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The Application of Graphic Methods and the DEA in Predicting the Risk of Bankruptcy

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Abstract: The paper deals with the issue of analyzing the financial failure of businesses. The aim was to select key performance indicators entering the DEA model. The research was carried out on a sample of 343 Slovak heat management companies. When addressing the research problem, we made use of multidimensional scaling (MDS) and principal component analysis (PCA), which pointed out the areas of financial health of companies that may predict their financial failure. The core of our interest and research was the data envelopment analysis (DEA) method, which represents a more exact approach to the assessment of financial health. The important finding is that the statistical graphical methods—PCA and MDS—are very helpful in identifying outliers and selecting key performance indicators entering the DEA model. The benefit of the paper is the identification of companies that are at risk of bankruptcy using the DEA method. The originality is the selection of key inputs and outputs to the DEA model by the PCA method.

Keywords: data envelopment analysis; financial distress; multidimensional scaling; prediction; principal component analysis; risk

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1. Introduction

At present, the importance of predicting the risk of business failure is increasing. There are a number of methods, both theoretical and practical, that measure the financial health of companies, reveal the likelihood of their possible financial failure, and propose solutions to avoid bankruptcy. Several of these methods are based on mathematical and statistical methods, most of which involve the application of regression models and models of discriminant analysis. Authors researching this issue classify these methods and models from different perspectives. According to Ghodrati and Moghaddam (2012) (in Klieštik et al. 2019), models for predicting business bankruptcy are divided into statistical models, artificial intelligence models, and theoretical models. In their study, Balcaen and Ooghe (2004) addressed the issue of methods that can be applied to detect possible business failure. They investigated whether sophisticated alternative methods yielded better results than classic statistical methods. In their conclusion they pointed out the significant benefits of these methods. In their work they classified alternative prediction models (Klieštik et al. 2019) to models based on the fuzzy principle, multi logit model, CUSUM model, DEHA (dynamic event history analysis), models of chaos theory and catastrophic scenarios, models of multidimensional scaling, linear goal programming, multicriteria decision making, models focused on analysis of rough sets, expert systems, and self-organizing maps.

Alternative methods include multidimensional survey techniques, which analyze relationships between groups of variables, relationships within groups of variables, and differences in the behavior of variables in different subgroups. With regard to that, we would like to point out the MDS and PCA methods, as well as the DEA, which are addressed in this paper. The DEA method is one of the non-statistical and non-parametric methods of analyzing the efficiency and financial health of companies as well as predicting possible bankruptcy. It is the opposite of the above-mentioned statistical methods of evaluation and prediction of possible financial distress of companies.

The paper follows our previous research (Horváthová and Mokrišová 2020; Štefko et al. 2020) in which we used the same sample of businesses. However, this paper deals with different methods and approaches to the selection of the indicators entering the DEA model. This is related to the identified gap in the research, which is finding a suitable method for selecting indicators to the DEA model. Whereas in the previous studies, we selected indicators based on the correlation matrix or we took over indicators from recognized authors, in this paper, we used the PCA method to point out the predictors of bankruptcy. We also compared the classification ability of the DEA model achieved in this study with the previous research.

Based on the above, the aim of the presented research was to select the inputs and outputs to the DEA model with the use of the PCA method. These inputs and outputs were selected based on the research of Serrano-Cinca and Mar Molinero (2004) and the research of Nasser (2019). The partial goal was to predict the financial distress of the analyzed sample of businesses using the DEA method and to graphically display the results using MDS and PCA methods. Another partial goal was to compare the results of the applied methods and draw conclusions that affect the applicability of these methods (statistical, but also non-statistical) in the field of predicting the financial distress of companies. In line with the above-mentioned, we set the following hypothesis: We assume that the use of the PCA method to select inputs and outputs to the DEA model will increase the classification ability of the model.

The remainder of the paper is structured as follows. Section 1 lists the link with the previous research, research gap, aim of the research, and the hypothesis. Section 2 outlines the theoretical basis of the studied problem. This part of the paper describes the use of the DEA, the PCA, and the MDS in financial health analysis and bankruptcy prediction. Section 3 describes the data, the analyzed sample of businesses, and the processing methods. When addressing the research problem, we made use of selected financial indicators, the MDS, the PCA, and the DEA. Section 4 lists the results of the PCA and the MDS, which were used to analyze the data and select the input parameters to the DEA model. This section also includes the results of the input-oriented BCC DEA model and compares them with the results of our previous research as well as the results of other authors. Section 5 summarizes the essential conclusions resulting from the research and presents the significant findings.

2. Literature Review

Fitzpatrick (1931; in Klieštik et al. 2019) was the first to deal with the prediction of bankruptcy. In his study, he compared the financial indicators of solvent and insolvent companies. The use of simple financial indicators in predicting bankruptcy was questioned by Beaver (1966), who used one-dimensional discriminant analysis. After one-dimensional discriminant analysis, methods of multidimensional discriminant analysis were introduced. Altman (1968) developed a model of multidimensional discriminant analysis (MDA), known as the Z-score. Based on the shortcomings of discriminant analysis, the next step in the theory of bankruptcy prediction was the development of methods and models that would be able to provide information on the probability of business bankruptcy (Mihalovič 2015). Therefore, logistic regression began to be preferred. Recently, more than 30 different methods have emerged, which mainly use mathematical programming, operations research, or artificial intelligence in forecasting the financial situation of companies. These methods have been used by Mahmood and Lawrence (1987), Gupta et al. (1990), Chen et al. (2009), and Cho et al. (2010).

One of the appropriate mathematical programming methods for predicting bankruptcy that are applied in this research is DEA. This method is based on the work of Farrell (1957), who proposed an approach to measuring production efficiency. The pioneers of DEA were Charnes et al. (1978), who introduced the first DEA model based on the concept of Farrell's efficiency. Their model is abbreviated CCR after the names of its founders, and it is based on the assumption of constant returns to scale (CRS). The CCR model defines the relative efficiency for any DMU as the weighted sum of the outputs divided by the weighted sum of the inputs, whereas the efficiency score can only be between 0 and 1. A score less than 1 means that the production unit is below the production frontier (Thanassoulis 2001). Assuming constant returns to scale, the limit of production possibilities is linear, which means that outputs grow at the same rate as inputs. If we use the CCR model, the estimated efficiency score of the production unit is the same for the input-oriented and output-oriented model.

The second basic model is the BCC DEA model designed by Banker et al. (1984). The model is also named after its creators. It assumes variable returns to scale (VRS). The difference between the CCR and BCC model is in the addition of the constraint. Production frontiers of the BCC model are spanned by the convex hull of the existing DMUs (Vishkaeii et al. 2020). It results in a larger number of units being marked as effective when using this model (Kočišová 2012).

The first idea associated with the use of the DEA method for predicting bankruptcy was recorded by Simak (1997), who first compared its results with the results of Altman's Z-score. It has been found that this research has various application opportunities. Other authors who predicted bankruptcy using DEA were Cielen et al. (2004). These authors used the radial DEA model to predict bankruptcy and compared its results with the results of discriminant analysis (DA). In the same year, Paradi et al. (2004) applied the additive and radial model applying peeling technique. This model achieved 100% success in predicting bankrupt companies. Premachandra et al. (2009) used the additive DEA model and compared its results with the results of logistic regression. The outcome of this research was a satisfactory level of correct prediction of bankruptcy. The prediction rate for financially healthy businesses was less accurate. Sueyoshi and Goto (2009) applied the additive DEA model and created a financial distress frontier. They compared the results of the DEA model with the DEA–DA approach. Premachandra et al. (2011) joined the radial and additive DEA model to create a DEA assessment index. Shetty et al. (2012) applied the DEA model to determine the assumption of bankruptcy of the analyzed sample of businesses. The result of their study was the proposal of indicators that should be applied as predictors of bankruptcy (in Horváthová and Mokrišová 2020). It is also possible to mention the research of Ouenniche and Tone (2017), who proposed an out-of-sample evaluation framework for DEA and confirmed that DEA as a classifier is a very suitable contender for discriminant analysis. In connection with the research of the application of DEA models, it is necessary to mention the authors Mousavi et al. (2015), who proposed an orientation-free super-efficiency DEA model as a multi-criteria assessment framework. They also addressed the issue of to what extent the selection and design of explanatory variables and their nature affect the outcome of modeling frameworks. They confirmed that the choice of explanatory variables affects the results of individual modeling frameworks and even improves them in the case of a mixture of account-based and market-based information.

