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Article

Information Entropy-Based Metrics for Measuring Emergences in Artificial Societies

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Abstract: Emergence is a common phenomenon, and it is also a general and important concept in complex dynamic systems like artificial societies. Usually, artificial societies are used for assisting in resolving several complex social issues (e.g., emergency management, intelligent transportation system) with the aid of computer science. The levels of an emergence may have an effect on decisions making, and the occurrence and degree of an emergence are generally perceived by human observers. However, due to the ambiguity and inaccuracy of human observers, to propose a quantitative method to measure emergences in artificial societies is a meaningful and challenging task. This article mainly concentrates upon three kinds of emergences in artificial societies, including emergence of attribution, emergence of behavior, and emergence of structure. Based on information entropy, three metrics have been proposed to measure emergences in a quantitative way. Meanwhile, the correctness of these metrics has been verified through three case studies (the spread of an infectious influenza, a dynamic microblog network, and a flock of birds) with several experimental simulations on the Netlogo platform. These experimental results confirm that these metrics increase with the rising degree of emergences. In addition, this article also has discussed the limitations and extended applications of these metrics.

Keywords: emergence; entropy; dynamic system; artificial society; information theory

PACS Codes: 05.10.-a; 05.65.+b; 02.70.-c; 07.05.Tp; 89.75.-k; 89.65.-s; 07.05.Pj

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1. Introduction

An emergence is a general phenomenon which is also an important feature of complex dynamic systems [1]. The emergence means the global properties, behaviors, structures or patterns of a complex system arising from localized individual behaviors [2]. Usually, an emergence is always connected with the concept of self-organization [3], which may be due to the local nonlinear self-organized interactions between individuals in a complex dynamic system. In the natural world, there are a large number of emergences like fish schools and flocks of birds, and such emergences can assist the individuals in avoiding dangers or achieving a macro-level goal. However, in the real society, an emergency represents an event or situation that could pose an immediate risk to health, life, property or the environment [4], and an emergency may give rise to some emergences, which may have a negative impact on the individuals in the society and even result in a great loss of life and property, e.g., an outbreak of H7N9, H1N1 or SARS. In order to avoid such terrible emergences, emergency management is put in place to control and manage this kind of emergency. Because of the nonrepeatability and high-cost of experiments in the real society, borrowing the idea of artificial society and parallel societies, social computing [5] is emerging to aid in meeting the challenge in emergency management. Artificial society is taken as the foundation of social computing, which is an abstract and mapping of the real society and comprises multiple agents from bottom-up. Hence, emergency management experiments could be performed in artificial societies, where emergency management policies could also be evaluated and refined. As one knows, various emergencies may cause different emergences if without effective emergency management policies. The degree of an emergence can reflect the effectiveness of an emergency management policy. According to different levels of the emergence, experts could adopt corresponding emergency management policies to respond to emergencies. Usually, an emergence is perceived by human observers, even though the meaning of the emergence observed by humans is not clear enough and levels of the emergence may be ambiguous. There is a lack of a general metric to measure the degree of an emergence in artificial societies. Therefore, the motivation and purpose of this paper are to propose quantitative metrics to measure the levels of emergences.

Some research has attempted to provide effective methods to measure emergences. The idea of entropy is one of these approaches. It is well known that the concept of entropy has been widely applied in various disciplines with distinct definitions. Generally, the entropy is conceived as a metric of the disorder in a complex dynamic system [6,7]. There are several entropies in various domains such as thermodynamic entropy [8], Boltzmann's entropy [9,10], social entropy [11,12], information entropy [13], and generalized entropy [14], *etc.* Moreover, entropy could also be used to measure the self-organization in complex dynamic systems [15]. Self-organization generally could result in emergences, but the entropy for self-organization is not the same as the metric of emergence [16]. Reference [16] has proposed a relative entropy redundancy only to measure emergences of attributes in complex dynamic systems but not the emergence of behaviors or structures. Evolution in the biosphere could be taken as an emergence (a particular macroscopic behavior) derived from micro-level organismic interactions [17], and evolutionary activity is introduced as an objective, quantitative, and observable metric for evolution (emergence). A simple approach with the usage of statistics could measure the degree of evolutionary activity (*i.e.*, the levels of the emergence), where persistent usage of a new gene could be seen as an evolutionary activity. Emergence can also be considered as a matter of degree, which can be defined by the amount of simulations to get a fact [18]. Meanwhile, a metric using interaction statistics from the agent-based simulations [19] has been used for detecting emergences of attributes and behaviors in the simulations. When continuous attributes are considered, and some new technical metrics like divergence measures for probability densities to measure emergences [20]. In the same way, these methods in [20] only focus on emergences of attributes or behaviors but not emergences of structure.

Due to the diversity of agents, artificial societies and emergences, proposing a general metric for measuring emergences in artificial societies is a challenging task. Generally, emergences can be divided into two kinds: weak emergences and strong emergences. As we know, a strong emergence means the macro-level property of a system, which cannot be found in the properties of the system's parts or in the local interactions between the systems' parts [21]. A strong emergence is a macro-level property and in principle it cannot be identified from the sum of micro-level properties. The strong emergence cannot be simulated by a computer, however, the emergent macro-level properties have irreducible causal powers [22–24]. A classic case of strong emergence is the emergence of consciousness such as the qualia of pain from the neurobiological processes [25]. Its hard to measure such an strong emergence. Compared with strong emergences, weak emergences are in principle identifiable from micro-level individuals [26], and weak emergences are derivable and the micro-to-macro derivable tracks should be non-trivial, which can be amenable to computer simulations [23]. In addition, there is another form of definition for weak emergence, *i.e.*, macrostate P of a system S with micro-level dynamic state D is weakly emergent iff P can be derived from D and S's external conditions but only by simulation [18]. Hence, weak emergence is the focus of this paper, and we will not discuss from strong emergence to weak emergence. Aiming to measuring emergences in artificial societies, this paper introduces a general artificial society model—the zombie-city [27] to overcome the diversity of artificial societies by modeling artificial societies with this general model, and borrows the idea of information entropy to propose metrics for measuring the emergences, including the emergences of attribute, behavior and structure. Through three case studies with experimental simulations on Netlogo, these metrics can correctly measure the levels of attribute, behavior and structure emergences in artificial societies, and the results show that these metrics increase with the rising levels of emergences.

The remaining sections of this article are organized as follows. Section 2 introduces the metrics for measuring emergences in artificial societies constructed with the zombie-city model, and this section presents the definitions of agents, entropy of disorder in artificial societies, and metrics for measuring emergences. Section 3 studies three cases to confirm the effectiveness and correctness of the metrics. Section 4 discusses the limitations of these metrics and extended applications of these metrics for emergency management in artificial societies. Section 5 summarizes the results of this paper, and foresees the future works.

