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# Analysis of the Complexity Entropy and Chaos Control of the Bullwhip Effect Considering Price of Evolutionary Game between Two Retailers

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**Abstract:** In this research, a model is established to represent a supply chain, which consists of one manufacturer and two retailers. The price-sensitive demand model is considered and the price game system is built according to the rule of bounded rationality as well as the entropy theory. With the increase of the price adjustment speed, the game system may go into chaos from the stable and periodic state. The bullwhip effect and inventory variance ratio of different stages that the system falls in are compared in real time. We also employ the delayed feedback control method to control the system and succeed in mitigating the bullwhip effect of the system. On the whole, the bullwhip effect and inventory variance ratio in the stable state are smaller than those in period-doubling and chaos. In the stable state, there is an optimal price adjustment speed to obtain both the lowest bullwhip effect and inventory variance ratio.

**Keywords:** bullwhip effect; price game; inventory variance ratio; measure entropy; two retailers; bifurcation; chaos control

## 1. Introduction

Bullwhip effect is a fluctuation difference amplification phenomenon as the information of orders goes up in the supply chain step by step. Forrester [1] was the first to find the causes of this fluctuation difference amplification phenomenon. Sterman [2] studied the well-known beer game at Massachusetts Institute of Technology (MIT), which made a great contribution to the research of the bullwhip effect. Lee et al. [3] were the first to give the definition of this demand volatility amplification phenomenon. For the sake of the mitigation of this phenomenon, a lot of work has been done.

Lots of different demand processes are investigated by research in the literature that studies the bullwhip effect. Lee et al. [3] analyzed the four sources of the bullwhip effect by constructing a first order autoregressive (AR(1)) demand model. Chen et al. [4,5] used the AR(1) demand model to quantify the bullwhip effect in a simple supply chain, and to investigate the impact of the forecasting, lead time and information. Forecast methods like the minimum mean-squared error (MMSE), moving average (MA) and exponential smoothing (ES) were employed to quantify the bullwhip effect. Furthermore, the first-order autoregressive process and order-up-to inventory policy were used by Zhang [6]. Luong [7] has explored the quantification of the bullwhip effect in a two echelon supply chain containing just one retailer and one supplier.

Agrawal et al. [8] and Chatfield et al. [9] studied the influence of fixed and stochastic lead time and information sharing on the bullwhip effect. Cannella et al. [10–14] investigated the inventory policies in multi-echelon supply chains by simulation, and the case with collaboration in supply chains

is also discussed. Furthermore, they conducted research of the bullwhip effect in closed-loop supply chains. Not only the information sharing was discussed, but the information exchange was also studied by Trapero et al. [15].

Ciaccimino et al. [16] presented a new mathematical model of a Synchronised Supply Chain to mitigate the bullwhip effect. Dominguez et al. [17,18] found that the bullwhip can be considerably reduced by collaboration or the smoothing replenishment rules in divergent supply chain networks.

Luong and Phien measured the bullwhip effect with the AR(2) and AR(p) demand model. For the two-stage supply chain, Duc et al. [19] applied the first-order auto-regressive and moving average (ARMA (1, 1)) demand process to the two echelon supply chain model, and the effects of parameters on the bullwhip effect were analyzed.

Ma et al. [20] investigated a two-level supply chain in which the demand is price sensitive, while the price follows a first-order autoregressive pricing process. Wang et al. [21] further investigated the case with price-sensitive demand and information sharing to probe into the bullwhip effect. Ma et al. [22] expanded the single supply chain to the two parallel supply chains and gave the expressions of these two kinds of bullwhip effects.

A large amount of literature on supply chain chaos sprang up. Larsen et al. [23] showed how the cascaded structure of a production–distribution chain can produce a wide variety of dynamic behaviors. Hwang [24,25] interpreted the supply chain dynamic from a perspective of chaos. Mosekilde et al. [26] devoted their research to the non-linear dynamic phenomena of the beer model. Wang and Disney et al. [27,28] discussed the stability and oscillatory dynamics of the inventory system.

Similar to what Ma et al. [20,22] have done, we also import in price-sensitive demand in the supply chain. First, we assume that the two retailers both employ the AR (1) demand process, and both use the MMSE method, which is easy to be used to forecast the lead-time demand. Then, we build the price game model according to the marginal profits and the bounded rationality adjustment mechanism. The evolution characteristics of the price game model are investigated. The game system goes through the stable state, period-doubling, and chaos with the increase of the price adjustment speed. The bullwhip effect and inventory variance ratio in three states are presented, respectively, through the results of experiment design. We also research the effect of the price adjustment speed on the bullwhip effect and inventory variance ratio.

There are also a large amount of references that have explored the price based on entropy theory in the oligopoly market. Gao et al. [29] established an electric energy procurement decision-making model for the large consumers as price fluctuations from the spot market, and they verified the feasibility and effectiveness of the model. Zou et al. [30] analyzed the crude oil price dynamics according to the wavelet entropy theory. Ma and Si [31] studied a continuous Bertrand duopoly game model considering two-stage delay to investigate the influence of delay and weight on the complex dynamic characteristics of the system.

The structure of this paper is as follows: this section shows the theoretical basis and the related literature. Section 1 explains the basic model in a two echelon supply chain containing two retailers that both face demands that conform to the same demand function, and further establishes the price game model. Then, we analyze the complexity of the price game model in this section. Chaos control is conducted to mitigate the bullwhip effect in Section 2. Finally, Section 3 provides the conclusions for this paper.

