Supplement information
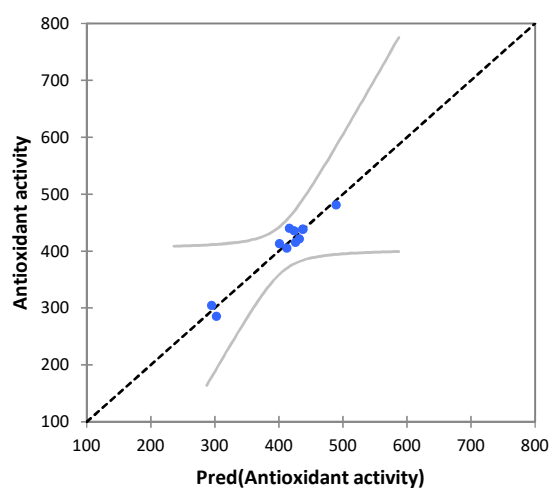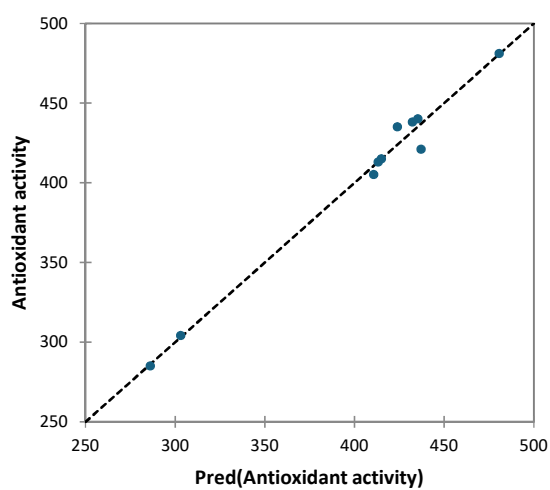

MLR

MNLR

Figure S1. displays a validation graph depicting the predicted antioxidant activity of the marine microalga *I. galbana.*
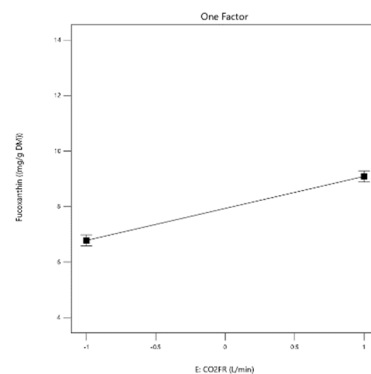
Figure S2. Illustrates the representations of fucoxanthin across one-dimensional, two-dimensional, and three-dimensional formats.

