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The Changing Dynamics of Board Independence: A Copula Based Quantile Regression Approach

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Abstract: This paper examines the effect of board characteristics, especially board independence, on firm performance from a dynamic perspective through copula-based quantile regression approaches, which allow us to focus on changes at different points in the distribution of board characteristics. We find that the effect of board independence on Tobin's Q, a proxy of firm value, is negatively associated with firm value, using ordinary least squares (OLS) regression. This negative effect using the conditional mean of the firm value does not hold across the conditional quantiles of the distribution of Tobin's Q, and this finding is still held under both the linear and the nonlinear quantile regressions. We even lessen the assumption of distributions of multivariate board variables by employing parametric copula-based quantile regressions as well as nonparametric ones. The results support our findings. Our results suggest that estimating the quantile effect of board variables on firm value can provide more meaningful insight than just examining the conditional mean effect.

Keywords: causal inference; board structure; corporate governance; linear quantile regression; copula based quantile regression

1. Introduction

A considerable body of work examines the association between board characteristics and aspects of firm structure (e.g., governance characteristics), corporate conduct (specific events or decisions), or firm value (performance) in terms of the causal relationship by using the mean linear regression method. Therefore, the objective of these studies is to deliver overall insights into which aspects of board governance matter for firm performance. That is, to understand the causal interpretation of which board characteristics matter for firm performance in detail, we need some other statistical method which is more than the sensitivity analysis studies that became popular in the empirical economic growth literature in the late 1990s (Levine and David 1992; Sala-I-Martin 1997).

Bhagat and Bolton (2008) argued that using seven different governance measures, better corporate governance measured by the GIM and BCF indices, stock ownership of board members, and CEO chair separation are significantly positively correlated with better contemporaneous and subsequent operating performance. Pellegrini and Sironi (2017) investigate whether companies who switched to a one-tier board corporate governance system had a lower performance for a sample of unlisted Italian joint-stock companies, and they find that the adoption of a one-tier model of corporate governance between 2003 and 2013 negatively affected firm's performances measured by return on equity (ROE), return on assets (ROA), and revenues from sales. Bhagat and Black (2001) examine the causal relationship between the degree of board independence and the variability of long-term firm performance. The traditional brief on board independence is that an independent board may enhance the financial performance of a firm. However, using board composition data of 957 large U.S. public corporations, they find no evidence that greater board independence leads to improved firm performance. Calabrese and Osmetti (2013) suggest a new GLM regression model that could overcome the drawbacks of the logistic regression model in the probability of default (PD) in credit risk analysis. Although the logistic regression model is the most widely used approach to estimate the PD, it has some drawbacks such as underestimation of default probability which will be critical for banks. Using the quantile function of the GEV distribution that is suitable for modeling for extreme values or rare event data as a link function, they show that the accuracy of the GEV model to identify defaults improves by reducing the sample. During the recent COVID-19 pandemic era, the degree of the stock market crash is also related to some corporate governance measures. Ding et al. (2020) find that the stock prices of firms with anti-takeover devices fall faster during this period, implying that the market reacts differently based on governance structure.

To investigate these issues, we focus on a representative analysis from the board literature: the relation between board characteristics and Tobin's Q (which arguably provides insights as to the association between firm value and the board's advising skills).[1] For these two key issues, we examine the association between the dependent variable and eight board characteristics that are readily available in commercial databases, many of which have been used in prior work. Our study contributes to the corporate governance literature in several ways. First, we include eight well-known board characteristics and find which board variables remain significant to firm value in each quantile stage by employing the quantile regression method. We are sure that the endogeneity issue can be solved by including an overall set of board structures and firm characteristics along with employing the advanced statistical methods, the copula parametric and nonparametric quantile regression methods. A copula is a statistical method that does not require normality, linearity, and independence assumptions to be satisfied in an OLS regression method. Given that it can model dependent structures between various complex correlated variables, copula models are being paid attention in recent years in all areas of human endeavors including energy, environment, social, natural, and physical sciences (Bhatti and Do 2019). The copula method has also been getting popular in finance research areas since Cherubini et al. (2004) published a book titled *Copula Methods in Finance*. At each quantile stage from the bottom five percentile to the top five percentile of the response variable Tobin's Q, we can suggest which board structures are robust and sustainable in the relationship between firm value and board structure. By doing so, we find that the relationships between Tobin's Q and board variables vary across different quantiles and show recognizable non-linear patterns. Especially, the v_outsidepct, the percentage of independent directors in a board, shows that the relationships with Tobin's Q are drastically changed from significantly positive ones to significantly negative ones. These findings no doubt shed light on the relationships between firm value and corporate governance, especially captured by board characteristics.

Second, we identify the quantile causal inferences among board structures and firm characteristics through linear quantile regression and nonlinear copula quantile regression methods. This study is the first to analyze and investigate quantile causal relationships within board-level and firm characteristics using copula nonlinear quantile regression methods. By employing the quantile regression methods, we can explain the controversial evidence of the association between board/director characteristics and firm value by considering eight main board/director characteristics from prior literature.

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[1] We recognize that there is debate in the literature as to exactly what Tobin's Q measures, and that aggregate pay is but one aspect of governance. We simply use these as representative analyses to illustrate the issues.

This article is organized as follows: Section 2 briefly does the literature review for the board of directors. Section 3 proposes the literature review of our research method. Section 4 proposes a linear quantile regression model, copula, and D-vine copulas. Section 5 shows the empirical result of the proposed linear and nonlinear quantile regression model with real data. A brief discussion then follows in Section 6.

2. Previous Literature on Board of Directors

The topics on boards of directors have been widely researched by many corporate governance studies (Bhagat and Bolton 2008; Garner et al. 2017). The major interest in finance literature regarding this topic is which types of boards may maximize shareholders' wealth. As boards are formed by directors, director characteristics have received the most attention.

2.1. Board Size and Independence

Previous studies show that the size of a board of directors is one of the key determinants of firm value. Jensen (1993) theoretically predicts that board size is negatively related to firm value since oversized boards may not effectively monitor CEOs, empirically confirmed by Yermack (1996) and Eisenberg et al. (1998)Eisenberg et al. Similarly, Bhagat and Black (2001) study whether board independence correlates with the long-term performance of large U.S. firms and finds no evidence that firms with more independent boards do not perform better than other firms.

Yet, these studies overlook that the relationship between board size and firm value may be nonlinear. As Raheja (2005) theoretically suggests, better-informed insiders can increase firm value. Hence, the optimal size of the board can be related to effective monitoring between insiders and outsiders as many studies show (Fama and Jensen 1983; Huson et al. 2001). Shareholders also usually prefer outsiders who can be better monitors (Rosenstein and Wyatt 1990; Brickley et al. 1994; Byrd and Hickman 1992).

Outside directors sometimes may not be good monitors, especially if they are appointed by a CEO, they are called gray directors. Core et al. (1999) find that gray directors can act like insiders by showing the relationship between executive compensation and various board characteristics. Hallock (1997) also posits that CEO compensation is high when two CEOs sit on each other's board. Faleye et al. (2011) show that the boards with better expertise of independent directors can prohibit executives from extracting shareholders' wealth.

Yet, if a director with expertise sits on too many boards, they may not be a good monitor since they do not have enough time to monitor all firms. This is called the "busyness" of directors. Fich and Shivdasani (2006) show that busy outside directors, those who sit on more than three boards, may not be good monitors, exhibiting lower profitability. However Field et al. (2013) suggest that busy boards may be beneficial since busy directors are generally more experienced and have more expertise, enabling them to be good advisors.

Firm characteristics may be interacted with board size to determine the monitoring and advising effectiveness. Coles et al. (2008) suggest that complexity plays a crucial role, finding that large and complex firms may benefit from large boards when they are composed of many expert outsiders. Linck et al. (2008) also find that smaller firms with high growth opportunities generally have smaller and less independent boards. Additionally, they show that different regulatory environments may be another determinant for board size and independence. These studies suggest that smaller and more independent boards are not always beneficial (Cheng 2008; Harris and Raviv 2008).

2.2. Board Diversity

A number of studies find that more diverse boards may be beneficial for firm value maximization. Shrader et al. (1997) find that the proportion of female directors is positively related to firm performance measured by accounting profitability. Carter et al. (2003) show that more diverse boards composed of more women, African Americans, Asians, and Hispanics may improve firm values measured by

Tobin's Q. Generally, female directors attend board meeting more frequently and more likely join the monitoring committee (Adams and Ferreira 2009). In addition, female directors bring unique skills to corporate boards more than male directors (Kim and Starks 2016). Furthermore, boards with more female directors improve the informativeness of the firms because female directors are more interested in monitoring the firms (Gul et al. 2011).

And yet, Adams and Ferreira (2009) suggest that gender quotas may negatively impact firm values if the firms have good corporate governance systems. A study by Ahern and Dittmar (2012) finds that gender quotas imposed by the Norwegian government decrease Tobin'Q and lead to negative market reactions. This suggests that too strict regulation may encourage firms to hire younger and less expert directors.

Overall, empirical findings show that a diverse board is generally beneficially especially if the firms are more complex and have weak governance systems.

