Journal of
Theoretical and Applied
Electronic Commerce Research

MDPI

Article

Personalized Recommendation in a Retail Platform Under the Hybrid Selling Mode

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Abstract: Retail platforms have widely implemented recommender systems to provide personalized recommendations to consumers, influencing sales significantly. However, under the hybrid selling mode where platforms offer both their products and third-party sellers' products, the profitability of a recommender system and the optimal allocation of recommendations become critical considerations. This paper introduces a game-theoretic model to investigate these issues and unveil how a recommender system and its characteristics influence prices and profits. A key finding is that the recommender system increases prices and profits only if the commission rate is high and the system is profit-oriented or inaccurate. Surprisingly, higher recommendation accuracy does not always translate into higher profits; it is advantageous only in a consumer-oriented system. Moreover, the retail platform tends to allocate more recommendations to its own product than to the third-party seller's product, a strategy known as self-preferencing. This strategy gives the platform a competitive edge and boosts its profit compared to the third-party seller. Furthermore, the degree of self-preferencing varies with the accuracy and orientation of the recommendation system. Specifically, in a consumer-oriented system, self-preferencing increases with accuracy, while in a profit-oriented system, it decreases with accuracy.

Keywords: recommender systems; retail platform; selling mode; personalization; game theory

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Citation: Wang, W.; Han, X.; Ma, Y.; Li, G. Personalized Recommendation in a Retail Platform Under the Hybrid Selling Mode. *J. Theor. Appl. Electron. Commer. Res.* **2024**, *19*, 3606–3631. https://doi.org/10.3390/jtaer19040175

Academic Editor: Mirjana Pejić-Bach

Received: 26 April 2024
Revised: 16 October 2024
Accepted: 17 December 2024
Published: 20 December 2024

1. Introduction

Online retail platforms typically operate in two selling modes. One is the reselling mode, where the platform acquires products from suppliers and sells them to consumers with a markup. The other is the agency selling mode, where the platform acts as a marketplace for suppliers or third-party sellers to sell products, earning commission fees without taking ownership of the products or setting final retail prices. Retail platforms often utilize both selling modes simultaneously, a practice known as the hybrid selling mode [1]. A prominent example is Amazon, a leading retail platform in the global market. Additionally, this mode is widely adopted by several major retail platforms in various local markets, including JD.com in China, Lazada in Southeast Asia, Zalando in Europe, and Kogan in Australia. In this mode, the platform often sells products that directly compete with those of third-party sellers, creating a co-competition scenario in product sales. This dynamic implies that an increase in third-party sellers' product sales could reduce demand for the platform's own products but also result in higher commission earnings. Thus, a key challenge for platforms adopting the hybrid selling mode is to strike a balance between revenue from their own products and that from third-party sellers' products.

Personalized recommendation is a widely-used strategy on retail platforms, proving effective in boosting consumer awareness and driving sales. For instance, Amazon employs the Amazon Personalize tool, powered by a sophisticated algorithm integrating artificial intelligence and machine learning. This algorithm recommends products to each consumer based on their past purchases and interactions, and ratings of displayed items. It also takes into account similar items viewed by consumers with comparable preferences and interests [2]. According to McKinsey's research, as much as 35% of Amazon's sales are generated through personalized recommendations [3]. While personalized recommendation is thought to benefit consumers by recommending products aligning with their preferences and needs (consumer-oriented), there is evidence of biased recommendations from algorithms that push consumers towards purchasing products, such as high-markup items, benefiting the retail platforms or sellers (profit-oriented) [4]. Moreover, recommendations may not be entirely accurate due to factors like methodological flaws, data sparsity, and privacy concerns [5]. These aspects are critical for retail platforms when designing recommender systems. However, the effects of these recommender system characteristics (i.e., orientation and accuracy) on recommendation strategies and firm profits are still unclear. This creates uncertainty for retail platforms when making decisions about adopting recommender systems and allocating recommendations.

Furthermore, research indicates that personalized recommendation can inform consumers of products that they would be unaware of otherwise and enlarge their consideration sets [6,7]. Therefore, it is a powerful tool for a retail platform to manipulate the information configuration of the market through recommendation, thus influencing price competition and product demands. Then, the question of how recommender systems affect price competition between competitive products becomes crucial for the decision-making of retail platforms and sellers [8]. Particularly, in the hybrid selling mode, a retail platform faces a fundamental decision regarding allocating recommendations between its own products and third-party sellers' products. Practical evidence shows that some retail platforms, such as Amazon, often allocate more recommendations to their own products than to third-party sellers' products, a practice known as self-preferencing [9,10]. However, whether this strategy is profitable and how it is influenced by the characteristics of recommender systems remain unclear in both theory and practice. This uncertainty leads to hesitation in platforms' decision-making. For instance, Amazon has recently been adjusting its strategy by offering more advertising resources to third-party sellers [11]. While a growing number of studies have focused on this issue, examining platforms' incentives for self-preferencing and its outcomes through theoretical and empirical methods [12,13], there is still a lack of consensus. Further research is needed to better understand self-preferencing in the context of personalized recommendation.

In summary, the hybrid selling mode and personalized recommendation have gained significant popularity in online retailing. In this context, retail platforms face a trade-off when allocating recommendations between their own products and those of third-party sellers, taking into account the orientation and accuracy of the recommender systems. This raises several key questions that this paper seeks to address: (1) What is the impact of a recommender system on price competition and firm profitability in a retail platform with the hybrid selling mode? (2) How should a retail platform allocate recommendations between its own product and a third-party seller's product? (3) What are the roles of commission rate and recommender system characteristics (i.e., accuracy and orientation) in recommendation strategy and market outcomes?

In our analysis, we employ a game-theoretic model involving a retail platform (referred to as firm R) and a third-party seller (referred to as firm S) who sell horizontally differentiated products to a continuum of consumers uniformly distributed on a Hotelling line. Consumers are classified into several segments based on their initial awareness of the products: captive consumers of each firm who only know about that firm's existence and selective consumers who are aware of both products. Firm R utilizes a recommender system to recommend products to consumers, thereby informing them about the products' existence. Firm R decides which product to recommend to each consumer using a recommendation score, which comprises the expected consumer utility and expected profit if the product is recommended. The relative weight of profit in the recommendation score determines the orientation of the recommender system, with a higher (lower) weight indicating a more profit-oriented (consumer-oriented) system. Additionally, the recommender system is not perfectly accurate: firm R lacks precise knowledge of each consumer's location and can only access a noisy signal indicating consumer location. We define recommendation accuracy as the probability of the signal accurately representing the consumer's true location.

Under the model setup, we first ascertain the impact of the recommender system on equilibrium prices and profits. A key finding is that the presence of the recommender system can either increase or decrease both firms' prices and profits, depending on the commission rate and recommendation characteristics. We summarize the impact in two aspects: the reconfiguration effect caused by converting captive consumers to selective consumers and the recommendation effect resulting from firms' strategic price adjustments to compete for recommendations for their products. Notably, we find that two conditions must be met for the recommender system to weaken price competition and enhance profitability: a sufficiently high commission rate and a recommender system that is sufficiently profit-oriented or inaccurate. This is because both the reconfiguration effect and the recommendation effect are positive under these conditions. Additionally, we observe that firm R is more likely to benefit from the recommender system than firm S, indicating a potential disagreement between the firms regarding the introduction of the recommender system.

Second, we analyze firm R's recommendation strategy in equilibrium and unveil the impacts of the commission rate and recommender system characteristics on this strategy. A key finding is that firm R always allocates more recommendations to its own product compared to firm S's product. Even in a scenario where the commission rate approaches zero (indicating near-symmetric competition between the firms), the asymmetric allocation of recommendations persists, enabling firm R to achieve a higher profit than firm S. This result suggests that firm R can use the recommender system to gain a competitive edge over firm S, providing theoretical backing for the self-preferencing strategy often observed in retail platforms. Additionally, we observe that the likelihood of firm R's product receiving recommendations (i.e., the extent of self-preferencing) increases with recommendation accuracy if the recommender system is consumer-oriented, and decreases if the system is profit-oriented. Furthermore, firm R exhibits higher self-preferencing in recommendations if the recommender system is more profit-oriented or if the commission rate is lower.

Finally, we shed light on the impacts of recommender system characteristics on equilibrium prices and profits. Importantly, we highlight that the effect of recommendation accuracy on prices and profits depends on the recommendation orientation. Higher recommendation accuracy does not necessarily translate to higher profitability for the firms. Specifically, it raises both firms' prices and profits if the recommender system is consumer-oriented, but it has the opposite effect if the system is profit-oriented. Moreover, a more profit-oriented recommender system consistently increases firm R's price and profit, while its impact on firm S's price and profit is modulated by the commission rate: a higher commission rate amplifies the positive impact of a profit-oriented recommender system. Therefore, in a scenario where the commission rate approaches zero, recommendation orientation has no effect on firm S's price or profit.

Our results offer several theoretical contributions. First, although there is extensive literature on retail platform operations, research on the hybrid selling mode—particularly self-preferencing behavior—remains limited. Our study contributes to this field by demonstrating the presence of self-preferencing in a new context, namely personalized recommendations, and revealing the impact of the recommender system characteristics on the degree of self-preferencing in the hybrid selling mode. Second, while many studies have explored the effects of recommender systems on marketing strategies and performance, they primarily focus on the system as a whole. In contrast, our research takes a more detailed perspective, examining the economic effects of key recommender system characteristics (i.e., orientation and accuracy), which also provides valuable insights into recommender system design. Third, most existing analytical studies on recommendation strategy focus on the profitability of recommender systems within the pure agency selling mode. We show that these findings can differ in the hybrid selling mode. For instance, in the pure agency selling mode, profit-oriented recommendations may reduce profits for platforms and sellers, but in the hybrid selling mode, they consistently benefit both.

Altogether, this study enhances our comprehension of the economic effects of a recommender system and its characteristics on a retail platform utilizing the hybrid selling mode, contributing to the literature on recommendation systems and e-marketplace management. Furthermore, our findings provide practical insights for firms' decision-making processes, such as assessing the effectiveness of a recommender system, designing recommender system characteristics, and adjusting pricing and recommendation strategies based on the system's characteristics and the product category's commission rate.

The organization of the remainder of the paper is as follows: Section 2 reviews related literature and positions our research. Section 3 outlines the model setup. In Section 4, we derive the equilibrium strategies with and without the recommender system. Section 5 delves into the impact of the recommender system, followed by Sections 6 and 7, which present the impacts of recommender system characteristics and commission rate, respectively. Finally, the paper concludes in Section 8, summarizing the key findings and suggesting potential avenues for future research. All mathematical proofs are included in the Appendixes A–C.

2. Literature Review

Our research fits within the literature on the operations of online retail platforms. Specifically, we review the related literature from the following three aspects: hybrid selling mode, recommender systems, and recommendation strategy.

2.1. Hybrid Selling Mode

The literature on online retail platforms focuses on key issues like why platforms open to third-party sellers and which selling mode (i.e., agency selling, reselling, and hybrid) to choose. Existing studies have identified crucial factors influencing selling mode choice, such as demand patterns [14], bargaining power [15], control rights over marketing decisions [16], supplier competition [1,17], leader-follower relationships among channel members [18], cross-channel spillovers [19] and spillover effects of consumer awareness [20]. In the context of these studies, price competition is the core element that affects firms' decision-making process. The literature also explores interactions between selling modes and other operational strategies like information sharing [21,22], advertising [23], demand-enhancing services [24–27], and personalized pricing [28]. The context in these studies is more complex for a retail platform to make decisions because it needs to balance among multiple streams of revenues, such as product sales revenue, commission fees, information fees, and advertising fees, for maximizing the profit.

