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Dynamic Conditional Bias-Adjusted Carry Cost Rate Futures Hedge Ratios

Dean Leistikow, Yi Tang * and **Wei Zhang**

Gabelli School of Business, Fordham University, 140 West 62nd Street, New York, NY 10023, USA; leistikow@fordham.edu (D.L.); wzhang167@fordham.edu (W.Z.)
* Correspondence: ytang@fordham.edu; Tel.: +1-646-312-8292

Abstract: This paper proposes new dynamic conditional futures hedge ratios and compares their hedging performances along with those of common benchmark hedge ratios across three broad asset classes. Three of the hedge ratios are based on the upward-biased carry cost rate hedge ratio, where each is augmented in a different bias-mitigating way. The carry cost rate hedge ratio augmented with the dynamic conditional correlation between spot and futures price changes generally: (1) provides the highest hedging effectiveness and (2) has a statistically significantly higher hedging effectiveness than the other hedge ratios across assets, sub-periods, and rolling window sizes.

1. Introduction

The goal of futures hedging is to reduce the firm's risk and increase its value. A key consideration for a futures hedger is the ratio of futures assets to short over the number of spot assets long, i.e., the futures hedge ratio. Not surprisingly, a large literature has evolved regarding the best hedge ratio estimation technique, i.e., that which is expected to reduce the hedger's risk the most. Wang et al. (2015) empirically tested many hedge ratio estimation methods and found that they fail to beat the naïve hedge ratio.

This paper examines dynamic conditional futures hedge ratios. It focuses on: (1) the economics-based carry-cost rate hedge ratio introduced in Leistikow et al. (2020) and employed in Leistikow and Chen (2019), (2) a hedge ratio based on the Engle (2002) statistics-based dynamic conditional correlation model, and (3) three bias-adjusted versions of the carry-cost rate hedge ratio, one of which incorporates the Engle (2002) dynamic conditional correlation model. It compares the hedge ratios' hedging performances across three broad asset classes to those of each other and those of two common benchmark hedge ratios. The carry-cost rate hedge ratio when augmented with the dynamic conditional correlation between the spot and futures price changes generally (1) provides the highest hedging effectiveness and (2) has a statistically significantly higher hedging effectiveness than either that of the common benchmark hedge ratios or the other approaches across: assets, sub-periods, and rolling window sizes. Moreover, it explains why the carry-cost rate hedge ratio, when augmented with the dynamic conditional correlation between the spot and futures price changes, performs better than the naïve hedge ratio.

Section 2 of the paper discusses the hedge ratios studied. Section 3 specifies the data employed. Section 4 presents statistics regarding the hedge ratios, where the hedge ratios are calculated using a 1008 trading day (i.e., 4 years) rolling window. Alternative rolling window sizes (of 2 and 6 years) are analyzed and found to yield similar results; these results are available in Appendices that will be provided upon request from the authors. Section 5 shows the hedge ratios' hedging performances and relative hedging performances. Section 6 concludes the paper.

2. Risk-Minimizing Hedge Ratio Estimation

For each unit of the spot asset, the hedge ratio (h) for the one-unit futures contract that minimizes the future period's risk is:

$$h = \frac{\rho \sigma_s}{\sigma_F},\tag{1}$$

where: σ_s is the standard deviation of the spot price change (ΔS)[1], σ_F is the standard deviation of the futures price change (ΔF) and ρ is the correlation between ΔS and ΔF. In this section, we discuss alternative empirical hedge ratio measures.

2.1. The Traditional Hedge Ratio

The traditional way to estimate the risk-minimizing hedge ratio on day d is to run the ordinary least square regression:

$$\Delta S_d = a + h_t \Delta F_d + \epsilon_d,\tag{2}$$

where the slope coefficient of ΔF, h_t, is the "traditional" hedge ratio. In this study, we estimate Equation (2) on day d using a rolling-window out-of-sample procedure, where ΔS_d and ΔF_d are daily spot and future price changes over the past 1008 trading days (i.e., day d-1007 to day d), respectively.[2] The 1008-trading days, i.e., 4-year, rolling window is updated daily. Moreover, we use the same 1008-trading days rolling window when estimating alternative models (discussed below) for the hedge ratio estimated on d to ensure that they are compared on an equal basis. This "traditional" hedge ratio is discussed in mainstream textbooks, e.g., Hull (2018) and has traditionally been used as the benchmark for alternative hedge ratios that have been proposed over the last 40 years.

2.2. The "Dynamic Conditional" Hedge Ratio

Equation (2) is often considered a static model in that the historical data are assumed to be equally informative about future realizations. In this subsection, we relax this assumption by estimating an autoregressive-based model that allows more recent observations to be weighted more heavily. Our rationale is that if the dynamics of the underlying variable are indeed time-varying, an appropriately chosen dynamic conditional model is likely to outperform the static model. Engle (2002) proposes a new class of multivariate models called dynamic conditional correlation (DCC) models that are used to predict the time-varying correlations between two financial assets. These models have the flexibility of univariate GARCH models coupled with parsimonious parametric models for the correlations.[3] Following Engle (2002), we estimate the conditional covariance between ΔS and ΔF:

$$\Delta S_d = \alpha_0^{\Delta S} + \varepsilon_{\Delta S,d},\tag{3}$$

$$\Delta F_d = \alpha_0^{\Delta F} + \varepsilon_{\Delta F,d},\tag{4}$$

$$E_d\left[\varepsilon_{\Delta S,d+1}^2\right] \equiv \sigma_{\Delta S,d+1}^2 = \beta_0^{\Delta S} + \beta_1^{\Delta S}\varepsilon_{\Delta S,d}^2 + \beta_2^{\Delta S}\sigma_{\Delta S,d}^2,\tag{5}$$

