

Article

Further Tests of the ZCAPM Asset Pricing Model

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Abstract: In a recent book, Kolari et al. developed a new theoretical capital asset pricing model dubbed the ZCAPM. Based on out-of-sample cross-sectional tests using U.S. stocks, the ZCAPM consistently outperformed well-known multifactor models popular in the finance literature. This paper presents further evidence that expands their sample period from 1927 to 2020. Results are provided for the subperiods 1927 to 1964 and 1965 to 2020. Our results corroborate those of KLH. In cross-sectional tests, the ZCAPM outperforms the CAPM as well as the Fama and French three-factor model and Carhart four-factor model. Outperformance is found in terms of both higher goodness of fit and the statistical significance of factor loadings. Interestingly, the earlier subperiod results highlight problems with the endogeneity of test assets in cross-sectional tests of multifactor models.

Keywords: asset pricing; zero-beta CAPM; return dispersion; expectation-maximization (EM) regression; latent variable



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1. Introduction

This paper extends the recent work by Kolari et al. (2021) (hereafter KLH) in which they developed a new theoretical model of capital asset prices dubbed the ZCAPM. The authors derived the ZCAPM from Black's (1972) renowned zero-beta CAPM as a special case based on two unique efficient and inefficient orthogonal portfolios. This special case enabled the derivation of an alternative specification of the zero-beta CAPM. The ZCAPM is a parsimonious two-factor model comprised of beta risk associated with average market returns and zeta risk related to the cross-sectional return dispersion of assets in the market.¹ Based on the theoretical ZCAPM, an innovative empirical ZCAPM was developed using expectation-maximization (EM) regression methods.²

Subsequent empirical tests by KLH demonstrated that the ZCAPM is a superior asset pricing model that outperforms the CAPM as well as popular multifactor models, including the Fama and French (1992, 1993, 1995, 2015, 2018, 2020) three-, five-, and six-factor models in addition to the Carhart (1997), Hou et al. (2015), and Stambaugh and Yuan (2017) four-factor models. In their empirical tests of U.S. stock returns, out-of-sample Fama and MacBeth (1973) cross-sectional regression tests were conducted for the sample period 1965 to 2018. The ZCAPM consistently outperformed the aforementioned models in terms of both goodness-of-fit and statistical significance of zeta risk factor loadings. In some test asset portfolios, the empirical ZCAPM was able to achieve cross-sectional R^2 estimates as high as 95 percent and normally had values exceeding 70 percent: this goodness-of-fit is near perfect. It means that estimated risk parameters in an earlier period almost completely explain out-of-sample (next month) returns in the cross section of average stock returns. By comparison, other popular multifactor models typically had R^2 values noticeably lower than the ZCAPM in different test asset portfolios and sample periods. Regarding the statistical significance of factor loadings, zeta risk loadings associated with cross-sectional return dispersion in the ZCAPM almost always had t statistics in the range of

3 to 6. However, factors in popular multifactor models did not reach this level of statistical significance in cross-sectional tests. These findings are important in light of the recent work by [Harvey et al. \(2016\)](#) and [Chordia et al. \(2020\)](#), who found that factor loadings should attain t statistics of three or more to avoid false discoveries in asset pricing studies. The ZCAPM was the only model that passed the recommended validity tests.

Where does the ZCAPM fit into the prior literature? From a theoretical perspective, it is based on the general equilibrium framework of the capital asset pricing model (CAPM) of [Treynor \(1961, 1962\)](#), [Sharpe \(1964\)](#), [Lintner \(1965\)](#), and [Mossin \(1966\)](#). It applies the foundational mean–variance [Markowitz \(1959\)](#) portfolio theory and the [Tobin \(1958\)](#) equilibrium pricing methods to derive an alternative form of Black’s zero-beta CAPM. Compared to extant asset pricing models that have been tested in the literature, the empirical ZCAPM is a new econometric model based on EM regression and mixture model methods. No previous asset pricing studies employ these methods in the estimation of factor models. The two factors in the empirical ZCAPM have precedent in the financial literature. Mean market returns are used to estimate beta risk. In addition, cross-sectional return dispersion is used to estimate zeta risk. Regarding this factor, a limited number of studies by [Jiang \(2010\)](#), [Demirer and Jategaonkar \(2013\)](#), [Garcia et al. \(2014\)](#), and [Chichernea et al. \(2015\)](#) augmented the market model form of the CAPM with a return dispersion factor. However, they used standard ordinary least squares (OLS) regression methods to estimate a coefficient related to return dispersion, rather than EM regression in a mixture model. The ZCAPM is different in its empirical estimation of this coefficient in that it explicitly models both positive and negative sensitivity to changes in return dispersion over time. A signal variable denoted $D_{jt} = +1, -1$ for asset j at time t (e.g., one day) is introduced to capture the potential two-sided effects of return dispersion on asset returns. As cross-sectional return dispersion increases in the population of assets at a point t in time, assets in the upper part of the distribution of returns experience increasing returns, and conversely those in the lower part of the distribution experience decreasing returns. If the return dispersion decreases, the opposite return effects occur for assets in the upper and lower parts of the distribution of returns. Since D_{jt} is a latent, unobservable variable, KLH estimated its probability using EM regression. This probability is multiplied by the coefficient on the return dispersion to obtain an estimate of the zeta risk, which is different from other previous studies that incorporated a return dispersion factor.

It is important to distinguish between cross-sectional return dispersion from time-series return dispersion. An example of the latter is the work of [Bekaert et al. \(2012\)](#), who employed the time-series standard deviation of returns for stocks as a factor in an asset pricing model. They used daily returns in a one-month period to compute monthly time-series standard deviations of returns for individual stocks and then averaged this idiosyncratic risk metric for N firms in the market to compute an aggregate idiosyncratic variance measure. Numerous studies have utilized a time-series market volatility factor, including those of [Ang et al. \(2006b, 2009\)](#), [Adrian and Rosenberg \(2008\)](#), [Da and Schaumburg \(2011\)](#), [Chang et al. \(2013\)](#), [Bansal et al. \(2014\)](#), [Bollerslev et al. \(2016\)](#), and [Chen et al. \(2021\)](#), among others.³ Relevant to the ZCAPM, cross-sectional return dispersion is quite different from time-series dispersion. Earlier work by [Jiang \(2010\)](#) showed that, for U.S. stock returns in the period of 1963 to 2005, these two measures of return volatility are uncorrelated with one another in many sample periods. This evidence led Jiang to conclude that time-series and cross-sectional return dispersion are different market risk measures. Hence, the ZCAPM extends the small set of studies that incorporate cross-sectional return dispersion in an asset pricing model but has little or no connection to the larger body of time-series volatility studies.

The present study contributes further evidence on the ZCAPM. First, using U.S. stock return series available on Kenneth French’s data library website⁴, we extend the analysis period back to the 1928 to 1964 period. Second, we update their analyses to the period 1965 to 2020. ZCAPM results are benchmarked against the CAPM as well as the [Fama and French \(1992, 1993, 1995\)](#) three-factor model and [Carhart \(1997\)](#) four-factor model.

We do not test other multifactor models for which factors and test asset portfolios are not available on French's website. Test assets include 25 size and book-market equity ratio (BM) sorted, 25 size and momentum sorted, and 40 industry portfolios. We report results for out-of-sample, cross-sectional Fama and MacBeth tests. In general, our results support those of KLH. The empirical ZCAPM outperforms the CAPM as well as three- and four-factor models, in some cases by large margins. Zeta risk loadings are highly significant, with t statistics exceeding the recommended three hurdle rate in all cases. While multifactor loadings in the three- and four-factor models have t statistics exceeding 3.0 in the 1928 to 1964 subperiod, they generally do not in the more recent 1965 to 2020 subperiod. Additionally, we find that estimated zeta risk premiums are economically meaningful with a range from 0.47 percent to 1.29 percent per month per unit estimated zeta coefficient.

Graphical analyses of the ZCAPM, CAPM, and three- and four-factor models are also provided. In these cross-sectional analyses, fitted (or predicted) one-month-ahead excess stock returns are compared to realized (or actual) excess stock returns of test asset portfolios. Hence, these analyses are out-of-sample investable strategies. In general, we find that the ZCAPM outperforms other models. When industry portfolios are included in the test assets, the ZCAPM outperforms other models by considerable margins. These analyses demonstrate a major problem in testing the three- and four-factor models with endogenous test asset portfolios created from sorts on the same firm-level variables (i.e., size and BM) used to construct the size and value factors. In the earlier period of 1928 to 1964, this endogeneity problem worsened relative to the more recent 1965 to 2020 period due to smaller sample sizes of stocks, as the data go back in time. Our results support [Lewellen et al. \(2010\)](#), [Daniel and Titman \(2012\)](#), and others who have advocated for combining exogenous industry portfolios with other portfolios in asset pricing tests. In sum, our graphical analyses confirm the findings of KLH in support of the ZCAPM over the CAPM as well as three- and four-factor models, using long/short zero-investment portfolios as multifactors.

We conclude from these findings that the ZCAPM dominates other popular asset pricing models. Given that size, BM, and momentum sorted portfolios as test assets are exogenous to the ZCAPM's mean market return and return dispersion factors, this dominance is remarkable. Further research is recommended for applying the ZCAPM to different countries and asset classes (e.g., bonds, commodities, and real estate) to assess its performance relative to the existing asset pricing models. Additionally, applications to event studies, mutual and hedge funds, investment analysis, and other areas of finance are recommended.

The plan of this study is as follows. Section 2 overviews the ZCAPM. Section 3 describes our methodology, including data and empirical tests. Section 4 presents the empirical results. The last Section 5 gives the conclusion.

