(1-d)g^2]\}+\gamma\rho[\sigma_u^2+3(1-d)^2g^2\sigma_d^2]\right\rangle}{\left\langle 1+2g^2(1-d)(1-\gamma)+2\rho[(1-d)^2g^2\sigma_d^2+\sigma_u^2]\{1-g^2[1-d-\gamma(1-d)]\}\right\rangle^2}$$

First, one needs to identify the relevant zero in the domain of ρ. The relevant part of $\partial e^*/\partial d$ that must be solved is given by

$$-1+\gamma-\rho\{\sigma_u^2-\sigma_d^2(1-d)[2-3(1-d)g^2]\}+\gamma\rho[\sigma_u^2+3(1-d)^2g^2\sigma_d^2] = 0$$

such that

$$\rho = \widehat{\rho} := \frac{1-\gamma}{\sigma_d^2(1-d)[2-3g^2(1-d)(1-\gamma)]-\sigma_u^2(1-\gamma)}$$

as claimed in Lemma 3. Based on the behavior of the polynomial function $-1+\gamma-\rho\{\sigma_u^2-\sigma_d^2(1-d)[2-3(1-d)g^2]\}+\gamma\rho[\sigma_u^2+3(1-d)^2g^2\sigma_d^2]$ in the domain of ρ, it is also straightforward to check that

$$\frac{\partial e^*}{\partial d} > (<) 0 \Leftrightarrow \rho > (<) \widehat{\rho}.$$

Then, one needs to confirm that the corresponding parameter space is non-empty (i.e. that it holds in equilibrium). Since $\widehat{\rho}$ is strictly positive for any $0<\gamma<1$, one needs to show that $\rho \in [\widehat{\rho}, \overline{\rho}]$ exists. Define $\Delta\rho := \overline{\rho} - \widehat{\rho}$ and solve

$$\Delta\rho = 0 \Leftrightarrow \sigma_d^2 = \widetilde{\sigma}_d := \frac{g^2\gamma\sigma_u^2(1-\gamma)^2}{(1-d)\{2-g^2(1-d)(1-\gamma)[4-g^2(1-\gamma)(2-2d+\gamma)+2g^4\gamma(1-d)(1-\gamma)^2\}}$$

Since $\widetilde{\sigma}_d$ is strictly positive in the relevant region of parameters follows that

$$\Delta\rho > 0 \Leftrightarrow \sigma_d^2 > \widetilde{\sigma}_d \ \ \forall 0<d<1,\ 0<\rho\leq\overline{\rho},\ \sigma_d>0,\ 0<\sigma_u\leq\overline{\sigma}_u,\ 0<g<\overline{g},\ 0<\gamma<1.$$

thereby confirming Lemma 3. With a simple rearrangement one can alternatively consider that

$$\Delta\rho > 0 \ \ \Leftrightarrow \sigma_d > \sqrt{\widetilde{\sigma}_d} \ \ \forall 0<d<1,\ 0<\rho\leq\overline{\rho},\ \sigma_d>0,\ 0<\sigma_u\leq\overline{\sigma}_u,\ 0<g<\overline{g},\ 0<\gamma<1$$

and the proof is finalized. □

G. Proof of Corollary 3

After defining $\Delta n := n_u^* - n_d^*$ follows

$$\Delta n = \frac{[1+g^2\gamma(1-\gamma)][1-g(1-\gamma)][1-\rho(\theta^2\sigma_d^2+\sigma_u^2)]}{1-2g\theta(1-\gamma)+2\rho[1-g\theta(1-\gamma)](\theta^2\sigma_d^2+\sigma_u^2)}$$

such that

$$\Delta n > 0 \ \ \forall\theta>0,\ 0<\rho\leq\overline{\rho},\ \sigma_d>0,\ 0<\sigma_u\leq\overline{\sigma}_u,\ 0<g<\overline{g},\ 0<\gamma<1$$

thereby allowing us to validate the claim of Corollary 3. □

H. Proof of Lemma 4

Differentiating n_u^* with respect to parameter g

$$\frac{\partial n_u^*}{\partial g} = \frac{2(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\{\theta+\theta g\gamma[1-g(1-\gamma)]\}-\rho(\theta^2\sigma_d^2+\sigma_u^2)\{\theta+\theta g\gamma[1-g(1-\gamma)]\}}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial n_u^*}{\partial g} > 0 \ \ \forall\theta>0,\ 0<\rho\leq\overline{\rho},\ \sigma_d>0,\ 0<\sigma_u\leq\overline{\sigma}_u,\ 0<g<\overline{g},\ 0<\gamma<1.$$

