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Article

Adaptive Switched Generalized Function Projective Synchronization between Two Hyperchaotic Systems with Unknown Parameters

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Abstract: In this paper, we investigate adaptive switched generalized function projective synchronization between two new different hyperchaotic systems with unknown parameters, which is an extension of the switched modified function projective synchronization scheme. Based on the Lyapunov stability theory, corresponding adaptive controllers with appropriate parameter update laws are constructed to achieve adaptive switched generalized function projective synchronization between two different hyperchaotic systems. A numerical simulation is conducted to illustrate the validity and feasibility of the proposed synchronization scheme.

Keywords: generalized function projective synchronization; switched state; hyperchaotic system; stability

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1. Introduction

Hyperchaos, which was first introduced by Rössler [1], is usually characterized as a chaotic attractor with more than one positive Lyapunov exponent. The degree of chaos of a system can be measured by a generalization of the concept of entropy for state space dynamics [2,3]. It is a highly desired property to ensure security in a chaos encryption scheme that the larger the entropy, the larger the unpredictability of the system [4]. After the hyperchaotic Rössler system, many other hyperchaotic systems have been reported, including the hyperchaotic Lorenz system [5], hyperchaotic Chen system [6], hyperchaotic Lü system [7]. In [8], the positive topological entropy was calculated, which indicated that the system from two coupled Wien-bridge oscillators was hyperchaotic.

Since the concept of synchronizing two identical chaotic systems from different initial conditions was introduced by Pecora and Carroll in 1990 [9], synchronization in chaotic systems has been extensively investigated over the last two decades. Many synchronization schemes have been proposed, which include complete synchronization [10,11], lag synchronization [12], generalized synchronization [13], phase synchronization [14], anti-synchronization [15,16], partial synchronization [17,18], Q-S synchronization [19,20], projective synchronization [21–32], anticipating synchronization [33], inverse lag synchronization [34] and inverse π-lag synchronization [35,36].

Among the above-mentioned synchronization phenomena, projective synchronization has been investigated with increasing interest in recent years due to the fact that it can obtain faster communication with its proportional feature [23–26]. The concept of projective synchronization was first introduced by Mainieri and Rehacek in 1999 [27], in which the drive and response systems could be synchronized up to a constant scaling factor. Later on, Li [28] proposed a new synchronization scheme called modified projective synchronization (MPS), where the drive and response dynamical states synchronize up to a constant scaling matrix. Afterwards, Chen *et al.* [29] extended the modified projective synchronization and proposed function projective synchronization (FPS), where the drive and response dynamical states synchronize up to a scaling function matrix, but not a constant one. Recently, Du *et al.* [30] discussed a new type of synchronization phenomenon, modified function projective synchronization (MFPS), in which the drive and response systems could be synchronized up to a desired scaling function matrix. Many of these synchronization schemes have been applied to investigate chaotic or fractional chaotic systems [37–44]. More recently, Yu and Li [31] have proposed a new synchronization scheme by choosing a more generalized scaling function matrix, called generalized function projective synchronization (GFPS), which is an extension of all the aforementioned projective synchronization schemes. Lately, Sudheer and Sabir [32] reported switched modified function projective synchronization (SMFPS) in hyperchaotic Qi system using adaptive control method, in which a state variable of the drive system synchronize with a different state variable of the response system up to a desired scaling function matrix.

Inspired by the previous works, in this paper, we propose the switched generalized function projective synchronization (SGFPS) between two different hyperchaotic systems using adaptive control method by extending the GFPS and SMFPS schemes, in which a state variable of the drive system synchronizes with a different state variable of the response system up to a more generalized scaling function matrix. Due to the unpredictability of the switched states and scaling function matrix, this synchronization scheme can provide additional security in secure communication.

The rest of this paper is organized as follows. Section 2 gives a brief description of the SGFPS scheme and two new hyperchaotic systems. In Section 3, we propose appropriate adaptive controllers and parameter update laws for the adaptive switched generalized function projective synchronization of two different hyperchaotic systems. Section 4 presents a numerical example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

2. Description of the Switched Generalized Function Projective Synchronization and Two New Hyperchaotic Systems

Consider the following drive and response systems:

$$\begin{cases} \dot{x} = f(x) \\ \dot{y} = g(y) + u(t, x, y) \end{cases} \tag{1}$$

where $x, y \in R^n$ are the state vectors, $f(x), g(x) : R^n \to R^n$ are differentiable vector functions, and $u(t, x, y)$ is the controller vector to be designed.