In the framework of the hybrid selling mode, some studies focus on self-preferencing strategies of retail platforms, which is most closely related with our work. Self-preferencing is conventionally achieved through methods like withholding consumer information [28], manipulating search rankings [29,30], distorting product attractiveness [31] and directly blocking third-party sellers [32]. Kittaka et al. [12] offer a systematic review for this. Notably, research has investigated the effects of self-preferencing. Empirical findings from Lam [9] and Lee and Musolff [10] suggest that Amazon's self-preferencing may benefit consumers because they prefer products offered by Amazon. Using a game-theoretic model, Wang and Qiu [33] explore the platform's self-preferencing in product featuring strategy, indicating that the dual role of the retail platform can sometimes benefit consumers by motivating the platform to feature better merchants and encouraging price competition among sellers, allowing more of them to find satisfied product at lower prices. Zou and Zhou [30] find that search neutrality can weaken the price competition between the platform and third-party sellers, which will hurt consumers if most of them ex-ante prefer the third-party product. Long and Amaldoss [34] point out it is not always optimal for the platform to self-preference its private label in sponsored advertising and place it in a prominent position, instead it should concede the ad slot to third-party sellers when the commission rate is large. Our study complements this stream of literature by demonstrating the existence of self-preferencing in a new context, i.e., personalized recommendation, and unveil the impact of recommender system characteristics on the degree of self-referencing in the hybrid selling mode.

2.2. Recommender Systems

Our study is also closely related to the literature on recommender systems. A significant portion of this stream of literature focuses on developing recommendation algorithms using techniques like collaborative filtering, content-based filtering, machine learning, and hybrid methods [35–39]. Notably, recommendation orientation is a crucial consideration in algorithm development. Some algorithms aim to match consumer preferences or enhance consumer utility [40,41], assuming that consumer-oriented recommendations consistently improve firms' profitability. However, this assumption may not hold in cases where there are conflicts of interest between consumers and firms [4]. Hence, some researchers develop algorithms that prioritize increasing firm profits [42,43]. Hosanagar et al. [44] propose a recommendation algorithm that balances consumer utility and firm profits. Aligned with this stream of literature, our study models two factors, consumer utility and firm profit, in the recommendation score of products. From a theoretical standpoint, we examine how the relative weights of these factors affect recommendation strategy and performance in the context of retail platforms operating under the hybrid selling mode.

The impact of recommender systems has been extensively explored in the literature. Some studies investigate how personalized recommendations influence consumer purchase behaviors. For instance, Senecal and Nantel [45] find that consumers are twice as likely to choose recommended products over non-recommended ones. Lee and Hosanagar [46] demonstrate that recommender systems increase consumers' views of advertisements and final conversion rates. Yoon and Lee [47] shed light on consumers' acceptance of AI-driven recommendations. Wan et al. [48] and Donnelly et al. [49] show that personalized recommendations encourage consumers to conduct more searches. He et al. [50] discover that accurate recommendations can enhance consumer satisfaction by triggering a sense of feeling right. Furthermore, the literature explores the impact of recommender systems on sales. For example, Brynjolfsson et al. [51] find that consumers' use of recommendation engines can increase sales diversity in online channels compared to offline channels. Oestreicher-Singer and Sundararajan [52] demonstrate that recommendations lead to flatter demand and revenue distributions. Hosanagar et al. [53] show that recommendations encourage consumers to purchase more similar products. Lee and Hosanagar [54] highlight that recommender systems can have varied effects on sales diversity at the individual-consumer level and aggregate-sales level. Wan et al. [55] distinguish recommendations into retargeted and generic types and identify their different roles in product sales. Overall, much of the literature in this field employs empirical or experimental methods. In contrast, we use an analytical approach, providing a theoretical perspective for understanding the economic impact of recommender systems.

2.3. Recommendation Strategy

There are some analytical studies on product recommendation strategies in the context of retailing. Inderst and Ottaviani [56] and Teh and Wright [57] examine recommendation strategies in two-sided markets, showing that sellers have incentives to pay higher commissions for more recommendations. The key difference from our framework is that sellers cannot determine commission fees on retail platforms, leading to variations in the driving forces behind sellers' and the platform's price adjustments in response to a recommender system. In a typical channel with two competing manufacturers and a common retail platform, Zhou et al. [58] find that product recommendations benefit the platform if the commission rate is high, and the impact on prices depends on both the commission rate and product substitutability. However, they regard recommendations mainly as demand enhancers, lacking personalized features. In contrast, we use a Hotelling-type model to incorporate preference-based recommendations and identify the critical location where the platform is indifferent between recommending two products to consumers. Li et al. [7] explore the interplay between advertising and recommendations, finding that recommender systems can harm sellers and platforms by intensifying price competition and reducing advertising investment. In our model, recommendations are the only source of consumer information, and we emphasize recommendation characteristics' role. Zhou and Zou [59] show that sellers' strategic adjustments in pricing are crucial for a retail platform's recommender system performance. They find that higher recommendation accuracy and profit-oriented recommendations may reduce profits for platforms and sellers. The major difference is our consideration of the hybrid selling mode compared to their pure agency selling mode, leading to different conclusions where profit-oriented recommendations are consistently beneficial for both sellers and platforms.

Finally, our framework closely aligns with that of Li et al. [8], where the recommendation target is determined by a recommendation score combining consumer utility and profit factors. However, several key differences exist between our model and theirs. First, we focus on the hybrid selling mode, while they examine the agency selling mode. Second, they incorporate loyal consumers who exclusively consider purchasing from one firm, essential for their model's equilibrium structure but absent in our setting. These differences lead to unique findings in our study. For instance, we highlight the critical role of the commission rate, where the recommender system's benefits are significant only at high commission rates, and the impact of recommendation accuracy is contingent on the commission rate. In contrast, in the agency selling mode of Li et al. [8], the commission rate is not influential since the retail platform lacks competitive ties with sellers and the sellers are totally symmetric. Additionally, Li et al. [8] find that highly profit-oriented recommender systems may not be profitable due to demand effects in loyal consumer segments. However, in our scenario without loyal consumers, this finding is not applicable.

3. Model Setup

In this section, we introduce the setting of the game-theoretic model.

3.1. Market Structure

We consider a market comprises a retail platform (denoted as "firm R"), a third-party seller (denoted as "firm S"), and a continuum of consumers with unit mass (refer to Figure 1). Firms R and S offer two horizontally differentiated products with a base value of v. For simplicity and model tractability, we assume that v is sufficiently large to ensure full market coverage, meaning that all consumers will purchase a product. The specific range of v is detailed in Appendix A. Firm R sells its product directly to consumers at a price of p_R, while firm S sells its product through firm R's platform at a price of p_S. Firm S pays a commission fee to firm R, with a fraction $\alpha \in (0, 1)$ of p_S on each sale. Firms R and S are positioned at the ends of a unit-length Hotelling line, with R at 0 and S at 1.

Consumers are uniformly distributed on the Hotelling line, signifying heterogeneity in their product preferences [60]. Specifically, a consumer located at location $x \in [0, 1]$ on the Hotelling line receives utilities of $U_R = v - tx - p_R$ and $U_S = v - t(1 - x) - p_S$ from purchasing products R and S, respectively. Here, location x represents the horizontal attributes of the consumer's ideal product. The distance between the consumer and product $i \in \{R, S\}$ reflects the degree of misfit between the consumer's ideal product and product i. This misfit imposes a cost (or disutility) on the consumer, which is proportional to the degree of misfit when considering whether to purchase the product. Therefore, a consumer located at x incurs a total misfit cost of tx for product R and $t(1 - x)$ for product S, where $t > 0$ is the unit misfit cost. Since the market is fully covered with a sufficiently large v, the equilibrium prices and profits are proportional to t. Thus, t does not affect the relative sizes of the equilibrium prices or profits across different cases, allowing us to normalize it to 1 for simplicity. Additionally, we assume that each consumer has unit demand for the product.

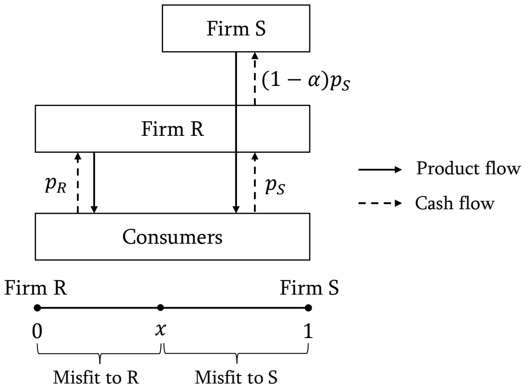

Figure 1. Market structure.

3.2. Consumer Awareness

Consumers are divided into the following segments based on their initial product awareness: "captive consumers" of firm $i \in \{R, S\}$ who are aware only of product i (each segment with a proportion of $\theta_i \in (0,1)$), and "selective consumers" who are aware of both products (with a proportion of $1-\theta_R-\theta_S$). We assume that consumers who are completely uninformed about either product are not relevant to personalized recommendation. This is because in reality, if a consumer has no knowledge of a product category, they likely have not searched or browsed relevant information about it, resulting in limited data for profiling by firms. Furthermore, the information configuration with captive and selective consumers is considered exogenous, influenced by factors such as brand popularity and consumers' prior knowledge, which are outside this study's scope. Specifically, we focus on a representative case where the firms initially have equal brand awareness, meaning that their captive consumer segments are of equal size, i.e., $\theta_R = \theta_S = \theta \in (0, \frac{1}{2})$. This is not uncommon in practice for brands that are equally competitive in the market. Moreover, this assumption is crucial for ensuring the concavity of the profit functions and the existence of closed-form solutions [8]. It also enables us to focus on the driving role of the hybrid selling mode in firm R's self-preferencing behavior in the personalized recommendation context. Additionally, consumers' awareness of the products is independent of their preferences. Thus, each consumer segment is uniformly distributed on the Hotelling line. When making purchase decisions, selective consumers compare product utilities and choose the one offering higher utility, while captive consumers purchase the product they are aware of.

3.3. Recommendation Accuracy

Firm R utilizes a recommender system to offer personalized recommendations to consumers, leveraging data like past purchases and browsing records to predict their preferences. When a captive consumer receives a recommendation for a product they are not aware of, they become a selective consumer as they gain knowledge of both products. We assume that firm R incurs no variable costs for providing recommendations to consumers. Additionally, we model the recommendation system as imperfectly accurate, following a standard approach from related literature [7,8,61]. Specifically, firm R observes a signal s that indicates a consumer's location or preference. This signal represents the output of a predictive algorithm, e.g., Amazon's "item-based collaborative filtering" [62]. The signal matches the consumer's true location with a probability of $\beta \in (0,1)$, while being uninformative and uniformly distributed on $[0,1]$ with a probability of $1-\beta$. We term β as recommendation accuracy. For a consumer whose true location is at $y \in [0,1]$, the probabilities are $\mathbb{P}(s = y | x = y) = \beta$ and $\mathbb{P}(s \neq y | x = y) = 1-\beta$. Using Bayesian updating, we derive the consumer's expected location conditional on the signal as follows:

$$\mathbb{E}(x | s = y) = \frac{1-\beta}{2} + \beta y. \tag{1}$$

Notably, we assume that firm S does not incur variable costs for recommendations. In practice, sellers do not "buy" or explicitly pay for recommendations, which is a key distinction between recommendations and traditional advertising [7]. Even in cases where sellers pay for recommendations, the payment is typically performance-based, such as cost-per-purchase [63]. In this context, the variable costs for recommendations are proportional to product demand, since in our model, consumers purchase the products recommended to them by the platform. Therefore, the variable costs for recommendations can be mathematically reduced to the commission fee.

3.4. Recommendation Orientation

To facilitate comparison with previous literature, we adopt the same recommendation mechanism as in Li et al. [8]. Specifically, firm R decides which product to recommend to a consumer based on two key factors: the consumer's utility and the profit that the consumer can generate for firm R. Since firm R lacks knowledge of a consumer's true location, it determines the recommendation of product i according to the following score:

$$\begin{aligned} C_i = & \mathbb{E}(\text{the consumer's utility}|i \text{ is recommended}) \\ & + \tau\mathbb{E}(\text{firm R's profit from the consumer}|i \text{ is recommended}) \end{aligned} \quad (2)$$

Here, $\tau > 0$ represents the relative weight of firm R's profit in the score. A higher (lower) value of τ indicates a more profit- (consumer-) oriented recommendation system. A consumer with a signal of $s = y$ expects utilities of $v - \mathbb{E}(x|s = y) - p_R$ and $v - [1 - \mathbb{E}(x|s = y)] - p_S$ from products R and S, respectively. The expected profit for firm R from the consumer is p_R if the consumer buys product R, or αp_S if the consumer buys product S. Firm R will recommend product R (S) to the consumer if and only if $C_R \geq C_S$ ($C_R < C_S$). Additionally, we assume that recommendation orientation (value of τ) is known by firm S.

