

Article

# The Isolated Electron: De Broglie's Hidden Thermodynamics, SU(2) Quantum Yang-Mills Theory, and a Strongly Perturbed BPS Monopole

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**Abstract:** Based on a recent numerical simulation of the temporal evolution of a spherically perturbed BPS monopole, SU(2) Yang-Mills thermodynamics, Louis de Broglie's deliberations on the disparate Lorentz transformations of the frequency of an internal "clock" on one hand and the associated quantum energy on the other hand, and postulating that the electron is represented by a figure-eight shaped, self-intersecting center vortex loop in SU(2) Quantum Yang-Mills theory we estimate the spatial radius  $R_0$  of this self-intersection region in terms of the electron's Compton wave length  $\lambda_C$ . This region, which is immersed into the confining phase, constitutes a blob of deconfining phase of temperature  $T_0$  mildly above the critical temperature  $T_c$  carrying a frequently perturbed BPS monopole (with a magnetic-electric dual interpretation of its charge w.r.t.  $U(1) \subset SU(2)$ ). We also establish a quantitative relation between rest mass  $m_0$  of the electron and SU(2) Yang-Mills scale  $\Lambda$ , which in turn is defined via  $T_c$ . Surprisingly,  $R_0$  turns out to be comparable to the Bohr radius while the core size of the monopole matches  $\lambda_C$ , and the correction to the mass of the electron due to Coulomb energy is about 2%.

**Keywords:** wave; particle; Lorentz boost; excited BPS monopole; Harrington-Shepard caloron; quantum of action

## 1. Introduction

The electron and other charged leptons are considered fundamental particles in the present Standard Model of particle physics. According to the overwhelming majority of data on (high-energy) electron-electron/electron-positron collisions and atomic physics such a classification rests on the fact that no structural charge-density and spin distributions are revealed by typical scattering processes and spectroscopic experiments. Moreover, in many condensed-matter systems the notion of the Quantum Mechanics (QM) of a point-like, spinning particle together with the exclusion principle do describe the collective behavior of electrons in a highly satisfactory and realistic way [1]. However, the quantitative description of strongly correlated two-dimensional electrons associated, e.g., with high-temperature superconductivity, seems to demand a deviating treatment. Here a spatial separation between charge and spin is a serious option favored by the data [2].

To describe the spatial probability density for the presence of a point-like electron in terms of the square of a wave function based on de Broglie's particle-wave duality [3], whose time evolution is governed by the Hamiltonian of the isolated system, is an extremely successful and useful concept: About a century ago, it started to revolutionize our understanding of atomic stability and of the discreteness of the spectra of light emitted by excited atoms [4–17], the chemical bond [18], and the role of and interplay between electrons in condensed matter and hot plasmas thanks to the development of

efficient calculational procedures for (extended) multi-electron systems. Moreover, new computational and conceptual horizons, enabled by quantum field theoretical (QFT) second quantization in the framework of Quantum Electrodynamics (QED) [19–21] and its electroweak generalization [22–25], have yielded accurate predictions of radiative corrections to atomic energy levels, to the electron’s magnetic moment, and in electroweak scattering cross sections.

Why then is there a need for a deeper understanding of the nature of the free electron and its properties as revealed by the application of external forces, the implied (classical) radiation reaction, and tree-level as well as radiative quantum behavior playing out in scattering processes? The answer to this question touches a number of basic problems.

First, already on the classical level Maxwell’s equation and the Lorentz force equation do not describe the phenomenon of radiation reaction: This local field theory ignores the back-reaction onto an electron induced by the radiation emitted by this particle when under acceleration in an external field. Various proposals on how to modify the force equation were made in the literature, see, e.g., [26,27], but a basic theory yielding an effective, unique correction is not available. As was shown in [26], the effects of radiation reaction can dominate the dynamics of the electron for strong accelerations posited by high-power laser pulses.