Several authors have applied the DEA method in combination with PCA in their papers. Mar Molinero and Ezzamel (1991) were among the first authors to apply this method in the field of business failure prediction. Mar-Molinero and Serrano-Cinca (2001) extended that work and proposed a way to use MDS models as an alternative to discriminant analysis or a logit model to classify companies as failed or continuing.

As early as 1998, Zhu researched the performance of selected Chinese cities using the DEA and PCA methods. In his research he used the non-statistical DEA method, which is based on the use of the linear programming. The results were compared with the results of a multidimensional statistical method. He tested the results of both methods in order to determine their match using selected nonparametric statistical tests. Premachandra (2001) followed up on Zhu's research. He pointed out the need to apply additional parameters to ensure the match of the results of the DEA and PCA model. These are parameters that Zhu (1998) did not use in his research. At the same time, he pointed out the fact that if there are few effective units in the analyzed sample, the concordance of the results of both methods is confirmed. However, if we have a sample with a large number of effective units, it is necessary to apply a modified approach when testing the match of the results of given methods.

In their study, Serrano-Cinca and Mar Molinero (2004) used the PCA method in selecting the DEA model. They pointed out that the choice of input parameters is very important, as the parameters are crucial to whether the DMU will be effective or not. The authors developed various specifications of the DEA model and analyzed their results using the PCA method. Using this method, it is possible to assess the equivalence, but also the difference of individual DEA models, and at the same time create a ranking of DMUs in terms of their effectiveness. The performance of Internet banking using the DEA and PCA methods was analyzed by Ho and Wu (2009). In 2018, the DEA method and the PCA method were used in the field of education by the authors Jakaitiené et al. (2018). The PCA and DEA methods were used by Rahimpour et al. (2020) to examine the indicators of employee loyalty and intellectual capital.

Another author who used the DEA method and the PCA method is Nasser (2019). He used these methods in the analysis of hospitals' performance. He concluded that PCA plays an important role in reducing the number of input and output variables in performance analysis, helping to identify effective DMUs while improving the classification ability of DEA models.

Garcia-Lacalle and Martin (2010) (in Shen 2017) used DEA and MDS techniques in health-care research and compared the results of rural hospitals in Spain with the results of urban hospitals in terms of efficiency and perceived quality. According to these authors, the combination of DEA and MDS can help clarify the map drawn up using MDS observed points, making the relationship between each aspect much clearer. The MDS method in combination with the DEA method was applied by Lozano and Gutiérrez (2011) (in Shen 2017) in order to analyze the effectiveness of the EU-25 countries in terms of tourism.

The DEA and MDS methods were used in the research by Sagarra et al. (2017) in the analysis of the effectiveness of selected universities. Other authors who used this method to analyze the effectiveness of universities were Torre et al. (2018).

Based on the above starting points, it can be stated that performance analysis with the application of the DEA method and multidimensional analysis techniques can be of a great importance in this area. Therefore, we focused our research on the use of the PCA method as well as the MDS method in relation to the DEA method. In our research, we evaluate the benefits of these methods in predicting the financial distress of companies.

3. Methodology and Data

3.1. Description of the Research Sample

The input database of the empirical study was created from the data of 497 companies that do business in Slovakia in the field of heat supply. The financial statements for 2016, which were the source of data for the analysis, were provided by the Slovak analytical agency CRIF—Slovak Credit Bureau, s.r.o. (CRIF 2016). According to the branch classification of economic activities SK NACE Rev. 2, the analyzed sample of companies belong to Section D: "Electricity, gas, steam and cold air supply."

The Slovak Republic is a country with a well-developed central heating system. The majority of heat sources and heat distribution were built and developed at the time the urban agglomerations were being built (mainly housing, municipal constructions, and civic amenities)—prior to 1990. The centralized heat supply systems supply heat to people's homes, industry, and the service sector. These systems have a potential to benefit from the highly efficient heat and power generation system. In terms of the amount of heat supplied, the heat market has been seeing a decline in the supply of heat to district heating systems for several years. This is mainly due to the reduction in heat consumption in residential buildings, thermal insulation, and the implementation of rationalization measures. Given the large range of measures implemented in residential buildings throughout Slovakia, it is assumed that this trend from previous years will stabilize in the coming years.

The sector analyzed is important from both an economic and a social point of view and plays an important role in the daily lives of society and consumers. Despite some similarities with other energy sectors, heat as a commodity cannot be traded between countries and, due to significant heat losses in transmission and distribution, cannot be traded between networks existing in different locations. The overall nature and structure of heat management, as well as the method of heat and hot water supply in individual geographical areas, are determined by various factors, including, in particular, the climate and fragmentation, historical development, demographic conditions and territorial division, character of housing, commercial and industrial construction, economic activity, and availability of fuel sources for heat production. Based on the above factors, in each larger city or municipality we encountered a different structure and system of heat supply. Each specific system also consists of its own system of heating equipment (Antimonopoly Office of the Slovak Republic 2013). These facts are a prerequisite for the existence of risk factors that affect the financial health and performance of the analyzed companies.

3.2. Input Analysis of the Data

The input analysis was carried out on a sample of 343 companies. In this sample, we identified 37 businesses threatened with bankruptcy. This information was used to confirm the classification ability of the DEA model.

For the analysis, we selected indicators that are most commonly applied in the analysis of businesses' financial distress (Cielen et al. 2004)—equity ratio, retained earnings/total assets, expired taxes, cash ratio, inventories, financial debt ratio, gross return, coverage of debt, net return, current ratio, quick ratio, leverage (Feruś 2010)—net profit indicator, asset return indicator, equity capital return indicator, liquidity ratio, daily return indicator, total liabilities indicator (Mukhopadhyay et al. 2012)—current ratio, net working capital to total assets, return on assets, total debt ratio, market value to book value, earnings before interest and tax to total assets, interest coverage ratio, current assets to total assets, current liabilities to total assets (Premachandra et al. 2011)—cash flow to total assets, net income to total assets, working capital to total assets, current assets to total assets, earnings before interest and taxes to total assets, earnings before interest and taxes to interest expense, market value of equity to book value of common equity (Mendelová and Bieliková 2017)—current ratio, net working capital to total assets, cash flow from operations to current liabilities, equity ratio, return on sales, return on assets, profit/loss from operations to sales, cash flow from operations to total assets, current trade payables to sales, current liabilities to total assets, long-term liabilities to total assets, operating expenses to operating revenues, interest expense to revenues). The selected indicators are listed in Table 1.

Table 1. Selected parameters for input analysis.

Indicator	Sign	Formula	Average	Median	Standard Deviation
Current ratio	CL	$\frac{short-term\ assets}{short-term\ liabilities}$	3.8	0.877	13
Quick ratio	QR	$\frac{(short-term\ receivables+financial\ assets)}{short-term\ liabilities}$	3.7	0.774	12
Average collection period	ACP	$current\ receivables/sales \times 360$	6223.1	58.931	97,685
Inventory turnover	IT	$inventory/sales \times 360$	98.7	0.165	1135
Creditors payment period	CPP	$current\ liabilities/sales \times 360$	60,245.5	197.895	1,021,468
Cash-to-cash cycle	CTC	$ACP + IT - CPP$	−53,923.6	−119.747	925,078
Total assets turnover ratio	TATR	$sales/assets$	0.8	0.255	1
Return on assets	ROA	$EBIT/assets \times 100$	0.1	0.046	0
Return on equity	ROE	$EAT/equity \times 100$	0.1	0.130	2

Table 1. *Cont.*

Indicator	Sign	Formula	Average	Median	Standard Deviation
Return on sales	ROS	$EAT/sales \times 100$	−27.5	0.037	496
Equity ratio	ER	$equity/assets \times 100$	0.2	0.149	0
Debt-to-equity ratio	DER	$debt/equity$	0.8	0.851	0
Equity-to-fixed assets ratio	EFAR	$equity/fixed\ assets$	4.3	0.234	63
Cost ratio	CR	$costs/revenues$	15.0	0.957	258
Material intensity	MI	$material\ costs/revenues$	0.3	0.051	0
Labor-to-revenue ratio	LR	$labor\ costs/revenues$	0.1	0.005	0
EVA ROS	EVA ROS	$EVA/sales$	−20.2	−0.003	365

Notes: EBIT—earnings before interest and taxes, EAT—earnings after taxes, EVA—economic value added.