3.5. Game Sequence

The game unfolds in the following sequence (refer to Figure 2): Stage 1: Firms R and S simultaneously set their prices p_i. Stage 2: Firm R determines the product to recommend to each consumer. Stage 3: Consumers make purchase decisions.

In our subsequent analysis, we examine two cases: one without the recommender system, denoted as Case $\mathcal{N}$, and the other with the recommender system, denoted as Case $\mathcal{R}$. This comparison enables us to reveal the distinct impact of the recommender system on the market outcomes. The notation of the model is presented in Table 1.

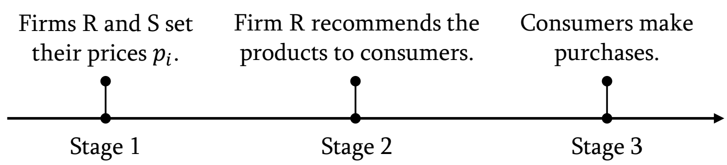

Figure 2. Game sequence.

Table 1. Notation of the model.

Notation	Explanation
$i \in \{R, S\}$	Index of firm R or S
$\mathcal{N}, \mathcal{R}$	Index of the cases with and without the recommender system
v	Base value of the product
x	True consumer location
y	Consumer location indicated by the signal
β	Recommendation accuracy
θ	Initial proportion of each firm's captive consumers
α	Commission rate
C_i	Recommendation score of product i
τ	Relative weight of profit in the recommendation score
p_i	Price of firm i
π_i	Profit of firm i
D_i	Total demand of product i

4. Equilibrium Analysis

In this section, we analyze consumers' purchase decisions and derive the profit functions for the firms in Cases $\mathcal{N}$ and $\mathcal{R}$. Using this information, we then determine the equilibrium outcomes.

4.1. Pricing Strategy in Case $\mathcal{N}$

The products' demand from selective consumers is determined by their utility maximization, where they choose the product offering higher utility. Solving the utility equation $v - x - p_R = v - (1-x) - p_S$, we have $x_0 = \frac{1-p_R+p_S}{2}$, which represents the location of the marginal selective consumer who is indifferent between the two products. Selective consumers located at $x \leq x_0$ choose product R, while those at $x > x_0$ choose product S. Therefore, the demands for products R and S from selective consumers are $(1-2\theta)x_0$ and $(1-2\theta)(1-x_0)$, respectively. Captive consumers, on the other hand, purchase the products they are aware of, contributing a demand of θ to each firm. Hence, the total demands for the firms are as follows:

$$D_R = (1-2\theta)x_0 + \theta \text{ and } D_S = (1-2\theta)(1-x_0) + \theta. \tag{3}$$

The firms' profits are:

$$\pi_R = p_R D_R + \alpha p_S D_S \text{ and } \pi_S = (1-\alpha) p_S D_S. \tag{4}$$

The firms determine their prices to maximize their profits. Using first-order condition, we obtain the equilibrium outcomes and present them in the following lemma.

Lemma 1. *In Case $\mathcal{N}$, the equilibrium outcomes are as follows:*

(a) *Prices:*

$$p_R^{\mathcal{N}} = \frac{3+\alpha}{(3-\alpha)(1-2\theta)} \text{ and } p_S^{\mathcal{N}} = \frac{3}{(3-\alpha)(1-2\theta)}.$$

(b) *Profits:*

$$\pi_R^{\mathcal{N}} = \frac{9-2\alpha^2+6\alpha}{2(3-\alpha)^2(1-2\theta)} \text{ and } \pi_S^{\mathcal{N}} = \frac{9(1-\alpha)}{2(3-\alpha)^2(1-2\theta)}.$$

4.2. Pricing and Recommendation Strategies in Case $\mathcal{R}$

Suppose that product R is recommended to a consumer, three possible scenarios exist regarding the consumer's awareness after recommendation, as outlined in Table 2.

Table 2. A consumer's awareness if product R is recommended.

Original Type of the Consumer	Probability	Awareness After Recommendation
Selective Consumer	$1-2\theta$	Both products
R's Captive Consumer	θ	Only product R
S's Captive Consumer	θ	Both products

Table 2 illustrates that after the recommendation of product R, the consumer's awareness changes in two scenarios: being aware of both products with a probability of $1-\theta$ and being aware of only product R with a probability of θ. Similarly, recommending product S will also result in two scenarios: being aware of both products with a probability of $1-\theta$ and being aware of only product S with a probability of θ. Notably, the recommendation scores for the two products vary only when a consumer is aware of just one product after recommendation. Specifically, the difference in the recommendation scores is as follows:

$$\begin{aligned} C_R - C_S &= \theta[v - \mathbb{E}(x|s=y) - p_R + \tau p_R] - \theta[v - (1-\mathbb{E}(x|s=y)) - p_S + \tau\alpha p_S] \\ &= \theta[(1-2y)\beta + (1-\alpha\tau)p_S - (1-\tau)p_R] \end{aligned}. \tag{5}$$

Let us define y_0 as the threshold signal at which the recommendation scores of the two products are equal. Then, using Equation (5) and setting $C_R - C_S = 0$, we can solve for y_0 as follows:

$$y_0 = \frac{(1-\alpha\tau)p_S - (1-\tau)p_R + \beta}{2\beta}. \tag{6}$$

When a consumer's signal is $y \le y_0$, firm R recommends product R to that consumer. Conversely, if the signal is $y > y_0$, firm R recommends product S to the consumer.

Next, we derive the demand functions. Notably, it can be proven that the demand functions are independent of the relationship between y_0 and x_0. Therefore, we focus on illustrating the derivation process when $y_0 \ge x_0$. For consumers who were already selective prior to the recommendation, their awareness and purchase decisions remain unaffected by the recommendation. Consequently, they will buy products R and S if they are located at $x \in [0, x_0)$ and $x \in (x_0, 1]$, respectively. Mathematically, the demands of products R and S from these consumers are $D_{Rs} = (1-2\theta)x_0$ and $D_{Ss} = (1-2\theta)(1-x_0)$, respectively.

For firm R's captive consumers before recommendation, they will buy product R if they are located at $x \in [0, x_0)$, regardless of whether they are recommended product R or S. However, if they are located at $x \in [x_0, 1]$, they will buy the recommended product. Thus, the demands for products R and S from these consumers are:

$$\begin{aligned} D_{Rcr} &= \theta[x_0 + \int_{x_0}^{y_0} \mathbb{P}(s \le y_0|x)dx + \int_{y_0}^{1} \mathbb{P}(s \le y_0|x)dx] \\ &= \theta[y_0 + x_0(1-y_0)(1-\beta)], \end{aligned} \tag{7}$$

$$D_{Scr} = \theta - D_{Rcr}. \tag{8}$$

For firm S's captive consumers before recommendation, they will buy product S if they are located at $x \in (x_0, 1]$, regardless of whether they are recommended product R or S. However, if they are located at $x \in [0, x_0)$, they will buy the recommended product. Hence, the demands for products S and R from these consumers are:

$$D_{Scs} = \theta[1 - x_0 + \int_0^{x_0} \mathbb{P}(s > y_0|x)dx] = \theta[1 - \beta x_0 - (1-\beta)x_0 y_0], \tag{9}$$

$$D_{Rcs} = \theta - D_{Scs}. \tag{10}$$

The total demands for products R and S are summarized as follows:

$$D_R = D_{Rs} + D_{Rcr} + D_{Rcs} = x_0(1-\theta) + y_0\theta, \tag{11}$$

$$D_S = D_{Ss} + D_{Scr} + D_{Scs} = 1 - x_0(1-\theta) - y_0\theta. \tag{12}$$

The firms' profits are as follows:

$$\pi_R = p_R D_R + \alpha p_S D_S \text{ and } \pi_S = (1-\alpha) p_S D_S. \tag{13}$$

The firms determine their prices to maximize their profits. Using first-order condition, we obtain the equilibrium outcomes and present them in the following lemma.

Lemma 2. *In Case $\mathcal{R}$, the equilibrium outcomes are as follows:*

(a) *Prices:*

$$p_R^{\mathcal{R}} = \frac{\beta[(\alpha+3)\beta(\theta-1)+\theta(\alpha(4\tau-1)-3)]}{[\beta(\theta-1)+\theta(\tau-1)][(\alpha-3)\beta(\theta-1)-\theta(2\alpha\tau+\alpha-3)]},$$

$$p_S^{\mathcal{R}} = \frac{3\beta}{(\alpha-3)\beta(\theta-1)-\theta(2\alpha\tau+\alpha-3)}.$$

(b) *Indifferent location of recommendation:*

$$y_0^{\mathcal{R}} = \frac{\begin{array}{l}\alpha\beta^2(\theta-1)^2 - \alpha\beta(\theta-1)[\theta(\tau+2)+2\tau+1] - \alpha\theta(\tau-1)(2\theta\tau+\theta-\tau+1) \\ -3\beta^2(\theta-1)^2 - 3\beta(\theta-1)[\theta(\tau-2)-\tau] + 3\theta^2(\tau-1)\end{array}}{2[\beta(\theta-1)+\theta(\tau-1)][(\alpha-3)\beta(\theta-1)-\theta(2\alpha\tau+\alpha-3)]}.$$

(c) *Profits:*

$$\pi_R^{\mathcal{R}} = \frac{\beta\begin{pmatrix}(2\alpha^2-6\alpha-9)\beta^2(\theta-1)^2 - 2\beta\theta(\theta-1)[\alpha^2(\tau+2)+6\alpha(2\tau-1)-9] \\ +\theta^2[\alpha^2(-13\tau^2+2\tau+2)+6\alpha(4\tau-1)-9]\end{pmatrix}}{2[\beta(\theta-1)+\theta(\tau-1)][(\alpha-3)\beta(\theta-1)-\theta(2\alpha\tau+\alpha-3)]^2},$$

$$\pi_S^{\mathcal{R}} = \frac{9(\alpha-1)\beta[\theta(\alpha\tau-1)+\beta(\theta-1)]}{2[(\alpha-3)\beta(\theta-1)-\theta(2\alpha\tau+\alpha-3)]^2}.$$

Due to the complexity of outcomes in Case $\mathcal{R}$, we narrow our focus to two representative scenarios in the subsequent analysis. One scenario is characterized by a low commission rate, where $\alpha \to 0^+$, and the other scenario represents a high commission rate, where $\alpha \to 1^-$. This comparison allows us to succinctly illustrate the role of α. Additionally, we delve deeper into the impact of α in Section 7 as a supplement. By substituting $\alpha \to 0^+$ or $\alpha \to 1^-$ into the expressions for the equilibrium outcomes derived in Lemmas 1 and 2, we obtain simplified equilibrium outcomes for each scenario, as presented in Table 3. It is important to note that we impose certain bounds on the parameters v, β, θ, and τ in each scenario to ensure internal solutions and the validity of the equilibrium. Further details on the parameter ranges can be found in Appendix A.

Table 3. Equilibrium outcomes in two representative scenarios.

Item	Low Commission Scenario	High Commission Scenario
$p_R^{\mathcal{N}}$	$\frac{1}{1-2\theta}$	$\frac{2}{1-2\theta}$
$p_S^{\mathcal{N}}$	$\frac{1}{1-2\theta}$	$\frac{3}{2(1-2\theta)}$
$\pi_R^{\mathcal{N}}$	$\frac{1}{2(1-2\theta)}$	$\frac{13}{8(1-2\theta)}$
$\pi_S^{\mathcal{N}}$	$\frac{1}{2(1-2\theta)}$	0
$p_R^{\mathcal{R}}$	$\frac{\beta}{\beta(1-\theta)+\theta(1-\tau)}$	$\frac{2\beta}{\beta(1-\theta)+\theta(1-\tau)}$
$p_S^{\mathcal{R}}$	$\frac{\beta}{\beta(1-\theta)+\theta}$	$\frac{3\beta}{2[\beta(1-\theta)+\theta(1-\tau)]}$
$\pi_R^{\mathcal{R}}$	$\frac{\beta}{2[\beta(1-\theta)+\theta(1-\tau)]}$	$\frac{13\beta}{8[\beta(1-\theta)+\theta(1-\tau)]}$
$\pi_S^{\mathcal{R}}$	$\frac{\beta}{2[\beta(1-\theta)+\theta]}$	0

5. Impact of Recommender System

In this section, we explore how the recommender system influences equilibrium prices and profits in both low and high commission scenarios.