$$E_d\left[\varepsilon_{\Delta F,d+1}^2\right] \equiv \sigma_{\Delta F,d+1}^2 = \beta_0^{\Delta F} + \beta_1^{\Delta F}\varepsilon_{\Delta F,d}^2 + \beta_2^{\Delta F}\sigma_{\Delta F,d}^2,\tag{6}$$

$$E_d[\varepsilon_{\Delta S,d+1}\varepsilon_{\Delta F,d+1}] \equiv \sigma_{\Delta S\Delta F,d+1} = \rho_{\Delta S\Delta F,d+1} \cdot \sigma_{\Delta S,d+1} \cdot \sigma_{\Delta F,d+1},\tag{7}$$

$$\rho_{\Delta S\Delta F,d+1} = \frac{q_{\Delta S\Delta F,d+1}}{\sqrt{q_{\Delta S\Delta S,d+1} \cdot q_{\Delta F\Delta F,d+1}}},\tag{8}$$

$$q_{\Delta S\Delta F,d+1} = \overline{\rho}_{\Delta S\Delta F} + a_1 \cdot (\varepsilon_{\Delta S,d} \cdot \varepsilon_{\Delta F,d} - \overline{\rho}_{\Delta S\Delta F}) + a_2 \cdot (q_{\Delta S\Delta F,d} - \overline{\rho}_{\Delta S\Delta F})\tag{9}$$

In the above system of equations, ΔS_d and ΔF_d denote the daily spot and futures price changes over the past 1008 trading days up to day d, respectively, and $E_d[.]$ denotes the expectation operator conditional on day d information. $\sigma_{\Delta S,d+1}^2$ is the day-d expected condi-

tional variance of daily spot price changes (ΔS), $\sigma^2_{\Delta F,d+1}$ is the day-d expected conditional variance of daily futures price changes (ΔF), and $\sigma^2_{\Delta S\Delta F,d+1}$ is the day-d expected conditional covariance between ΔS and ΔF. $\rho_{\Delta S\Delta F,d+1} = q_{\Delta S\Delta F,d+1}/\sqrt{q_{\Delta S\Delta S,d+1} \cdot q_{\Delta F\Delta F,d+1}}$ is the day-d expected conditional correlation between ΔS and ΔF; $\overline{\rho}_{\Delta S\Delta F}$ is the unconditional correlation between ΔS and ΔF. To ease the parameter convergence, we follow Bali and Engle (2010) and Bali et al. (2017) and use correlation targeting, assuming that the time-varying correlation mean reverts to the sample correlation, $\overline{\rho}_{\Delta S\Delta F}$. For each day $d + 1$, the DCC hedge ratio is defined as the ratio of Equations (6) and (7): $h_{dcc} = \sigma_{\Delta S\Delta F,d+1}/\sigma^2_{\Delta F,d+1}$. Therefore, h_{dcc} is purely out-of-sample, estimated using the information available up to the formation of the hedged portfolio (i.e., day d).

2.3. The Simple Carry-Cost-Rate Hedge Ratio

Unfortunately, the traditional hedge ratio ignores the economic connection between the spot and futures prices. Leistikow et al. (2020) derived a hedge ratio that incorporates the carry-cost hypothesis that links spot and futures prices as follows:

$$S(1+c)^T = F, \tag{10}$$

where c is the annualized spot asset's carry-cost rate (hereinafter CCR) to the futures' maturity,[4] T is the years to the futures contract's maturity, and S and F are the asset's contemporaneous spot and futures prices, respectively.

From the carry-cost hypothesis, it follows that $\frac{\partial S}{\partial F} = (1+c)^{-T}$; thus, $\frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = (1+c)^{-T}$. Assuming ΔS and ΔF are perfectly correlated, the day d instantaneous hedge ratio (h_{ccr}) is derived as[5]:

$$h_{ccr} = (1+c)^{-T} \tag{11}$$

Therefore, h_{ccr} is calculated quickly and simply, since it only requires knowledge of c and T on day d.

2.4. The Augmented Carry-Cost-Rate Hedge Ratios

The simple CCR-based hedge ratio discussed in the previous subsection is easier to implement and has a firmer economic foundation than the statistics-based approaches discussed earlier. However, h_{ccr} is biased upward because the correlation between ΔS and ΔF is almost certainly less than its assumed level of one.[6] In this subsection, we introduce several ways to augment h_{ccr} to correct its upward bias.

First, we augment h_{ccr} with the dynamic conditional correlation between ΔS and ΔF. It was estimated as part of the dynamic conditional hedge ratio discussed earlier in Section 2.2. We calculate this hedge ratio, denoted h_{ccr_dcc}, on day d as:

$$h_{ccr\_dcc} = h_{ccr} \cdot \rho_{dcc}, \tag{12}$$

where ρ_{dcc} comes from Equation (8) based on daily data over the period from d-1007 to day d.

Second, we augmented h_{ccr} with the correlation (ρ) between daily ΔS and ΔF over the period from d-1007 to day d. We calculated this hedge ratio, denoted h_{ccr_corr}, on day d as:

$$h_{ccr\_corr} = h_{ccr} \cdot \rho \tag{13}$$

Finally, we use the Leistikow et al. (2020) BAM bias-adjusted h_{ccr} on day d, denoted h_{ccr_bam}:

$$h_{ccr\_bam} = h_{ccr} \cdot BAM \tag{14}$$

where BAM is the bias-adjustment multiplier, defined as the average ratio of the average daily h_t and average h_{ccr} for each futures contract employed in the hedges over the 1008 trading days between day d-1007 to day d. In this way, each of the three bias-adjustment factors:

ρ_{dcc}, ρ, and BAM, proxy h_{ccr}'s bias based on the same amount of data, 1008 trading days for now.