The median of the current ratio indicates that half of the analyzed sample of companies achieved a liquidity value higher than 0.9, which can be considered appropriate in relation to the characteristics of the industry. These businesses struggled with a high creditors payment period (median 198 days), resulting in a negative cash-to-cash cycle. The total assets turnover ratio of the analyzed sample of companies was low. Its average value was 0.8, and the median of this indicator was even lower: −0.3. The median of the return on assets was 4.6%, so it can be assumed that half of the analyzed companies reached an ROA value higher than 4.5%. The median of return on equity was 13%, which we evaluated positively. The capital structure of these companies was, on average, 20%: 80% in favor of external sources, which may be the reason for the lower stability of these companies. From the point of view of the average value and the median of the equity-to-fixed assets ratio, we classified these companies as overcapitalized. The median of the cost ratio was 96%. It follows that the average profitability of these companies was EUR 0.04 cents. Material intensity accounted for 28% of costs. The labor-to-revenue ratio accounted only for 4% of costs. We have to evaluate the performance of the analyzed sample of companies negatively as the value of the EVAROS indicator was negative.

3.3. Applied Methods

With regard to the processing methods that were used in the research, we outline important points. The MDS and PCA methods were chosen to analyze the input data of selected companies and to choose the inputs and outputs to the DEA model. The output of the application of these methods was a set of financial indicators, which represented the input to the DEA model. The process of the research is illustrated in Figure 1.

The PCA and MDS methods have their strengths rooted in the graphic design and display of the position of companies according to their achieved financial results in space. On the one hand, the MDS method captures similarities companies share, and on the other hand, the PCA method captures the position of companies in terms of the achieved values of financial indicators, which are then incorporated into the value of the calculated principal components. These methods can be used for the adequate selection of input parameters to the DEA model.

3.3.1. Multidimensional Scaling

The aim of the MDS method is to find hidden dimensions that will make it possible to explain the similarities or differences between companies. With MDS it is possible to analyze any kind of similarity or distance on the basis of the so-called proximity matrix.

In MDS, it is important to determine two parameters, namely, the number of coordinates and the criterion of maximum likelihood. A very important output of MDS is the MDS map of objects, which has been used by several authors (Van der Maaten and Hinton 2012; Mar Molinero and Ezzamel 1991). To express how well the data is represented by the MDS map, we used Kruskal's stress, which is the most common goodness-of-fit-statistic. Stress is calculated according to Formula (1) (Kruskal 1964):

$$Stress = \sqrt{\frac{\sum_{k=1}^{m}\left(d_{ij} - \hat{d}_{ij}\right)^2}{\sum_{k=1}^{m} d_{ij}^2}} \tag{1}$$

where $\hat{d}_{ij}$ expresses the predicted distance between objects i and j and d_{ij} is the actual distance between objects i and j.

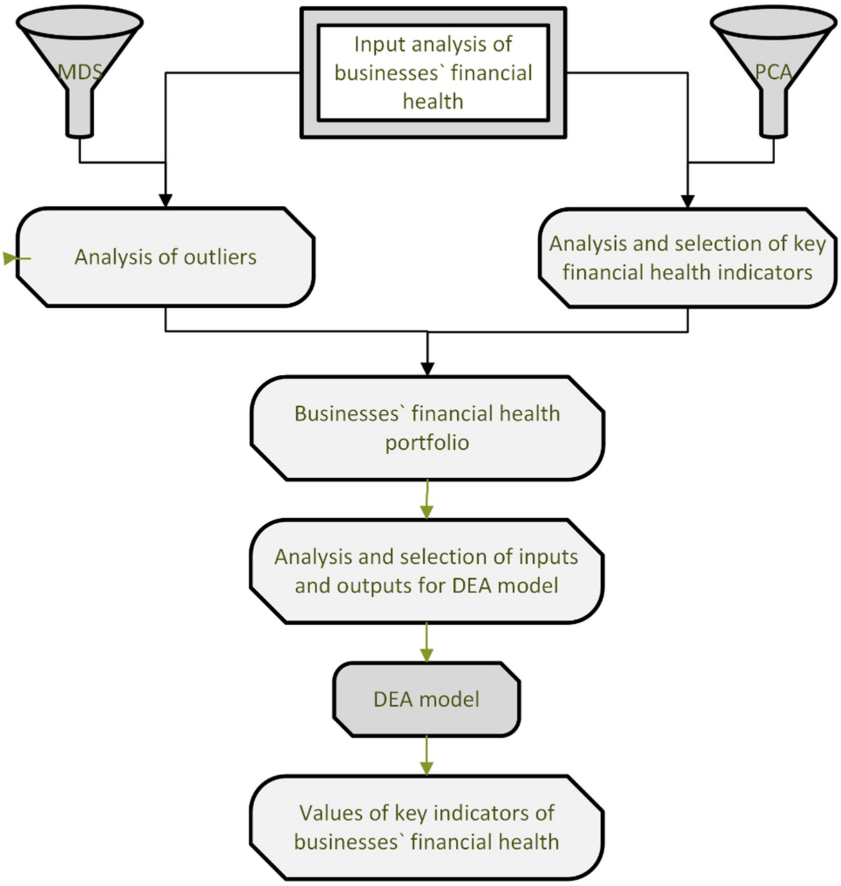

Figure 1. Flowchart of the research.

A small value of stress indicates a good fitting solution, whereas a high value indicates a bad fit. According to Kruskal (1964) a stress of around 0.20 means insufficient overlap, 0.10 sufficient, 0.05 good, 0.025 excellent, and 0.00 perfect fit. Based on the lowest possible value of stress, we can determine the number of dimensions. The aim is to keep the number of dimensions as small as possible (we usually choose two-dimensional, or maximum three-dimensional space).

Unlike other multidimensional techniques, multidimensional scaling does not use random variables. MDS creates its own "random variable," the so-called subjective co-ordinate, which is suitable for mutual comparison of objects. Subjective coordinates are derived from the overall degrees of similarity between objects. We talk about choosing a dependent variable (similarity between objects). Multidimensional scaling of objects reduces human influence by not requiring a precise definition of the variables used in the comparison, such as in cluster analysis. The disadvantage is that the user is not sure which variables the respondent used to compare objects (Meloun et al. 2017).

3.3.2. Principal Component Analysis

In the case of the PCA method, there are several rules for determining the optimal number of main components (Smith 2002). This number can be determined on the basis of one's own consideration of the need to preserve information, taking into account the eigenvalues that explain, e.g., 90% variability. In this case, Kaiser's rule is used. According to this rule, the main components whose eigenvalue is greater than the average of all eigenvalues are taken into account. For standardized data, the average is equal to 1, which means that we take into account the principal components whose eigenvalue is greater than 1. The principal components that together account for at least 70% of the total variance shall be applied. This is based on a graphic display, from the so-called Cattel's index graph of eigenvalues. A break is found in the figure and the main components laying prior to this break are taken into account. Anderson's rule (sphericity test), which assumes that only the first q of eigenvalues is uniquely determined, with the others being the same, is applied.

The principal components must meet the following criteria (Labudová et al. 2010; Meloun 2011): They are a linear combination of the original standardized variables, and the number of main components is equal to or less than the number of original variables—they do not correlate (they are independent). The first two or three principal components are used primarily as techniques for displaying multidimensional data in a graphical representation in a space or a plane. The first new coordinate axis (first principal component) is guided in the direction of maximum variance between objects. The second axis (second principal component) is perpendicular to the first axis and is guided in the direction of the second largest variance between objects, etc. The relative position of objects in the original space and in the space given by the principal components is the same. During the transformation to the principal components, the original coordinate system rotates in the direction of maximum variance between objects, while the Euclidean distances between objects are maintained (Meloun 2011; Jolliffe 2002).

The important part of the PCA analysis is the explanation of the relationship between the original variables and principal components. Let X denote the $n \times m$ original dataset matrix, the rows of which corresponds to individual businesses and columns to financial indicators. The output of PCA contains an $n \times m$ matrix of score Z, an $m \times m$ rotation matrix V, and the standard deviations of the projections along the PCs $\lambda_1, \ldots, \lambda_m$. Principal components are defined by the columns $v_1, \ldots, v_n$ of the rotation matrix V. The ith principal component can be written with the following linear function: $f(x) = v_i^T x$. The matrix Z includes the values of each principal component function evaluated at each observation in the dataset $Z = XV$ (Reris and Brooks 2015).