2. Method for Measuring Emergences

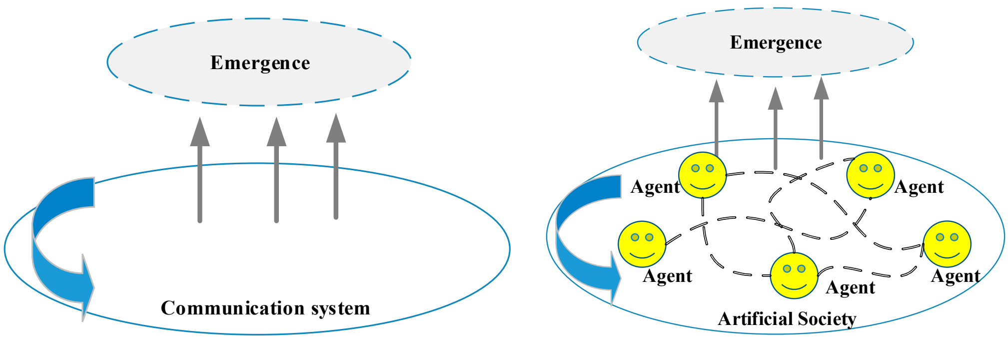
2.1. Prerequisite Knowledge

In information theory, it can be taken as an emergence that larger information is produced in a communication system. The metric of emergence [28] could be simply described by Equation (1):

$$E = I_{out} / I_{in} . \quad (1)$$

where I_{in} represents the input information and I_{out} denotes the output information. It is well known, however, that an artificial society is different from a communication system. A communication system is considered as a whole, but an artificial society comprises multiple agents. An emergence in a communication system is generally considered as the whole behavior of the system and caused by the complexity of the communication system, while the emergence in an artificial society is created by individuals and their nonlinear interactions or other behaviors, as shown in Figure 1. Therefore, the metric of emergences in the communication system cannot be directly used to measure emergences in artificial societies.

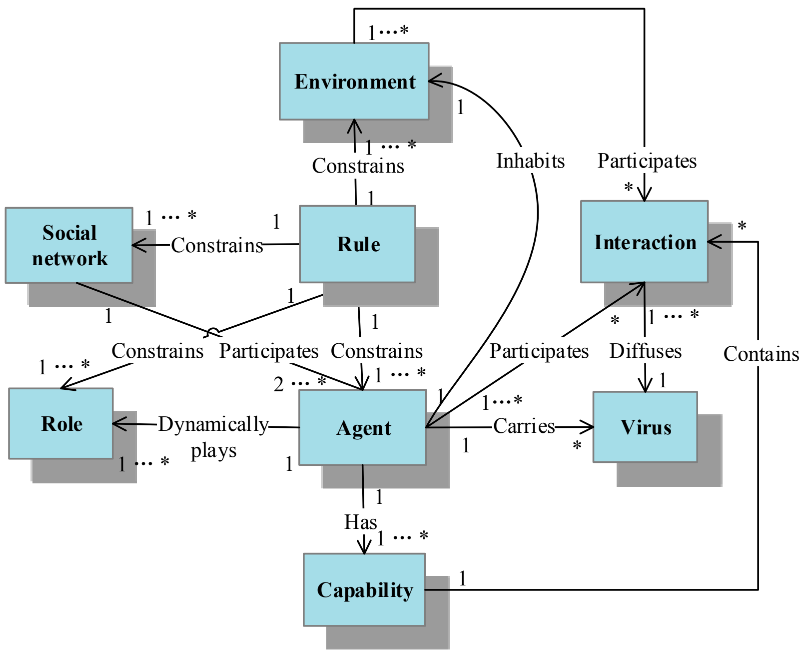
Figure 1. (**a**) Emergence in a communication system; (**b**) Emergence in an artificial society.



(**a**) (**b**)

The social system is a special complex dynamic system, and an artificial society is the mapping of the social system with several complex features like diversity. This paper focuses on the artificial societies modeled with the zombie-city model, which is a general artificial society model and can be used in social computing for emergency management. Figure 2 presents the meta-model of the zombie-city model, including agents, roles, environment, social network and rules, *etc.* Agents have their own capabilities, one of which is the interaction. Meanwhile, agents may carry viruses, and a virus is an abstract concept, which could not only denote an infectious virus but also indicate information and other concepts. Based-on the zombie-city model, an artificial society *AS* could be defined as: *AS*:: = < *AG*, *RO*, *EN*, *SN*, *RU* >, where *AG* means the set of agents in this artificial society, *RO* represents the set of roles, *EN* means the environment made up of grids, *SN* is the set of social relationships between agents, and *RU* denotes the set of rules in the artificial society [27]. The emergence in the artificial society is initiated by agents and their nonlinear interactions or behaviors from the micro-level to the macro-level. Here, we should give a general definition of an agent in the artificial society as *Agent*:: = < *ID*, *AT*, *CAP*, *SR*, *Ro*, *Beh*, *Env* >, where *ID* is the identification number of an agent, *AT* denotes the values of attributes that the agent owns, *CAP* indicates the capabilities that the agent has, *SR* means the *ID* set of agents who have social relationships with this agent, *Ro* means the role that this agent is playing at the current moment, *Beh* represents the behavior that the agent is performing at the current moment, and *Env* means the place that the agent is inhabiting at the current moment. All these concepts could be expressed by a serial of digits. For example, one agent *i* could be defined as *Agent_i* :: = < *i*, {01101010, 11111010}, {10101000, 11011101}, {l, k, j}, 00, 10101000, (10, 12) >. As agents in the artificial society are numerically expressed, the physical statistical method could be used to aid in measuring emergences in artificial societies.

Figure 2. Meta-model of the zombie-city model.



2.2. Metrics of Various Emergences

Without metrics for measuring emergences, emergences are usually ambiguously perceived by human observers. In an artificial society modeled based-on the zombie-city model, emergences could be divided into three kinds of emergences, including attribute emergence, behavior emergence, and structure emergence, and these emergences all belong to the weak emergence class. Various applications may concentrate diverse kinds of emergences, and emergences in the artificial society represent the meaning of "order" but not "disorder". The order is also depending on the selection of certain attributes, behaviors, or structures. As one knows, entropy can express the disorder levels of a complex dynamic system. Emergences in artificial societies can also be measured with the assist of the entropy in information theory. Artificial society as a complex system has its own attributes, behaviors and structures. Therefore, emergences in artificial societies can also be subdivided into emergence of attribute, emergence of behavior and emergence of structure. Table 1 shows the classifications, applications and metrics of emergences.

Emergence of an attribute means the emergent macro-level attribute of the system, which is reducible from micro-level individuals' attributes or local interactions between individuals, and one single individual does not hold the emergent attribute. For example, outbreak of an infectious influenza could be seen as an emergence of attribute, but the individual's attribute could not reflect the emergence. Meanwhile, the emergence of behavior means that the macro-level emergent behavior of the system derived from the micro-level individuals' behaviors, and one single individual's behavior cannot represent the whole emergent behavior of the system. For instance, emergence of interactions is an emergence of behavior, and the individual's interaction behavior could not show the emergence of interactions. Emergence of behavior could be seen as emergence of attribute, e.g., emergence of interactions could be seen as an emergence of attribute (interaction can be denoted this attribute). In addition, emergence of structure means the emergent macro-level whole structure, which is derived from the micro-level individuals' attributes, behaviors, interactions between individuals or local structures, and the emergent structure of the system cannot be hold by any individuals. For example, the social network structure of an artificial society follows scale-free, and the social relationships of any individuals could not represent the property of this emergence of structure.

Table 1. Classifications, applications and metrics of emergences.

