

Appendix

Equilibrium result

$$w^{NU\#} = \frac{2t}{4t-\eta^{2'}}, e^{NU\#} = \frac{\eta}{4t-\eta^{2'}}, p_b^{NU\#} = \frac{3t}{4t-\eta^{2'}}, D_b^{NU\#} = \frac{t}{4t-\eta^{2'}}, \pi_m^{NU\#} = \frac{t}{8t-2\eta^{2'}}, \pi_b^{NU\#} = \frac{t^2}{(4t-\eta^2)^2}.$$

$$w^{NR\#} = \frac{2t-p_e(p_e+\eta+e_o p_e \eta - e_o(2t-\eta^2))}{4t-(p_e+\eta)^2}, \quad e^{NR\#} = \frac{(1-e_o p_e)(p_e+\eta)}{4t-(p_e+\eta)^2}, \quad p_b^{NR\#} = \frac{3t-p_e(p_e+\eta+e_o p_e \eta - e_o(t-\eta^2))}{4t-(p_e+\eta)^2},$$

$$D_b^{NR\#} = \frac{(1-e_o p_e)t}{4t-(p_e+\eta)^2}, \quad \pi_m^{NR\#} = \frac{(1-e_o p_e)^2 t}{2(4t-(p_e+\eta)^2)} + p_e S, \quad \pi_b^{NR\#} = \frac{(1-e_o p_e)^2 t^2}{(4t-(p_e+\eta)^2)^2}.$$

$$e^{EU\#} =$$

$$\frac{k(-1+\delta)\eta\theta(-1+k\theta)(\delta(-6+k\theta)(-2+k\theta)-2(8+k\theta(-5+k\theta)))}{kt\theta(4k(13-\delta)(1-\delta)\theta-32(1-\delta)-4k^2(6-\delta)(1-\delta)\theta^2+k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(k\theta(5-k\theta)-8+\delta(4-k\theta)(2-k\theta))},$$

$$w^{EU\#} =$$

$$\frac{(-1+\delta)(-1+k\theta)((-1+\delta)\eta^2(-4+k\theta)(-2+k\theta)(-1+k\theta)+2kt\theta(-8+k\theta(2(3+\delta)-k(2+\delta)\theta)))}{kt\theta(4k(13-\delta)(1-\delta)\theta-32(1-\delta)-4k^2(6-\delta)(1-\delta)\theta^2+k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(k\theta(5-k\theta)-8+\delta(4-k\theta)(2-k\theta))},$$

$$D_m^{EU\#} =$$

$$\frac{(2-k\theta)(2-\delta(2-k\theta))(kt\theta(2-k(2-\delta)\theta)-(1-\delta)\eta^2(1-k\theta))}{kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))},$$

$$D_b^{EU\#} =$$

$$\frac{(1-\delta)\eta^2(3-k\theta)(2-k\theta)(1-k\theta)+2kt\theta(2-2k\theta-\delta(2-k\theta)(3-2k\theta))}{kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))},$$

$$\pi_b^{EU\#} =$$

$$\frac{2(2-k\theta)(1-k\theta)(2-\delta(2-k\theta))^2((1-\delta)\eta^2(1-k\theta)-kt\theta(2-k(2-\delta)\theta))^2}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)+2(1-\delta)\eta^2(1-k\theta)(-8+k\theta(5-k\theta)+\delta(4-k\theta)(2-k\theta)))^2},$$

$$\pi_m^{EU\#} =$$

$$\frac{(1-\delta)(1-k\theta)((1-\delta)\eta^2(2-k\theta)^2(1-k\theta)-4kt\theta(2-k\delta\theta(2-k\theta)))}{4(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))-2kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)},$$

$$\pi_p^{EU\#} =$$

$$\frac{k\delta\theta(1-k\theta)((1-\delta)\eta^2(2-k\theta)(1-k\theta)+2kt\theta(6\delta+k(8-5\delta)\theta-k^2(2-\delta)\theta^2-10))(2kt\theta(2k\theta-2+\delta(2-k\theta)(3-2k\theta))-(1-\delta)\eta^2(3-k\theta)(2-k\theta)(1-k\theta))}{(2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))-kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3))^2}$$