## 2. Price Game Model

### 2.1. Model Description

A two-level supply chain with a manufacturer and two retailers is considered, as Figure 1 shows. Two retailers share the same market. Let  $d_{i,t}$  note the customer demand faced by the retailer  $i$  in period  $t$ . Retailer  $i$  decides and places the order quantity  $q_i$  with the manufacturer. The demand model

considering prices of two retailers will be constructed. Two retailers both employ the order-up-to inventory policy and use the same Exponential Smoothing method to estimate the lead-time demand.

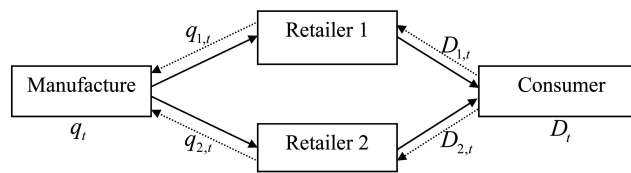


Figure 1. The supply chain model.

Two retailers both choose the price as the decision variable, and they sell the same product in the same market. Therefore, the demand functions of them are as follows:

$$d_{1,t} = a_1 - b_1 p_{1,t} + c_1 p_{2,t}, \tag{1}$$

$$d_{2,t} = a_2 - b_2 p_{2,t} + c_2 p_{1,t}. \tag{2}$$

Here,  $a_i, b_i, c_i > 0, i = 1, 2$ .  $d_{1,t}$  and  $d_{2,t}$  represent the demands of retailer 1 and retailer 2, respectively.  $p_{1,t}$  and  $p_{2,t}$  represent the prices of the two retailers. The wholesale price of the product is  $w$  ( $w > 0$ ). Then, the profit functions of two retailers are as follows:

$$\Pi_{1,t} = (p_{1,t} - w)(a_1 - b_1 p_{1,t} + c_1 p_{2,t}), \tag{3}$$

$$\Pi_{2,t} = (p_{2,t} - w)(a_2 - b_2 p_{2,t} + c_2 p_{1,t}). \tag{4}$$

The marginal profits of two retailers are as follows:

$$\frac{\partial \Pi_{1,t}}{\partial p_{1,t}} = a_1 + w b_1 - 2 b_1 p_{1,t} + c_1 p_{2,t}, \tag{5}$$

$$\frac{\partial \Pi_{2,t}}{\partial p_{2,t}} = a_2 + w b_2 - 2 b_2 p_{2,t} + c_2 p_{1,t}. \tag{6}$$

The market information has more commercial value in the real market, but retailers do not have enough market information. It is possible for either retailer to know the information of the opponent. Under bounded rationality, they make decisions of the price of the next period on the basis of their marginal profits as the following patterns:

$$\begin{cases} p_{1,t+1} = p_{1,t} + \alpha p_{1,t}(a_1 + w b_1 - 2 b_1 p_{1,t} + c_1 p_{2,t}), \\ p_{2,t+1} = p_{2,t} + \beta p_{2,t}(a_2 + w b_2 - 2 b_2 p_{2,t} + c_2 p_{1,t}). \end{cases} \tag{7}$$

Here,  $\alpha$  and  $\beta$  are the price adjustment speeds of retailer 1 and retailer 2, accordingly, and  $\alpha > 0, \beta > 0$ . If the marginal profit of one of the retailers at the current period is positive, this retailer will raise its price at the next period to make more profit. Conversely, if the marginal profit of the retailer at the current period is negative, this retailer will reduce its price in the next period for more sales, and then achieve more profit as a result.

In this work, the retailers both employ the order-up-to inventory policy. The order-up-to policy has been studied many times in the bullwhip effect literature [3,4]. The first retailer knows the exact consumer demand  $d_{1,t-1}$  at the end of period  $t - 1$ . According to past demand data, this retailer estimates the order-up-to point  $y_{1,t}$  in the period of  $t$ . The order-up-to level contains the lead-time demand and a safety stock showing the risk preference of the decision maker. The order-up-to point,  $y_{1,t}$  is given by:

$$\widehat{D}_t^L + z \widehat{\sigma}_{e,t}^L, \tag{8}$$

where  $\hat{\sigma}_{e,t}^L = \sqrt{\text{Var}(D_t^L - \hat{D}_t^L)}$ , and  $z$  is the safety factor set to meet a desired service level.

Then, an order of quantity  $q_{1,t}$  will be shipped by the retailer to the manufacturer for satisfying the order-up-to point  $y_{1,t}$  in the period of  $t$ . After lead-time  $t + L$ , the retailer gets the products from the manufacturer at the beginning of the period  $t + L$ . Therefore:

$$q_{1,t} = y_{1,t} - y_{1,t-1} + d_{1,t-1} = (\hat{D}_{1,t-1}^L - \hat{d}_{1,t}^L) + z(\hat{\sigma}_{1,t}^L - \hat{\sigma}_{1,t-1}^L) + d_{1,t-1}. \tag{9}$$

At the beginning of period  $t$ , the consumer demand  $d_{1,t-1}$  has been known. However, the demand  $d_{1,t}, d_{1,t+1}, \dots$ , and  $d_{1,t+L-1}$  have not been known and need to be estimated.  $\hat{D}_t, \hat{d}_{t+1}, \dots, \hat{D}_{t+L-1}$  is the prediction of  $d_{1,t}, d_{1,t+1}, \dots, d_{1,t+L-1}$  accordingly. The lead-time demand  $\hat{D}_t^L$  can be given by

$$\hat{D}_{1,t}^L = \hat{D}_{1,t} + \hat{D}_{1,t+1} + \dots + \hat{D}_{1,t+L-1} = \sum_{i=0}^{L-1} \hat{D}_{1,t+i}. \tag{10}$$

The exponential smoothing (ES) method is one of the most popular forecasting techniques because of its convenience, flexibility, and robustness. ES has been used in much bullwhip effect literature, such as Zhang [6]. In this paper, ES is used to the forecast lead-time demand. Therefore,

$$\hat{D}_{1,t} = \gamma d_{1,t-1} + (1 - \gamma)\hat{D}_{1,t-1}, \tag{11}$$

where  $\gamma \in (0, 1)$ .