Second, even though QM and QFT provide an efficient and reliable computational framework there are conceptual questions. Why does the electron as a particle have a modest rest mass of

$$m_0 = 0.511 \text{ MeV}/c^2 \quad (1)$$

if probing its structure by high-energetic scattering experiments at four-momentum transfers  $Q$  up to several hundreds of  $\text{GeV}/c$  does not indicate any deviation from structurelessness or point-particle behavior? Classically speaking, such a point-likeness would imply the spatial integral of its Coulombic electric field energy density and hence its rest mass  $m_0$  to be of order  $Q$ . Namely, in natural units  $c = \hbar = 1$  one has

$$m_0 \sim \int_{Q^{-1}}^{\infty} dr \frac{r^2}{r^4} = Q \quad (2)$$

which contradicts the value in Equation (1). Notice that radiative corrections to  $m_0$ , computable in QED in terms of powers of a small coupling constant  $\alpha \sim 1/137$ , must, by the very definition of a renormalized perturbation series, be much smaller than  $m_0$ . Concerning another aspect of the electron’s potential structure, one may ask how a finite magnetic moment can possibly be related to the spin of a point particle? Yet, although (or possibly because) it ignores this basic question, representation theory in QM is immensely successful in classifying the effect of coupled angular momentum on energy levels and on the overall dynamics of composite systems.

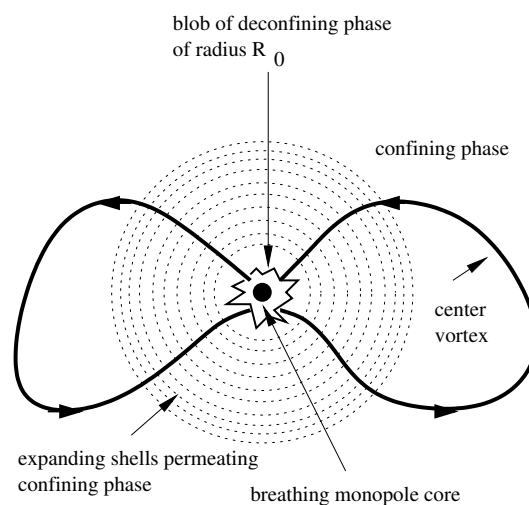
Third, Louis de Broglie’s deep ideas, underlying the proposal of wave-particle duality for the electron [3], in their original form imply that the electron is anything but a particle of vanishing spatial extent. Strictly speaking, this contradicts Born’s interpretation of the square of the wave function describing the probability density for the spatial occurrence of a point particle. De Broglie argues [28] by considering respective changes, under a Lorentz boost at velocity  $v$ , of the electron’s rest mass  $m_0$ , associated with the frequency  $\nu_0$  of an internal “clock” oscillation via Planck’s quantum of action  $h$  as

$$m_0 c^2 = h \nu_0, \quad (3)$$

and viewed as the zero component of the electron’s four-momentum. Subsequently, he contrasts this with the disparate changes of the same *internal* frequency as implied by time dilatation. As a consequence, the increase of particle energy from  $m_0 c^2$  to  $mc^2 = \frac{m_0 c^2}{\sqrt{1-\beta^2}}$  can be decomposed into a reduction of internal heat from  $m_0 c^2 \equiv Q_0$  to  $Q = Q_0 \sqrt{1-\beta^2}$  plus an increase of (quasi-)translational energy from zero to  $vp$ :

$$mc^2 = \frac{m_0c^2}{\sqrt{1-\beta^2}} = Q + vp \equiv m_0c^2\sqrt{1-\beta^2} + vp, \quad (4)$$

where  $\beta \equiv v/c$ , the relativistic spatial momentum is given as  $p \equiv \frac{m_0v}{\sqrt{1-\beta^2}}$ , and  $c$  denotes the speed of light in vacuum. Notice that  $Q = Q_0\sqrt{1-\beta^2}$  refers to Planck's formulation of relativistic thermodynamics which, in a straight forward way and without directly addressing dissipative processes, assures proper Lorentz-transformation behavior of an exhaustive set of thermodynamical quantities. This may be contrasted with Ott's formulation [29,30] whose justification appears to be rather mysterious to the present author. Notice also that the second term on the right-hand side of Equation (4)—the (quasi-)translational energy—reduces to nonrelativistic kinetic energy  $\frac{1}{2}m_0v^2$  only modulo a factor of two for  $v \ll c$ . This is an unexplained point which eventually needs to be clarified. It is absurd, however, to assign an internal heat  $Q$  to a point particle. The present work proposes that the thermodynamics of an isolated electron at rest and the existence of a preferred internal frequency  $\nu_0 = \frac{m_0c^2}{h}$  are consequences of the existence of a compact, spatial region containing an electric, BPS-like monopole (after an electric-magnetically dual interpretation [31,32]), which accommodates the electron's charge and mass, representing a blob of deconfining phase of SU(2) Quantum Yang-Mills theory immersed into a confining-phase environment. Macroscopically seen, this blob represents the self-intersection of an SU(2) center vortex loop, that is, (again, after an electric-magnetically dual interpretation [31,32]) a figure-eight shaped, one-fold knotted loop of electric center flux inducing a magnetic moment twice as large as the fundamental unit provided by a single center-vortex loop, see Figure 1.