Differentiating n_u^* with respect to parameter $t := \sigma_d^2$

$$\frac{\partial n_u^*}{\partial t} = -\frac{\theta^2\rho[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial n_u^*}{\partial t} < 0 \ \ \forall\theta > 0,\ 0 < \rho \leq \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \leq \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1.$$

Differentiating n_d^* with respect to parameter ρ

$$\frac{\partial n_d^*}{\partial \rho} = -\frac{g(1-\gamma)(\theta^2\sigma_d^2+\sigma_u^2)[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial n_d^*}{\partial \rho} < 0 \ \ \forall\theta > 0,\ 0 < \rho \leq \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \leq \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1.$$

Differentiating n_d^* with respect to parameter θ

$$\frac{\partial n_d^*}{\partial \theta} = \frac{2g(1-\gamma)[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}\left\{g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2-\theta\rho\sigma_d^2\right\}$$

such that

$$\frac{\partial n_d^*}{\partial \theta} > (<) 0 \Leftrightarrow g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)] > (<)\theta\rho\sigma_d^2$$

thereby confirming the ambiguity mentioned in Lemma 4.
Indeed, note that in the absence of uncertainty

$$n_d^*|_{(\sigma_d^2,\sigma_u^2)=(0,0)} = \frac{g(1-\gamma)[1+g^2\gamma(1-\gamma)]}{1-2g\theta(1-\gamma)}$$

and

$$\left.\frac{\partial n_d^*}{\partial \theta}\right|_{(\sigma_d^2,\sigma_u^2)=(0,0)} = \frac{2g^2(1-\gamma)^2[1+g^2\gamma(1-\gamma)]}{[1-2g\theta(1-\gamma)]^2}$$

such that

$$\left.\frac{\partial n_d^*}{\partial \theta}\right|_{(\sigma_d^2,\sigma_u^2)=(0,0)} > 0 \ \ \forall\theta > 0,\ 0 < g < \overline{g},\ 0 < \gamma < 1.$$

In the absence of uncertainty, a higher externality of online gamers on viewers has a positive effect on the equilibrium number of online gamers. As a result, this impact is dissuaded as the uncertainty in the opposite side of the market increases.

Differentiating n_d^* with respect to parameter $s := \sigma_u^2$

$$\frac{\partial n_d^*}{\partial s} = -\frac{g(1-\gamma)\rho[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial n_d^*}{\partial s} < 0 \ \ \forall \theta > 0,\ 0 < \rho \leq \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \leq \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1.$$

which completes the proof. □

I. Proof of Lemma 5

Focus on part (I) of Lemma 5. Recall that $CS_d^* = f^{*2}/2$. Therefore, the impact of a parameter change on CS_d^* is directly proportional to the same impact on f^*. For the sake of simplicity, one exposes the comparative statics with respect to f^*. It follows that

$$\frac{\partial f^*}{\partial \theta} = \frac{2g(1-\gamma)[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}\left\{g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2-\theta\rho\sigma_d^2\right\}$$

such that

$$\frac{\partial f^*}{\partial \theta} > (<) 0 \Leftrightarrow g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2 > (<) \theta\rho\sigma_d^2.$$

Therefore

$$\frac{\partial CS_d^*}{\partial g} = \frac{\partial CS_d^*}{\partial f^*} \times \frac{\partial f^*}{\partial g} = f^* \times \frac{\partial f^*}{\partial g} > (<) 0 \Leftrightarrow g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2 > (<) \theta\rho\sigma_d^2$$

$$\forall \theta > 0,\ 0 < \rho \leq \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \leq \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1$$

thereby confirming the claim expressed in Lemma 5.

Recall that $CS_u^* = 1/2 - v^{*2}/2$, thus, the impact of a parameter change on CS_u^* is inversely related to the same impact on v^*. For the sake of simplicity, one exposes the comparative statics with respect to v^*. It follows that

$$\frac{\partial v^*}{\partial \theta} = \frac{2[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}\left\{\theta\rho\sigma_d^2-g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2\right\}$$

such that

$$\frac{\partial v^*}{\partial \theta} > (<) 0 \Leftrightarrow \theta\rho\sigma_d^2 > (<) g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2$$

$$\forall \theta > 0,\ 0 < \rho \leq \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \leq \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1.$$

Consequently

$$\frac{\partial CS_u^*}{\partial \theta} = \frac{\partial CS_u^*}{\partial v^*} \times \frac{\partial v^*}{\partial \theta} = -v^* \times \frac{\partial f^*}{\partial \theta} > (<) 0 \Leftrightarrow g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2 > (<) \theta\rho\sigma_d^2$$

$$\forall \theta > 0,\ 0 < \rho \leq \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \leq \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1$$

thereby confirming the claim expressed in Lemma 5.