Classifications			**Applications**	**Metrics**
Weak emergence	Emergence of attribute		Outbreak of infectious disease	Relative entropy, e.g., E(t), E_S(t), E_C(t).
	Emergence of behavior		Emergence of interactions	
	Emergence of structure	Emergence of distribution	Matthew effect in the wealth distribution, power-law distribution	
		Emergence of cluster	Flocking birds, fish school	
Strong emergence			emergence of consciousness like qualia from the neurobiological processes	Multi-scale variety [21]

2.2.1. Emergence of Attribute and Behavior

Due to the similarity between attribute emergence and behavior emergence, we could give the definition of metrics for measuring both emergence of attribute and emergence of behavior. Before presenting the computation of entropy, we should declare some notations and parameters. *AS* means an artificial society, *N* denotes the number of agents in the artificial society, *m* indicates the number of an attribute's values that agents possibly have or the number of behaviors that agents possibly perform (in different scenes, the values of *m* may be different), and $p_i(t)$ means the statistical probability of agents owning the i^{th} value of an attribute or performing the i^{th} behavior at *t* moment. Then, based on the information entropy, we could give the entropy of the artificial society in Equation (2):

$$H(t) = -\sum_{1}^{m} p_i(t) \log_2 p_i(t) , \tag{2}$$

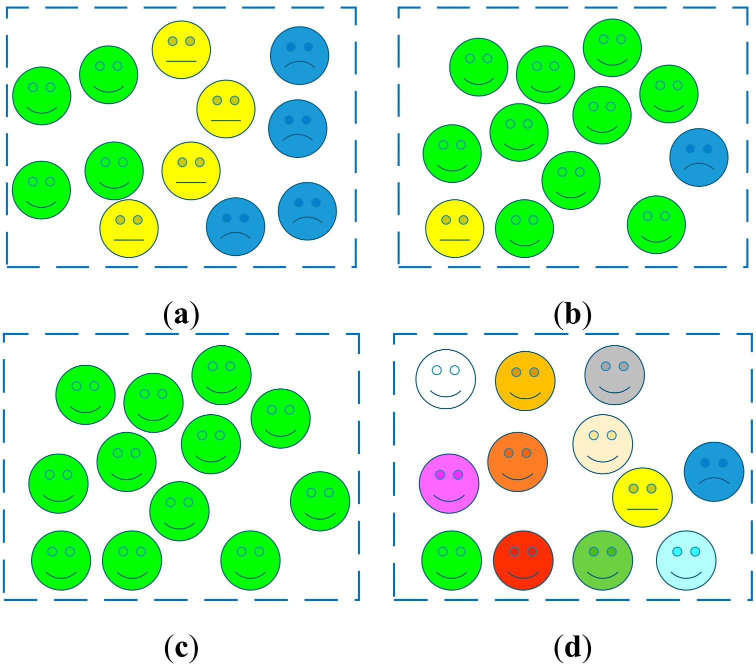
As we know, the entropy of an artificial society as defined in Equation (2) will reach the maximum value presented in Equation (3), while all values of these $p_i(t)$ are the same at *t* moment, *i.e.*, any $p_i(t) = 1/m$. If the entropy of an artificial society reaches the maximum value, we could conclude that this artificial society is in an absolute disorder:

$$H_{MAX} = -\sum \frac{1}{m} \log_2 \frac{1}{m}, \tag{3}$$

$$E(t) = \frac{H_{MAX} - H(t)}{H_{MAX}}. \tag{4}$$

Entropy increases with decreasing order, but emergence increases with increasing order. Thus, emergence increases with decreasing entropy. A metric for measuring emergences of attribute and behavior could be defined as in Equation (4). When $H(t) = H_{Max}$, the value of metric $E(t)$ is 0, which means the artificial society absolutely has no emergences of attribute or behavior. When the value of an attribute of all agents is the same or all agents perform a same behavior, $H(t)$ is equal to 0 and then $E(t)$ is 1, which means the emergence of an attribute or behavior has appeared in the artificial society. Metric $E(t)$ denotes the levels of an emergence, and $E(t)$ increases with the growing levels of the emergence. As shown in Figure 3, there are four scenes (different moments) of an artificial society, and this artificial society contains 12 agents.

Figure 3. (a) Agents play three roles at moment *a*; **(b)** Agents play three roles at moment *b*; **(c)** All agents play the same role at moment *c*; **(d)** All agents play different roles at moment *d*.



When agents play different roles, they will have different colors. The emergence in which agents play the same role is not only an emergence of behavior but also an emergence of role attribute (the role could be considered as one attribute of agents). Through human observing, we could see the absolute emergence of playing the same role appears at moment *c*, and at moment *d* the system is in an utter disorder. Hence, we could calculate their entropies. The entropy of the artificial society in Scene (a) (*i.e.*, at moment *a*) $H(a)$ is about 1.585, *i.e.*, $H(a) = -\sum 1/3\log_2 1/3 \approx 1.585$. As the same, $H(b) = -(5/6\log_2 5/6 + 1/12 \log_2 1/12 + 1/12\log_2 1/12) \approx 0.817$, $H(c) = 0$, $H(d) = -\sum 1/12\log_2 1/12 \approx 3.585$, and $H_{Max} = -\sum 1/12\log_2 1/12 \approx 3.585$. Then, we could acquire the metric values of metric $E(t)$ for measuring emergences in these four scenes: $E(a) \approx 0.558$, $E(b) \approx 0.772$, $E(c) = 1$, and $E(d) = 0$. These results have clearly and correctly reflected the levels of emergences of attribute and behavior.

2.2.2. Emergence of Structure

The meaning of structure emergence in artificial societies is also as diverse as flocks of birds, fish schools, and the Matthew effect (that means the rich get richer and the poor get poorer) in society, *etc.* The structure emergence depends on the observers' views. For instance, it is assumed that we focus on whether or not the degree distribution follows the power-law. If the degree distribution of a social network of an artificial society follows the power-law, we can say that there is an emergence of structure in the artificial society. Moreover, emergences like flocks of birds and fish schools are both structure emergences. Hence, we could divide structure emergences into two kinds: emergent distribution of an attribute or behavior, and emergent clustering or community.

First, we will discuss a structure emergence of distribution like wealth distribution (Matthew effect), degree distribution (scale-free), *etc*. Usually, such emergences are perceived by human observers in an equivocal way. Without any quantitative evaluations, humans directly estimate and judge whether or not this kind of emergence appears. Human observers should have the memory of the objective distribution, and then estimate if the real distribution follows the objective distribution. In order to quantitatively measure this kind of emergence, some notations and parameters should be defined. N denotes the number of agents, M means the maximum number of value intervals of an item (like wealth and degree), r_i indicates the probability (in the objective distribution) that the value of this item is in the i^{th} value interval, and $p_i(t)$ represents the estimated probability that the value of this item is during the i^{th} value interval at moment t. In addition, $\frac{|r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}$ means the i^{th} relative probability redundancy. As presented in Equation (5), the entropy $H_S(t)$ means the disorder between the estimated distribution and the objective distribution at moment t. In addition, the entropy H_S could get the maximum value (signed as H_{S_Max}), when the values of all $\frac{|r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}$ are the same and equal to $1 - \frac{\sum_{i=1}^{M} |r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}$. Equation (6) is the definition of H_{S_Max}. Above all, we could calculate the value of metric $E_S(t)$ of distribution emergence with $E_S(t)$ and H_{S_Max}, as shown in Equation (7). $1 - \frac{\sum_{i=1}^{M} |r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}$ means the whole probability with the same distribution. When two distributions are the same, the value of $1 - \frac{\sum_{i=1}^{M} |r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}$ is 1, thus the value of $H_S(t)$ is 0 and $E_S(t)$ is equal to 1.