The error states between the drive and response systems are defined as

$$e_i = y_i - \phi_i(x)x_j, (i, j = 1, 2, ..., n,\ i \neq j) \tag{2}$$

where $\phi_i(x) : R^n \to R(i = 1, 2, ..., n)$ are scaling function factors, and are continuous differentiable bounded , which compose the scaling function matrix $\phi(x)$, $\phi(x) = diag\{\phi_1(x), \phi_2(x), ..., \phi_n(x)\}$.

Definition 1. For the two systems described in Equation (1), we say that they are switched generalized function projective synchronous with respect to the scaling function matrix $\phi(x)$ if there exists a controller vector $u(t, x, y)$ such that

$$\lim_{t\to\infty} \|e_i\| = \lim_{t\to\infty} \|y_i - \phi_i(x)x_j\| = 0, (i, j = 1, 2, ..., n, i \neq j) \tag{3}$$

which implies that the error dynamic system (2) between the drive and response systems is globally asymptotically stable.

Remark 1. For the SGFPS, we define $i \neq j$ in the above Equation (3). If $i = j$, the SGFPS degenerates to the GFPS [25].

Recently, Li *et al.* [45] proposed a new hyperchaotic Lorenz-type system described by

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = bx - xz - cy + w \\ \dot{z} = xy - dz \\ \dot{w} = -ky - rw \end{cases} \tag{4}$$

where a, b, c, d, k and r are positive constant system parameters. When $a = 12, b = 23, c = 1$, $d = 2.1, k = 6$ and $r = 0.2$, and with the initial condition $[1, 2, 3, 4]^T$, system (4) is hyperchaotic and its attractor is shown in Figure 1.

Lately, Dadras *et al.* [46] reported the following four-wing hyperchaotic system, which has only one unstable equilibrium

$$\begin{cases} \dot{x} = ax - yz + w \\ \dot{y} = xz - by \\ \dot{z} = xy - cz + xw \\ \dot{w} = -y \end{cases} \tag{5}$$

where a, b and c are positive constant system parameters. When $a = 8, b = 40$ and $r = 14.9$, and with the initial condition $[10, 1, 10, 1]^T$, system (5) is hyperchaotic and its attractor is shown in Figure 2.

Figure 1. Hyperchaotic attractor of system (4) with $a = 12, b = 23, c = 1, d = 2.1, k = 6$ and $r = 0.2$: (**a**) $x - y - z$ space; (**b**) $x - y$ plane; (**c**) $x - z$ plane; (**d**) $x - w$ plane.