Figure S3. Calibration curve of fucoxanthin by HPLC-UV.
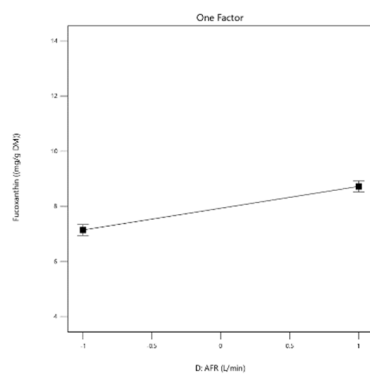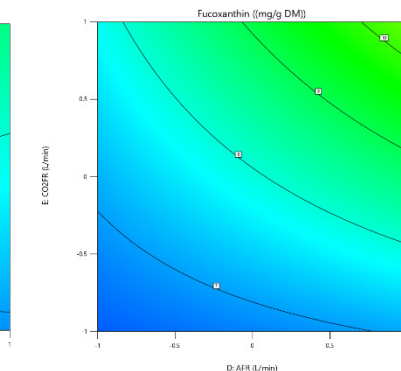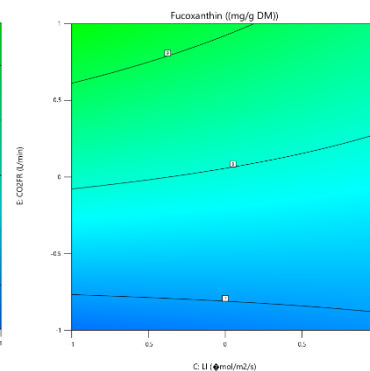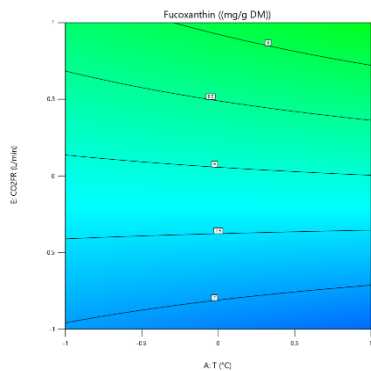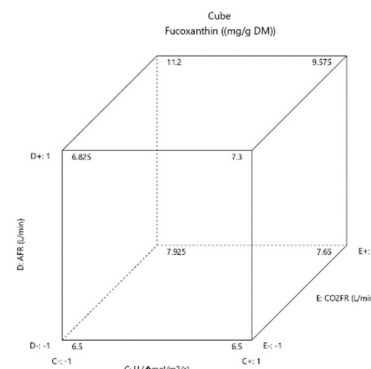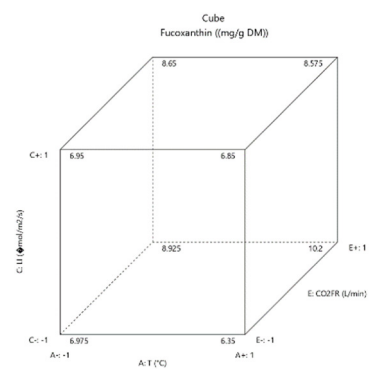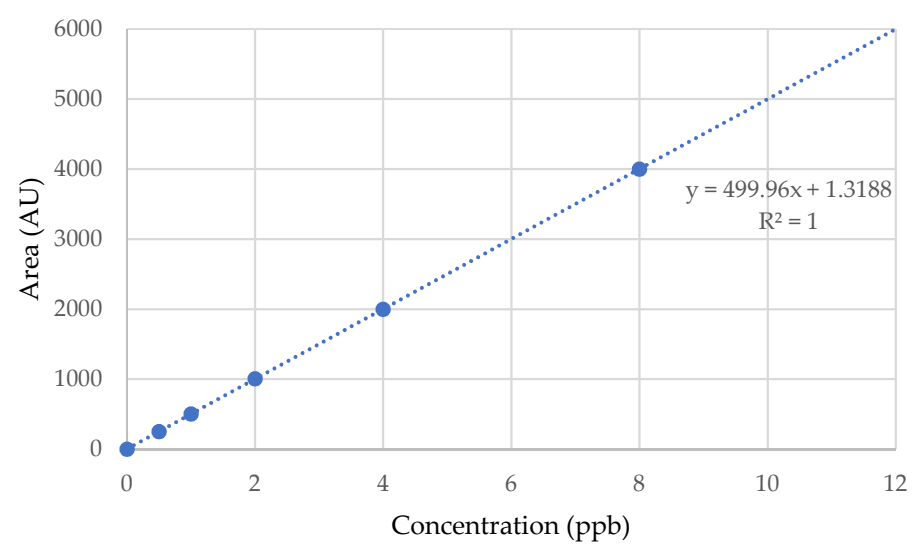
Table S1. Statistical parameters of the multiple regression of antioxidant activity as a function of the content of carotenoids, total phenolic compounds and fucoxanthin in cultures of the microalga *I. galbana*.

Parameters	MLR	MNLR
Equation	$Z1 = 533.08 + (5.27 \times Z2) + (4.51 \times Z3) - (53.61 \times Z4)$	$Z1 = 2343.15 - (272.13 \times Z2) - (96.80 \times Z3) + (293.17 \times Z4) + (10.49 \times Z2^2) + (2.63 \times Z3^2) - (29.94 * Z4^2)$
R^2	0.954	0.986
MCE	262.950	156.573
RMCE	16.216	12.513

Table S2. Fucoxanthin is used for each test in all experiments according to the design of the experiments.

