"Wave-Packet Reduction" and the Quantum Character of the Actualization of Potentia

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Abstract: Werner Heisenberg introduced the notion of quantum *potentia* in order to accommodate the indeterminism associated with quantum measurement. Potentia captures the capacity of the system to be found to possess a property upon a corresponding sharp measurement in which it is actualized. The specific potentiae of the individual system are represented formally by the complex amplitudes in the measurement bases of the eigenstate in which it is prepared. All predictions for future values of system properties can be made by an experimenter using the probabilities which are the squared moduli of these amplitudes that are the diagonal elements of the density matrix description of the pure ensemble to which the system, so prepared, belongs. Heisenberg considered the change of the ensemble attribution following quantum measurement to be analogous to the classical change in Gibbs' thermodynamics when measurement of the canonical ensemble enables a microcanonical ensemble description. This analogy, presented by Heisenberg as operating at the epistemic level, is analyzed here. It has led some to claim not only that the change of the state in measurement is *classical mechanical*, bringing its quantum character into question, but also that Heisenberg held this to be the case. Here, these claims are shown to be incorrect, because *the analogy concerns the change of ensemble attribution by the experimenter upon learning the result of the measurement*, not the actualization of the potentia responsible for the change of the individual system state which—in Heisenberg's interpretation of quantum mechanics—is objective in nature and independent of the experimenter's knowledge.

1. Introduction

A quantum physical system may be seen upon measurement manifestly to possess a specific property that it was not certain to possess beforehand, even though its state had been fully specified by its preparation. This novelty, first clearly appearing in quantum physics, was explained by Werner Heisenberg via the notion of quantum *potentia* [1,2], which he introduced and related to an earlier notion of Aristotle—*dynamis* [3], that aspect of an entity's being which reflects its capacity to be found later in a different state. Heisenberg innovated this notion by adding a quantitative dimension to it by representing potentiae as the components of the state $|\psi\rangle$, a complex vector in Hilbert space with amplitudes $\{c_i\}$, the squared magnitudes of which provide the probabilities $p_i = |c_i|^2$ for the system—upon measurement—to possess the respective values of its physical properties, which are then actual [4]. In this interpretation of the quantum state, a dynamical property of a quantum system S becomes actual upon precise measurement when the measuring apparatus, A, is in contact with the greater physical environment, which is macroscopic and unavoidable [5].

The quantum ensemble to which the system is then associated by the experimenter is a mixture of quantum states weighted by the p_i until the outcome of the measurement, and hence, the actual

value is known to him. At the completion of an exact measurement of its dynamical properties, the system is once again isolated from the measuring apparatus and its state has generally changed discontinuously from $|\psi\rangle$ to a different Hilbert-state vector $|\phi_k\rangle$ corresponding to its actual value; upon awareness of the actual value as that of the kth possible outcome, an experimenter assigns the system for predictive purposes to the pure ensemble described by the density matrix $\rho_k = |\phi_k\rangle\langle\phi_k|$ [6]. Heisenberg viewed this change of ensemble assignment as "exactly analogous" to that occurring in classical statistical thermodynamics upon the measurement of a Gibbs ensemble with an outcome associated with a given microstate. This has been interpreted as an indication that the change of state $|\psi\rangle$ upon measurement is *classical mechanical*; for example, Karl Popper claimed on the basis of this exact analogy that the " 'transition from the possible to the actual' ... Heisenberg himself ... admits ... is not a quantum effect" [7]. Here, it is shown that such an understanding of actualization is incorrect. In particular, it is shown that the actualization of potentia is distinctly quantum and occurs in the quantum state space, which is a complex Hilbert space, even though the two theories involve analogous changes—which correspond to the knowledge provided by measurements in each case—in the *ensembles* associated with the systems used for predictive purposes.