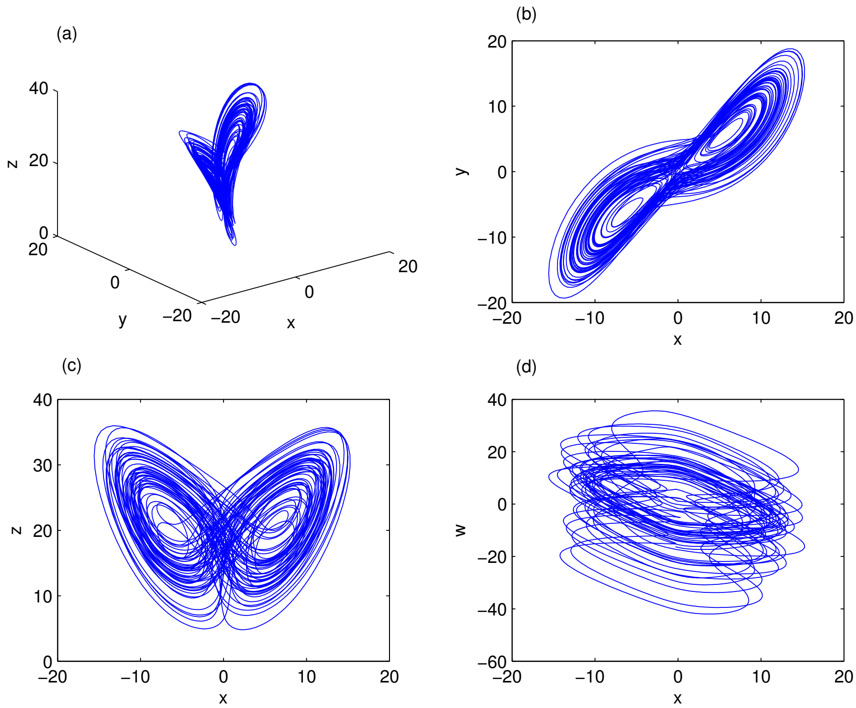
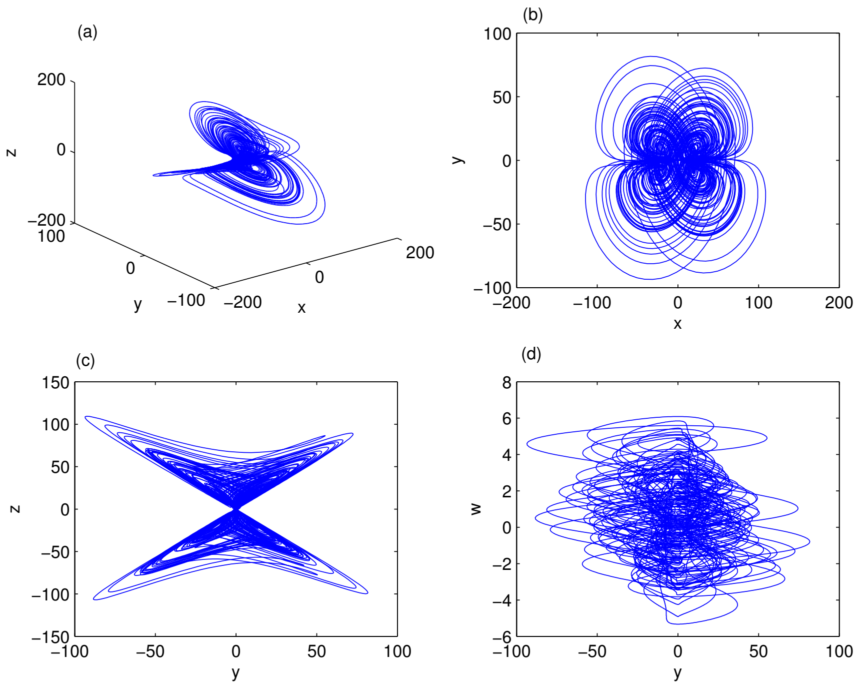


Figure 2. Hyperchaotic attractor of system (5) with $a = 8, b = 40$ and $r = 14.9$: (**a**) $x-y-z$ space; (**b**) $x - y$ plane; (**c**) $x - z$ plane; (**d**) $y - w$ plane.



For more information on the dynamical behaviors of these two systems, please refer to [45,46].

3. Switched Generalized Function Projective Synchronization between Two Hyperchaotic Systems

In this section, we investigate the adaptive SGFPS between systems (4) and (5) with fully unknown parameters.

Suppose that system (4) is the drive system whose four variables are denoted by subscript 1 and system (5) is the response system whose variables are denoted by subscript 2. Then the drive and response systems are described by the following equations, respectively,

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) \\ \dot{y}_1 = b_1 x_1 - x_1 z_1 - c_1 y_1 + w_1 \\ \dot{z}_1 = x_1 y_1 - d_1 z_1 \\ \dot{w}_1 = -k_1 y_1 - r_1 w_1 \end{cases} \tag{6}$$

and

$$\begin{cases} \dot{x}_2 = a_2 x_2 - y_2 z_2 + w_2 + u_1 \\ \dot{y}_2 = x_2 z_2 - b_2 y_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - c_2 z_2 + x_2 w_2 + u_3 \\ \dot{w}_2 = -y_2 + u_4 \end{cases} \tag{7}$$

where $a_1, b_1, c_1, d_1, k_1, r_1, a_2, b_2$ and c_2 are unknown parameters to be identified, and $u_i (i = 1, 2, 3, 4)$ are controllers to be determined such that the two hyperchaotic systems can achieve SGFPS, in the sense that