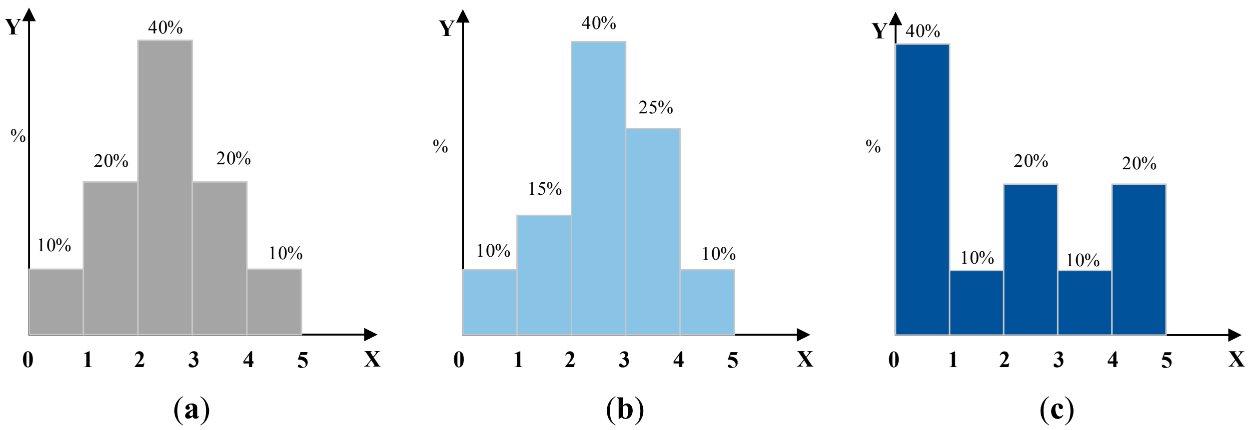
$$H_S(t) = -\left(\sum_{i=1}^{M} \frac{|r_i - p_i(t)|}{\sum_{i=1}^{M} r_i} \log_2 \frac{|r_i - p_i(t)|}{\sum_{i=1}^{M} r_i} + \left(1 - \frac{\sum_{i=1}^{M} |r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}\right) \log_2 \left(1 - \frac{\sum_{i=1}^{M} |r_i - p_i(t)|}{\sum_{i=1}^{M} r_i}\right)\right), \quad (5)$$

$$H_{S\_MAX} = -\sum \frac{1}{\mathrm{M+1}} \log_2 \frac{1}{\mathrm{M+1}}, \quad (6)$$

$$E_S(t) = \frac{H_{S\_MAX} - H_S(t)}{H_{S\_MAX}}. \quad (7)$$

As shown in Figure 4, there are three distributions of an artificial society, and its assumed that the first distribution in the left chart is also the objective distribution. Thus, the parameter M is 5, and the value intervals are [0, 1], (1, 2], (2, 3], (3, 4], and (4, 5]. For the distribution at moment b, this distribution is similar to the objective distribution at moment a, and the entropy $H_S(b)$ of the artificial society at moment b is about 0.569. In the same way, $H_S(a) = 0$, $H_S(c) \approx 2.446$, and $H_{S\_Max} = -\sum 1/6 \log_2 1/6 \approx 2.585$. Then, we could calculate all these metrics of the artificial society in these three scenes, *i.e.*, $E_S(a) = 1$, $E_S(b) \approx 0.780$, and $E_S(c) \approx 0.054$. The metric values clearly and correctly reflect the real levels of emergences in these three scenes.

Figure 4. (a) The distribution at moment *a* (Scene *a*) and also the objective distribution; **(b)** The distribution at moment *b* (Scene *b*); **(c)** The distribution at moment *c* (Scene *c*).



Another structure emergence is the emergent clustering or community, and flocking is a classic case of this kind of emergence. A cluster is a general concept, which represents a set of individuals, objects or agents in the same group (called a cluster) that are more similar to each other than those in other groups (clusters). It's assumed that the set $Clusters = \{C_1, C_2, \ldots, C_M\}$, where C_i denotes a cluster, and each agent should belong to one of these clusters. These clusters may be based-on the physical environment or social network. $|C_i|$ means the number of agents that belong to the cluster C_i. In addition, N represents the number of agents in the artificial society. Then, $|C_i|/N$ means the probability that agents belong to the cluster C_i. The entropy $H_C(t)$ could be defined in Equation (8), and Equations (9) and (10) present the maximum entropy and metric of emergence, respectively. $E_C(t)$ could measure the levels of the clustering emergence at moment t:

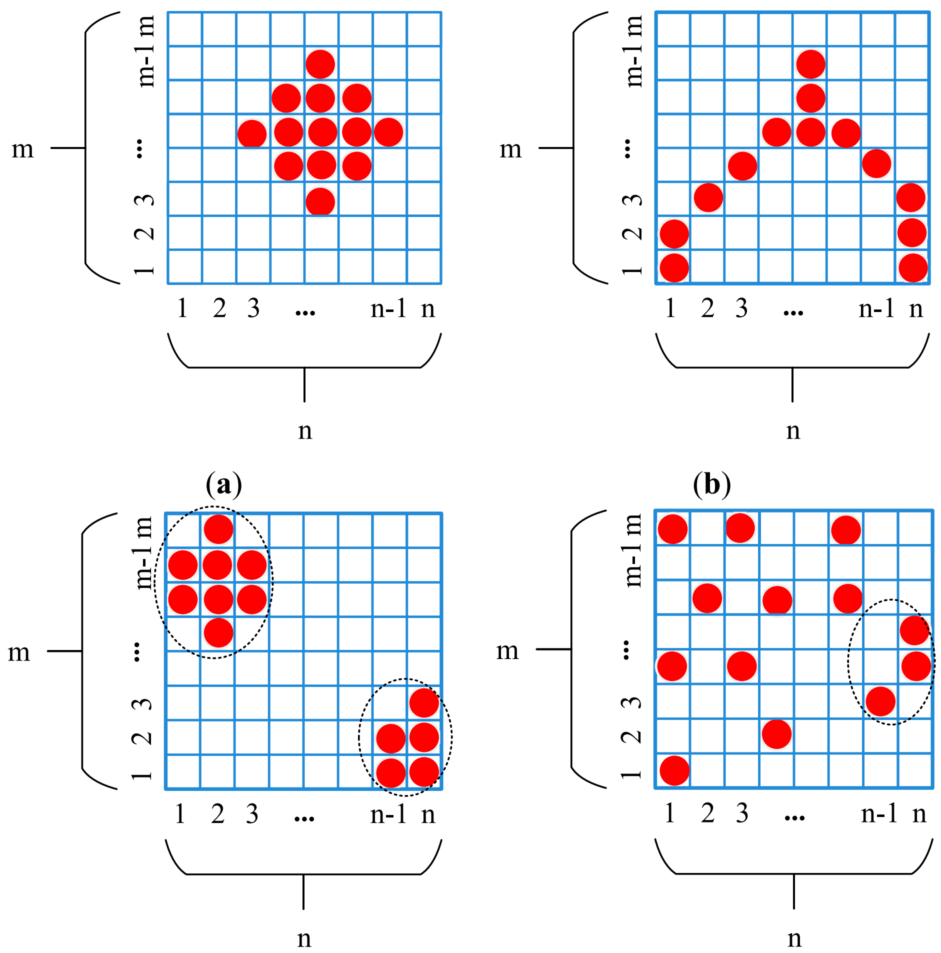
$$H_C(t) = -\sum_{i=1}^{M} \frac{|C_i|}{N} \log_2 \frac{|C_i|}{N}, \tag{8}$$