For forecasting lead-time demand  $\hat{D}_{1,t}^L$ , the demand forecasts  $d_{1,t}, d_{1,t+1}, \dots, d_{1,t+L-1}$  that are estimated at the beginning of period  $t$  are equal. Thus,

$$\hat{D}_{1,t} = \hat{d}_{1,t+1} = \dots = \hat{d}_{1,t+L-1} = \gamma d_{1,t-1} + (1 - \gamma)\hat{d}_{1,t-1}. \tag{12}$$

Substituting Equation (12) into Equation (10), the lead-time demand  $\hat{d}_{1,t}^L$  becomes:

$$\hat{d}_{1,t}^L = \sum_{i=0}^{L-1} \hat{d}_{1,t+i} = L\hat{d}_{1,t} = L(\gamma d_{1,t-1} + (1 - \gamma)\hat{d}_{1,t-1}). \tag{13}$$

Substituting Equation (13) into Equation (9), the order quantity  $q_{1,t}$  becomes:

$$\begin{aligned} q_{1,t} &= L(\hat{d}_{1,t} - \hat{d}_{1,t-1}) + z(\hat{\sigma}_{1,t}^L - \hat{\sigma}_{1,t-1}^L) + d_{1,t-1} \\ &= L[(\gamma d_{1,t-1} + (1 - \gamma)\hat{d}_{1,t-1}) - \hat{d}_{1,t-1}] + z(\hat{\sigma}_{1,t}^L - \hat{\sigma}_{1,t-1}^L) + d_{1,t-1} \\ &= (1 + \gamma L)d_{1,t-1} - \gamma L\hat{d}_{1,t-1} + z(\hat{\sigma}_{1,t}^L - \hat{\sigma}_{1,t-1}^L). \end{aligned} \tag{14}$$

For simplicity, we further assume the safety factor  $z = 0$ . Thus,

$$q_{1,t} = (1 + \gamma L)d_{1,t-1} - \gamma L\hat{d}_{1,t-1}. \tag{15}$$

The second retailer also employs order-up-to inventory policy and the same ES forecasting method, has same lead-time. Thus, its order quantity  $q_{2,t}$  is

$$q_{2,t} = (1 + \gamma L)d_{2,t-1} - \gamma L\hat{d}_{2,t-1}. \tag{16}$$

### 2.2. The Dynamics of the Price Game Model

Let  $p_{i,t+1} = p_{i,t}, i = 1, 2$ . The system (7) has four equilibrium points:

$$E_0(0, 0), E_1(0, \frac{a_2 + cb_2}{2b_2}), E_2(\frac{a_1 + cb_1}{2b_1}, 0), E^*(\frac{2a_1b_2 + a_2c_1 + 2cb_1b_2 + cb_2c_1}{4b_1b_2 - c_1c_2}, \frac{2a_2b_1 + a_1c_2 + 2cb_1b_2 + cb_1c_2}{4b_1b_2 - c_1c_2}).$$

$E_0, E_1, E_2$  are the boundary equilibrium points, and  $E^*$  is the unique Nash equilibrium point. To ensure  $E^* > 0$ , there should be  $4b_1b_2 - c_1c_2 > 0$ . Otherwise, one of retailers will be out of the market.

**Proposition** *The boundary equilibrium points  $E_0, E_1, E_2$  are unstable equilibrium points.*

**Proof.** (See Appendix A). □

From the view of economics, we are more interested in studying the local stability properties of the Nash equilibrium point  $E^*$ . With respect to the boundary equilibrium points, it is more difficult to explicitly calculate the characteristics values of the Nash equilibrium, but it still possible to evaluate its stability by using the Jury conditions.

At the Nash equilibrium point  $E^*$ , the Jacobi matrix is as follows:

$$J(E^*) = \begin{pmatrix} 1 + \alpha(a_1 + b_1c + \frac{c_1f_1 - 4b_1f_2}{f_3}) & \frac{\alpha c_1 f_2}{f_3} \\ \frac{\beta c_2 f_1}{f_3} & 1 + \beta(a_2 + b_2c + \frac{c_2f_2 - 4b_2f_1}{f_3}) \end{pmatrix}, \tag{17}$$

where

$$\begin{aligned} f_1 &= 2a_2b_1 + a_1c_2 + 2b_1b_2c + b_1cc_2, \\ f_2 &= 2a_1b_2 + a_2c_1 + 2b_1b_2c + b_2cc_1, \\ f_3 &= 4b_1b_2 - c_1c_2. \end{aligned}$$