2. Overview of the ZCAPM

Here, we overview the theoretical ZCAPM and its companion empirical ZCAPM. Again, [Kolari et al. \(2021\)](#) (KLH) derived the ZCAPM as a special case of Black's zero-beta CAPM.⁵ In their derivation, they focused on two orthogonal portfolios on the boundary of the mean–variance investment parabola—one that is efficient and one that is inefficient—with the same time-series variance of returns. Formulas of the expected returns for these two portfolios are written based on new insights concerning the mean–variance parabola. Upon substituting these expected returns into the zero-beta CAPM, the theoretical ZCAPM is obtained. Subsequently, the authors proposed a novel empirical ZCAPM for estimation purposes, using real world data. Unlike prior asset pricing models that use ordinary least squares (OLS) regression for estimation, the empirical ZCAPM utilizes expectation–maximization (EM) regression methods. In the forthcoming discussion, we abbreviate the derivations in the work of KLH to conserve space and highlight the main ideas of the theoretical and empirical ZCAPM. Readers interested in more details are referred to their book.

2.1. Theoretical ZCAPM

KLH mathematically proved two new insights about the Markowitz mean–variance investment parabola. First, they provided two mathematical proofs⁶ to show that the width or span of the parabola is largely determined by the cross-sectional standard deviation of returns of all assets’ returns. Second, given that this *return dispersion* defines the width of the parabola, the mean return of all assets should lie somewhere in the middle of the parabola on its axis of symmetry. The latter finding implies that the mean market portfolio used to proxy the market portfolio is inefficient. Regardless of whether all assets are equal- or value-weighted to form portfolios, the market portfolio in the CAPM, which lies on the efficient frontier, is far above the mean market portfolio that is located on the axis of symmetry. Consistent with the Roll (1977) critique, because the CAPM cannot be tested without an efficient portfolio, previous empirical tests of the CAPM using the mean market model returns to proxy market portfolio returns are invalid. The CAPM cannot be declared dead because it was never legitimately tested using efficient portfolios (see Fama and French 1996, 2004).

Figure 1 illustrates the return dispersion and mean market return characteristics of the mean–variance parabola. The x -axis is the time-series variance of returns for an asset or portfolio denoted as $\tilde{\sigma}_p^2$. In a one-day period of time, this variance can be measured by computing returns in, say, 10 min intervals during the day. On the y -axis is the expected returns of assets. The cross-sectional variance of returns of all assets in the market during the day is denoted as $\tilde{\sigma}_a$. Naturally, the mean market return denoted $E(\tilde{R}_a)$ must be located in the middle of the cross-sectional distribution of asset returns. In turn, it must be true that $E(\tilde{R}_a) \approx E(\tilde{R}_G)$, where the latter is the expected return on the global minimum variance portfolio G . Clearly, the mean market portfolio a is located far below the efficient frontier in Figure 1.

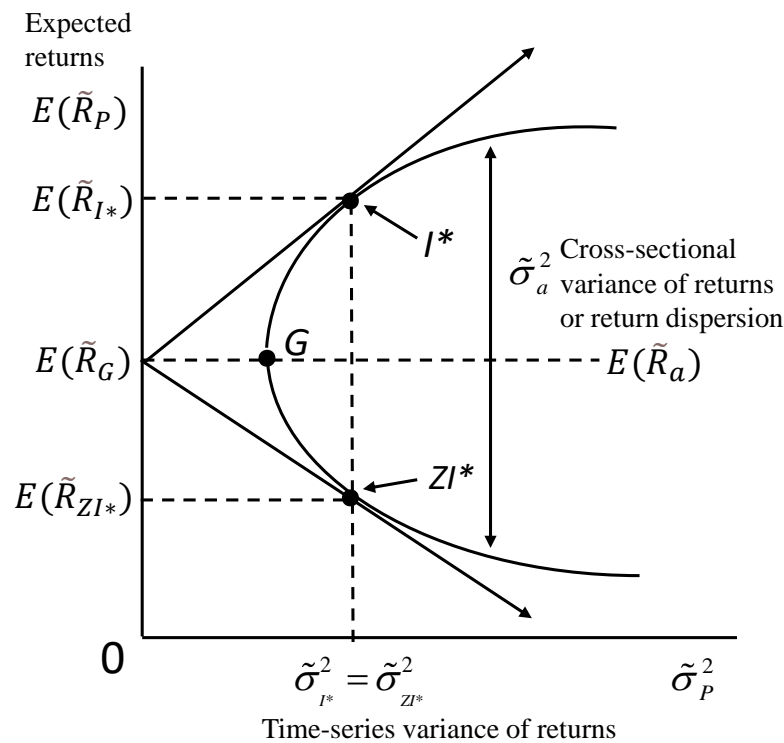


Figure 1. Geometric approach of the theoretical ZCAPM based on the Markowitz mean–variance investment parabola.

Next, KLH used this framework to identify two unique portfolios, I^* and ZI^* , that are uncorrelated with one another. These portfolios have the same time-series variance of returns or total risk, i.e., $\tilde{\sigma}_{I^*}^2 = \tilde{\sigma}_{ZI^*}^2$.⁷ Notice that portfolio I^* is on the efficient frontier,

and portfolio ZI^* is inefficient on the parabola's lower boundary. A new geometry is introduced in this analysis. In the CAPM, the market portfolio M is geometrically located at the tangent point from a ray extending from the riskless rate to the efficient frontier. In the ZCAPM, portfolios I^* and ZI^* are located by moving along the axis of symmetry at the expected rate $E(\tilde{R}_a)$ and then up or down, respectively, by the cross-sectional return dispersion $\tilde{\sigma}_a$. Using this geometry, KLH defined the expected returns for portfolios I^* and ZI^* as follows:

$$E(\tilde{R}_{I^*}) \approx E(\tilde{R}_a) + f(\theta)\sigma_a \tag{1}$$

$$E(\tilde{R}_{ZI^*}) \approx E(\tilde{R}_a) - f(\theta)\sigma_a, \tag{2}$$

where $f(\theta)$ is a complex expression approximately equal to one (due to almost completely random risky asset returns⁸).

Assuming $f(\theta) = 1$, KLH substituted the expected returns for portfolios I^* and ZI^* into Black's zero-beta CAPM to derive the theoretical ZCAPM without a riskless asset. The zero-beta CAPM specifies the expected return for the i th asset as

$$E(\tilde{R}_i) = E(\tilde{R}_{ZM}) + \beta_{i,M}[E(\tilde{R}_M) - E(\tilde{R}_{ZM})] \tag{3}$$

$$E(\tilde{R}_i) = \beta_{i,M}E(\tilde{R}_M) + (1 - \beta_{i,M})[E(\tilde{R}_{ZM})], \tag{4}$$

where $\beta_{i,M}$ is the sensitivity or beta risk of asset i 's return with respect to the excess return of the expected market portfolio return, $E(\tilde{R}_M)$ and its zero-beta (uncorrelated) portfolio expected return, or $E(\tilde{R}_{ZM})$. The latter is the borrowing rate in Black's model, unlike the riskless rate R_f in the CAPM.⁹

For portfolios I^* and ZI^* , their expected returns are

$$E(\tilde{R}_{I^*}) = \beta_{I^*,M}E(\tilde{R}_M) + (1 - \beta_{I^*,M})E(\tilde{R}_{ZM}) \tag{5}$$

$$E(\tilde{R}_{ZI^*}) = \beta_{ZI^*,M}E(\tilde{R}_M) + (1 - \beta_{ZI^*,M})E(\tilde{R}_{ZM}), \tag{6}$$

where $\beta_{I^*,M}$ and $\beta_{ZI^*,M}$ are beta risks of portfolios I^* and ZI^* associated with market portfolio M , respectively. Solving these equations¹⁰, we obtain the general form of the zero-beta CAPM:

$$E(\tilde{R}_i) = \beta_{i,I^*}E(\tilde{R}_{I^*}) + (1 - \beta_{i,I^*})E(\tilde{R}_{ZI^*}), \tag{7}$$

where $\beta_{i,I^*} = (\beta_{i,M} - \beta_{ZI^*,M}) / (\beta_{I^*,M} - \beta_{ZI^*,M})$. As observed by Roll (1980), the above expression shows that the zero-beta CAPM can be specified in terms of any efficient portfolio and its orthogonal zero-beta (inefficient) counterpart on the mean-variance parabola. Here, KLH re-wrote the zero-beta CAPM using the unique portfolios I^* and ZI^* .

Upon substituting $E(\tilde{R}_{I^*})$ and $E(\tilde{R}_{ZI^*})$ in Equations (1) and (2) into Equation (7), the theoretical ZCAPM can be specified as follows:

$$\begin{aligned} E(\tilde{R}_i) &= \beta_{i,I^*}E(\tilde{R}_{I^*}) + (1 - \beta_{i,I^*})E(\tilde{R}_{ZI^*}) \\ &= E(\tilde{R}_{ZI^*}) + \beta_{i,I^*}[E(\tilde{R}_{I^*}) - E(\tilde{R}_{ZI^*})] \\ &= E(\tilde{R}_a) - \sigma_a + \beta_{i,I^*}\{[E(\tilde{R}_a) + \sigma_a] - [E(\tilde{R}_a) - \sigma_a]\} \\ &= E(\tilde{R}_a) + (2\beta_{i,I^*} - 1)\sigma_a \\ E(\tilde{R}_i) &= E(\tilde{R}_a) + Z_{i,a}^*\sigma_a, \end{aligned} \tag{8}$$

where $Z_{i,a}^* = 2\beta_{i,I^*} - 1$.

