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A Threshold GARCH Model for Chilean Economic Uncertainty

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Abstract: In this paper, an autoregressive moving average (ARMA) model with threshold generalized autoregressive conditional heteroscedasticity (TGARCH) innovations is considered to model Chilean economic uncertainty time series. Uncertainty is measured through the Business Confidence Index (BCI) and Consumer Perception Index (CPI). The BCI time series provide useful information about industry; commerce; the finance, mining, construction, and agricultural sectors; and the global economic situation and the general business situation. As a counterpart, the CPI time series measure the perception of consumers regarding the state of the Chilean economy, evaluating their economic situation and expectations. The ARMA-TGARCH model is compared with the classical seasonal ARIMA and threshold AR ones. The results show that the ARMA-TGARCH model explains the regime changes in economic uncertainty better than the others, given that negative shocks are associated with statistically significant and quantitatively larger levels of volatility produced by the COVID-19 pandemic. In addition, a diagnostic analysis and prediction performance illustrates the suitability of the proposed model. Using a cross-validation analysis for the forecasting performance, a proposed heteroscedastic model may effectively help improve the forecasting accuracy for observations related to pessimism periods like the social uprising and the COVID-19 crisis which produced volatility in the Chilean uncertainty indexes.

Keywords: TGARCH; economic uncertainty; time-series analysis; COVID-19 pandemic; Chile

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1. Introduction

In financial studies, it is common to model stochastic processes from temporal indexes. In particular, indexes are related to values created by a company's actions or a group of companies included in an index, such as the S&P500 (Vătămănescu et al. 2020). These examples are based on indexes related to companies, but indexes may also have a more general scope, such as a country's macroeconomic information, which includes indexes such as the unemployment rate and inflation. In addition, these indexes may change due to announcements, laws, or decrees, which may spark a crisis to be reflected in time-series behavior, producing more volatility.

Two economic actors are key. One is the business community, or companies, and the other consumers (Barbu et al. 2021). The interaction between them generates trade. In Chile, two indexes indicate the behavior of both actors. One is the Business Confidence Index (BCI), developed by the Center for Studies in Economics and Business of Universidad del Desarrollo (CEEN-UDD) (ICARE 2004). This index is based on a regular survey of around 300 CEOs, responding to questions about their company and industry and general economic aspects. The index gauges the business community's mood, allowing for estimating possible economic scenarios that enable talk about investments or raise an early alarm when economic decline is on the horizon. The Consumer Perception Index (CPI), on the other hand, also created by the CEEN-UDD, involves a survey among around 380 consumers in several Chilean commercial centers (Acuña 2017). This index gauges consumer perception about the economy, which involves respondents' current economic situation, unemployment, expectations on the economic situation, and income.

Uncertainty indexes are deeply linked to political decisions (Cerda et al. 2016), so the indexes are assumed to be influential because they capture the relationship between economics and politics. They served for the interpretation of crises following highly relevant tax reforms since the return to democracy in 1991. Their implementation was followed by events that accentuated uncertainty. For example, the social uprising of 2019 (Jara-Labarthé and Cisneros 2021), the health crisis of 2020–2022 (Mena et al. 2021), the accumulation of large macroeconomic imbalances in 2021 (Idrovo-Aguirre and Contreras-Reyes 2021b), and the persistent inflation of 2022. In addition, uncertainty was used to detect construction investors' confidence, which decreased permanently starting in 2014 (Idrovo-Aguirre and Contreras-Reyes 2019). The accumulation of negative shocks to investment has almost permanently deteriorated investor confidence in the construction sector. Specifically, the period of a more persistent increase in local uncertainty and the constant loss of business confidence (measured by the BCI) coincided with the structural change involving a lower construction investment (CChC 2022). In addition, consumer perception was considered in the study to better predict US consumption, as reported in Lahiri et al. (2016).