2.5. The "Naive" Hedge Ratio

Despite the widespread acceptance of h_t and the development of more complex statistical variants of it, Wang et al. (2015) find that h_t and its more complex statistical variants do not reduce risk better than does the "naive" hedge ratio of 1 (h_1). Therefore, we use h_1 as well as the regression-based hedge ratio (h_t) as benchmarks.

3. Data

We study the S&P500, Japanese Yen (JY), and gold since they represent a broad cross section of asset classes, their carry-cost rates are known and objectively calculable, and their spot and futures markets trade actively. All data are downloaded via a Bloomberg terminal, which are updated daily during our sample period.

CME e-mini S&P500 futures contracts mature on the 3rd Friday of their maturity month and have been the most liquid equity futures since about 2000 when they overtook the regular S&P500 futures. We study their March, June, September, and December contract maturities because they are the most liquid contract maturities and are very liquid (except when they are far from and very near to their maturities). These futures contract maturity months are alternatingly used as the hedging instrument where the hedging instrument is the near maturity futures contract until it is one week to its maturity at which point it switches to the second near maturity futures contract. The JY futures contracts mature two business days before the 3rd Wednesday in the maturity month. For liquidity reasons, the near futures maturity contract is used as the hedging instrument for the IMM Japanese Yen, until the 1st Friday of the maturity month. For COMEX gold, we use the liquid February, April, June, August, October, and December futures contract maturities and roll the hedging instrument from the near to the second near futures maturity two weeks earlier than for the JY due to gold's earlier liquidity shift from the near to the second near futures due to gold futures' delivery options. The gold futures mature the 3rd last business day of the maturity month.

The S&P500 data begins with the Mar '98, while the JY data begins with the Mar '97, and the gold data begins with the Feb '91 futures contract as the hedging instrument. The Jun '20 futures is the last futures used as the hedging instrument for all the assets.

The carry-cost rate is the US risk-free short-term interest rate[7] minus the S&P500 dividend yield, the excess of the US over the JY risk-free short-term interest rate, and the US risk-free short-term interest rate, for the S&P500, JY and gold, respectively.

4. Statistics for the Hedge Ratios and Their Differences from the Benchmarks

In this section, we provide statistics for the various daily profit hedge ratios (and their differences from the benchmark hedge ratios) discussed in Section 2. After the first 4 years (given the 4 years rolling window), for each subsequent S&P500 and JY futures contract an average hedge ratio is calculated based on the 252/4 trading days that the contract is the hedging instrument since their contracts follow a quarterly rotation. The corresponding gold average hedge ratio is calculated based on the 252/6 trading days, since gold futures contracts follow a bimonthly rotation. These futures' average hedge ratios represent the underlying data for the Table 1 statistics. Table 1A reports the descriptive statistics for the average hedge ratios for the S&P500 in Panel A, JY in Panel B, and gold in Panel C.[8] In each panel, the first column presents the results of the traditional regression hedge ratio (h_t), which serves as Table 1A's benchmark hedge ratio. Columns 2–6 represent the descriptive statistics for the differences between h_t and the: dynamic conditional hedge ratio (h_{dcc}), the simple carry-cost-rate hedge ratio (h_{ccr}), and, finally, the carry-cost-rate hedge ratio augmented with (i) the dynamic conditional correlation estimated from the DCC model (h_{ccr_dcc}), (ii) the simple correlation between the daily spot and futures price changes (h_{ccr_corr}), and (iii) the bias-adjustment multiplier (h_{ccr_bam}), respectively.

Column (1) of Table 1A shows that the mean traditional hedge ratios (h_t) for the S&P500, JY, and gold are between 0.8975 and 0.9568. Their standard deviations are very small, ranging from 0.0151 to 0.0469. Thus, as expected, their daily hedge ratios are highly persistent since they are estimated using 1008 trading-days rolling windows.

Table 1. The Benchmark Hedge Ratios and Differences from them.

	A. The Benchmark Hedge Ratio and Differences from it When h_t is the Benchmark.					
	Panel A. S&P500					
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9568	0.0073	0.0441	0.0133	0.0153	−0.0012
std	0.0151	0.0323	0.0156	0.0164	0.014	0.0144
count	74	74	74	74	74	58
t-stat		1.9382	24.3608	6.9605	9.3919	−0.614
signif lev		10%	1%	1%	1%	
	Panel B. JY					
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9554	−0.01	0.0423	0	−0.0065	−0.0036
std	0.018	0.0389	0.0177	0.0232	0.0208	0.0162
count	78	78	78	78	78	62
t-stat		−2.2673	21.1487	0.0174	−2.757	−1.7318
signif lev		5%	1%		1%	10%
	Panel C. Gold					
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.8975	0.0112	0.0982	0.0233	0.0098	0.0058
std	0.0469	0.0582	0.0483	0.0387	0.0189	0.0287
count	153	153	153	153	153	129
t-stat		2.3805	25.1193	7.4559	6.3829	2.3166
signif lev		5%	1%	1%	1%	5%
	Panel D. HR Differences Aggregated across Assets					
		$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean		0.0002	0.0715	0.0123	0.0084	0.0015
std		0.0429	0.0451	0.0238	0.0215	0.0156
count		222	222	222	222	174
t-stat		0.0775	23.619	7.6897	5.8467	1.2684
signif lev			1%	1%	1%	

The count is the number of futures contracts used as hedge instruments, or, alternatively, the number of hedges since each hedge has a single hedging instrument. For each futures, the "average hedge ratio" is determined from the ≈252/4 dynamic daily hedge ratios calculated for a quarterly maturing futures contract (e.g., as for the S&P500 and JY, whereas for gold there are ≈252/6 dynamic daily hedge ratios since the futures maturities are bimonthly). Since the BAM is calculated from 4 years of futures contracts, the h_{ccr_bam} count is 4 × 4 (6 × 4) for the quarterly futures contract maturities of the S&P500 and JY (bimonthly maturities of gold).