Recall that

$$\frac{\partial n_d^*}{\partial \theta} = \frac{2g(1-\gamma)[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}\left\{g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2-\theta\rho\sigma_d^2\right\}.$$

Then compute

$$\frac{\partial \Pi^*}{\partial \theta} = \frac{[1+g^2\gamma(1-\gamma)]^2}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}\left\{g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]^2-\theta\rho\sigma_d^2\right\}$$

such that a similar conclusion is obtained relative to that observed for n_d^*

$$\frac{\partial \Pi^*}{\partial \theta} > (<) 0 \Leftrightarrow g(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)] > (<) \theta\rho\sigma_d^2$$

thereby allowing to confirm the claim expressed in Lemma 5.

Focus on part (II) of Lemma 5. Recall that $CS_d^* = f^{*2}/2$. Therefore, the impact on CS_d^* is directly proportional to the impact on f^*. For the sake of simplicity, one exposes the comparative statics with respect to f^*. It follows that

$$\frac{\partial f^*}{\partial g} = \frac{2g(1-\gamma)[1+g^2\gamma(1-\gamma)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial f^*}{\partial g} > 0 \;\; \forall \theta > 0,\; 0 < \rho \leq \overline{\rho},\; \sigma_d > 0,\; 0 < \sigma_u \leq \overline{\sigma}_u,\; 0 < g < \overline{g},\; 0 < \gamma < 1.$$

Therefore

$$\frac{\partial CS_d^*}{\partial g} = \frac{\partial CS_d^*}{\partial f^*} \times \frac{\partial f^*}{\partial g} = f^* \times \frac{\partial f^*}{\partial g} > 0 \;\; \forall \theta > 0,\; 0 < \rho \leq \overline{\rho},\; \sigma_d > 0,\; 0 < \sigma_u \leq \overline{\sigma}_u,\; 0 < g < \overline{g},\; 0 < \gamma < 1.$$

Moreover, recall that $CS_u^* = 1/2 - v^{*2}/2$. Therefore, the impact of a parameter change on CS_u^* is inversely related to the same impact on v^*. For the sake of simplicity, one exposes the comparative statics with respect to v^*. It follows that

$$\frac{\partial v^*}{\partial g} = -\frac{2(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\left\langle \theta + g\gamma[1-g\theta(1-\gamma)] + \rho(\theta^2\sigma_d^2+\sigma_u^2)\{\theta + g\gamma[2-g\theta(1-\gamma)]\}\right\rangle}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial v^*}{\partial g} < 0 \;\; \forall \theta > 0,\; 0 < \rho \leq \overline{\rho},\; \sigma_d > 0,\; 0 < \sigma_u \leq \overline{\sigma}_u,\; 0 < g < \overline{g},\; 0 < \gamma < 1.$$

Therefore

$$\frac{\partial CS_u^*}{\partial g} = \frac{\partial CS_u^*}{\partial v^*} \times \frac{\partial v^*}{\partial g} = -v^* \times \frac{\partial f^*}{\partial g} > 0 \;\; \forall \theta > 0,\; 0 < \rho \leq \overline{\rho},\; \sigma_d > 0,\; 0 < \sigma_u \leq \overline{\sigma}_u,\; 0 < g < \overline{g},\; 0 < \gamma < 1.$$