$$\begin{cases} \lim\limits_{t\to\infty} \|e_1\| = \lim\limits_{t\to\infty} \|x_2 - \phi_1(x) z_1\| = 0 \\ \lim\limits_{t\to\infty} \|e_2\| = \lim\limits_{t\to\infty} \|y_2 - \phi_2(x) w_1\| = 0 \\ \lim\limits_{t\to\infty} \|e_3\| = \lim\limits_{t\to\infty} \|z_2 - \phi_3(x) x_1\| = 0 \\ \lim\limits_{t\to\infty} \|e_4\| = \lim\limits_{t\to\infty} \|w_2 - \phi_4(x) y_1\| = 0 \end{cases} \tag{8}$$

where $\phi_i(x) (i = 1, 2, 3, 4)$ are scaling functions.

So the SGFPS error dynamical system is determined as follows

$$\begin{cases} \dot{e}_1 = a_2 x_2 - y_2 z_2 + w_2 - \dot{\phi}_1(x) z_1 - \phi_1(x)(x_1 y_1 - d_1 z_1) + u_1 \\ \dot{e}_2 = x_2 z_2 - b_2 y_2 - \dot{\phi}_2(x) w_1 - \phi_2(x)(-k_1 y_1 - r_1 w_1) + u_2 \\ \dot{e}_3 = x_2 y_2 - c_2 z_2 + x_2 w_2 - \dot{\phi}_3(x) x_1 - \phi_3(x) a_1 (y_1 - x_1) + u_3 \\ \dot{e}_4 = -y_2 - \dot{\phi}_4(x) y_1 - \phi_4(x)(b_1 x_1 - x_1 z_1 - c_1 y_1 + w_1) + u_4 \end{cases} \tag{9}$$

Without loss of generality, the scaling functions can be chosen as $\phi_1(x) = m_{11} x_1 + m_{12}$, $\phi_2(x) = m_{21} y_1 + m_{22}$, $\phi_3(x) = m_{31} z_1 + m_{32}$ and $\phi_4(x) = m_{41} w_1 + m_{42}$, where $m_{ij} (i = 1, 2, 3, 4; j = 1, 2)$ are constant numbers. And substituting systems (6) and (7) into system (9), yields the following form:

$$\begin{cases} \dot{e}_1 = a_2x_2 - y_2z_2 + w_2 - m_{11}a_1(y_1 - x_1)z_1 - \phi_1(x)(x_1y_1 - d_1z_1) + u_1 \\ \dot{e}_2 = x_2z_2 - b_2y_2 - m_{21}(b_1x_1 - x_1z_1 - c_1y_1 + w_1)w_1 - \phi_2(x)(-k_1y_1 - r_1w_1) + u_2 \\ \dot{e}_3 = x_2y_2 - c_2z_2 + x_2w_2 - m_{31}(x_1y_1 - d_1z_1)x_1 - \phi_3(x)a_1(y_1 - x_1) + u_3 \\ \dot{e}_4 = -y_2 - m_{41}(-k_1y_1 - r_1w_1)y_1 - \phi_4(x)(b_1x_1 - x_1z_1 - c_1y_1 + w_1) + u_4 \end{cases} \tag{10}$$

Our goal is to find the appropriate controllers $u_i(i = 1, 2, 3, 4)$ to stabilize the error variables of system (10) at the origin. For this purpose, we propose the following controllers for system (10)

$$\begin{cases} u_1 = -\bar{a}_2x_2 + y_2z_2 - w_2 + m_{11}\bar{a}_1(y_1 - x_1)z_1 + \phi_1(x)(x_1y_1 - \bar{d}_1z_1) - l_1e_1 \\ u_2 = -x_2z_2 + \bar{b}_2y_2 + m_{21}(\bar{b}_1x_1 - x_1z_1 - \bar{c}_1y_1 + w_1)w_1 + \phi_2(x)(-\bar{k}_1y_1 - \bar{r}_1w_1) - l_2e_2 \\ u_3 = -x_2y_2 + \bar{c}_2z_2 - x_2w_2 + m_{31}(x_1y_1 - \bar{d}_1z_1)x_1 - \phi_3(x)\bar{a}_1(y_1 - x_1) - l_3e3 \\ u_4 = y_2 + m_{41}(-\bar{k}_1y_1 - \bar{r}_1w_1)y_1 + \phi_4(x)(\bar{b}_1x_1 - x_1z_1 - \bar{c}_1y_1 + w_1) - l_4e_4 \end{cases} \tag{11}$$

where $L = diag(l_1, l_2, l_3, l_4)$ is a positive gain matrix for each state controller. In practical applications the synchronization process can be sped up by increasing the gain matrix L.