The "aggregated across assets" count is equal-weighted. It uses the most recent differences for each asset for roughly the same period as the e−mini S&P500 (since its data series is the shortest). Since the gold futures data started earlier and there are 50% more gold futures maturities/years, a little less than the 1st half of the gold futures contract results do not enter into the aggregation across assets.

Table 1. *Cont.*

B. The Benchmark Hedge Ratio and Differences from it When h_1 is the Benchmark					
Panel A. S&P500					
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	1	−0.0359	0.0009	−0.0299	−0.0279	−0.0396
std	0	0.0321	0.0024	0.0058	0.0054	0.0106
count	74	74	74	74	74	58
t-stat		−9.6302	3.3772	−44.7285	−44.1444	−28.4181
signif lev		1%	1%	1%	1%	1%
Panel B. JY					
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	1	−0.0546	−0.0023	−0.0446	−0.0511	−0.0431
std	0	0.0391	0.0025	0.0156	0.0132	0.0123
count	78	78	78	78	78	62
t-stat		−12.3317	−8.2224	−25.2679	−34.2633	−27.5771
signif lev		1%	1%	1%	1%	1%
Panel C. Gold					
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	1	−0.0913	−0.0043	−0.0791	−0.0927	−0.0913
std	0	0.0671	0.0039	0.0385	0.0465	0.0362
count	153	153	153	153	153	129
t-stat		−16.8137	−13.8224	−25.4034	−24.6593	−28.6446
signif lev		1%	1%	1%	1%	1%
Panel D. HR Differences Aggregated across Assets					
		$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean		−0.0721	−0.0008	−0.06	−0.0639	−0.0694
std		0.059	0.0024	0.0368	0.039	0.0422
count		222	222	222	222	174
t-stat		−18.2127	−4.9113	−24.2818	−24.4085	−21.6855
signif lev		1%	1%	1%	1%	1%

The count is the number of futures contracts used as hedge instruments, or, alternatively, the number of hedges since each hedge has a single hedging instrument. For each futures, the "average hedge ratio" is determined from the ≈252/4 dynamic daily hedge ratios calculated for a quarterly maturing futures contract (e.g., as for the S&P500 and JY, whereas for gold there are ≈252/6 dynamic daily hedge ratios since the futures maturities are bimonthly). Since the BAM is calculated from 4 years of futures contracts, the h_ccr_bam count is 4 × 4 (6 × 4) for the quarterly futures contract maturities of the S&P500 and JY (bimonthly maturities of gold).

The "aggregated across assets" count is equal-weighted. It uses the most recent differences for each asset for roughly the same period as the e−mini S&P500 (since its data series is the shortest). Since the gold futures data started earlier and there are 50% more gold futures maturities/years, a little less than the 1st half of the gold futures contract results do not enter into the aggregation across assets.

Columns 2–6 of Table 1A show that the mean differences between the other hedge ratios and h_t are small (<0.0233 in absolute value across the assets); except for the upward-biased h_{ccr}; its maximum mean difference is 0.0982 (for gold). Therefore, the bias adjustments to the CCR hedge ratio largely mitigate the simple CCR hedge ratio's upward bias. Similar to h_{ccr}, both h_ccr_dcc, and h_{ccr_corr} are larger than h_t and the difference is significant at the 1% level; however, this does not hold for the JY. The paper's statistical significance

tests are two-tailed tests, except for the one-tailed $h_{ccr} - h_t$ test in that h_{ccr} is upward biased relative to h_t.

Table 1A, Panel D has equal-weighted, roughly contemporaneous, aggregated across asset results that end with the assets' June 2020 futures contracts. To generate these results, since the S&P500 data/results start later than the other assets' (starting with the Mar 2002 futures), all of its results are included but only the contemporaneous JY (starting with the Mar 2002 futures) and roughly contemporaneous gold results are included in the aggregation (starting with the April 2008 futures). The gold results included in the equal-weighted aggregation begin later, since gold has 50% more futures maturities/years than the others and therefore gets the same number of futures contracts in 2/3 the time. The aggregated across assets mean $h_{ccr} - h_t$ was above 0.07, while the means for the other differences were not more than 0.0123 in absolute value. Hedge ratios h_{ccr}, h_{ccr_dcc}, and h_{ccr_corr} are all larger than h_t at the 1% significance level.

Table 1B below is the same as Table 1A, except that Table 1B's benchmark hedge ratio is h_1. The other hedge ratios' means, except for the S&P500 h_{ccr}, are significantly less than 1 at the 1% confidence level for all assets. For the Panel D, aggregations across assets, while h_{ccr}'s mean is barely below 1, the other hedge ratios' means are between 0.06 and 0.0721 below 1. All of the hedge ratio means are below h_1 at the 1% significance level. The fact that, for this study period and these assets, the upward biased h_{ccr} is significantly below 1 portends poor h_1 hedging performance; this conjecture is strongly supported by the results in the next section.

5. The Benchmark Hedge Ratios' Hedging Performances and Hedging Performance Differences from Those of the Benchmark Hedge Ratios

Next, we calculated each hedge ratio's out-of-sample hedge effectiveness (*HE*) and compared it to those of the two benchmark hedge ratios. The *HE* is defined as the percentage profit variance reduction for each contract over our sample period:

$$HE = 1 - \frac{Var(\text{hedged profits})}{Var(\text{unhedged profits})} \tag{15}$$

$$\text{Hedged profit} = \Delta S - h \cdot \Delta F \tag{16}$$

$$\text{Unhedged profit} = \Delta S \tag{17}$$

where $Var(.)$ denotes variance, ΔS and ΔF are, respectively, the daily spot and futures price changes on day $d + 1$, and h denotes a hedge ratio on day d calculated from one of the hedge ratios described in Section 2. Therefore, the HE of a hedge ratio increases with the risk reduced.