**Figure 1.** Schematics of a one-fold self-intersecting center-vortex loop, immersed into the confining phase of SU(2) Yang-Mills theory. SU(2) Yang-Mills theory is to be interpreted in an electric-magnetically dual way, and therefore the breathing core of the perturbed BPS monopole, in turn immersed into the (fuzzy) intersection region of radius  $R_0$  endowed with deconfining energy density  $\rho$ , actually represents an electric charge. Also, the magnetic center flux, residing in the two wings of the vortex-loop, actually is an electric one, giving rise to a magnetic moment twice that of a single vortex loop. Spin-1/2 is represented by the two possible directions of (dually interpreted)  $Z_2$  center flux. The associated local  $Z_2$  symmetry is broken dynamically in the confining phase by the 't Hooft loop acquiring a finite expectation [31,32]. The concentric circles indicate the dilution of the confining phase by expanding high-frequency shells carried by massive off-Cartan modes of the monopole. These shells are induced by the action of (anti)caloron centers perturbing the BPS monopole.

Finally, deeper insight into the functioning of the electron as a quantum particle is destined to shed light on the “sweet mysteries” of Quantum Mechanics (e.g., instantaneous state reduction by measurement) and the renormalization problem in perturbative Quantum Field Theory.

The present work offers a conceptual scheme for the description of the unaccelerated electron which supports de Broglie’s ideas. This scheme draws on the following developments: (i) Phase structure of SU(2) Yang-Mills thermodynamics [31,32], composition and wave/particle like excitability of the deconfining thermal ground state [33,34], and principle nature of solitonic excitations in the confining phase [35,36]; (ii) Recent results on the classical dynamics of the strongly (and isotropically) perturbed BPS monopole [37]. Pairing with the work in [37], there are insightful and supporting treatments of linear perturbations of the BPS monopole (spectrum of quasi-normal modes in [38]).

As a consequence of (i) and (ii) the quantum thermodynamics within the self-intersection region of a center-vortex loop manifests itself as follows (in natural units): Created by the dissociation of a large-holonomy caloron, a BPS monopole, with the asymptotic value of its Higgs field modulus at temperature  $T_0$  given as

$$H_\infty(T_0) = \pi T_0 \quad (5)$$

due to maximum (anti)caloron holonomy (Temperature  $T_0$  of the intersection region is slightly higher than the critical temperature  $T_c$  of the deconfining-preconfining phase transition:  $T_0 = 1.315 T_c$ , see Section 3) [39–46] and the value of the coupling constant  $e$  in the adjoint Higgs model roughly determined by the plateau value  $e = \sqrt{8}\pi$  [31,32] is immersed into deconfining SU(2) Yang-Mills thermodynamics. At  $T_0 = 1.315 T_c$ , see Section 2, the pressure of deconfining SU(2) Yang-Mills thermodynamics vanishes. The contribution to the total pressure from a static (noninteracting) BPS monopole [47,48] is also nil. In the absence of interactions between such an explicit monopole and deconfining SU(2) Yang-Mills thermodynamics the intersection region would thus be static at temperature  $T_0$ , that is, non-expanding and non-contracting.