Adding a third riskless asset rate R_f , and again using the definitions of $E(\tilde{R}_{I^*})$ and $E(\tilde{R}_{ZI^*})$ in Equations (1) and (2), the expected return of the i th asset is

$$\begin{aligned} E(\tilde{R}_i) &= w_{I^*}E(\tilde{R}_{I^*}) + w_{ZI^*}E(\tilde{R}_{ZI^*}) + w_fR_f \\ &= w_{I^*}[E(\tilde{R}_a) + \sigma_a] + w_{ZI^*}[E(\tilde{R}_a) - \sigma_a] + w_fR_f \\ &= (w_{I^*} + w_{ZI^*})E(\tilde{R}_a) + (w_{I^*} - w_{ZI^*})\sigma_a + w_fR_f, \end{aligned} \tag{9}$$

where I^* , ZI^* , and f are orthogonal assets with corresponding weights w_{I^*} , w_{ZI^*} , and w_f that sum to one with both long and short positions in the assets allowed. By rearranging terms and using Equation (9), the final form of the theoretical ZCAPM becomes

$$E(\tilde{R}_i) - R_f = (w_{I^*} + w_{ZI^*})[E(\tilde{R}_a) - R_f] + (w_{I^*} - w_{ZI^*})\sigma_a \tag{10}$$

$$E(\tilde{R}_i) - R_f = \beta_{i,a}[E(\tilde{R}_a) - R_f] + Z_{i,a}^*\sigma_a, \tag{11}$$

where beta risk coefficient $\beta_{i,a} = w_{I^*} + w_{ZI^*}$ measures the sensitivity of the i th asset's excess returns to average market excess returns of all assets, and zeta risk coefficient $Z_{i,a}^* = w_{I^*} - w_{ZI^*}$ measures the sensitivity of an asset's excess returns to the market return dispersion of all assets.¹¹ KLM used the notation $\beta_{i,a}$ to denote beta risk with respect to the average returns on the portfolio of n assets in the market. This beta is distinguished from CAPM market beta $\beta_{i,M}$ with respect to the market portfolio (typically denoted simply as β_i).

Returning to the mean–variance parabola, it is interesting that beta risk and zeta risk in the theoretical ZCAPM can be used to describe its architecture, including not only boundary portfolios, but locations of assets and portfolios within the parabola. Assets and portfolios with positive (negative) zeta risk lie in the upper (lower) portion of the investment parabola. On any zeta risk curve in the parabola, as beta risk increases, the time-series variance of returns increases. As shown in [Kolari et al. \(2021, Figure 10.1, p. 272 and Figure 10.2, p. 274\)](#), an interlocking web of beta and zeta risks result that shape the parabola with zeta risk increasing vertically and beta risk increasing horizontally. Hence, the parabola contains a risk structure based on the systematic risks of assets with respect to average market returns and market return dispersion. Interestingly, in Chapter 10 of their book, KLH confirmed this architecture using out-of-sample (next month) empirical evidence for U.S. stock portfolios. Portfolios along the highest zeta risk curve comprise the efficient frontier.¹² Additionally, the mean market portfolio a lies approximately on the axis of symmetry of the parabola. Supporting this conjecture, in Chapter 9 of their book, KLH constructed relatively efficient portfolios and showed that the CRSP market index lies along the axis of symmetry of the parabola.

2.2. Empirical ZCAPM

Figure 2 shows how assets in the upper and lower portions of the mean–variance parabola are affected over time in response to changes in the mean market returns and cross-sectional return dispersion of all assets in the market.¹³ Comparing $t = 1$ to $t = 2$, when the average market returns do not change, we see that asset returns in the upper (lower) portion of the parabola experience increasing (decreasing) returns as the return dispersion increases. At $t = 3$, the return dispersion decreases, which tends to decrease (increase) the asset returns in the upper (lower) portion of the parabola. Of course, as mean market returns decrease in this period, all asset returns decrease in concert with lower mean market returns. In period $t = 4$, mean market returns increase but the return dispersion changes little, if at all. In this period, all assets' returns tend to increase under these conditions.

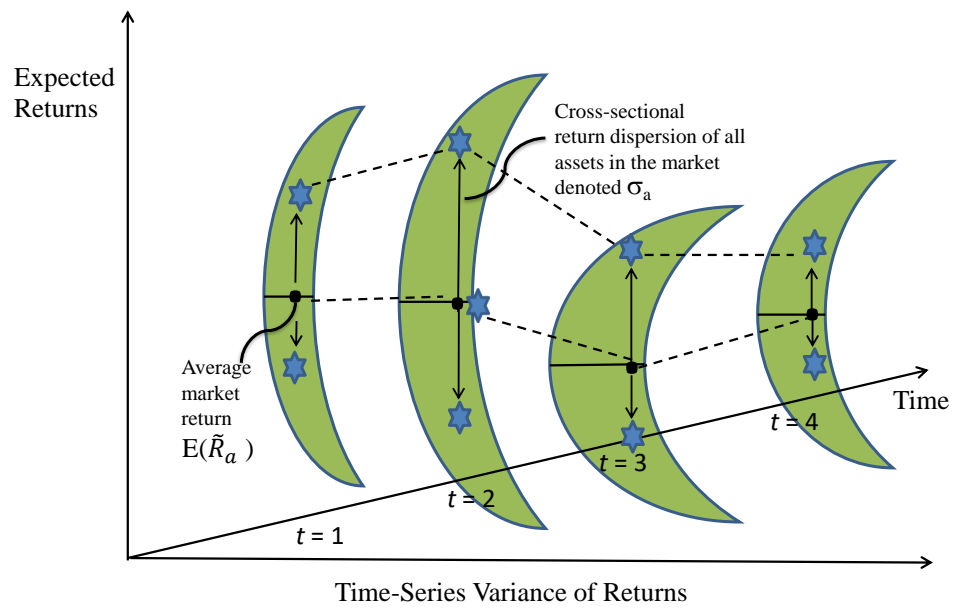


Figure 2. As the investment parabola moves over time t , its level and width change. The level changes with average market returns, and the width changes with cross-sectional return dispersion of all assets in the market. Assets in the upper (lower) half of the parabola experience opposite return effects of changing return dispersion, whereas as all assets’ returns move up and down in concert with the average market returns.

How did KLH empirically model the time-series behavior of the mean–variance parabola depicted in Figure 2? The positive and negative effects of return dispersion on asset returns in the upper and lower portions of the parabola need to be taken into account. To solve this problem, they introduced a dummy signal variable denoted D_{it} for each i th asset. The following novel empirical ZCAPM is proposed:

$$\tilde{R}_{it} - R_{ft} = \alpha_i + \beta_i(\tilde{R}_{at} - R_{ft}) + Z_i D_{it} \tilde{\sigma}_{at} + \tilde{u}_{it}, \quad t = 1, \dots, T \tag{12}$$

where $R_{it} - R_{ft}$ is the excess return for the i th asset over the riskless rate at time t , β_i measures sensitivity to excess average market returns equal to $R_{at} - R_{ft}$, Z_i measures sensitivity to return dispersion σ_{at} , D_{it} is a signal variable with values $+1$ and -1 representing positive and negative return dispersion effects on stock returns at time t , respectively, and $u_{it} \sim \text{iid } N(0, \sigma_i^2)$. No previous studies modeled two-sided return dispersion risk using positive and negative risk loadings. A previous study by Ang et al. (2006a) estimated downside and upside market betas, i.e., β^- and β^+ , using excess market returns over time, below and above the mean market return, but did not introduce a dummy variable in their analyses. Similarly, Lettau et al. (2014) found that the cross section of currency returns can be explained by downside market beta risk. More recently, Bollerslev et al. (2016) proxied good and bad stock return volatility by utilizing a relative difference in the semi-variance measure but again did not use a dummy variable approach to simultaneously model their effects to predict returns.

Departing from previous literature, signal variable D_{it} is modeled by KLH as an unknown or latent (hidden) variable. They defined D_{it} as an independent random variable with the following two-point distribution:

$$D_{it} = \begin{cases} +1 & \text{with probability } p_i \\ -1 & \text{with probability } 1 - p_i, \end{cases} \tag{13}$$

where p_i (or $1 - p_i$) is the probability of a positive (or negative) return dispersion effect, and D_{it} are independent of u_{it} .¹⁴

To estimate the empirical ZCAPM’s parameters $\theta_i = (\beta_i, Z_i, p_i)$, KLH employed the expectation–maximization (EM) algorithm of Dempster et al. (1977) (See also Jones and McLachlan 1990; McLachlan and Peel 2000; McLachlan and Krishnan 2008). Their book gives detailed step-by-step estimation procedures. Unlike any previous asset pricing model, the empirical ZCAPM can be characterized as a probabilistic mixture model with two mixture components. Each component itself is a two-factor regression model (see Equations in Note 14). Hidden dummy variable D_{it} determines the operative regression model.

Notice that the coefficient of the return dispersion in regression Equation (12) is a random variable $Z_{i,a}D_{i,t}$ with two possible values, $+Z_{i,a}$ or $-Z_{i,a}$, based on the sign of signal variable $D_{i,t}$. Here, the signal variable has mean $E(D_{it}) = 2p_i - 1$ and variance $\text{Var}(D_{it}) = 4p_i(1 - p_i)$. They separate the mean from the random coefficient $Z_{i,a}D_{it}$ associated with σ_{at} as follows:

$$Z_{i,a}D_{it} = Z_{i,a}(2p_i - 1) + Z_{i,a}[D_{it} - (2p_i - 1)]. \tag{14}$$

Thus, using definitions $Z_{i,a}^* = Z_{i,a}(2p_i - 1)$ and $u_{it}^* = Z_{i,a}[D_{it} - (2p_i - 1)]\sigma_{at} + u_{it}$, the marginal form of the empirical ZCAPM relation (12) becomes

$$R_{it} - R_{ft} = \beta_{i,a}(R_{at} - R_{ft}) + Z_{i,a}^*\sigma_{at} + u_{it}^*, \quad t = 1, \dots, T. \tag{15}$$

where the term $Z_{i,a}^*\sigma_{at}$ results from integrating out the probability distribution of the unobservable signal variable in the term $Z_{i,a}D_{it}\sigma_{at}$ in model (12). Regression parameter $Z_{i,a}^*$ represents the zeta risk loading in the theoretical ZCAPM as specified in Equation (11).

It should be mentioned that there is no mispricing error term (i.e., $\alpha_i = 0$) in empirical ZCAPM relation (15). In tests using U.S. stock returns, KLH found that introducing an α_i term did not lower the residual variance and therefore did not improve in-sample data fitting. In the present study, the α term is not needed, as we test the empirical ZCAPM using standard Fama and MacBeth (1973) cross-sectional regression analyses, to be discussed shortly.¹⁵

The positive or negative sign of zeta risk loading $Z_{i,a}^*$ is determined by the probability p_i of signal variable D_{it} in sample period $t = 1, \dots, T$. If $p_i > 1/2$ (or $< 1/2$), $Z_{i,a}^*$ has a positive (or negative) sign. By way of interpretation, $Z_{i,a}^*$ measures the average increase or decrease in asset returns in response to a one unit change in market return dispersion σ_{at} .