3. Data & Variables

In this section, we outline our data and sample selection.

3.1. Sample Construction

Our sample is comprised of the intersection of RiskMetrics, Center for Research in Security Prices (hereafter CRSP), and COMPUSTAT from 1998–2008. RiskMetrics provides director and board level information. CRSP provides stock market data, while COMPUSTAT provides firm-level accounting and financial data. We exclude financial and utility companies using 2-digit SIC codes and observations with missing data. This results in a sample of 10,787 firm-year observations.

3.2. Dependent Variables

As we are somewhat agnostic with respect to the specific dependent variable of interest, we select Tobin's Q that commonly appears in the empirical board literature as a holistic measure of firm performance (Yermack 1996; Core et al. 1999; Anderson and Reeb 2003; Ferris et al. 2003). Tobin's Q measures the firm value arguably linked to the board's role as advisors. Tobin's Q is the ratio between a physical asset's market value and its replacement value. We define Tobin's Q as the year-end book value of total, assets less the book value of equity plus the market value of equity all scaled by the book value of total assets as follows[2].

$$Tobin's\ Q = \frac{Total\ Asset - Book\ Value\ of\ Equity + Market\ Value\ of\ Equity}{Total\ Asset}$$

A ratio of 1 or higher generally implies that the firm's market value exceeds that of its book value of assets recorded.

3.3. Board Characteristics

We identify eight commonly available board variables, many of which have appeared in the literature.[3] These include board size, tenure, expertise, gender, etc. Table A1 in Appendix A provides a detailed description of the variables, their construction, and selected papers where they have been used.

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2 Compustat data: Tobin's Q = (data6 − data60 + (data25 × data199))/(data6). data6: Total assets; data25: common shares outstanding; data199: Price close fiscal year; data60: Common equity;

3 For example, we examine variables used by: Yermack (1996); Core et al. (1999); Anderson and Reeb (2003); Ferris et al. (2003); Adams et al. (2005); Fich and Shivdasani (2006); Faleye (2011); Guner et al. (2008); Adams and Ferreira (2009); Agrawal and Nasser (2019); Barnea and Guedj (2009); Chhaochharia and Grinstein (2009); Masulis and Mobbs (2011); Masulis et al. (2012). Appendix A provides a detailed description of the variables, their construction, and selected papers which study each variable.

Table 1 shows summary statistics for Tobin's Q, as well as board and firm characteristics used in this study. The average firm in the sample reports a Tobin's Q of 2.026 with a median score of 1.627. With respect to the number of directors on the board, firms have about nine directors on the board, of whom 72% are independent, 8.8% are outside CEOs. Furthermore, the average age of board members in our sample is about 60, and firms tend to have no directors with financial expertise. There are on average 10.4% of female directors, and 2% are directors whose primary employers' country of origin is not the U.S. in our sample. Lastly, our sample firms tend to be listed on average for 24 years and have more than two business segments. Skewness and Kurtosis show that most of the variables tend to deviate from normality. We remedy this non-normality using the copula approach addressed later.

Table 1. Summary Statistics.

Variables	Min	Q1	Median	Mean	Q3	Max	SD	Skewness	Kurtosis
Q	0.748	1.243	1.627	2.026	2.338	7.917	0.721	1.407	12.094
Size	3	7	9	9.003	10	21	0.989	0.265	5.509
Indepen	0	0.625	0.75	0.720	0.857	1	0.089	−0.513	5.237
Dage	40.333	58	60.667	60.501	63.182	78	1.993	−0.147	4.524
Family	0	0	0	0.088	0	1	0.183	1.338	12.170
Outside	0	0	0	0.088	0.143	0.714	0.077	0.307	4.577
Female	0	0	0.1	0.104	0.167	0.667	0.047	0.069	5.548
Foreign	0	0	0	0.020	0	0.714	0.036	1.226	12.482
Financial	0	0	0	0.000	0	0.167	0.004	11.087	504.405
Sales	4.012	6.373	7.317	7.422	8.387	11.333	0.319	−0.305	6.880
Capx	0.002	0.022	0.037	0.072	0.066	0.926	0.049	1.091	34.786
ROA	−0.188	0.084	0.138	0.140	0.195	0.455	0.067	−0.043	6.867
Fage	1	11	18	24.546	34	88	2.696	−0.494	54.110
Segment	0	1	2	2.349	4	12	0.886	0.258	9.481

This table presents the summary statistics of 14 variables including firm value, firm characteristics, and board characteristics. The sample size is 13,954 and firm and board characteristics are collected from Risk Metrics and COMPUSTAT from 1998 to 2013. The sample data range from "time period." Tobin's Q measures the firm value and is defined as the book value of total assets less book value of equity plus the market value of equity all scaled by the book value of total assets *((data6 − data60 + data25 × data199)/data6 (COMPUSTAT))*, which can be obtained from COMPUSTAT. Variables for firm characteristics are log(Sales), Capital expenditure/sales, Return on assets, firm age, and Business segments. Variables for board characteristics are board size, independent directors, director age, family directors (1/0), independent directors with financial expertise, female directors, foreign directors, and outside-CEO directors. Variable definitions are available in Appendix A.

Table 2 reports the Pearson correlation matrix of our variables.[4] The direction and size of the correlation coefficients for most variables are significantly different from zero at the 1% level. We observe that firms with more directors have higher numbers of female directors, are older, have higher accounting performance (as measured by returns on assets), but are associated with smaller market values (proxied by Tobin's Q) and lower market share. In addition, older firms experience more outside directors on the board, lower market share, smaller market values, and larger capital expenditures. These relations are generally consistent with those documented in the existing literature.

[4] The skewness of our sample in Table 1 suggests that the distribution of mean across our sample is not normal. We provide Kendall and Spearman's two non-parametric correlation coefficients matrix of 14 variables in Table A2 in Appendix A. Although we observe some temporal variation, the relation remains relatively stable.

Table 2. Pearson Correlation Matrix.

Var.	Q	Size	Indepen	Dage	Family	Outside	Female	Foreign	Financial	Sales	Capx	ROA	Fage	Segments
Q	1.000													
Size	−0.095	1.000												
Indepen	−0.071	0.108	1.000											
Dage	−0.129	0.132	0.193	1.000										
Family	−0.024	0.101	−0.265	0.068	1.000									
Outside	0.034	0.128	−0.025	−0.238	−0.030	1.000								
Female	0.001	0.325	0.239	−0.033	0.003	0.018	1.000							
Foreign	0.003	0.091	0.067	0.010	−0.004	−0.035	0.051	1.000						
Financial	0.009	0.009	0.016	0.010	0.014	−0.025	0.018	0.041	1.000					
Sales	−0.085	0.589	0.201	0.140	0.016	0.116	0.378	0.091	0.013	1.000				
Capx	−0.021	−0.037	−0.046	−0.010	−0.006	−0.016	−0.124	0.030	−0.008	−0.108	1.000			
ROA	0.397	0.046	−0.004	−0.020	0.009	0.019	0.089	−0.012	0.003	0.118	0.017	1.000		
Fage	−0.103	0.407	0.241	0.231	0.005	0.148	0.222	0.066	0.016	0.446	−0.049	0.024	1.000	
Segment	−0.134	0.184	0.088	0.103	0.009	0.075	0.051	−0.004	0.016	0.202	−0.092	−0.060	0.240	1.000

This table presents the correlation matrix of 14 variables including firm value, firm characteristics, and board characteristics. The sample size is 13,954 and firm and board characteristics are collected from Risk Metrics and COMPUSTAT. The sample data range from "time period." Tobin's Q measures the firm value and is defined as the book value of total assets less book value of equity plus the market value of equity all scaled by the book value of total assets *((data6 − data60 + data25 × data199)/data6 (COMPUSTAT))*, which can be obtained from COMPUSTAT. Variables for firm characteristics are log(Sales), Capital expenditure/sales, Return on assets, firm age, and business segments. Variables for board characteristics are board size, independent directors, director age, family directors (1/0), independent directors with financial expertise, female directors, foreign directors, and outside-CEO directors. Variable definitions are available in Appendix A.

4. Research Methods

One of the most popular methods in financial analysis as well as corporate governance is linear regression. This method, however, may not be suitable for most financial data which usually are skewed and fat-tailed. In addition, the OLS explores the relationship between the dependent variable, Y, and independent variables, Xs, based on the central tendency. This approach often does not reflect the reality of corporate governance data structure where the dependent variable ranges between upper and lower values and therefore may be heterogeneous across different percentiles of the dependent variables. For example, a board variable having a negative impact based on the central tendency for firm performance may not have the same effect on the firm performance in the upper or lower bounds in terms of the magnitude (even the direction). The failure of the OLS to capture the heterogeneity in the estimated relationship is one of the pieces of evidence that prior literature on corporate governance shows mixed results on the effect of various board variables on dependent variables such as firm's performance, CEO's compensations, and/or firm's events.

Therefore, we employ the quantile regression as a remedy of the OLS. It allows us to generate various estimated coefficients at certain quantile of dependent variables and thus shed light on the impact of board variables on performance indicators under the heterogeneous influences of these variables across quantiles.

