

Supplementary Materials: Theoretical Design of a Depolarized Interferometric Fiber-Optic Gyroscope (IFOG) on SMF-28 Single-Mode Standard Optical Fiber Based on Closed-Loop Sinusoidal Phase Modulation with Serrodyne Feedback Phase Modulation Using Simulation Tools for Tactical and Industrial Grade Applications

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1. Calculations of Noise Sources

Contribution Due Only to Photon-Shot-Noise (Thresold Sensitivity) before Correction

$$I_{sn}^2 = \frac{e^2 q \lambda}{h c} P_{\max-\text{detector}} \Delta f = \frac{(1.6 \times 10^{-19})^2 \times 0.65 \times (1310 \times 10^{-9})}{6.624 \times 10^{-34} \times (3 \times 10^8)} \times (100 \times 10^{-6}) \times 1 = 1.096940 \times 10^{-23} \text{ A}^2$$

then we have:

$$I_{sn} = \sqrt{1.096940 \times 10^{-23}} = 3.312008 \times 10^{-12} \text{ A}$$

$$\Delta\Omega = \Omega_{\text{lim}} \cong \left(\frac{hc^2}{\pi e q L D P_{\max}} \right) I_{sn} = \left[\frac{6.624 \times 10^{-34} \times (3 \times 10^8)^2}{\pi \times (1.6 \times 10^{-19}) \times 0.65 \times 300 \times 0.08 \times (100 \times 10^{-6})} \right] \times 3.312008 \times 10^{-12} =$$

$$\approx 2,5180234 \times 10^{-7} \text{ [rad / s]} = 2,5180234 \times 10^{-7} \times \left(\frac{180^\circ}{\pi} \right) \times \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 0,05193796 \left[\frac{^\circ}{\text{h}} \right]$$

Contribution Due Only to Excess Relative-Intensity-Noise (Rin) before Correction(*)

$$\Delta\Omega = \frac{2}{K_0} \sqrt{\frac{\lambda^2}{4c\Gamma t}} = \frac{1}{K_0} \frac{\lambda}{\sqrt{c\Gamma t}} \cong \frac{1}{0.3837} \frac{1310 \times 10^{-9}}{\sqrt{(3 \times 10^8) \times (30 \times 10^{-9}) \times 1}} \cong 1.1387 \times 10^{-6} \left[\frac{\text{rad}}{\text{s}} \right] \approx 0.2349 \left[\frac{^\circ}{\text{h}} \right]$$

Full Contribution Due to Photon-Shot-Noise + Excess Relative-Intensity-Noise (Rin) before Correction (*)

$$\Delta\Omega = \frac{2}{K_0} \sqrt{\frac{e}{P_d R t} + \frac{\lambda^2}{4c\Gamma t}} = \frac{2}{0.3837} \sqrt{\frac{1.60 \times 10^{-19}}{(145.61 \times 10^{-6}) \times 0.68678 \times 1} + \frac{(1310 \times 10^{-9})^2}{4 \times (3 \times 10^8) \times (30 \times 10^{-9}) \times 1}} =$$

$$\cong 1.1567 \times 10^{-6} \left[\frac{\text{rad}}{\text{s}} \right] \approx 0.2386 \left[\frac{^\circ}{\text{h}} \right] \quad (1)$$

(*) here:

$$K_0 = \text{open-loop scale-factor (calculated by } K_0 = \frac{2\pi L D}{\lambda c} \text{)}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

P_d = average power at the detector = 145.61 μW

R = responsivity (InGaAs photodetector) $\cong 0.68678 \mu\text{A}/\mu\text{W}$

t = averaging time $\cong 1 \text{ s}$

$$\lambda = 1310 \text{ nm}$$

c = speed of light in vacuum $\approx 3 \times 10^8 \text{ m/s}$

Γ = spectral linewidth of broadband source = $(\Delta f)_{FWHM} \approx 30 \text{ nm}$ for $\lambda = 1310 \text{ nm}$

2. Calculations of Noise Sources (Continuation)

Contribution due only to Photon-Shot-Noise (threshold sensitivity) after correction

$$\Delta\Omega = \frac{2}{K_0} \sqrt{\frac{e}{P_d R t}} = \frac{2}{0.3837} \sqrt{\frac{1.60 \times 10^{-19}}{(145.61 \times 10^{-6}) \times 0.68678 \times 1}} \approx 0.2084941 \times 10^{-6} \left[\frac{\text{rad}}{\text{s}} \right] = 0.043 \left[\frac{\circ}{\text{h}} \right]$$

Contribution Due Only to Excess Relative-Intensity-Noise (Rin) after Correction

$$\Delta\Omega = \frac{2}{K_0} \sqrt{\frac{\lambda^2}{4c\Gamma t}} \times 0.063 \cong 1.1387 \times 10^{-6} \left[\frac{\text{rad}}{\text{s}} \right] \times 0.063 \approx 0.2349 \times 0.063 \left[\frac{\circ}{\text{h}} \right] = 0.015 \left[\frac{\circ}{\text{h}} \right]$$