Classical time-series models used for economic uncertainty include the autoregressive integrated moving average (ARIMA) and self-exciting threshold autoregressive (SETAR) processes (Tong 1993; Tsay 1989). A SETAR process is the precursor of the threshold autoregressive one and was considered in several studies. For example, Cao and Tsay (1992) measured the volatility of stock returns, analyzing the S&P500 from January 1928 to December 1989, comparing the results obtained through a TAR model and with other models, such as the ARMA and generalized autoregressive conditional heteroscedastic (GARCH) ones (Bollerslev 1986). They concluded that monthly stock return volatilities are non-linear and TAR models produce a higher forecast accuracy than ARMA models (see similar results in Djeddour and Boularouk 2013; Moreno and Nieto 2014). Pérez and Velásquez (2004) studied the dynamic behavior of Spain's quarterly GDP from 1970 to 1998 using a TAR model, also concluding this model's predictions are more accurate than those of linear models, such as the ARMA ones (see similar results in Gibson and Nur 2011; Uribe 2015). Hansen (2011) carried out a complete literature review on the influence of SETAR models in economic research.

The present work was motivated by the former, where time-series modeling of economic uncertainty was extended to a threshold GARCH (TGARCH) model (Zakoian 1994). A TGARCH model involves a threshold component defined by regimes and one defined by the variance modeled conditionally in time. The model was commonly used to study interactions between the information of stock and foreign exchange markets to find asymmetric reactions of stock returns and the associated variability Yang and Chang (2008), while Wu (2010) used TGARCH for volatility index modeling as a threshold variable, analyzing 20 stocks of the Major Market Index and concluding that the threshold model with an exogenous trigger fit the data well. Posteriorly, Korap (2011) modeled inflation for a study on Turkish economic uncertainty. Because the BCI and CPI define expectation scenarios related to agents' future behavior, these scenarios have not been modeled yet based on Chilean economic uncertainty. It is expected that observations related to pessimism periods (social uprising and the COVID-19 crisis) produced BCI and CPI volatility. Therefore, two questions to be answered are as follows: (1) Can the ARMA-TGARCH model adjust well to periods of economic crisis? (2) How effective can the ARMA-TGARCH model be in predicting future (optimism/pessimism) periods?

This work attempts to model the behavior of these indicators with a non-linear functional, enabling projections of volatility behavior, and as an alternative of expected scenarios. In addition, an ARMA-TGARCH model could identify the BCI and CPI regimes, which must be consistent with financial crises, announcements of tax measures/reforms, and monetary policy decisions, among others. The volatility of these perceptions could be explained by the existence of threshold values that produce (non-linear) regime changes in time series. Finally, it is expected that the ARMA-TGARCH model is more suitable for these indexes than their competitors (such as the ARIMA or TAR models). Specifically, it was intended to develop an ARMA-TGARCH model in several steps. In a first instance, and given the arguments related to the literature review, a classical seasonal ARIMA was carried out. Subsequently, a TAR model was considered under the assumption of the non-linear behavior of observations. A suitable non-linearity test (Tsay 1989) is useful to support the TAR model. Finally, and given the presence of volatility in the observations, the ARMA-TGARCH model was considered. Estimates were obtained regarding both the threshold values and autoregressive orders (produced by volatility). Subsequently, a residual diagnostic and cross-validation analysis were carried out for the model and forecasting performances. Importantly, this methodology was not applied to analyze the behavior of the expectation indicators. In general, they were applied to time series with past information, such as the GDP and other indicators whose records depend on a lag.

The paper is organized as follows. Section 2 presents a description of the survey and data related to the BCI and CPI. Section 3 presents the seasonal ARIMA, TAR, TGARCH, and ARMA-TGARCH models and their main properties (stationarity, estimation methods, and diagnostics). Section 4 presents the main results from applying the models to the indexes. Finally, the conclusions and discussions are presented in Section 5.

2. Data

Chilean economic perception time series are measured via the BCI[1] and CPI[2], both provided by CEEN-UDD. The BCI measures investors' confidence in industry, commerce, finance, mining, construction, and agriculture. BCI time series provide useful information about these sectors, the global economic and the general business situations. As counterpart, CPI time series measure the perception of consumers regarding the state of the economy, evaluating the economic situation and their expectations.