We obtain that the Trace (Tr) and Determinant (Det) of  $J(E^*)$  is:

$$\begin{aligned} Tr(J(E^*)) &= (1 + \alpha(a_1 + b_1c + \frac{c_1f_1 - 4b_1f_2}{f_3})) + (1 + \beta(a_2 + b_2c + \frac{c_2f_2 - 4b_2f_1}{f_3})), \\ Det(J(E^*)) &= (1 + \alpha(a_1 + b_1c + \frac{c_1f_1 - 4b_1f_2}{f_3}))(1 + \beta(a_2 + b_2c + \frac{c_2f_2 - 4b_2f_1}{f_3})) - (\frac{\alpha c_1 f_2}{f_3})(\frac{\beta c_2 f_1}{f_3}). \end{aligned}$$

According to Jury's conditions, the necessary and sufficient condition for the locally stability of Nash equilibrium point  $E^*$  is as follows:

$$\begin{aligned} (i) : 1 + Tr(J(E^*)) + Det(J(E^*)) &> 0, \\ (ii) : 1 - Tr(J(E^*)) + Det(J(E^*)) &> 0, \\ (iii) : 1 - Det(J(E^*)) &> 0. \end{aligned} \tag{18}$$

The values of all parameters are restricted by Equation (18), and the stable region with respect to the price adjustment speed can also be determined by the above conditions. In this paper, we are interested in the complexity of system (7) affected by parameters  $\alpha$  and  $\beta$ . Let  $a_1 = 0.7, a_2 = 0.8, b_1 = 1, b_2 = 1.1, w = 0.1, c_1 = 0.3,$  and  $c_2 = 0.4$ . The Nash equilibrium point is  $(p_1^*, p_2^*) = (0.47, 0.5)$ . Figure 2 gives the stability and instability region of the Nash equilibrium point  $E^*$ . It means that the system will converge to the point  $E^*$  after the long-term evolutionary when the parameters  $\alpha$  and  $\beta$  take values in the stable regions.

In this section, the complex behaviors of system (7) will be discussed by using parameter basin plots, bifurcation diagrams and strange attractor. The initial values are chosen as  $(p_1(0), p_2(0)) = (0.2, 0.25)$ .

Figure 3 is the parameter basin plots of system (7), in which different colors represent different states. The green means the stable region, we have dark blue for the stable cycles of period 2, light blue for period 4, yellow for cycles of odd period, red for chaos, and gray for divergence.

When parameters  $(\alpha, \beta)$  move from the green region and pass through dark blue and light blue regions and come into the red region, system (7) goes into chaos through flip bifurcation. Figure 4 shows the bifurcation diagrams of system (7) for  $\beta = 1.5$  and  $\alpha \in (0, 3)$ . Figure 5 gives the corresponding largest Lyapunov exponent (LLE). When  $\alpha = 2.8$  and  $\beta = 1.5$ , the LLE of the system (7) is positive, which shows that the system is in chaos. From the managerial point of view, the retailers' price adjustment speed  $\alpha$  and  $\beta$  should be in a certain range; otherwise, the system will goes into the chaos that is irregular, unpredictable, sensitive to initial values and bad for retailers. It also shows that the

adjustment range of  $\alpha$  is larger than that of  $\beta$ , which means that retailer 2 has a more sensitive price adjustment speed.

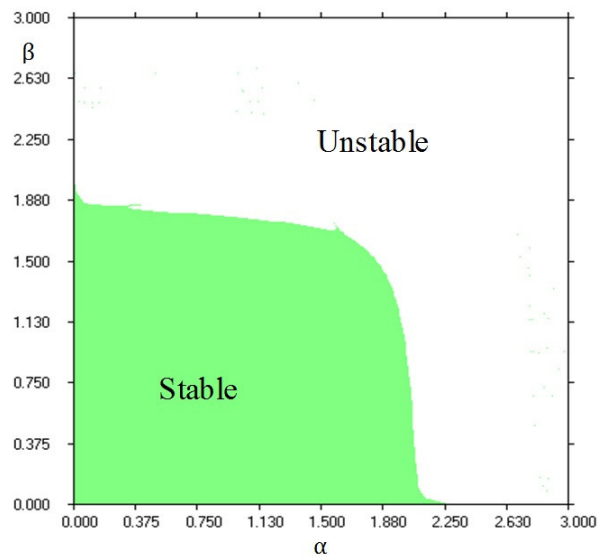


Figure 2. The stability and instability region at the Nash equilibrium point.

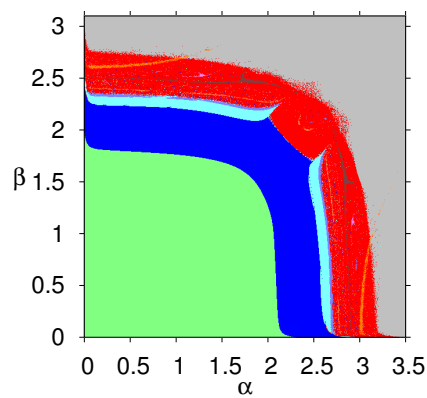


Figure 3. The parameter basin plots.