2. Heisenberg's Analogy

Heisenberg consistently held that "all the concepts used in classical theory for the description of a mechanical system can also be defined exactly for atomic processes in analogy to the classical concept" [8]. The key to understanding the analogy of the change of ensemble attribution in quantum mechanics to a similar change in classical mechanics offered by Heisenberg is noting that the latter change—the transition from the canonical ensemble description to the microcanonical description—is due to *the gaining of knowledge of measurement outcomes*, rather than any physical mechanism acting on states in the Hilbert space; the analogy was itself inspired in part by Niels Bohr's use of Gibbs' thermodynamics as an illustration of the epistemological aspects of statistical physics in general: "Bohr has always pointed to [it] as an especially clear application of the theory of knowledge in physics" ([1], p. 25). Heisenberg saw this analogy as due to the similarity of *information that can be obtained* in a class of measurements in Gibbs' thermodynamics; like Bohr, Heisenberg saw a strong similarity between that aspect of quantum measurements and this set of measurements in classical mechanics because in both cases the measuring apparatus must come into contact with both the measured system *and the greater environment* in measurements, limiting the experimenter's knowledge of the state of the physical system, and so preventing a standard causal description (cf. [9]).

The analogy in question was offered in the article "The development of the interpretation of the quantum theory," in a volume in honor of Bohr and edited by Wolfgang Pauli [1], wherein Heisenberg defended the understanding of quantum mechanics in the "Copenhagen spirit" from a broad range of criticisms with a particular emphasis on the objectivity of quantum physics, among other aspects [10–12]; there, after reviewing what he considered to be the resolution of various paradoxes, Heisenberg provides "a further analysis, showing to what extent this basis of all physics[—a world 'which would be present, essentially unchanged if he (man) were not there'—]has been maintained in the Copenhagen interpretation of quantum theory" ([1], p. 25). Heisenberg begins this analysis by discussing the statistical aspects of measurements of temperature and energy in Gibbs' thermodynamics and setting up the analogy, which serves to demonstrate the similarity of the process of optimization of objectivity in quantum mechanics to that in classical theory—namely, the removal of the experimenter's (subjective) ignorance of the physical state.

The analysis of quantum measurement that then follows focuses on the relationship between the knowledge of the properties of quantum systems available and the actualization of a quantum potentiality that enables it by the production of actual values. It is pointed out that the fundamental limitations of the predetermination of the set of properties of the individual quantum system measured are reflected in the need, in general, to predict its future properties probabilistically. In particular, it is noted that the change *by the experimenter* of the ensemble assignment to a pure one upon learning

the value found in a sharp quantum measurement is analogous to that of an experimenter in Gibbs'
thermodynamics upon his identifying a microcanonical ensemble.

The specific measurements considered by Heisenberg in his discussion involve the temperature
T, energy E, and the sampling of emissions from a metal initially described by a classical canonical
ensemble, where complete equilibrium is present and the temperature is fixed; then, T is an objective
property in that any proper thermometer would provide the same value for it which does not
depend on an observer to arise. If in contact with its greater environment, the system of metal plus
thermometer would have a *fluctuating* energy, with fluctuation δE in a nearly Gaussian distribution
about a mean value, E_m, according to the canonical distribution, with a density of states $\rho(E)$, with
total differential probability

$$W(E)\mathrm{d}E = P(E)\rho(E)\mathrm{d}E = K\exp(-E/kT)\rho(E)\mathrm{d}E \,. \tag{1}$$

The specific example class of such measurements considered is that by a thermometer wherein the
measuring system is a proportional counter of (classically modeled) electrons entering with detectable
velocities and detected by an irreversible means (e.g., the blackening of a photographic emulsion).
Heisenberg points out in his discussion that, in classical mechanics, the probability of the thermometer
to respond and provide a value could be calculated in principle in that case, but an instant t when this
occurs could not be exactly found: if complete equilibrium is present in the phase space available to
the system, then there is no variation in time, which in that sense is indeterminate; in order to perform
an analysis of the measurement process, one must compare equilibria before and after it, as with any
physical process. Alternatively, the joint system might be considered microcanonically as having a total
energy E_{TOT}, (cf. [13]). Heisenberg then points out that if the metal plus thermometer system were
fully isolated from the external world and every physical detail of it was known at the outset, then the
instant t of recording could in principle be calculated beforehand. However, the temperature thus
calculated would not correspond to what is read out, because the reading out of T requires *contact
with the world beyond the thermometer*, making it different, in general. (Note, however, that this does not
immediately constitute grounds for a quantitative complementarity relation here (cf. [14]).).

