The Optimal Power Allocation for Sum Rate and Energy Efficiency of Full-Duplex Two-Way Communication Network

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Abstract: Full-duplex (FD) transmission holds a great potential of improving the sum data rate of wireless communication systems. However, the self-interference introduced by the full-duplex transmitter brings a big challenge to enhance the energy efficiency. This paper investigates the power allocation problem in a full-duplex two-way (FDTW) communication network over an OFDM channel, aiming at improving the sum data rate and energy efficiency. We first characterize the sum rate and energy efficiency achieved in a single-carrier FDTW system. The optimal transmit power that achieves the maximal sum data rate is presented. The energy efficiency maximization problem is solved by using fractional programming. Then we further formulate sum rate and energy efficiency maximization problem in a multi-subcarrier FDTW system. In particular, the sub-optimal transmit power allocation which achieves a decent sum rate improvement is found by using a proposed iterative algorithm. By combining the iterative algorithm and fractional programming, we further maximize the energy efficiency of the multi-subcarrier system. With our proposed algorithm, we can easily obtain an optimal transmit power that approximates the global optimal solution. Simulation results show that using the obtained optimal transmit power allocation algorithm can significantly improve the sum rate and energy efficiency in both single-carrier and multi-subcarrier systems.

Keywords: full-duplex two-way communication; power allocation; sum data rate; energy efficiency

1. Introduction

With the rapidly growing throughput requirements of user equipment (UE) and the ever-increasing number of connected devices, the next major breakthrough in wireless networking is proceeding at full speed. Full-duplex has been well studied because of the potential of improving the spectral efficiency [1,2]. However, the self-interference (SI) obstructs FD from being widely applied in practice [3]. Specifically, analyses presented in [4,5] showed that when the target spectral efficiency (SE) is high, SI deteriorates the performance of energy efficiency sharply. Along with the development of self-interference cancellation technologies, FD wireless transmission gradually becomes feasible and is well positioned to meet the urgent demand on transmission throughput [6]. Recent advances showed a great potential of canceling the SI up to the receiver noise floor [7–9]. The authors in [8] mentioned that 78 dB of interference suppression can be achieved by using multiple self-interference cancellation (SIC) techniques. Thus, the FD communication system has a huge application prospect.

Among the study of FD transmission, the FDTW communication system has attracted great attention from researchers. In [10], the authors studied the energy efficiency (EE) of two-way channels operating in the full-duplex mode, where the corresponding optimal power allocation is derived by using fractional programming. To improve the SE, the mode selection problem for FD-enabled two-way D2D communications is investigated in [11], where the authors maximized the system SE under the conditions of satisfying

the minimum rate and maximum power constraints of cellular and D2D users in the communication system. These works both considered the transmit power allocation to achieve higher energy efficiency or SE. This is because the transmit power of the node does not only affect its own transmit data rate, but also generates interference to receivers, leading to the decline of received SNR. Therefore, the power allocation strategy of the nodes plays the most critical role in FDTW communication system.

OFDM technology is widely used to reduce frequency selective fading in broadband communication systems, and the combination of FD technology can effectively improve the transmission rate of the system. The authors of [12] designed a resource allocation scheme for a multi-user FDTW system by rationally allocating available power and carriers, which greatly improves the maximum throughput under a single power constraint. The energy efficiency was also discussed in an OFDM cellular network with a FD relay [13]. The authors considered the joint bandwidth sharing and power allocation problem, and solved it as a three-stage Stackelberg game. In the work [14], a joint optimal power allocation and relay selection scheme is proposed to obtain the optimal transmit power and the cooperative relay under the given time switching factor. The authors considered two cases that the relay nodes harvest energy with single antenna and dual antennas.

The authors of [15] applied big data-based Apache Spark to investigate the 5G wireless communication network. Particularly, they studied the Apache Spark suppression filters and proposed the feedback adaptive filtering for relay systems with multiple-input multiple-out (MIMO) antennas. Another work [16] investigated the downlink outage performance of full-duplex (FD) multi-cell networks where interference cancellation is considered at the receivers. In particular, the authors of [16] put forward a practical scheme to cancel the interference when the power of the interference is very large. The energy efficiency of a decode-and-forward (DF) based standard relay channel was considered in Ref. [17], where the power of residual self-interference and the extra circuit power consumption are modeled as linearly increasing with the relay transmission power. The authors of [18] investigated the energy transmission and computation resource allocation problem in a federated learning-oriented wireless communication network. By adjusting the source precoder and the IRS phase-shift matrix, the authors of [19] maximized the system sum rate of a full-duplex MIMO two-way communication system. Specifically, by decoupling a complex nonconvex problem, the authors formed three subproblems for which closed-form solutions can be obtained.

Both OFDM and FD can improve system transmission spectrum efficiency, and they are widely studied in the sixth generation mobile communications. This is because both schemes can significantly improve data rate and energy efficiency. The authors of [20] proposed a two-step method in which the power is optimally assigned to each subcarrier firstly, and then the assigned power is allocated to the two nodes at each subcarrier. Resource allocation in a full-duplex OFDMA wireless network had been discussed by using matching theory in [21]. However, there is no research on combining the two techniques to improve system data rate and energy efficiency for a two-way communication system. In this paper, we focus on improving the sum data rate and energy efficiency of a FDTW system based on OFDM channel. In this FDTW system, the nodes are more tightly coupled in terms of power, which makes the self-interference of the system more complicated. Hence, it is more difficult to efficiently allocate the power such that a decent sum data rate and energy efficiency can be achieved. We first formulate the optimal problem of selecting optimal power allocation strategy to maximize sum data rate and energy efficiency in a FDTW communication system based on OFDM channel, and each transmit node should satisfy the different power constraints. Then, the fractional programming and iteration algorithm are applied to obtain a suboptimal transmit power strategy. In particular, the proposed algorithms can approach the global optimal solution and significantly improve the sum data rate and energy efficiency in the FDTW system with an OFDM channel. The main contributions of this work are summarized as follows:

(1) We consider a single-carrier FDTW system where the nodes have different power constraints. In order to investigate the maximization of the sum data rate and EE in this single-carrier FDTW system, we first analyze how the transmit power, carrier gain, and self-interference coefficient affect the sum data rate. We further optimize the non-convex problem about energy efficiency by applying fractional programming.