The unperturbed monopole’s rest mass is given as [47,48]

$$m_{\text{mon},0} = \frac{8\pi^2}{e^2} H_\infty \sim H_\infty, \quad (6)$$

where  $H_\infty$  denotes its asymptotic adjoint Higgs field, and the energy density  $\rho$  of deconfining SU(2) Yang-Mills thermodynamics at  $T_0$  reads

$$\rho(T_0) = 8.31 \rho^{\text{gs}}(T_0), \quad (7)$$

see Section 2, where  $\rho^{\text{gs}} = 4\pi\Lambda^3 T$  denotes the energy density of the thermal ground state, and  $\Lambda$  is the Yang-Mills scale, related to  $T_c$  as

$$T_c = \frac{13.87}{2\pi} \Lambda = \frac{1}{1.32} T_0. \quad (8)$$

The fact that the explicit monopole is subject to perturbations, issued by quantum kicks due to caloron/anticaloron centers [33], leads to a fluctuating re-distribution of stress-energy within the self-intersection region. This situation is characterized, e.g., by the existence of a breathing mode of the monopole core whose (circular) frequency  $\omega_0$  essentially represents  $m_0$  [28] and is given by the mass  $m_w = eH_\infty$  of the vector modes [37].

The remainder of this paper is intended to corroborate and explain the above-sketched model of the electron in more quantitative terms, thereby considering a few (rather small) corrections to the results obtained in the simplified treatment of [49] where complete ground-state dominance of the deconfining thermodynamics within the intersection region was assumed. Since our derivations heavily rest on a grasp of the phase structure of an SU(2) Yang-Mills theory we will briefly review it in Section 2. Section 3 discusses the deconfining pressure  $P$  and derives the ratio  $\rho(T_0)/\rho^{\text{gs}}(T_0)$  and the value of the coupling  $e$  (slightly higher than  $\sqrt{8}\pi$ ) at the zero  $T_0$  of  $P$ . The mass of the noninteracting BPS monopole, given by the first of Equation (6), thus is slightly decreased compared to the value given

by the expression to the far right, and the mass  $m_w$  of the vector modes is mildly increased. The results obtained in [37] on the dynamics of the spherically perturbed monopole are reviewed in Section 4. Finally, in Section 5 we combine the results of Sections 2–4 to propose a model for the electron as a genuine quantum particle of finite extent in Section 5. Section 6 summarizes this work and gives an outlook to future activity.

## 2. Review on Phase Structure of SU(2) Yang-Mills Thermodynamics

SU(2) Yang-Mills thermodynamics occurs in three distinct phases [50]. The high-temperature, deconfining phase is characterized by a thermal ground state, composed of overlapping, topological-charge-modulus unity calorons and anticalorons. At a given temperature  $T \geq T_c$ , the spatial extent of their densely packed centers is determined by a radius  $\rho_u = |\phi|^{-1} = \sqrt{2\pi T/\Lambda^3}$  which also sets the preferred (anti)caloron scale parameter for the spatial coarse-graining process involving a homogeneously and adjointly transforming two-point function of the fundamental Yang-Mills field-strength tensor  $F_{\mu\nu}$  evaluated on a Harrington-Shepard (HS) (that is, trivial-holonomy) caloron and its anticaloron [51,52]. Here  $|\phi|$  denotes the modulus of the emergent, adjoint, and inert scalar field representing coarse-grained (anti)caloron centers,  $T$  refers to temperature as defined by the inverse temporal period of the HS-(anti)caloron, and  $\Lambda$  is the Yang-Mills scale. The latter emerges as an integration constant when solving a first-order, ordinary differential equation for  $\phi$ 's potential. This first-order equation expresses the intactness of (Euclidean) BPS saturation within (anti)caloron centers. A departure from (anti)selfduality in the field configuration, reflecting the overlap of (anti)caloron peripheries, is manifested by a finite ground-state pressure  $P^{\text{gs}}$  and energy density  $\rho^{\text{gs}} = -P^{\text{gs}} = 4\pi\Lambda^3 T$ . The associated, effective gauge-field configuration  $a_\mu^{\text{gs}}$  is obtained as a zero-curvature solution to the effective Yang-Mills equation  $D_\mu G_{\mu\nu} = 2ie[\phi, D_\nu\phi]$  where  $G_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu + ie[a_\mu, a_\nu]$  denotes the effective Yang-Mills field-strength tensor.

Excitations of the deconfining thermal ground state are partially massive (mass  $m = 2e|\phi|$ ) by virtue of the adjoint Higgs mechanism invoked by the field  $\phi$ . These massive excitations never propagate in a wavelike way: their high would-be frequencies probe the interior of (anti)caloron centers to provoke an indeterministic, quantum-like response. On the level of free thermal quasi-particles (that is, not taking into account feeble radiative effects [31,32,53]) the massive sector thus merely represents uncorrelated, Bose-Einstein distributed energy and momentum fluctuations. The *massless* sector, on the other hand, may propagate in a wavelike fashion if a constraint on intensity and frequency is satisfied [33], governed by the Yang-Mills scale  $\Lambda$  of the theory.