Heisenberg argues that in order to bring the temperature measurement process to an end, the value
of a thermometer must ultimately be read by an experimenter who is no longer in contact with the metal
and who—as a result of the reading—identifies a new microcanonical ensemble assignment for the
system alone, namely, that corresponding to the outcome of the energy measurement. This knowledge
changes the statistical description the *experimenter* can provide for its future values. Heisenberg then
points out that the use of an ensemble description *involves a subjective element* in this classical statistical
context that is exactly analogous to that taking place in quantum mechanics, due both to *using statistical
descriptions in making predictions of outcomes and to changing the ensemble under consideration upon learning
new information*. In the present cases, prediction via a large ensemble allowing a range of possible
values is replaced by prediction via a smaller one (cf. [13], Section 2.2).

Heisenberg relates the change of ensemble assignments in quantum measurement by analogy
to the above measurements in Gibbs' thermodynamical situation as follows. In quantum mechanics,
assuming an initially pure ensemble assignment,

> "If the observer later registers a certain behavior of the measuring apparatus as actual, then
> the mathematical representation changes discontinuously because a certain one among the
> various possibilities has proved to be the real one. The discontinuous "reduction of the wave
> packets," which cannot be derived from Schrödinger's equation, is a consequence of the
> transition from the possible to the actual. It is exactly analogous in Gibbs' thermodynamics
> to a measurement restricting a system from a large ensemble to a smaller one." ([1], p. 27)

Citing this comment, Karl Popper later claimed that the " 'transition from the possible to the actual'
…Heisenberg himself …admits …is not a quantum effect" [7]. We may now ask: Is this so? Is there

a basis in Heisenberg's treatment of quantum mechanics for such a claim about the *actualization of potentia*?

3. Is the Actualization of Potentia Classical?

On the basis of the above analysis of measurement with its analogy to Gibbs' thermodynamics, Popper concluded that the actualization of quantum potentia (a.k.a. potentiality) is *classical mechanical*. He was led to this conclusion by assuming that the above statement of Heisenberg which mentions the "reduction of the wave packet"—that is, for the latter, the assignment of the new ensemble description by the experimenter—was *also* referring to the actualization of potentiality; i.e., by conflating the two: Popper states that the analogy is concerned with "the famous, or rather notorious, 'transition from the possible to the actual'—that is to say, with the 'reduction of the wave packet'" [7]. However, this assumption is erroneous: the actualization of potentiality is distinct from his wave-packet reduction.

Heisenberg considered actualization to be the basis for *the experimenter's coming to change the ensemble assignment*, because actualization provides the experimenter with a measurement outcome *to know*. Although in standard terminology "wave packet" is short-hand for a spatial wavefuntion that is non-negligible only within a limited space-time region (cf. [15]), Heisenberg uses that phrase synonymously with a *distribution of quantum probability*, a "probability wave" ([1], pp. 13–14, 24) here, where it involves the "characterization of a system by an ensemble" ([1], p. 26). The inference that the actualization of potentiality is classical is invalid because the transition from the possible to the actual is not a consequence of, or identical to the subjective gaining of information: The obtaining of knowledge of the quantum measurement outcome by the observer is a "*consequence* of the transition from the possible to the actual", and not the other way around. Heisenberg considered actualization independent of the observer's knowledge of the result, so that for actualization, "it does not matter whether the observer is an apparatus or a human being; but *the registration*, i.e., *the transition from the possible to the actual*, is absolutely necessary here, and cannot be omitted from the interpretation of the quantum theory" ([10], p. 137) (emphasis mine); the actualization of the outcome-indicating property in the measuring apparatus and of the property of the individual quantum system measured *both* take place upon registration, which then allows the experimenter to learn the actual outcome [3]—immediately so only if his attention is already directed toward the apparatus. As Heisenberg puts it, upon registration, the observer learns that "a certain one among the various possibilities *has* proved to be the real one" (emphasis mine) [1]. Clearly, in an objective physics the experimenter can only associate the system to a new ensemble upon coming to know the measurement outcome and according to its actual value—that is, according to the actual state.