$$,$$

$$\begin{aligned} & a_0^{20} * \\ & \left(k(1-\delta)\theta \left(p_e(-16+20\delta+26k\theta-4k(9-\delta)\delta\theta-k^2(4-\delta)(3-4\delta)\theta^2+k^3(2-\delta)(1-\delta)\theta^3)+(1-\delta)\eta(1-k\theta)(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta))) \right) + \right. \\ & \quad \left. e_o p_e(2P_e(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))-(1-\delta)\eta(k\delta^2\theta(2-k\theta)^2+2\delta(4-k\theta)(1-k\theta)(2-k\theta)(8-k\theta(5-k\theta)))) \right) / \\ & \left(2P_e^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))+(1-\delta)(2(1-\delta)\eta^2(1-k\theta)(8(1-\delta)-k(5-6\delta)\theta+k^2(1-\delta)\theta^2)-k\theta\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3))- \right. \\ & \quad \left. 2P_e(1-\delta)\eta(k\delta^2\theta(2-k\theta)^2+2\delta(4-k\theta)(2-k\theta)(1-k\theta)-2(1-k\theta)(8-k\theta(5-k\theta))) \right); \\ & a_0^{18} * \\ & \left((1-k\theta) \left(P_e^2(4(1-\delta)(3+e_o(4-\delta)\eta)-2k(1-2\delta^2+e_o(5-2(4-\delta)\delta)\eta)\theta-k^2(1-\delta)(3-4\delta-e_o(2-\delta)\eta)\theta^2+k^3(1-\delta)^2\theta^3) - \right. \right. \\ & \quad \left. P_e(e_o(1-\delta)\eta^3(-8(2-\delta)+2k(5-3\delta)\theta-k^2(2-\delta)\theta^2)+e_o k\theta\theta(16-2k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))-(1-\delta)\eta(16-k\theta(12-k\theta(4-\delta(2-k\theta)))) \right) / \\ & \left(2P_e(1-\delta)\eta(k\delta^2\theta(2-k\theta)^2+2\delta(4-k\theta)(2-k\theta)(1-k\theta)-2(1-k\theta)(8-k\theta(5-k\theta))) - \right. \\ & \quad \left(2P_e^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))+(1-\delta)(2(1-\delta)\eta^2(1-k\theta)(8(1-\delta)-k(5-6\delta)\theta+k^2(1-\delta)\theta^2)-k\theta\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3))) \right); \\ & p_{\theta}^{20} * \\ & \left((1-k\theta) \left(P_e^2(4(1-\delta)(3+e_o(4-\delta)\eta)-2k(1-2\delta^2+e_o(5-2(4-\delta)\delta)\eta)\theta-k^2(1-\delta)(3-4\delta-e_o(2-\delta)\eta)\theta^2+k^3(1-\delta)^2\theta^3) - \right. \right. \\ & \quad \left. (1-\delta)(2k\theta\theta(2-k\theta)(6-2k\theta-3\delta(2-k\theta))-(1-\delta)\eta^2(1-k\theta)(12-k\theta(6-k\theta)-\delta(6-k\theta)(2-k\theta))) \right) + \\ & \quad P_e(e_o(1-\delta)\eta^3(4(4-\delta)(1-\delta)-2k(5-2(4-\delta)\delta)\theta+k^2(2-\delta)(1-\delta)\theta^2)-e_o k\theta\theta(4(4-\delta)(1-\delta)-2k(5-2(4-\delta)\delta)\theta+k^2(2-\delta)(1-\delta)\theta^2)+(1-\delta)\eta(2-k\theta)(12-4k\theta-\delta(2-k\theta)(6-k(1-\delta)\theta)))) \right) / \\ & \left(2P_e^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))+(1-\delta)(2(1-\delta)\eta^2(1-k\theta)(8(1-\delta)-k(5-6\delta)\theta+k^2(1-\delta)\theta^2)-k\theta\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3))- \right. \\ & \quad \left. 2P_e(1-\delta)\eta(k\delta^2\theta(2-k\theta)^2+2\delta(4-k\theta)(2-k\theta)(1-k\theta)-2(1-k\theta)(8-k\theta(5-k\theta))) \right); \\ & p_{\theta}^{18} * \\ & \left((1-k\theta) \left(P_e(k\theta(1-\delta)\eta(20-4k\theta(4-k\theta)-\delta(4-k\theta)(2-k\theta))+P_e^2(18-k\theta(13-3k\theta)-\delta(2-k\theta)(8-3k\theta))+(1-\delta)((1-\delta)\eta^2(2-k\theta)(1-k\theta)-2k\theta\theta(18-6\delta-k(8-5\delta)\theta+k^2(2-\delta)\theta^2))- \right. \right. \\ & \quad \left. e_o p_e\eta(\eta(P_e+\eta-\delta\eta)-k\theta\theta)(\delta(2-k\theta)(8-3k\theta)-2(8-k\theta(5-k\theta)))) \right) / \\ & \left(2P_e^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))+(1-\delta)(2(1-\delta)\eta^2(1-k\theta)(8(1-\delta)-k(5-6\delta)\theta+k^2(1-\delta)\theta^2)-k\theta\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3))- \right. \\ & \quad \left. 2P_e(1-\delta)\eta(k\delta^2\theta(2-k\theta)^2+2\delta(4-k\theta)(2-k\theta)(1-k\theta)-2(1-k\theta)(8-k\theta(5-k\theta))) \right); \end{aligned}$$

$$D_b = \frac{-\left\{ \left[-P_e^2(-2+k\theta)(3-2\delta+k(-1+\delta)\theta)(1+e_0\delta\eta+k(-1+\delta)\theta)+(-1+\delta)\left[(-1+\delta)\eta^2(-3+k\theta)(-2+k\theta)(-1+k\theta)+2k\theta\theta(-2+2k\theta+\delta(-2+k\theta)(-3+2k\theta)) \right] + \right. \right. \\ \left. P_e(-2(-1+\delta)\eta(-2+k\theta)(-3+\delta+k(4-3\delta)\theta+k^2(-1+\delta)\theta^2)+e_0\left[(-1+\delta)\delta\eta^2(-3+k\theta)(-2+k\theta)+\eta\left(k\delta^2\theta(-2+k\theta)^2+2(-1+k\theta)(8+k\theta(-5+k\theta))-\delta(-2+k\theta)(8+k\theta(-13+3k\theta)) \right) \right] \right\}}{2P_e^2(-1+k\theta)(-8+k\theta(5-k\theta)+\delta(-4+k\theta)(-2+k\theta))+(-1+\delta)\left[2(-1+\delta)\eta^2(-1+k\theta)(8(-1+\delta)+k(5-6\delta)\theta+k^2(-1+\delta)\theta^2)+k\theta\theta(-32(-1+\delta)-4k(-13+\delta)(-1+\delta)\theta+4k^2(-6+\delta)(-1+\delta)\theta^2-k^2(-2+\delta)^2\theta^2) \right] +} \\ \left. 2P_e(-1+\delta)\eta\left(k\delta^2\theta(-2+k\theta)^2+2\delta(-4+k\theta)(-2+k\theta)+2(-1+k\theta)(8+k\theta(-5+k\theta)) \right) \right\}}{2P_e((2-k\theta)(2-\delta(2-k\theta))\left[P_e(1-\delta)\eta(2+e_0\delta\eta)-kP_e(e_0\delta\eta+(2-\delta)(1-\delta)\eta)\theta+P_e^2(1+e_0\delta\eta-k(1-\delta)\theta)+(1-\delta)\left((1-\delta)\eta^2(1-k\theta)-k\theta\theta(2-k(2-\delta)\theta) \right) \right] +} \\ \left. \left[2P_e^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))+(-1+\delta)\left[2(1-\delta)\eta^2(1-k\theta)(8(1-\delta)-k(5-6\delta)\theta+k^2(1-\delta)\theta^2)-k\theta\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^2(2-\delta)^2\theta^2) \right] - \right. \right. \\ \left. \left. 2P_e(1-\delta)\eta\left(k\delta^2\theta(-2+k\theta)^2+2\delta(4-k\theta)(2-k\theta)(1-k\theta)-2(1-k\theta)(8-k\theta(5-k\theta)) \right) \right] \right\}}{2P_e(1-\delta)\eta(k\delta^2\theta(-2+k\theta)^2+2\delta(4-k\theta)(2-k\theta)(1-k\theta)-2(1-k\theta)(8-k\theta(5-k\theta)))};$$

Proof of Lemma 2 Given the profit π_m^{NR} showed in Eq 1, we have $\frac{\partial^2 \pi_b^{NR}}{\partial (p_b^{NR})^2} = -2.$, thus π_b^{NR} is

strictly concave in p_b^{NR} . Using the FOC yields $p_b^{NR}(w^{NR}, e^{NR}) = \frac{1}{2}(1 + w + e\eta)$. (A1)

Substituting Eq. (A1) into π_m^{NR} , we have, when $\frac{\partial^2 \pi_m^{NR}}{\partial (w^{NR})^2} = -1$ and $\frac{\partial^2 \pi_m^{NR}}{\partial (e^{NR})^2} = \eta p_e - t < 0$. Let H be

a Hessian of π_m^{NR} is $H = \begin{bmatrix} \eta p_e - t & -\frac{1}{2}(S - \eta) \\ -\frac{1}{2}(S - \eta) & -1 \end{bmatrix} = t - \frac{1}{4}(p_e + \eta)^2 > 0$. H is a negative definite.