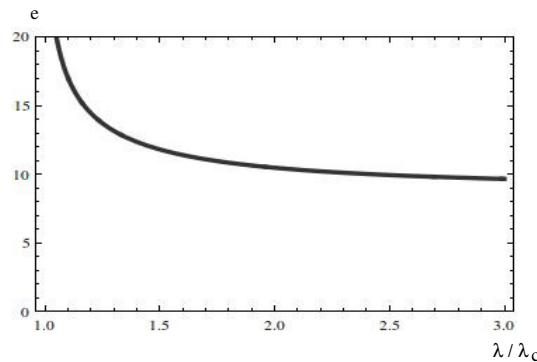
The thermal ground state of the deconfining phase rearranges itself into a different structure at the critical temperature  $T_c = \frac{13.87}{2\pi} \Lambda$  where screened BPS monopoles and antimonopoles, released by rarely occurring large-holonomy calorons and anticalorons, become massless, therefore abundant, and, as a consequence, form a condensate. This marks the onset of the preconfining phase. Energetically, the condensate of monopoles and antimonopoles together with its massive gauge-mode excitations (Meissner-Ochsenfeld effect or Abelian Higgs mechanism [54]) is not stable immediately at  $T_c$  since the energy density of the deconfining phase is lower within the following temperature range [31,32]:

$$T_* \equiv 0.88T_c \leq T \leq T_c. \quad (9)$$

Shortly below  $T_*$  the entropy density of the system approaches zero, and the thermal ground state of the preconfining phase decays into (spatial)  $n$ -fold self-intersecting center-vortex loops ( $n = 0, 1, 2, \dots$ ). Since their masses scale as  $n\Lambda$  but their multiplicities (number of distinct topologies at given  $n$ ) scale more than factorially in  $n$  [55,56] the very concept of a partition function is inapplicable. This is the characteristics of a highly nonthermal Hagedorn transition: the homogeneity of pressure, energy density, and other “would-be” thermodynamical quantities is strongly violated, and the SU(2) Yang-Mills plasma exhibits a highly turbulent behavior.

### 3. Pressureless Deconfining SU(2) Thermodynamics at $T_0$

For accuracies on the 1% level it is sufficient to consider thermodynamical quantities in one-loop approximation. Thermodynamical consistency up to this loop order implies an evolution equation for the coupling  $e$  [31,32]. Its solution is depicted in Figure 2 as a function of  $\lambda/\lambda_c$  where  $\lambda \equiv \frac{2\pi T}{\Lambda}$  and  $\lambda_c = 13.87$ .



**Figure 2.** Evolution of the coupling  $e$  in the deconfining phase of SU(2) Yang-Mills thermodynamics as a function of  $1 \leq \lambda/\lambda_c \leq 3$ . At  $\lambda \sim \lambda_c = 13.87$  the theory undergoes a second-order like phase transition towards the preconfining phase which is signalled by a (logarithmically thin) divergence of  $e$ .

The deconfining-phase pressure  $P$  and energy density  $\rho$  are then given as [31,32]

$$\begin{aligned}
 P(\lambda) &= -\Lambda^4 \left\{ \frac{2\lambda^4}{(2\pi)^6} [2\bar{P}(0) + 6\bar{P}(2a)] + 2\lambda \right\}, \\
 \rho(\lambda) &= \Lambda^4 \left\{ \frac{2\lambda^4}{(2\pi)^6} [2\bar{\rho}(0) + 6\bar{\rho}(2a)] + 2\lambda \right\},
 \end{aligned}
 \tag{10}$$

where

$$\begin{aligned}
 \bar{P}(y) &\equiv \int_0^\infty dx x^2 \log \left[ 1 - \exp(-\sqrt{x^2 + y^2}) \right], \\
 \bar{\rho}(y) &\equiv \int_0^\infty dx x^2 \frac{\sqrt{x^2 + y^2}}{\exp(\sqrt{x^2 + y^2}) - 1},
 \end{aligned}
 \tag{11}$$