Table 2A reports the hedge effectiveness results, where h_t's HE is the benchmark. Column 1 is h_t's HE while the remaining columns represent the hedge effectiveness differences in between those for the various hedge ratios and for h_t.

Table 2A, column (1) shows that h_t's average HE for the S&P500, JY, and gold are between 0.8464 and 0.9388. Their corresponding standard deviations are small, ranging from 0.0378 to 0.1477. These results indicate that h_t's out-of-sample HE is highly persistent.

The HE for h_t is higher than that for either h_{dcc} or h_{ccr}. The latter result suggests that the benefit of h_{ccr} through recognizing the economic link between the spot and the future prices is less than the cost of its upward bias. The finding that h_{dcc} does not outperform h_t is consistent with previous studies that show that complicated time-series hedge ratios do not yield superior HE performance.[9] Therefore, modeling the dynamic conditional hedge ratio based on spot and futures price changes alone is insufficient to improve hedge effectiveness.

Columns 4–6 of Table 2A compare the augmented h_{ccr}s' HEs with h_t's HE where the h_{ccr} is augmented with the conditional correlation (h_{ccr_dcc}), the unconditional correlation (h_{ccr_corr}), and the bias-adjustment multiplier (h_{ccr_bam}), respectively. Column (4) shows that the HE for h_{ccr_dcc} is higher than that for h_t for each asset, where the difference in HE is generally statistically significant at the 5% level. When the HE differences are aggregated across assets,

the HE for h_{ccr_dcc} is statistically significantly higher than that for h_t at the 1% significant level. Columns (5) and (6) show that the HEs for h_{ccr_corr} and h_{ccr_bam} are not typically significantly different from that for h_t. Therefore, the h_{ccr_dcc} result suggests that incorporating the economic relation between the spot and futures prices and properly modeling their time-varying conditional correlations jointly produce the best hedging performance.

Table 2. The Benchmark HRs' Hedge Effectiveness and Differences from them.

	A. The Benchmark HR's Hedge Effectiveness and Differences from it When h_t is the Benchmark					
			Panel A. S&P500			
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9388	−0.0014	−0.0011	0.0035	0.0003	0.0001
std	0.0378	0.0215	0.005	0.0137	0.0022	0.002
count	74	74	74	74	74	58
t-stat		−0.5447	−1.9024	2.1813	1.1068	0.2169
signif lev			10%	5%		
			Panel B. JY			
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9107	−0.0023	−0.0018	0.0003	0.0008	0.0001
std	0.0479	0.0098	0.0051	0.0028	0.003	0.0022
count	78	78	78	78	78	62
t-stat		−2.0244	−3.1242	1.0111	2.2843	0.5062
signif lev		5%	1%		5%	
			Panel C. Gold			
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.8464	−0.0184	−0.0115	0.0055	−0.0006	−0.0006
std	0.1477	0.1955	0.026	0.0259	0.0046	0.0071
count	153	153	153	153	153	129
t-stat		−1.1639	−5.4726	2.6309	−1.5674	−0.9196
signif lev			1%	1%		
		Panel D. HE Differences Aggregated across Assets				
	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$	
mean	−0.0023	−0.0085	0.0032	0	0.0001	
std	0.0189	0.0197	0.0124	0.0041	0.0022	
count	222	222	222	222	174	
t-stat	−1.8015	−6.4151	3.8774	−0.1053	0.4399	
signif lev	10%	1%	1%			
	B. The Benchmark HR's Hedge Effectiveness and Differences from it When h_1 is the Benchmark					
			Panel A. S&P500			
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.9378	−0.0004	−0.0002	0.0044	0.0012	0.0015
std	0.0406	0.0228	0.0005	0.0167	0.003	0.0047
count	74	74	74	74	74	58
t-stat		−0.1566	−2.6257	2.2837	3.5766	2.3963
signif lev			5%	5%	1%	5%

Table 2. *Cont.*

Panel B. JY						
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.9087	−0.0003	0.0002	0.0023	0.0028	0.0019
std	0.0492	0.0126	0.0004	0.0057	0.0071	0.0055
count	78	78	78	78	78	62
t-stat		−0.1944	4.2624	3.554	3.4394	2.7357
signif lev			1%	1%	1%	1%
Panel C. Gold						
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.8343	−0.0063	0.0006	0.0176	0.0115	0.014
std	0.1629	0.1958	0.0011	0.0373	0.0251	0.0219
count	153	153	153	153	153	129
t-stat		−0.3963	7.3019	5.8403	5.692	7.2647
signif lev			1%	1%	1%	1%
Panel D. HE Differences Aggregated across Assets						
		$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean		0.0063	0.0001	0.0118	0.0086	0.0096
std		0.0296	0.0005	0.0249	0.0184	0.0185
count		222	222	222	222	174
t-stat		3.1852	4.0726	7.0864	6.9614	6.8561
signif lev		1%	1%	1%	1%	1%

Table 2B reports the same results; however, h_1's HE is the benchmark. The HE for h_1 is statistically significantly lower (generally at the 1% level) than that for all the other hedge ratios except for h_{dcc} and, for the S&P500, h_{ccr}. Our finding differs from that of Wang et al. (2015) and neither should be interpreted as a general result. The low HE for h_1 was anticipated earlier when we noted that the upward biased h_{ccr} was significantly less than 1. Our low HE for h_1 is not a general result since the carry-cost rate varies across assets, currency denominations, and time; thus, in the very improbable case that the upward biased h_{ccr} exceeded 1 by its bias, the risk minimizing hedge ratio would be h_1.