Hence, π_m^{NR} is concave in w^{NR} and e^{NR} , we derive the following by using the FOCs $w^{NR} =$

$\frac{p_e(p_e + \eta + e_0 p_e \eta - e_0(2t - \eta^2)) - 2t}{(p_e + \eta)^2 - 4t}$, $e^{NR} = \frac{(e_0 p_e - 1)(p_e + \eta)}{(p_e + \eta)^2 - 4t}$. Substituting w^{NR} and e^{NR} into (A1) yields the

equilibrium $p_b^{NR} = \frac{p_e(p_e + \eta + e_0 p_e \eta - e_0(2t - \eta^2)) - 3t}{(p_e + \eta)^2 - 4t}$.

Proof of Lemma 3 $\frac{\partial e^{NR}}{\partial \eta} = \frac{(1 - e_0 p_e)(4t + (p_e + \eta)^2)}{(4t - (p_e + \eta)^2)^2} > 0$, $\frac{\partial w^{NR}}{\partial \eta} = \frac{(1 - e_0 p_e)(4t\eta - p_e^3 - 2p_e^2\eta - p_e\eta^2)}{(4t - (p_e + \eta)^2)^2} > 0$

$\frac{\partial D_b^{NR}}{\partial \eta} = \frac{2(1 - e_0 p_e)t(p_e + \eta)}{(4t - (p_e + \eta)^2)^2} > 0$, $\frac{\partial \pi_m^{NR}}{\partial \eta} = \frac{(1 - e_0 p_e)^2 t(p_e + \eta)}{(4t - (p_e + \eta)^2)^2} > 0$, $\frac{\partial \pi_b^{NR}}{\partial \eta} = \frac{4(1 - e_0 p_e)^2 t^2(p_e + \eta)}{(4t - (p_e + \eta)^2)^3} > 0$.

Proof of Lemma 4 Given the profit π_m^{EU} showed in Eq. , we have $\frac{\partial^2 \pi_m^{EU}}{\partial (p_m^{EU})^2} = -\frac{2(1-\delta)}{k\theta(1-k\theta)}$, Because

$0 \leq k\theta < 1$, thus π_m^{EU} is strictly concave in p_m^{EU} . Using the FOC yields $p_m^{EU}(w, e, p_b) = \frac{1}{2} \left[\frac{k w \theta}{1 - \delta} + e(\eta - k\eta\theta) + k\theta p_b \right]$.

Substituting $p_m^{EU}(w, e, p_b)$ into $\pi_b^{EU}(w, e, p_b)$, we have $\frac{\partial^2 \pi_b^{EU}}{\partial (p_b^{EU})^2} = -1 + \frac{1}{-1 + k\theta} < 0$. Thus, we

derive the following by using the FOCs, $p_b^{EU}(w^{EU}, e^{EU}) = \frac{(1-\delta)(2+e\eta)(1-k\theta)+w(2-\delta(2-k\theta))}{2(1-\delta)(2-k\theta)}$. Then,

substituting $p_b^{EU}(w, e)$ and $p_m^{EU}(w, e, p_b)$ into $\pi_m^{EU}(w, e)$ we have $\frac{\partial^2 \pi_m^{EU}}{\partial (e^{EU})^2} = -t +$

$\frac{(1-\delta)\eta^2(4-k\theta)^2(1-k\theta)}{8k\theta(2-k\theta)^2} < 0$, $\frac{\partial^2 \pi_m^{EU}}{\partial (w^{EU})^2} = \frac{-32(1-\delta)+4k(13-\delta)(1-\delta)\theta-4k^2(6-\delta)(1-\delta)\theta^2+k^3(2-\delta)^2\theta^3}{8(1-\delta)(2-k\theta)^2(1-k\theta)} < 0$. Since the

dual channel demand constraint are the linear function of w^{EU}, e^{EU} , $\pi_m^{EU}(w^{EU}, e^{EU})$ are jointly

concave in w^{EU}, e^{EU} , thus the Kuhn–Tucker conditions are necessary and sufficient for solving

problem of optimal profit. The K-T conditions are as follows:

$$\begin{aligned} \frac{\partial \pi_m^{EU}}{\partial e^{EU}} + \lambda_1 \frac{\partial \left(p_b^{EU} - \frac{p_m^{EU} - (1-k\theta)\eta e^{EU}}{k\theta} \right)}{\partial e^{EU}} + \lambda_2 &= 0; \\ \frac{\partial \pi_m^{EU}}{\partial w^{EU}} + \lambda_1 \frac{\partial \left(p_b^{EU} - \frac{p_m^{EU} - (1-k\theta)\eta e^{EU}}{k\theta} \right)}{\partial w^{EU}} + \lambda_2 &= 0; \\ \lambda_1 \frac{\partial \left(p_b^{EU} - \frac{p_m^{EU} - (1-k\theta)\eta e^{EU}}{k\theta} \right)}{\partial w^{EU}} &= 0; \\ \lambda_2 \frac{\partial (1 + p_m^{EU} - k\theta - p_b^{EU})}{\partial w^{EU}} &= 0; \end{aligned}$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, $\lambda_3 \geq 0$ and $\lambda_4 \geq 0$ are the Lagrange multipliers.