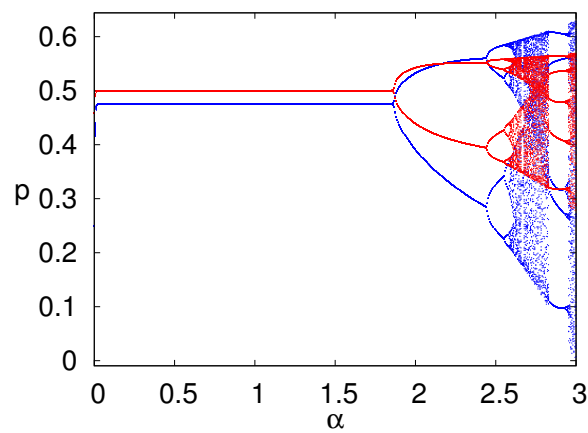
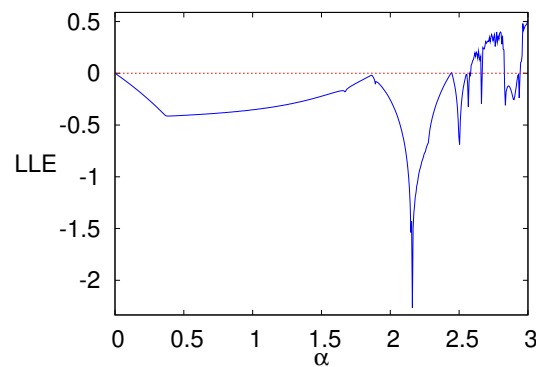


Figure 4. The bifurcation diagrams of the system.



**Figure 5.** The largest Lyapunov exponent of the system.

### 2.3. Experimental Design, Numerical Result and Discussion

We take the Order Variance Ratio (OVR) applied by Chen et al. [4,5] to measure the bullwhip effect of the supply chain, and the Inventory Variance Ratio (IVR) proposed by Disney and Towill [32] to measure the amplification of demands on the inventory. In this section, the OVR and IVR of the whole supply chain will be investigated under different states:

$$\text{OrderVarianceRatio} = \frac{\sigma_q^2 / \mu_q}{\sigma_d^2 / \mu_d},$$

$$\text{InventoryVarianceRatio} = \frac{\sigma_I^2 / \mu_I}{\sigma_d^2 / \mu_d}.$$

As shown in Figure 3 with the increase of  $\alpha$ , system (7) goes into chaos from stable state through period-doubling by flip bifurcation when  $\beta = 1.5$ . Therefore, the game system is in different states when the price adjustment parameters are set as follows:

- (a) Stable state:  $\alpha = 1.5, \beta = 1.5$ ;
- (b) Period doubling:  $\alpha = 2.3, \beta = 1.5$ ;
- (c) Chaos:  $\alpha = 2.8, \beta = 1.5$ .

Referring to the relevant literature like that of Duc [19] which employed the value of lead time from 1 to 6 and Zhang [6] that employed the value of lead time from 1 to 3, the lead time was set as three. Meanwhile, 52 weeks is the common research cycle of the bullwhip effect experiment design according to Costantino [33]. Hence, the time length of numerical experiment was set as 52 weeks. In order to calculate OVR and IVR, we set the parameters of inventory and forecasting method as follows:

- (a) The lead-time of two retailers:  $L = 3$ ;
- (b) The safety stock factor:  $z = 0$ ;
- (c) The demand smoothing index:  $\gamma = 0.3$ ;
- (d) The time length of numerical experiments:  $T = 52$ .

The following two experiments are designed to investigate bullwhip effect and inventory fluctuation of the supply chain from the perspectives of time evolution and price adjustment.

Experiment 1: Real-time OVR ( $T \in [0, 52]$ ) and IVR in different states will be calculated and compared.

Experiment 2: How does the price adjustment speed of retailer 1 affect the OVR and IVR?

In Figure 6, the curves with different colors represent the OVR in different states. When the system (7) is stable (the green curve), the bullwhip effect sharply increases to the maximum value, and then slowly declines. When the system is in the period-doubling (the blue curve), the bullwhip effect also sharply increases to the maximum value, and then vibrates down with smaller and smaller

amplitude. When the system is in chaos (the red curve), the bullwhip effect also sharply increases to the maximum value, and then vibrates acutely. The figure shows that the bullwhip effect in the stable state is the smallest, and the bullwhip effect in the chaos is the largest. The bullwhip effects in different states all reach the maximum value in the beginning. This means that when retailers begin to sell a kind of product, the manager can not forecast the demand in the market because of lacking experience, and the bullwhip effect is the most serious. As time goes on, the manager can forecast the demand more accurately than before with the support of more experience, and the bullwhip effect gradually declines.

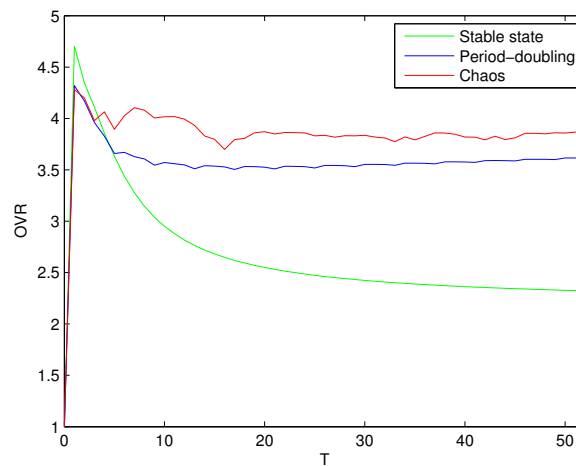


Figure 6. The timing diagram of the Order Variance Ratio .

In Figure 7, the curves with different colors represent the IVR in different states. When system (7) is stable (the green curve), the IVR decreases to the minimum value, and then smoothly increases. When system (7) is in the period-doubling (the blue curve), the IVR also decreases to the minimum value, and smoothly increases with small amplitude. When system (7) is in chaos (the red curve), the IVR also decreases to the minimum value, and then sharply increases with large amplitude. The figure shows that the IVR in the stable state is smaller than that in the period-doubling and chaos. The IVR in the chaos is not bigger than that in the period-doubling, but the amplitude in the chaos is larger than it in the period-doubling. The IVR in different states all reach the minimum value in the beginning. This means that when retailers begin to sell a kind of product, the IVR is the smallest because of lacking inventory. As time goes on, the inventory backlogs gradually because of the bullwhip effect, and the IVR smoothly increases.