The update laws for the unknown parameters $a_1, b_1, c_1, d_1, k_1, r_1, a_2, b_2$ and c_2 are given as follows

$$\begin{cases} \dot{\bar{a}}_1 = -m_{11}(y_1 - x_1)z_1e_1 - \phi_3(x)(y_1 - x_1)e_3 + (a_1 - \bar{a}_1) \\ \dot{\bar{b}}_1 = -m_{21}x_1w_1e_2 - \phi_4(x)x_1e_4 + (b_1 - \bar{b}_1) \\ \dot{\bar{c}}_1 = \phi_4(x)y_1e_4 + m_{21}y_1w_1e_2 + (c_1 - \bar{c}_1) \\ \dot{\bar{d}}_1 = \phi_1(x)z_1e_1 + m_{31}z_1x_1e_3 + (d_1 - \bar{d}_1) \\ \dot{\bar{k}}_1 = \phi_2(x)y_1e_2 + m_{41}y_1^2e_4 + (k_1 - \bar{k}_1) \\ \dot{\bar{r}}_1 = \phi_2(x)w_1e_2 + m_{41}w_1y_1e_4 + (r_1 - \bar{r}_1) \\ \dot{\bar{a}}_2 = x_2e_1 + (a_2 - \bar{a}_2) \\ \dot{\bar{b}}_2 = -y_2e_2 + (b_2 - \bar{b}_2) \\ \dot{\bar{c}}_2 = -z_2e_3 + (c_2 - \bar{c}_2) \end{cases} \tag{12}$$

where $\bar{a}_1, \bar{b}_1, \bar{c}_1, \bar{d}_1, \bar{k}_1, \bar{r}_1, \bar{a}_2, \bar{b}_2$ and $\bar{c}_2$ are the estimate values for these unknown parameters, respectively. Then, we have the following main result.

Theorem 1. For a given continuous differential scaling function matrix $\phi(x) = diag\{\phi_1(x), \phi_2(x), \phi_3(x), \phi_4(x)\}$, and any initial values, the SGFPS between systems (6) and (7) can be achieved by the adaptive controllers (11) and the parameter update laws (12).

Proof. Choose the following Lyapunov function,

$$\begin{aligned} V =& \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + (\bar{a}_2 - a_2)^2 + (\bar{b}_2 - b_2)^2 + (\bar{c}_2 - c_2)^2) \\ &+ \frac{1}{2}((\bar{a}_1 - a_1)^2 + (\bar{b}_1 - b_1)^2 + (\bar{c}_1 - c_1)^2 + (\bar{d}_1 - d_1)^2 + (\bar{k}_1 - k_1)^2 + (\bar{r}_1 - r_1)^2) \end{aligned} \tag{13}$$

Taking the time derivative of V along the trajectory of the error dynamical system (10) yields