(2) We extend the studies of single-carrier FDTW system to multi-subcarrier FDTW system. We approach the optimal sum data rate of the system by proposing the cooperative iterative multi-subcarrier system algorithm (CMS). A joint fractional programming and iterative cooperative algorithm (FPIC) is proposed to achieve a suboptimal solution of the energy efficiency of multi-subcarrier system.

The rest of this paper is organized as follows. We introduce the system model in Section 2. Section 3 studies the power allocation in the single-carrier case for sum data rate and energy efficiency. Section 4 extends the studies to multi-subcarrier systems. Simulation results are presented in Section 5. Finally, in Section 6 we conclude this paper.

2. System Model

Let us consider a FDTW orthogonal frequency division multiplexing communication system consisting of two nodes N_1 and N_2, as shown in Figure 1. There are M parallel channels in the system, each of which uses a dedicated OFDM subcarrier with bandwidth W Hz. Besides this, when $M = 1$ the system is a single carrier communication system with only one channel; when $M > 1$ the system becomes a multi-subcarrier system with M subcarriers and each subcarrier is perfectly orthogonal to each other without inter-subcarrier interference.
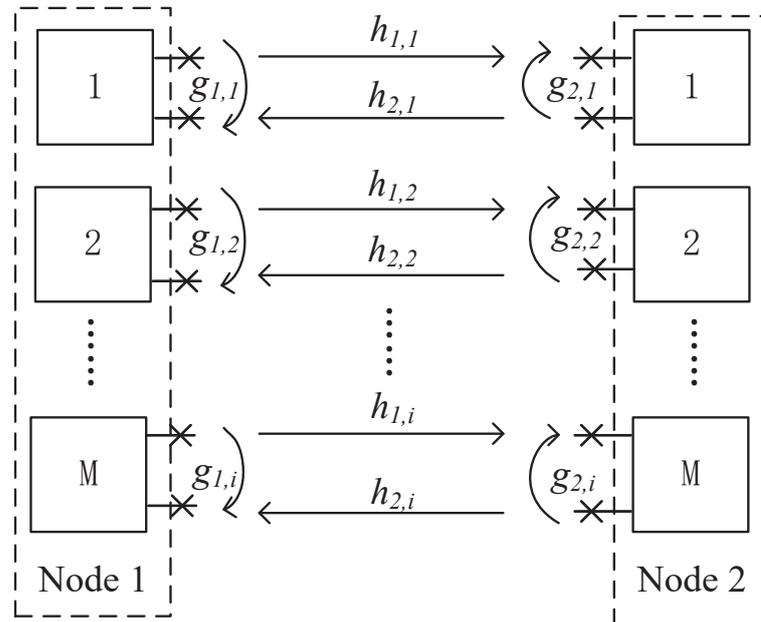
Figure 1. A full-duplex two-way communication system.

Each node receives and transmits its signal by using the M subcarriers simultaneously, which causes severe self-interference. After SI cancellation, we can obtain the signal for the i-th subcarrier as

$$Y_{1,i} = h_{1,i}\sqrt{P_{2,i}}X_{2,i} + g_{1,i}\sqrt{P_{1,i}}X_{1,i} + Z_i, \tag{1}$$

$$Y_{2,i} = h_{2,i}\sqrt{P_{1,i}}X_{1,i} + g_{2,i}\sqrt{P_{2,i}}X_{2,i} + Z_i, \tag{2}$$

where $X_{j,i}$ for $j = 1, 2$; $i = 1, 2, \cdots, M$ denotes the signal transmitted from node j over the i-th subcarrier, and the transmit power of node j over the i-th subcarrier is denoted as $P_{j,i}$ for $j = 1, 2$. The $h_{j,i}$ represents the subcarrier gain, and $g_{j,i}$ represents the coefficient of the i-th subcarrier from node j to node $-j$. The Z_i denotes additive white Gaussian noise received at node j over the i-th subcarrier. In this work we assume that the two nodes have the same SIC ability in each subcarrier, thus $g_{1,i} = g_{2,i} = g_i$. We consider normalized bandwidth W Hz for each subcarrier where the data rate of each subcarrier would be equal to the spectral efficiency of that subcarrier. Let N_0 denote the power density of the noise.

In this paper, we focus on how the transmission power affects energy efficiency of the system with different power constraints.

Firstly, let us formulate the sum data rate. The signal to interference and noise ratio (SINR) for the two nodes over the i-th subcarrier are respectively denoted as $\gamma_{1,i}$ and $\gamma_{2,i}$, i.e.,

$$\gamma_{1,i} = \frac{h_{1,i} \cdot P_{2,i}}{1 + g_{1,i} \cdot P_{1,i}}, \ \gamma_{2,i} = \frac{h_{2,i} \cdot P_{1,i}}{1 + g_{2,i} \cdot P_{2,i}}. \tag{3}$$

Let us further denote the sum data rate of the system as $R(P_{1,i}, P_{2,i})$. Then it can be computed by using the Shannon formula, $\gamma_{1,i}$ and $\gamma_{2,i}$.

$$R(P_{1,i}, P_{2,i}) = \sum_{i=1}^{M} W(\log_2(1 + \gamma_{1,i}) + \log_2(1 + \gamma_{2,i})). \tag{4}$$

Note that both the $\gamma_{1,i}$ and $\gamma_{2,i}$ are strongly related to the transmit power $P_{1,i}$ and $P_{2,i}$.