Finally

$$\frac{\partial \Pi^*}{\partial g} = \frac{(1-\gamma)[1+g^2\gamma(1-\gamma)][1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2} \times \frac{\left\langle \theta + g\gamma[2-3g\theta(1-\gamma)] + \rho(\theta^2\sigma_d^2+\sigma_u^2)\{\theta+g\gamma[4-3g\theta(1-\gamma)]\}\right\rangle}{\{1+2\rho(\theta^2\sigma_d^2+\sigma_u^2)-2g\theta(1-\gamma)[1+\rho(\theta^2\sigma_d^2+\sigma_u^2)]\}^2}$$

such that

$$\frac{\partial \Pi^*}{\partial g} > 0 \quad \forall \theta > 0,\ 0 < \rho \le \overline{\rho},\ \sigma_d > 0,\ 0 < \sigma_u \le \overline{\sigma}_u,\ 0 < g < \overline{g},\ 0 < \gamma < 1.$$

which finalizes the proof. $\square$

Part B. Figures and Tables

Figure S1. Optimal number of principal components for each dependent variable

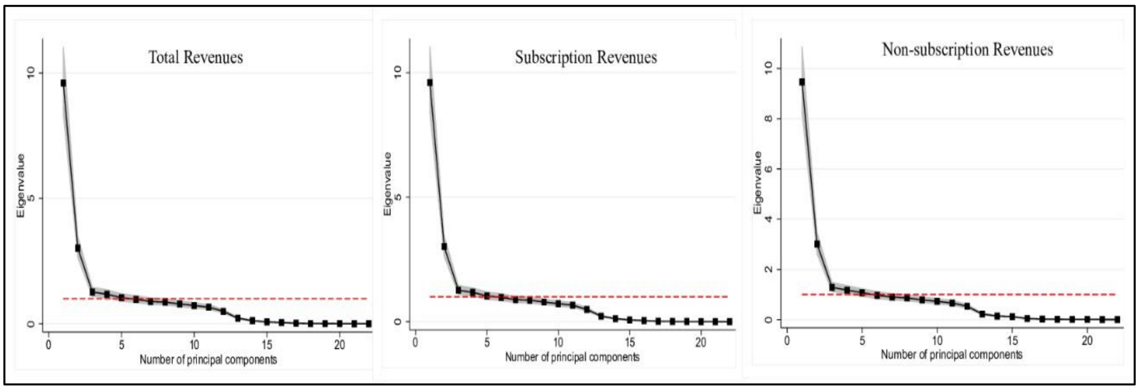

Table S1. Influence of indirect network effects on effectiveness and risk

	Null network effect	Indirect network effects
Effectiveness[2]	1	$\frac{1}{[1 - g\theta(1-\gamma)]^2}$
Risk	σ^2	$\frac{\theta^2\sigma_d^2 + \sigma_u^2}{[1 - g\theta(1-\gamma)]^2}$

Table S2. Summary statistics

Acronym	Description	Mean	Std. Dev.	Max.	Min.
Dependent variables					
zTR	Total revenues	0	1	−1.704	1.395
zSR	Subscription revenues	0	1	−1.697	1.398
zNSR	Non-subscription revenues	0	1	−1.986	1.224
Covariates					
zNewFollowers	New followers	0	1	−1.426	1.886
zSubscribers	Subscribers of the type I, II and III	0	1	−1.901	1.205
zLiveViews	Live views	0	1	−1.179	2.042
zAvgViewers	Average number of viewers	0	1	−2.392	3.494
zMaxViewers	Maximum number of viewers	0	1	−1.768	4.782
zUniqViewers	Unique number of viewers	0	1	−2.028	5.533
zHostRaidViewers	Percentage of host/raid viewers	0	1	−1.720	4.807
zTimeStreamed	Total streaming time in minutes	0	1	−3.421	4.332
zChatAud	Number of unique viewers who chatted with the gamer	0	1	−2.554	4.790
zChatMess	Total number of messages sent	0	1	−2.463	5.134
zClipsCreated	Number of clips created from streams	0	1	−1.386	3.982
zClipViews	Total views of clips created from streams	0	1	−2.470	5.137
zAdBreaks	Time of ad breaks ran by the gamer during streams in minutes	0	1	−2.197	4.196
zAdTimeHour	Average time per hour of ads running during streams in minutes	0	1	−1.458	6.248
zNotif	Number of notifications to the gamer	0	1	−1.695	3.394
zIntSpeed	Average download speed in day t	0	1	−2.061	5.806
PsyC	Psychological state of the online gamer in day $t-1$	0.572	0.495	0	1
zTR_L1	Total revenues lagged one day	−0.004	0.999	−1.704	1.394
zTR_L2	Total revenues lagged two days	−0.007	0.998	−1.704	1.393
zTR_L3	Total revenues lagged three days	−0.011	0.996	−1.704	1.393
zTR_L4	Total revenues lagged four days	−0.014	0.995	−1.704	1.392
zTR_L5	Total revenues lagged five days	−0.018	0.994	−1.704	1.390
zSR_L1	Subscription revenues lagged one day	−0.004	0.999	−1.697	1.397
zSR_L2	Subscription revenues lagged two days	−0.007	0.998	−1.697	1.397
zSR_L3	Subscription revenues lagged three days	−0.011	0.996	−1.697	1.396
zSR_L4	Subscription revenues lagged four days	−0.014	0.995	−1.697	1.395
zSR_L5	Subscription revenues lagged five days	−0.018	0.994	−1.697	1.393
zNSR_L1	Non-subscription revenues lagged one day	−0.003	0.999	−1.986	1.222
zNSR_L2	Non-subscription revenues lagged two days	−0.006	0.999	−1.986	1.221
zNSR_L3	Non-subscription revenues lagged three days	−0.009	0.998	−1.986	1.219
zNSR_L4	Non-subscription revenues lagged four days	−0.012	0.997	−1.986	1.219
zNSR_L5	Non-subscription revenues lagged five days	−0.016	0.997	−1.986	1.219