According to the K-T conditions, we can get the following situations:

$$(1) \quad e^{EU} = e^{EU\#} =$$

$$\frac{k(-1+\delta)\eta\theta(-1+k\theta)(\delta(-6+k\theta)(-2+k\theta)-2(8+k\theta)(-5+k\theta))}{kt\theta(4k(13-\delta)(1-\delta)\theta-32(1-\delta)-4k^2(6-\delta)(1-\delta)\theta^2+k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(k\theta(5-k\theta)-8+\delta(4-k\theta)(2-k\theta))},$$

$$w^{EU} = w^{EU\#} =$$

$$\frac{(-1+\delta)(-1+k\theta)((-1+\delta)\eta^2(-4+k\theta)(-2+k\theta)(-1+k\theta)+2kt\theta(-8+k\theta(2(3+\delta)-k(2+\delta)\theta)))}{kt\theta(4k(13-\delta)(1-\delta)\theta-32(1-\delta)-4k^2(6-\delta)(1-\delta)\theta^2+k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(k\theta(5-k\theta)-8+\delta(4-k\theta)(2-k\theta))} \quad \text{for}$$

$$\frac{p_m - (1-k\theta)\eta e}{k\theta} < p_b \leq 1 + p_m - k\theta, \text{ when } \lambda_1 = 0 \text{ and } \lambda_2 = 0.$$

$$(2) \text{ When } \lambda_1 = 0, \lambda_2 = -\frac{2(-1+\delta)(2+\delta)(-2+k\theta)((-1+\delta)\eta^2(-1+k\theta))+kt\theta(2+k(-2+\delta)\theta))}{(-2+k\theta)(-2(-1+\delta)^2\eta^2(-1+k\theta)(-1+\delta+k\theta)+kt\theta(2-2k\theta+\delta(-2+k\theta))^2)} < 0, \text{ therefore}$$

the solution is not the optimal one.

$$(3) \text{ When } \lambda_1 = \frac{2k(-1+\delta)\theta((-1+\delta)\eta^2(-3+k\theta)(-2+k\theta)(-1+k\theta)-2kt\theta(-2+2k\theta+\delta(-2+k\theta)(-3+2k\theta)))}{-2(-1+\delta)\eta^2(-4+k\theta)(-2+k\theta)(-1+k\theta)(-1+\delta+k\theta)+k^2t\theta^2(-2(1+\delta)+k(2+\delta)\theta)^2} < 0, \lambda_2 = 0,$$

therefore the solution is not the optimal one.

$$(4) \text{ When } \lambda_1 = -\frac{k\theta\left((1-\delta)(1-\delta-k\theta))-\frac{kt\delta\theta(-2+2k\theta+\delta(2-k\theta))}{\eta^2(1-k\theta)}\right)}{(1-\delta-k\theta)^2}, \lambda_2 = -\frac{((1-\delta)(1-\delta-k\theta))+\frac{k^2t\delta\theta^2(2(1-\delta)-k(2+\delta)\theta)}{\eta^2(2-k\theta)(1-k\theta)}}{(1-\delta-k\theta)^2},$$

in this case we can also get $\lambda_1 < 0$, $\lambda_2 < 0$, therefore the solution is not the optimal one.

Substituting $e^{EU\#}$, $p_m^{EU\#}$ and $p_b^{EU\#}$ into constraint $\frac{p_m - (1-k\theta)\eta e}{k\theta} < p_b \leq 1 + p_m - k\theta$, We can get

threshold t_1 , when $t > t_1$, dual channel case exists, it means that manufacturer always chooses to launch FB products, when $t < t_1$, brand product exit the market, this result tends to be impossible, so omit it.

Then, substituting w^{EU} and e^{EU} into (A1) yields the equilibrium as follows.

$$D_m^{EU} = D_m^{EU\#} =$$

$$\frac{(2-k\theta)(2-\delta(2-k\theta))(kt\theta(2-k(2-\delta)\theta)-(1-\delta)\eta^2(1-k\theta))}{kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))},$$

$$D_b^{EU} = D_b^{EU\#} =$$

$$\frac{(1-\delta)\eta^2(3-k\theta)(2-k\theta)(1-k\theta)+2kt\theta(2-2k\theta-\delta(2-k\theta)(3-2k\theta))}{kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))} ;$$

Substituting $e^{EU\#}$, $w^{EU\#}$, $D_m^{EU\#}$ and $D_b^{EU\#}$ into π_b^{EU} , π_m^{EU} and π_p^{EU} , we have

$$\pi_b^{EU} = \pi_b^{EU\#} =$$

$$\frac{2(2-k\theta)(1-k\theta)(2-\delta(2-k\theta))^2((1-\delta)\eta^2(1-k\theta)-kt\theta(2-k(2-\delta)\theta))^2}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)+2(1-\delta)\eta^2(1-k\theta)(-8+k\theta(5-k\theta)+\delta(4-k\theta)(2-k\theta)))^2}.$$

$$\pi_m^{EU} = \pi_m^{EU\#} =$$

$$\frac{(1-\delta)(1-k\theta)((1-\delta)\eta^2(2-k\theta)^2(1-k\theta)-4kt\theta(2-k\delta\theta(2-k\theta)))}{4(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))-2kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)}.$$

$$\pi_p^{EU} = \pi_p^{EU\#} =$$

$$\frac{k\delta\theta(1-k\theta)((1-\delta)\eta^2(2-k\theta)(1-k\theta)+2kt\theta(6\delta+k(8-5\delta)\theta-k^2(2-\delta)\theta^2-10))(2kt\theta(2k\theta-2+\delta(2-k\theta)(3-2k\theta))-(1-\delta)\eta^2(3-k\theta)(2-k\theta)(1-k\theta))}{(2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta))-kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3))^2}$$

.

Proof of Lemma 5.