5.1. Low Commission Scenario

Comparing the equilibrium prices and profits of Cases $\mathcal{N}$ and $\mathcal{R}$ with $\alpha \to 0^+$, we can assert the following proposition.

Proposition 1. *In the low commission scenario, the introduction of the recommender system always reduces both firms' equilibrium prices and profits. Specifically,* $p_i^{\mathcal{R}} < p_i^{\mathcal{N}}$ *and* $\pi_i^{\mathcal{R}} < \pi_i^{\mathcal{N}}$.

Proposition 1 illustrates that the recommender system exacerbates price competition and negatively affects both firms' profitability when the commission rate is low (refer to Figure 3a). This can be explained by the system's alteration of the market structure, converting some captive consumers into selective ones. This transformation triggers two opposing effects: heightened consumer price sensitivity, intensifying price competition and adversely affecting prices and profits; and an increase in potential consumers for each firm due to product recommendations, boosting demands and thereby positively affecting prices and profits. This dual effect is termed as *the reconfiguration effect*.

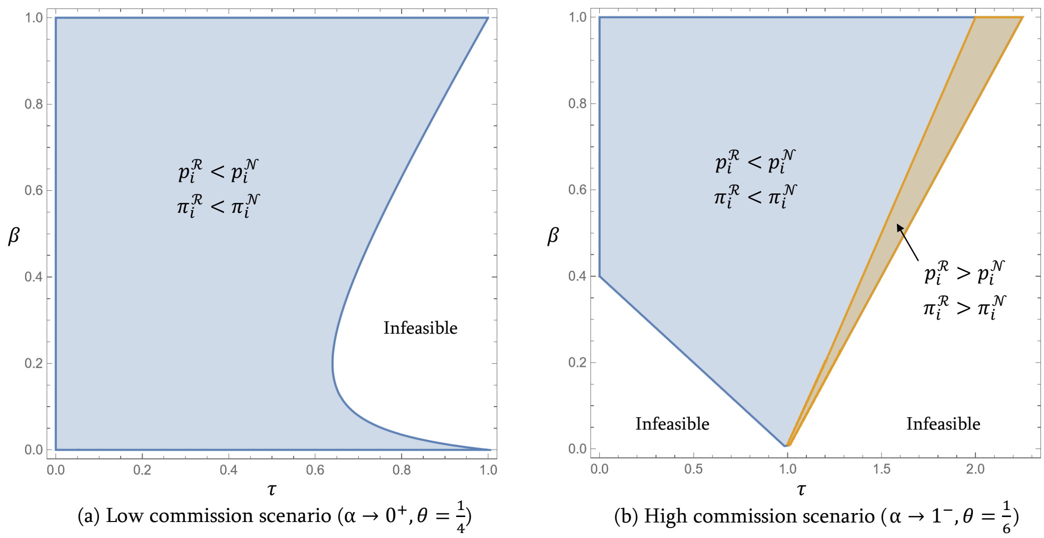

Figure 3. Recommender system impact.

It is important to note that the sign of the reconfiguration effect is tied to the value of α. Revenue-sharing, represented by the commission fee in the hybrid selling mode, serves as a fundamental mechanism to moderate competition and foster collaboration between a retail platform and a third-party seller. In our model, when the commission rate is low, the firms operate more like pure competitors because the cooperative aspect of revenue-sharing is minimized. Consequently, the firms primarily face intensified competition arising from recommendations. Particularly, from firm R's perspective, the increased demand for product S due to recommendations bolsters firm S's competitiveness while yielding minimal benefits for firm R's profit, posing a challenge for firm R. In essence, the predominance of the reconfiguration effect's negative aspect becomes evident with a low commission rate.

Equation (5) highlights another significant change induced by the recommender system: the firms strategically increase the recommendation scores of their products (i.e., the chance of their products being recommended) by adjusting their product prices. However, whether they should raise or lower prices to enhance their recommendation scores hinges largely on the value of τ. A higher τ places greater emphasis on the profit factor in the recommendation score, motivating the firms to raise prices. Conversely, a lower τ prioritizes the consumer utility factor, prompting the firms to lower prices. Additionally, for firm S, its pricing incentives are also influenced by α. A decrease in α reduces the profit factor's weight in product S's recommendation score, making firm S less willing to increase its price. This strategic pricing adjustment for recommendation competition is termed as *the recommendation effect*. In the low commission scenario, firm S can only boost its recommendation score by enhancing consumer utility, compelling it to aggressively lower its price in the presence of the recommender system. This intensifies price competition, driving firm R to also reduce its price.

Proposition 1 provides a managerial insight that the intense price competition driven by the recommender system should be approached with caution. A retail platform should avoid introducing the system when the commission rate for third-party sellers is very low.

Corollary 1. *In the low commission scenario, firms R and S have equal prices and profits in the absence of the recommender system. However, firm R has a higher price and profit than firm S in the presence of the recommender system. Mathematically,* $p_R^{\mathcal{N}} = p_S^{\mathcal{N}}$, $\pi_R^{\mathcal{N}} = \pi_S^{\mathcal{N}}$, $p_R^{\mathcal{R}} > p_S^{\mathcal{R}}$, *and* $\pi_R^{\mathcal{R}} > \pi_S^{\mathcal{R}}$.

The result in Corollary 1 is straightforwardly deduced from the outcomes listed in Table 3, so we do not delve into the detailed proof here. When $\alpha \to 0^+$, the two firms are nearly symmetric, leading to equal prices and profits in the absence of the recommender system. Interestingly, the introduction of the recommender system creates an asymmetry in the equilibrium, resulting in different prices and profits for the firms. Specifically, firm R emerges with a higher price and profit compared to firm S. This discrepancy arises as firm R leverages its control over the recommendation system to bolster its competitive edge over firm S. For example, we observe that $y_0^{\mathcal{R}} > \frac{1}{2}$, indicating that firm R directs more recommendations towards its own product than firm S's product. The finding in Corollary 1 sheds light on the theoretical rationale behind practices such as Amazon's promotion of its private-label products (e.g., Amazon Basics) over third-party sellers' products, commonly known as self-preferencing [12,13].

5.2. High Commission Scenario

In the high commission scenario, as $\alpha \to 1^-$, firm S's profit diminishes to zero in equilibrium. Therefore, our analysis focuses on both firms' prices and solely on firm R's profit in this context. Comparing the outcomes of Cases $\mathcal{N}$ and $\mathcal{R}$, we present the following proposition.

Proposition 2. *In the high commission scenario, the presence of the recommender system can either increase or decrease both firms' equilibrium prices and firm R's profit, contingent upon the relationship between τ and β. Specifically, $p_i^{\mathcal{R}} \geq p_i^{\mathcal{N}}$ and $\pi_R^{\mathcal{R}} \geq \pi_R^{\mathcal{N}}$ if and only if $\tau \geq 1 + \beta$.*

Proposition 2 elucidates that the influence of the recommender system in the high commission scenario hinges on two characteristics of the system: recommendation orientation, denoted by τ, and recommendation accuracy, represented by β (see Figure 3b). When considering recommendation orientation, if the system is profit-oriented (i.e., $\tau \geq 1 + \beta$), it can elevate both firms' prices and bolster firm R's profit. Conversely, if it is consumer-oriented (i.e., $\tau < 1 + \beta$), prices will decrease, negatively affecting firm R's profitability.

This dynamic is driven by the positive aspects of both the recommendation effect and reconfiguration effect on prices and profits. Specifically, in scenarios with high α and τ, fostering a robust cooperation between firms R and S, firm R can capitalize on increased demand for both products due to recommendation. Additionally, both firms may raise their recommendation scores by adjusting prices upward. Consequently, the recommender system mitigates price competition, leading to higher prices and increased profit for firm R. This finding suggests that retail platforms employing the hybrid selling mode and charging high commissions should prioritize a profit-oriented recommender system over a consumer-oriented one for optimal results.

Proposition 2 can also be interpreted intriguingly from the lens of recommendation accuracy: With $\tau > 1$, the profitability of the recommender system for firm R hinges on its accuracy, being more profitable when it is inaccurate (i.e., $\beta \leq \tau - 1$) rather than it is accurate ($\beta > \tau - 1$). This finding may seem counter-intuitive since conventional wisdom suggests that higher recommendation accuracy is superior, prompting firms to strive for accuracy improvements in practice. However, our result unveils a different dynamic.

The rationale lies in the shifting importance of the consumer utility factor in the recommendation score with varying recommendation accuracy. When accuracy is higher, the consumer utility factor, reliant on accurate prediction of consumer location, gains prominence in the recommendation score. Consequently, the firms are incentivized to lower prices to enhance the likelihood of their products being recommended. In essence, the recommendation effect acts negatively on prices and profits with high recommendation accuracy and $\tau > 1$. Later in Proposition 3, we delve deeper into the impact of β, providing additional insights. The practical implication derived from Proposition 2 is noteworthy: an inaccurate recommender system could yield higher profitability for a retail platform operating under the hybrid selling mode. This suggests that the pursuit of accuracy improvements, even if it is costless, may not always translate into meaningful gains for a retail platform.

Combining Propositions 1 and 2 underscores the pivotal role of the commission rate α in determining the impact of the recommender system within the hybrid selling mode. Specifically, the profitability of the recommender system hinges on a high commission rate. This is elucidated by Proposition 1, where both the reconfiguration effect and the recommendation effect of the recommender system are positive only when α is high. Consequently, a retail platform is advised to leverage personalized recommendation strategies when imposing relatively high commission rates on third-party sellers or when focusing on product categories with high commission rates. Additionally, the platform should not blindly pursue high recommendation accuracy. In fact, an inaccurate recommender system can sometimes be more advantageous than an accurate one in the hybrid selling mode.

It is worth noting that our findings diverge from those of Li et al. [8]. In their pure agency selling framework, the commission rate's impact is less pronounced: the presence of a recommender system can yield higher prices and profits even with a low commission rate, since there is no direct competition between the retail platform and sellers. Consequently, the commission rate fails to negate the positive reconfiguration effect of the recommender system in their framework. This comparison implies that a recommender system might be less profitable under the hybrid selling mode compared to the pure agency selling mode due to the stricter conditions for profitability in the former. Retail platforms operating with the hybrid selling mode should be mindful of this potential drawback associated with recommender systems.

6. Impacts of Recommender System Characteristics

In this section, we explore the impacts of recommendation accuracy and recommendation orientation on the equilibrium outcomes in Case $\mathcal{R}$. To begin, we conduct a comparative statics analysis with respect to β, leading to the following propositions.

Proposition 3. *(1) In the low commission scenario, both equilibrium prices and profits increase with β. Mathematically, $\frac{\partial p_i^{\mathcal{R}}}{\partial \beta} > 0$ and $\frac{\partial \pi_i^{\mathcal{R}}}{\partial \beta} > 0$. (2) In the high commission scenario, the equilibrium prices of both firms and the profit of firm R increase with β when τ is low, but they decrease with β when τ is high. Specifically, $\frac{\partial p_i^{\mathcal{R}}}{\partial \beta} \geq 0$ and $\frac{\partial \pi_R^{\mathcal{R}}}{\partial \beta} \geq 0$ if and only if $\tau \leq 1$.*

It is worth noting that in the low commission scenario (as detailed in Appendix A), τ is assumed to be less than 1. As a result, the outcome in the low commission scenario aligns with that in the high commission scenario when $\tau \leq 1$. Proposition 3 essentially shows that higher recommendation accuracy increases equilibrium prices and profits when the recommender system is consumer-oriented (i.e., $\tau \leq 1$), regardless of the commission rate. However, in the high commission scenario, τ can exceed 1. In such cases, higher recommendation accuracy may reduce equilibrium prices and profits if the recommender system is profit-oriented (i.e., $\tau > 1$).

The rationale behind Proposition 3 can be explained through two opposing effects of higher recommendation accuracy on the firms' pricing strategies. First, as discussed in Propositions 1 and 2, higher accuracy prompts the firms to lower prices in order to compete for more recommendations for their products. However, higher accuracy also improves the match between recommended products and consumer preferences, increasing the likelihood of product purchases and thus boosting demand. It is notable that the balance between these effects hinges on recommendation orientation. In a consumer-oriented system (i.e., $\tau < 1$), the positive effect tends to outweigh the negative effect, resulting in higher prices and profits. Conversely, in a profit-oriented system (i.e., $\tau > 1$), the negative effect may dominate, leading to lower prices and profits.