In the practical scenarios, the energy supply of the device is not a constant, or the energy supply does not sustain the continuous operation of the device with high power consumption. Different devices are limited by different energy constraints and a part of devices has demanding requirements about energy consumption. Both data rate and energy efficiency are important for those energy-constrained scenarios. Hence, we consider that the nodes in the FDTW system are working with different power constraints.

Through the above analyses, we can formulate the sum data rate maximization problem as $\mathcal{P}_1$:

$$\mathcal{P}_1: \quad \max_{P_{1,i}, P_{2,i}} R = \sum_{i=1}^{M} W(\log_2(1 + \gamma_{1,i}) \tag{5}$$
$$+ \log_2(1 + \gamma_{2,i})).$$

$$\text{s.t.} \ \sum_{i=1}^{M} P_{1,i} \leq P_1^{\max}, \tag{6}$$

$$\sum_{i=1}^{M} P_{2,i} \leq P_2^{\max}. \tag{7}$$

We assume that the devices of both nodes have the same power consumption denoted as P_c. Based on the formulation of R, the energy efficiency of a FDTW system which is denoted by η can be expressed as follows:

$$\eta = \frac{R}{\sum\limits_{i=1}^{M} (P_{1,i} + P_{2,i}) + 2 \cdot P_c}. \tag{8}$$

Therefore, let us consider the optimal problem of maximizing the EE as $\mathcal{P}_2$:

$$\mathcal{P}_2 : \max_{P_{1,i}, P_{2,i}} \eta = \frac{R}{\sum\limits_{i=1}^{M}(P_{1,i} + P_{2,i}) + 2 \cdot P_c}, \tag{9}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} P_{1,i} \leq P_1^{\max}, \tag{10}$$

$$\sum_{i=1}^{M} P_{2,i} \leq P_2^{\max}. \tag{11}$$

From $\mathcal{P}_1$ and $\mathcal{P}_2$, one can find that both $P_{1,i}$ and $P_{2,i}$ affect the sum data rate and energy efficiency, where are tightly coupled transmit power. In order to indicate how transmit power affects both R and η, let us first explore the the optimal problem in single-carrier system, and then extend the study to multi-subcarrier system.

3. Single Carrier System

To gain some insights on the optimal power allocation, let us first study the effect of node transmit power in a single-carrier system. Specifically, we assume that the two nodes have the same carrier gain and self-interference coefficient. We remove the subscript of the $h_{j,i}$ and $g_{j,i}$ for notational simplicity in this section. That is, $h_{1,i} = h_{2,i} = h$ and $g_{1,i} = g_{2,i} = g$. Without loss of generality, let us further assume that $P_1^{\max} \leq P_2^{\max}$.

3.1. Sum Data Rate Maximization

For a single-carrier system, it is given that $M = 1$. Then, one can rewrite the optimal problem $\mathcal{P}_1$ as $\mathcal{P}_{R_s}$ as follows:

$$\mathcal{P}_{R_s} : \max_{P_1, P_2} \quad R_s = \sum_{i=1}^{M} W\left(\log_2\left(1 + \frac{h \cdot P_2}{1 + g \cdot P_1}\right)\right.$$

$$\left. + \log_2\left(1 + \frac{h \cdot P_1}{1 + g \cdot P_2}\right)\right), \tag{12}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} P_1 \leq P_1^{\max}, \tag{13}$$

$$\sum_{i=1}^{M} P_2 \leq P_2^{\max}. \tag{14}$$

To solve $\mathcal{P}_{R_s}$, let us first analyze the derivative of R_s with respect to P_1 and P_2, which can be expressed as follows:

$$\frac{\partial R_s}{\partial P_1} =$$

$$\frac{hgP_2}{gP_2(hP_1 + 1 + gP_2)} - \frac{hgP_2}{(hP_2 + 1 + gP_1)(1 + gP_1)}, \tag{15}$$

$$\frac{\partial R_s}{\partial P_2} =$$

$$\frac{hgP_1}{gP_1(hP_2 + 1 + gP_1)} - \frac{hgP_1}{(hP_1 + 1 + gP_2)(1 + gP_2)}. \tag{16}$$

For conciseness, let us define the function φ_1 and φ_2:

$$
\begin{aligned}
\varphi_1 &= gP_2(hP_1 + 1 + gP_2) - (hP_2 + 1 + gP_1)(1 + gP_1) \\
&= (P_2 - 2P_1) + g^2(P_2^2 - P_1^2) - 1 - hP_2 \\
&= -g^2 P_1^2 - 2P_1 - 1 - hP_2 + g^2 P_2^2 + P_2,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\varphi_2 &= (P_1 - 2P_2) + g^2(P_1^2 - P_2^2) - 1 - hP_1 \\
&= (P_1 - 2P_2) + g^2(P_1^2 - P_2^2) - 1 - hP_1 \\
&= -g^2 P_2^2 - 2P_2 - 1 - hP_1 + g^2 P_1^2 + P_1.
\end{aligned}
\tag{18}
$$

By comparing Equations (15) and (17), Equations (16) and (18), one can find that φ_j would determine the monotonicity of the function $\frac{\partial R_s}{\partial P_j}$, i.e., if $\varphi_j < 0$, then $\frac{\partial R_s}{\partial P_j} > 0$; if $\varphi_j > 0$, then $\frac{\partial R_s}{\partial P_j} < 0$. According to Equation (18), it indicates that when $P_2 > P_1$, that is $P_2^2 > P_1^2$ and $2P_2 > P_1$, then $\varphi_2 < 0$, $\frac{\partial R_s}{\partial P_2} > 0$. This shows that R_s increases with P_2 when $P_2 > P_1$. Let P_1^* and P_2^* denote the best transmit power of N_1 and N_2 for system. Then we can obtain the conclusion that $R_s^* = R_s(P_1^*, P_2^{\max})$ when $P_1^{\max} < P_2^{\max}$.