At the core of the empirical study presented in this paper is the usage of the DEA method. It is a non-statistical method that uses its own calculation algorithm to calculate the efficiency of companies and to create the ranking of companies in terms of efficiency and performance. The DEA method calculates the efficiency of companies, determines their order from the point of view of efficiency, and proposes target values of applied indicators.

3.3.3. Data Envelopment Analysis

In this paper, we applied the BCC DEA model. This model is similar to the CCR model. It is based on mathematical programming, much like the CCR model. As mentioned in the theoretical part of the paper, it was published by Banker et al. (1984). The CCR model assumes that each unit of input brings the same unit of output, i.e., constant returns to scale. The BCC model assumes variable returns to scale. We solved the dual input-oriented BCC DEA model, which can be stated as follows (2):

$$\text{Minimize} \qquad \theta_o - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$\text{Subject to} \qquad \sum_{j=1}^{n} x_{ij}\lambda_j + s_i^- = \theta_o x_{io}, i = 1, 2, \ldots, m,$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro}, \qquad r = 1, 2, \ldots, s, \tag{2}$$

$$\lambda_j = 1, \; j = 1, 2, \ldots, n.$$

$$s_i^- \geq 0, s_r^+ \geq 0.$$

where θ_o and ϕ_o are the values of objective functions, ε is the non-Archimedean infinitesimal value, x_{ij}, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ is the value of i input for DMU$_i$, y_{rj}, $r = 1, 2, \ldots, s$, $j = 1, 2, \ldots, n$ is ste value of k output for DMU$_j$, m and s are the number of inputs and outputs, respectively, n is the number of enterprises, λ_j is convex coefficient, and s_i^- and s_r^+ are input and output slack variables that express input excesses and output shortfalls, respectively.

The evaluation of the efficiency of companies with the application of the DEA model is based on the value of the objective function and the value of slacks. Slacks are the distances from the efficiency frontier. If the value of the objective function is equal to 1 and the values of all slacks are equal to zero, it can be stated that the company is efficient. Otherwise, the business is inefficient.

In addition to the efficiency of companies, the DEA model was also applied in terms of solving and revealing the bankruptcy of companies. When solving the models, the distances from the financial health and bankruptcy frontier as well as the target values were calculated. The models were calculated in the software DEA Frontier (Zhu 2019).

4. Results and Discussion

4.1. The Results of Principal Component Analysis

The projection of cases applied in a given research study allows the placement of individual DMUs in two-dimensional space or three-dimensional space, with each DMU and its position being determined by the values of all applied indicators simultaneously. The projection of cases processed by the PCA method indicates that the entire analyzed sample of companies creates a significant cluster in the space around the starting point of the coordinate system. This cluster is given by the coordinates (x: 5, −5; y: −5, 5). Outside this cluster are companies that achieve different results. The space for cluster analysis was defined by factors 1 and 2. Factor 1 captures information on a CPP indicator, and this indicator is inversely proportional to the first factor. Factor 1 has an equally strong and inversely proportional relationship with the indicator (CR). It shows a strong directly proportional relationship with the ROS and EVAROS indicators.

The second factor shows a strong directly proportional relationship with the indicators CL and QR. The third factor captures information on the capital structure of the company through indicators ER and DER.

A more detailed analysis of the projection of cases (Figure 2) shows that two significant sub-clusters of companies were formed within the main cluster. One was created in the space (x: 0, −5; y: 0, 5), and in this space there were companies that showed problems in terms of the CR and profitability. The second significant cluster was in the space (x: 0, 5; y: 0, 5), and in this space there were companies that achieved good results in terms of liquidity as well as profitability. Companies with extreme values that were outside the cluster were excluded from the following analyses. These were the companies TP122, TP176, TP142, TP284, and TP134.

Figure 3 shows the projection of the applied indicators. Using the above projection, it is possible to select indicators that are the basis of the following analyses. As the strong directly proportional relationship between the indicators CL and QR was confirmed in this analysis, only one indicator from these indicators should be selected for the following analyses. The quick ratio indicator was chosen, as this indicator carries financial risk. With regard to measuring the company's profitability, the ROS indicator was chosen. The third indicator that emerged from this initial analysis was the CPP indicator. The fourth indicator we chose for bankruptcy analysis was the CR indicator.

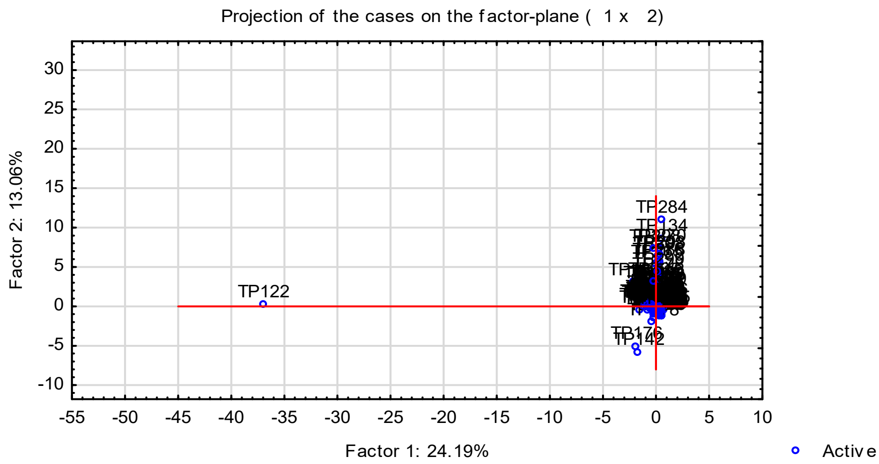

Figure 2. Projection of cases.

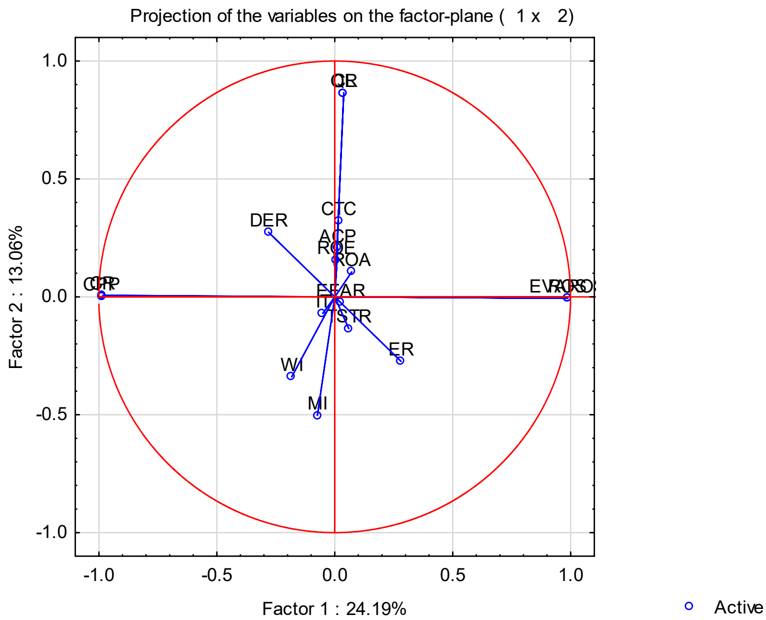

Figure 3. Projection of variables.

The projection of cases after the exclusion of TP122, TP176, TP142, TP284, and TP134 is shown in Figure 4. Again, a significant cluster of companies was formed, which was concentrated in the space (x: 3, −3; y: 3, −4). The space of occurrence of companies narrowed after the selected companies were excluded.

In the space (x: 0, −3; y: 0, 3) there were companies having a problem with profitability. In the space (x: 0, 3; y: 0, −4) there were companies that had a problem with liquidity. The best companies in terms of the analyzed indicators were in the space (x: 0, 5; y: 0, 3); the worst companies were in the space (x: 0, −3; y: 0, −4). After excluding companies with extreme values, the significance of the IT indicator in relation to the first factor was confirmed. In relation to the second factor, the significance of the ER and DER indicators was confirmed. However, companies reaching extreme values were again visible in the sample and were therefore excluded: TP 25, TP278, TP129, TP305, TP280, TP282, TP268, TP49, TP291, TP267, TP275, TP279, TP173, TP204, TP199, TP312, and TP85.