$$\frac{\partial e^{EU}}{\partial \eta} =$$

$$\frac{k(1-\delta)\theta(1-k\theta)(2(8-k\theta(5-k\theta))-\delta(6-k\theta)(2-k\theta))(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)+2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2}$$

$$> 0 ; \quad \frac{\partial e^{EU}}{\partial t} =$$

$$\frac{k^2(1-\delta)\eta\theta^2(1-k\theta)(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} <$$

$$0 ; \quad \frac{\partial w^{EU}}{\partial \eta} =$$

$$\frac{2k^2t(1-\delta)^2\eta\theta^2(1-k\theta)^2(8(2-\delta)-2k(5-3\delta)\theta+k^2(2-\delta)\theta^2)(2(8-k\theta(5-k\theta))-\delta(6-k\theta)(2-k\theta))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} >$$

$$0 . \quad \frac{\partial w^{EU}}{\partial t} =$$

$$\frac{k^2(1-\delta)^2\eta^2\theta^2(1-k\theta)^2(8(2-\delta)-2k(5-3\delta)\theta+k^2(2-\delta)\theta^2)(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} <$$

$$0 ; \quad \frac{\partial D_b^{EU}}{\partial \eta} =$$

$$\frac{2kt(1-\delta)\eta\theta(1-k\theta)(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))(\delta(2-k\theta)(8-k\theta(7-k\theta))-2(1-k\theta)(8-k\theta(5-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} <$$

$$0 ; \quad \frac{\partial D_b^{EU}}{\partial t} =$$

$$\frac{k(1-\delta)\eta^2\theta(1-k\theta)(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))(2(1-k\theta)(8-k\theta(5-k\theta))-\delta(2-k\theta)(8-k\theta(7-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} <$$

$$0 ; \quad \frac{\partial D_m^{EU}}{\partial \eta} =$$

$$\frac{2k^2t(1-\delta)\delta\eta\theta^2(2-k\theta)(1-k\theta)(2-\delta(2-k\theta))(2(8-k\theta(5-k\theta))-\delta(6-k\theta)(2-k\theta))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} >$$

$$0 \quad ; \quad \frac{\partial D_m^{EU}}{\partial t} =$$

$$\frac{k^2(1-\delta)\delta\eta^2\theta^2(2-k\theta)(1-k\theta)(2-\delta(2-k\theta))(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} <$$

$$0 \quad ; \quad \frac{\partial \pi_m^{EU}}{\partial \eta} =$$

$$\frac{k^2t(1-\delta)^2\eta\theta^2(1-k\theta)^2(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))^2}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} > 0;$$

$$\frac{\partial \pi_m^{EU}}{\partial t} = - \frac{k^2(1-\delta)^2\eta^2\theta^2(1-k\theta)^2(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))^2}{2(kt\theta(32(-1+\delta)+4k(13-\delta)(1-\delta)\theta-4k^2(6-\delta)(1-\delta)\theta^2+k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(-8+k\theta(5-k\theta)+\delta(4-k\theta)(2-k\theta)))^2} < 0;$$

$$\frac{\partial \pi_b^{EU}}{\partial \eta} = \frac{8k^2t(1-\delta)\delta\eta\theta^2(2-k\theta)(1-k\theta)^2(2-\delta(2-k\theta))^2(((1-\delta)\eta^2(1-k\theta))-kt\theta(2-k(2-\delta)\theta))(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^3} >$$

$$0 \quad ; \quad \frac{\partial \pi_b^{EU}}{\partial t} =$$

$$\frac{4k^2(1-\delta)\delta\eta^2\theta^2(2-k\theta)(1-k\theta)^2(2-\delta(2-k\theta))^2(((1-\delta)\eta^2(1-k\theta))-kt\theta(2-k(2-\delta)\theta))(2(8-k\theta(5-k\theta))-\delta(6-k\theta)(2-k\theta))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^3} > 0;$$

$$\frac{\partial \pi_p^{EU}}{\partial \eta} = \frac{4k^2t(1-\delta)\delta\eta\theta^2(1-k\theta)^2(\delta(6-k\theta)(2-k\theta)-2(8-k\theta(5-k\theta)))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^2} * \\ (kt\theta(4\delta(2-k\theta)(6-k\theta(6-k\theta))(8-k\theta(5-k\theta))-4(1-k\theta)(8-k\theta(5-k\theta))(6-k\theta(4-k\theta))-\delta^2(2-k\theta)^2(48-k\theta(54-k\theta(16-k\theta))))-(1-\delta)\eta^2(2-k\theta)(1-k\theta)(8-k\theta(5-k\theta))(2-\delta(2-k\theta))) < 0;$$

$$\frac{\partial \pi_p^{EU}}{\partial t} = \frac{2k^2(1-\delta)\delta\eta^2\theta^2(1-k\theta)^2(2(8-k\theta(5-k\theta))-\delta(6-k\theta)(2-k\theta))}{(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta+4k^2(6-\delta)(1-\delta)\theta^2-k^3(2-\delta)^2\theta^3)-2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta)-\delta(4-k\theta)(2-k\theta)))^3} * \\ (kt\theta(4\delta(2-k\theta)(6-k\theta(6-k\theta))(8-k\theta(5-k\theta))-4(1-k\theta)(8-k\theta(5-k\theta))(6-k\theta(4-k\theta))-\delta^2(2-k\theta)^2(48-k\theta(54-k\theta(16-k\theta))))-(1-\delta)\eta^2(2-k\theta)(1-k\theta)(8-k\theta(5-k\theta))(2-\delta(2-k\theta))) < 0.$$

Proof of Lemma 6. Given the profit π_m^{ER} showed in Eq. , we have $\frac{\partial^2 \pi_m^{ER}}{\partial (p_m^{EU})^2} = -\frac{2(1-\delta)}{k\theta(1-k\theta)}$, Because

$0 \leq k\theta < 1$, thus π_m^{EU} is strictly concave in p_m^{EU} . Using the FOC yields $p_m^{EU}(w, e, p_b) =$

$$\frac{1}{2} \left[\frac{e_0 p_e - k(e_0 p_e - w)\theta - e(p_e - (1-\delta)\eta)(1-k\theta) + k(1-\delta)\theta p_b}{(1-\delta)} \right].$$

$$\begin{aligned} \frac{\partial \pi_m^{ER}}{\partial e^{ER}} + \lambda_1 \frac{\partial \left(p_b^{ER} - \frac{p_m^{ER} - (1-k\theta)\eta e^{ER}}{k\theta} \right)}{\partial e^{ER}} + \lambda_2 &= 0; \\ \frac{\partial \pi_m^{ER}}{\partial w^{ER}} + \lambda_1 \frac{\partial \left(p_b^{ER} - \frac{p_m^{ER} - (1-k\theta)\eta e^{ER}}{k\theta} \right)}{\partial w^{ER}} + \lambda_2 &= 0; \\ \lambda_1 \frac{\partial \left(p_b^{ER} - \frac{p_m^{ER} - (1-k\theta)\eta e^{ER}}{k\theta} \right)}{\partial w^{EU}} &= 0; \\ \lambda_2 \frac{\partial (1 + p_m^{ER} - k\theta - p_b^{ER})}{\partial w^{ER}} &= 0; \end{aligned}$$

(1) $e^{ER} = e^{ER\#1}$, $w^{ER} = w^{ER\#}$; for $\frac{p_m - (1-k\theta)\eta e}{k\theta} < p_b \leq 1 + p_m - k\theta$, when $\lambda_1 = 0$ and $\lambda_2 = 0$.