$$\begin{aligned}
\dot{V} =&\dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + (\bar{a}_2 - a_2)\dot{\bar{a}}_2 + (\bar{b}_2 - b_2)\dot{\bar{b}}_2 + (\bar{c}_2 - c_2)\dot{\bar{c}}_2 \\
&+ (\bar{a}_1 - a_1)\dot{\bar{a}}_1 + (\bar{b}_1 - b_1)\dot{\bar{b}}_1 + (\bar{c}_1 - c_1)\dot{\bar{c}}_1 + (\bar{d}_1 - d_1)\dot{\bar{d}}_1 + (\bar{k}_1 - k_1)\dot{\bar{k}}_1 + (\bar{r}_1 - r_1)\dot{\bar{r}}_1 \\
=&e_1(a_2 x_2 - y_2 z_2 + w_2 - m_{11} a_1 (y_1 - x_1) z_1 - \phi_1(x)(x_1 y_1 - d_1 z_1) + u_1) \\
&+ e_2(x_2 z_2 - b_2 y_2 - m_{21}(b_1 x_1 - x_1 z_1 - c_1 y_1 + w_1) w_1 - \phi_2(x)(-k_1 y_1 - r_1 w_1) + u_2) \\
&+ e_3(x_2 y_2 - c_2 z_2 + x_2 w_2 - m_{31}(x_1 y_1 - d_1 z_1) x_1 - \phi_3(x) a_1 (y_1 - x_1) + u_3) \\
&+ e_4(-y_2 - m_{41}(-k_1 y_1 - r_1 w_1) y_1 - \phi_4(x)(b_1 x_1 - x_1 z_1 - c_1 y_1 + w_1) + u_4) \\
&+ (\bar{a}_2 - a_2)\dot{\bar{a}}_2 + (\bar{b}_2 - b_2)\dot{\bar{b}}_2 + (\bar{c}_2 - c_2)\dot{\bar{c}}_2 \\
&+ (\bar{a}_1 - a_1)\dot{\bar{a}}_1 + (\bar{b}_1 - b_1)\dot{\bar{b}}_1 + (\bar{c}_1 - c_1)\dot{\bar{c}}_1 + (\bar{d}_1 - d_1)\dot{\bar{d}}_1 + (\bar{k}_1 - k_1)\dot{\bar{k}}_1 + (\bar{r}_1 - r_1)\dot{\bar{r}}_1
\end{aligned} \tag{14}$$

Substituting Equation (11) into Equation (14) yields

$$\begin{aligned}
\dot{V} =& - l_1 e_1^2 - l_2 e_2^2 - l_3 e_3^2 - l_4 e_4^2 \\
&- (\bar{a}_1 - a_1)^2 - (\bar{b}_1 - b_1)^2 - (\bar{c}_1 - c_1)^2 - (\bar{d}_1 - d_1)^2 - (\bar{k}_1 - k_1)^2 - (\bar{r}_1 - r_1)^2 \\
&- (\bar{a}_2 - a_2)^2 - (\bar{b}_2 - b_2)^2 - (\bar{c}_2 - c_2)^2 \\
<&0
\end{aligned} \tag{15}$$

Since the Lyapunov function V is positive definite and its derivative $\dot{V}$ is negative definite in the neighborhood of the zero solution for system (10). According to the Lyapunov stability theory, the error dynamical system (10) can converge to the origin asymptotically. Therefore, the SGFPS between the two hyperchaotic systems (6) and (7) is achieved with the adaptive controllers (11) and the parameter update laws (12).

This completes the proof.

4. Numerical Simulation

In this section, to verify and demonstrate the effectiveness of the proposed method we consider a numerical example. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. The true values of the "unknown" parameters of systems (6) and (7) are chosen as $a_1 = 12,\ b_1 = 23,\ c_1 = 1,\ d_1 = 2.1,\ k_1 = 6,\ r_1 = 0.2,\ a_2 = 8,\ b_2 = 40,\ c_2 = 14.9$, so that the two systems exhibit hyperchaotic behavior, respectively. The initial values for the drive and response systems are $x_1(0) = 8.3,\ y_1(0) = 10.8,\ z_1(0) = 17.4,\ w_1(0) = -11.1,\ x_2(0) = -0.2,\ y_2(0) = -0.1,\ z_2(0) = 16.9$ and $w_2(0) = -0.7$, and the estimated parameters have initial conditions 0.1. Given that the function factors are $\phi_1(x) = 2x_1 - 0.3$, $\phi_2(x) = 2y_1 + 0.5,\ \phi_3(x) = 0.5z_1 + 0.03,\ \phi_4(x) = -0.5w_1 + 0.03$, and the gain matrix L is given as $diag\{10, 10, 10, 10\}$. The simulation results are shown in Figures 3–5. Figure 3 demonstrates the SGFPS errors of the drive system (6) and response system (7). From this figure, it can be seen that the SGFPS errors converge to zero, *i.e.*, these two systems achieved SGFPS. And Figures 4 and 5 show that the unknown system parameters approach the true values.

Figure 3. The time evolution of SGFPS errors for the drive system (6) and response system (7) with controllers (11) and parameter update laws (12), where $e_1 = x_2-(2x_1-0.3)z_1, e_2 = y_2-(2y_1+0.5)w_1,\ e_3 = z_2-(0.5z_1+0.03)x_1,\ e_4 = w_2-(-0.5w_1+0.03)y_1$.