2.1. Business Confidence Index

The BCI is prepared through phone and email surveys among around 300 CEOs or other high-level executives and business owners. The index seeks to visualize the proportion of optimistic and pessimistic business actors with respect to some aspects of their company, industry, and the general economy. The sample has a panel structure because the individuals surveyed are repeated over time. In addition, it considered business actors' economic situation and expectations, classifying answers as optimistic, neutral, and pessimistic.

Regarding index building, a subindex X_i was generated, corresponding to the sum of the number of optimistic (OPT_i), neutral (NEU_i), and pessimistic (PES_i) answers, in line with economic sector and company size ($i = 1$: small, 2: medium, 3: large), to obtain

$$X_i = \frac{OPT_i - PES_i}{TOT_i},$$

where $TOT_i = OPT_i + NEU_i + PES_i$, $i = 1, 2, 3$. Then, X_i was used for a weighted average according to company size to obtain subindex Z_i that depends on both questions and economic sector, given by

$$Z_i = \sum_{j=1}^{3} w_j X_i,$$

where

$$w_j = \begin{cases} 0.540, & \text{for large companies;} \\ 0.175, & \text{for medium companies;} \\ 0.285, & \text{for small companies.} \end{cases}$$

A sectorial index S_i was obtained by the average of subindexes of questions related to each economic sector

$$S_i = \frac{1}{18} \sum_{j=1}^{18} Z_j,$$

where total number 18 is obtained by multiplication of three w_j (company sizes) and the six g_j (economic sector[3]) weights,

$$g_j = \begin{cases} 0.064, & \text{for the agricultural sector;} \\ 0.201, & \text{for the trade sector;} \\ 0.142, & \text{for the construction sector;} \\ 0.112, & \text{for the financial sector;} \\ 0.240, & \text{for the industrial sector;} \\ 0.241, & \text{for the mining sector.} \end{cases}$$

Finally, monthly BCI was obtained by

$$BCI = \sum_{i=1}^{6} g_i S_i.$$

The survey developed in (Acuña 2017) considered the main sector of Chile's economy in g_j. However, a survey related to small and medium-sized enterprises in economies such as Europe's could be developed in terms of other factors (see, e.g., Vătămănescu et al. 2020). Table 1 illustrates a qualitative interpretation of BCI values, where neutrality is defined with respect to 0. Optimistic and pessimistic categories are defined on a qualitative scale based on possible BCI values.

Table 1. Business Confidence Index qualitative scale.

Category	Interval
Extraordinarily optimistic	≥ 45
Very optimistic	$[35, 45)$
Optimistic	$[25, 35)$
Moderately optimistic	$[15, 25)$
Slightly optimistic	$[5, 15)$
Neutral	$[-5.5)$
Slightly pessimistic	$[-25, -15)$
Moderately pessimistic	$[-35, -25)$
Very pessimistic	$[-45, -35)$
Extraordinarily pessimistic	< -45

2.2. Consumer Perception Index

This survey is answered in person, that is, a pollster is asking questions and collecting the answers. The questionnaire measures:

- Current economic situation compared to the previous year: Would you say that your current economic situation is worse, the same, or better?
- Current unemployment in relation to the previous year: Today, unemployment in the country is higher, equal, or lower?
- Future economic situation: Would you say that in a year or more your economic situation will be worse, the same, or better?
- Future unemployment: Would you say that in a year or more unemployment in the country will be higher, the same, or lower?
- Future income: Do you think your total family income in the next year will be more, the same, or less?