The findings from Proposition 3 provide theoretical guidance for retail platforms and third-party sellers on setting their product prices based on the characteristics of the recommender system. In practical terms, retail platforms often invest significantly in data collection and algorithm development to improve the accuracy of their recommender systems. However, our analysis shows that the benefits of high accuracy depend on the system's orientation. While a consumer-oriented system can effectively harness high accuracy, a profit-oriented system may encounter drawbacks from increased accuracy. Therefore, retail platforms should carefully weigh both orientation and accuracy when designing recommender systems. An accurate, consumer-oriented system or an inaccurate, profit-oriented system may be profitable for platforms operating in the hybrid selling mode.

Proposition 4. *The impact of β on the indifferent location of recommendation is as follows:*

(1) *In the low commission scenario, $\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} \geq 0$ if and only if*

$$\tau \leq min\{-\frac{(\beta\theta-\beta-\theta)^2}{\beta\theta^2-\beta-\theta^2}, \frac{\theta^2-\beta^2(\theta-1)^2}{\theta^2}\}.$$

(2) *In the high commission scenario, $\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} \geq 0$ if and only if $\tau \leq 1$.*

We can depict the essence of Proposition 4 using Figure 4. This proposition emphasizes the pivotal role of recommendation orientation in how recommendation accuracy influences the indifferent location of recommendation. Specifically, in both low and high commission scenarios, higher recommendation accuracy leads firm R to recommend its own product to more consumers while recommending firm S's product to fewer consumers if and only if the recommendation system is consumer-oriented enough (i.e., τ is small). Furthermore, in the low commission scenario, $\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} \geq 0$ is observed when both β and τ are low. This signifies that the likelihood of recommendations for product R initially increases with recommendation accuracy and then decreases, given that the recommender system is consumer-oriented and the commission rate is low.

The rationale behind Proposition 4 can be elucidated as follows. When τ is low, higher recommendation accuracy elevates the firms' prices, as demonstrated by Proposition 3. Since firm R garners the entire revenue from p_R but only a fraction α from firm S's price p_S, raising y_0 enables firm R to leverage the positive impact of increased p_R on profitability more effectively. Consequently, $\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} \geq 0$ holds when τ is low. Conversely, if τ is high, higher recommendation accuracy reduces the firms' prices. Firm R lowers y_0 to entice more consumers to purchase product S, leveraging the commission rate to weaken the adverse effect of reduced prices on profitability. Thus, $\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} < 0$ is observed when τ is high.

In the symmetric setting presented by Li et al. [8], the equilibrium y_0 remains constant at $\frac{1}{2}$ regardless of market parameters. However, in our asymmetric setting in the hybrid selling mode, the equilibrium y_0 shifts from the midpoint of the Hotelling line, influenced by the characteristics of the recommender system and the commission rate. This advancement in our work highlights how the attributes of a recommendation system impact y_0, offering practical insights into how a retail platform should adjust its recommendation strategy when the characteristics of its recommender system evolve. For instance, when the system becomes more accurate, the platform should allocate more (or fewer) recommendations to its own products if the system is highly consumer-oriented (or profit-oriented).

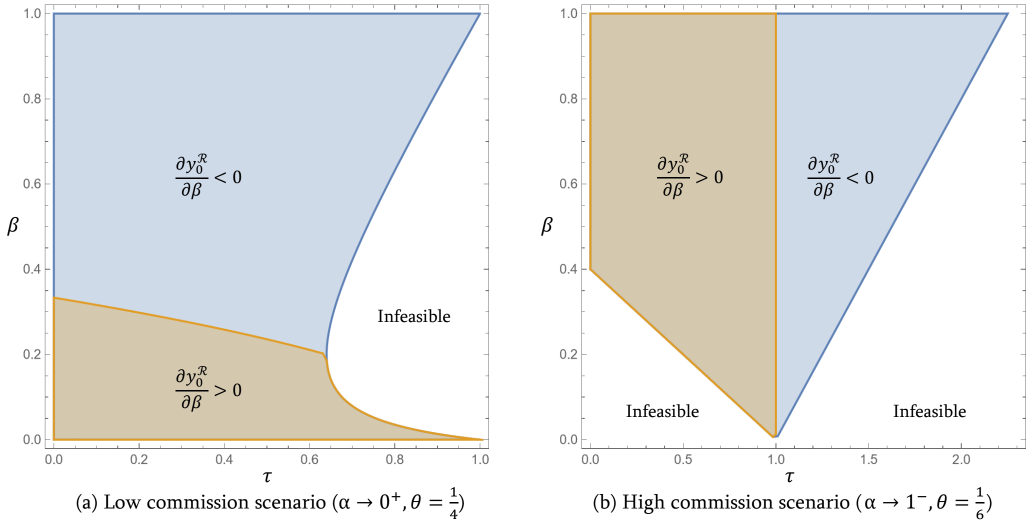

Figure 4. Impact of β on $y_0^{\mathcal{R}}$.

Next, Remark 1 highlights several observations regarding the impact of τ on the equilibrium prices and profits, based on Table 3:

Remark 1. *(1) In both low and high commission scenarios,* $\frac{\partial p_R^{\mathcal{R}}}{\partial \tau} > 0$ *and* $\frac{\partial \pi_R^{\mathcal{R}}}{\partial \tau} > 0$*. (2) In the low commission scenario,* $\frac{\partial p_S^{\mathcal{R}}}{\partial \tau} = \frac{\partial \pi_S^{\mathcal{R}}}{\partial \tau} = 0$*. However, in the high commission scenario,* $\frac{\partial p_S^{\mathcal{R}}}{\partial \tau} > 0$*.*

Remark 1 reveals that when the recommender system becomes more profit-oriented (i.e., higher τ), both firms incline towards increasing their chances of product recommendations by raising prices. This positive recommendation effect mitigates price competition, leading to higher prices and profits for both firms. However, in the low commission scenario where $\alpha \to 0^+$, the adjusted weight of profit in the recommendation score for product S, denoted as $\alpha\tau$, becomes extremely small regardless of the value of τ (see Equation (5)). Consequently, τ exerts minimal influence on firm S's pricing decisions, rendering the equilibrium price and profit of firm S independent of τ. This observation suggests that τ is more likely to affect firm R's pricing and profitability than firm S's. It underscores that attempting to prompt third-party sellers to adjust prices by manipulating the recommender system's orientation may not be effective when the commission rate is low. Instead, altering recommendation orientation emerges as a more effective strategy for influencing the platform's own pricing decisions and overall performance.

Finally, we examine the impact of τ on the indifferent location of recommendation, leading to the following proposition.

Proposition 5. *In both low and high commission scenarios,* $y_0^{\mathcal{R}}$ *consistently increases with* τ*, i.e.,* $\frac{\partial y_0^{\mathcal{R}}}{\partial \tau} > 0$*.*

Proposition 5 elucidates that when the recommendation system is more profit-oriented, firm R prioritizes recommending its own product while diminishing the recommendation likelihood for firm S's product. This shift is driven by a strategic response: as τ increases, firm R is inclined to raise its price p_R to vie for recommendation opportunities. However, this price increase may diminish firm R's market share competitiveness, leading to a reduction in x_0. Consequently, firm R compensates for this by increasing y_0 to maintain its market share.

This proposition provides strategic guidance for retail platforms on adjusting recommendation strategies based on changes in recommendation orientation. Specifically, it suggests that platforms operating under the hybrid selling mode should adopt a more self-preferencing approach when the recommender system shifts towards a profit-oriented focus. For third-party sellers, this means their products may receive fewer recommendations if the platform prioritizes profit considerations in the recommendation process.

7. Impact of Commission Rate

In this section, we delve into the impact of α on the equilibrium outcomes, complementing the analysis from previous sections. To simplify the analysis, we assume $\beta = 1$ and $\theta = \frac{1}{6}$, focusing primarily on the roles of α and τ. This choice is representative and insightful, and we have verified that the qualitative results remain consistent across different combinations of β and θ.

By substituting $\beta = 1$ and $\theta = \frac{1}{6}$ into the mathematical expressions for the equilibrium outcomes, as presented in Lemmas 1 and 2, we obtain the equilibrium outcomes summarized in Table 4. Additionally, to ensure the validity of these equilibrium outcomes, we impose constraints on the parameters v, α, and τ. For further details, please refer to Appendix B.

Table 4. Equilibrium outcomes with $\beta = 1$ and $\theta = \frac{1}{6}$.

Item	Case $\mathcal{N}$	Case $\mathcal{R}$
p_R	$\frac{3(\alpha+3)}{2(3-\alpha)}$	$\frac{6[\alpha(3-2\tau)+9]}{(\tau-6)[\alpha(\tau+3)-9]}$
p_S	$\frac{9}{2(3-\alpha)}$	$\frac{9}{9-\alpha(\tau+3)}$
π_R	$\frac{3(9-2\alpha^2+6\alpha)}{4(\alpha-3)^2}$	$\frac{\alpha^2(216-39\tau^2+36\tau)+216\alpha(2\tau-3)-972}{4(\tau-6)[\alpha(\tau+3)-9]^2}$
π_S	$\frac{27(1-\alpha)}{4(\alpha-3)^2}$	$\frac{27(\alpha-1)(\alpha\tau-6)}{4[\alpha(\tau+3)-9]^2}$

We compare the firms' equilibrium prices and profits in Cases $\mathcal{N}$ and $\mathcal{R}$, providing insights into the impact of the recommender system across a broader range of α. The findings are summarized in the following proposition.

Proposition 6. *The presence of the recommender system leads to increased (decreased) equilibrium prices and profits when both α and τ are high (low). Mathematically, $\pi_i^{\mathcal{R}} \geq \pi_i^{\mathcal{N}}$ if and only if $\tau \geq \tau_{\pi i}(\alpha)$, and $p_i^{\mathcal{R}} \geq p_i^{\mathcal{N}}$ if and only if $\tau \geq \tau_{pi}(\alpha)$.*

The detailed expressions for the critical points $\tau_{\pi i}(\alpha)$ and $\tau_{pi}(\alpha)$ are provided in Appendix C. The essence of Proposition 6 is illustrated in Figure 5. It confirms that the firms set higher prices and achieve higher profits in the presence of the recommender system if and only if the system is profit-oriented and the commission rate is high. This aligns with the insights from Proposition 2, showcasing the robustness of the findings in Sections 5 and 6. Additionally, when α approaches 1^-, the critical points for $\pi_i^{\mathcal{R}} = \pi_i^{\mathcal{N}}$ and $p_i^{\mathcal{R}} = p_i^{\mathcal{N}}$ become equal, i.e., $\tau_{\pi i}(1^-) = \tau_{pi}(1^-)$. However, for a general value of α, these critical points can differ. Specifically, we observe that $\tau_{\pi S}(\alpha) > \tau_{pS}(\alpha) > \tau_{\pi R}(\alpha) > \tau_{pR}(\alpha)$ with $\alpha \in (0, 1)$. This leads to several new insights, as presented in the following corollaries.

Corollary 2. *(1) If $\tau < \tau_{pR}(\alpha)$, $p_i^{\mathcal{R}} < p_i^{\mathcal{N}}$. (2) If $\tau > \tau_{pS}(\alpha)$, $p_i^{\mathcal{R}} > p_i^{\mathcal{N}}$. (3) If $\tau_{pR}(\alpha) \leq \tau \leq \tau_{pS}(\alpha)$, $p_R^{\mathcal{R}} \geq p_R^{\mathcal{N}}$ while $p_S^{\mathcal{R}} \leq p_S^{\mathcal{N}}$.*

Corollary 2 highlights that in the presence of the recommender system, both firms reduce prices if τ falls below $\tau_{pR}(\alpha)$ (Region 1 in Figure 5), but they increase prices if τ exceeds $\tau_{pS}(\alpha)$ (Regions 4 and 5 in Figure 5). In the moderate τ range, i.e., $\tau_{pR}(\alpha) \leq \tau \leq \tau_{pS}(\alpha)$ (Regions 2 and 3 in Figure 5), firm R increases its price while firm S decreases its price.