4. Quantum State Change and Measurement

Recall that for the individual system and discrete quantities, the change of state from that before measurement to that after measurement, where the kth potentia $\langle\psi|\phi_k\rangle$ is actualized, in general, one from an initial $|\psi\rangle$ to a final $|\phi_k\rangle$ is

$$|\psi\rangle \rightarrow |\phi_k\rangle\,,\tag{2}$$

regardless of the initial state $|\psi\rangle$ of the system. In the formal treatment of measurement, the measurement process in quantum theory is now sometimes also described in terms of the joint state evolution of the measuring system and measured system which, in general, is

$$|\psi\rangle|\chi_0\rangle \rightarrow |\eta_k\rangle|\phi_k\rangle\tag{3}$$

(n.b.: some standard requirements of such a treatment are stated below). However, in Heisenberg's approach this is generally inappropriate, because the measuring apparatus must be considered under the unavoidable and unaccountable influence of its greater environment from the beginning of any proper measurement; in point of fact, the joint system cannot be accurately described by the state $|\psi\rangle|\chi_0\rangle$ with actualization, but only *before* their interaction and while it is genuinely closed—that

is, only if it is exceptionally well isolated from the greater external world, which is not the case for measurements in the approach.

Indeed, this joint-system treatment leads to the quantum measurement problem and the appearance of the paradox of Wigner's friend [16], as discussed below in the following sections. It would be like a situation in Gibbs' thermodynamics where a microcanonical description was used for the ensemble for the joint system consisting of the large and the small system discussed in the previous section, imagining the greater environment to be irrelevant. A joint state-vector description of the system and apparatus would be generally allowed in Heisenberg's approach only provided that the system identified as the measuring apparatus would *itself* be the system to be measured by the experimenter. Consider the succession of interacting physical systems linking this newly delimited system—call it S′—to the remainder of the physical world—which could include everything involved, except the experimenter's seat of awareness (i.e., his brain B)—within a "chain" of interaction

$$\text{S–A–X–Y–}\cdots \ ;$$

the originally considered system S interacts with other systems X, Y, . . . , in the original apparatus A's environment forming this new to-be-measured "system" $S' \equiv S + A + X + Y + \cdots$.

The freedom, under the proper conditions regarding the environment (see below), to formally consider any subset of systems from this chain taken jointly as the "system" ($S'' \equiv S + \cdots$) under measurement—with the remaining "apparatus" portion being what measures it and is not included in the formal description but only identified conceptually as the complement of the measured system in this chain of measurement interaction—is known as "the mobility of the 'Heisenberg cut'" (*Schnitt*); it reflects a new species of "relativity" of acceptable physical description, according to Heisenberg. Such changes of delineation in the formal description make no difference to the initial predictions provided by the application of the Born rule because—due to the perfect correlation in a successful measurement between the state of observed system S and other interacting parts of the measurement chain—they must be identical for all choices of location of the cut; see Equation (4) and what follows it, below. A measurement description under all these delineations must—after contact with the greater world has been made—involve a change of state incompatible with the Schrödinger evolution, because an irreversible non-linear change resulting in the final state $|\phi_k\rangle$ of S must occur in each of them for a correct description of the final outcome to be possible. Based on the value of the resulting outcome, new probability distributions—corresponding to the density matrix description for the range of possible outcomes—are assigned for different future sets of identical measurements on members of an ensemble of systems so prepared; that is, having that very same measurement outcome.