Notes: Standardized values follow a $N \sim (0,1)$ distribution. Hence, mean (standard deviation) is equal to 0 (1) for all variables except for the psychological state of the online gamer and lagged dependent variables, respectively.

Table S3. Component matrix with rotation and KMO measure of sampling adequacy

Component	Type of dependent variable	Variance	Difference				Proportion of variance explained	Cumulative
PC1	Total revenues	9.477	6.445				0.431	0.431
	Subscription revenues	9.479	6.447				0.431	0.431
	Non-subscription revenues	9.301	6.270				0.423	0.423
PC2	Total revenues	3.032	1.777				0.138	0.569
	Subscription revenues	3.032	1.777				0.138	0.569
	Non-subscription revenues	3.031	1.775				0.138	0.561
PC3	Total revenues	1.255	0.084				0.057	0.626
	Subscription revenues	1.255	0.085				0.057	0.626
	Non-subscription revenues	1.256	0.019				0.057	0.618
PC4	Total revenues	1.170	0.008				0.053	0.679
	Subscription revenues	1.170	0.007				0.053	0.679
	Non-subscription revenues	1.237	0.065				0.056	0.674
PC5	Total revenues	1.163	0				0.053	0.732
	Subscription revenues	1.163	0				0.053	0.732
	Non-subscription revenues	1.172	0				0.053	0.727
	Variable	**PC1**	**PC2**	**PC3**	**PC4**	**PC5**	**UV (%)**	**KMO**
Total Revenues								
	zNewFollowers	0.317					0.029	0.879
	zSubscribers	0.315					0.062	0.881
	zLiveViews	0.311					0.052	0.856
	zAvgViewers						0.266	0.856
	zMaxViewers						0.259	0.847
	zUniqViewers		0.537				0.118	0.592
	zHostRaidViewers					−0.680	0.434	0.565
	zTimeStreamed		0.510				0.186	0.850
	zChatAud					0.401	0.656	0.657
	zChatMess		0.339				0.591	0.905
	zClipsCreated						0.648	0.888
	zClipViews		0.538				0.084	0.635
	zAdBreaks			0.630			0.384	0.713
	zAdTimeHour			0.647			0.400	0.413
	zNotif				0.497		0.604	0.704
	zIntSpeed				0.697		0.374	0.669
	PsyC					0.494	0.640	0.808
	zTR_L1	0.322					0.019	0.906
	zTR_L2	0.322					0.019	0.933
	zTR_L3	0.322					0.022	0.940
	zTR_L4	0.322					0.026	0.914
	zTR_L5	0.320					0.031	0.928
Overall KMO measure of sampling adequacy								0.870
Average interitem covariance								0.260
Number of items in the scale								22
Scale reliability coefficient (Cronbach's α)								0.891
Subscription Revenues								
	zNewFollowers	0.317					0.043	0.862
	zSubscribers	0.315					0.047	0.941
	zLiveViews	0.312					0.085	0.869
	zAvgViewers						0.256	0.843
	zMaxViewers						0.253	0.839