Setting the empirical ZCAPM apart from other studies that include a return dispersion factor cited earlier in Section 1, the variance of the error term u_{it}^* in relation (15) is not constant. This heterogeneity of error variance can be defined as follows:

$$\text{Var}(u_{it}^*) = 4 p_i(1 - p_i) Z_{i,a}^2 \sigma_{at}^2 + \text{Var}(u_{it}). \tag{16}$$

Due to this property, other studies incorporating return dispersion as a factor are misspecified.

KLH provided Matlab, R, and Python programs for EM estimation of the empirical ZCAPM at GitHub (<https://github.com/zcapm> (accessed on 1 September 2021)): Programs to run cross-sectional Fama and MacBeth regression tests are provided also. In this study, we employ their R programs due to the faster estimation speed relative to the Matlab and Python programs.

3. Cross-Sectional Tests

3.1. Data

U.S. stock returns for all common stocks on the Center for Research in Security Prices (CRSP) database are used. Daily stock returns are gathered for two subperiods: (1) January 1928 to December 1964, and (2) January 1965 to December 2020. CRSP value-weighted index returns and 30-day U.S. Treasury bill rates, in addition to size, value, and momentum factors, are downloaded from Kenneth French’s online database website.

We compute the return dispersion factor for the ZCAPM as the daily cross-sectional standard deviation of returns of all stocks in the market:

$$\sigma_{at} = \sqrt{\frac{n}{n-1} \sum_{i=1}^n w_{it-1} (R_{it} - R_{at})^2}, \tag{17}$$

where n is the total number of stocks, w_{it-1} is the previous day’s market value weight for the i th stock, R_{it} is the return of the i th stock on day t , and R_{at} is the value-weighted average return of all available stocks in the country on day t .

To benchmark the performance of the ZCAPM in our empirical tests, we employ the following asset pricing models:

- CAPM in market model form (See [Sharpe 1963](#); [Fama 1968](#)). with an excess market return factor (*MKT-RF*) defined as the value-weighted CRSP return minus the U.S. Treasury bill rate;
- The [Fama and French \(1992, 1993, 1995\)](#) three-factor model based on augmenting the CAPM with a size factor (viz. small minus large firms’ stock returns denoted as *SMB*) and a value factor (viz. high value minus low value firms’ stock returns denoted *HML*);
- The [Carhart \(1997\)](#) four-factor model based on augmenting the three-factor model with a momentum factor (viz. stocks with high past returns minus stocks with low past returns denoted *MOM*).

French’s website contains construction details for the multifactors *SMB*, *HML*, and *MOM*. As defined there, based on portfolio deciles, *SMB* is the average return on the three small portfolios minus the average return on the three big portfolios. *HML* is the average return on the two value portfolios minus the average return on the two growth portfolios. Additionally, *MOM* is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios.

Descriptive statistics for our data are provided in [Table 1](#). Compared to the market, size, value, and momentum factors, the magnitude of the cross-sectional return dispersion is much larger at 1.56 percent, compared to a range of only 0.003 percent to 0.03 percent for the other factors. In addition, with the exception of return dispersion, notice that the standard deviations of factors are quite large relative to their mean values; hence, these factors can fluctuate widely over time.

Table 1. Descriptive statistics for U.S. stock returns in the sample period of January 1928 to December 2020.

Panel A. 1928 to 1964						
	90 Portfolios	<i>MKT-RF</i>	<i>SMB</i>	<i>HML</i>	<i>MOM</i>	<i>Ret Disp</i>
Mean	0.09	0.03	0.003	0.02	0.02	1.56
Std dev	1.80	1.16	0.66	0.68	0.79	0.86
Panel B. 1965 to 2020						
	90 Portfolios	<i>MKT-RF</i>	<i>SMB</i>	<i>HML</i>	<i>MOM</i>	<i>Ret Disp</i>
Mean	0.07	0.03	0.007	0.01	0.03	1.79
Std dev	1.21	1.03	0.54	0.55	0.76	0.60

[Table 1](#) gives the means and standard deviations of returns for test asset portfolios and asset pricing factors also. The 90 test assets are formed by combining 25 size and book-to-market ratio (BM) portfolios, 25 size and momentum portfolios, and 40 industry portfolios. Two subperiods are used: (1) January 1928 to December 1964, and (2) January 1965 to December 2020.

3.2. Cross-Sectional Regression Tests

We provide two different cross-sectional regression tests. The first test is based on the standard two-step Fama and MacBeth (1973) regression analyses. Step one estimates the time-series regression equation for the asset pricing model, using daily returns in a one-year period for each of the test asset portfolios. Estimated factor loadings for beta and zeta coefficients are retained for use in the next step. Step two is a cross-sectional regression with one-month-ahead returns for test asset portfolios as the dependent variable and beta and zeta risk factor loadings from the previous year as the independent variables. This procedure is rolled forward one month at a time until the end of the sample period. This procedure represents an investable strategy in the sense that an investor could implement it in the real world. Out-of-sample returns are related to prior risk parameter estimates to assess the validity of models. No tampering or manipulation is possible in this setup. In this regard, Simin (2008, p. 356) commented that the use of step-ahead (e.g., one-month-ahead) returns in this procedure to assess the predictive ability of asset pricing models mitigates a number of evaluation problems, including data snooping, the use of R^2 as a measure of goodness-of-fit, and efficiency issues. Likewise, Ferson et al. (2013) argued that the practical value of asset pricing models should be assessed using out-of-sample tests as in the two-step Fama and MacBeth procedure discussed above.

In the second step of the Fama–MacBeth procedure, we run the following cross-sectional regression to test the empirical ZCAPM based on estimates of beta and zeta risk coefficients (or loadings) from time-series regression (15):

$$R_{i,T+1} - R_{fT+1} = \lambda_0 + \lambda_\beta \hat{\beta}_i + \lambda_{Z^*} \hat{Z}_i^* + u_{it}, i = 1, \dots, N, \tag{18}$$

where λ_β and λ_{Z^*} are coefficient estimates of the market price of beta risk (associated with sensitivity to mean market returns) and the market price of zeta risk (associated with sensitivity to cross-sectional market volatility or return dispersion) in percent terms, respectively, and the other notation is as before. According to Ferson (2019), estimated risk premiums λ_β and λ_{Z^*} approximate mimicking portfolio returns that are long stocks with higher betas or zetas and short stocks with lower betas or zetas. As observed by Cochrane (2005, pp. 250–51), t -statistics associated with estimated factor prices $\hat{\lambda}_k$ using the monthly rolling approach are corrected for cross-sectional correlation of residual errors (and therefore, are similar to Shanken (1992) corrected OLS standard errors).

It should be noted that beta loadings ($\hat{\beta}_i$) are time invariant for the most part with similar values, using daily or monthly returns. The reason for this invariance is that they are benchmarked to one corresponding to the beta risk of the average market return of all assets. By contrast, KLH noted that zeta risk loadings (\hat{Z}_i^*) are time variant (i.e., the holding period can affect their estimated values) due to not being benchmarked to one. By way of interpretation, the estimated market price λ_{Z^*} related to the return dispersion measures the risk premium per unit zeta risk. Given that time-series regression (15) is used to estimate risk parameters with daily returns, and the cross-sectional regression Equation (18) uses one-month-ahead excess returns as the dependent variable, \hat{Z}_i^* can be rescaled from a daily to monthly basis as follows:

$$R_{i,T+1} - R_{fT+1} = \lambda_0 + \lambda_\beta \hat{\beta}_i + \lambda_{Z^*} \hat{Z}_i^* N_{T+1} + u_{it}, i = 1, \dots, N, \tag{19}$$

where N_{T+1} is the number of trading days in month $T + 1$ (i.e., 21 days), $Z_i^* N_{T+1}$ is the monthly estimated zeta risk, and λ_{Z^*} is the monthly risk premium associated with zeta risk. This rescaling does not change the risk premium $\hat{\lambda}_{Z^*}$ per unit zeta risk.¹⁶

Another important statistic in the cross-sectional regressions is the R^2 estimate. Following Jagannathan and Wang (1996) and Lettau and Ludvigson (2001, footnote 17, p. 1254), we compute this goodness-of-fit measure by using the R^2 statistic from a single regression approach. Using the 1928 to 1964 (1965 to 2020) subperiod, we obtain 444 (672) monthly estimates of $\hat{\lambda}_k$ for each test asset portfolio as we roll forward month by month to the end of the analysis subperiod. We also have the same number of one-month-ahead realized excess

returns for each portfolio. After taking the averages of the $\hat{\lambda}_{kS}$ and realized excess returns for each portfolio, the average realized excess returns for the n portfolios are regressed on the average $\hat{\lambda}_{kS}$ to obtain an estimate of R^2 .

The above discussion of R^2 estimation leads to a second cross-sectional test. In this test, we compute the one-month-ahead average realized excess returns for the n portfolios as before. Additionally, we compute the one-month-ahead average fitted excess returns for each portfolio. To do this, for each portfolio, the empirical ZCAPM is estimated, and β_i and Z_i^* risk parameters are retained. In the next month $T + 1$, these risk parameters are multiplied by estimated factor prices of risk, or λ_{kS} to compute the fitted excess return for each portfolio. Rolling forward month by month to the end of the subperiod, a series of fitted excess returns are available to compute the average fitted excess return. Finally, plots of average realized excess returns (x -axis) and average fitted excess returns (y -axis) for the n portfolios are created. If the model works perfectly, all points will lie on a 45-degree line from the origin.

4. Cross-Sectional Regression Results

Tables 2 and 3 report the results for the out-of-sample Fama and MacBeth cross-sectional regression tests in subperiods 1928 to 1964 and 1965 to 2020, respectively.

Table 2. Out-of-sample Fama–MacBeth cross-sectional regressions for U.S. stocks: January 1928 to December 1964.