Returning to the central point here, recall that Heisenberg's analogy operates at the epistemic level—something evident at the beginning of his analysis, where he describes Gibbs' thermodynamics as a theory "which Bohr has always pointed to as an especially clear application of the theory of knowledge in physics" ([1], p. 25). The crux is that there is a distinction made by Heisenberg that Popper failed to recognize, between the transition from the possible to the actual and the "reduction of the wave packet": the latter for Heisenberg here is a change of ensemble assigned to the system by the observer *based on its knowledge of its actual properties*, whereas the former is a change *in the system itself* exhibited in that of its state vector which is distinctly quantum mechanical. This distinction is subtle, not least because of the fact—in correspondence with the choice of cut location just discussed above—that the quantum mechanical formalism allows some freedom of assignment of the *time* at which measurement can be consistently taken as having been completed:

> "Of course, it is entirely justified to imagine this transition, from the possible to the actual,
> moved to an earlier point of time, for the observer himself does not produce the transition;
> but it cannot be moved back to a time when the compound system was still separate
> from the external world [, which would] not be compatible with the validity of quantum
> mechanics for the closed system" ([1], p. 27).

5. Measurement without the "Cut"

In the now-standard quantum theory of measurement (cf., e.g., [17], p. 28), contrary to Heisenberg, the process of measurement and its elements are typically treated via a two-component joint system form, as follows. A system S is initially prepared through a series of physical interactions, such as filtering, in some well-defined quantum state $|\eta\rangle$, after which it is measured through interaction with a measurement apparatus A. The apparatus—after beginning in a similarly prepared, fiducial initial state $|\chi_0\rangle$—is also required to enter a final state corresponding to the value of a pointer property Z (the strictness of the—at least implicit—eigenvalue–eigenstate relation varies from one to another interpretation of the formalism) correlated with the value of the measured property (non-degenerate observable) E of the system. Thus, both system and apparatus are given a full formal quantum mechanical representation. For simplicity, consider the measured property to be discrete; say,

$$E = \sum_i e_i |\psi_i\rangle\langle\psi_i| \, ,$$

where $\{|\psi_i\rangle\}$ is a countable orthonormal basis for the system Hilbert space $\mathcal{H}$ corresponding to its eigenvalues $\{e_i\}$. A minimal additional requirement for a successful measurement is that a "calibration condition" be satisfied—namely, that if a measured property is real, then it must exhibit its value properly, unambiguously, and with certainty; that is, if system S is an eigenstate $|\psi_k\rangle$ of E, then the state of apparatus A after the interaction of the two is an eigenstate of Z (with eigenbasis $\{|\phi_i\rangle\}$ associated with pointer readings z_i), which serves to indicate the specific value of E present (the free-Hamiltonian function contribution to the evolution of the system is considered negligible relative to the effect of the measurement interaction contribution). Accordingly, for measurable properties represented by Hermitian operators, the calibration condition can be considered in the form of a probability reproducibility condition—namely, that a probability measure E_T for a property be "transcribed" onto that of the corresponding apparatus pointer property, thereby "objectifying" it. Furthermore, the *registration* of the measured property by the measurement apparatus is assumed to include the physical reading out of the registered value.

By iteration of the above, one can similarly consider a fully formal sketch of the *entire chain* of interacting objects involved in measurement, from the system to be measured up to and including the brain of the experimenter himself. Such application of the quantum formalism to the set of physical subsystems, for example X, Y, ..., in the environment in addition to the original measurement system S and the experimenter's apparatus A—such as a proportional counter, cables, computer, and output display—would involve all these becoming correlated in their properties for the result of the outcome to be physically indicated. Under the Schrödinger state evolution (which is unitary), upon measurement, one must then have

$$|\Psi\rangle = \sum c_i |s_i\rangle |a_i\rangle |x_i\rangle |y_i\rangle \ldots \, , \tag{4}$$

with $\{s_i\}$, $\{a_i\}$, $\{x_i\}$, $\{y_i\}$ etc. being the Hilbert space eigenbases for S, A, X, Y, ..., respectively.