zUniqViewers		0.537				0.118	0.590
zHostRaidViewers					−0.680	0.434	0.594
zTimeStreamed		0.510				0.188	0.872
zChatAud					0.402	0.611	0. 710
zChatMess		0.340				0.593	0.890
zClipsCreated						0.653	0.893
zClipViews		0.538				0.083	0.629
zAdBreaks			0.630			0.378	0.765
zAdTimeHour			0.647			0.409	0.379
zNotif				0.499		0.653	0.814
zIntSpeed				0.696		0.353	0.631
PsyC					0.494	0.687	0.853
zSR_L1	0.323					0.028	0.915
zSR_L2	0.322					0.028	0.925
zSR_L3	0.322					0.029	0.938
zSR_L4	0.322					0.034	0.910
zSR_L5	0.320					0.039	0.926
Overall KMO measure of sampling adequacy							0.873
Average interitem covariance							0.259
Number of items in the scale							22
Scale reliability coefficient (Cronbach's α)							0.890
Non-subscription Revenues							
zNewFollowers	0.313					0.043	0.862
zSubscribers	0.323					0.047	0.941
zLiveViews						0.085	0.869
zAvgViewers						0.256	0.843
zMaxViewers						0.253	0.839
zUniqViewers		0.537				0.118	0.590
zHostRaidViewers					−0.684	0.434	0.594
zTimeStreamed		0.510				0.188	0.872
zChatAud					0.408	0.611	0. 710
zChatMess		0.340				0.593	0.890
zClipsCreated						0.653	0.893
zClipViews		0.539				0.083	0.629
zAdBreaks			0.617			0.378	0.765
zAdTimeHour			0.643			0.409	0.379
zNotif				0.395		0.653	0.814
zIntSpeed				0.707		0.353	0.631
PsyC					0.452	0.687	0.853
zNSR_L1	0.325					0.028	0.915
zNSR_L2	0.324					0.028	0.925
zNSR_L3	0.324					0.029	0.938
zNSR_L4	0.324					0.034	0.910
zNSR_L5	0.322					0.039	0.926
Overall KMO measure of sampling adequacy							0.873
Average interitem covariance							0.259
Number of items in the scale							22
Scale reliability coefficient (Cronbach's α)							0.890

Notes: method of extraction is PCA. Method of rotation is orthogonal varimax (Kaiser off). Rotation has converged with $n = 397$, trace = 22 and $\rho \approx 0.7$ with 5 PCs being the optimal outcome for all possible dependent variables. Blank spaces correspond to the absolute value of loadings below threshold 0.3. UV stands for unexplained variance, whereas KMO stands for Kaiser-Mayer-Olkin.

Table S4. Estimated coefficients with PCA

Component	Coefficient (Std. Error)		
	M1 – Total Revenues	**M2 – Subscription Revenues**	**M3 – Non-subscription Revenues**
PC1 (Committed viewers)	0.318***	0.318***	0.318***
	(0.002)	(0.002)	(0.003)
PC2 (Non-committed viewers)	−0.032***	−0.032***	−0.031***
	(0.005)	(0.005)	(0.005)
PC3 (Publicity dimension)	−0.001	−0.001	−0.041***
	(0.007)	(0.007)	(0.008)
PC4 (Structural dimension)	−0.013	−0.013	−0.009
	(0.010)	(0.010)	(0.013)
PC5 (Emotional dimension)	−0.024*	−0.023*	−0.060***
	(0.013)	(0.013)	(0.016)
R^2	0.976	0.976	0.963
AIC (BIC)	−345.099 (−325.180)	−348.003 (−328.083)	−176.725 (−156.806)
RMSE	0.156	0.155	0.192

Notes: Symbol *** (**) [*] represents 1% (5%) [10%] of significance level, respectively. The regression includes robust standard errors and the constant term was omitted.