Panel A: 25 Size and BM Sorted Portfolios							
Model	$\hat{\lambda}_0$	$\hat{\lambda}_\beta$	$\hat{\lambda}_{Z^*}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2
CAPM	2.57 (4.89)	−0.73 (−2.12)					0.00
Three-factor	1.86 (4.71)	−1.05 (−2.65)		1.37 (5.44)	1.24 (5.19)		0.79
Four-factor	1.82 (5.20)	−1.12 (−2.88)		1.25 (5.07)	1.37 (5.25)	−1.44 (−2.84)	0.92
ZCAPM	1.42 (3.80)	−0.18 (−0.54)	0.92 (6.19)				0.98
Panel B: 25 Size and Momentum Sorted Portfolios							
Model	$\hat{\lambda}_0$	$\hat{\lambda}_\beta$	$\hat{\lambda}_{Z^*}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2
CAPM	4.64 (4.60)	−2.68 (−3.38)					0.07
Three-factor	3.47 (5.07)	−3.27 (−4.86)		1.57 (4.62)	3.28 (3.68)		0.85
Four-factor	4.46 (5.89)	−4.17 (−5.68)		1.44 (4.32)	3.22 (3.93)	0.22 (0.76)	0.94
ZCAPM	1.73 (3.44)	−0.39 (−0.93)	1.29 (10.22)				0.98
Panel C: 90 Total Portfolios							
Model	$\hat{\lambda}_0$	$\hat{\lambda}_\beta$	$\hat{\lambda}_{Z^*}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2
CAPM	2.86 (5.62)	−0.95 (−2.88)					0.00
Three-factor	1.92 (5.45)	−1.27 (−3.70)		1.65 (6.80)	1.33 (4.48)		0.59
Four-factor	1.97 (5.36)	−1.33 (−3.73)		1.57 (6.62)	1.41 (4.83)	−0.05 (−0.20)	0.59
ZCAPM	1.65 (5.18)	−0.28 (−1.06)	1.00 (9.53)				0.95

The portfolio *MKT* in the ZCAPM is the value-weighted mean market portfolio rather than a proxy for the market portfolio *M* as in the CAPM.

Table 3. Out-of-sample Fama–MacBeth cross-sectional regressions for U.S. stocks: January 1965 to December 2020.

Panel A: 25 Size and BM Sorted Portfolios							
Model	$\hat{\lambda}_0$	$\hat{\lambda}_\beta$	$\hat{\lambda}_{Z^*}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2
CCAPM	2.18 (8.82)	−1.41 (−6.11)					0.72
Three-factor	2.45 (10.78)	−1.75 (−7.93)		0.36 (2.46)	0.26 (1.93)		0.83
Four-factor	2.45 (10.43)	−1.77 (−7.79)		0.36 (2.50)	0.23 (1.74)	−0.43 (−1.47)	0.86
ZCAPM	1.48 (5.82)	−0.77 (−3.25)	0.47 (4.00)				0.98
Panel B: 25 Size and Momentum Sorted Portfolios							
Model	$\hat{\lambda}_0$	$\hat{\lambda}_\beta$	$\hat{\lambda}_{Z^*}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2
CCAPM	1.53 (6.90)	−0.78 (−3.52)					0.38
Three-factor	1.81 (7.77)	−1.23 (−5.41)		0.54 (3.75)	0.03 (0.11)		0.76
Four-factor	2.06 (9.05)	−1.42 (−6.70)		0.52 (3.61)	−0.20 (−0.91)	0.47 (2.48)	0.80
ZCAPM	0.87 (4.06)	−0.19 (−0.85)	0.69 (6.07)				0.93
Panel C: 90 Total Portfolios							
Model	$\hat{\lambda}_0$	$\hat{\lambda}_\beta$	$\hat{\lambda}_{Z^*}$	$\hat{\lambda}_{SMB}$	$\hat{\lambda}_{HML}$	$\hat{\lambda}_{MOM}$	R^2
CAPM	1.82 (9.06)	−0.94 (−4.45)					0.26
Three-factor	1.52 (9.42)	−0.93 (−5.04)		0.57 (3.72)	0.26 (1.85)		0.48
Four-factor	1.58 (9.91)	−0.97 (−5.37)		0.53 (3.37)	0.23 (1.82)	0.26 (1.39)	0.54
ZCAPM	1.22 (6.81)	−0.43 (−2.29)	0.63 (8.50)				0.87

The portfolio *MKT* in the ZCAPM is the value-weighted mean market portfolio rather than a proxy for the market portfolio *M* as in the CAPM.

4.1. Subperiod 1928 to 1964

The cross-sectional regression results for the 1928 to 1964 subperiod based on three different test asset portfolios are shown in Panels A to C in Table 2. In Panel A for the 25 size and BM portfolios, we see that the CAPM performed the worst with virtually no explanatory power at $R^2 = 0$ and a marginally significant negative market price for beta $\hat{\lambda}_\beta = -0.73$ ($t = -2.12$). Since beta risk should be positively priced in the CAPM, our results confirm those of Fama and French (1992, 1993, 1995) and many others that do not support the CAPM. By contrast, the best performing models are the four-factor model and ZCAPM with estimated R^2 values of 92 percent and 98 percent, respectively. In assessing the relative performance of different models, it is important to recognize that the three- and four-factor models use endogenous test asset portfolios; that is, size, BM, and momentum test asset portfolios are constructed from the same firm-level characteristics as the respective factors. Notably, these firm-level characteristics are exogenous to the ZCAPM, which makes the almost perfect goodness-of-fit of the ZCAPM quite remarkable.

In terms of the significance of factor loadings, again the ZCAPM outperforms the other models. The $\hat{\lambda}_{Z^*}$ market price of zeta risk loadings is 1.29 percent per month with a very high t -statistic of 6.19. The t -statistics for $\hat{\lambda}_{SMB}$ and $\hat{\lambda}_{HML}$ are very high also in the

range of 5.07 to 5.44. As discussed earlier, Harvey et al. (2016) and Chordia et al. (2020) recommended that asset pricing factors have t statistics greater than 3.0 to avoid false discoveries. Our findings suggest that the size and value factors of Fama and French are not false discoveries. This inference holds for the return dispersion factor in the ZCAPM also. The estimated market price of momentum risk loadings $\hat{\lambda}_{MOM}$ with $t = -2.98$, which is borderline significant relative to the threshold hurdle rate. However, its market price has a negative sign that is difficult to explain (i.e., higher risk should imply higher risk premiums). Even so, adding momentum to the three-factor model noticeably boosts its goodness-of-fit from 79 percent to 92 percent in the four-factor model.

The results in Panel B for the 25 size and momentum portfolios are similar to those in Panel A. Again, the ZCAPM has almost perfect goodness-of-fit at a 98 percent R^2 estimate, and $\hat{\lambda}_{Z^*}$ has the highest t statistic of all factors tested. Regarding the latter, the t statistic equals 10.22, which is extremely high. No previous studies to our knowledge have reported a t statistic this high in cross-sectional regression tests. As before, the four-factor model outperforms the three-factor model, and the CAPM does the worst in terms of very low goodness-of-fit.

Lastly, Panel C contains the results for 90 combined portfolios including 40 industry portfolios. These results mitigate endogeneity problems by incorporating exogenous industry test assets. Upon doing so, the three- and four-factor models' performance diminishes substantially compared to Panels A and B. Now their R^2 values only reach 59 percent, which is far below that of the ZCAPM with near perfect goodness-of-fit at 95 percent. While the t statistics for $\hat{\lambda}_{SMB}$ and $\hat{\lambda}_{HML}$ are high in the range of 4.48 to 6.80, they are well below that of the market price of zeta risk $\hat{\lambda}_{Z^*}$ at 9.53.

Another finding in Panels A to C of Table 2 is that the intercept term $\hat{\lambda}_0$ is somewhat lower for the ZCAPM compared to the other models. This pattern is most clearly seen in Panel B, wherein $\hat{\lambda}_0 = 1.73$ percent per month ($t = 3.44$) for the ZCAPM, compared to estimates in the range of 3.47 percent to 4.64 percent for the other models. This lower mispricing error further supports the ZCAPM.

In sum, the ZCAPM outperforms the popular three- and four-factor models, even when endogenous test assets are used (which are exogenous to the ZCAPM factors). Consistent with earlier studies, the CAPM performs poorly in cross-sectional tests. When exogenous industry portfolios are added to the test assets, the ZCAPM outperforms other models by a large margin. The latter results are the most reliable and highlight the dominance of the ZCAPM, compared to often-used multifactor models.

4.2. Subperiod 1965 to 2020

In Table 3, we repeat the cross-sectional analyses in Table 2 for the subperiod 1965 to 2020. The results are similar to those in the earlier subperiod, with the exception that the three- and four-factor models' performance diminishes noticeably. For example, these models now have R^2 values of 83 percent and 86 percent, compared to 72 percent and 92 percent in Panel A of Table 2. None of the t statistics for these multifactor models breaks the recommended 3.0 threshold—namely, they range from 1.74 to 2.50. In addition, the market price of momentum loadings $\hat{\lambda}_{MOM}$ is insignificant with a negative sign. By contrast, the ZCAPM has a near perfect R^2 value of 98 percent, and the market price of zeta risk loadings $\hat{\lambda}_{Z^*}$ is highly significant with $t = 4.00$. The CAPM performs better in this subperiod with $R^2 = 72$ percent, but the market price of beta risk is again significantly negative at $\hat{\lambda}_\beta = -1.41$ percent ($t = -6.11$).

In Panel B, the results using 25 size and momentum portfolios are little changed. The ZCAPM continues to outperform the multifactor models, even with endogenous assets (that are exogenous to the ZCAPM). Now the market prices of size loadings $\hat{\lambda}_{SMB}$ have t statistics exceeding the 3.0 threshold at 3.75 and 3.61 in the three- and four-factor models. The market price of momentum loadings $\hat{\lambda}_{MOM}$ is positive and significant at $t = 2.48$, but falls below the 3.0 threshold. Recall that it was negative and significant in the earlier subperiod for the 25 size and BM portfolios. So here, we see some instability in the

momentum factor results over time. By comparison, the ZCAPM's $\hat{\lambda}_{Z^*} = 0.47$ percent has $t = 6.07$, which is much higher than the size loadings. As in Panel A, the goodness-of-fit of the ZCAPM surpasses the other models by a larger margin than in the earlier subperiod. These results suggest that the earlier subperiod has greater endogeneity problems than the later subperiod due to smaller sample sizes as you go back in time before 1965. Hence, the earlier subperiod findings underscore the endogeneity problem in the three- and four factor models.