Next, let us further find the P_1^* that achieves the largest sum data rate of the system. Recall that $P_1 \in [0, P_1^{\max}]$. Note that $\varphi_1 = 0$ is a quadratic equation regarding of P_1; therefore, we can obtain that

$$
\triangle = 4g^2(1 + gP_2^{\max} + (gP_2^{\max})^2 - hP_2 - 1),
\tag{19}
$$

where $\triangle$ denote the discriminant of quadratic equation of $\varphi_1 = 0$ w.r.t. P_1. We can find $\frac{\partial \varphi_1}{\partial P_1} = -2g - 2gP_1 < 0$ all the time if $P_1 \in [0, P_1^{\max}]$. Hence, there are two cases based on the sign of $\triangle$: $\triangle \geq 0$ or $\triangle < 0$.

In this case, $\triangle \geq 0$. Let us denote the positive solution of equation $\varphi_1 = 0$ as P_1'. We can obtain

$$
P_1' = \frac{\sqrt{4g^2(1 + gP_2^{\max} + (gP_2^{\max})^2 - hP_2 - 1)} - 1}{g}.
\tag{20}
$$

On the basis of Equation (17), one can find that $\varphi_1 > 0$ when $P_1 \in [0, P_1']$, $\varphi_1 < 0$ when $P_1 \in [P_1', P_1^{\max}]$. This indicates that $P_1^* = 0$ or $P_1^{\max}$. In order to determine the largest sum data rate, we define the function $\chi = R_s(0, P_2^{\max}) - R_s(P_1, P_2^{\max})$ to identify P_1^*. Through $\triangle \geq 0$, we can obtain that

$$
h \leq \frac{1 + gP_2^{\max} + (gP_2^{\max})^2}{P_2^{\max}} := h_0.
\tag{21}
$$

Let us denote the positive solution of $\chi = 0$ as P_1''. Then,

$$
P_1'' = \frac{6P_2^{\max} + 4hP_2^{\max}}{2 + hP_2^{\max}}.
\tag{22}
$$

Analyzing Equation (22) based on $P_1^{\max} < P_2^{\max}$ and $0 < h \leq h_0$, $P_1'' > P_2^{\max} > P_1^{\max}$ all the time. Therefore, if $h \leq h_0$ and $P_1^{\max} < P_2^{\max}$, then $P_1^* = 0$, and $P_2^* = P_2^{\max}$, and the optimal R_s achieves at $R_s^* = R_s(0, P_2^{\max})$.

In this case $\triangle < 0$, we can obtain the relationship between carrier gain h and h_0:

$$
h > \frac{1 + gP_2^{\max} + (gP_2^{\max})^2}{P_2^{\max}} := h_0.
\tag{23}
$$

Since $\triangle < 0$, the sum data rate increases as P_1 increases. Therefore, if $h > h_0$, then $P_1^* = P_1^{\max}$, and $P_2^* = P_2^{\max}$, and the optimal R_s achieves at $R_s^* = R_s(P_1^{\max}, P_2^{\max})$.

3.2. Energy Efficiency Maximization

In this section, we further maximize the energy efficiency of a single-carrier FDTW system. Specifically, in this case, $M = 1$. Let us obtain the problem degradation as $\mathcal{P}_{\eta_s}$ from $\mathcal{P}_2$ for a single-carrier FDTW system as follows:

$$\mathcal{P}_{\eta_s}: \quad \max_{P_1, P_2} \; \eta_s = \frac{R_s}{P_1 + P_2 + 2 \cdot P_c}, \tag{24}$$

$$\text{s.t.} \; P_1 \leq P_1^{\max}, \tag{25}$$

$$P_2 \leq P_2^{\max}. \tag{26}$$

Note from Equation (24) that the transmit power occurs in both the numerator and denominator, which indicates that the transmit power of different nodes are coupled with each other and difficult to be solved directly. Hence, let us optimize this non-convex problem by using fractional programming mentioned in [22–24]. The optimal solution of $\mathcal{P}_{\eta_s}$ can be achieved if and only if

$$R_s^* - \eta_s^* \cdot P_j^* = 0, \tag{27}$$

where R_s^* denotes the optimal sum data rate, P_j^* denote the corresponding optimal transmit power, and η_s^* is the maximum energy efficiency of $\mathcal{P}_{\eta_s}$ [25].

Let us define $F(\eta_s) = R_s - \eta_s P_j$, and we can easily find the optimal EE η_s^* by using bisection search because the function $F(\eta_s)$ is monotonically decreasing. Furthermore, the equation $F(\eta_s) = R_s^* - \eta_s^* \cdot P_j^* = 0$ can be obtained when (27) follows and the optimal EE is achieved. In each searching step, we solve the following problem to get the optimal transmit power with a initial energy efficiency (η_{mid}).

$$\mathcal{P}_3: \quad \max_{P_j} \; F(\eta_s), \tag{28}$$

$$\text{s.t.} \; P_j \leq P_j^{\max}. \tag{29}$$

The objective function has a convex feasible region and is continuous with respect to P_j. Therefore, the maximum value of $F(\eta_s)$ can be found when transmit power meet the criteria (29). In a single-carrier FDTW system, to obtain the optimal transmit power is a convex problem for any given η_s as an initial η_{mid}. Hence, by several iterations, we can obtain the optimal transmit power P_j^* and energy efficiency η_s^* until $F(\eta_s^*) = 0$.

We design the Algorithm 1 based on above analyses to search the optimal energy efficiency (η_s^*).

Algorithm 1 Fractional programming algorithm for $\mathcal{P}_{\eta_s}$

Input: The initial energy efficiency η_{mid} which is defined on the closed interval [a,b], the margin of error δ.

Output: the optimal energy efficiency η_s^*.

1: Initializing system parameters.

2: $j \leftarrow 1$.