Table S5. Estimated coefficients with LASSO

Period	**One day step-ahead**						**Thirty days steps-ahead**	
Technique	**k-fold CV**				**Rolling *h*-step ahead CV**		**Rolling *h*-step ahead CV**	
Model	***LASSO***	***Post-est OLS***	***LASSO***	***Post-est OLS***	***LASSO***	***Post-est OLS***	***LASSO***	***Post-est OLS***
M1	**(λ^*_{LOPT} = 1.841; α^*_{LOPT} = 1)**		**(λ^*_{LSE} = 11.836; α^*_{LSE} = 1)**		**(λ^*_{LOPT} = 39.668)**		**(λ^*_{LOPT} = 52.439)**	
zSubscribers	0.614***	0.611***	0.619***	0.613***	0.616***	0.642***	0.608***	0.642***
zLiveViewers	0.384***	0.394***	0.370***	0.379***	0.349***	0.375***	0.340***	0.375***
zAvgViewers	0.026***	0.041***	0.007***	0.016***				
zMaxViewers	−0.021***	−0.045***						
zHostRaidViewer	0.002***	0.003***						
zChatAud	−0.002***	−0.002***						
zChatMess	−0.003***	−0.005***						
zClipsCreated	−0.019***	−0.021***	−0.010***	−0.021***				
zClipViews	−0.001***	−0.003***						
zAdBreaks	−0.028***	−0.030***	0.015***	0.028***				
zAdTimeH	−0.008***	−0.009***						
zNotif	−0.010***	−0.013***						
zIntSpeed	0.011***	0.012***						
M2	**(λ^*_{LOPT} = 1.840; α^*_{LOPT} = 1)**		**(λ^*_{LSE} = 11.829; α^*_{LSE} = 1)**		**(λ^*_{LOPT} = 43.511)**		**(λ^*_{LOPT} = 52.409)**	
zSubscribers	0.606***	0.603***	0.611***	0.605***	0.605***	0.634***	0.599***	0.634***
zLiveViewers	0.393***	0.403***	0.379***	0.387***	0.355***	0.383***	0.349***	0.384***
zAvgViewers	0.026***	0.042***	0.007***	0.016***				
zMaxViewers	−0.022***	−0.046***						
zHostRaidViewer	0.002***	0.003***						
zChatAud	−0.002***	−0.002***						
zChatMess	−0.003***	−0.005***						
zClipsCreated	−0.018***	−0.021***	−0.010***	−0.021***				
zClipViews	−0.002***	−0.003***						
zAdBreaks	0.027***	0.030***	0.014***	0.027***				
zAdTimeH	−0.008***	−0.009***						
zNotif	−0.010***	−0.014***						
zIntSpeed	0.011***	0.012***						
M3	**(λ^*_{LOPT} = 3.547; α^*_{LOPT} = 1)**		**(λ^*_{LSE} = 15.714; α^*_{LSE} = 1)**		**(λ^*_{LOPT} = 47.989)**		**(λ^*_{LOPT} = 33.077)**	
zNewFollowers	0.050***	0.048***	0.046***	0.056***	0.009***	0.065***	0.026***	0.065***
zSubscribers	0.905***	0.907***	0.907***	0.902***	0.916***	0.912***	0.915***	0.912***
zUniqViewers	0.020***	0.023***	0.007***	0.024***				
zChatAud	0.018***	0.022***	0.004***	0.022***				
zChatMess	0.0003***	0.004***						
zClipsCreated	−0.026***	−0.030***	−0.014***	−0.030***				
zAdBreaks	0.052***	0.057***	0.039***	0.056***	0.005***	0.057***	0.021***	0.057***
zAdTimeH	−0.004***	−0.011***						
zIntSpeed	0.005***	0.010***						
PsyC	−0.014***	−0.020***						

Notes: M1 is the model whose dependent variable is the Total Revenue, M2 is the model whose dependent variable corresponds to Subscription Revenues and M3 is the model whose dependent variable corresponds to Non-subscription Revenues. CV stands for cross-validation. Under k-fold CV considering 10 folds by assumption, α equal to 1 means that the LASSO is preferred to elastic net and ridge regressions. LOPT stands for the λ that minimizes the mean square prediction error (MSPE). LSE stands for largest λ for which MSPE is within one standard error of the minimal MSPE. *** $p < 0.01$.

Table S6. CL model: first-step descriptive statistics of the predicted values of each dependent variable with RF and second-step estimated coefficients with OLS and ARIMA

First-step statistics	$\bar{x}_{RF}$	σ_{RF}	$Variance_{RF}$	Min_{RF}	Max_{RF}	$Skewness_{RF}$	$Kurtosis_{RF}$
M1	−0.083	0.882	0.779	−1.541	0.872	−0.284	1.580
M2	−0.085	0.879	0.773	−1.501	0.873	−0.277	1.571
M3	−0.084	0.894	0.773	−1.747	0.710	−0.706	1.859