6. The Actualization of Potentia Is Quantum

In Heisenberg's approach, a treatment with the entire measurement chain, or even simply the system and apparatus in direct contact with it, described within the state-vector formalism as above, is considered implausible. However, a description of systems involved in measurement can be related to his approach via the notion of a cut or split (*Schnitt*) between the measured portion of this chain of systems and the measuring part, which includes the brain of the experimenter. The "chain of statistical correlations" appearing in the state $|\Psi\rangle$ is considered to be "cut" in two, into a system S$'$ and the remainder A$'$, somewhere along this chain of interacting systems, with subsystems to the left of the cut collectively considered the system to be measured: S$'$ subsumes S together with all other subsystems left of the cut, and A$'$ = A + W is the collective of those systems right of the cut—that is, the "apparatus

plus the rest of the world" W. Then, A′ is removed from the formal description because it is not part of the system being considered as measured, which is all that need be described. The cut must be made somewhere within S–X–Y–···–A before the influence of the environment makes the description via Equation (4) implausible; that is, some time before a macroscopic number of degrees of freedom becomes involved. The actualization of potentia requires an interaction of S′ ≥ S with the measuring apparatus, itself in interaction with the rest of the world; A′ is not to be described formally via a vector state, as *its objective state is held to be unknowable both in practice and in principle* to any experimenter, because of this interaction with the world at large, which is unaccountable.

Upon the actualization of potentia, if the experimenter knows an outcome must have occurred as well as the basis corresponding to the measurement but is not yet aware of the result, he must describe S in terms of a statistical mixture of states,

$$\rho = \sum_{i=1}^{n} p_i \rho_i \,, \tag{5}$$

each of the ρ_i corresponding to one the elements of the set $\{s_i\}$ of n possible distinct values for the measured property of S, weighted by the probability $p_i = |c_i|^2$. This is so *no matter where the measurement chain is cut, so long as the apparatus has at that point not interacted with a macroscopic portion of the greater world* and correspondingly—because there is a temporal ordering of the sequence of interactions in the measurement chain from left to right—*no matter when after the measurement the chain has been so cut*; the actual value will occur with the same likelihood p_i for all choices due to the linearity of the Schrödinger state evolution and by the associativity of the vector multiplication in Equation (4).

When the measurement has ended, actualization of the, say, $i = k$th potentiality will have given rise to the new system state $|\psi_k\rangle$. With an awareness, upon attending to it, of the actual outcome k of the experiment (according to (2))—that is, with his *having come to know* the actualized value—the *experimenter* must assign the system to a new ensemble, namely, that with

$$\rho_k = |\psi_k\rangle\langle\psi_k|.$$

The knowledge of the measurement outcome that corresponds to the "collapse of the wave packet",

$$\rho \rightarrow \rho_k \,,$$

is distinct from the actualization of potentia, that is, from the change of state-vector amplitude $c_k \rightarrow 1$ that enables it—and so from the state change

$$|\psi\rangle \rightarrow |\phi_k\rangle \,.$$

If instead one were to simply proceed without a cut, a difficulty known as the quantum measurement problem would arise: if the initial state of the system is a state $|\eta\rangle$ that is not an eigenstate of E, so that the joint system S′ + A′ begins in the state $|\eta\rangle|\chi_0\rangle$, and if the joint evolution is linear (i.e., does not itself depend on the system state, and described by a unitary operator U), a joint state results that is a non-trivial superposition of joint eigenstates of E and Z having the form

$$U|\eta\rangle|\chi_0\rangle = \sum_i c_i |\psi_i\rangle|\phi_i\rangle \,. \tag{6}$$