Another major obstacle in corporate governance is to control the endogeneity between dependent variables and independent variables. In this paper, we use the vine copula approach to address the potential endogeneity issue (see Section 4.2 for detail). Copula approaches are well-known to address the dependencies by including an overall set of independent variables and dependent variables. One of the issues in practical applications of this approach is how to identify copula functions with larger numbers of variables. Though standard copulas such as Gaussian or Student-t copulas offer some improvement of modeling the dependence among a large number of variables, they have limitations such as parameter restriction. Copula-based modeling is a popular tool in the presence of dependence among variables but is usually applied to pairs of securities. We resolve this potential limitation using the vine copulas. Vine copulas (Bhatti and Do 2020; Kurowicka and Cooke 2007) are a flexible graphical model for addressing multivariate copulas based on bivariate copulas, so-called pair-copulas. Decomposing multivariate probability density into bivariate copulas by independently choosing pair-copulas from the other, a vine copula allows vast flexibility in modeling dependence. This approach is especially useful when the data structure shows asymmetries and tail dependence. The typical issues raised in the quantile regression such as quantile crossing or transformation are naturally taken care of under the vine copulas. Also, interactions among variables as well as the collinearity of variables are automatically assumed in the model, let alone fast and accurate variable selection by maximizing the conditional log-likelihood. In sum, vine copulas implement the benefits of multivariate copula modeling where the modeling of the marginal distributions can be easily separated from the dependence modeling in a bivariate copula setting where a vast variety of copula families is studied.

4.1. Quantile Regression

A quantile regression method is an extension of the classical regression that offers information on the whole conditional distribution of the response variable. While in the classical regression case, the goal could be to approximate the conditional mean, in quantile regression, the focus could be to approximate the conditional quantile functions of a response variable Y given a set of variables $\mathbf{X}$. The quantile regression model can capture the information associated with the location, scale, and shapeshift of the conditional distribution. The method is especially useful when heteroskedasticity is involved and when the usual parametric assumptions do not hold in homogeneous regression models.

It is also well known that no error distribution is imposed in quantile regression. We start with an equivalent definition of the θ-quantiles (Davino et al. 2013).

$$\mathrm{q}_{\theta} = \operatorname{argmin}_{c} E[\rho_{\theta}(\mathrm{Y}-\mathrm{c})]$$

where

$$\mathrm{q}_{\theta}(y) = [(1-\theta)\mathbf{I}(y \leq 0) + \theta\mathbf{I}(y > 0)]\big|y\big|$$

denotes an asymmetric absolute loss function. The conditional quantile is defined as

$$\hat{q}_{\theta}(\theta, \mathbf{X}) = \operatorname{argmin}_{Q_Y(\theta, \mathbf{X})} E[\rho_{\theta}(Y - Q_Y(\theta, \mathbf{X}))]$$

where $Q_Y(\theta, \mathbf{X}) = Q_{\theta}[Y|\mathbf{X} = \mathbf{x}]$ denotes the conditional quantile function for the θ-quantile. For the linear model case $Q_{\theta}(Y|\mathbf{X}) = \mathbf{X}\beta(\theta)$ we have

$$\hat{\beta}_{\theta}(\theta) = \operatorname{argmin}_{\beta} E[\rho_{\theta}(Y - \mathbf{X}\beta)]$$

where the parameters and the estimators correspond to the specific quantile. In the quantile regression linear model, the parameter estimates have the same interpretation as in the classical linear models. The coefficient

$$\hat{\beta}_{\theta}(\theta) = \frac{\partial Q_{\theta}(Y|\mathbf{X})}{\partial \mathbf{x}_i}$$

indicates the change rate of the quantile in the dependent variable distribution per unit change of the i-th independent variable. Quantile regression estimators have the same equivariance property as the OLS estimators, but the equivariance to monotone transformations is specific only to quantile regression (see Koenker 2005 for more details).

4.2. Vine Copula Based Quantile Regression

4.2.1. Copula

A copula is a multivariate distribution function defined on the unit hypercube, with uniformly distributed marginals. As an illustration, we focus on a bivariate copula $C(\cdot,\cdot)$.

Any bivariate distribution function, $F_{XY}(X,Y)$, can be represented as a function of its marginal distribution of X and Y, $F_X(X)$ and $F_Y(Y)$, by using a two-dimensional copula.

$$F_{XY}(X,Y) = C(F_X(X), F_Y(Y)) = C(u,v)$$

Selected examples

$$Clayton:\ C^{C}(u,\ v,\theta\ ) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{\frac{-1}{\theta}}, \qquad for\ \theta > 0$$

$$Gumbel:\ C^{C}(u,\ v,\theta\ ) = exp\left[-\left((-\log u)^{\theta} + (-\log v)^{\theta}\right)^{\frac{1}{\theta}}\right], \qquad for\ \theta \geq 1$$

$$Joe:\ C^{J}(u,\ v,\theta\ ) = 1 - \left[(1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta}\right], \quad for\ \theta \geq 1$$

4.2.2. D-Vine Quantile Regression (DVQR)

The main purpose of a D-vine copula based on quantile regression is to predict the quantile of a response variable Y given the outcome of some predictor variables $X_1, \ldots, X_d,\ d = 1,$ where $Y \sim F_Y$

and $X_j \sim F_j,\ j = 1, \ldots, d$. Hence, the focus of interest lies on the joint modeling of Y and X and in particular on the conditional quantile function for $\alpha \in (0,\ 1)$:

$$q_\alpha(X_1, \ldots, X_d) = F^{-1}_{Y|X_1,\ldots,X_d}(\alpha|X_1, \ldots, X_d)$$

Using the probability integral transforms (PIT) $V := F_Y(Y)$ and $U_j := F_j\big(X_j\big)$ with corresponding PIT values $V := F_Y(y)$ and $u_j := F_j\big(x_j\big)$, it follows that

$$F_{Y|X_1,\ldots,X_d}(y|X_1, \ldots, X_d) = C_{V|U_1,\ldots,U_d}(V|U_1, \ldots, U_d)$$

Therefore, inversion yields

$$F^{-1}_{Y|X_1,\ldots,X_d}(\alpha|X_1, \ldots, X_d) = F^{-1}_Y\Big(C^{-1}_{V|U_1,\ldots,U_d}(\alpha|U_1, \ldots, U_d)\Big)$$

Now, we can obtain an estimate of the conditional quantile function by estimating the marginals F_y and $F_{j,}\ j = 1, \ldots$, d, as well as the copula $C_{v,}\ u_1, \ldots u_d$ and plugging them in:

$$\hat{q}_\alpha(X_1, \ldots, X_d) := \widehat{F}^{-1}_Y\Big(\hat{C}^{-1}_{V|U_1,\ldots,U_d}\Big(\alpha\Big|\widehat{U}_1, \ldots, \widehat{U}_d\Big)\Big),$$

where $\widehat{U}_j = \widehat{F}_j\big(x_j\big)$ is the estimated PIT of $X_{j,}\ j = 1, \ldots, d$.

5. Empirical Analysis and Results

We begin our analysis by examining the relationship between Tobin's Q and corporate board characteristics using the sample of RiskMetrics firms with available data. We regress firms' Tobin's Q on a set of observable firm-level board characteristic variables that have previously been found to be significant determinants of firm's Tobin's Q. Before testing the multivariate analysis, we first run a quantile regression for each of the individual variables in Figure 1. The figures show that the coefficient of each variable across quantile points has drastically changed. The findings indicate that the relationships between Tobin's Q and each variable vary across the different quantile values. Except for the v_financialoutpct, the absolute magnitudes of the coefficients of board variables are increased as the quantile increases. For the variables of firm characteristics, c_capx_sale and c_fichroa even show that the signs of coefficients are changed from negative ones to positives while the coefficients of c_lnsale, c_firmage, and c_segment_bus are near zero in lower quantiles to significant negative values as quantiles increase.

As for the board variables, v_bsize, v_age, and v_relativeflag show a near-zero value of their coefficients in the lower quantiles and have significant negative values toward upper quantiles. Especially, the coefficients of v_outsidepct, the percentage independent directors, are changed from significant positive ones to negatives with statistical significance. We argue that these results somewhat resolve the contradictory relationships between proxies of firm value and the role of independent directors.

Table 3 examines this dynamic relationship of board variables with Tobin's Q on the multivariate level and reports the results of both OLS regression and quantile linear regressions. All regression models include year-fixed effects and firm-fixed effects to control unobservable variation in firm-specific environments and other plausible year differences related to a firm's Tobin's Q. In general, the multivariate analysis in Table 3 shows similar results with the quantile regressions with individual variables. The OLS regression shows that the impact of board governance factors on firm performance tends to be evident in most of the variables in relation to Tobin's Q, except the coefficient of v_femalepct and v_financialoutpct, suggesting that sitting on more female committees does not add value for the firm's performance. Specifically, v_bisze, v_outsidepct, v_age, and v_relativeflag are statistically negatively associated with Tobin's Q while v_ceodirector and v_foreignpct show a statistically positive association. These OLS results address the relationship between Tobin's Q and board variables by regressing Tobin's Q on the conditional mean on board variables. From the descriptive statistics in Table 1, we are aware of the non-normality of these variables and expect quantile regressions to be a possible improvement to treat non-normality, which provides the understanding of relationships between variables outside of the mean of the data.