3: Find optimal transmit power P_j^{temp} by standard convex optimization algorithm and get $F^{\max}(\eta_{mid})$.

4: **if** $F^{\max}(\eta_{mid}) < 0$. **then**

5: 　　$a \leftarrow \eta_{mid}$.

6: **else**

7: 　　$b \leftarrow \eta_{mid}$.

8: **end if**

9: **if** $F^{\max}(\eta_{mid}) = 0$ or $|\eta_{mid} - (a+b)/2| < \delta$. **then**

10: 　　$\eta_s^* \leftarrow \eta_{mid}$, $P_j^* \leftarrow P_j^{\text{temp}}$, $j \leftarrow 2$.

11: 　　Go to 3, and take the same steps for the other node.

12: **else**

13: 　　$\eta_{mid} \leftarrow (a+b)/2$.

14: 　　Go to 4.

15: **end if**

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4. Multi-Subcarrier System

In this section, we analyze the sum data rate and EE of a FDTW system with multi-subcarrier. Since the number of subcarrier increases, the optimal power allocation problem becomes more complicated.

4.1. Sum Data Rate Maximization

For the multi-subcarrier condition, let us assume that different nodes can share their system information which have been mentioned in the last section. Nodes can send their messages and consider how much impact they made for this system, such as sum data rate, SI. Furthermore, Nodes can choose an appropriate transmit power to achieve a balance between higher transmit data rate and stronger self-interference. The data rate of each node is jointly influenced by the matching full-duplex two-way nodes. We can rewrite the optimization problem $\mathcal{P}_1$ for this condition as follows:

$$\mathcal{P}_{R_m}: \quad \max_{P_{j,i}, P_{-j,i}} \quad R_m(P_{j,i}, P_{-j,i}), \tag{30}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} P_{j,i} \le P_j^{\max}, \tag{31}$$

$$\sum_{i=1}^{M} P_{-j,i} \le P_{-j}^{\max}. \tag{32}$$

A node will consider the other's transmit strategy to select the optimal transmit power, but it does not know the transmit strategy chosen by the other node. Let us first define a Lagrange function to solve the problem based on $\mathcal{P}_{R_m}$:

$$L_2(P_{j,i}, P_{-j,i}, \lambda_j, \lambda_{-j}) = R_m$$
$$- \lambda_j (\sum_{i=1}^{M} P_{j,i} - P_j^{\max}) - \lambda_{-j} (\sum_{i=1}^{M} P_{-j,i} - P_{-j}^{\max}). \tag{33}$$

One should note that the solution of $\mathcal{P}_{R_m}$ is following the equations through Equation (33), as follows:

$$\frac{\partial L_2(P_{j,i}, P_{-j,i}, \lambda_j, \lambda_{-j})}{\partial P_{j,i}} = 0, \tag{34}$$

$$\frac{\partial L_2(P_{j,i}, P_{-j,i}, \lambda_j, \lambda_{-j})}{\partial P_{-j,i}} = 0. \tag{35}$$

Because Equations (34) and (35) are transcendental equations, and the transmit power of two nodes are coupled together, it is difficult to obtain the closed form solution as $P_{j,i}^*$ of the two nodes' best transmit power at the same time. We propose a cooperative iterative multi-subcarrier scheme (CMS) to obtain suboptimal R_m.

We assume that node selects the transmit strategy through the previous decisions of the other node; hence, we can obtain the optimal transmit power in each round iteration process as $P_{j,i}^{(n)}$ as follows:

$$P_{j,i}^{(n)} = \left[\frac{\sqrt{b_{j,i}^2 - 4a_{j,i}(c_{j,i} - \lambda_j)} - b_{j,i}}{2a_{j,i}} \right]^+, \forall i. \tag{36}$$

In Equation (36), the parameters are defined as

$$\begin{cases} a_{j,i} = 2\lambda_j h_{j,i} g_i, \\ b_{j,i} = \lambda_j h_{j,i} + \lambda_j g_i + \lambda_j g_i^2 P_{-j,i}^{(n-1)}, \\ c_{j,i} = \lambda_j g_i P_{-j,i}^{(n-1)} + g_i^2 P_{-j,i}^{(n-1)} + \lambda_j - h_{j,i}. \end{cases} \tag{37}$$

We fix transmit power at the beginning and obtain the other node's transmit power, then repeat the same operation for the other node as shown in Figure 2. We search for an optimal transmit power $P_{1,i}^{(1)}$ with (34) satisfied, by given initial transmit power $P_{2,i}^{(0)}$ of N_2. By substituting $P_{1,i}^{(1)}$ into (35), we can obtain a $P_{2,i}^{(1)}$ which satisfies (35) through using a standard convex optimization algorithm. After that, iteratively substituting $P_{2,i}^{(1)}$ into (34), one can obtain a new optimal power for N_1, $P_{1,i}^{(2)}$. Then repeat in this way until $|P_{1,i}^{(n)} - P_{1,i}^{(n-1)}| < \delta$ and $|P_{2,i}^{(n)} - P_{2,i}^{(n-1)}| < \delta$. The suboptimal transmit strategy can be obtained after n-round iteration to approach the suboptimal solution as $(P_{1,i}^*, P_{2,i}^*)$. This iterative algorithm greatly reduces the complexity of the calculation.
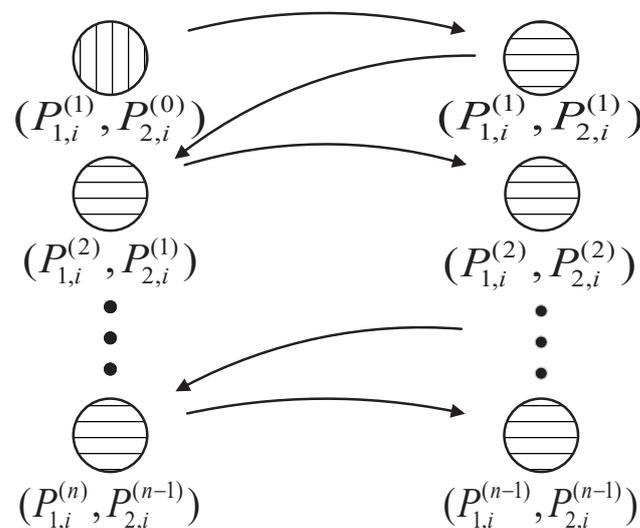
Figure 2. The iterative process of Algorithm 2.