$$H_{C\_MAX} = -\sum \frac{1}{N} \log_2 \frac{1}{N}, \tag{9}$$

$$E_C(t) = \frac{H_{C\_MAX} - H_C(t)}{H_{C\_MAX}}. \tag{10}$$

Figure 5 presents four scenes of an artificial society, and agents inhabit the physical environment (which comprises grids). In the Scene *a* (at moment *a*) and Scene *b* (at moment *b*), all agents are in one cluster. Hence, both the entropy values (*i.e.*, values of $H_C(a)$ and $H_C(b)$) in these two scenes are 0, and then the metric value of clustering emergence are 1, *i.e.*, $E_C(a) = 1$ and $E_C(b) = 1$. In the scene c, there are two clusters, and one cluster has eight agents and another one contains five agents. Then, $H_C(c) = -(5/13\log_2 5/13 + 8/13\log_2 8/13) \approx 0.961$, and $H_{C\_Max} = -\sum 1/13 \log_2 1/13 \approx 3.7$. Thus, $E_C(c) \approx 0.74$. In the same way, $E_C(d) \approx 0.099$. The value of $E_C(t)$ increases with the decreasing number of clusters.

Figure 5. (**a**) Agents are in one cluster at *a* moment (Scene *a*); (**b**) Agents are in one cluster at *b* moment (Scene *b*); (**c**) Agents are in two clusters at *c* moment (Scene *c*); (**d**) Agents are in eleven clusters at *d* moment (Scene *d*).



(c) (d)

3. Experiments

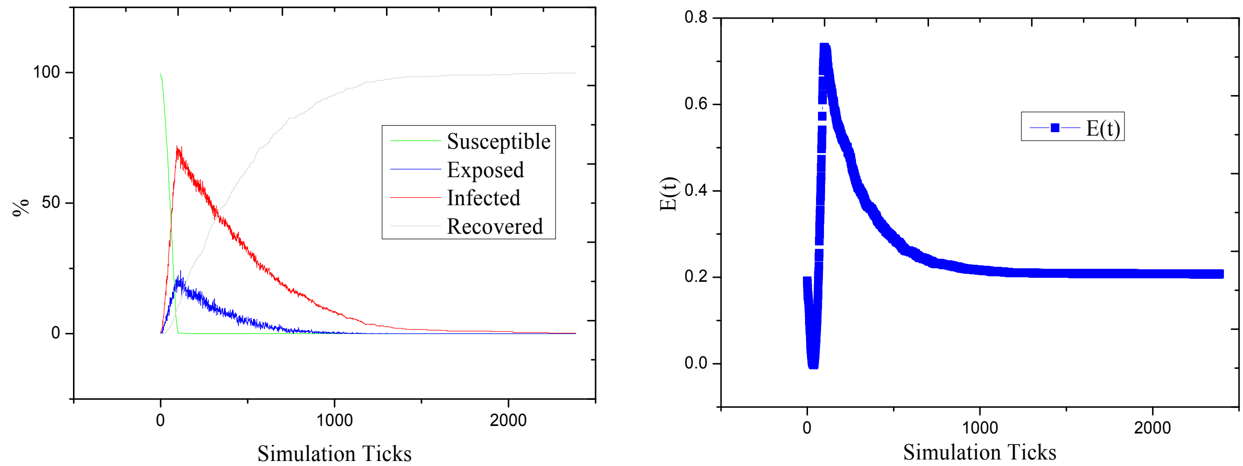
Section 2 presents the definitions of three metrics for diverse emergences, and then in this section we will study three cases with experimental simulations to further verify and confirm the effectiveness and correctness of these metrics. Through these experiments we could observe whether or not these metrics are consistent with the levels of emergences. All simulation experiments in this section are performed in the Netlogo platform, which is a widely applied MAS (Multi-agents systems) simulation platform. In addition, all these experiments are performed on a workstation with a 2.60 GHz Intel Core i5 processor and 4 GB RAM running the Win 8 operating system.

Case 1: The Spread of an Infectious Influenza:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\omega E(t)S(t) \\ \dfrac{dE(t)}{dt} = \omega E(t)S(t) - \mu E(t) \\ \dfrac{dI(t)}{dt} = \mu E(t) - \lambda R(t) \\ \dfrac{dR(t)}{dt} = \lambda I(t) \end{cases} \tag{11}$$

The first case is about the spread of an infectious influenza. As one knows, the spread of an infectious influenza may result in a terrible emergence (the outbreak of this infectious influenza), e.g., the outbreak of H1N1, H7N9, or SARS. In this case, it is assumed that the spread model of the infectious influenza is the SEIR model, and its mathematical model is depicted by Equation (11). Equation (11) defines four states: healthy and susceptible (S), exposed (E), infected (I), healthy and recovered (R), and each individual should be in only one of these states [29]. Meanwhile, we will discuss the spread of the infectious influenza in a school. It's assumed that 40% individuals are students, 30% individuals are teachers, and 30% individuals are administrators. When an individual is infected by the infectious influenza to become exposed or infected, he/she will play patient role. In this case, we will concentrate upon the emergence of role attribute.

Figure 6. (a) The spread status of the infectious influenza in the artificial school; **(b)** The metric $E(t)$ of role attribute emergence changes with time.



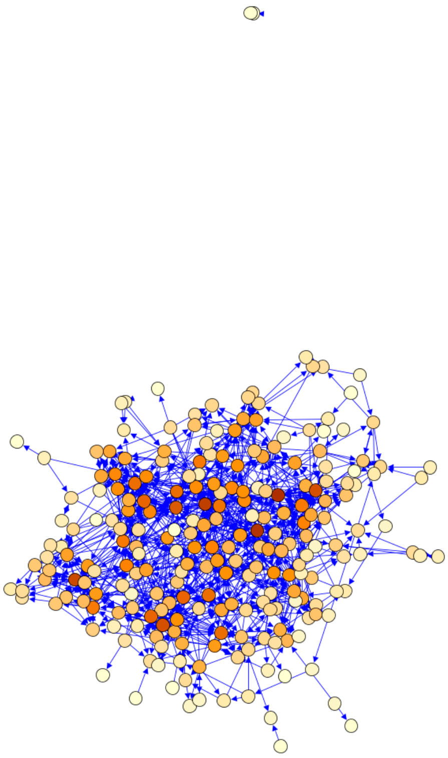
(a) **(b)**

At first, we can construct an artificial society with the zombie-city model, and perform the experiments on Netlogo. The population of agents in this artificial school is assumed to be 1000, and there are four roles: Student, Teacher, Administrator, and Patient. The Patient role represents the agents in the infected state or the exposed state. *Susceptible* agent may become *Exposed* agent with the probability of ω after interacting with other infected agents. The *Exposed* agent may become the *Infected* agent with a probability of μ. The *Infected* agent can become a *Recovered* agent with the probability of λ. Here, it's assumed the parameters $\omega = 0.1$, $\mu = 0.5$, and $\lambda = 0.25$. Figure 6a shows the spread status of this infectious influenza in this artificial school. In addition, the levels of the emergence of role attribute could be computed with Equations (2), (3), and (4). The parameter $p_i(t)$ denotes the percent of the i^{th} role at moment t, m is 4 in this case, and then H_{Max} equals 2. $E(t)$ changes with the spread status of the infectious influenza. As presented in Figure 6b, the metric $E(t)$ reaches the peak point, while almost agents are in the infected or exposed states. Therefore, this metric has clearly and correctly reflected the degree of emergence of role attribute.