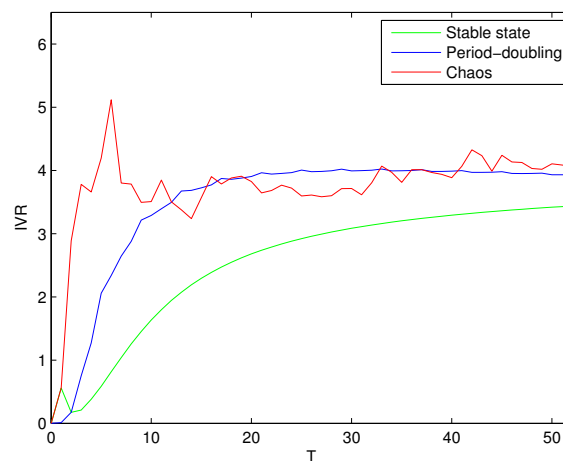


Figure 7. The timing diagram of the Inventory Variance Ratio.



Figure 8 represents the influence of price adjustment speed of retailer 1 on the bullwhip effect. When system (7) is stable, the bullwhip effect drops to the minimum value rapidly and then smoothly increases with the increase of the price adjustment speed. The bullwhip effect continues to increase when system (7) goes into the period-doubling. The bullwhip effect begins to vibrate violently when system (7) is in chaos. Figure 8 shows that the bullwhip effect in the chaos is bigger than that in the stable state and period-doubling. For retailer 1, there is an optimal price adjustment speed to get the smallest bullwhip effect.

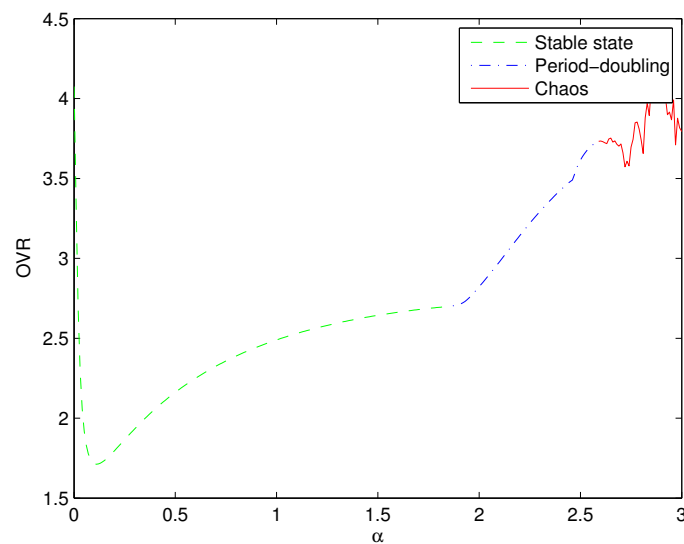


Figure 8. The effect of the price adjustment speed of retailer 1 on the OVR.

Figure 9 shows the impact of the price adjustment speed of retailer 1 on the IVR. When system (7) is stable, IVR drops to the minimum value rapidly and then smoothly increases with the increase of the price adjustment speed. The IVR fluctuates within a narrow range when system (7) goes into period-doubling. The IVR begins to vibrate violently when system (7) is in chaos. Different from Figure 8, the IVR in the chaos is not always bigger than that in the stable state and period-doubling. The same as Figure 8, there is an optimal price adjustment speed for the smallest IVR for retailer 1.

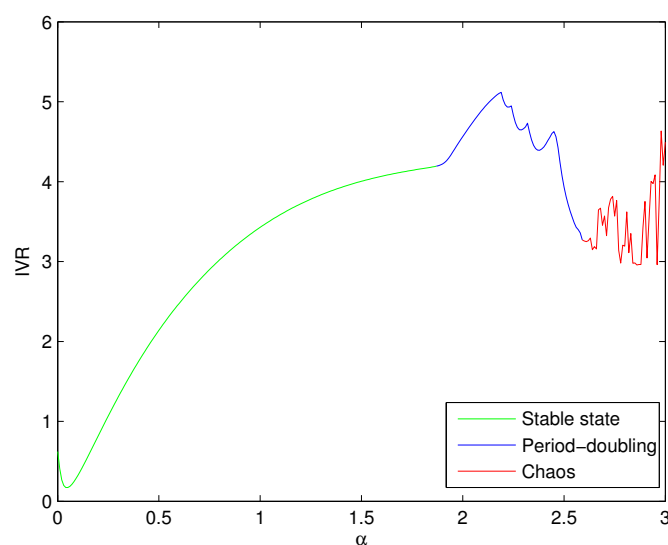


Figure 9. The effect of the price adjustment speed of retailer 1 on the IVR.