The final projection of cases after excluding the values of these companies is shown in Figure 5.

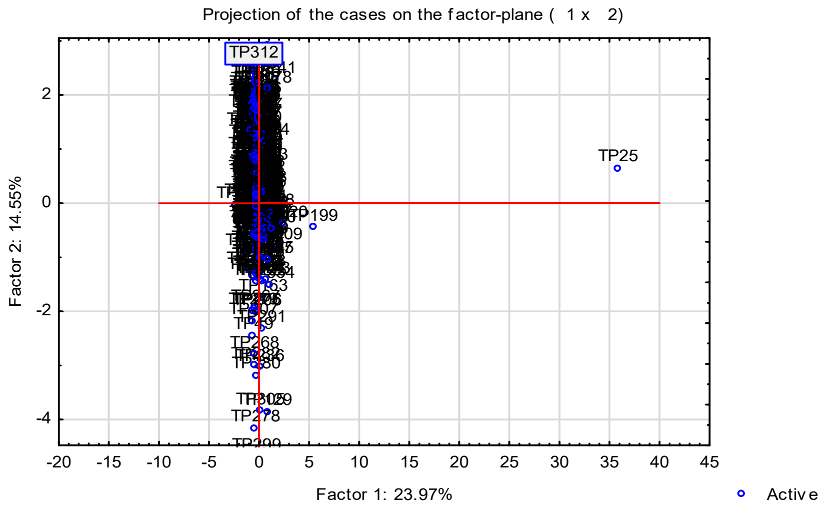

Figure 4. Projection of cases after excluding businesses with extreme values of indicators.

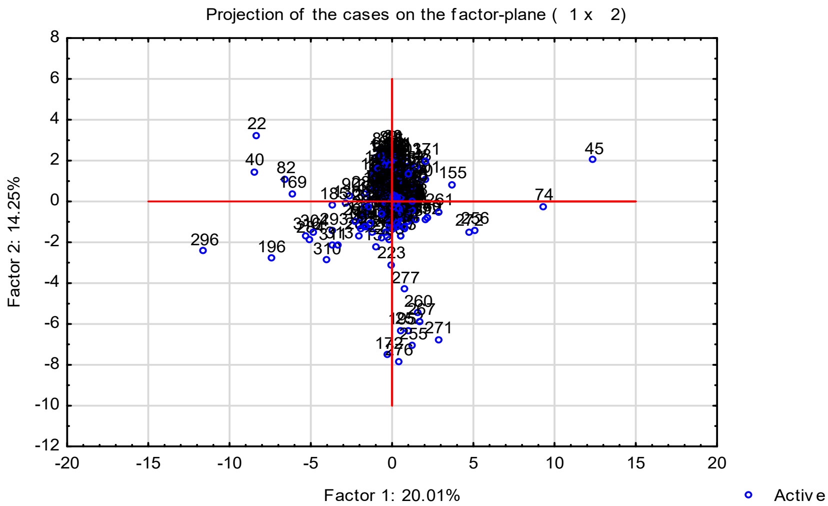

Figure 5. Final projection of cases after excluding businesses with extreme values of indicators.

The projection (Figure 4) shows that it was necessary to exclude the companies TP22, TP40, TP82, TP169, TP296, TP196, TP277, TP260, TP267, TP252, TP195, TP255, TP271, TP276, TP172, TP 45, TP74, and TP256.

Based on Figure 6 we can say that after the exclusion of above-mentioned companies, a significant directly proportional relationship between the first factor and ROA was confirmed.

The PCA method confirmed a small number of companies that were likely to fail in the analyzed sample. A large number of companies was located in an area grouping the companies that were not likely to fail but still reported certain problems in one of the analyzed financial areas. Using the results of this method, it was possible to select significant indicators that described the analyzed sample of companies. Individual factors thus provided sufficient information about the analyzed sample of companies.

Significant indicators that described the financial situation of the analyzed sample of companies in great detail were the following indicators: QR, ROS, and ROA—outputs. The indicators CPP, IT, and CR were confirmed as significant inputs. As outputs, we chose indicators that need to be increased to increase efficiency. Indicators that need to be reduced to increase efficiency were chosen as inputs. The benefit of the analysis (using the PCA method) is a set of financial indicators that can be applied in the analysis of the financial health of companies and in bankruptcy predictions.

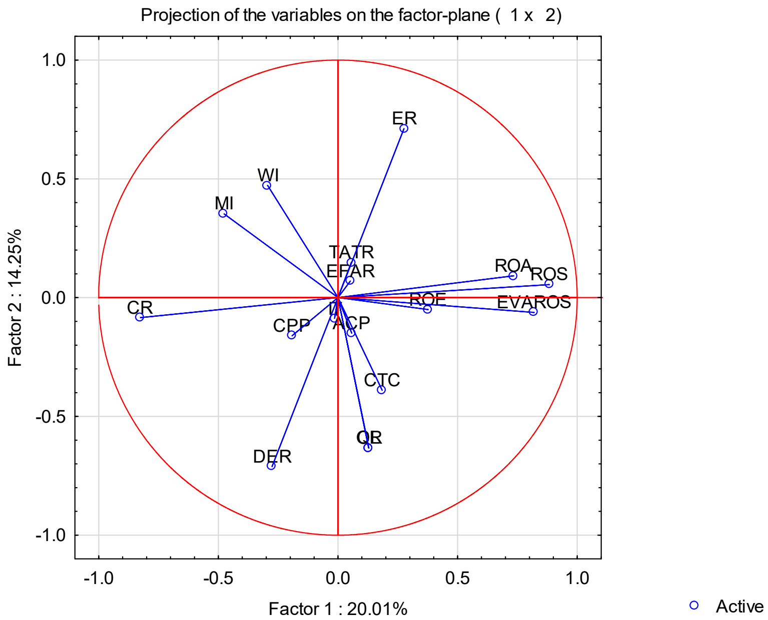

Figure 6. Final projection of analyzed indicators.

This method made it possible to analyze the position of companies in terms of selected indicators as well as create the preconditions for improving the position of companies by improving selected financial indicators. This method should be applied as a benchmarking tool, as it provides a quick overview of the financial health of companies.

4.2. The Results of Multidimensional Scaling

The results of the analysis by the PCA method are also confirmed by Figure 7, which was processed using the MDS method. The suitability of the model with selected indicators was confirmed by the value of stress, namely, 0.07, which means a sufficient overlap between the calculated and entered coordinates of objects. This represented a significant improvement over the case where all indicators and all companies were applied. In this case the value of stress was 0.22, which represented an insufficient overlap between the calculated and entered object coordinates. Figure 6 shows the similarities of the analyzed companies.

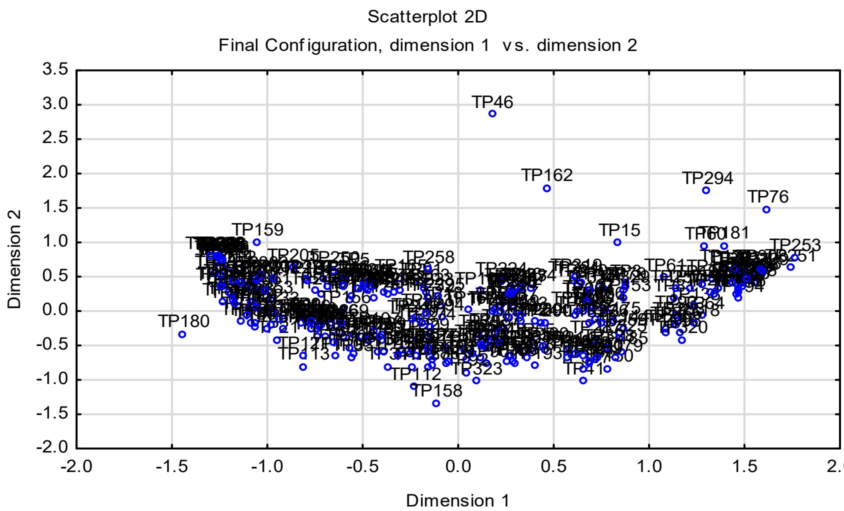

Figure 7. Graphic representation of businesses in space using the MDS.