Full Contribution Due to Photon-Shot-Noise + Excess Relative-Intensity-Noise (Rin) after Correction

$$\begin{aligned} \Delta\Omega &= \frac{2}{K_0} \sqrt{\frac{e}{P_d R t} + \frac{\lambda^2}{4c\Gamma t}} \cong \frac{2}{0.3837} \sqrt{\frac{1.60 \times 10^{-19}}{(145.61 \times 10^{-6}) \times 0.68678 \times 1} + \frac{(1310 \times 10^{-9})^2 \times 0.063^2}{4 \times (3 \times 10^8) \times (30 \times 10^{-9}) \times 1}} \\ &\cong 0.24267 \times 10^{-6} \left[\frac{\text{rad}}{\text{s}} \right] \approx 0.0501 \left[\frac{\circ}{\text{h}} \right] \end{aligned}$$

3. Estimations (from Simulation Results)

Threshold Sensitivity (Estimated from SNR Measurement at the Detector)

$$SNR = 10 \log \left(\frac{S}{N} \right) \Rightarrow N = S \times 10^{-\frac{SNR}{10}}$$

$$N = 103.805 \times 10^{-\frac{190.162192}{10}} \approx 1 \times 10^{-17} \mu\text{W}$$

$$(P_{\text{detector}} = 103.805 \mu\text{W}, SNR = -190.162192 \text{ dB})$$

$$I_{\text{source-noise}} = 0.68678 \times 1 \times 10^{-17} \mu\text{A} = (6.8678 \times 10^{-6}) \times 10^{-12} \text{ A}$$

$$I_{\text{sn}} = 3.312008 \times 10^{-12} \text{ A (formerly calculated)}$$

$$I_{\text{dark}} = 0.00001 \text{ nA} = 0.01 \times 10^{-12} \text{ A}$$

$$\Omega_{\text{lim}} \cong \left(\frac{hc^2}{\pi e q L D P_{\text{max-detector}}} \right) \sqrt{I_{\text{sn}}^2 + I_{\text{source-noise}}^2 + I_{\text{dark}}^2} =$$

$$= \left[\frac{6.624 \times 10^{-34} \times (3 \times 10^8)^2}{\pi \times (1.6 \times 10^{-19}) \times 0.65 \times 300 \times 0.08 \times (100 \times 10^{-6})} \right] \times \sqrt{(3.312008 \times 10^{-12})^2 + (6.8678 \times 10^{-18})^2 + (1 \times 10^{-14})^2} \approx$$

$$\approx 2.518034848 \times 10^{-7} \left[\frac{\text{rad}}{\text{s}} \right] = 2.518034848 \times 10^{-7} \times \left(\frac{180^\circ}{\pi} \right) \times \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = 0.051938197 \left[\frac{\circ}{\text{h}} \right]$$

Dynamic Range (for $\phi_s = \pm \frac{\pi}{6}$)

$$\Omega_{\text{max}} = \frac{\lambda c}{12 L D} = \frac{(1310 \times 10^{-9}) \times (3 \times 10^8)}{12 \times 300 \times 0.08} = 1.364583 \frac{\text{rad}}{\text{s}} = 1.364583 \frac{\text{rad}}{\text{s}} \times \frac{180^\circ}{\pi \text{ rad}} = 78.185 \left[\frac{\circ}{\text{s}} \right]$$

$$\Omega_{\text{min}} \approx \frac{\sqrt{h L_b}}{L D} = \frac{\sqrt{10^{-5} \times (7.8 \times 10^{-3})}}{300 \times 0.08} = 1.16369 \times 10^{-5} \frac{\text{rad}}{\text{s}}$$

$$DR = 20 \log \left(\frac{\Omega_{\text{max}}}{\Omega_{\text{min}}} \right) = 20 \log \left(\frac{1.364583}{1.16369 \times 10^{-5}} \right) = 101.38 \text{ dB}$$

Open-Loop Scale Factor

$$K_0 = SF = \frac{2\pi LD}{\lambda c} = \frac{2\pi \times 300 \times 0.08}{(1310 \times 10^{-9}) \times (3 \times 10^8)} = 0.3837 \text{ s}$$

for $\phi_s = \pm \frac{\pi}{6}$ due to sine-function non-linearity, we have:

$$(SF)_{\text{non-linearity}} = \frac{2\pi LD}{\lambda c} \times \frac{\sin\left(\frac{\pi}{6}\right)}{\left(\frac{\pi}{6}\right)} = \frac{2\pi \times 300 \times 0.08}{(1310 \times 10^{-9}) \times (3 \times 10^8)} \times \frac{\sin\left(\frac{\pi}{6}\right)}{\left(\frac{\pi}{6}\right)} = 0.3837 \text{ s} \times 0.954929658 = 0.3664 \text{ s}$$