Table 3 repeats the HE analyses, except that it analyzes the first and second halves of our sample period separately to see if the HE and HE difference results are stable across sub-periods. The only HE difference from the benchmark's HE that is nearly always statistically significantly positive across both benchmarks, both halves, and all 3 assets is that for h_{ccr_dcc} and the JY in the 1st half when h_t is the benchmark is the main exception. The HEs for the other two augmented h_{ccr}s are statistically significantly higher than those for h_1 for 8 of the 12 individual asset results. However, their HEs are not generally higher for those of h_t. Table 3, Panel D, shows for the aggregations across assets that the HE for h_{ccr_dcc} is statistically significantly higher than that for either benchmark in both sub-periods at the 1% level. The HEs for the other two augmented h_{ccr}s are also statistically significantly higher than those for h_1 in both halves at the 1% level.

Table 3. Stability of the Benchmark HRs' HEs and Differences from them.

			Panel A. S&P500			
			1st half: HE and HE differences when the benchmark is h_t			
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9427	−0.0007	0.0003	0.0013	0.0008	0.0001
std	0.0248	0.0045	0.0036	0.0028	0.0017	0.0022
count	37	37	37	37	37	21
t-stat		−0.9852	0.493	2.8216	3.0666	0.2467
signif lev				1%	1%	
			2nd half: HE and HE differences when the benchmark is h_t			
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9348	−0.002	−0.0025	0.0057	−0.0003	0
std	0.0475	0.0303	0.0058	0.019	0.0026	0.0019
count	37	37	37	37	37	37
t-stat		−0.3998	−2.6145	1.8057	−0.6361	0.0712
signif lev			5%	10%		
			1st half: HE and HE differences when the benchmark is h_1			
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.9429	−0.001	0.0001	0.0011	0.0006	0.0003
std	0.0256	0.0042	0.0003	0.0039	0.0026	0.0037
count	37	37	37	37	37	21
t-stat		−1.4123	1.0029	1.6317	1.4325	0.3594
signif lev						
			2nd half: HE and HE differences when the benchmark is h_1			
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.9327	0.0001	−0.0004	0.0078	0.0019	0.0022
std	0.0513	0.0322	0.0005	0.0229	0.0032	0.0051
count	37	37	37	37	37	37
t-stat		0.0259	−3.9637	2.0694	3.5117	2.5698
signif lev			1%	5%	1%	5%
			Panel B. JY			
			1st half: HE and HE differences when the benchmark is h_t			
	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	0.9072	−0.0019	−0.002	0	0.0008	−0.0002
std	0.0553	0.0117	0.0059	0.0034	0.0036	0.0013
count	39	39	39	39	39	23
t-stat		−0.9886	−2.1761	−0.065	1.3773	−0.8266
signif lev			5%			

Table 3. *Cont.*

	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
2nd half: HE and HE differences when the benchmark is h_t						
mean	0.9142	−0.0026	−0.0015	0.0007	0.0008	0.0004
std	0.0397	0.0076	0.0041	0.002	0.0023	0.0026
count	39	39	39	39	39	39
t-stat		−2.1798	−2.3064	2.0882	2.0626	0.8715
signif lev		5%	5%	5%	5%	

	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
1st half: HE and HE differences when the benchmark is h_1						
mean	0.9049	0.0004	0.0002	0.0022	0.0031	0.0017
std	0.0574	0.0148	0.0005	0.0067	0.0085	0.0066
count	39	39	39	39	39	23
t-stat		0.1703	2.804	2.0646	2.2411	1.248
signif lev			1%	5%	5%	

	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
2nd half: HE and HE differences when the benchmark is h_1						
mean	0.9126	−0.001	0.0002	0.0024	0.0024	0.002
std	0.0398	0.01	0.0003	0.0046	0.0053	0.0049
count	39	39	39	39	39	39
t-stat		−0.601	3.7447	3.2522	2.8669	2.6087
signif lev			1%	1%	1%	5%

Panel C. Gold

	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
1st half: HE and HE differences when the benchmark is h_t						
mean	0.9013	−0.0333	−0.0012	0.0053	−0.0001	−0.0007
std	0.155	0.2766	0.018	0.0334	0.0029	0.0051
count	76	76	76	76	76	52
t-stat		−1.051	−0.5814	1.3761	−0.2459	−0.9525
signif lev						

	h_t	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
2nd half: HE and HE differences when the benchmark is h_t						
mean	0.7922	−0.0036	−0.0216	0.0058	−0.0011	−0.0005
std	0.118	0.0226	0.0286	0.0156	0.0057	0.0082
count	77	77	77	77	77	77
t-stat		−1.4157	−6.6423	3.2315	−1.6364	−0.5428
signif lev			1%	1%		

	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
1st half: HE and HE differences when the benchmark is h_1						
mean	0.8993	−0.0313	0.0009	0.0073	0.002	0.0028
std	0.1648	0.2734	0.0014	0.0385	0.0198	0.0108

Table 3. *Cont.*

	1st half: HE and HE differences when the benchmark is h_1					
count	76	76	76	76	76	52
t-stat		−0.9975	5.4563	1.6599	0.8745	1.8718
signif lev			1%			10%
	2nd half: HE and HE differences when the benchmark is h_1					
	h_1	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.7701	0.0184	0.0004	0.0278	0.021	0.0215
std	0.1337	0.0411	0.0006	0.0334	0.0263	0.0242
count	77	77	77	77	77	77
t-stat		3.9315	6.0318	7.3037	6.9998	7.8227
signif lev		1%	1%	1%	1%	1%