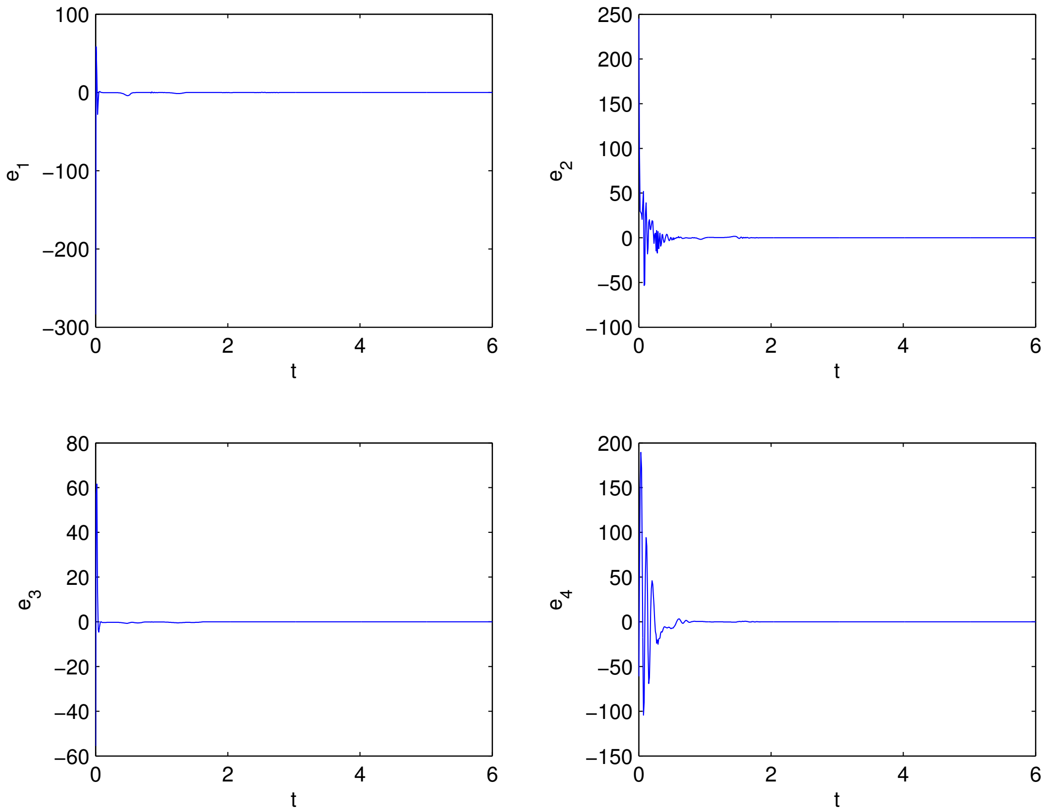


Figure 4. The time evolution of the estimated unknown parameters of system (6).