In addition, socioeconomic characterization questions are included. The calculation of this index is based on consumer perceptions (optimistic, neutral, pessimistic), so the percentage is calculated for each question. The result is constructed based on dividing the number of optimistic consumer responses (OR) by the sum of optimistic and pessimistic (PR) responses as

$$IP_i = \frac{OR}{OR + PR}\%,$$

where i is the ith question. Finally, the five perception indexes are averaged to obtain the CPI as

$$CPI = \frac{1}{5} \sum_{i=1}^{5} IP_i.$$

In addition, the short-term index is related to the current economic situation, which is why it is calculated through the average of economic situation and current unemployment perceptions. The future is considered via the expectations index that is calculated as the average of perceptions about the future economy, future unemployment, and future income. Finally, each index is divided by its initial value and multiplied by 100 to obtain a percentage.

Figure 1 illustrates the BCI and CPI time series with $n = 212$ observations measured from May 2005 to June 2022, where CPI values below and above zero are interpreted as low and high economic optimism. We observed three pessimism events related to the Euro-zone crisis (2007–2009), change in public policies (2014–2016), and the COVID-19 crisis (2020–2022). A mismatch between the indexes was produced, where BCI typically anticipated a CPI crisis (Contreras-Reyes 2023).

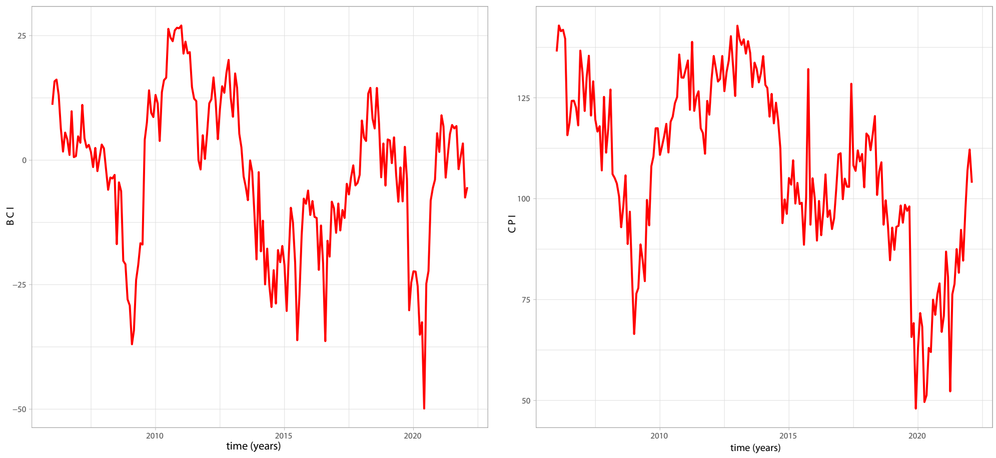

Figure 1. Chilean economic perception from May 2005–June 2022 with BCI (**left**) and CPI (**right**) time series.

3. Statistical Modeling

3.1. ARIMA Model

Let y_t be an ARMA(p, q) process (Box et al. 2015) with p autoregressive and q moving average parameters and innovations $\epsilon_t \sim RB(0, \sigma^2)$. Thus, y_t is represented by

$$y_t = \epsilon_t + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta_i \epsilon_{t-i}, \quad (1)$$

or in equivalent form, $\Phi_p(L) y_t = \Theta_q(L) \epsilon_t$, where $\Phi_p(L)$ and $\Theta_q(L)$ are the autoregressive and moving average polynomials, with backshift operator L such as $L^d y_t = y_{t-d}$, $d \in \mathbb{N}$. In particular, ϵ_t is independent and identically normal distributed with location 0 and scale σ^2. Stationarity of (1) is given when the roots of $\Phi_p(L)$ are outside the unit circle. This way, an integrated ARMA model of order (p, d, q) can be expressed as

$$\Phi_p(L)(1 - L)^d y_t = \Theta_q(L) \epsilon_t, \quad (2)$$

where $\Theta_q(L)$ does not have common roots.