	Panel D. HE Differences Aggregated across Assets:				
	1st half: HE differences when the benchmark is h_t				
	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
Mean	−0.0045	−0.0066	0.0011	0.0005	0.0002
std	0.016	0.0189	0.0062	0.0036	0.002
count	111	111	111	111	63
t-stat	−2.9776	−3.7015	1.9475	1.3209	0.7654
signif lev	1%	1%	10%		
	2nd half: HE differences when the benchmark is h_t				
	$h_{dcc} - h_t$	$h_{ccr} - h_t$	$h_{ccr\_dcc} - h_t$	$h_{ccr\_corr} - h_t$	$h_{ccr\_bam} - h_t$
mean	−0.0001	−0.0103	0.0053	−0.0005	0
std	0.0213	0.0204	0.0162	0.0044	0.0024
count	111	111	111	111	111
t-stat	−0.028	−5.3351	3.4564	−1.2058	0.0331
signif lev		1%	1%		
	1st half: HE differences when the benchmark is h_1				
	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.0023	0.0001	0.0079	0.0072	0.0082
std	0.0255	0.0003	0.0198	0.0193	0.0165
count	111	111	111	111	63
t-stat	0.931	4.322	4.2188	3.9366	3.9247
signif lev		1%	1%	1%	1%
	2nd half: HE differences when the benchmark is h_1				
	$h_{dcc} - h_1$	$h_{ccr} - h_1$	$h_{ccr\_dcc} - h_1$	$h_{ccr\_corr} - h_1$	$h_{ccr\_bam} - h_1$
mean	0.0104	0.0001	0.0158	0.01	0.0105
std	0.0329	0.0006	0.0287	0.0174	0.0196
count	111	111	111	111	111
t-stat	3.3385	2.4052	5.7886	6.0397	5.6234
signif lev	1%	5%	1%	1%	1%

Table 4 directly tests the HE differences across the various bias-adjusted h_{ccr} approaches. The HE is statistically significantly higher for h_{ccr_dcc} than it is for the other bias-adjustment approaches for the individual assets (other than for the JY) and for the aggregation across assets.

Table 4. Hedge Effectiveness Differences across h_{ccr} Bias-Adjustment Methods.

	$h_{ccr\_dcc} - h_{ccr\_corr}$	$h_{ccr\_dcc} - h_{ccr\_bam}$	$h_{ccr\_corr} - h_{ccr\_bam}$
Panel A. S&P500			
mean	0.0032	0.0038	−0.0001
std	0.0152	0.0164	0.0018
count	74	58	58
t-stat	1.8087	1.7788	−0.4114
signif lev	10%	10%	
Panel B. JY			
mean	−0.0005	0	0.0008
std	0.0037	0.0028	0.0035
count	78	62	62
t-stat	−1.1004	0.1176	1.6953
signif lev			10%
Panel C. Gold			
mean	0.0061	0.0064	0.0001
std	0.0261	0.0254	0.0087
count	153	129	129
t-stat	2.8874	2.8625	0.162
signif lev	1%	1%	
Panel D. HE Differences Aggregated across Assets			
mean	0.0033	0.0036	−0.0004
std	0.0137	0.0141	0.0039
count	222	174	174
t-stat	3.5345	3.3777	−1.5283
signif lev	1%	1%	

Table 5 repeats the analyses of Table 4 except that it analyzes the first and second halves separately to see if the HE difference results are stable across halves. Though the sample sizes in each half are not large, the HE is generally higher for h_{ccr_dcc} than for the other bias-adjustment approaches across assets and halves (the JY is an exception). While the HE aggregation across assets for the 1st half is not statistically significant, it is significant at the 1% level for the 2nd half.

Table 5. Stability of the HE Differences across h_{ccr} Bias-Adjustment Methods.

	$h_{ccr\_dcc} - h_{ccr\_corr}$	$h_{ccr\_dcc} - h_{ccr\_bam}$	$h_{ccr\_corr} - h_{ccr\_bam}$
Panel A. S&P500 1st Half			
mean	0.0004	0.0007	0.0002
std	0.0025	0.0033	0.0016
count	37	21	21
t-stat	1.0708	0.9055	0.7225
signif lev			
S&P500 2nd Half			
mean	0.0059	0.0056	−0.0003
std	0.0211	0.0203	0.0019
count	37	37	37
t-stat	1.7108	1.6902	−0.9254
signif lev	10%	10%	
Panel B. JY 1st Half			
mean	−0.0008	−0.0004	0.0014
std	0.0049	0.0039	0.0053
count	39	23	23
t-stat	−1.0677	−0.5421	1.2426
signif lev			
JY 2nd Half			
mean	−0.0001	0.0003	0.0004
std	0.0018	0.0019	0.0019
count	39	39	39
t-stat	−0.2797	1.0794	1.3287
signif lev			10%
Panel C. Gold 1st Half			
mean	0.0053	0.0066	0.0011
std	0.033	0.0346	0.0039
count	76	52	52
t-stat	1.4141	1.3841	2.1034
signif lev	10%	10%	5%
Gold 2nd Half			
mean	0.0068	0.0063	−0.0006
std	0.0169	0.0169	0.0107
count	77	77	77
t-stat	3.5346	3.2453	−0.4598
signif lev	1%	1%	

Table 5. *Cont.*

	Panel D. Aggregated across Assets 1st Half		
	$h_{ccr\_dcc} - h_{ccr\_corr}$	$h_{ccr\_dcc} - h_{ccr\_bam}$	$h_{ccr\_corr} - h_{ccr\_bam}$
mean	0.0007	0.0006	−0.0003
std	0.007	0.0071	0.0048
count	111	63	63
t-stat	1.0398	0.7008	−0.5409
signif lev			
	Aggregated across Assets 2nd Half		
mean	0.0058	0.0053	−0.0005
std	0.0177	0.0166	0.0032
count	111	111	111
t-stat	3.4493	3.3615	−1.6791
signif lev	1%	1%	10%

6. Conclusions

The goal of futures hedging is to reduce the firm's risk and increase its value. A key consideration for a futures hedger is the ratio of futures assets to short over the number of spot assets long, i.e., the futures hedge ratio.