### 3. The Mitigation of the Bullwhip Effect by Chaos Control

The above analysis has shown that once the system is in the period-doubling or chaos, the bullwhip effect of the supply chain is much more than that when the supply chain is in the stable state. Thus, the supply chain system should be kept in a stable state as long as possible, and try to avoid the period-doubling or chaos. Therefore, it is necessary to establish effective control measures that can help retailers to extend the magnitude of stable state and weaken the bullwhip effect. In this paper, we employ the delayed feedback control method to control the system because this method has been widely applied in the chaos of supply chain system [34,35]. Without loss of generality, we make retailer 1 take the control measures when making price decisions. By introducing a control parameter  $K$ , the expression of the first retailer's price decision can be rewritten as:

$$p_{1,t+1} = p_{1,t} + \alpha p_{1,t}(a_1 + wb_1 - 2b_1 p_{1,t} + c_1 p_{2,t}) + K(p_{1,t+1-T} - p_{1,t+1}).$$

Here,  $T$  is the length of the lag time, and  $K$  is the control parameter. Let  $T = 1$ , and then a new price game system under the control of retailer 1 can be written as:

$$\begin{aligned} p_{1,t+1} &= p_{1,t} + \alpha p_{1,t}(a_1 + wb_1 - 2b_1 p_{1,t} + c_1 p_{2,t}) + K(p_{1,t} - p_{1,t+1}), \\ p_{1,t+1} &= p_{1,t} + \alpha p_{1,t}(a_2 + wb_2 - 2b_2 p_{2,t} + c_2 p_{1,t}). \end{aligned}$$

Based on the scenario in Figure 4, we observe the behavior of the system under control. In order to investigate the effect of the delayed feedback control method, we make the control parameter  $K = 0.6$  and study the bifurcation diagram, parameter basin plots and bullwhip effect of the controlled system for the first step.

From Figures 10 and 11, we can find that the stable state of retailer 1 is extended after controlling. Figure 12 shows the comparison of the bullwhip effect between the original system and the controlled system. We can find that the bullwhip effect of the controlled system is mitigated for the same price adjustment speed of retailer 1.

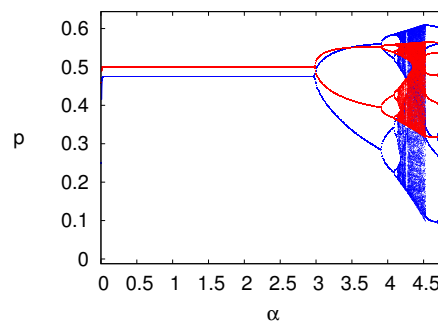


Figure 10. Bifurcation diagram of the system with  $\beta = 1.5$ ,  $K = 0.6$  and  $\alpha$  varying from 0 to 4.8.

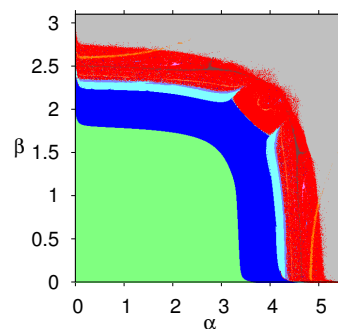
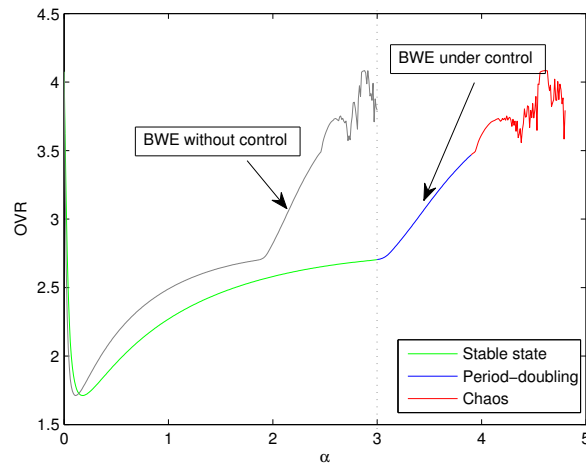


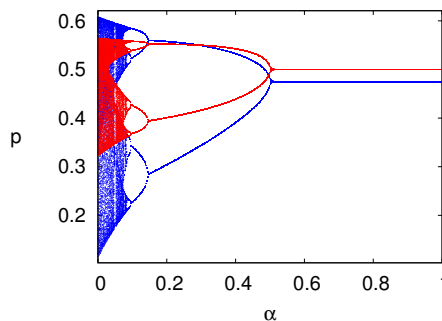
Figure 11. The parameter basin plots of the system under control.



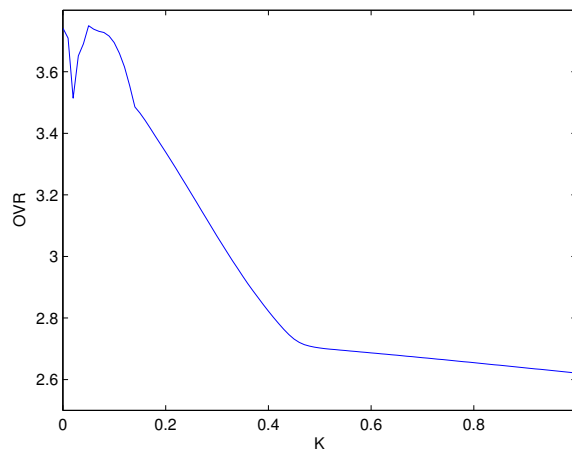
**Figure 12.** Bullwhip effect of the system without control and under control with  $\alpha$  varying from 0 to 5.

Secondly, we set a scenario which is in the chaotic state, and then investigate the impact of the control parameter  $K$  on the system and bullwhip effect. Let  $\alpha = 2.8$  and  $\beta = 1.5$ , and we then plot the bifurcation diagram and the bullwhip effect as  $K$  varying from 0 to 1.