Culture microalgae	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Response
Parameters	A:T	B:pH	C:LI	D:AFR	E:CO₂FR	Fucoxanthin
Unit	°C		$\mu mol/m^2/s$	L/min	L/min	(mg/g)
1	-1	-1	-1	-1	-1	7.8
2	1	-1	-1	-1	-1	6.2
3	-1	1	-1	-1	-1	5.8
4	1	1	-1	-1	-1	6.2
5	-1	-1	1	-1	-1	6.7
6	1	-1	1	-1	-1	6
7	-1	1	1	-1	-1	6.5
8	1	1	1	-1	-1	6.8
9	-1	-1	-1	1	-1	6.9
10	1	-1	-1	1	-1	6.2
11	-1	1	-1	1	-1	7.4
12	1	1	-1	1	-1	6.8
13	-1	-1	1	1	-1	7.2
14	1	-1	1	1	-1	7.2
15	-1	1	1	1	-1	7.4
16	1	1	1	1	-1	7.4
17	-1	-1	-1	-1	1	7.9
18	1	-1	-1	-1	1	7.9
19	-1	1	-1	-1	1	8.1
20	1	1	-1	-1	1	7.8
21	-1	-1	1	-1	1	7.7
22	1	-1	1	-1	1	7.6
23	-1	1	1	-1	1	7.7
24	1	1	1	-1	1	7.6
25	-1	-1	-1	1	1	10.5
26	1	-1	-1	1	1	13.4
27	-1	1	-1	1	1	9.2
28	1	1	-1	1	1	11.7
29	-1	-1	1	1	1	9.7
30	1	-1	1	1	1	9
31	-1	1	1	1	1	9.5
32	1	1	1	1	1	10.1

Table S3. Coefficients of optimized models for fucoxanthin production.

Factor	Coefficient Estimate
Intercept	7.930
A-T	0.059
B-pH	-0.059
C-LI	-0.178
D-AFR	0.790
E-CO$_2$FR	1.150
AB	0.115
AC	-0.103
AD	0.190
AE	0.240
BC	0.178
BD	0.021
BE	-0.065
CD	-0.109
CE	-0.296
DE	0.509
ABC	0.028
ABD	-0.053
ABE	-0.078
ACD	-0.159
ACE	-0.234
ADE	0.171
BCD	0.021
BCE	0.059
BDE	-0.159
CDE	-0.228
ABCD	0.071
ABCE	0.096
ABDE	0.128
ACDE	-0.190
BCDE	0.228

<p style="text-align:center">Table S4. Statistical studies of the model.</p>

Mean	7.930	**R²**	0.99
standard deviation	0.088	**Adjusted R²**	0.99
Variance	0.007	**Predicted R²**	0.91
Coefficient of variation	1.11%	**Adequate precision P**	87.36%

<p style="text-align:center">Table S5. ANOVA for the selected factorial model.</p>

Source	Sum of Squares	df	Mean Square	F-value	p-value	Significance
Model	90.50	30	3.02	386.15	0.04	Significant
A-T	0.11	1	0.11	14.44	0.16	Not significant
B-pH	0.11	1	0.11	14.44	0.16	Not significant
C-LI	1.02	1	1.02	129.96	0.06	Not significant
D-AFR	20.00	1	20.00	2560.36	0.01	Significant
E-CO₂FR	42.55	1	42.55	5446.44	0.01	significant
AB	0.42	1	0.42	54.76	0.08	Not significant
AC	0.34	1	0.34	43.56	0.09	Not significant
AD	1.16	1	1.16	148.84	0.06	Not significant
AE	1.85	1	1.85	237.16	0.04	Significant
BC	1.02	1	1.02	129.96	0.06	Not significant
BD	0.01	1	0.01	1.96	0.39	Not significant
BE	0.13	1	0.13	17.64	0.14	Not significant
CD	0.38	1	0.38	49.00	0.09	Not significant
CE	2.82	1	2.82	361.00	0.03	Significant
DE	8.30	1	8.30	1062.76	0.01	Significant
ABC	0.02	1	0.02	3.24	0.32	Not significant
ABD	0.09	1	0.09	11.56	0.18	Not significant
ABE	0.19	1	0.19	25.00	0.12	Not significant
ACD	0.81	1	0.81	104.04	0.06	Not significant
ACE	1.76	1	1.76	225.00	0.04	Significant
ADE	0.94	1	0.94	121.00	0.06	Not significant
BCD	0.01	1	0.01	1.96	0.39	Not significant
BCE	0.11	1	0.11	14.44	0.16	Not significant
BDE	0.81	1	0.81	104.04	0.06	Not significant
CDE	1.67	1	1.67	213.16	0.04	Significant
ABCD	0.16	1	0.16	21.16	0.13	Not significant
ABCE	0.30	1	0.30	38.44	0.10	Not significant
ABDE	0.52	1	0.52	67.24	0.07	Not significant
ACDE	1.16	1	1.16	148.84	0.06	Not significant
BCDE	1.67	1	1.67	213.16	0.04	Significant