$$a \equiv \frac{m}{2T}.
 \tag{12}$$

In Equation (12)  $m = 2e\sqrt{\frac{\Lambda^3}{2\pi T}}$  denotes the thermal quasi-particle mass of the vector modes (unitary gauge) as induced by the adjoint Higgs mechanism effectively inaugurated by (anti)caloron centers. In Figure 3  $P/\Lambda^4$ ,  $\rho/\Lambda^4$ , and  $\rho/\rho^{gs}$  are depicted for  $\lambda_c = 13.87 \leq \lambda \leq 1.5\lambda_c$ .

Notice the zero of  $P$  (positive pressure of (quasi-particle) excitations balanced by negative ground-state pressure) at

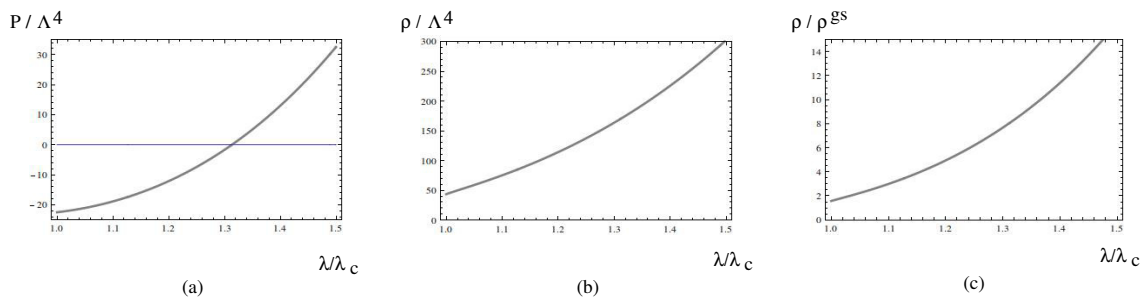
$$\lambda_0 = 1.32\lambda_c = 18.31
 \tag{13}$$

in Figure 3a where

$$\rho/\rho^{gs}(1.32) = 8.31,
 \tag{14}$$

see Figure 3c, meaning that (quasi-particle) excitations already dominate the energy density at  $\lambda_0$ . On the other hand, one has  $\rho/\rho^{gs}(1) = 1.56$ , indicating the importance of the ground state energy density at the deconfining-preconfining phase boundary, see Figure 3c. Finally, at  $\lambda_0$  the coupling is

$$e(1.32) = 12.96.
 \tag{15}$$



**Figure 3.** Thermodynamical quantities of deconfining SU(2) Yang-Mills thermodynamics close to the deconfining-preconfining phase transition. (a) dimensionless pressure  $P/\Lambda^4$ ; (b) dimensionless energy density  $\rho/\Lambda^4$ ; and (c)  $\rho/\rho^{GS}$  all as functions of  $\frac{\lambda}{\lambda_c}$ . At  $\lambda \sim \lambda_c = 13.87$  the theory undergoes a second-order like phase transition towards the preconfining phase.

#### 4. Review on Strongly Perturbed BPS Monopole

The normal-mode spectrum of a 't Hooft-Polyakov monopole in the BPS limit (considering small field fluctuations only) was investigated in [38]. In [37] a spherically symmetric, strong perturbation of the BPS monopole in the SU(2) Yang-Mills-adjoint-Higgs model was analysed numerically as a nonlinear initial-value problem by virtue of a hyperboloidal conformal transformation of the original field equations for the profiles  $H(r, t) = h(r, t)/r + H_\infty$  and  $w(r, t)$  of the adjoint Higgs and off-Cartan fields, respectively. Their results can be summarized as follows. Considering a spherically symmetric, localized initial pulse as a strong perturbation of the static BPS monopole, the typical dynamical response did not depend on the parameter values of this pulse within a wide range. There are high-frequency oscillations in  $w$  which form expanding shells decaying in time as  $t^{-1/2}$ . (The further away from the monopole core the shell is the higher the frequencies are that build it.) Moreover, a localized breathing state appears in association with the energy density of the monopole core region whose frequency  $\omega_0$  approaches the mass  $m_w = eH_\infty$  ( $e$  denoting the gauge coupling) of the two off-Cartan modes in a power-like way in time (natural units):

$$\omega_0 = eH_\infty - C_w t^{-2/3} \Rightarrow \lim_{t \rightarrow \infty} \omega_0 = eH_\infty, \tag{16}$$

where  $C_w$  is a positive constant. The amplitude of the oscillation in energy density decays like  $C_a t^{-5/6}$ ,  $C_a$  again denoting a positive constant. Disregarding for now the question what the physics of the exciting initial condition is, an internal clock of (circular) frequency  $\omega_0 \sim m_w$  is therefore run within the core region of the perturbed monopole.