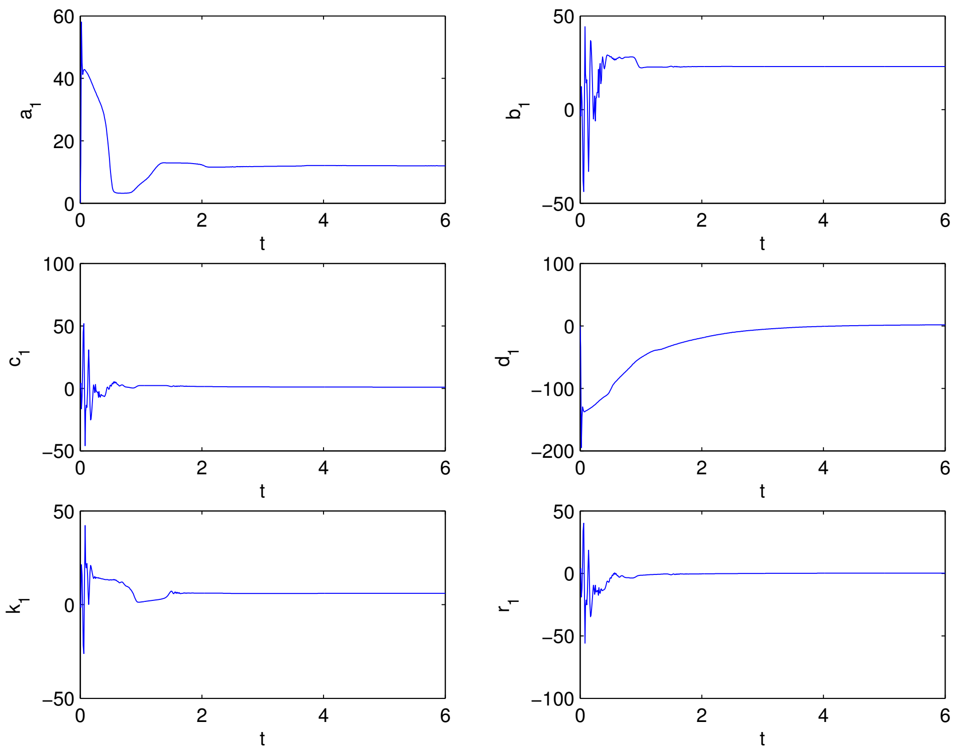
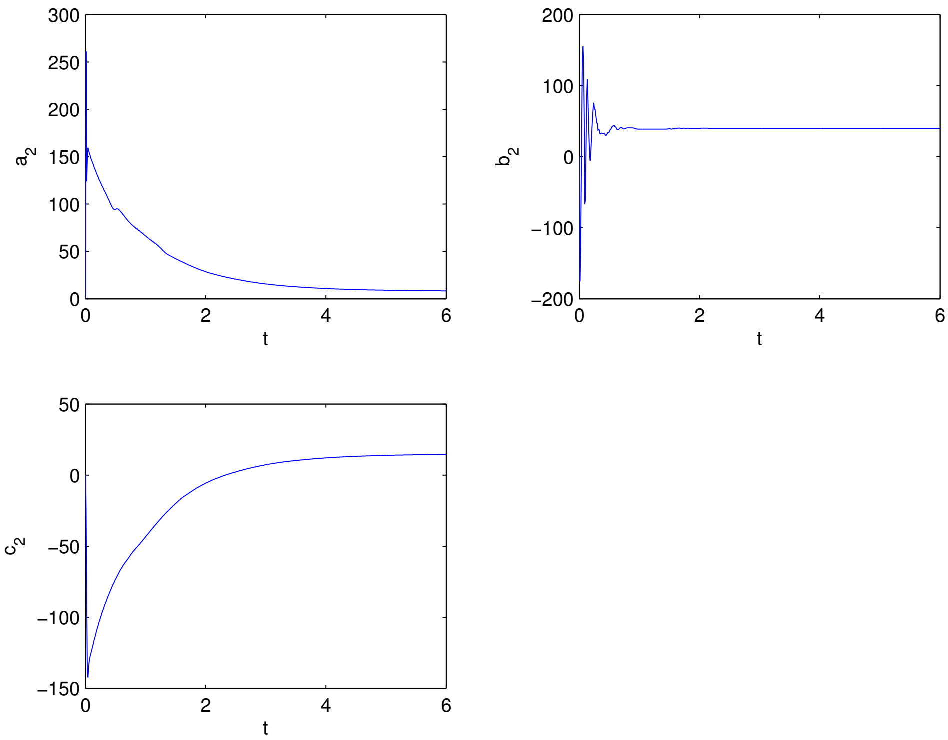


Figure 5. The time evolution of the estimated unknown parameters of system (7).



5. Conclusions

In this paper, we have investigated switched generalized function projective synchronization between two new different hyperchaotic systems with fully unknown parameters, which extended the switched modified function projective synchronization scheme. In this synchronization scheme, a state variable of the drive system synchronizes with a different state variable of the response system up to a generalized scaling function matrix. Due to the unpredictability of the switched states and scaling function matrix, this synchronization scheme can provide additional security in secure communication. By applying the adaptive control theory and Lyapunov stability theory, the appropriate adaptive controllers with parameter update laws are proposed to achieve SGFPS between two different hyperchaotic systems. A numerical simulation was conducted to illustrate the validity and feasibility of the proposed synchronization scheme.

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Conflicts of Interest

The authors declare no conflict of interest.

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