Case 2: A Dynamic Microblog Network:

Nowadays, social media have become a hot research topic which also involve complex dynamic systems. A microblog is one of the kinds of social media, whose social network is a complex dynamic network. In our previous work, a dynamic microblog network has been proposed. In this case, we will evaluate and analyze the levels of a structure emergence, *i.e.*, whether or not the degree distribution of this microblog network follows the power-law. One model of this dynamic microblog network is detailed as the follows [30]:

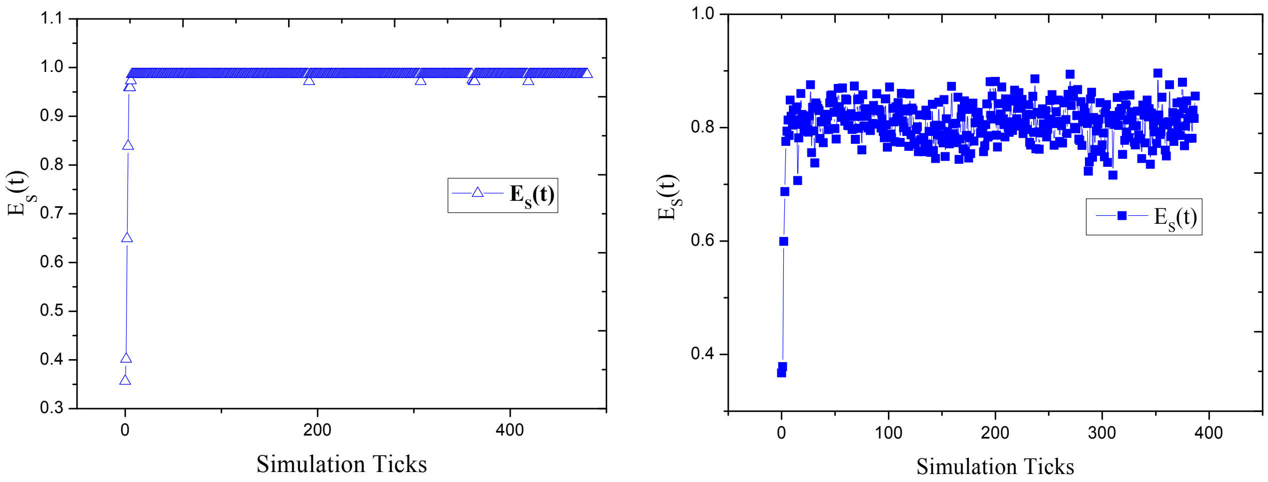
Figure 7. A snapshot of the dynamic microblog network, where N = 250, $\mu = 5$, $r_0 = 0.0015$, $r_1 = 0.1$, $r_2 = 0.0015$, $r_3 = 2$, and $\gamma = 0.1$.



Let N indicate the number of nodes in the microblog network, the total out-degree number (n_p) of the network is $N(N-1)$, and the existing total out-degree number is $n_e = \frac{1}{2}\sum z_{i\rightarrow}$ ($z_{i\rightarrow}$ represents the out-degree of node i). Meanwhile, $ownz^*_{i\rightarrow}$ denotes the limited out-degree of node i (*i.e.*, maximum out-degree of node i), and maximum out-degree of all nodes follows a power-law as the exponential distribution ($p(k) \propto e^{-\frac{k}{\mu}}$):

(1) In each time, randomly select $n_p r_0$ pairs of nodes. For each pair of nodes, randomly choose one of these two nodes signed as node i. If out-degree of node i is smaller than $ownz^*_{i\rightarrow}$, then node i will connect to the other one.
(2) In each time, randomly choose $n_p r_1$ pairs of nodes. For each pair of nodes, if one of the chose nodes (node j) connects to the other (node i), and node i does not connect to node j and out-degree of the node i is less than $ownz^*_{i\rightarrow}$, then node i will connect to node j.
(3) In each time, randomly select $n_p r_2$ pairs of nodes. For each pair of nodes, if one of the selected nodes (node i) with the smaller in-degree does not connect to the other node and out-degree of node i is less than $ownz^*_{i\rightarrow}$, then node i will connect to the other node.
(4) In each time, randomly choose $n_m r_3$ nodes ($n_m = \frac{1}{2}\sum z_{i\rightarrow}(z_{i\rightarrow} - 1)$). For each node, randomly select one of nodes from its in-neighbor nodes (called node i), and randomly choose one of nodes from its out-neighbor nodes (signed as node j). If node i does not connect to node j and out-degree of node i is smaller than $ownz^*_{i\rightarrow}$, and then node i will connect to node j.
(5) In each time, randomly choose $n_e \gamma$ nodes (γ is a constant). For each node, randomly select one of its out-links and cancel this link.

Figure 8. (a) The metric $E_S(t)$ of structure emergence changes with time, where γ = 0.001; **(b)** The metric $E_S(t)$ changes with time, where γ=0.1.