The result, in its full generality, is an absence of strict determination of the pointer observable: the superposition is non-trivial (that is, has more than one summand), leaving the outcome of the measurement underdetermined, violating the requirement of an unambiguous certain value of $|\phi_k\rangle$ (cf. [1], p. 22). This is the essence of the quantum measurement problem; it arises when the Heisenberg cut is *not* made and A′ is retained formally (cf. [18], Section 2.5). There have been a number of attempts to achieve such a result by reference to the mechanism of quantum decoherence (cf. [19]), but these all require some degree of approximation and raise other interpretational issues (cf. [18], Section 3.5). Other approaches, such as GRW and Bohmian mechanics, require modification of the quantum

theory—either formally or ontologically, and so fall outside of the the standard theory—in order to fully capture quantum entanglement or to be applicable to all known quantum fields. In general, some sudden state change would be required, in addition to any evolution of the type of Equation (6) that might take place, for a joint state $|\psi_k\rangle|\phi_k\rangle$ which would formally accurately reflect the state of the joint system after measurement to result; such a sudden change cannot arise consistently with the Schrödinger equation.

In Heisenberg's treatment, the ensemble described by Equation (5) is the appropriate one for prediction because the measurement of a quantum system involves its interaction with the measuring apparatus which in turn interacts with its environment, preventing one from being able to know the state of system S precisely; it must be learned from direct experience of the measurement outcome after actualization. The situation that the experimenter is temporarily not in possession of the corresponding most precise individual state description and then learns it through awareness of the measurement result is responsible for the similarity of what Heisenberg here calls "wavepacket collapse" to the situation in Gibbs' thermodynamics explicated above, rather than any classicality of the process of actualization, which occurs in the state of the measured systems independently of the experimenter's knowledge and is entirely quantum mechanical. Thus, Heisenberg explained, the parallel between the two theories arises from the relationship between the possible and the factual in relation to the statistically described quantities of quantum mechanics when only probabilistic predictions for future values are available. In his German-language memoir, *Der Teil und das Ganze*, he explained it as follows: "*Vielleicht könnte man sagen, sie stellen ein Zwischending zwischen Möglichem und Faktischem dar, das objektiv höchstens im gleichen Sinne genannt werden kann wie etwa die Temperatur in der statistischen Wärmlehre. Diese bestimmte Erkenntnis des Möglichen läßt zwar einige sichere und scharfe Prognosen zu, in der Regel aber erlaubt sie nur Schlüsse auf die Warscheinlichkeit eines zukünftigen Ereignisses.*"

> "Perhaps one might say that [potentiae] represent an intermediate between the possible and the factual, which can be called objective at most in the same sense as the temperature in the statistical theory of heat. This definite knowledge of the possible gives some definite predictions, but as a rule conclusions can only be drawn about the probability of a future event." ([20], pp. 54–55)

The nonetheless nonclassical character of the quantum state transition itself was in fact pointed out by Heisenberg in his analysis—and directly after that seized upon by Popper as shown at the end of our Section 2—wherein he noted that the quantum (i.e., Hilbert-space) description is *complementary* to any classical one:

> "The characterization of a system by a Hilbert vector is complementary to its description in classical terms, similar to the way that a microscopic state is complementary in Gibbs' thermodynamics to the statement of the temperature." ([1], p. 27)

In both cases, it is also that there is no deterministic connection between the discovered state of the measured system and its previous description that is accessible via the fundamental deterministic equations of the respective theories.

> "The discontinuous change is naturally *not* contained in the mechanical equations of the system or of the ensemble which characterizes the system...the characterization of a system by an ensemble not only specifies the properties of this system, but also contains information about the extent of the observer's knowledge of the system" ([1], p. 26).

Upon the completion of measurement, new information is available to the observer, allowing a better fitting ensemble assignment and, therefore, more accurate predictions than provided via Equation (5).