(3) When $\lambda_1 = 0$ and $\lambda_2 \neq 0$, in this case $\lambda_2 < 0$, thus, this solution is not the optimal one.

$$\frac{(-2+k\theta)(k\theta(2(-1+3\delta)(P+\eta-\delta\eta)+k(P(-4+\delta)(-1+2\delta)+(-1+\delta)(-4+7\delta)\eta)\theta-k^2(-2+\delta)(-1+\delta)(P+\eta)\theta^2))+jP(-8(-1+\delta)\eta+2P(-1+k\theta)(-4+k\theta(5-2\delta+k(-$$

$$2(-1+\delta)\eta^2(-4+k\theta)(-2+k\theta)(-1+k\theta)(-1+\delta+k\theta)-k^2t\theta^2(-2(1+\delta)+k(2+\delta)\theta)^2+2P\eta(-2+k\theta)(k\delta^2\theta(-2+k\theta)+2\delta(-2+k\theta)^2(-1+k\theta)-2(-4+k\theta)(-1+$$

$$;$$
$$((-1+k\theta)(P^2(-2+k\theta)(-4+4j(-2+\delta)\eta)+k(3+j(10-7\delta)\eta)\theta+k^2(-1+\delta)(-1+2j\eta)\theta^2))+(-1+\delta)(-((-1-k\theta)(P^2(-2-k\theta)(-4-4j(-2-\delta)\eta)-k(3-j(10+7\delta)\eta)\theta-k^2(-1-\delta)(-1-2j\eta)\theta^2))$$

In this case, we can get $D_m^{ER\#2} = 0$, which means although factory open the online channel, but there is no factory product in the market, customer can only buy brand products from retailer channel.

The constant parameters X_1 to X_{10} are defined as below to simplify the presentation of equilibrium results:

Proposition 1 Subtracting π_m^{NU} from π_m^{EU} yields $\pi_m^{EU} - \pi_m^{NU} = \frac{A_1 e_0^2 + B_1 e_0 + C_1}{D_1} = \frac{L_1}{D_1}$, given $0 < \theta < 1$, $0 < k < 1$ and $0 < \delta < 1$, we have $\frac{\partial^2 L_1}{\partial e_0^2} > 0$, $\frac{\partial L_1(e_0)}{\partial e_0} > 0$. Thus, $L_1 > 0$ when $0 < \theta < 1$, $0 < k < 1$ and $0 < \delta < 1$ which implies that $\pi_m^{EU} > \pi_m^{NU}$ is established. Therefore, Part (i) of Proposition 1 holds.

Proposition 2 Subtracting π_m^{NR} from π_m^{ER} yields $\pi_m^{ER} - \pi_m^{NR} = \frac{A_2 e_0^2 - B_2 e_0 + C_2}{D_2} = \frac{L_2}{D_2}$. It is easy to verify $A_2 > 0$ and $D_2 > 0$ always exist. For $\pi_m^{ER} - \pi_m^{NR} = 0$, there are two root $e_0^{\#1} = \frac{B_2 - \sqrt{B_2^2 - 4A_2 C_2}}{2A_2}$; $e_0^{\#2} = \frac{B_2 + \sqrt{B_2^2 - 4A_2 C_2}}{2A_2}$; thus in the interval of $e_0^{\#1} < e_0 < e_0^{\#2}$, $\pi_m^{ER} - \pi_m^{NR} < 0$, when $e_0^{\#1} > e_0$ or $e_0^{\#2} < e_0$, $\pi_m^{ER} - \pi_m^{NR} > 0$ which implies that $\pi_m^{ER} > \pi_m^{NR}$ is established.

Proposition 3 Subtracting π_m^{NU} from π_m^{NR} yields $\pi_m^{NR} - \pi_m^{NU} = p_e S + \frac{(1 - e_0 p_e)^2 t}{2(4t - (p_e + \eta)^2)} - \frac{t}{8t - 2\eta^2} = L_3$, we have $\frac{\partial L_3}{\partial e_0} = \frac{p_e(1 - e_0 p_e)t}{-4t + (p_e + \eta)^2}$, and $\frac{\partial^2 L_3}{\partial e_0^2} = \frac{p_e^2 t}{4t - (p_e + \eta)^2} > 0$, When $p_e > \frac{1}{e_0}$, $\frac{\partial L_3}{\partial e_0} > 0$ and when $0 < p_e < \frac{1}{e_0}$, $\frac{\partial L_3}{\partial e_0} < 0$. Thus when the $L_3\left(\frac{1}{p_e}\right) \geq 0$, that is $S \geq S_1 = \frac{t}{8p_e t - 2p_e \eta^2}$, the government carbon quota is enough, in this case, for $\forall e_0 > 0$, $L_3 \geq 0$ exists, and $\pi_m^{NR} \geq \pi_m^{NU}$.

When $L_3\left(\frac{1}{p_e}\right) < 0$, by $L_3 = 0$ we can get two root, $e_0^{\#1}$ and $e_0^{\#2}$, now we should compare with $e_0^{\#1}$, $e_0^{\#2}$ and $\frac{1}{p_e}$, respectively. When $0 < p_e < \frac{1}{e_0}$, if $e_0 \in (e_0^{\#1}, \frac{1}{p_e})$, $L_3 < 0$, $\pi_m^{NR} < \pi_m^{NU}$, if $e_0 \in (0, e_0^{\#1})$, $L_3 > 0$, $\pi_m^{NR} > \pi_m^{NU}$. Similar, when $p_e > \frac{1}{e_0}$, if $e_0 \in (\frac{1}{p_e}, e_0^{\#2})$, $L_3 < 0$, $\pi_m^{NR} < \pi_m^{NU}$, if $e_0 \in (\frac{1}{p_e}, e_0^{\#2})$, $L_3 < 0$, $\pi_m^{NR} < \pi_m^{NU}$; finally, if $e_0 > e_0^{\#2}$, $L_3 > 0$, $\pi_m^{NR} > \pi_m^{NU}$.