Thus, we next test the relationship in our baseline regression utilizing quantile regression. As mentioned before, OLS gives the relationship around the conditional mean of the distribution and hence leaves out the extreme cases. We, thus, specify various quantiles, from 5th to 95th incremented by 5th percentage, using the same list of explanatory variables for each of these quantiles, and examine whether the relationship between board characteristics and performance is unique across all scales of performance. The second column to the last present the estimates for the linear quantile regression with intercept and coefficients regarding the impact of board and firm characteristics on firm performance.

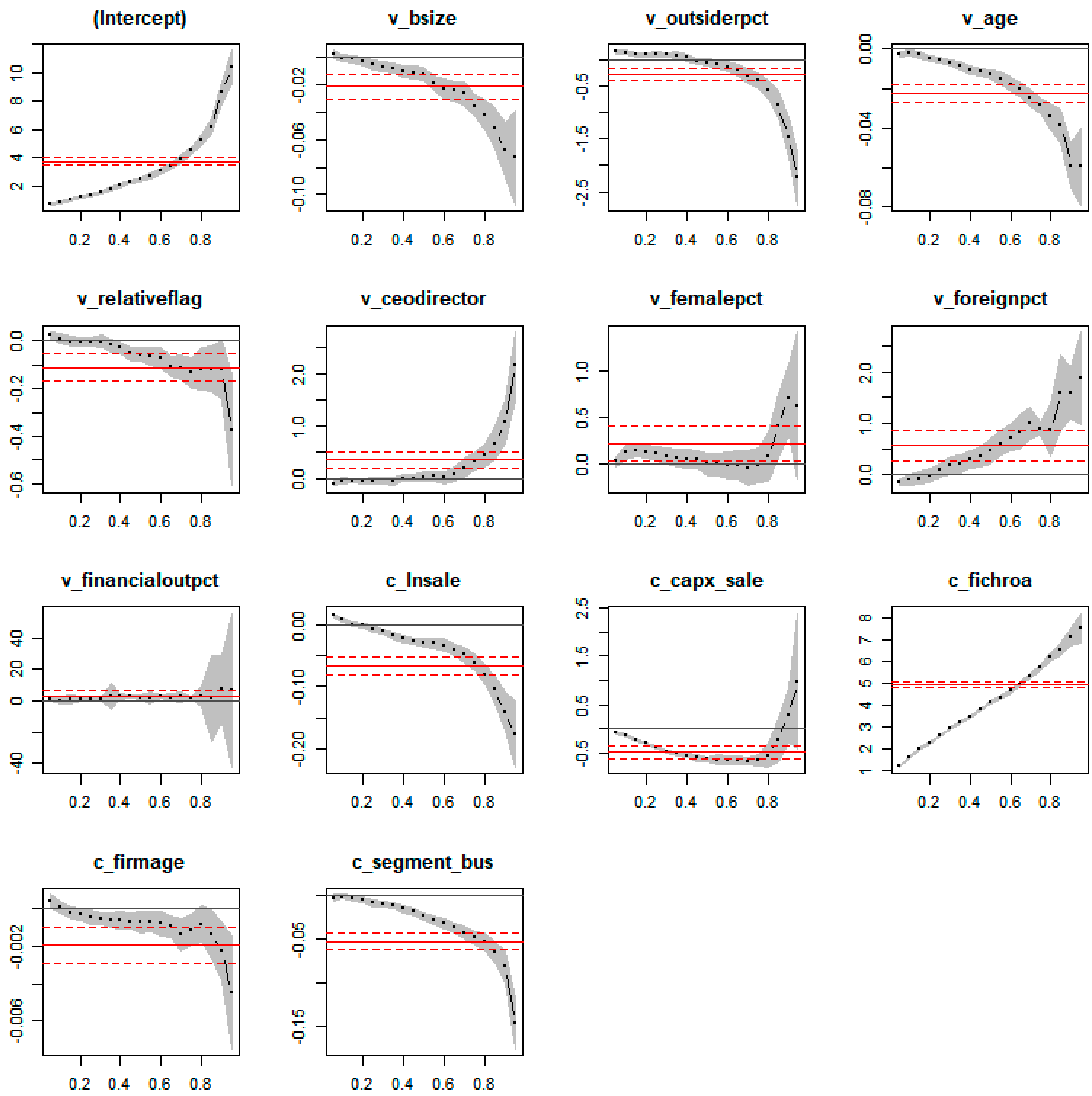

Figure 1. Quantile Regression Plots for Individual variables (tau = seq (0.05, 0.95, by = 0.05)).

The most interesting finding of Table 4 is the results of v_outsidepct (percentage of independent directors in a board). While the OLS regression coefficient of the variable shows a significant negative value, the coefficients of a series of quantile regressions show significant positive values up to the 35th quantile, an insignificant one until the median, and significantly negative values in the quantiles above the median. Furthermore, the magnitude of the coefficients monotonically decreased from 0.144 to −2.246. According to the dynamics of the coefficients of v_outsidepct, we argue that the empirical outcomes of the value-enhancement of independent directors are not uniform but are affected by how many seats are occupied by independent directors in a board. The lower percentage of independent directors plays a role to enhance firm value, but this positive relationship will disappear as the portion reaches half of all outside board members, and it is detrimental to the firm value as the independent directors are dominant directors on the board.

We first observe that v_size, v_age, c_capx_sale, and c_segment_bus generally show a negative and significant impact on Tobin's Q, and this negative impact is stronger for the upper quantiles. On the other hand, when considering profitability, v_fichroa shows a positive and significant impact with an increasing pattern across quantiles. We also see that while v_relativeflag (v_foreignpct) has no significant effect on Tobin's Q at lower quantiles, its effect is negative (positive) and significant when the Tobin's Q ratio starts attaining the 40th (30th) quantile.

With respect to gender diversity (v_femalepct), it has a significant positive impact on the lower quantiles but becomes a non-significant and decreasing pattern for the upper quantile. Sale (c_lnsale) shows in general a positive and significant impact on Tobin's Q in lower quantiles. However, in fact, the pattern shows a strong decreasing pattern as the quantile changes. The results for linear quantile regression models can be confirmed in Figure 1. The estimated value of intercept and c_fichora coefficients increased over the unit of quantile increasing. The estimated value of the v_age coefficient decreased over the unit of quantile increasing. The estimated value of the c_capx_sale coefficient decreased and then increased around the 80 percentile over the unit of quantile increasing.

Table 4 shows the summary statistics with the fitted values of Quantile regressions, Table 5 shows the summary statistics with the fitted values of parametric Copula based Quantile regressions.

Table 6 shows the summary statistics with the fitted values of nonparametric Copula based Quantile regressions. Comparing the summary statistics in Tables 4–6, we can conclude that the spread (standard deviation and range) of the fitted values by parametric (nonparametric) Copula based Quantile regressions is much higher than the spread (standard deviation and range) of the fitted values by linear Quantile regressions even though the mean and median of the fitted values by parametric (nonparametric) Copula based Quantile regressions are slightly smaller than the mean and median of the fitted values by linear Quantile regressions. Figures 2–4 confirm the findings from Tables 4–6. Figures 5 and 6 show the estimated coefficients of the parametric (nonparametric) Copula based Quantile regressions. They show that the x-axis values are the estimated coefficient of the covariate, and the colored nonlinear lines are copula quantile regression lines. The pattern of the estimated coefficients of the statistically significant variables over the unit of quantile increasing is very similar to the result by linear Quantile regressions. The two meaningful results obtained by employing the copula quantile regression are that first, we can make sure that the result by linear quantile regression is reliable and robust, and second, we can have a better understanding of corporate finance by using a graphical display of each covariate, which is statistically significant to our target response firm's Tobin's Q variable over each quantile plot.