7. The Role of the Actual

In quantum mechanics, just as in the classical case, there is an objective matter of fact in actuality. The experimenter may become aware of this, providing new information relating to the best ensemble

description; in the quantum case, this awareness corresponds to the change of his description from the density operator description ρ of a mixed ensemble to that of the appropriate pure one, ρ_k. The commonality between the two cases is the role of the *factual* in physics: knowledge of the factual gained through observation influences the accuracy of description and prediction. The analogy under consideration here is between measurements of properties influencing the choice of ensemble descriptions for different types of mechanical system. Heisenberg emphasized that *the fact that in quantum mechanics knowledge plays a role in finding the state of the individual system does not drive the notion of objective reality entirely from physics.*

> "The criticism of the Copenhagen interpretation of quantum theory reflects an anxiety that with this interpretation, the concept of 'objective reality' which forms the basis of classical physics might be driven out of physics. As we have shown here, this anxiety is groundless, since the 'actual' plays the same decisive part in quantum theory as it does in classical physics." [1]

It is only that such a false impression may arise because constraints on our ability to measure the full set of properties of quantum systems render its state *incompletely inaccessible*; our knowledge of their objective being at any given instant is always limited.

The commonality between classical and quantum physics is found in *the role of actual, rather than that of actualization*. Heisenberg made it clear that he recognized that "the statistical character of quantum theory is in many respects fundamentally different from that used in the kinematic interpretation of thermodynamics" because the knowledge of the objective state is *fundamentally* limited in the quantum case:

> "...in the kinematic interpretation of thermodynamics [the degree of accuracy] always expresses [only] our lack of knowledge of the system concerned. But in quantum theory, ignorance of the result of future experiments *can be* compatible with the complete understanding—in the usual accepted sense—of the state of the system concerned. ...in Heat, [by contrast,] ignorance about the results of certain experiments is always identified with ignorance of the true state of the system [itself] and is shown up in all experiments." ([21], pp. 50–51)

What makes the quantum mechanical situation different from the classical one is that there are inherent limitations in one's ability as a physicist to describe system properties via the completely described physical state: the quantum eigenstate of no Hermitian operator provides definite values to all measurable quantities—even a maximally specified one. This fundamental difference between the two theories prevents the common significance of the actual to them from amounting to their identification.

Thus, one sees that it is not the case, as Popper claims, that Heisenberg's "exact" analogy between wave-function collapse in quantum mechanics suffices for the *process of actualization of quantum potentiality* to be deemed classical; actualization is a change of objective state that takes place for each individual quantum system upon its measurement independently of epistemic considerations.

8. Conclusions

It has been shown here that the analogy made by Heisenberg under consideration here is one between, on the one hand, the change in quantum mechanics of assignment of a system from a mixed ensemble described by a diagonal density matrix to a pure quantum ensemble enabled by a sharp measurement, and on the other hand, the change in Gibbs' thermodynamics from a classical system's assignment to canonical ensemble to its assignment to a microcanonical ensemble enabled by a precise measurement.

In Gibbs' thermodynamics, the result of a given measurement is the restriction of the statistical description under consideration from that of a large ensemble to a smaller one, due to the newly

obtained knowledge of the system state. The system in question can be a system of any size, but the heat bath must be very large, and it is isolated in the sense the only energy exchange allowed is that with the bath. A similar situation is obtained in the quantum case, in the sense that the assignment to a pure ensemble of the conjunction of the system to be measured together with the measuring apparatus to be used is valid so long as the greater environment does not interact with this joint system. However, quantum measurement only takes place when these two systems interact with the greater world external to them, at which point the measured value is actualized.

When actualization occurs, the attribution of the measured system by any experimenter carrying out the measurement—so long as he has has *not* paid attention to its result—is to a mixed ensemble described by a diagonal density matrix. The consequence of the actualization is that there is an actual value to be attended to, so that when awareness of the measurement result occurs the observer's newly obtained knowledge of that value also provides him with the most precise description of the individual system—namely, the Hilbert eigenvector and so with the associated eigenvalue that is shared by members of the corresponding pure ensemble as well.

It has been claimed that the above analogy is grounds for considering the actualization of potentia as classical in nature, rather than uniquely quantum. It is shown here that this is not the case and that, in the version of this claim as argued for by Popper, it is based on the conflation of actualization of a property (reflected in the change of state vector of the individual system) with the change of the ensemble to which the system is later generally assigned by the experimenter due to an increase in his knowledge of that actualized property.

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