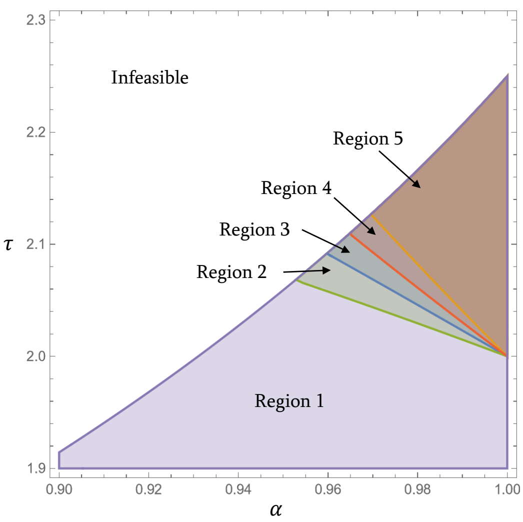

Figure 5. Impact of recommender system with a general range of α.

Corollary 2 underscores that firm R is inclined to raise its price more than firm S when the recommendation system is in play. This stems from two factors. First, firm R benefits from dual revenue streams—its own product revenue and commission revenue—unlike firm S, which relies solely on product revenue. Second, firm R can leverage its control over the recommendation strategy (i.e., y_0, as discussed in Corollary 1) to bolster its competitive edge against firm S. Consequently, firm R faces less pressure to reduce prices in response to the negative reconfiguration effect of the recommender system compared to firm S.

Corollary 3. *(1) If $\tau < \tau_{\pi R}(\alpha)$, $\pi_i^{\mathcal{R}} < \pi_i^{\mathcal{N}}$. (2) If $\tau > \tau_{\pi S}(\alpha)$, $\pi_i^{\mathcal{R}} > \pi_i^{\mathcal{N}}$. (3) If $\tau_{\pi R}(\alpha) \leq \tau \leq \tau_{\pi S}(\alpha)$, $\pi_R^{\mathcal{R}} \geq \pi_R^{\mathcal{N}}$ while $\pi_S^{\mathcal{R}} \leq \pi_S^{\mathcal{N}}$.*

Corollary 3 reveals that firm R stands to benefit more from the recommender system compared to firm S. Specifically, in the presence of the recommender system, both firms experience lower profits if τ falls below $\tau_{\pi R}(\alpha)$ (Regions 1 and 2 in Figure 5), but they have higher profits if τ exceeds $\tau_{\pi S}(\alpha)$ (Region 5 in Figure 5). In the moderate τ range, i.e., $\tau_{\pi R}(\alpha) \leq \tau \leq \tau_{\pi S}(\alpha)$ (Regions 3 and 4 in Figure 5), firm R gains a higher profit while firm S experiences a lower profit.

The intuition behind Corollary 3 echoes that of Corollary 2, hence we do not reiterate it. However, this result offers an insight into potential disagreements between a retail platform and a third-party seller regarding the adoption of a recommender system. Additionally, we note that $\frac{\partial[\tau_{\pi S}(\alpha) - \tau_{\pi R}(\alpha)]}{\partial \alpha} < 0$, indicating that the likelihood of the disagreement diminishes (increases) with higher (lower) commission rates.

The impact of α on the equilibrium prices and profits is also examined. We observe that irrespective of the presence of the recommender system, both firms experience price increases with a higher value of α, as evidenced by $\frac{\partial p_i^{\mathcal{N}}}{\partial \alpha} > 0$ and $\frac{\partial p_i^{\mathcal{R}}}{\partial \alpha} > 0$. This trend arises due to the reduced competition intensity between the firms as the commission rate α rises. Additionally, firm R's profit has an increase with α ($\frac{\partial \pi_R^{\mathcal{N}}}{\partial \alpha} > 0$ and $\frac{\partial \pi_R^{\mathcal{R}}}{\partial \alpha} > 0$), while firm S's profit decreases ($\frac{\partial \pi_S^{\mathcal{N}}}{\partial \alpha} < 0$ and $\frac{\partial \pi_S^{\mathcal{R}}}{\partial \alpha} < 0$). These outcomes align with common intuition and industry dynamics, where higher commission rates typically benefit the platform (firm R) but may lead to reduced profit margins for third-party sellers (firm S). Given the straightforward nature of these findings, we omit the detailed derivation process. Notably, we highlight another significant result regarding the impact of α on firm R's allocation of recommendations, as detailed in the following remark.

Remark 2. *In Case $\mathcal{R}$, the indifferent location of recommendation decreases with α, i.e.,*

$$\frac{\partial y_0^{\mathcal{R}}}{\partial \alpha} = -\frac{9(2\tau+3)}{2[\alpha(\tau+3)-9]^2} < 0.$$

Remark 2 underscores the relationship between the commission rate α and firm R's allocation of recommendations. Specifically, as α increases, firm R allocates more recommendations to product S. This behavior is driven by the shifting revenue dynamics within firm R's profit structure. With a higher α, the revenue derived from product S becomes a more significant contributor to firm R's overall profit. Consequently, firm R strategically adjusts its recommendation strategy by decreasing the parameter y_0, aiming to boost the demand for product S.

8. Conclusions

The adoption of personalized recommendation systems is prevalent among retail platforms, yet their precise impact on price competition and firm profitability remains unclear to both researchers and practitioners. This lack of clarity poses challenges for retail platforms in deciding whether to implement recommender systems. This issue is particularly pronounced in the context of the hybrid selling mode, where retail platforms must navigate the optimal allocation of recommendations between their proprietary products and those of third-party sellers. To address these challenges, we develop a game-theoretic model that delves into the strategic dynamics of personalized recommendation strategies within a channel comprising a retail platform with its own product and a third-party seller offering a horizontally differentiated product through the platform. Our analysis yields several key findings that illuminate this complex landscape. Moreover, these findings directly address the research questions we initially posed and offer both theoretical and managerial insights.

8.1. Key Findings

(1) What is the impact of a recommender system on price competition and firm profitability in a retail platform with the hybrid selling mode? We identify two primary effects of the recommender system: the reconfiguration effect and the recommendation effect. The reconfiguration effect involves converting some captive consumers into selective consumers, which increases the firms' demands while also making consumers more price-sensitive. On the other hand, the recommendation effect drives the firms to adjust prices to compete for recommendations. The overall impact of the recommender system hinges on the interplay between these effects, influenced by three critical factors: the commission rate, recommendation orientation, and recommendation accuracy. Specifically, when the commission rate is high and the system is profit-oriented or inaccurate, the recommender system can mitigate price competition, leading to higher prices and profits. Conversely, if these conditions are not met, the system can intensify price competition, harming both firms. It is worth noting that the retail platform tends to benefit more from the recommender system compared to the third-party seller.

(2) How should a retail platform allocate recommendations between its own product and a third-party seller's product? We demonstrate that the commission rate and characteristics of the recommender system influence the retail platform's recommendation strategy. Particularly, our analysis reveals that the retail platform consistently prioritizes its own product over the third-party seller's product, a strategy known as self-preferencing. This asymmetric equilibrium persists even when the firms are nearly symmetric, such as when the commission rate approaches zero. The self-preferencing strategy gives the retail platform a competitive edge and leads to higher

profits compared to the third-party seller. Furthermore, we observe that the extent of self-preferencing (i.e., the likelihood of recommending the retail platform's product) increases (or decreases) with recommendation accuracy when the system is consumer-oriented (profit-oriented). Additionally, it rises with a higher level of system profit-orientation or a lower commission rate.

(3) What are the roles of commission rate and recommender system characteristics (i.e., accuracy and orientation) in recommendation strategy and market outcomes? We ascertain the impacts of the recommender system characteristics on equilibrium prices and profits. Surprisingly, higher recommendation accuracy does not always benefit the firms. Its impact on prices and profits is positive if and only if the system is consumer-oriented. Additionally, a more profit-oriented recommender system always increases the price and profit of the retail platform. However, its impact on the third-party seller is contingent on the commission rate. A higher commission rate magnifies the positive impact of a profit-oriented system. Therefore, when the commission rate is near zero, the system's orientation does not affect the third-party seller's prices or profits.

8.2. Theoretical and Managerial Insights

Our findings contribute to the literature on retail platforms and recommender systems management, providing insights into the impacts of recommender systems within the hybrid selling mode. Specifically, our study enhances the literature on the hybrid selling mode by demonstrating the existence of self-preferencing in the context of personalized recommendations and revealing how recommender system characteristics influence the degree of self-preferencing. Furthermore, we explore the economic effects of key recommender system characteristics (i.e., orientation and accuracy), offering a micro-level perspective on how recommender systems affect marketing strategies and performance and enriching the literature on recommender system design. Additionally, our work is among the first to analytically examine the profitability of recommender systems in the hybrid selling mode, indicating that the results in this context may contradict conventional wisdom derived from the purely agency selling mode.

Our findings also have practical implications for firms' decision-making processes. First, we provide insights into a retail platform's adoption of a recommender system, emphasizing that platforms should not solely prioritize recommendation accuracy but also consider recommendation orientation. Both accurate consumer-oriented and inaccurate profit-oriented systems can be beneficial for retail platforms and third-party sellers in the hybrid selling mode. Moreover, a recommender system can be more profitable for both retail platforms and third-party sellers when the commission rate is higher. Second, we offer insights into how retail platforms should allocate recommendations between their own products and those of third-party sellers. For instance, we suggest that it is advantageous for a retail platform to allocate more recommendations to its own products with higher recommendation accuracy, but only if the recommender system is consumer-oriented. Otherwise, platforms should adopt a less self-preferencing approach when the recommendation accuracy is higher. Additionally, our results provide theoretical guidance on pricing adjustments for retail platforms and third-party sellers in response to varying recommender system characteristics. For example, we recommend that retail platforms and sellers increase product prices when the recommender system is consumer-oriented and more accurate.

8.3. Future Research

Although our study offers valuable insights, it also presents certain limitations and areas for future exploration. First, we do not consider recommendation fees, an aspect in practical scenarios where third-party sellers may incur recommendation costs, such as cost-per-click, on a retail platform when their products are recommended to consumers. This may introduce a new dimension of trade-offs between product revenue and advertising revenue for retail platforms, warranting further investigation. Second, due to the complexity of our current model, we overlook uninformed consumers and assume equal captive consumer segments across the firms. Consequently, the proportion of captive consumers (i.e., θ) does not exert a significant influence. Future research could explore the impact of consumer information configuration in greater depth. Finally, our study focuses on a fully-covered market, and there may be novel insights to uncover in a partially-covered market setting.

Author Contributions: Conceptualization, W.W.; methodology, W.W.; software, W.W.; validation, W.W., X.H., Y.M. and G.L.; formal analysis, W.W.; writing—original draft, W.W., X.H. and G.L.; writing—revised version, W.W., X.H. and Y.M.; funding acquisition, W.W. and G.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research is funded by the National Natural Science Foundation of China, grant number 72202175.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Parameter Range of Section 5

Several conditions must be satisfied to ensure the validity of the equilibrium outcomes presented in Table 3. First, the profit functions should exhibit concavity or the Hessian matrix should be negative definite, mathematically expressed as:

$$A_i = \frac{\partial \pi_i^2}{\partial p_i^2} < 0 \text{ and } B = \frac{\partial \pi_R^2}{\partial p_R^2}\frac{\partial \pi_S^2}{\partial p_S^2} - \frac{\partial \pi_R^2}{\partial p_R \partial p_S}\frac{\partial \pi_S^2}{\partial p_S \partial p_R} > 0. \tag{A1}$$

In Case $\mathcal{N}$, $A_R = -1 + 2\theta$, $A_S = -(1-\alpha)(1-2\theta)$, and $B = \frac{1}{4}(3-\alpha)(1-\alpha)(2\theta-1)^2$. Equation (A1) holds if $0 < \theta < \frac{1}{2}$ and $0 < \alpha < 1$.

In Case $\mathcal{R}$, $A_R = \frac{\beta\theta - \beta + \theta\tau - \theta}{\beta}$, $A_S = -\frac{(\alpha-1)(\alpha\theta\tau + \beta\theta - \beta - \theta)}{\beta}$, and

$$B = \frac{(\alpha-1)(\beta\theta - \beta + \theta\tau - \theta)(\alpha\beta\theta - \alpha\beta - 2\alpha\theta\tau - \alpha\theta - 3\beta\theta + 3\beta + 3\theta)}{4\beta^2}. \tag{A2}$$

Equation (A1) holds if $0 < \theta < \frac{1}{2}$, $0 < \alpha < 1$, $0 < \beta < 1$, and $0 < \tau < \frac{\beta - \beta\theta + \theta}{\theta}$.