When we marked the clusters of similar companies, we could state that in the given figure there were five significant clusters of companies (Figure 8). The best companies in terms of their financial health created a cluster in the space (x: 0, 2; y: 0, 2.5) and the worst companies created a cluster in the space (x: 0, −1.5; y: 0, −2). For these companies, it was possible to anticipate problems in the area of their financial health.

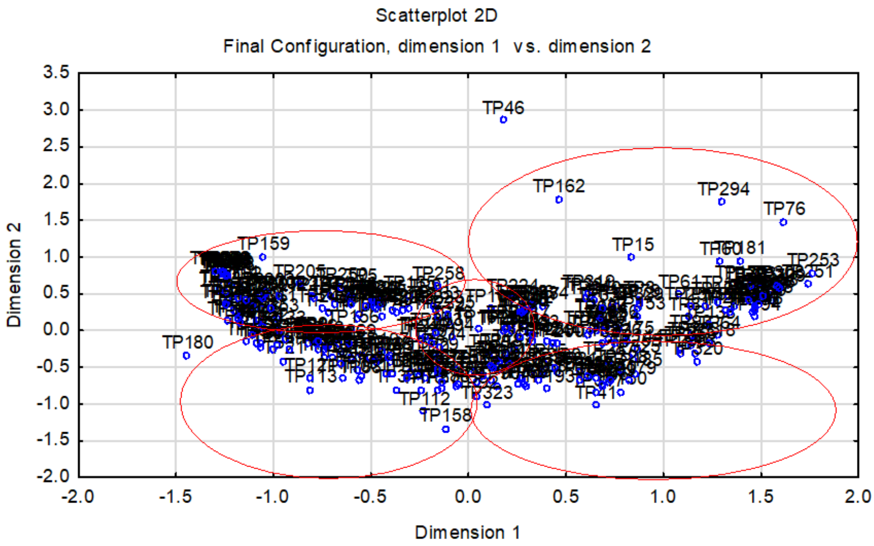

Figure 8. Graphic representation of important clusters.

4.3. The Results of Data Envelopment Analysis

However, these statistical methods did not offer us a precise definition of companies that were expected to go bankrupt. For this purpose, it was appropriate to apply the non-statistical DEA method, which was described in the theoretical part of the paper. In our research, we applied the input-oriented BCC DEA model. We dealt with two versions of the model, namely the model of financial health and the model of bankruptcy. Table 2 provides a summary of businesses that were expected to go bankrupt. It can be stated that the BCC DEA model confirmed eight companies. These companies reached a cost ratio of over 1 and a quick ratio of 0.16.

Table 2. Results of the input-oriented BCC DEA model of bankruptcy.

DMU No.	DMU Name	Bankruptcy
38	TP41	1.00000
77	TP85	1.00000
88	TP96	1.00000
110	TP118	1.00000
227	TP251	1.00000
262	TP307	1.00000
264	TP309	1.00000
272	TP320	1.00000

The advantage of the DEA model solution is the design of target values—benchmarks. If a company is able to reach them, it could become financially sound and competitive. The results of the BCC DEA model, which was aimed at revealing financially sound companies, yielded the results shown in Table 3. From the analyzed sample, 13 companies were on the financial health frontier. As already mentioned, there were eight companies on the bankruptcy frontier. All other companies were in a production possibility set, which means that it would be necessary for them to optimize their inputs or outputs to become financially sound businesses.

Table 3. Results of the input-oriented BCC DEA model of financial health.

DMU No.	DMU Name	Financial Health
42	TP46	1.00000
70	TP76	1.00000
86	TP94	1.00000
90	TP98	1.00000
165	TP183	1.00000
184	TP208	1.00000
225	TP249	1.00000
240	TP270	1.00000
245	TP285	1.00000
248	TP288	1.00000
252	TP294	1.00000
256	TP299	1.00000
274	TP322	1.00000

The predictive and classification ability of the DEA model in the case of failing companies reached 73%, which can be considered a significant classification ability compared to other studies (Premachandra et al. (2009) achieved 84.89% accuracy in predicting failing companies, Mendelová and Stachová (2016) 10–42.86%, and Cielen et al. (2004) 74.4–75.7%).

Since companies not facing bankruptcy were located not only on the financial health frontier, but also in a production possibility set, we can state that the classification ability in the case of non-bankrupt companies was 98%.

4.4. Comparison of the Results with the Authors' Previous Studies

The comparison of the results achieved in this paper with our previous studies is given in Table 2.

Table 4 shows a comparison of the research carried out in this paper with the results obtained in the authors' previous papers. The indicators, which were applied in all studies, were approximately the same, and the differences between them can be seen in Table 2. The table also lists methods that were applied in the selection of indicators. An important part of the results presented in the table is the comparison of the classification ability of the models within the given studies. It can be stated that in the case of bankrupt businesses, the hypothesis was confirmed. These businesses achieved 98% classification ability in this research. The classification ability for bankrupt businesses was higher when selecting indicators based on the correlation matrix (Štefko et al. 2020) compared to the PCA method applied in this paper.

The results of the DEA model can be compared with the results of the logit model, which we processed in our previous research.

Table 5 shows that the classification ability for bankrupt businesses was higher in the case of DEA model than the logit model.

Table 4. Comparison of the achieved results with the results of the previous studies.

Papers	Sample	Modifications of the Sample—Exclusion	DEA Model	Software	Indicators		Method of Indicator Selection	Classification Ability	
					Inputs	Outputs		Bankrupt Businesses	Non-Bankrupt Businesses
(Štefko et al. 2020)	343	Yes	ADD	EMS	LLTA CLTA	TRTA CL WCTA CATA EBTA EBIE ED	Correlation matrix	86%	56%

Table 4. *Cont.*

Papers	Sample	Modifications of the Sample—Exclusion	DEA Model	Software	Indicators		Method of Indicator Selection	Classification Ability	
					Inputs	Outputs		Bankrupt Businesses	Non-Bankrupt Businesses
(Horváthová and Mokrišová 2020)	343	Yes	BCC	Zhu -DEAFrontier	TDTA CLTA	CFTA NITA WCTA CATA EBTA EBIE ED	Premachandra and Altman	41%	96%
(Štefko et al. 2021)	343	Yes	BCC	Zhu -DEAFrontier	CR IT CPP	CL ROA ROS	PCA and graphical representation using MDS	73%	98%

Notes: ADD—additive model, EMS—efficiency measurement systems, LLTA—long-term liabilities/total assets, CLT—current liabilities/total assets, TRTA—total revenue/total assets, WCT—working capital/total assets, CAT—current assets/total assets, EBT—earnings before interest and taxes/total assets, EBI—earnings before interest and taxes/interest expense, ET—equity total debt, TDTA—total debt/total assets, CFT—cash flow/total assets, NIT—net income/total assets, E—equity/debt.

Table 5. Comparison of the results of the DEA model and logit model.

Model	Classification Ability for Bankrupt Businesses	Classification Ability for Non-Bankrupt Businesses
Logit	45.45	98.88
DEA	73	98

5. Conclusions

The production, distribution, and supply of heat in Slovakia belong to the field of network regulation. Within this issue, it is possible to argue about whether to regulate heat prices or not. Price regulation helps to set low margins. Countries use them to protect risk groups in the population from existential problems. Low energy prices have a positive effect on inflation as well as on business development. However, on the other hand, heat management companies in many cases cannot adjust their variable and fixed costs to the level of regulated prices. They are not able to pay the costs; therefore, they go bankrupt. However, they could be beneficial for the state and its people, as they provide alternative options for heat production and heat supply. It should be pointed out that unregulated prices would instead motivate competition to enter the market and ensure price competition even without the necessary regulation. In addition, the regulator can set prices incorrectly and cause losses to many heat management companies. The regulator may also be subject to political pressure to not adapt heat prices to reality. As a result, many heat management companies do not continue to operate due to losses that have prevented them from operating in the business environment. Businesses themselves are not able to change these conditions. Therefore, in order to ensure the survival of these businesses and their competitiveness, it would be appropriate to revise certain regulatory measures. The effects of these measures are more or less reflected in all financial indicators of businesses.

In this paper, we focused on the identification of specific financial bankruptcy indicators of analyzed businesses. Knowledge of these indicators can help them to overcome their unfavorable position in the country's economy, survive, and provide their product to the population.