Finally, Panel C provides the results for the 90 combined portfolios with industry portfolios included. As before, the inclusion of exogenous test assets reduces the performance of the multifactor models. The ZCAPM has $R^2 = 87$ percent and $t = 8.50$ with respect to the market price of zeta risk $\hat{\lambda}_{Z^*} = 0.63$ percent, which exceeds the R^2 values of the three- and four-factor models at 0.48 percent and 0.54 percent, respectively, and t -statistics of 3.72 and 3.37 for $\hat{\lambda}_{SMB}$ at 0.57 percent and 0.53 percent. The market prices of value and momentum loadings are insignificant at the 5 percent level in these test assets. The inability of these factors to consistently be significant from over time and across test assets suggests that they are false discoveries. Only the size factor continues to pass the 3.0 threshold, even when exogenous assets are included in the test assets.

We should mention that previously cited studies that incorporated a return dispersion factor in an OLS time-series regression model obtain much weaker and ambiguous findings than the EM ZCAPM regression model with a dummy latent variable. Unlike the present study, Verousis and Voukelators (2015) found that return dispersion loadings are negatively priced. Other studies by Jiang (2010), Demirer and Jategaonkar (2013), Garcia et al. (2014), and Chichernea et al. (2015) reported positive prices of return dispersion loadings, but the significance levels did not consistently exceed the 3.0 threshold. In this regard, because they used in-sample cross-sectional tests rather than out-of-sample tests as in the present study, their results cannot be directly compared to our results.

Another noteworthy finding is that, as in the earlier subperiod, the intercept terms $\hat{\lambda}_0$ for the ZCAPM are lower than those for the other models in Panels A to C in Table 3. We infer that this is likely due to the better goodness-of-fit of the ZCAPM compared to the other models.

In sum, the ZCAPM outperforms the CAPM and commonly used multifactor models in terms of both goodness-of-fit and significance of return dispersion factor loadings. Differences in performance are greater, using exogenous industry assets, which call attention to the endogeneity problem in using tests assets sorted on firm-level characteristics that are used to construct long-short, zero-investment factors. We infer that our results corroborate those in KLH—that is, the ZCAPM consistently dominates multifactor models in out-of-sample cross-sectional regression tests. According to KLH, the return dispersion factor outperforms multifactors due to the fact that the latter are actually rough return dispersion measures that capture different slices within the total return dispersion. The size factor is long small stocks' returns and short big stocks' returns, and so captures a portion of the total return dispersion. Sometimes, multifactors switch from positive to negative market prices of risk in cross-sectional tests; for example, the market prices of momentum factor loadings switch from negative in the earlier subperiod to negative in the later subperiod in Tables 2 and 3, respectively. The reason for this erratic pricing behavior is that momentum was capturing negative zeta risk in the earlier subperiod and positive zeta risk in the later subperiod. Surely, momentum is a return dispersion measure, as it is defined as past winner stocks' turns minus past loser stocks' returns. These multifactors can shift around within the total return dispersion of stocks over time and at times become insignificant, which is what we find in our results in Tables 2 and 3. Because multifactors are proxies for different slices of return dispersion, they are related to the ZCAPM. As the theoretical ZCAPM posits, return dispersion is needed to span the return and risk dimensions of the mean–variance Markowitz investment parabola. To locate efficient and orthogonal inefficient portfolios per Black's zero-beta CAPM, the return dispersion is a critical asset pricing factor that is needed to augment the mean market return factor.

4.3. Cross-Sectional Fitted and Realized Excess Returns

Fama and MacBeth (1973, p. 613) observed that: “As a normative theory the model only has content if there is some relation between future returns and estimates of risk that can be made on the basis of current information”. Following this logic, Lettau and Ludvigson (2001) and other researchers typically generated graphs of cross-sectional fitted excess returns and realized excess returns for different test asset portfolios. We discussed details for computing these out-of-sample returns in the previous section. We next display in Figures 3–10 illustrations of the relation between actual and fitted excess returns.

In Figures 3–6 corresponding to the earlier subperiod, we provide the cross-sectional results for the CAPM, three-factor model, four-factor model, and ZCAPM. In Figure 3, the CAPM demonstrates no relation between past beta estimates (used to compute future fitted excess returns) and future realized excess returns. The three- and four-factor models in Figures 4 and 5, respectively, do a much better job. Even so, they have difficulties with portfolios with average realized excess returns greater than about 4 percent per month. For these higher risk portfolios, fitted excess returns tend to underestimate the realized excess returns. We infer that some portion of risk is left out of these models which explains this downward bias. By contrast, the ZCAPM in Figure 6 correctly prices these higher risk portfolios. Hence, the ZCAPM more completely measures risk than the multifactor models. Additionally, and of major importance as a normative theory, based on the ZCAPM, portfolios fall fairly close to the 45-degree line from the origin for fitted and realized excess returns.

Turning to Figures 7–10 related to the later subperiod, we find similar patterns in fitted versus realized excess returns. In Figure 7, the CAPM does better than in the previous subperiod but a markedly flat relation is obvious that fails to capture a linear relation between risk and return. The three- and four-factor models in Figures 8 and 9, respectively, do a much better job than the CAPM but again have difficulties with underestimating the fitted excess returns of high return (risk) portfolios. In Figure 10, the ZCAPM clearly demonstrates a closer fit between fitted and realized excess returns than the other models in the cross section of average stock returns. Additionally, the ZCAPM better prices high return (risk) portfolios.

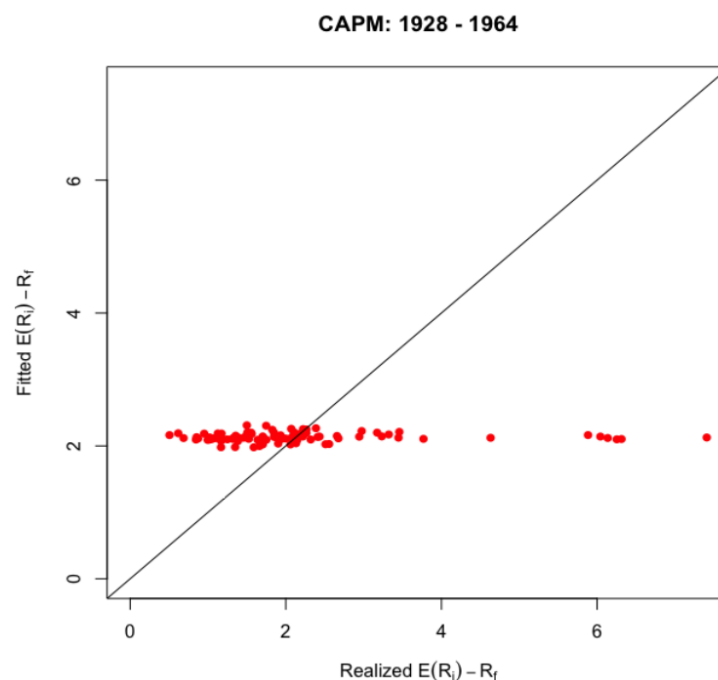


Figure 3. Out-of-sample cross-sectional relationships for the CAPM between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1928 to December 1964.

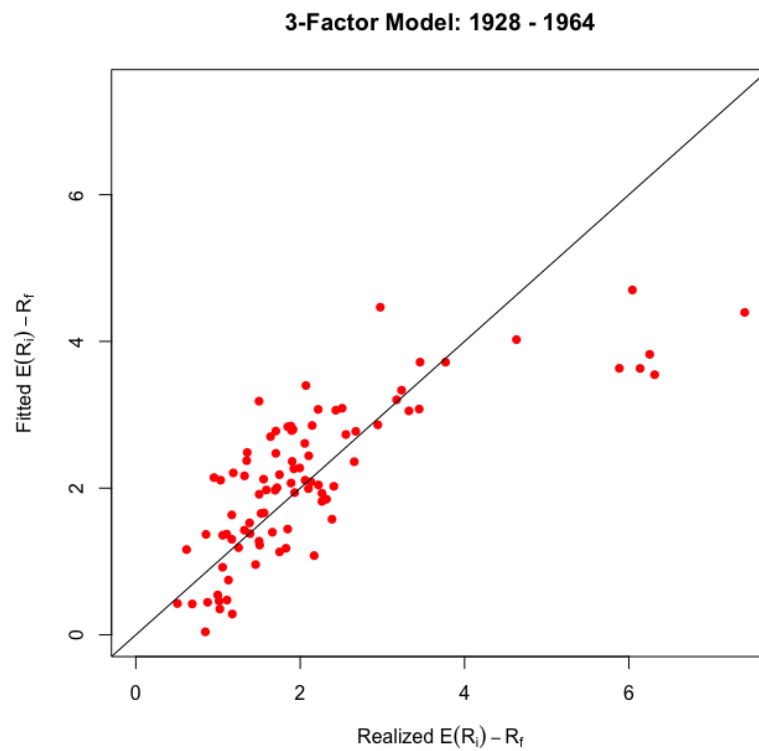


Figure 4. Out-of-sample cross-sectional relationships for the Fama and French three-factor model between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1928 to December 1964.