Based on Figure 13, we can find that a chaotic system is gradually changed from chaos to stable state as the feedback control parameter increases from 0 to 1. From Figure 14, an obvious reduction of the bullwhip effect can be found due to the entrance of the control parameter. Thus, we can reach the conclusion that the delayed feedback control has achieved a good effect, and it can effectively alleviate the bullwhip effect of the supply chain.



**Figure 13.** Bifurcation diagram of the system with  $\alpha = 2.8$  and  $\beta = 1.5$ ,  $K$  varying from 0 to 1.



**Figure 14.** Bullwhip effect of the system with  $\alpha = 2.8$  and  $\beta = 1.5$ ,  $K$  varying from 0 to 1.

#### 4. Conclusions

In this research, a supply chain that consists of one manufacturer and two retailers is studied. We build the price game model according to the marginal profits and the bounded rationality adjustment mechanism. The evolution characteristics of the price game model are investigated. The game system goes through the stable state, period-doubling, and chaos as the price adjustment speed increases. The bullwhip effect and inventory variance ratio are presented in real time, respectively, in three states by carrying out experimental designs. We also research the effect of the price adjustment speed on the bullwhip effect and inventory variance ratio. In the end, we employ the delayed feedback control method to control the system and succeed in mitigating the bullwhip effect of the system. From the complexity analysis and the experimental designs, we can find some important conclusions, such as: (1) the system stability, the bullwhip effect and inventory variance ratio are all affected by the price adjustment speed of retailer; (2) the bullwhip effect and inventory variance ratio in the stable state are smaller than those in period-doubling and chaos; and (3) in the stable state, there is an optimal price adjustment speed for the lowest bullwhip effect and inventory variance ratio, respectively.

From the perspective of management, we also propose some useful insights for firms in supply chains: (1) retailers of supply chains can increase their price adjustment speeds in order to gain more profit, but the values of the adjusting parameters cannot exceed the stable region of the price game system; (2) manufacturers that play leader roles in supply chains can take the delayed feedback control method on aggressive retailers to keep the price system stable and totally mitigate the aggressive retailers.

However, there are limitations in our current research. For example, this paper fails to derive an analytic expression of the bullwhip effect on the basis of the price competition in the supply chain. Some conclusions are drawn from the numerical simulations and experiments. For the future, analytic research about the bullwhip effect in a price competitive supply chain system is very promising in this field.

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**Author Contributions:** Junhai Ma and Xiaogang Ma built the supply chain model. Junhai Ma provided economic interpretation of the conclusions; Xiaogang Ma performed mathematical derivation and Wandong Lou operated numerical simulation analysis. They wrote this research article together. All authors have read and approved the final manuscript.

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#### Appendix A. The Proof of Proposition

The local stability of the equilibrium point is determined by the characteristic values of the Jacobi matrix calculated at this equilibrium point. The Jacobi matrix of system (7) at point  $(p_1, p_2)$  is as follows:

$$J(p_1, p_2) = \begin{pmatrix} 1 + \alpha(a_1c_1p_2 + b_1w) - 4\alpha b_1p_1 & \alpha c_1p_1 \\ \beta c_2p_2 & 1 + \beta(a_2c_2p_1 + b_2w) - 4\beta b_2p_2 \end{pmatrix}.$$

The Jacobi matrix of system (7) at point  $E_0$  is as follows:

$$J(E_0) = \begin{pmatrix} 1 + \alpha(a_1 + b_1w) & 0 \\ 0 & 1 + \beta(a_2 + b_2w) \end{pmatrix}.$$

The characteristics' values of  $J(E_0)$  are as follows:

$$\lambda_1 = 1 + \alpha(a_1 + b_1w), \lambda_2 = 1 + \beta(a_2 + b_2w).$$

It is easily concluded that  $\lambda_1 > 1, \lambda_2 > 1$ . Thus,  $E_0$  is not stable.

The Jacobi matrix of system (7) at point  $E_1$  is as follows:

$$J(E_1) = \begin{pmatrix} 1 + \alpha(a_1 + c_1(\frac{a_2 + wb_2}{2b_2}) + b_1w) & 0 \\ \frac{\beta c_2(a_2 + wb_2)}{2b_2} & 1 - \beta(2a_2 + wb_2) \end{pmatrix}.$$

The characteristics values of  $J(E_1)$  are as follows:

$$\lambda_1 = 1 + \alpha(a_1 + c_1(\frac{a_2 + wb_2}{2b_2}) + b_1w), \lambda_2 = 1 - \beta(2a_2 + wb_2).$$

It is easily concluded that  $\lambda_1 > 1, \lambda_2 < 1$ . Thus,  $E_1$  is not stable.

The Jacobian matrix of system (7) at point  $E_2$  is as follows:

$$J(E_2) = \begin{pmatrix} 1 - \alpha(2a_1 + b_1w) & \frac{\alpha c_1(a_1 + wb_1)}{2b_1} \\ 0 & 1 + \beta(a_2 + c_2(\frac{a_1 + wb_1}{2b_1}) + b_2w) \end{pmatrix}.$$

The characteristics' values of  $J(E_2)$  are as follows:

$$\lambda_1 = 1 - \alpha(2a_1 + b_1w), \lambda_2 = 1 + \beta(a_2 + c_2(\frac{a_1 + wb_1}{2b_1}) + b_2w).$$

It is easily concluded that  $\lambda_1 < 1, \lambda_2 > 1$ . Thus,  $E_2$  is not stable.

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