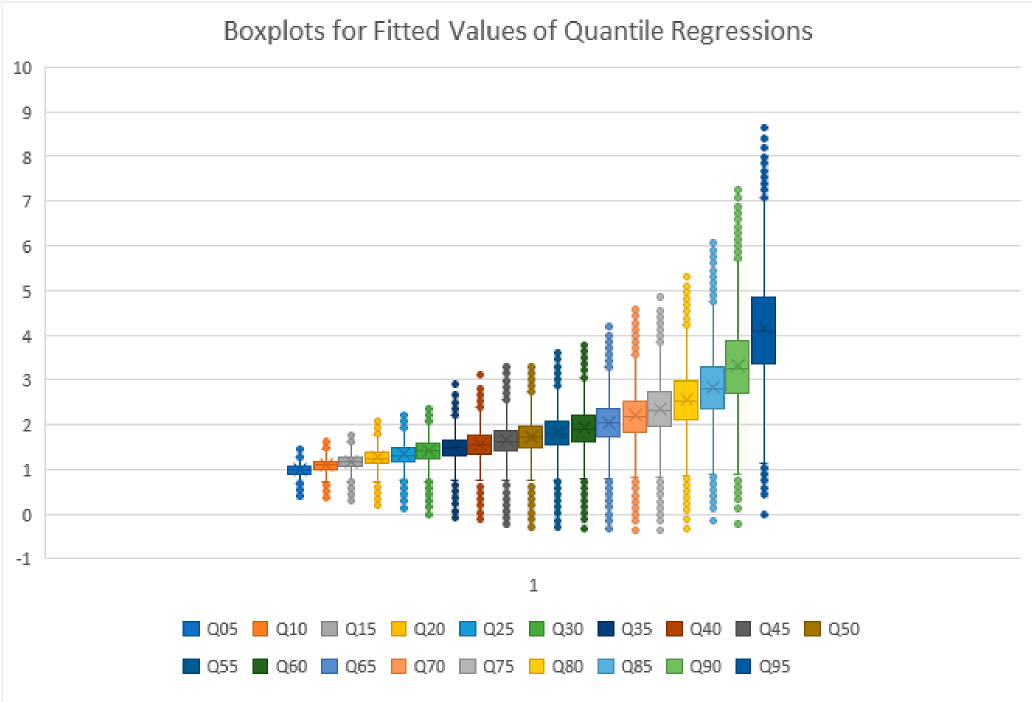

Figure 2. Plots for Fitted values of Quantile Regressions.

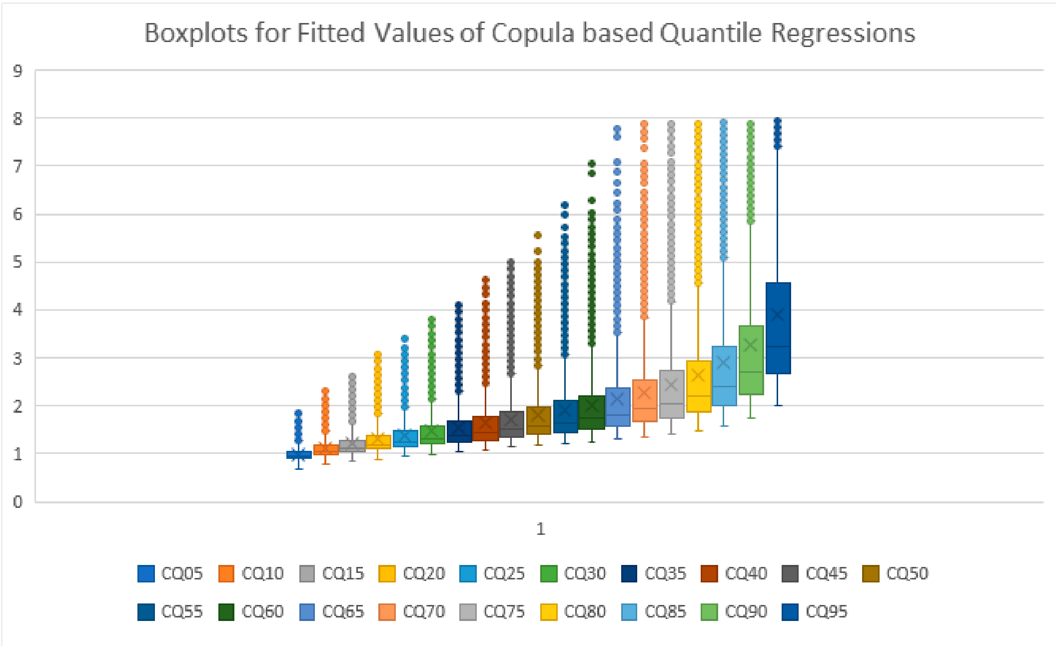

Figure 3. Plots for Fitted values of Copula based Quantile Regressions (Parametric).

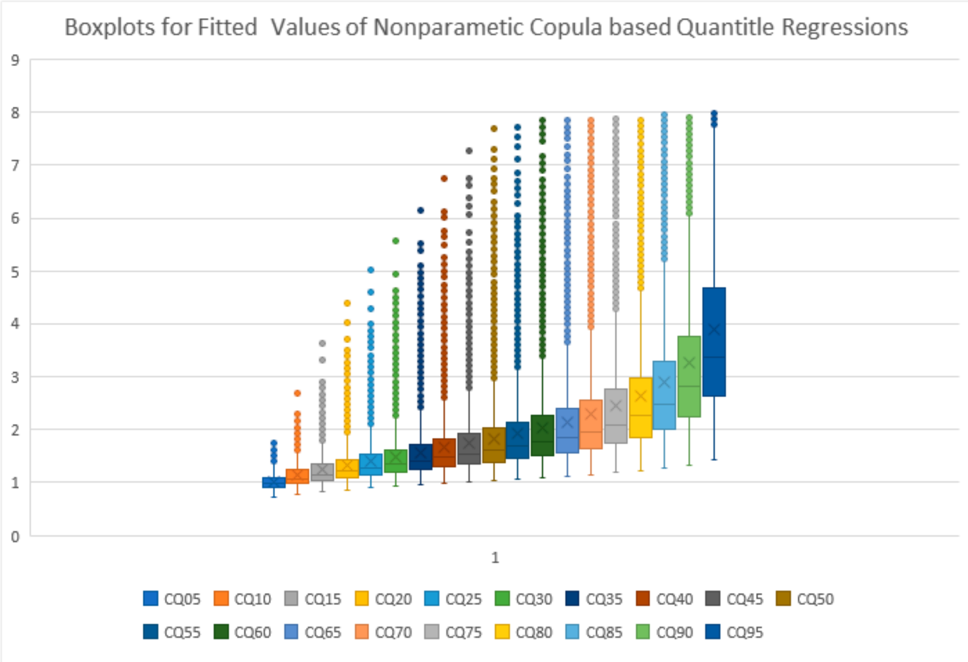

Figure 4. Plots for Fitted values of Nonparametric Copula based Quantile Regressions.

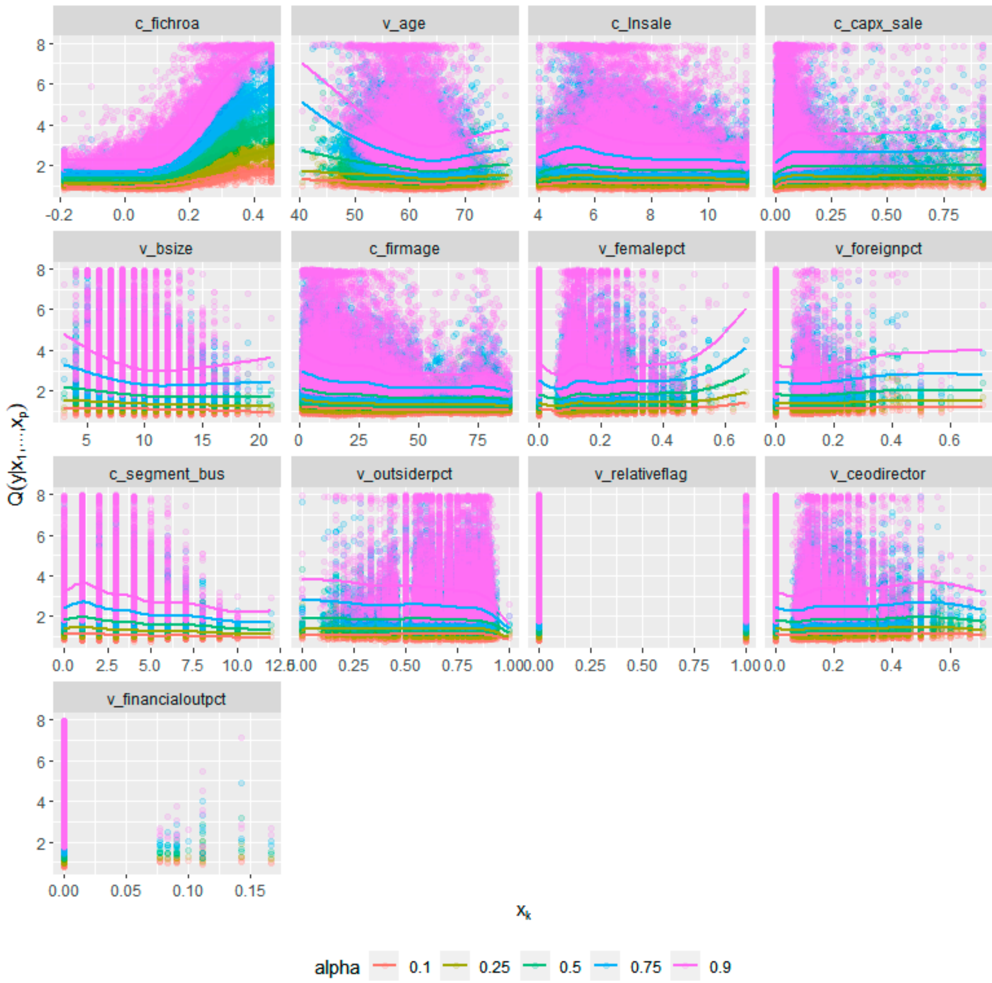

Figure 5. Plots for Copula based Quantile Regressions (Parametric).