Second, the market must be fully covered at the equilibrium prices, which necessitates lower bounds on the prices. Specifically, the farthest captive consumer of a firm will purchase the product if and only if their utility is positive, expressed as $v - p_i^j - 1 > 0$, leading to $p_i^j < v - 1$, where $j \in \{\mathcal{N}, \mathcal{R}\}$. Third, the equilibrium prices should be internal solutions, implying that $0 < x_0^j < 1$ and $0 < y_0^{\mathcal{R}} < 1$.

The final range of the parameters is determined by the intersection of the three conditions. Specifically, in the low commission scenario, the range is:

$$\begin{aligned} &0 < \theta < \frac{1}{2},\ 0 < \beta < 1,\ 0 < \tau < -\frac{(\beta\theta - \beta - \theta)^2}{\beta\theta^2 - \beta - \theta^2}, \\ &v > \max\{\frac{\beta\theta - 2\beta + \theta\tau - \theta}{\beta\theta - \beta + \theta\tau - \theta}, \frac{2\theta - 2}{2\theta - 1}\}. \end{aligned} \tag{A3}$$

In the high commission scenario, the range is:

$$\begin{aligned} &0<\theta<\frac{1}{4}, \ 0<\beta<1, \ \max\{0, \frac{-2\beta\theta+2\beta+2\theta-1}{2\theta-1}\}<\tau<\frac{-2\beta\theta+2\beta+2\theta+1}{2\theta+1}, \\ &v>\max\{\frac{\beta\theta-3\beta+\theta\tau-\theta}{\beta\theta-\beta+\theta\tau-\theta}, \frac{2\theta-3}{2\theta-1}\}. \end{aligned} \tag{A4}$$

Notably, in the low commission scenario, the upper bound of τ is less than 1. Conversely, in the high commission scenario, the upper bound of τ is larger than 1, while the lower bound is less than 1.

Appendix B. Parameter Range of Section 7

Similar to Appendix A, three conditions must be met for the equilibrium outcomes to hold: the concavity of the profit functions, full market coverage, and internal solutions. These conditions define the following parameter range:

$$\begin{aligned} &0<\alpha<1, 0<\tau<\frac{3\left(\sqrt{49\alpha^2-348\alpha+324}+7\alpha-18\right)}{8\alpha}, \\ &v\geq\max\{\frac{\alpha\tau^2-15\alpha\tau-9\tau+108}{\alpha\tau^2-3\alpha\tau-18\alpha-9\tau+54}, \frac{\alpha+15}{2(3-\alpha)}\}. \end{aligned} \tag{A5}$$

Appendix C. Mathematical Proofs of Propositions

Proof of Proposition 1. With $\alpha \rightarrow 0^+$ and within the parameter range defined by Equation (A3), we compare the equilibrium prices and profits of Cases $\mathcal{R}$ and $\mathcal{N}$, resulting in:

$$p_R^{\mathcal{R}}-p_R^{\mathcal{N}}=2(\pi_R^{\mathcal{R}}-\pi_R^{\mathcal{N}})=\frac{\theta(\beta-\tau+1)}{(1-2\theta)(\beta\theta-\beta+\theta\tau-\theta)}<0, \tag{A6}$$

$$p_S^{\mathcal{R}}-p_S^{\mathcal{N}}=2(\pi_S^{\mathcal{R}}-\pi_S^{\mathcal{N}})=\frac{(\beta+1)\theta}{(1-2\theta)(\beta\theta-\beta-\theta)}<0. \tag{A7}$$

□

Proof of Proposition 2. Comparing the prices and profits in Cases $\mathcal{R}$ and $\mathcal{N}$ with $\alpha \rightarrow 1^-$, we have:

$$p_R^{\mathcal{R}}-p_R^{\mathcal{N}}=\frac{4}{3}(p_S^{\mathcal{R}}-p_S^{\mathcal{N}})=\frac{16}{13}(\pi_R^{\mathcal{R}}-\pi_R^{\mathcal{N}})=\frac{2\theta(\beta-\tau+1)}{(1-2\theta)(\beta\theta-\beta+\theta\tau-\theta)}. \tag{A8}$$

According to Equation (A4), we can know that Equation (A8) is non-negative if and only if $\tau \leq 1+\beta$. □

Proof of Proposition 3. In the low commission scenario and within the parameter range defined by Equation (A3), we have:

$$\frac{\partial p_R^{\mathcal{R}}}{\partial\beta}=2\frac{\partial\pi_R^{\mathcal{R}}}{\partial\beta}=\frac{\theta(1-\tau)}{(\beta\theta-\beta+\theta\tau-\theta)^2}>0, \tag{A9}$$

$$\frac{\partial p_S^{\mathcal{R}}}{\partial\beta}=2\frac{\partial\pi_S^{\mathcal{R}}}{\partial\beta}=\frac{\theta}{(\beta\theta-\beta-\theta)^2}>0. \tag{A10}$$

Additionally, in the high commission scenario, we have:

$$\frac{\partial p_R^{\mathcal{R}}}{\partial\beta}=\frac{4}{3}\frac{\partial p_S^{\mathcal{R}}}{\partial\beta}=\frac{16}{13}\frac{\partial\pi_R^{\mathcal{R}}}{\partial\beta}=\frac{2\theta(1-\tau)}{(\beta\theta-\beta+\theta\tau-\theta)^2}. \tag{A11}$$

According to the parameter range defined by Equation (A4), we can know that Equation (A11) is non-negative if and only if $\tau \leq 1$. □

Proof of Proposition 4. In the low commission scenario, we have:

$$\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} = -\frac{(1-\theta)\tau\left(\beta^2\theta^2 - 2\beta^2\theta + \beta^2 + \theta^2\tau - \theta^2\right)}{2(\beta\theta - \beta - \theta)^2(\beta\theta - \beta + \theta\tau - \theta)^2}. \quad \text{(A12)}$$

Equation (A12) is non-negative if and only if $\beta^2\theta^2 - 2\beta^2\theta + \beta^2 + \theta^2\tau - \theta^2 \leq 0$, which yields $\tau \leq \frac{\theta^2 - \beta^2(\theta-1)^2}{\theta^2}$. Because $\tau < -\frac{(\beta\theta-\beta-\theta)^2}{\beta\theta^2-\beta-\theta^2}$ according to Equation (A3), the condition for $\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} \geq 0$ is $\tau \leq \min\{-\frac{(\beta\theta-\beta-\theta)^2}{\beta\theta^2-\beta-\theta^2}, \frac{\theta^2-\beta^2(\theta-1)^2}{\theta^2}\}$.

In the high commission scenario, we have:

$$\frac{\partial y_0^{\mathcal{R}}}{\partial \beta} = \frac{(\theta-1)(\tau-1)}{4(\beta\theta - \beta + \theta\tau - \theta)^2}. \quad \text{(A13)}$$

According to the parameter range defined by Equation (A4), we can know that Equation (A13) is non-negative if and only if $\tau \leq 1$. □

Proof of Proposition 5. In the low commission scenario, we have:

$$\frac{\partial y_0^{\mathcal{R}}}{\partial \tau} = \frac{\beta(1-\theta)}{2(\beta\theta - \beta + \theta\tau - \theta)^2} > 0. \quad \text{(A14)}$$

In the high commission scenario, we have:

$$\frac{\partial y_0^{\mathcal{R}}}{\partial \tau} = \frac{\beta(1-\theta)}{4(\beta\theta - \beta + \theta\tau - \theta)^2} > 0. \quad \text{(A15)}$$

□

Proof of Proposition 6. Comparing the outcomes in Cases $\mathcal{R}$ and $\mathcal{N}$, we have the following results:

$$\pi_R^{\mathcal{R}} - \pi_R^{\mathcal{N}} = \frac{3\begin{pmatrix} 1458 - 810\alpha^2 + 324\alpha^3 - 36\alpha^4 - 729\tau + 324\alpha\tau - 675\alpha^2\tau + 342\alpha^3\tau \\ -42\alpha^4\tau + 162\alpha\tau^2 - 9\alpha^2\tau^2 + 42\alpha^3\tau^2 - 13\alpha^4\tau^2 - 9\alpha^2\tau^3 - 6\alpha^3\tau^3 + 2\alpha^4\tau^3 \end{pmatrix}}{4(3-\alpha)^2(\tau-6)(9-3\alpha-\alpha\tau)^2}. \quad \text{(A16)}$$

Within the parameter range defined in Equation (A5), Equation (A16) is positive if and only if $\alpha > \alpha_{\pi R} \approx 0.960$ and $\tau > \tau_{\pi R}$, where $\tau_{\pi R}$ is the unique feasible root of the following equation:

$$\begin{aligned} &\tau^3\left(2\alpha^4 - 6\alpha^3 - 9\alpha^2\right) + \tau^2\left(-13\alpha^4 + 42\alpha^3 - 9\alpha^2 + 162\alpha\right) \\ &+ \tau\left(-42\alpha^4 + 342\alpha^3 - 675\alpha^2 + 324\alpha - 729\right) - 36\alpha^4 + 324\alpha^3 - 810\alpha^2 + 1458 = 0 \end{aligned}. \quad \text{(A17)}$$

$$\pi_S^{\mathcal{R}} - \pi_S^{\mathcal{N}} = \frac{27(\alpha-1)\left(\alpha^3\tau + \alpha^2\tau^2 + 3\alpha^2 - 9\alpha\tau - 18\alpha + 27\right)}{4(\alpha-3)^2(\alpha\tau + 3\alpha - 9)^2}. \quad \text{(A18)}$$

Within the parameter range defined in Equation (A5), Equation (A18) is positive if and only if $\alpha > \alpha_{\pi S} \approx 0.970$ and $\tau > \tau_{\pi S}$, where

$$\tau_{\pi S} = \frac{(\alpha-3)\left(\sqrt{\alpha^2 + 6\alpha - 3} - \alpha - 3\right)}{2\alpha}. \quad \text{(A19)}$$

$$p_R^{\mathcal{R}} - p_R^{\mathcal{N}} = \frac{3\left(\alpha^2\tau^2 - 11\alpha^2\tau - 6\alpha^2 + 3\alpha\tau^2 + 6\alpha\tau - 27\tau + 54\right)}{2(\alpha-3)(\tau-6)(\alpha\tau + 3\alpha - 9)}. \quad \text{(A20)}$$

Within the parameter range defined in Equation (A5), Equation (A20) is positive if and only if $\alpha > \alpha_{pR} \approx 0.957$ and $\tau > \tau_{pR}$, where

$$\tau_{pR} = \frac{11\alpha^2 - \sqrt{145\alpha^4 - 60\alpha^3 + 414\alpha^2 - 972\alpha + 729} - 6\alpha + 27}{2\alpha^2 + 6\alpha}. \tag{A21}$$

$$p_S^{\mathcal{R}} - p_S^{\mathcal{N}} = \frac{9(\alpha\tau + \alpha - 3)}{2(\alpha - 3)(\alpha\tau + 3\alpha - 9)}. \tag{A22}$$

Within the parameter range defined in Equation (A5), Equation (A22) is positive if and only if $\alpha > \alpha_{pS} = \frac{3}{34}\left(49 - 3\sqrt{161}\right) \approx 0.965$ and $\tau > \tau_{pS} = \frac{3-\alpha}{\alpha}$. $\square$