Algorithm 2 Search for the optimal solution of $\mathcal{P}_{R_m}$

Input: Maximum transmit power $P_j^{\max}$, algorithm total iteration times N, the initial trans-
   mit power at i-th subcarrier of j-th node $P_{j,i}^{\text{temp}}$, the margin of error δ.

Output: optimal transmit power $P_{1,i}^*$ and $P_{2,i}^*$.

1: Initializing system parameters.
2: $n \leftarrow 1$.
3: Obtaining the optimal transmit power $P_{1,i}^{(n)}$ according to Equation (36).
4: **if** $|P_{1,i}^{(n)} - P_{1,i}^{(n-1)}| < \delta$ & $|P_{2,i}^{(n)} - P_{2,i}^{(n-1)}| < \delta$ **then**
5:    **return** $P_{1,i}^* \leftarrow P_{1,i}^{(n)}, P_{2,i}^* \leftarrow P_{2,i}^{(n-1)}$
6: **end if**
7: Obtaining the optimal transmit power $P_{2,i}^{(n)}$ according to Equation (36).
8: **if** $|P_{1,i}^{(n)} - P_{1,i}^{(n-1)}| < \delta$ & $|P_{2,i}^{(n)} - P_{2,i}^{(n-1)}| < \delta$ **then**
9:    **return** $P_{2,i}^* \leftarrow P_{2,i}^{(n-1)}, P_{1,i}^* \leftarrow P_{1,i}^{(n)}$
10: **else**
11:   **if** $n < N$ **then**
12:     $n \leftarrow n + 1$, go to 3.
13:   **end if**
14: **end if**

We design Algorithm 2 based on these analyses to search the suboptimal solution at $(P_{1,i}^*, P_{2,i}^*)$. The suboptimal sum data rate could be achieved as $R_m(P_{1,i}^*, P_{2,i}^*)$.

4.2. Energy Efficiency Maximization

Through our analyses in Section 3, we have used fractional programming to optimize the single-carrier energy efficiency problem, and the iterative scheme mentioned in the previous section to achieve the optimal sum data rate. We rewrite the optimal problem $\mathcal{P}_2$ for multi-subcarrier condition as $\mathcal{P}_{\eta_m}$.

$$\mathcal{P}_{\eta_m}: \quad \max_{P_{1,i}, P_{2,i}} \quad \eta_m = \frac{R_m}{\sum\limits_{i=1}^{M} P_{1,i} + P_{2,i} + 2 \cdot P_c}, \tag{38}$$

$$\text{s.t.} \quad \sum_{i=1}^{M} P_{1,i} \leq P_1^{\max}, \tag{39}$$

$$\sum_{i=1}^{M} P_{2,i} \leq P_2^{\max}. \tag{40}$$

As the number of subcarriers M increases, the transmit power between the subcarriers is more tightly coupled, and it becomes more difficult to solve $\mathcal{P}_{\eta_m}$. Therefore, we propose the combining fractional programming and iterative cooperative algorithm (FPIC). The iterative cooperation scheme is used to reduce the complexity of a single calculation in the multi-subcarrier system, and the non-convex problem of energy efficiency is solved by fractional programming. By using the proposed FPIC scheme, the sub-optimal solution is continuously approached to the optimal solution of the system.

In our proposed FPIC scheme, we first assume that N_2 allocates equal-power $P_{2,i}^{\text{temp}}$ in each subcarrier for N_1 in the beginning. The optimal transmit power $P_{1,1}^{\text{temp}}$ and η_m^{temp} of the first subcarrier can be found by applying fractional programming with a given initial η_{mid}. Then, let η_m^{temp} be the η_{mid} for next subcarrier, we can achieve the $P_{1,2}^{\text{temp}}$ and refresh the η_m^{temp}. Until all subcarriers of N_1 are optimized, we note the latest η_m^{temp} as $\eta_m^{*(1)}$ and $P_{1,i}^{\text{temp}}$ as $P_{1,i}^{*(1)}$. The N_2 can obtain the transmit power $P_{1,i}^{\text{temp}}$ and the latest η_m^{temp}. Then, we fix the transmit power of N_1 to $P_{1,i}^{*(1)}$. Hence, N_2 can repeat the same optimal progress as N_1

for each subcarrier to refresh η_m^{temp} and transmit power $P_{2,i}^{\text{temp}}$. Until all subcarriers of N_2 are optimized, we note the latest η_m^{temp} as $\eta_m^{*(2)}$ and $P_{2,i}^{\text{temp}}$ as $P_{2,i}^{*(2)}$, and substitute them to N_1. Repeating in this way after n-round of iteration until $\eta_m^{*(j)}$ is converges, we can achieve the optimal energy efficiency η_m^*.

We design Algorithm 3 based on the above analyses to search for the optimal energy efficiency (η_m).

Algorithm 3 Search for the optimal solution of $\mathcal{P}_{\eta_m}$

Input: Maximum transmit power $P_j^{\max}$, algorithm total iteration times N, the initial transmit power at i-th subcarrier of j-th node $P_{j,i}^{\text{temp}}$, the initial iteration energy efficiency η_{mid}.