Second-step coefficients	**M1 – Total Revenues**			**M2 – Subscription Revenues**			**M3 – Non-subscription Revenues**		
	OLS		**ARIMA**	**OLS**		**ARIMA**	**OLS**		**ARIMA**
Predicted_Viewers_RF	−0.001***	−0.002***	−0.002	−0.001***	−0.002***	−0.002	−0.003***	−0.008***	−0.001
	(0.0003)	(0.0003)	(0.002)	(0.0003)	(0.0003)	(0.003)	(0.0003)	(0.001)	(0.006)
Predicted__Viewers_RF^2		−0.002***			−0.002***			−0.006***	
		(0.0005)			(0.0005)			(0.001)	
Const	0.008***	0.010***		0.008***	0.010***		0.008***	0.012***	
	(0.0003)	(0.0005)		(0.0003)	(0.0005)		(0.0003)	(0.001)	
*dz*TR_AR_L1			0.981***						
			(0.019)						
*dz*SR_AR_L1						0.981***			
						(0.020)			
*dz*NSR_AR_L1									1.134***
									(0.096)
*dz*NSR_AR_L2									−0.140
									(0.095)
*dz*NSR_MA_L1									−0.817***
									(0.069)
Sigma			0.002***			0.002***			0.005***
			(0.0004)			(0.0004)			(0.0003)
AIC			−3843.999			−3820.519			−3127.604
(BIC)			(−3832.046)			(−3808.575)			(−3107.697)
Log pseudo-likelihood			1924.995			1913.260			1568.802

Notes: In the first step, RF is applied to predict the different types of dependent variables by relying on representative covariates of active and passive data covering the opposite side of the market: zNewFollowers, zSubscribers, zLiveViews, zAvgViewers, zMaxViewers, zUniqViewers, zHostRaidViewers and lagged dependent variables up to the fifth lag. In the second step, we regress effective values of each type of revenue enjoyed by the online gamer as a function of the predicted values previously estimated in the first-step which, in turn, depend on active and passive data related to the opposite side of the market, thereby meaning that in the second-step one obtains OLS estimations for the different types of dependent variables as a function of information exclusively related to viewers. As a robustness check, we also consider ARIMA models. For these, the dependent variable corresponds to the first difference of the original variable given the results obtained from Augmented Dickey-Fuller tests. Focusing on M1 and considering the original dependent variable, it follows that: ($z(t)$ = 3.757; p-value = 1.000) without trend nor lags, ($z(t)$ = −1.560; p-value = 0.808) with trend and one period lag, and ($z(t)$ = −1.700; p-value = 0.751) with trend and five lagged periods. Once considering the first difference, one obtains: ($z(t)$ = −3.627; p-value = 0.028) without trend nor lags, ($z(t)$ = −3.719; p-value = 0.021) with trend and one period lag, and ($z(t)$ = −3.317; p-value = 0.064) with trend and five lagged periods. Therefore, the null hypothesis of a unit root is certainly rejected for a significant level of 10% when the first difference of the original variable is adopted. Focusing on M2 and considering the original dependent variable, it follows that: ($z(t)$ = 3.572; p-value = 1.000) without trend nor lags, ($z(t)$ = −1.628; p-value = 0.781) with trend and one period lag, and ($z(t)$ = −1.763; p-value = 0.722) with trend and five lagged periods. Once considering the first difference, one obtains: ($z(t)$ = −3.676; p-value = 0.024) without trend nor lags, ($z(t)$ = −3.689; p-value = 0.023) with trend and one period lag, and ($z(t)$ = −3.315; p-value = 0.064) with trend and five lagged periods. Therefore, the null hypothesis of a unit root is certainly rejected for a significant level of 10% when the first difference of the original variable is adopted. Focusing on M3 and considering the original dependent variable, it follows that: ($z(t)$ = 1.973; p-value = 1.000) without trend nor lags, ($z(t)$ = 0.812; p-value = 1.000) with trend and one period lag, and ($z(t)$ = −1.700; p-value = 0.751) with trend and five lagged periods. Once considering the first difference, one obtains: ($z(t)$ = −12.035; p-value = 0.000) without trend nor lags, ($z(t)$ = −8.274; p-value = 0.000) with trend and one period lag, and ($z(t)$ = −4.516; p-value = 0.001) with trend and five lagged periods. Therefore, the null hypothesis of a unit root is certainly rejected for a significant level of 1% when the first difference of the original variable is adopted. *Mutatis mutandis*, similar outcomes are found with Phillips-Perron, Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) and Elliott, Rothenberg and Stock (ERS) tests. Based on the highest log pseudo-likelihood value and lowest Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC) values, only the best autoregressive models are presented here for each one of the possible dependent variables. Both M1 and M2 are characterized by the presence of one autoregressive (AR) component and by the absence of moving average (MA) components. M3 is characterized by two AR components and one MA component. Symbol *** (**) [*] represents 1% (5%) [10%] of significance level, respectively. The regression includes robust standard errors and the constant term was omitted for ARIMA models.