#### 5. Size Estimates

Based on [49] we would now like to perform an improved estimate of the radius  $R_0$  associated with the region of self-intersection. The idea is to prescribe at  $T_0$ , where deconfining SU(2) thermodynamics does not exert any pressure, that the rest mass of the quantum particle electron,

$$m_0 \sim e(T_0)H_\infty(T_0) = 12.96 H_\infty(T_0), \tag{17}$$

see Equations (15) and (16), decomposes into that of a static monopole

$$m_{\text{mon},0} = \frac{8\pi^2}{e^2(T_0)}(T_0)H_\infty(T_0) = \frac{8\pi^2}{12.96^2}H_\infty, \tag{18}$$

compare with Equations (6) and (15), and the energy  $E_0$  of deconfining SU(2) thermodynamics contained in the volume  $V = \frac{4}{3}R_0^3$ :

$$\begin{aligned} m_0 &= 12.96 H_\infty(T_0) = m_m + E_0 = \frac{8\pi^2}{12.96^2} H_\infty(T_0) + \frac{4}{3} \pi R_0^3 \rho(T_0) \\ &= H_\infty(T_0) \left( \frac{8\pi^2}{12.96^2} + 8.31 \times \frac{128\pi}{3} \left( \frac{R_0}{18.31} \right)^3 H_\infty^3(T_0) \right). \end{aligned} \tag{19}$$

In writing Equation (19), Equation (9) of [49] and Equations (13) and (14) were used. Interactions between the monopole and deconfining SU(2) thermodynamics (caloron and anticaloron centers [33] invoking energy transfer from quasi-particle excitations to the monopole which partially radiates this energy back into the Yang-Mills plasma) will effectively introduce spatial fluctuations of temperature  $T$  about the equilibrium value  $T_0$ , but they won't change the energy content of the overall system. Solving Equation (19) for  $R_0$ , we obtain the mean radial extent of the system as

$$R_0 = 4.10 H_\infty^{-1}(T_0). \tag{20}$$

To ensure that the use of infinite-volume thermodynamics, which has led to Equation (20), is consistent we need to compare  $R_0$  to the correlation length  $|\phi|^{-1}(T_0) = \sqrt{2\pi T_0/\Lambda^3}$  of the thermal ground state [31,32]. Appealing to Equations (5) and (24) below, one easily derives

$$|\phi|^{-1}(T_0) = H_\infty^{-1}(T_0) \sqrt{2 \left( \frac{118.6}{12.96} \right)^3}. \tag{21}$$

Thus

$$R_0 |\phi|(T_0) = 160.5, \tag{22}$$

which justifies the use of infinite-volume thermodynamics in deriving Equation (20).

Comparing  $R_0$  to the Compton wave length  $\lambda_C = m_0^{-1} = 2.43 \times 10^{-12}$  m, we have

$$R_0 \sim 53.14 \lambda_C \sim 1.29 \times 10^{-10} \text{ m}. \tag{23}$$

Thus,  $R_0$  is 2.43 times larger than the Bohr radius  $a_0$  which, in turn, is  $4.10 \times 12.96/2.43 = 21.87$  times larger than the monopole core radius  $R_c = \frac{1}{e} H_\infty^{-1}(T_0) = 2.43 \times 10^{-12}$  m =  $\lambda_C$ . It is remarkable that according to Equation (16) the relation between electron mass  $m_0$  (that is, monopole-core breathing frequency  $\omega_0$  [37]) and  $H_\infty$ , is in agreement with  $R_c^{-1} = m_0$  (or  $R_c = \lambda_C$ ). Therefore, the electron is *not* a point particle in the present model: Its charge is distributed over a spatial region whose radius matches the Compton wave length  $\lambda_C$  but (quantum) moves within a much larger volume of radius  $R_0 \sim 10^{-10}$  m of deconfining phase. Scattering experiments do not reveal an inner structure because *quantum thermodynamics* within radius  $R_0$ —the state of maximum entropy—is structureless. Moreover, the Yang-Mills scale  $\Lambda$ , which is determined from  $H_\infty(T_0) = \pi T_0$  (monopole originated by dissociation of maximally non-trivial holonomy caloron) and Equation (19), reads