Output: Energy efficiency η_m^*.
1: Initializing system parameters.
2: $n \leftarrow 1, j \leftarrow 1$.
3: For each subcarrier of j-th node, obtaining the optimal transmit power at $P_{j,i}^{(n)}$ and the temporary optimal energy efficiency η_m^{temp} by Algorithm 1.
4: $\eta_{mid} \leftarrow \eta_m^{\text{temp}}$ and $P_{j,i}^{\text{temp}} \leftarrow P_{j,i}^{*(n)}$
5: **if** Computed all subcarrier of j-th node **then**
6:   $\eta_m^{*(j)} \leftarrow \eta_m^{temp}, \eta_{mid} \leftarrow \eta_m^{temp}, P_{j,i}^{*(j)} \leftarrow P_{j,i}^{temp}$.
7:   $j \leftarrow 2$, go to 3.
8: **end if**
9: **if** $n < N$ **then**
10:   $n \leftarrow n + 1$, go to 3.
11: **else**
12:   $\eta_m \leftarrow \eta_m^{*(j)}$
13: **end if**

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5. Simulation Results

In this section, we provide some numerical simulations of our proposed theory. Firstly, we analyze and compare the sum data rate of single-carrier system R_s under different assumptions of h. Then, we analyze the R_m among the proposed cooperative iterative multi-subcarrier scheme (CMS), the evenly distributed power multi-subcarrier scheme (EDMS) which be as the baseline, and the non-cooperative iterative multi-subcarrier scheme (NCMS) where each node distributes the power for each subcarrier through the water-filling method. Then, we compare the energy efficiency of the multi-subcarrier FDTW system when applying different schemes, i.e., EDMS, FPIC, and Max Value, which means that the multi-subcarrier system achieves the optimal theoretical value of EE. Finally, we analyze how the η_m achieved by the proposed FPIC scheme varies with the transmit power of each node.

Figure 3 shows how the sum data rate varies with the transmit power in the single-carrier with different single-carrier gains, e.g., $h = h_0/4$, $h = h_0/2$, $h = 2h_0$ or $h = 5h_0$. When h takes a low value, e.g., $h \leq h_0$, the R_s decreases first along with the increase of P_1. This manifests that there exists an optimal transmit power that achieves the best sum data rate. Furthermore, the system with higher h, e.g., $h = 2h_0$ or $h = 5h_0$, can obtain a higher sum data rate. This indicates that higher transmit power is not suitable to achieve a better sum data rate due to strong self-interference and poor single-carrier condition.
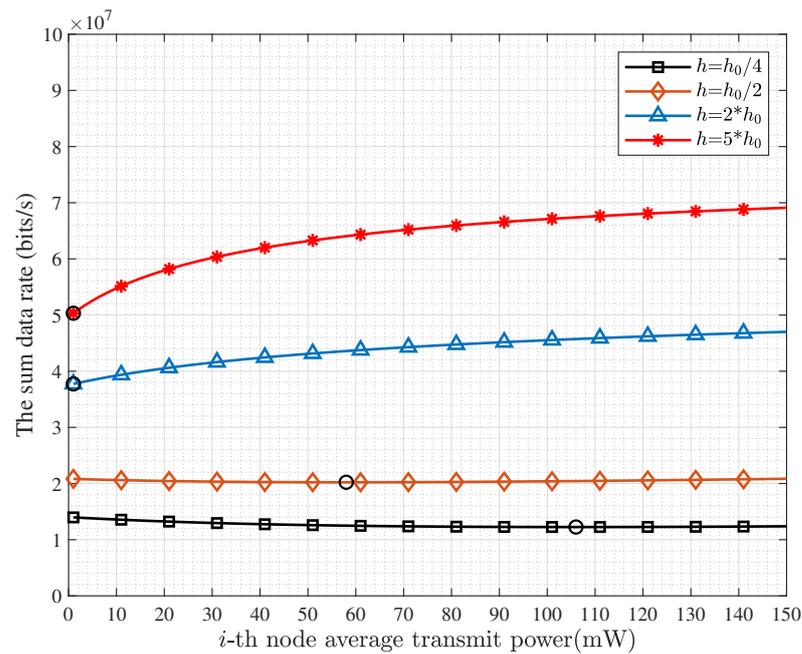
Figure 3. The R_s versus power of N_1 under different h.

Then, we compare the system sum data rate with transmit power between the CMS, NCMS and EMDS which are introduced at the beginning of Section 5 in Figure 4. We can see that both the CMS and NCMS schemes are able to improve the sum data rate of the system compared with the EDMS solution. Furthermore, because the CMS scheme can consider both the data rate gain and the interference caused by both sides when allocating power, it can have a maximum rate improvement of 10% at the same transmitting power.
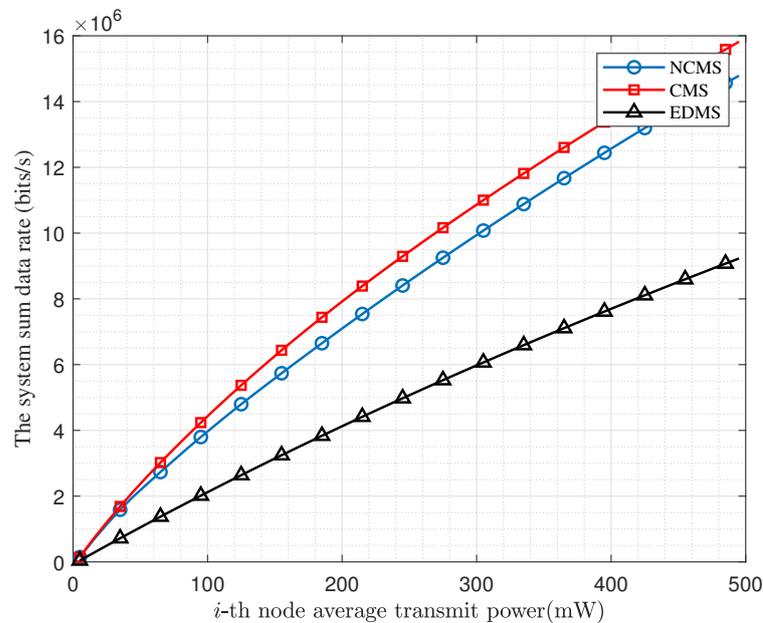
Figure 4. The R_m under different conditions.