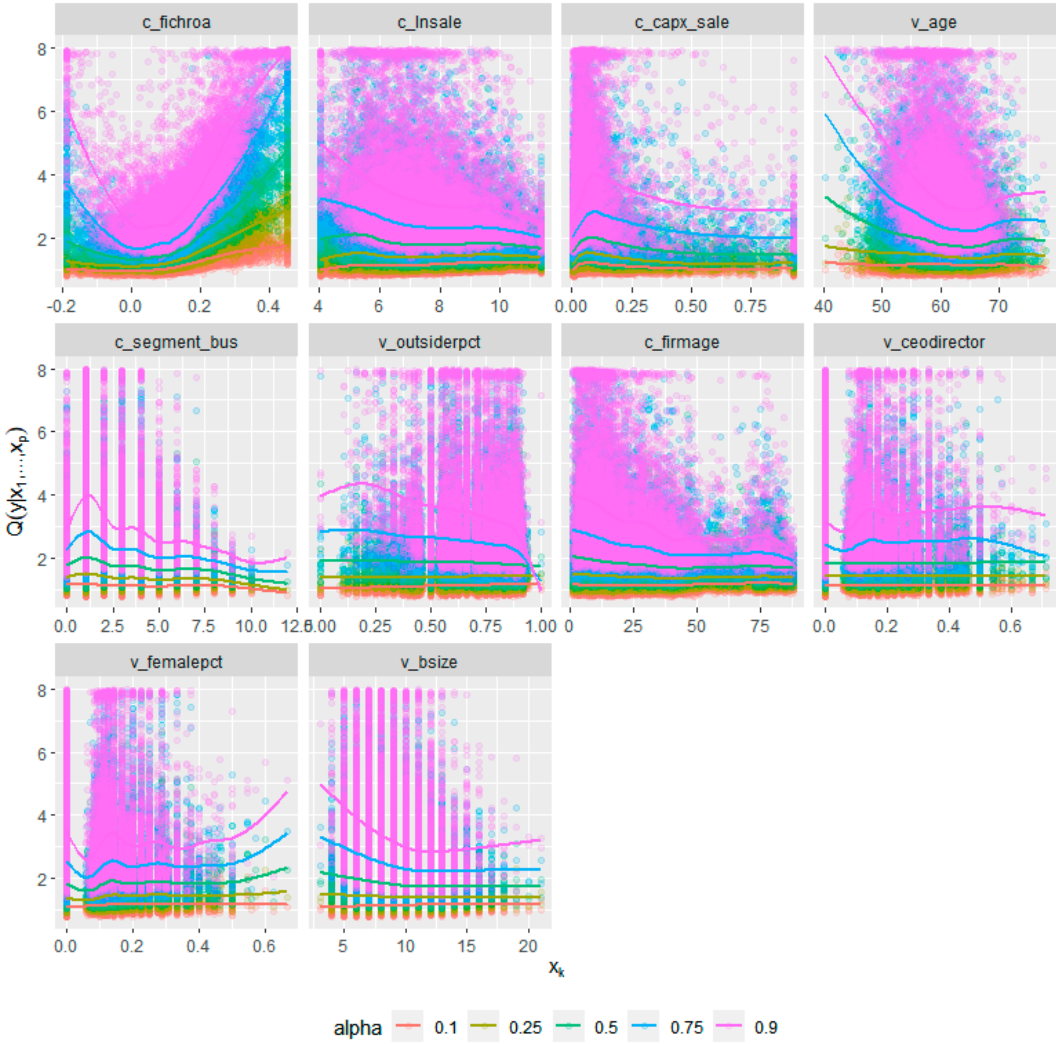

Figure 6. Plots for Nonparametric Copula based Quantile Regressions.

Table 3. The summary statistics of OLS and Quantile regression

	OLS	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
(Intercept)	3.741 ***	0.749 ***	0.843 ***	1.030 ***	1.228 ***	1.377 ***	1.558 ***	1.783 ***	2.039 ***	2.244 ***	2.424 ***	2.688 ***	3.053 ***	3.385 ***	3.908 ***	4.480 ***	5.268 ***	6.210 ***	8.625 ***	10.390 ***
	(0.157)	(0.056)	(0.059)	(0.061)	(0.055)	(0.062)	(0.066)	(0.087)	(0.086)	(0.074)	(0.099)	(0.125)	(0.143)	(0.144)	(0.177)	(0.156)	(0.246)	(0.331)	(0.471)	(0.739)
v_bsize	−0.021 ***	0.003	0.000	−0.001	−0.003 *	−0.005 **	−0.007 ***	−0.008 ***	−0.010 ***	−0.011 ***	−0.012 ***	−0.019 ***	−0.023 ***	−0.025 ***	−0.027 ***	−0.035 ***	−0.042 ***	−0.052 ***	−0.068 ***	−0.073 ***
	(0.005)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.005)	(0.007)	(0.009)	(0.012)	(0.021)
v_outsiderpct	−0.279 ***	0.144 ***	0.134 ***	0.098 ***	0.098 ***	0.105 ***	0.098 ***	0.081 **	0.040	−0.038	−0.053	−0.082 *	−0.133 **	−0.199 ***	−0.313 ***	−0.404 ***	−0.572 ***	−0.848 ***	−1.472 ***	−2.246 ***
	(0.068)	(0.024)	(0.025)	(0.026)	(0.024)	(0.028)	(0.026)	(0.040)	(0.040)	(0.040)	(0.042)	(0.050)	(0.064)	(0.061)	(0.080)	(0.087)	(0.112)	(0.152)	(0.226)	(0.307)
v_age	−0.023 ***	−0.003 ***	−0.002 **	−0.003 ***	−0.005 ***	−0.006 ***	−0.007 ***	−0.009 ***	−0.011 ***	−0.012 ***	−0.013 ***	−0.015 ***	−0.018 ***	−0.020 ***	−0.024 ***	−0.028 ***	−0.034 ***	−0.039 ***	−0.059 ***	−0.059 ***
	(0.003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.004)	(0.005)	(0.007)	(0.012)
v_relativeflag	−0.113 ***	0.024 **	0.010	−0.001	−0.004	−0.004	−0.002	−0.018	−0.027 *	−0.052 ***	−0.056 ***	−0.066 ***	−0.072 ***	−0.108 ***	−0.111 ***	−0.134 ***	−0.117 **	−0.118 **	−0.123	−0.374 ***
	(0.035)	(0.010)	(0.013)	(0.014)	(0.012)	(0.013)	(0.016)	(0.017)	(0.016)	(0.017)	(0.016)	(0.022)	(0.025)	(0.027)	(0.033)	(0.039)	(0.055)	(0.058)	(0.077)	(0.144)
v_ceodirector	0.341 ***	−0.115 ***	−0.052	−0.053 *	−0.055 *	−0.053	−0.043	−0.059	0.005	0.004	0.028	0.043	0.009	0.084	0.196 *	0.330 ***	0.430 ***	0.669 ***	1.077 ***	2.154 ***
	(0.086)	(0.027)	(0.037)	(0.031)	(0.033)	(0.038)	(0.041)	(0.047)	(0.049)	(0.038)	(0.057)	(0.061)	(0.073)	(0.081)	(0.104)	(0.099)	(0.133)	(0.200)	(0.260)	(0.396)
v_femalepct	0.218	0.026	0.128 ***	0.143 ***	0.127 ***	0.110 **	0.086 *	0.060	0.054	0.047	−0.002	−0.004	−0.017	−0.020	−0.048	−0.013	0.082	0.405 *	0.695 ***	0.626
	(0.111)	(0.036)	(0.045)	(0.041)	(0.042)	(0.049)	(0.052)	(0.055)	(0.060)	(0.058)	(0.071)	(0.068)	(0.084)	(0.094)	(0.116)	(0.115)	(0.160)	(0.212)	(0.232)	(0.481)
v_foreignpct	0.560 **	−0.155 ***	−0.097	−0.064	−0.025	0.084	0.177 *	0.201 *	0.292 ***	0.345 ***	0.465 ***	0.611 ***	0.726 ***	0.815 ***	1.008 ***	0.896 ***	0.859 ***	1.592 ***	1.588 ***	1.885 ***
	(0.179)	(0.047)	(0.069)	(0.072)	(0.089)	(0.076)	(0.096)	(0.109)	(0.098)	(0.116)	(0.155)	(0.141)	(0.159)	(0.207)	(0.188)	(0.078)	(0.308)	(0.449)	(0.314)	(0.549)
v_financialoutpct	3.268	1.277 *	0.573	1.177	1.288 *	1.100	0.715	3.053	3.149 ***	2.986 ***	2.363 ***	2.071	2.888 ***	2.189	2.544	1.829 ***	3.002	1.645	7.543	6.717
	(2.076)	(0.674)	(1.216)	(1.543)	(0.744)	(0.847)	(0.590)	(4.955)	(1.098)	(0.334)	(0.779)	(2.253)	(0.480)	(1.495)	(1.827)	(0.555)	(3.760)	(16.667)	(13.134)	(29.937)
c_lnsale	−0.067 ***	0.016 ***	0.008 ***	0.001	−0.001	−0.006 **	−0.011 ***	−0.017 ***	−0.023 ***	−0.027 ***	−0.030 ***	−0.030 ***	−0.032 ***	−0.041 ***	−0.048 ***	−0.060 ***	−0.080 ***	−0.105 ***	−0.140 ***	−0.178 ***
	(0.008)	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.004)	(0.003)	(0.004)	(0.005)	(0.006)	(0.006)	(0.008)	(0.007)	(0.010)	(0.014)	(0.019)	(0.033)
c_capx_sale	−0.475 ***	−0.087 ***	−0.142 ***	−0.217 ***	−0.297 ***	−0.362 ***	−0.450 ***	−0.501 ***	−0.542 ***	−0.596 ***	−0.624 ***	−0.635 ***	−0.639 ***	−0.655 ***	−0.661 ***	−0.637 ***	−0.540 ***	−0.221	0.287	0.986
	(0.077)	(0.019)	(0.019)	(0.017)	(0.016)	(0.017)	(0.018)	(0.025)	(0.023)	(0.042)	(0.025)	(0.060)	(0.059)	(0.053)	(0.051)	(0.075)	(0.157)	(0.278)	(0.341)	(0.854)
c_fichroa	4.951 ***	1.195 ***	1.621 ***	2.034 ***	2.309 ***	2.591 ***	2.956 ***	3.220 ***	3.485 ***	3.847 ***	4.158 ***	4.380 ***	4.665 ***	4.988 ***	5.365 ***	5.764 ***	6.213 ***	6.573 ***	7.161 ***	7.571 ***
	(0.095)	(0.030)	(0.036)	(0.033)	(0.034)	(0.037)	(0.038)	(0.041)	(0.044)	(0.042)	(0.047)	(0.054)	(0.066)	(0.066)	(0.085)	(0.075)	(0.126)	(0.190)	(0.243)	(0.414)
c_firmage	−0.002 **	0.000 *	0.000	0.000	0.000	0.000 *	−0.001 ***	−0.001 ***	−0.001 **	−0.001 ***	−0.001 **	−0.001 **	−0.001 **	−0.001 ***	−0.001 ***	−0.001 **	−0.001	−0.001 *	−0.002 **	−0.005 **
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.002)
c_segment_bus	−0.053 ***	−0.003	−0.002	−0.003 *	−0.006 ***	−0.009 ***	−0.011 ***	−0.012 ***	−0.016 ***	−0.019 ***	−0.024 ***	−0.029 ***	−0.033 ***	−0.038 ***	−0.043 ***	−0.049 ***	−0.053 ***	−0.065 ***	−0.082 ***	−0.147 ***
	(0.006)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.005)	(0.005)	(0.006)	(0.007)	(0.011)	(0.018)