References

1. Tian, L.; Vakharia, A.J.; Tan, Y.; Xu, Y. Marketplace, reseller, or hybrid: Strategic analysis of an emerging e-commerce model. *Prod. Oper. Manag.* **2018**, *27*, 1595–1610. [CrossRef]
2. Krysik, A. Amazon's Product Recommendation System in 2021: How Does the Algorithm of the Ecommerce Giant Work? 2021. Available online: https://recostream.com/blog/amazon-recommendation-system (accessed on 23 April 2024).
3. MacKenzie, I.; Meyer, C.; Noble, S. How Retailers Can Keep Up with Consumers. 2013. Available online: https://www.mckinsey.com/industries/retail/our-insights/how-retailers-can-keep-up-with-consumers#/ (accessed on 23 April 2024).
4. Xiao, B.; Benbasat, I. Designing warning messages for detecting biased online rroduct recommendations: An empirical investigation. *Inf. Syst. Res.* **2015**, *26*, 793–811. [CrossRef]
5. Ogunseyi, T.B.; Avoussoukpo, C.B.; Jiang, Y. A systematic review of privacy techniques in recommendation systems. *Int. J. Inf. Secur.* **2023**, *22*, 1651–1664. [CrossRef]
6. Tam, K.Y.; Ho, S.Y. Understanding the impact of web personalization on user information processing and decision outcomes. *MIS Q.* **2006**, *30*, 865–890. [CrossRef]
7. Li, L.; Chen, J.; Raghunathan, S. Informative role of recommender systems in electronic marketplaces: A boon or a bane for competing sellers. *MIS Q.* **2020**, *44*, 1957–1985. [CrossRef]
8. Li, L.; Chen, J.; Raghunathan, S. Recommender system rethink: Implications for an electronic marketplace with competing manufacturers. *Inf. Syst. Res.* **2018**, *29*, 1003–1023. [CrossRef]
9. Lam, H.T. *Platform Search Design and Market Power*; Job Market Paper; Northwestern University: Evanston, IL , USA, 2021.
10. Lee, K.H.; Musolff, L. *Entry into Two-Sided Markets Shaped by Platform-Guided Search*; Job Market Paper; Princeton University: Princeton, NJ , USA, 2021.
11. Dudley, R. Amazon's New Competitive Advantage: Putting Its WWN Products First. 2024. Available online: https://www.propublica.org/article/amazons-new-competitive-advantage-putting-its-own-products-first (accessed on 8 October 2024).
12. Kittaka, Y.; Sato, S.; Zennyo, Y. Self-preferencing by platforms: A literature review. *Jpn. World Econ.* **2023**, *66*, 101191. [CrossRef]
13. Etro, F. E-commerce platforms and self-preferencing. *J. Econ. Surv.* **2023**, *38*, 1516–1543. [CrossRef]
14. Jiang, B.; Jerath, K.; Srinivasan, K. Firm strategies in the "mid tail" of platform-based retailing. *Mark. Sci.* **2011**, *30*, 757–775. [CrossRef]
15. Mantin, B.; Krishnan, H.; Dhar, T. The strategic role of third-party marketplaces in retailing. *Prod. Oper. Manag.* **2014**, *23*, 1937–1949. [CrossRef]
16. Hagiu, A.; Wright, J. Marketplace or reseller? *Manag. Sci.* **2015**, *61*, 184–203. [CrossRef]
17. Xu, B.; Huang, J.; Zhang, X.; Alejandro, T.B. Strategic third-party product entry and mode choice under self-operating channels and marketplace competition: A game-theoretical analysis. *J. Theor. Appl. Electron. Commer. Res.* **2024**, *19*, 73–94. [CrossRef]
18. Wei, J.; Lu, J.; Zhao, J. Interactions of competing manufacturers' leader-follower relationship and sales format on online platforms. *Eur. J. Oper. Res.* **2020**, *280*, 508–522. [CrossRef]
19. Abhishek, V.; Jerath, K.; Zhang, Z.J. Agency selling or reselling? Channel structures in electronic retailing. *Manag. Sci.* **2016**, *62*, 2259–2280. [CrossRef]
20. Song, W.; Chen, J.; Li, W. Spillover effect of consumer awareness on third parties' selling strategies and retailers' platform openness. *Inf. Syst. Res.* **2020**, *32*, 172–193. [CrossRef]
21. Zha, Y.; Li, Q.; Huang, T.; Yu, Y. Strategic information sharing of online platforms as resellers or marketplaces. *Mark. Sci.* **2023**, *42*, 659–678. [CrossRef]
22. Li, G.; Tian, L.; Zheng, H. Information sharing in an online marketplace with co-opetitive sellers. *Prod. Oper. Manag.* **2021**, *30*, 3713–3734. [CrossRef]
23. Chen, J.; Guo, Z. New-media advertising and retail platform openness. *MIS Q.* **2022**, *46*, 431. [CrossRef]
24. Qin, X.; Liu, Z.; Tian, L. The optimal combination between selling mode and logistics service strategy in an e-commerce market. *Eur. J. Oper. Res.* **2021**, *289*, 639–651. [CrossRef]
25. Ha, A.Y.; Tong, S.; Wang, Y. Channel structures of online retail platforms. *Manuf. Serv. Oper. Manag.* **2022**, *24*, 1547–1561. [CrossRef]

26. Wang, X.; Chaolu, T. The impact of offline service effort strategy on sales mode selection in an e-commerce supply chain with showrooming effect. *J. Theor. Appl. Electron. Commer. Res.* **2022**, *17*, 893–908. [CrossRef]
27. Lai, G.; Liu, H.; Xiao, W.; Zhao, X. "Fulfilled by amazon": A strategic perspective of competition at the e-commerce platform. *Manuf. Serv. Oper. Manag.* **2022**, *24*, 1406–1420. [CrossRef]
28. Hu, Y.; Li, G.; Liu, M.; Qu, S. Information sharing and personalized pricing in online platforms. *Prod. Oper. Manag.* **2024** . [CrossRef]
29. Song, H. Prominence of store-brand products in an electronic platform. *J. Econ.* **2021**, *133*, 47–83. [CrossRef]
30. Zou, T.; Zhou, B. Self-preferencing and search neutrality in online retail platforms. *Manag. Sci.* **2024** . [CrossRef]
31. Long, F.; Liu, Y. Platform manipulation in online retail marketplace with sponsored advertising. *Mark. Sci.* **2024**, *43*, 317–345. [CrossRef]
32. Padilla, J.; Perkins, J.; Piccolo, S. Self-preferencing in markets with vertically integrated gatekeeper platforms. *J. Ind. Econ.* **2022**, *70*, 371–395. [CrossRef]
33. Wang, R.; Qiu, Y. Dual Role and Product Featuring Strategy of Digital Platform. *Mark. Sci.* **2024** , *43*, 1168-1187. [CrossRef]
34. Long, F.; Amaldoss, W. Self-preferencing in e-commerce marketplaces: The role of sponsored advertising and private labels. *Mark. Sci.* **2024** , *43*, 925-952. [CrossRef]
35. Afsar, M.M.; Crump, T.; Far, B. Reinforcement learning based recommender systems: A survey. *ACM Comput. Surv.* **2022**, *55*, 1–38. [CrossRef]
36. Kim, J.; Kim, J.H.; Kim, C.; Park, J. Decisions with ChatGPT: Reexamining choice overload in ChatGPT recommendations. *J. Retail. Consum. Serv.* **2023**, *75*, 103494. [CrossRef]
37. Zhang, Z. Exploration of Hybrid Recommendation Algorithms for Learning Nature Software. In Proceedings of the 2023 2nd International Conference on Data Analytics, Computing and Artificial Intelligence (ICDACAI), Zakopane, Poland, 17–19 October 2023; pp. 530–534.
38. Li, X.J.; Deng, G.S.; Wang, X.Z.; Wu, X.L.; Zeng, Q.W. A hybrid recommendation algorithm based on user comment sentiment and matrix decomposition. *Inf. Syst.* **2023**, *117*, 102244. [CrossRef]
39. Gao, M.; Wang, J.; Liu, O. Is UGC sentiment helpful for recommendation? An application of sentiment-based recommendation model. *Ind. Manag. Data Syst.* **2024**, *124*, 1356–1384. [CrossRef]
40. Hostler, R.E.; Yoon, V.Y.; Guimaraes, T. Assessing the impact of internet agent on end users' performance. *Decis. Support Syst.* **2005**, *41*, 313–323. [CrossRef]
41. Yıldız, E.; Güngör Şen, C.; Işık, E.E. A hyper-personalized product recommendation system focused on customer segmentation: An application in the fashion retail industry. *J. Theor. Appl. Electron. Commer. Res.* **2023**, *18*, 571–596. [CrossRef]
42. Shani, G.; Heckerman, D.; Brafman, R.I.; Boutilier, C. An MDP-based recommender system. *J. Mach. Learn. Res.* **2005**, *6*, 1265–1295.
43. Chen, L.S.; Hsu, F.H.; Chen, M.C.; Hsu, Y.C. Developing recommender systems with the consideration of product profitability for sellers. *Inf. Sci.* **2008**, *178*, 1032–1048. [CrossRef]
44. Hosanagar, K.; Krishnan, R.; Ma, L. Recomended for you: The impact of profit incentives on the relevance of online recommendations. In Proceedings of the International Conference on Information Systems, ICIS 2008, Paris, France, 14–17 December 2008; p. 31.
45. Senecal, S.; Nantel, J. The influence of online product recommendations on consumers' online choices. *J. Retail.* **2004**, *80*, 159–169. [CrossRef]
46. Lee, D.; Hosanagar, K. How do product attributes and reviews moderate the impact of recommender systems through purchase stages? *Manag. Sci.* **2021**, *67*, 524–546. [CrossRef]
47. Yoon, N.; Lee, H.K. AI recommendation service acceptance: Assessing the effects of perceived empathy and need for cognition. *J. Theor. Appl. Electron. Commer. Res.* **2021**, *16*, 1912–1928. [CrossRef]
48. Wan, X.; Kumar, A.; Li, X. How do product recommendations help consumers search? Evidence from a field experiment. *Manag. Sci.* **2023**, *70*, 5776–5794. [CrossRef]
49. Donnelly, R.; Kanodia, A.; Morozov, I. Welfare effects of personalized rankings. *Mark. Sci.* **2024**, *43*, 92–113. [CrossRef]
50. He, X.; Liu, Q.; Jung, S. The impact of recommendation system on user satisfaction: A moderated mediation approach. *J. Theor. Appl. Electron. Commer. Res.* **2024**, *19*, 448–466. [CrossRef]
51. Brynjolfsson, E.; Hu, Y.; Simester, D. Goodbye pareto principle, hello long tail: The effect of search costs on the concentration of product sales. *Manag. Sci.* **2011**, *57*, 1373–1386. [CrossRef]
52. Oestreicher-Singer, G.; Sundararajan, A. Recommendation networks and the long tail of electronic commerce. *MIS Q.* **2012**, *36*, 65–83. [CrossRef]
53. Hosanagar, K.; Fleder, D.; Lee, D.; Buja, A. Will the global village fracture into tribes? Recommender systems and their effects on consumer fragmentation. *Manag. Sci.* **2014**, *60*, 805–823. [CrossRef]
54. Lee, D.; Hosanagar, K. How do recommender systems affect sales diversity? A cross-category investigation via randomized field experiment. *Inf. Syst. Res.* **2019**, *30*, 239–259. [CrossRef]
55. Wan, X.; Kumar, A.; Li, X. Retargeted versus generic product recommendations: When is it valuable to present retargeted recommendations? *Inf. Syst. Res.* **2023**, *35*, 1403–1421. [CrossRef]
56. Inderst, R.; Ottaviani, M. Competition through commissions and kickbacks. *Am. Econ. Rev.* **2012**, *102*, 780–809. [CrossRef]

57. Teh, T.H.; Wright, J. Intermediation and steering: Competition in prices and commissions. *Am. Econ. J. Microecon.* **2022**, *14*, 281–321. [CrossRef]
58. Zhou, C.; Li, H.; Zhang, L.; Ren, Y. Optimal recommendation strategies for AI-powered e-commerce platforms: A study of duopoly manufacturers and market competition. *J. Theor. Appl. Electron. Commer. Res.* **2023**, *18*, 1086–1106. [CrossRef]
59. Zhou, B.; Zou, T. Competing for recommendations: The strategic impact of personalized product recommendations in online marketplaces. *Mark. Sci.* **2023**, *42*, 360–376. [CrossRef]
60. Hotelling, H. Stability in Competition. *Econ. J.* **1929**, *39*, 41–57. [CrossRef]
61. Kwark, Y.; Chen, J.; Raghunathan, S. Online product reviews: Implications for retailers and competing manufacturers. *Inf. Syst. Res.* **2014**, *25*, 93–110. [CrossRef]
62. Smith, B.; Linden, G. Two decades of recommender systems at Amazon. com. *IEEE Internet Comput.* **2017**, *21*, 12–18. [CrossRef]
63. Hu, Y.J.; Shin, J.; Tang, Z. Incentive Problems in Performance-Based Online Advertising Pricing: Cost per Click vs. Cost per Action. *Manag. Sci.* **2016**, *62*, 2022–2038. [CrossRef]