Apart from the external influences on this area of business, it is necessary to pay attention to the financial health of businesses, which is a prerequisite for their existence. Determining the state of financial health of a company as well as its competitive position in the industry is nowadays a significant challenge for all companies. In order to determine the real financial condition of the company, it is necessary to apply adequate methods and tools. In our research, statistical and graphical tools, as well as a non-statistical DEA model, were applied. These offered some important information on how to increase the company's performance and ensure its financial health.

We used the PCA method to select inputs and outputs to the DEA model that best described the financial health of the research sample. We also used this method to exclude companies, which caused a loss in the value of the research sample. The selected indicators were used as inputs and outputs to the DEA model. The result of the DEA model was the identification of businesses threatened with bankruptcy. In the graphical representation using the PCA and MDS methods, these companies appeared in the same cluster. MDS is an important graphical and benchmarking tool that helps managers understand where their company is in terms of financial health and with which companies it forms a cluster. These companies have similar values of indicators. They also can learn from other businesses how to become better. These are three methods that use their outputs to help each other to identify businesses threatened with bankruptcy, but also offer companies target values to improve their position in space.

The chosen research methods have both positives and negatives. DEA is an important benchmarking tool for improving financial indicators by learning from those in a better position. DEA models can be input or output oriented, and on this basis the target values of inputs or outputs can be calculated. Using the DEA method, it is possible to identify companies that are threatened with bankruptcy and are on the bankruptcy frontier as well as companies that are on the financial health frontier. Using the DEA method, it is possible to assign to each inefficient unit a peer unit or reference unit, which would serve as a pattern for this unit in determining its target values. It is also possible to find out which units are inefficient—we can measure this inefficiency and determine the probability of bankruptcy. A pattern for an inefficient unit is a unit with a similar combination of inputs and outputs. Using suitable software, it is possible to solve a large amount of data. DEA models eliminate subjectivity. A significant benefit of the DEA method is that the solution of DEA models is not based on prior probabilities of bankruptcy. DEA models accept financial and non-financial indicators, as well as environmental and social factors. The reliability of DEA models can be compared with the reliability of logit and probit models. In this regard, a high degree of comparability can be achieved while maintaining a uniform degree of indicators.

The disadvantages of the DEA model include the following: The DEA model is sensitive to extreme values; the financial distress frontier and financial health frontier are constructed of extreme values. As it is not possible for all healthy companies to form the financial health frontier, they can be found in a production possibility set together with companies in financial distress. The results of the DEA model need to be verified with the application of another method for the above reason, with logistic regression appearing to be the most appropriate.

The disadvantages of the DEA model can be eliminated by applying a multi-level DEA model, with the gradual exclusion of companies that are on the financial distress frontier. The advantage is the construction of the prediction coefficient from the two DEA models described above, namely, the model for determining financial health and the model for determining financial failure.

The applied graphical methods, MDS and PCA, facilitate and speed up the processing of large amounts of data, enable the reduction of the number of data dimensions, and thus create a precondition for the application of other important analytical procedures. Thanks to graphic display, the results can be interpreted in a simpler and clearer way. The selected group of variables can be replaced by two or three factors that capture information on the financial situation of a given sample of companies in great detail. These are factors that accumulate information about the company's profitability, liquidity, and capital structure, and belong to the symptoms of business bankruptcy. Managing them, it is possible to improve the financial position of companies in order to avoid bankruptcy. The MDS method makes it possible to identify clusters of companies that are not distant from each other and have similar characteristics. Based on the above, it is possible to identify a group of companies that is expected to go bankrupt.

In order to improve the interpretation of the results of these methods, it is very appropriate to supplement their results with the results of the DEA model.

The contribution of this study to the literature is the selection of inputs and outputs to the DEA model by the PCA method. We were also able to confirm the significance of ROA by gradually excluding companies with extreme values. At the same time, this study represents a connection between the graphical representation of companies in the financial health portfolio and the exact calculation of businesses' financial health. We found out that applied methods yield similar results. PCA and MDS can graphically capture the position of bankrupt businesses in space. There was a match between the results of these methods.

In conclusion, it is possible to point out the fact that the given research was significantly limited by the applied sample of data. Heat management companies show significant extreme values and deviations. The database was incomplete and contained a large number of outliers. In future research, we will focus on obtaining a database that is more comprehensive and for a longer period.

The research in this paper is linked to specific aspects of the journal's and Special Issue's scope. We applied special and modern methods in the field of bankruptcy prediction of small and medium-sized businesses. We also analyzed a specific regulated industry. In this industry, it is very difficult to find tools to stay in the market. As a result, many small businesses go bankrupt and only large ones survive, gaining an increasing monopoly position. We confirmed important indicators that are predictors of the bankruptcy of heat management companies operating in a regulated environment.

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References

Altman, Edward I. 1968. Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. *Journal of Finance* 23: 589–609. [CrossRef]

Antimonopoly Office of the Slovak Republic. 2013. Functioning and Problems in the Heat Management Sector in the Slovak Republic with a Focus on Local Central Heat Supply Systems from the Perspective of the Antimonopoly Office of the Slovak Republic. Available online: http://www.antimon.gov.sk/data/att/365.pdf (accessed on 21 June 2020). (In Slovak)

Balcaen, Sofie, and Hubert Ooghe. 2004. *35 Years of Studies on Business Failure: An Overview of the Classical Statistical Methodologies and Their Related Problems*. Working Papers of Faculty of Economics and Business Administration, Ghent University, Belgium 04/248. Ghent: Ghent University, Faculty of Economics and Business Administration.

Banker, Rajiv D., Abraham Charnes, and William W. Cooper. 1984. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* 30: 1078–92. [CrossRef]

Beaver, William H. 1966. Financial ratios as predictors of failure. *Journal of Accounting Research* 4: 71–111. [CrossRef]

Charnes, Abraham, William W. Cooper, and Edwardo L. Rhodes. 1978. Measuring the efficiency of decision making units. *European Journal of Operation Research* 2: 429–44. [CrossRef]

Chen, Hsueh-Ju, Shaio Yan Huang, and Chin-Shien Lin. 2009. Alternative diagnosis of corporate bankruptcy: A neuro fuzzy approach. *Expert Systems with Applications* 36: 7710–20. [CrossRef]

Cho, Sungbin, Hyojung Hong, and Byoung-Chun Ha. 2010. A hybrid approach based on the combination of variable selection using decision trees and case-based reasoning using the Mahalanobis distance: For bankruptcy prediction. *Expert Systems with Applications* 37: 3482–88. [CrossRef]

Cielen, Anja, Ludo Peeters, and Koen Vanhoof. 2004. Bankruptcy prediction using a data envelopment analysis. *European Journal of Operational Research* 154: 526–32. [CrossRef]

CRIF. 2016. *Financial Statements of Businesses*. Bratislava: Slovak Credit Bureau, s.r.o.

Farrell, Michael James. 1957. The Measurement of Productive Efficiency. *Journal of the Royal Statistical Society, Series A* 120: 253–90. [CrossRef]

Feruś, Anna. 2010. The Application of DEA Method in Evaluating Credit Risk of Companies. *Contemporary Economics* 4: 109–18. [CrossRef]

Fitzpatrick, Paul Joseph. 1931. A comparison of the ratios of successful industrial enterprises with those of failed companies. *Certified Public Accountant* 6: 727–31.

Garcia-Lacalle, Javier, and Emilio Martin. 2010. Rural vs. urban hospital performance in a 'competitive' public health service. *Social Science and Medicine* 71: 1131–40. [CrossRef] [PubMed]

Ghodrati, Hassan, and AmirHadi Manavi Moghaddam. 2012. A study of the Accuracy of Bankruptcy Prediction Models. *American Journal of Scientific Research* 59: 55–67.

Gupta, Yash P., Ramesh P. Rao, and Prabir K. Bagchi. 1990. Linear goal programming as an alternative to multivariate discriminant analysis: A note. *Journal of Business, Finance and Accounting* 17: 593–98. [CrossRef]

Ho, Chien-Ta Bruce, and Desheng Dash Wu. 2009. Online banking performance evaluation using data envelopment analysis and principal component analysis. *Computers & Operations Research* 36: 1835–42. [CrossRef]

Horváthová, Jarmila, and Martina Mokrišová. 2020. Comparison of the results of a Data Envelopment Analysis Model and Logit Model in Assessing Business Financial Health. *Information* 11: 160. [CrossRef]

Jakaitienė, Audronė, Antanas Zilinskas, and Dovile Stumbrienė. 2018. Analysis of Education Systems Performance in European Countries by Means of PCA-DEA. *Informatics in Education* 17: 245–63. [CrossRef]

Jolliffe, Ian T. 2002. *Principal Component Analysis*, 2nd ed. New York: Springer.