Note: ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels, respectively. Standard errors are in parentheses.

Table 4. Summary Statistics with Fitted values of Quantile Regression.

	Q05	Q10	Q15	Q20	Q25	Q30	Q35	Q40	Q45	Q50	Q55	Q60	Q65	Q70	Q75	Q80	Q85	Q90	Q95
Mean	0.981	1.089	1.180	1.258	1.334	1.412	1.486	1.567	1.654	1.742	1.829	1.932	2.054	2.198	2.356	2.563	2.847	3.319	4.142
Standard Error	0.001	0.001	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.006	0.007	0.008	0.010
Median	0.983	1.089	1.177	1.253	1.326	1.402	1.473	1.553	1.636	1.721	1.805	1.903	2.024	2.163	2.321	2.519	2.800	3.262	4.083
Standard Deviation	0.133	0.172	0.212	0.241	0.271	0.309	0.338	0.368	0.407	0.441	0.468	0.503	0.542	0.592	0.645	0.708	0.783	0.944	1.143
Sample Variance	0.018	0.030	0.045	0.058	0.073	0.096	0.114	0.135	0.166	0.195	0.219	0.253	0.294	0.350	0.416	0.501	0.612	0.891	1.306
Kurtosis	1.317	1.487	1.556	1.585	1.563	1.576	1.545	1.485	1.450	1.403	1.336	1.262	1.194	1.098	0.995	0.885	0.676	0.414	0.191
Skewness	−0.222	−0.122	−0.027	0.002	0.048	0.066	0.103	0.138	0.154	0.170	0.190	0.215	0.235	0.260	0.283	0.309	0.327	0.341	0.292
Range	1.063	1.312	1.622	1.893	2.120	2.491	2.979	3.237	3.534	3.699	3.903	4.211	4.520	4.948	5.320	5.758	6.213	7.467	8.671
Minimum	0.395	0.362	0.290	0.207	0.138	0.007	−0.067	−0.131	−0.232	−0.282	−0.290	−0.313	−0.340	−0.371	−0.370	−0.321	−0.133	−0.221	−0.018
Maximum	1.458	1.674	1.911	2.100	2.258	2.498	2.911	3.107	3.302	3.417	3.613	3.898	4.180	4.577	4.949	5.436	6.080	7.247	8.653

Table 5. Summary Statistics with Fitted values of Copula Quantile Regression.

	Q05	Q10	Q15	Q20	Q25	Q30	Q35	Q40	Q45	Q50	Q55	Q60	Q65	Q70	Q75	Q80	Q85	Q90	Q95
Mean	0.999	1.121	1.220	1.308	1.392	1.473	1.555	1.638	1.724	1.814	1.912	2.018	2.137	2.273	2.435	2.633	2.889	3.252	3.883
Standard Error	0.001	0.002	0.002	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.006	0.006	0.007	0.008	0.009	0.010	0.011	0.012	0.014
Median	0.956	1.053	1.129	1.196	1.258	1.319	1.380	1.441	1.506	1.576	1.651	1.734	1.827	1.934	2.057	2.212	2.419	2.710	3.239
Standard Deviation	0.146	0.207	0.263	0.316	0.367	0.417	0.467	0.517	0.570	0.626	0.687	0.754	0.829	0.916	1.019	1.142	1.287	1.456	1.651
Sample Variance	0.021	0.043	0.069	0.100	0.134	0.174	0.218	0.268	0.325	0.392	0.471	0.568	0.688	0.840	1.038	1.304	1.655	2.120	2.724
Kurtosis	3.037	3.918	4.216	4.228	4.121	3.998	3.908	3.873	3.888	3.953	4.088	4.296	4.514	4.592	4.520	4.167	3.365	2.144	0.435
Skewness	1.633	1.864	1.956	1.985	1.983	1.970	1.956	1.948	1.944	1.948	1.960	1.982	2.007	2.023	2.025	1.990	1.879	1.662	1.246
Range	1.152	1.552	1.910	2.225	2.531	2.822	3.174	3.548	3.946	4.400	5.079	5.876	6.461	6.506	6.487	6.440	6.363	6.238	5.987
Minimum	0.679	0.777	0.835	0.890	0.942	0.992	1.041	1.088	1.136	1.180	1.219	1.260	1.306	1.358	1.419	1.493	1.590	1.733	2.003
Maximum	1.831	2.329	2.745	3.114	3.473	3.814	4.214	4.636	5.082	5.581	6.298	7.136	7.767	7.864	7.905	7.932	7.953	7.971	7.990

Table 6. Summary Statistics with Fitted values of Nonparametric Copula based Quantile Regressions.

	Q05	Q10	Q15	Q20	Q25	Q30	Q35	Q40	Q45	Q50	Q55	Q60	Q65	Q70	Q75	Q80	Q85	Q90	Q95
Mean	1.017	1.139	1.237	1.326	1.410	1.491	1.572	1.654	1.739	1.829	1.926	2.031	2.150	2.286	2.448	2.646	2.904	3.270	3.901
Standard Error	0.001	0.002	0.002	0.003	0.003	0.004	0.004	0.005	0.005	0.006	0.006	0.007	0.008	0.008	0.009	0.010	0.011	0.012	0.014
Median	0.976	1.076	1.152	1.219	1.282	1.344	1.408	1.472	1.538	1.608	1.682	1.764	1.856	1.963	2.096	2.262	2.489	2.809	3.376
Standard Deviation	0.161	0.215	0.273	0.332	0.390	0.446	0.502	0.558	0.615	0.676	0.741	0.810	0.886	0.971	1.065	1.173	1.302	1.459	1.671
Sample Variance	0.026	0.046	0.074	0.110	0.152	0.199	0.252	0.311	0.379	0.457	0.549	0.656	0.786	0.943	1.134	1.376	1.695	2.127	2.791
Kurtosis	1.035	2.122	4.046	5.696	6.682	7.281	7.665	7.954	8.213	8.419	8.522	8.306	7.817	7.063	6.050	4.890	3.592	2.057	0.228
Skewness	1.078	1.379	1.726	1.994	2.155	2.254	2.316	2.359	2.394	2.422	2.441	2.431	2.394	2.324	2.213	2.064	1.864	1.567	1.085
Range	1.076	1.918	2.806	3.517	4.119	4.640	5.198	5.764	6.246	6.648	6.727	6.745	6.748	6.742	6.731	6.712	6.683	6.639	6.557
Minimum	0.713	0.773	0.822	0.863	0.898	0.932	0.959	0.984	1.011	1.039	1.071	1.100	1.126	1.153	1.186	1.224	1.276	1.340	1.440
Maximum	1.789	2.690	3.628	4.380	5.017	5.572	6.157	6.748	7.257	7.687	7.798	7.845	7.874	7.895	7.916	7.936	7.959	7.978	7.997

6. Conclusions

Many research studies focus on the association between specific board characteristics and firm value. A common approach in this field is to include a certain type of board characteristic as well as factors associated with firm value. Recent evidence shows contradictory results on the effects of some board characteristics on firm value. These could be due to different sample sizes, length of time periods, or model specifications.