Next, we compare the η_m of the FPIC scheme to that of the EDMS scheme in Figure 5. Along with the increase of N, the energy efficiency η_m of FPIC increases rapidly first, then stabilizes when $N = 6$. When the FPIC scheme has optimized the second subcarrier, that is $N = 2$, it can already achieve better energy efficiency than the EDMS scheme.
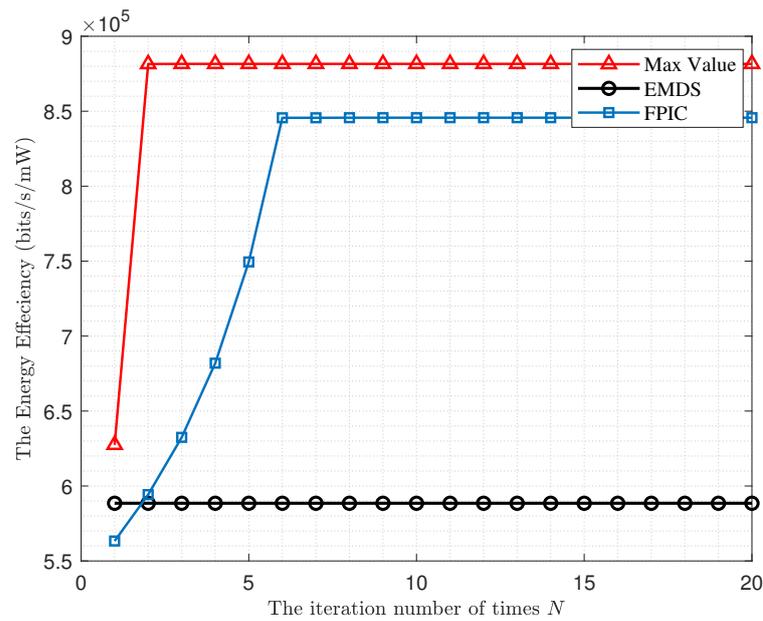
Figure 5. The η_m under different conditions versus iteration number of times N.

Finally, we compare the FPIC scheme and the EDMS scheme in Figure 6. In the simulation for η_m varies with $P_1^{\max}$ over different $P_2^{\max}$, we fix $P_2^{\max} = 200$ mW or 300 mW. Along with the increase of $P_1^{\max}$, the blue square curve and black circular curve represent the EDMS scheme, where they first gradually reach the optimal η_m and then gradually decrease. This shows that the blindly random allocation of power will lead to worse interference, which greatly reduces the energy efficiency of the FDTW system. This disadvantage does not occur in the proposed FPIC scheme. For maximizing energy efficiency, the algorithm maintains good consistency in the overflowed transmit power constraint system. The two transmitting nodes continuously adjust the transmission power through iterative cooperation to ensure that the transmit power of the system is always sub-optimal. Even if N_1 and N_2 both have sufficient energy supply, and neither party will selfishly increase its transmit power. The iterative and cooperation thoughts of the proposed FPIC scheme are well verified.
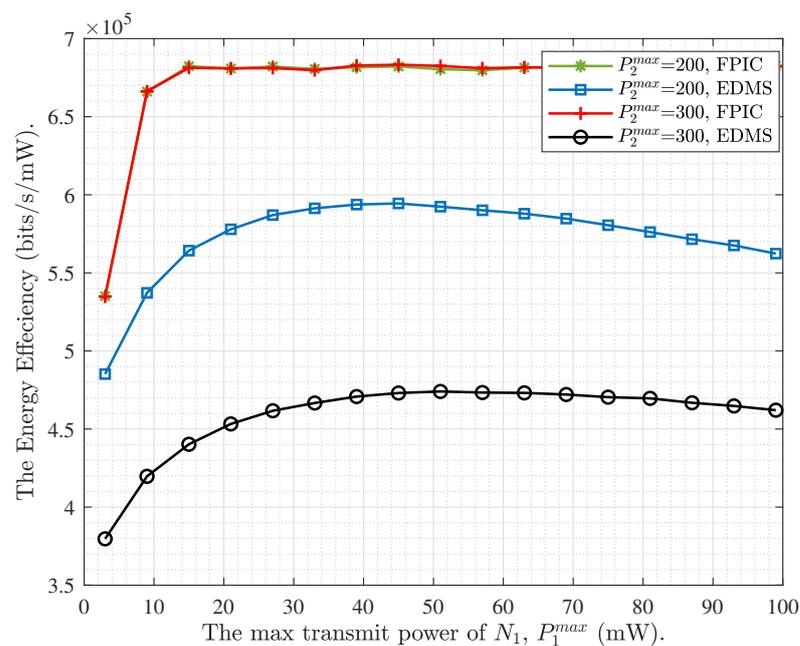
Figure 6. The η_m versus power of N_1 under different $P_2^{\max}$.

6. Conclusions

This work discussed the sum data rate and energy efficiency optimal for a full-duplex two-way communication system. We formulated and analyzed the optimization problem of the sum data rate and energy efficiency for single-carrier and multi-subcarrier conditions. Then, with the CMS and FPIC schemes, we achieved the optimal value of R and η of nodes in the energy-constrained FDTW system. By comparing these power allocation schemes, it is found that FPIC can improve EE both in single-carrier and multi-subcarrier conditions. Furthermore, under the multi-subcarrier cases, our CMS scheme can greatly increase the sum data rate of the system. Moreover, with our proposed FPIC and CMS schemes, It can easily obtain the optimal transmit power for each communication node in the system without a complicated solving process.

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