(a) **(b)**

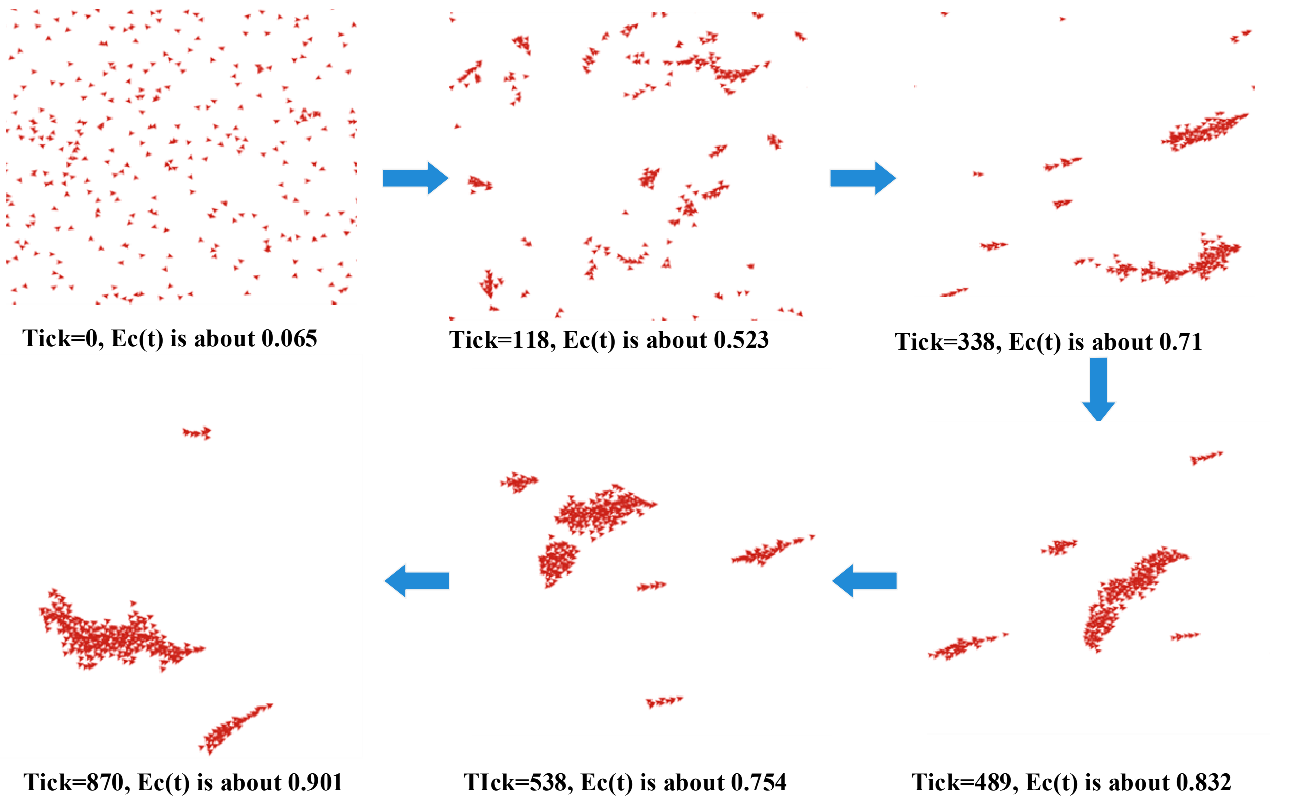
The microblog is similar to the society, and we could reconstruct an artificial microblog with the zombie-city model. Nodes in the microblog could be abstracted as agents, and links between these nodes could be considered as the social relationships between these agents. The simulation experiment in this case is also performed in Netlogo. In the experiment, these parameters are set as follows: N = 250,

μ = 5, r_0 = 0.0015, r_1 = 0.1, r_2 = 0.0015, r_3 = 2, and γ = 0.001. Figure 7 shows a snapshot of the dynamic microblog network, and we could observe that this network have the feature of clustering. In this case, we mainly focus on whether the out-degree distribution of this microblog follows the distribution of power-law ($p(k) \propto e^{-\frac{k}{\mu}}$), which is an emergence of structure. With Equations (5), (6), and (7), we could dynamically compute the metric $E_S(t)$ with time. As shown in Figure 8a, the value of E_S quickly grows to nearly 0.98, and then this value is stable with few fluctuations. Because with a smaller γ few relations or links will be deleted in each time. If we set γ from 0.001 to 0.1, in each time more relations or links will be deleted. As illustrated in Figure 8b, the metric $E_S(t)$ fluctuates around 0.8. The metric $E_S(t)$ has correctly and truly reflected the levels of this structure emergence in a quantitative way.

Case 3: Flock of Birds (Flocking Birds)

This case is about a classic emergence in the natural world, *i.e.*, a flock of birds. The phenomenon of flocking birds is based on some simple rules without external and central controls, and the process of flocking is a self-organized process. The flock of birds could also be taken as an artificial society, and we could construct an artificial society of flocking birds with the zombie-city model. The birds could be abstracted as agents. The simple rules of flocking could be described as follows [31,32]:

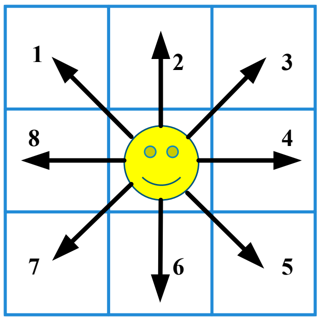
Figure 9. The processes of flocking.



(1) Cohesion: If an agent is far away from its nearest neighbor, and then this agent will turn towards its nearest neighbor.
(2) Separation: If an agent is too close to the nearest neighbor, and then this agent will turn away from the nearest neighbor.
(3) Alignment: All agents keep the average direction of all agents.

Its assumed that the population of agents in this artificial society is 300. The simulation experiment is also performed on Netlogo. At the initialization, all agents randomly live in the environment, which consists of 32 × 32 grids. Figure 9 presents the bird-flocking processes. In this case, we mainly concentrate on the levels of the emergence of flocking. The flocking is also a structure emergence, and the spatial clustering is the focus of this emergence. Hence, we could quantitatively and dynamically compute the metric of this structure emergence with Equations (8), (9), and (10).

Figure 10. All neighbors of an agent.

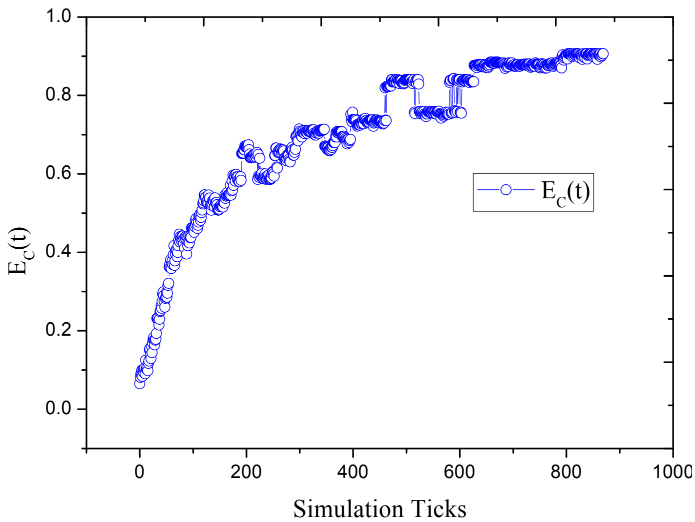


Algorithm 1: The algorithm of computing clusters of flocking birds.

```
Initialization:
j =1
N= |AG|
foreach  a_i ∈AG do
   a_i.cluster_id = 0
end
foreach a_i ∈AG do
   if a_i.cluster_id = 0 then
     a_i.cluster_id = j
   end if
   foreach a_k ∈a_i.Neighbors do
     a_k.cluster_id=j
   end
   p=0
   repeat
      foreach  a_m ∈AG do
         if a_m.cluster_id = j then
            foreach a_l ∈  a_m.Neighbors do
               if a_l.cluster_id != j then
                  a_l.cluster_id = j
               end if
            end
         end if
      end
      p++
   until  p =N-1
   j++
end
```

At first, we should design an algorithm to dynamically compute and then detect the clusters, because the clusters can change with time. As shown in Figure 10, an agent has eight neighbors. If an agent *A* has no neighbors, the agent *A* must be in a separate cluster and this cluster only has agent *A*. If agents are in a same cluster, then any one agent should be a neighbor of another agent belonging to this cluster. Hence, we could design an algorithm to compute the clusters in the artificial society, and the algorithm is presented in Algorithm 1. Each agent has an attribute *cluster_id* to denote ID of the cluster that this agent belongs to. The parameter j could record the total number of clusters, *i.e.*, $N = j$. In addition, the IDs of these clusters are from 1 to j. $|C_i|$ in Equation (8) indicates the number of agents whose *cluster_id* equals i. The entropy $H_C(t)$ could be calculated with Equation (8), and $H_{C\_Max} = -\log_2 1/300 \approx 5.044$. Real-time metric $E_C(t)$ of flocking emergence could be shown in Figure 11, and Figure 9 also presents the corresponding $E_C(t)$ according to different flocking processes at various simulation ticks. In the Figure 11, we could see that metric $E_C(t)$ is increasing with some fluctuations. For example, as shown in Figure 9, while the tick is 489, there are five clusters and the $E_C(t)$ is about 0.832. When the tick is 538, there are six clusters and then the $E_C(t)$ is about 0.754. However, the whole trend of the metric is rising. From these real data, we could conclude that this metric $E_C(t)$ has accurately reflected the levels of the clustering emergence.