In conclusion, when the manufacturer decides not to encroach, if $S \geq S_1 = \frac{t}{8p_e t - 2p_e \eta^2}$, $\pi_m^{NR} \geq \pi_m^{NU}$. Otherwise, there exist the interval of e_0 that if $e_0 \in (e_0^{\#1}, e_0^{\#2})$, $\pi_m^{NR} < \pi_m^{NU}$;

otherwise, $\pi_m^{NR} > \pi_m^{NU}$. In this proposition, $e_0^{\#1} = \frac{(4t - (p_e + \eta)^2)(\frac{p_e t}{4t - (p_e + \eta)^2} - \sqrt{\frac{p_e^2 t(t - 8p_e S t + 2p_e S \eta^2)}{(4t - \eta^2)(4t - (p_e + \eta)^2)}})}{p_e^2 t}$;

$$e_0^{\#2} = \frac{(4t - (p_e + \eta)^2)(\sqrt{\frac{p_e^2 t(t - 8p_e S t + 2p_e S \eta^2)}{(4t - \eta^2)(4t - (p_e + \eta)^2)}} + \frac{p_e t}{4t - (p_e + \eta)^2})}{p_e^2 t}.$$

Proposition 4 Subtracting π_m^{EU} from π_m^{ER} yields $\pi_m^{ER} - \pi_m^{EU} = L_4$, $L_4 = ae_0^2 + be_0 + c$,

we have $\frac{\partial L_4}{\partial e_0} = \left(p_e \left(2(1-\delta)((1-\delta)\delta\eta^2(2-k\theta)^2(1-k\theta) + kt\theta(16-20\delta-26k\theta+4k(9-\delta)\delta\theta + k^2(4-\delta)(3-4\delta)\theta^2 - k^3(2-\delta)(1-\delta)\theta^3)) + p_e \left(2e_0(1-\delta)\delta^2\eta^2(2-k\theta)^2 + 2(1-\delta)\delta\eta(2-k\theta)^2(1-k(1-\delta)\theta) - 4e_0t(1-k\theta)(8-k\theta(5-k\theta) - \delta(4-k\theta)(2-k\theta)) \right) \right) / (4p_e^2(1-k\theta)(8-k\theta(5-k\theta) - \delta(4-k\theta)(2-k\theta)) + 2(-1+\delta)(2(-1+\delta)\eta^2(1-k\theta)(8(1-\delta) - k(5-6\delta)\theta + k^2(1-\delta)\theta^2) + kt\theta(32(1-\delta) - 4k(13-\delta)(1-\delta)\theta + 4k^2(6-\delta)(1-\delta)\theta^2 - k^3(2-\delta)^2\theta^3)) + 4P(-1+\delta)\eta(k\delta^2\theta(2-k\theta)^2 + 2\delta(4-k\theta)(2-k\theta)(1-k\theta) - 2(1-k\theta)(8-k\theta(5-k\theta)))) \right)$, and $\frac{\partial^2 L_4}{\partial e_0^2} = (p_e^2(2(1-\delta)\delta^2\eta^2(2-k\theta)^2 - 4t(1-k\theta)(8-k\theta(5-k\theta) - \delta(4-k\theta)(2-k\theta)))) / (4p_e^2(1-k\theta)(8-k\theta(5-k\theta) - \delta(4-k\theta)(2-k\theta)) - 2(1-\delta)(2(1-\delta)\eta^2(1-k\theta)(8(-1+\delta) + k(5-6\delta)\theta - k^2(1-\delta)\theta^2) + kt\theta(32(1-\delta) - 4k(13-\delta)(1-\delta)\theta + 4k^2(6-\delta)(1-\delta)\theta^2 - k^3(2-\delta)^2\theta^3)) - 4p_e(1-\delta)\eta(k\delta^2\theta(2-k\theta)^2 + 2\delta(4-k\theta)(2-k\theta)(1-k\theta) - 2(1-k\theta)(8-k\theta(5-k\theta))))$, in this case, $\frac{\partial^2 L_4}{\partial e_0^2} = 2a$, $\frac{\partial L_4}{\partial e_0} = 2ae_0 + b$, thus by

$\frac{\partial L_4}{\partial e_0} - \frac{\partial^2 L_4}{\partial e_0^2} * e_0 = b$, put $-\frac{\frac{\partial L_4}{\partial e_0} - \frac{\partial^2 L_4}{\partial e_0^2} * e_0}{\frac{\partial^2 L_4}{\partial e_0^2}} = -\frac{b}{2a}$ into $L_4 = 0$ by Mathematica 13.0, we can get

$L_4(-\frac{b}{2a}) = 0$, $S = S_2$, similar to proposition 2, when the $S \geq S_2$, government carbon quota is enough, and for $\forall e_0 > 0$, $L_3 \geq 0$ exists, we can get $\pi_m^{ER} \geq \pi_m^{NR}$.

When $L_4(-\frac{b}{2a}) < 0$, by $L_4 = 0$ we can get two root, e_0^{*3} and e_0^{*4} , now we should compare

$L_4(e_0^{*3})$, $L_4(e_0^{*4})$ and $L_4(-\frac{b}{2a})$ with 0, respectively. When $e_0 < -\frac{b}{2a}$, if $e_0 \in (e_0^{*3}, -\frac{b}{2a})$, $L_4 < 0$,

$\pi_m^{NR} < \pi_m^{NU}$, if $e_0 \in (0, e_0^{*3})$, $L_4 > 0$, $\pi_m^{NR} > \pi_m^{NU}$. Similar, when $e_0 \geq -\frac{b}{2a}$, if $e_0 \in (-\frac{b}{2a}, e_0^{*4})$,

$L_4 < 0$, $\pi_m^{ER} < \pi_m^{EU}$, if $e_0 \in (-\frac{b}{2a}, e_0^{*4})$, $L_4 < 0$, $\pi_m^{ER} < \pi_m^{EU}$; finally, if $e_0 > e_0^{*4}$, $L_4 > 0$, $\pi_m^{NR} >$

π_m^{NU} .