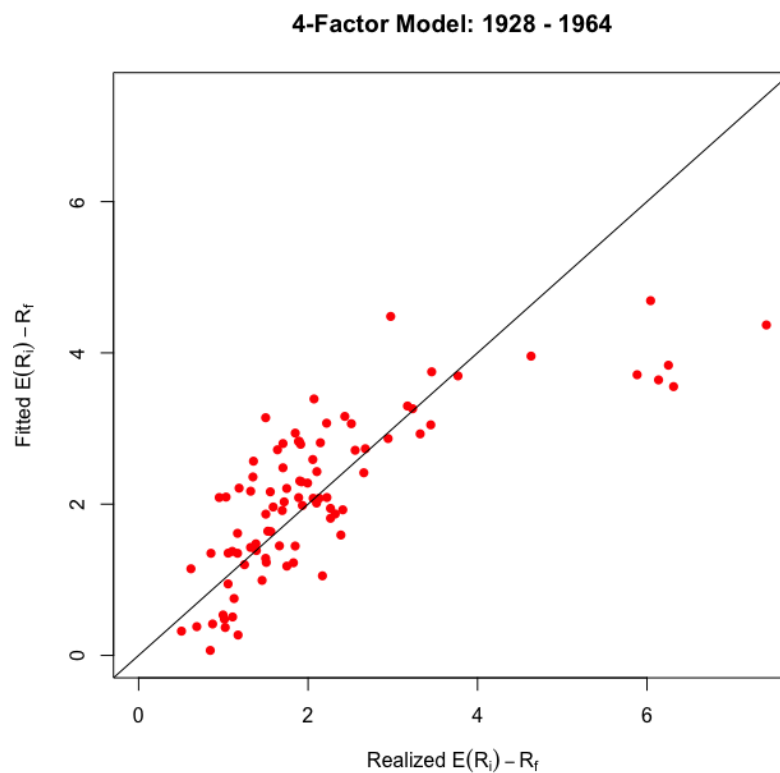


Figure 5. Out-of-sample cross-sectional relationships for the Fama and French four-factor model between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1928 to December 1964.

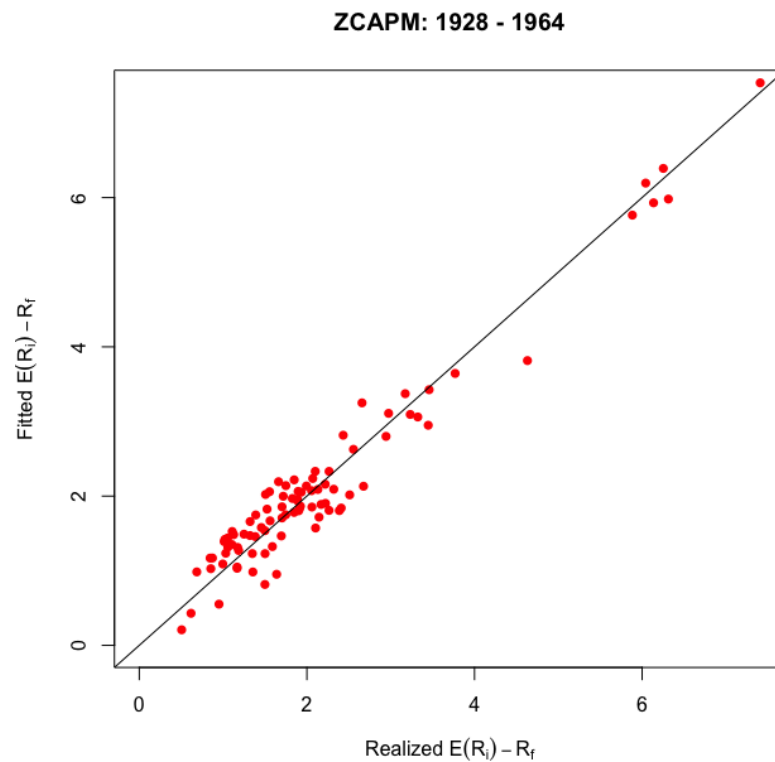


Figure 6. Out-of-sample cross-sectional relationships for the ZCAPM between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1928 to December 1964.

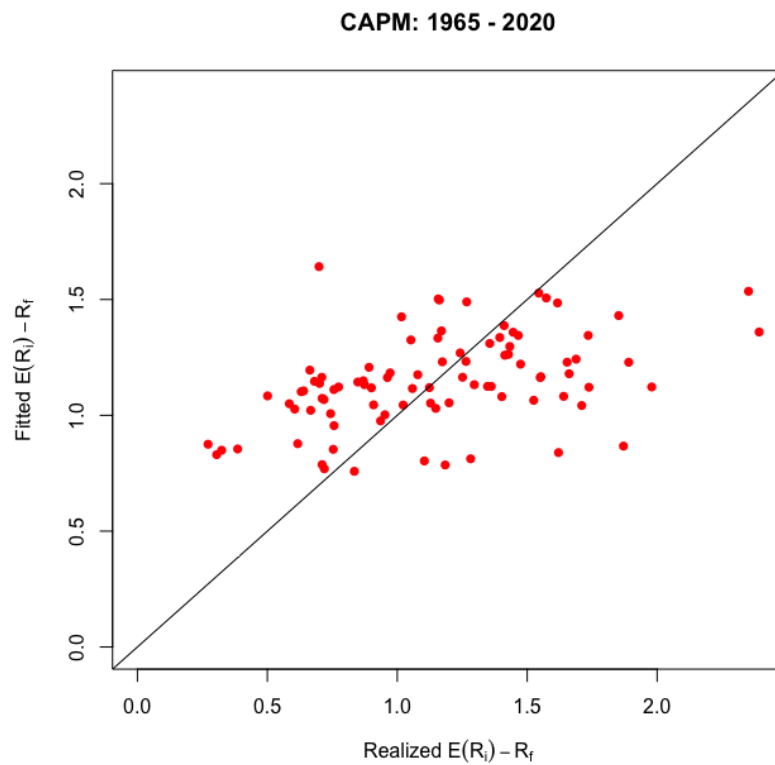


Figure 7. Out-of-sample cross-sectional relationships for the CAPM between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1965 to December 2020.

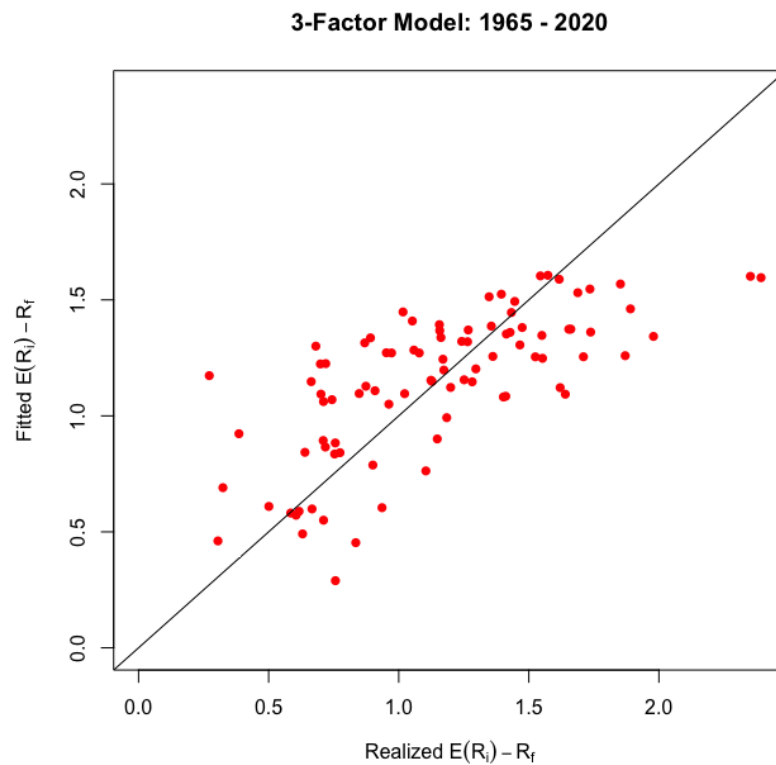


Figure 8. Out-of-sample cross-sectional relationships for the Fama and French three-factor model between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1965 to December 2020.

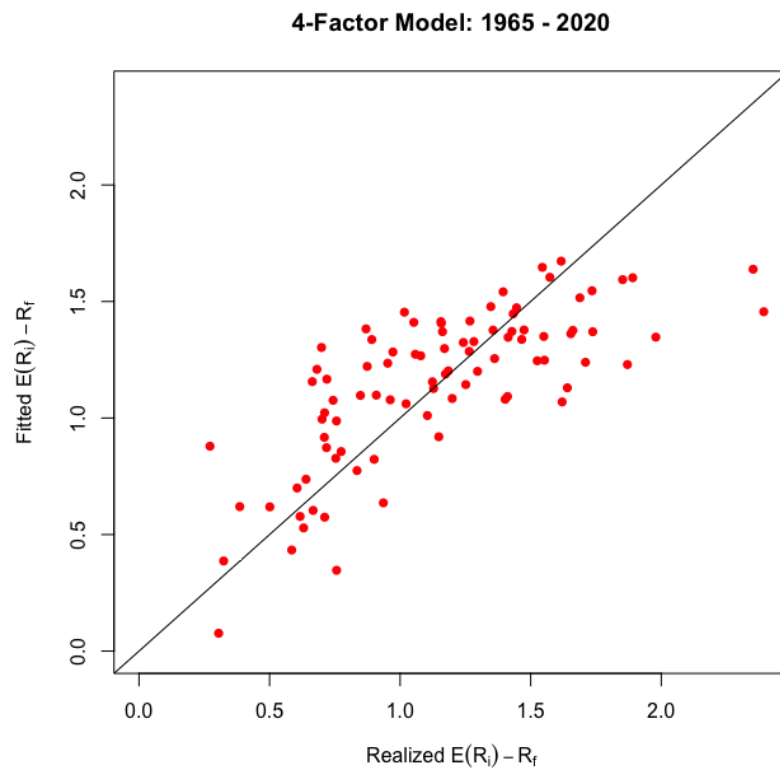


Figure 9. Out-of-sample cross-sectional relationships for the Fama and French four-factor model between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1965 to December 2020.