Kliestik, Tomáš, Katarína Valášková, Jana Klieštiková, Mária Kováčová, and Lucia Švábová. 2019. *Prediction of Financial Health of Business Entities in Transition Economies*. Žilina: EDIS. (In Slovak)

Kočišová, Kristína. 2012. Application of DEA models at the analysis of bank branches technical efficiency. *Ekonomický časopis* 60: 169–86. (In Slovak)

Kruskal, Joseph B. 1964. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika* 29: 1–27. [CrossRef]

Labudová, Viera, Mária Vojtková, and Bohdan Linda. 2010. The application of multidimensional methods in measuring poverty. *Ekonomie a Management* 10: 6–22. (In Slovak)

Lozano, Sebastián, and Ester Gutiérrez. 2011. Efficiency analysis of EU-25 member states as tourist destinations. *International Journal of Services, Technology and Management* 15: 69–88. [CrossRef]

Mahmood, Mo A., and Edward C. Lawrence. 1987. A performance analysis of parametric and nonparametric discriminant approaches to business decision making. *Decision Sciences* 18: 308–26. [CrossRef]

Mar Molinero, Cecilio, and Mahmoud Ezzamel. 1991. Multidimensional scaling applied to corporate failure. *Omega* 19: 259–74. [CrossRef]

Mar-Molinero, Cecilio, and Carlos Serrano-Cinca. 2001. Bank failure: A multidimensional scaling approach. *The European Journal of Finance* 7: 165–83. [CrossRef]

Meloun, Milan. 2011. Computer Analysis of Multidimensional Data in the Fields of Natural, Technical and Social Sciences. Available online: https://docplayer.cz/37127577-Pocitacova-analyza-vicerozmernych-dat-v-oborech-prirodnich-technickych-a-spolecenskych-ved.html (accessed on 21 November 2020). (In Czech)

Meloun, Milan, Jíří Militký, and Martin Hill. 2017. *Statistical Analysis of Multidimensional Data in Examples*. Praha: Karolinum. (In Czech)

Mendelová, Viera, and Tatiana Bieliková. 2017. Diagnosing of the Corporate Financial Health Using DEA: An Application to Companies in the Slovak Republic. *Political Economy* 65: 26–44. (In Slovak) [CrossRef]

Mendelová, Viera, and Mária Stachová. 2016. Comparing DEA and logistic regression in corporate financial distress prediction. Paper presented at International Scientific Conference FERNSTAT 2016, Banská Bystrica, Slovakia, September 22–23; pp. 95–104.

Mihalovič, Matúš. 2015. The Assessment of Corporate Financial Performance via Discriminant Analysis. *Acta Oeconomica Cassoviensia: Scientific Journal* 8: 57–69. [CrossRef]

Mousavi, Mohammad M., Jamal Ouenniche, and Bing Xu. 2015. Performance evaluation of bankruptcy prediction models: An orientation-free super-efficiency DEA-based framework. *International Review of Financial Analysis* 42: 64–75. [CrossRef]

Mukhopadhyay, Ayan, Suman Tiwari, Ankit Narsaria, and Bhaskar Roy Karmaker. 2012. New Approach to Predicting Bankruptcy: Combining DEA and Multi-Layer Perceptron. *International Journal of Computer Science Issues* 9: 71–8.

Nasser, Alissar. 2019. Measuring the Performance of Hospitals in Lebanese qadas Using PCA- DEA Model. *Computer and Information Science* 12: 23–32. [CrossRef]

Ouenniche, Jamal, and Kaoru Tone. 2017. An out-of-sample evaluation framework for DEA with application in bankruptcy prediction. *Annals of Operations Research* 254: 235–50. [CrossRef]

Paradi, Joseph C., Mette Asmild, and Paul C. Simak. 2004. Using DEA and worst practice DEA in credit risk evaluation. *Journal of Productivity Analysis* 21: 153–65. [CrossRef]

Premachandra, Inguruwatt M. 2001. A note on DEA vs. principal component analysis: An improvement to Joe Zhu's approach. *European Journal of Operational Research* 132: 553–60. [CrossRef]

Premachandra, Inguruwatt M., Gurmeet Singh Bhabra, and Toshiyuki Sueyoshi. 2009. DEA as a tool for bankruptcy assessment: A comparative study with logistic regression technique. *European Journal of Operational Research* 193: 412–24. [CrossRef]

Premachandra, Inguruwatt M., Yao Chen, and John Watson. 2011. DEA as a Tool for Predicting Corporate Failure and Success: A Case of Bankruptcy Assessment. *Omega* 3: 620–26. [CrossRef]

Rahimpour, Khojasteh, Hadi Shirouyehzad, Milad Asadpour, and Mehdi Karbasian. 2020. A PCA-DEA method for organizational performance evaluation based on intellectual capital and employee loyalty: A case study. *Journal of Modelling in Management* 15: 1479–513. [CrossRef]

Reris, Robert, and J. Paul Brooks. 2015. Principal component analysis and optimization: A tutorial. Paper presented at 14th INFORMS Computing Society Conference, Richmond, Virginia, January 11–13; pp. 212–25. [CrossRef]

Sagarra, Marti, Cecilio Mar-Molinero, and Tommaso Agasisti. 2017. Exploring the efficiency of Mexican universities: Integrating Data Envelopment Analysis and Multidimensional Scaling. *Omega* 67: 123–33. [CrossRef]

Serrano-Cinca, Carlos, and Cecilio Mar Molinero. 2004. Selecting DEA specifications and ranking units via PCA. *Journal of the Operational Research Society* 55: 521–28. [CrossRef]

Shen, Sheng-Lun. 2017. Efficiency of the healthcare system in Taiwan: An illustration with Multidimensional Scaling and Data Envelopment Analysis. *The International Journal of Organizational Innovation* 19: 48–63.

Shetty, Udaya, Tpm Pakkala, and Thathaiah Mallikarjunappa. 2012. A modified directional distance formulation of DEA to assess bankruptcy: An application to IT/ITES companies in India. *Expert Systems with Applications* 9: 1988–97. [CrossRef]

Simak, Paul C. 1997. DEA Based Analysis of Corporate Failure. Master thesis, Faculty of Applied Sciences and Enineering, University of Toronto, Toronto, ON, Canada.

Smith, Lindsay I. 2002. A Tutorial on Principal Component Analysis. Available online: https://ourarchive.otago.ac.nz/bitstream/handle/10523/7534/OUCS-2002-12.pdf (accessed on 17 November 2019).

Štefko, Róbert, Jarmila Horváthová, and Martina Mokrišová. 2020. Bankruptcy Prediction with the Use of Data Envelopment Analysis: An Empirical Study of Slovak Businesses. *Journal of Risk and Financial Management* 13: 212. [CrossRef]

Sueyoshi, Toshiyuki, and Mika Goto. 2009. Methodological comparison between DEA and DEA-DA from the perspective of bankruptcy assessment. *European Journal of Operational Research* 188: 561–75. [CrossRef]

Thanassoulis, Emmanuel. 2001. *Introduction to the Theory and Application of Data Envelopment Analysis*. New York: Springer.

Torre, Eva M., Fernando Casani, and Marti Sagarra. 2018. Defining typologies of universities through a DEA-MDS analysis: An institutional characterization for formative evaluation purposes. *Research Evaluation* 27: 388–403. [CrossRef]

Van der Maaten, Laurens, and Geoffrey Hinton. 2012. Visualizing non-metric similarities in multiple maps. *Machine Learning* 87: 33–55. [CrossRef]

Vishkaeii, Roghyeh Malekii, Behrouz Daneshian, and Farhad Hosseinzadeh Lotfi. 2020. Modifying the convexity condition in Data Envelopment Analysis (DEA). *Revista Científica de la Sociedad* 33: 454–67. [CrossRef]

Zhu, Joe. 1998. Data envelopment analysis vs. principal component analysis: An illustrative study of economic performance of Chinese cities. *European Journal of Operational Research* 111: 50–61. [CrossRef]

Zhu, Joe. 2019. *DEA Frontier Software*. Worcester: Foisie Business School, Worcester Polytechnic Institute.