This paper proposes new dynamic conditional futures hedge ratios. It studies: (1) the economics-based carry-cost rate hedge ratio introduced in Leistikow et al. (2020) and employed in Leistikow and Chen (2019), (2) a hedge ratio based on the Engle (2002) statistics-based dynamic conditional correlation model, and (3) three bias-adjusted versions of the carry-cost rate hedge ratio, where one uses the Engle (2002) dynamic conditional correlation model to adjust the bias. The hedge ratios' values and hedging performances are compared to those of two common benchmark hedge ratios (the traditional and "naive" hedge ratios) across three broad asset classes, two sub-periods, and three rolling window sizes.

The newly proposed economics-based carry-cost rate hedge ratio augmented with the Engle (2002) dynamic conditional correlation between the spot and futures prices nearly always (1) provides the highest hedging effectiveness and (2) has a statistically significantly higher hedging effectiveness than that of the other hedge ratio approaches across assets, sub-periods, and rolling window sizes.

All of the bias-adjusted carry-cost rate hedge ratios provide statistically significantly superior hedging performance across assets, sub-periods, and rolling window sizes to that of the "naive" hedge ratio of 1, h_1, advocated by Wang et al. (2015). However, this should not be interpreted as a general result. The poor hedging performance of h_1 is expected when the upward biased carry-cost rate hedge ratio, h_{ccr}, is significantly less than 1 as it was for these assets and time periods. Nevertheless, the carry-cost rate varies across assets, currency denominations, and time; thus, in the rare instances that the upward biased h_{ccr} is above 1 by its bias, the risk minimizing hedge ratio would also be h_1.

Unlike h_1, the statistics-based traditional hedge ratio, though inefficient compared to h_{ccr}, is correlated with the changing carry-cost rate. Of the three augmented carry-cost rate hedge ratios, only the one augmented with the Engle (2002) dynamic conditional correlation between the spot and futures prices nearly always provided higher hedging performance than the traditional hedge ratio.

Overall, our findings suggest that an effective hedge-ratio model needs to consider both the economic motivation of the model parameters and the dynamic nature of the correlations between a futures contract and its underlying asset. On the other hand, we acknowledge that our time-series models are estimated using historical data, which may

be susceptible to backward bias. In future research, more fruitful work can be done by incorporating forward-looking data (e.g., options data) as part of the information set into multivariate time-series models.

Notes

1. Many attribute this approach to Ederington (1979). Ferguson and Leistikow (1999) show that the ΔS term should be reduced by the period's carry-cost. Henceforth, we define ΔS as the carry-cost adjusted spot price change. Ferguson and Leistikow (1999) further shows that the carry-cost-adjusted spot price change hedge ratio calculation method is similar to, but theoretically superior to, the ECM hedge ratio advanced and studied by Kroner and Sultan (1993) and Ghosh and Clayton (1996).

2. The 1008 trading days, i.e., 4 years, rolling window size is arbitrary. For a robustness check, we replicate our analyses using 2- and 6-years rolling windows. Our results remain intact. The results are available upon request.

3. This modeling has been shown to produce superior empirical performance in a variety of situations. See, e.g., Engle (2002), Chiang et al. (2007), and Baur and Lucey (2010).

4. For a general carry-cost rate discussion, see Brennan (1958).

5. In discrete time hedging, T in Equation (11) is replaced by the hedge lift date's years to maturity, i.e., $T - t$.

6. For a more complete discussion of the advantages and disadvantages of the carry cost rate based hedge ratios, see Leistikow et al. (2020).

7. The goal was to get a long (at least back to 12 July 1990) series for the short term (ideally overnight for the daily data and weekly for the weekly data) US nearly riskless interest rate. Bloomberg's US 1-week Repo rate data begins on 23 July 1998 and has several gaps. Their overnight repo rate data begins at the same time as the 1-week repo rate data but is missing for about 100 more dates. Surprisingly, the 1-week rate average was about 2.5 basis points less than the overnight rate, but still this seems a minor difference. From these 2 series, we created a merged 1-week/1-day repo series: it is the 1-week repo rate unless it is unavailable, then it is the overnight repo rate −2.5 basis points. Bloomberg also has data on two 1-month interest rate series (repo rate and Libor) that go back farther than the above discussed (preferred, but unavailable) shorter (1-week/1-day) interest rates. The 1-month repo rate was about 20 basis points lower than the Libor on average. From these two 1-month rates, we created a 1 month merged rate series—it was the repo rate when available and Libor-20 basis points, otherwise. Finally, we created an overall series from the two merged series we just created (i.e., the 1-week/1-night series and the 1-month series). Since the merged 1 month series averaged 0.5 basis points less than the merged 1week/1night series, it is the merged 1 week/day repo series unless it is unavailable, in which case it is the 1 month series +0.5 basis points. This final merged series is the short term nearly riskless US interest rate used in CCR calculations for the various assets.

8. For the S&P500 and JY, which have quarterly maturities, we lose the first 16 contracts to construct the first 1008-days rolling window. The gold contracts have bimonthly maturities, so we lose the first 24 contracts to construct the first 1008-days rolling window. Therefore, in this section, the results of the S&P500, JY, and gold are based on 74, 78, and 153 contracts, respectively.

9. There are more statistics-based hedge ratios such as the GARCH method and its variants (e.g., Sarno and Valente (2000), Shaffer and Demaskey (2005), Alizadeh et al. (2008), Lee et al. (2009), and Wang et al. (2015). Alexander and Barbosa (2007) find the GARCH model hedging performance to be inferior to that for h_t, while Lien (2009) finds it is inferior except in small samples under special conditions.

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