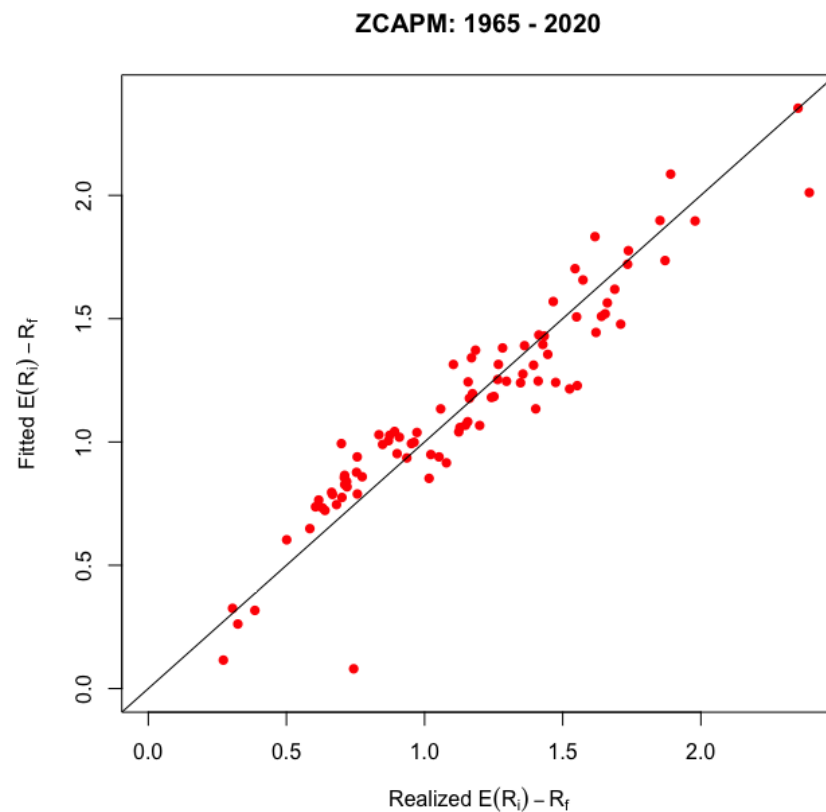


Figure 10. Out-of-sample cross-sectional relationships for the ZCAPM between average one-month-ahead fitted (predicted) excess returns in percent (y -axis) and average one-month-ahead realized excess returns in percent (x -axis): January 1965 to December 2020.

In sum, confirming [Fama and French \(1992, 1993, 1995\)](#) and others, graphical results for the CAPM suggest that there is no relation between the one-month-ahead fitted excess returns based on market beta risk and realized excess returns in the cross section of the average stock returns. Fama and French's three-factor model noticeably boosts the goodness-of-fit compared to the CAPM. Additionally, Carhart's four-factor model further improves the goodness-of-fit, especially in the earlier subperiod of 1928 to 1964. When industry portfolios are added to the test assets, the performance of the three- and four-factor models decreases considerably, whereas the ZCAPM continues to perform quite well. Graphs of average fitted and realized excess returns show that test asset portfolio returns fall closer to the 45-degree line from the origin for the ZCAPM, compared to the multifactor models. Unlike the ZCAPM, the latter multifactor models have difficulty in pricing higher return (risk) portfolios. Thus, the ZCAPM outperforms other models in cross-sectional analyses of stock returns.

5. Conclusions

This study extended the previous work by [Kolari et al. \(2021\)](#) (KLH) on tests of a new asset pricing model derived as a special case of [Black's \(1972\)](#) zero-beta CAPM, dubbed the ZCAPM. KLH investigated U.S. stock returns in the sample period 1965 to 2018. After reviewing the theoretical and empirical versions of the ZCAPM, we expanded their analyses by taking into account the earlier subperiod 1928 to 1964 as well as the later subperiod of 1965 to 2020. Standard out-of-sample [Fama and MacBeth \(1973\)](#) cross-sectional regression analyses were applied to a variety of test asset portfolios, including 25 size and BM portfolios, 25 size and momentum portfolios, and 90 combined portfolios with 40 industry portfolios. We benchmarked the ZCAPM results against the CAPM with a single market factor, the [Fama and French \(1992, 1993, 1995\)](#) three-factor model augmented

with size and value factors, and the Carhart (1997) four-factor model augmented with a momentum factor.

Our results corroborate those in KLH that the ZCAPM consistently dominates multifactor models, especially when using exogenous industry portfolios. Our CAPM results are similar to previous authors that find little or no support for the hypothesized positive relation between beta and average returns. The three- and four-factor models did much better than the CAPM but primarily showed strength using endogenous test asset portfolios based on size and momentum characteristics that are contained in the size and momentum factors. Even so, the ZCAPM outperformed these popular multifactor models. Interestingly, multifactor models did better in the earlier subperiod in all likelihood due to the smaller sample sizes of stocks relative to the later subperiod; that is, smaller sample sizes exacerbate the endogeneity problem in cross-sectional tests. When using exogenous industry portfolios, the multifactor models' performance declined substantially, whereas the ZCAPM continued to perform quite well in both earlier and later subperiods.

We conclude that, similar to the findings of KLH, the ZCAPM consistently outperformed multifactor models. A key reason for this outperformance is that the cross-sectional standard deviation of all stock's returns more comprehensively captures the return dispersion than the selected multifactors that are themselves rough measures of the return dispersion. While popular multifactors show significance in our tests, and can surpass the recommended 3.0 *t*-statistic thresholds in terms of the market prices of factor loadings in some test asset portfolios, their results tend to be inconsistent across subperiods and test asset portfolios. By contrast, *t* statistics associated with the market price of return dispersion loadings in the ZCAPM always exceed 3.0 in different subperiods and test asset portfolios. Additionally, near perfect goodness-of-fit was achieved by the ZCAPM for portfolios sorted on firm-level characteristics, even though these characteristics are exogenous to the mean market return and cross-sectional return dispersion factors of the ZCAPM.

Cochrane (2011, p. 1061) observed that, "...the world would be much simpler if betas on only a few factors, important in the covariance matrix of returns, accounted for a larger number of mean characteristics". The ZCAPM embodies a parsimonious two-factor model with mean market return and return dispersion factors. Regarding the latter return dispersion factor, the ZCAPM takes into account long/short, zero-investment factors based on firm-level characteristics that themselves are rough measures of total return dispersion. Future research is recommended on different countries¹⁷ as well as other asset classes, including bonds, commodities, real estate, etc. Moreover, further work on applications to other areas of finance is recommended, such as investment analysis, the cost of equity, event studies, etc.

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Notes

- 1 Numerous authors link return dispersion to economic fundamentals, including the business cycle, economic uncertainty, and macroeconomic shocks, including Loungani et al. (1990), Christie and Huang (1994), Bekaert and Harvey (1997, 2000), Connolly and Stivers (2003), Stivers (2003), Pastor and Veronesi (2009), Angelidis et al. (2015), and others.
- 2 See seminal work by Dempster et al. (1977) on the development of EM regression as well as applications in other areas of finance by Harvey and Liu (2016) and Chen et al. (2017). Wikipedia provides an excellent overview of EM regression and further citations to statistics literature.
- 3 These studies compute a variety of market volatility factors, including the time-series volatility index (VIX) of the Chicago Board of Options Exchange (CBOE), time-series variance of market returns, and volatility-of-volatility metrics. See (Ferson 2019, chp. 34) for an excellent discussion of studies using time-series volatility factors in the asset pricing literature.
- 4 See https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library (accessed on 1 September 2021)
- 5 See also their earlier work in Liu et al. (2012) and Liu (2013).
- 6 One proof uses random matrix mathematics, and the second proof is based on Markowitz mathematical methods.
- 7 This result was proved by KLH by means of two different geometric methods, including the Roll’s (1980) well-known geometric approach.
- 8 By contrast, the riskless rate is constant and therefore nonrandom.
- 9 In early CAPM studies, such as that of Black et al. (1972), it was found that $E(\tilde{R}_{ZM}) > R_f$, which implied that security market line (SML) had a higher intercept and lower slope than the theoretical CAPM.
- 10 To do this, KLH use the latent variable approach in conditional asset pricing (see Gibbons and Ferson 1985; Ferson and Locke 1998).
- 11 The following conditions hold: (1) assuming all funds are invested in either I^* or ZI^* , then $\beta_{I^*,a} = \beta_{ZI^*,a} = 1$ and $Z_{I^*,a}^* = 1$ or $Z_{ZI^*,a}^* = -1$, respectively; (2) assuming no riskless asset, Equation (11) reduces to Equation (8) (i.e., $\beta_{i,a} \equiv w_{I^*} + w_{ZI^*} = 1$); and (3) assuming the restriction $w_f > 0$ (i.e., no borrowing at the riskless rate is allowed), then $\beta_{i,a} < 1$.
- 12 The market portfolio M lies on the efficient frontier at the tangent point of a ray from the riskless rate.
- 13 This diagram is based on Figure 3.3 in Kolari et al. (2021, p. 68).
- 14 More specifically, KLH defined $\mathcal{T}_+ = \{t : 1 \leq t \leq T, D_{it} = +1\}$ and $\mathcal{T}_- = \{t : 1 \leq t \leq T, D_{it} = -1\}$ as sets of time indices associated with positive and negative signs of the signal variable. As such, the empirical ZCAPM Equation (12) becomes a two equation model:

$$R_{it} - R_{ft} = \beta_{i,a}(R_{at} - R_{ft}) + Z_{i,a}\sigma_{at} + u_{it}, \quad t \in \mathcal{T}_+$$

$$R_{it} - R_{ft} = \beta_{i,a}(R_{at} - R_{ft}) - Z_{i,a}\sigma_{at} + u_{it}, \quad t \in \mathcal{T}_-,$$

where first equation has probability p_i , and the second equation has probability $1 - p_i$.

- 15 As we will see, the goodness-of-fit of the empirical ZCAPM was exceptional, with estimated adjusted R^2 values as high as 98 percent in some tests. These results imply that the absence of a time-series α_i term in the empirical ZCAPM did not affect the cross-sectional results.
- 16 Without recalcing Z_i^* to a monthly basis, estimates of λ_{Z^*} would be much larger and not comparable to λ_β estimates related to beta loadings.
- 17 See the working paper by Kolari et al. (2021) on international stock market tests of the ZCAPM in Canada, France, Germany, Japan, the United Kingdom, and the United States.

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