Figure 11. Metric E_C for measuring flocking emergence changes with time.

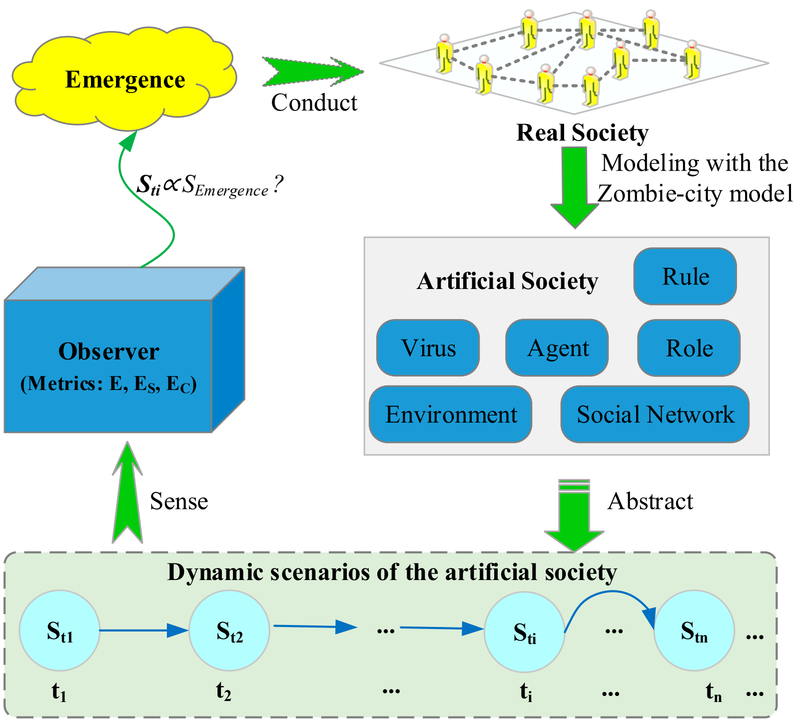


4. Discussion

For these metrics, there are several extended applications, e.g., metric $E_C(t)$ could also be used to measure the levels of the clustering or community emergence in a social network. However, there are still some limitations for these three metrics. Metric $E(t)$ for measuring emergences of attributes or behaviors could estimate the levels of one attribute or behavior emergence, but it cannot point out which attribute or behavior is emerging. In some applications, people may not know the number (m) of the possible values of attribute and $m < n$, which may be another limitation of the metric $E(t)$. In such scenario, we may assume that the value of m is equal to n. For metric $E_S(t)$, when the number of value intervals in objective distribution is very different from the estimated distribution, we should make previous processing to make these two numbers of value intervals as the same. Meanwhile, if people want to acquire an emergence that agents cluster into two or more similar clusters, and $E_C(t)$ may have a limitation to directly measure this kind of emergence, because the meaning of metric $E_C(t)$ is the level of structure emergence that agents should cluster into one cluster and community but not two or more. Another limitation of these metrics is that these metrics do not capture the time-dependent emergences, which may change from time to time [19]. Moreover, there may be some attributes with continuous values in practical applications and even some applications may have several continuous attributes [20]. These relative entropies for measuring emergences do not consider the continuous attributes or behaviors. However, if we measure emergences with these metrics by quantizing the continuous variables, then this may result in bad results. In the future work, we should consider the continuous attributes and behaviors, and even multiple continuous attribute and behaviors, which could be solved by borrowing and referencing the idea of differential entropy and joint entropy, even though a continuous entropy may result in negative values.

Emergences could be considered as special scenarios, and a scenario [33,34] is a runtime situation of a system to be observed, which describes the global properties, behaviors, and structures, *etc*. The scenario of emergence is signed as $S_{Emergence}$. At each moment t, an artificial society is in a scenario signed as S_t. If the scenario of the artificial society satisfies the scenario of the emergence at moment t, it could be depicted as $S_t \propto S_{Emergence}$. Figure 12 presents how to abstract dynamic scenarios of the artificial society and sense these scenarios by the observer with metrics for measuring emergences, and whether an emergence appears could conduct the real society.

Figure 12. Dynamic scenarios of the artificial society could be sensed by the observer with metrics to conduct the real society.



In this paper, we could define the $S_{Emergence}$ as the thresholds of metrics of emergences such as $E(t)$, $E_C(t)$, and $E_S(t)$, and S_t is seen as the real-time value of the metric of emergence. It is assumed that if

$S_t \geq S_{Emergence}$ then $S_t \propto S_{Emergence}$. As we know, an emergency may give rise to an emergence, which may result in the loss of health and property, and even endanger social stability and safety. Emergency management policies should be made to control and manage this emergency, and the metrics proposed in the paper could be used to evaluate the effectiveness of a policy in an artificial society. When a policy makes the corresponding metric with a smaller value, it means that this policy is more effective. Therefore, these metrics could be used to conduct the emergency management in the real society.

5. Conclusions

Emergence is an important concept in complex dynamic systems; meanwhile, emergences are diverse, including emergence of attribute, emergence of behavior, and emergence of structure. It is a meaningful task to propose metrics to detect emergences and measure their levels. For example, in order to control and manage the spread of H1N1 or H7N9 influenza, a series of emergency management policies should be made to respond to different serious or risky scenarios. The degree of an emergence (outbreak of H1N1 or H7N9 influenza) could affect and guide emergency management policy making experts to implement the corresponding suitable policies. Based on information entropy, this paper proposes metrics to measure various emergences in artificial societies modeled with the zombie-city model. Through three case studies, the effectiveness and correctness of these metrics have been verified. All these metrics decrease with the reduction of levels of emergences. We also have discussed the limitations of these metrics. The contributions of this article may be summarized as follows: we propose three metrics ($E(t)$, $E_S(t)$, and $E_C(t)$) for measuring various emergences, and study three complex cases to verify the correctness of these metrics; meanwhile, in the third case study, a spatial clustering algorithm has been proposed to calculate clusters in an artificial society.

Based on these works in this article and their limitations, future work can be foreseen as follows: the study more complex cases to further confirm these metrics; use of these metrics to aid in emergency management policies making, and then assist in evaluating the effectiveness of a policy; considering the continuous variables and multiple continuous variables, improvement of these metrics.

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Author Contributions

Mingsheng Tang designed and performed research and analyzed data; Mingsheng Tang and Xinjun Mao wrote this paper. Both authors have read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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