To mitigate such issues, this study first incorporates 13 well-known board and firm characteristics in our quantile regression models. We examine the relative strengths of their effects on firm value, as well as their consistency, using the linear and nonlinear quantile regression method. To reconcile some conflicting evidence from prior literature, we examined a linear and nonlinear quantile causal inference among firm value, board characteristics, and firm characteristics. Most studies in corporate governance examine a one-sided relationship: the impact of board traits on firm performance or firm value after controlling for firm characteristics.

We find that the effect of board independence on Tobin's Q, a proxy of firm value, is negatively associated with firm value, Tobin's Q using OLS regression. This negative effect using the conditional mean of the firm value does not hold across the conditional quantiles of the distribution of Tobin's Q, and this finding is still held under both the linear and the nonlinear quantile regressions. We even lessen the assumption of distributions of multivariate board variables by employing parametric copula-based quantile regressions as well as nonparametric ones. The results support our findings. According to the dynamics of the coefficients of board independence, we find that the lower percentage of independent directors plays a role to enhance firm value, but this positive relationship will disappear as the portion reaches half of all outside board members, and it is detrimental to the firm value as the independent directors are dominant directors in the board.

This study provides a novel perspective by considering the actual linear and nonlinear quantile causal relationship of the board and firm variables. By comparing the summary statistics of the fitted values of nonlinear copula parametric (nonparametric) quantile regression, we conclude that linear quantile regression is a useful and robust statistical method to corporate finance by comparing with nonlinear copula parametric (nonparametric) quantile regression. We can deliver meaningful and useful results to researchers in corporate finance.

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Appendix A

Table A1. Variable descriptions.

Variables	Descriptions (Sources)	Selected Studies
Dependent variable		
Q	(Book value of total assets − Book value of equity + Market value of equity)/Book value of total assets: (data6 − data60 + data25 × data199)/data6 (Compustat)	Yermack (1996)
Board Characteristics		
Size	The number of directors on the board (RiskMetrics)	Yermack (1996)
Indepen	The fraction of outside (independent) directors (RiskMetrics)	Yermack (1996)
Outside	The fraction of non-employee directors that are active CEOs (RiskMetrics)	Ferris et al. (2003)
Dage	The average age of directors on the board (RiskMetrics)	Faleye (2011)
Financial	The fraction of independent directors whose profession types are a banker (RiskMetrics)	Guner et al. (2008)
Foreign	The fraction of directors whose primary employer's country of origin is not the US (RiskMetrics)	Masulis et al. (2012)
Female	The fraction of directors who are female (RiskMetrics)	Adams and Ferreira (2009)
Family	One of the company's founding family is present on the board and zero otherwise (RiskMetrics)	Anderson and Reeb (2003)
Firm Characteristics		
Segments	The number of business segments (Compustat)	Fich and Shivdasani (2006)
Sales	The natural logarithm of Sales (data12) (Compustat)	Fich and Shivdasani (2006)
ROA	Net income/ book value of total assets: data172/data6 (Compustat)	Masulis et al. (2012)
Fage	Number of years the firm is listed in CRSP	Fich and Shivdasani (2006)
Capx	Capital expenditure to sales: data128/data12 (Compustat)	Anderson and Reeb (2003)

Table A2. Kendall & Spearman Correlation Matrix.

Panel A: Kendall Correlation Matrix														
Var.	**Q**	**Size**	**Indepen**	**Dage**	**Family**	**Outside**	**Female**	**Foreign**	**Financial**	**Sales**	**Capx**	**ROA**	**Fage**	**Segments**
Q	1.000	−0.055	−0.027	−0.066	−0.017	0.002	0.020	0.003	0.008	−0.032	0.070	0.326	−0.067	−0.080
Size	−0.055	1.000	0.148	0.095	0.080	0.100	0.214	0.114	0.018	0.441	0.013	0.033	0.288	0.122
Indepen	−0.027	0.148	1.000	0.111	−0.214	−0.034	0.183	0.072	0.016	0.160	−0.042	−0.008	0.158	0.063
Dage	−0.066	0.095	0.111	1.000	0.047	−0.194	−0.027	0.011	0.014	0.094	−0.055	−0.028	0.187	0.067
Family	−0.017	0.080	−0.214	0.047	1.000	−0.023	−0.002	0.000	0.017	0.012	0.000	0.010	0.030	0.002
Outside	0.002	0.100	−0.034	−0.194	−0.023	1.000	0.022	−0.024	−0.024	0.087	0.057	0.016	0.069	0.063
Female	0.020	0.214	0.183	−0.027	−0.002	0.022	1.000	0.074	0.017	0.271	−0.047	0.065	0.141	0.023
Foreign	0.003	0.114	0.072	0.011	0.000	−0.024	0.074	1.000	0.013	0.097	0.033	−0.016	0.061	−0.007
Financial	0.008	0.018	0.016	0.014	0.017	−0.024	0.017	0.013	1.000	0.012	−0.001	0.004	0.018	0.009
Sales	−0.032	0.441	0.160	0.094	0.012	0.087	0.271	0.097	0.012	1.000	−0.046	0.070	0.267	0.111
Capx	0.070	0.013	−0.042	−0.055	0.000	0.057	−0.047	0.033	−0.001	−0.046	1.000	0.093	−0.035	−0.047
ROA	0.326	0.033	−0.008	−0.028	0.010	0.016	0.065	−0.016	0.004	0.070	0.093	1.000	0.015	−0.050
Fage	−0.067	0.288	0.158	0.187	0.030	0.069	0.141	0.061	0.018	0.267	−0.035	0.015	1.000	0.145
Segment	−0.080	0.122	0.063	0.067	0.002	0.063	0.023	−0.007	0.009	0.111	−0.047	−0.050	0.145	1.000
Panel B: Spearman Correlation Matrix														
Var.	**Q**	**Size**	**Indepen**	**Dage**	**Family**	**Outside**	**Female**	**Foreign**	**Financial**	**Sales**	**Capx**	**ROA**	**Fage**	**Segments**
Q	1.000	−0.079	−0.039	−0.099	−0.020	0.003	0.028	0.004	0.010	−0.048	0.104	0.461	−0.100	−0.111
Size	−0.079	1.000	0.163	0.134	0.092	0.149	0.341	0.145	0.021	0.595	0.018	0.045	0.396	0.154
Indepen	−0.039	0.163	1.000	0.163	−0.257	−0.043	0.261	0.091	0.019	0.233	−0.062	−0.013	0.228	0.084
Dage	−0.099	0.134	0.163	1.000	0.057	−0.263	−0.035	0.014	0.017	0.140	−0.080	−0.040	0.278	0.091
Family	−0.020	0.092	−0.257	0.057	1.000	−0.026	−0.002	0.000	0.017	0.014	0.000	0.012	0.037	0.003
Outside	0.003	0.149	−0.043	−0.263	−0.026	1.000	0.023	−0.028	−0.027	0.120	0.077	0.022	0.094	0.079
Female	0.028	0.341	0.261	−0.035	−0.002	0.023	1.000	0.087	0.020	0.381	−0.065	0.091	0.195	0.029
Foreign	0.004	0.145	0.091	0.014	0.000	−0.028	0.087	1.000	0.013	0.123	0.041	−0.021	0.076	−0.009
Financial	0.010	0.021	0.019	0.017	0.017	−0.027	0.020	0.013	1.000	0.014	−0.002	0.005	0.022	0.011
Sales	−0.048	0.595	0.233	0.140	0.014	0.120	0.381	0.123	0.014	1.000	−0.068	0.105	0.385	0.146
Capx	0.104	0.018	−0.062	−0.080	0.000	0.077	−0.065	0.041	−0.002	−0.068	1.000	0.137	−0.053	−0.065
ROA	0.461	0.045	−0.013	−0.040	0.012	0.022	0.091	−0.021	0.005	0.105	0.137	1.000	0.023	−0.069
Fage	−0.100	0.396	0.228	0.278	0.037	0.094	0.195	0.076	0.022	0.385	−0.053	0.023	1.000	0.193
Segment	−0.111	0.154	0.084	0.091	0.003	0.079	0.029	−0.009	0.011	0.146	−0.065	−0.069	0.193	1.000

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