In conclusion, when the manufacturer decides to encroach, if $S \geq S_2$, $\pi_m^{ER} \geq \pi_m^{EU}$. Otherwise, there exist the interval of e_0 that if $e_0 \in (e_0^{*3}, e_0^{*4})$, $\pi_m^{ER} < \pi_m^{EU}$; otherwise, $\pi_m^{ER} \geq \pi_m^{EU}$.

$$t_1 = \frac{(1-\delta)\eta^2(1-k\theta)}{k\theta(2-k(2-\delta)\theta)}; t_2 = \frac{(P+\eta-\delta\eta)(P+\eta-\delta\eta+jP\delta\eta-k(1-\delta)(P+\eta)\theta)}{k\theta(2-2\delta+jP\delta-k(2-\delta)(1-\delta)\theta)};$$

$$t_3 = \frac{(-2+k\theta)(3P-2P\delta+3\eta-3\delta\eta-k(1-\delta)(P+\eta)\theta)(P+\eta-\delta\eta+jP\delta\eta-k(1-\delta)(P+\eta)\theta)}{2k(1-\delta)\theta(2-2k\theta-\delta(2-k\theta)(3-2k\theta))+jP(k\delta^2\theta(2-k\theta)^2-2(1-k\theta)(8-k\theta(5-k\theta))+\delta(2-k\theta)(8-k\theta(13-3k\theta)))}.$$

$$A_1 = k^2\theta^2(4(1-\delta)(5-9\delta) - 4k(1-\delta)(6-13\delta)\theta + k^2(4-(20-17\delta)\delta)\theta^2),$$

$$B_1 = 2k(1-\delta)\eta^2\theta(1-k\theta)(7-10\delta-k(9-13\delta)\theta + 2k^2(1-2\delta)\theta^2),$$

$$C_1 = (1-\delta)^2\eta^4(2-k\theta)^2(1-k\theta)^2,$$

$$D_1 = 2(4t-\eta^2)(kt\theta(32(1-\delta)-4k(13-\delta)(1-\delta)\theta + 4k^2(6-\delta)(1-\delta)\theta^2 - k^3(2-\delta)^2\theta^3) - 2(1-\delta)\eta^2(1-k\theta)(8-k\theta(5-k\theta) - \delta(4-k\theta)(2-k\theta))).$$

$$A_2 = P_e^2(((1-\delta)\delta^2\eta^2(P_e+\eta)^2(2-\theta k)^2) - 2t\delta\eta(16(1-\delta)(P_e+\eta) - (P_e(26-4(8-\delta)\delta) + (26-33\delta+6\delta^2)\eta)\theta k + (2P(6-\delta(9-2\delta)) + (12-\delta(18-5\delta))\eta)(\theta k)^2 - (2-\delta)(1-\delta)(P+\eta)(\theta k)^3) + t^2(64(1-\delta) - 8(17-22\delta+4\delta^2)\theta k + 4(25-\delta(41-(15-\delta)\delta))(\theta k)^2 - 4(1-\delta)(8-(7-\delta)\delta)(\theta k)^3 + (2-\delta)^2(1-\delta)(\theta k)^4));$$

$$B_2 = 2P_e(P_e^3(1-\delta)\delta\eta(2-\theta k)^2(1-(1-\delta)\theta k) + P_e^2((1-\delta)\delta\eta^2(2-\theta k)^2(3-\delta-3(1-\delta)\theta k) + t(1-(1-\delta)\theta k)(\delta^2(2-\theta k)^2\theta k - 2(1-\theta k)(8-(5-\theta k)\theta k) + \delta(2-\theta k)(8-3(4-\theta k)\theta k))) + P_e(1-\delta)\eta(\delta\eta^2(2-\theta k)^2(3-2\delta-3(1-\delta)\theta k) - 2t(\delta^2(2-\theta k)^2\theta k(1+\theta k) + 2(1-\theta k)^2(8-(5-\theta k)\theta k) - \delta(2-\theta k)(1-\theta k)(4-\theta k(10-3\theta k)))) + (1-\delta)((-1+\delta)\delta\eta^4(2-\theta k)^2(-1+\theta k) + t^2\theta k(-8\delta(-3+\theta k)(2-\theta k)(1-\theta k) + 3\delta^2(2-\theta k)^2\theta k - 4(1-\theta k)(8-(5-\theta k)\theta k)) - t\eta^2(2(1-\theta k)^2(8-(5-\theta k)\theta k) + \delta^2(2-\theta k)\theta k(2-(\theta k)^2) - \delta(1-\theta k)(16-\theta k(26-\theta k(16-3\theta k)))));$$

$$C_2 = P_e^4(1-\delta)(2-\theta k)^2(1-(1-\delta)\theta k)^2 + 2P_e^3(1-\delta)\eta(2-\theta k)^2(1-(1-\delta)\theta k)(2-\delta-2(1-\delta)\theta k) + (1-\delta)((1-\delta)^2\eta^4(2-3\theta k+(\theta k)^2)^2 + 2t(1-\delta)\eta^2(1-\theta k)\theta k(7-(9-2\theta k)\theta k - \delta(2-\theta k)(5-4\theta k)) + t^2(\theta k)^2(4(5-\theta k)(1-\theta k) - 4\delta(1-\theta k)(14-5\theta k) + \delta^2(2-\theta k)(18-17\theta k))) + 2P_e(-1+\delta)\eta(-((-1+\delta)\eta^2(2-\theta k)^2(-1+\theta k)(2-\delta-2(1-\delta)\theta k)) - t\theta k(2(1-\theta k)^2(7-2\theta k) - 2\delta(1-\theta k)(14-3\theta k(7-2\theta k)) + \delta^2(2-\theta k)(6-\theta k(15-8\theta k)))) + P_e^2(((1-\delta)\eta^2(2-\theta k)^2(6-6\delta+\delta^2-6(2-\delta)(1-\delta)\theta k + 6(1-\delta)^2(\theta k)^2)) + 2t\theta k(1-(1-\delta)\theta k)(7-9\theta k + 2((\theta k)^2 + \delta^2(2-\theta k)(3-2\theta k) - \delta(9-(11-3\theta k)\theta k))));$$

$$D_2 = P_e^2(((1-\delta)\delta^2\eta^2(P_e+\eta)^2(2-\theta k)^2) - 2t\delta\eta(16(1-\delta)(P_e+\eta) - (P_e(26-4(8-\delta)\delta) + (26-33\delta+6\delta^2)\eta)\theta k + (2P_e(6-\delta(9-2\delta)) + (12-\delta(18-5\delta))\eta)(\theta k)^2 - (2-\delta)(1-\delta)(P+\eta)(\theta k)^3) + t^2(64(1-\delta) - 8(17-22\delta+4\delta^2)\theta k + 4(25-\delta(41-(15-\delta)\delta))(\theta k)^2 - 4(1-\delta)(8-(7-\delta)\delta)(\theta k)^3 + (2-\delta)^2(1-\delta)(\theta k)^4));$$