$$\Lambda = \frac{1}{118.6} m_0. \tag{24}$$

Therefore,  $T_c = 13.87/(2\pi \times 118.6) m_0 = 0.019 m_0 = 9.49$  keV. For a comparison, the demonstrator tokamak ITER is envisaged to operate at an average electron temperature of 8.8 keV!

The important work [37] points out that high-frequency oscillations in the profile function  $w$  of the off-Cartan gauge fields, belonging to the perturbed monopole, develop expanding shells. If these shells are considered to penetrate the confining phase ( $r > R_0$ ) such that the static Coulomb field of the monopole, arising from a temporal average over many cycles of the monopole core-vibration,



can actually permeate this phase, then we may estimate the Coulomb-field correction  $\Delta m_0$  to the “thermodynamical” mass  $m_0$  as

$$\Delta m_0 = \int_{R_0}^{\infty} dr \frac{r^2}{r^4} = R_0^{-1} = \frac{1}{4.10} H_{\infty}(T_0) \ll 12.96 H_{\infty} = m_0 \quad \text{or} \quad m_0 \sim 53.14 \Delta m_0. \quad (25)$$

This is only a  $\sim 1.9\%$  correction to  $m_0$ . Notice the strong conceptual difference to the model of the classical electron whose radius defines the entire rest mass  $m_0$  via Coulomb self-energy.

Finally, we would like to mention that, in assigning half a Bohr magneton to the magnetic moment carried by a single center-vortex loop, a  $g$ -factor of two naturally arises by the electron being composed of two such center-vortex loops by virtue of the region of self-intersection, see Figure 1.

## 6. Summary

The present article’s purpose is a model of the electron along the lines proposed in [49] with more precise inputs from SU(2) Yang-Mills thermodynamics and relying on the work in [37]. This model posits the electron to be represented by a one-fold self-intersecting spatial center-vortex loop immersed into the confining phase. The intersection region—a blob of deconfining phase, whose center-of-mass position essentially is a modulus of this solitonic configuration, contains a BPS monopole responding to perturbations issued by the surrounding thermodynamics. This interplay between non-linear monopole dynamics and the thermal plasma gives rise to a core-breathing mode of the monopole which runs an internal clock. The existence of such a localized vibration was foreseen by de Broglie already ninety years ago [3]. That this assertion, which is at the heart of the proposal of the electron’s wave-particle duality, in turn leading to the development of wave mechanics by E. Schrödinger [11–15], was more or less forgotten during the century to follow definitely is owed to the success of QM as a quantitative, physical theory. This is a truly remarkable state of affairs.

The space external to the intersection region is subject to the confining phase which, however, is permeated by high-frequency expanding shells [37], thus providing a way for the Coulomb field of the monopole to reach beyond the self-intersection region. We have shown that the radius of self-intersection region is comparable to atomic dimensions with the monopole core size matching the electron’s Compton wave length. Coulomb self-energy turns out to be a small correction to the energy of the intersection region. That the bulk of the electron mass and its charge is represented by the “thermodynamics of an isolated particle” is an early and exceptionally deep insight by Louis de Broglie [28]. In the present work we have indicated that SU(2) Yang-Mills thermodynamics supports this idea.

We have not discussed the complicated situation of a radiating electron, which interacts with external fields, and the associated problem of radiation reaction. Also, it is not yet clear in detail how the weak interactions are accommodated into our model although the presence of massive vector modes with a hierarchically higher mass is assured by the confining phase [31,32]. Hopefully, all this can be worked out in the near future. From the present treatment it is obvious that other charged leptons—the muon and the  $\tau$ -lepton—should admit a description identical to that of the electron: One considers accordingly larger Yang-Mills scales of the associated SU(2) gauge theories and introduces mixing of Cartan algebras.

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