An extension of the ARIMA models considered a seasonal component for the AR and MA parts, the so-called SARIMA models, that include a set of lag parameters related to process $\{y_t\}$ and the seasonal cycles (Hyndman and Khandakar 2007). We say that $\{y_t\}$ follows a SARIMA model as it is created by its ARIMA part and the seasonal AR and MA components as

$$\Phi_s(L^s)(1 - L^s)^D \Phi_p(L)(1 - L)^d Y_t = \delta + \Theta_s(L^s) \Theta_q(L) \epsilon_t, \quad (3)$$

where the polynomial with P seasonal autoregressive parameters is $\Phi_s(L^s)$ and the polynomial with Q seasonal moving average parameters is $\Theta_s(L^s)$. We denoted this model as $\text{SARIMA}(p,d,q) \times (P,D,Q)_s$.

3.2. TAR Model

Let y_t be a $\text{TAR}(r; p_1, \ldots, p_r)$ process (Tong 1993) with a threshold variable z_t and r regimes, defined as

$$y_t = \alpha_{0,k} + \sum_{i=1}^{p_k} \alpha_{i,k} y_{t-i} + \epsilon_{k,t}, \quad \gamma_{k-1} \le z_{t-d} < \gamma_k, \tag{4}$$

with $k = 1, 2, \ldots, r$, and $p_1, \ldots, p_r$ corresponding to non-negative autoregressive orders of y_t in each regime. On the other hand, $\epsilon_{k,t}$ is i.i.d. with location 0 and variance σ_k^2. With respect to threshold values z_{t-d}, we have the ordered values $\gamma_0 < \gamma_1 < \ldots < \gamma_d$, where d regimes of the model are defined. When z_{t-d} is given by lags or functions of the same process y_t, we obtain the SETAR one. TAR processes accomplished stationarity and ergodicity properties Petruccelli and Woolford (1984), which are crucial for asymptotic properties of estimators.

3.2.1. Model Identification

In this section, we describe the identification procedure of a TAR model, i.e., if it is able to model the observations. To do this, the observations must follow non-linear behavior. The test of non-linearity proposed by Tsay (1989) contrasts the hypothesis

$$\begin{aligned} H_0: &\quad y_t \sim TAR(1), \\ H_1: &\quad y_t \sim TAR(r), \quad r > 1. \end{aligned}$$

Alternative hypothesis H_1 is explained by threshold presence; thus, the test's statistic was built through an ordered regression whose parameters are estimated by the Ordinary Least Square (OLS) error estimation method (see Section 3.4), which considered the predictive residuals for the test's statistic. If H_0 is rejected (observations follow non-linear behavior), the next step is identification of structural parameters such as the number of model regimes (r); the autoregressive orders in each partition ($p_1, \ldots, p_r$) in which an $AR(p_i)$ process is defined; threshold values ($\gamma_1, \ldots, \gamma_{r-1}$), where each regime is defined; and lag parameter (d) of threshold variable (z_{t-d}).

3.2.2. Ordinary Least Square Error Estimation Method

In this section, the OLS error estimation method is described for the computation of structural parameter lag d and threshold values γ_i. Without loss of generality, the procedure of Hansen (2011) for a $\text{TAR}(2, p_1, p_2)$ model is presented. The OLS method minimizes the square error sum, proves the existence of one or more threshold values, and determines the asymptotic distribution of the coefficients. Under the model

$$y_t = \begin{cases} X_t^\top \Phi_1 + \sigma_1 \epsilon_t, & \text{if} \quad z_{t-d} \le \gamma, \\ X_t^\top \Phi_2 + \sigma_2 \epsilon_t, & \text{if} \quad z_{t-d} > \gamma, \end{cases} \tag{5}$$

where $X_t^\top = (1, y_{t-1}, \ldots, y_{t-p})$, $\Phi_1 = (\phi_{10}, \phi_{11}, \ldots, \phi_{1p_1})$, $\Phi_2 = (\phi_{20}, \phi_{21}, \ldots, \phi_{2p_2})$, it is assumed that z_{t-d} is stationary. Then, the OLS estimator is

$$\hat{\Phi}_i(\gamma, d) = \left(\sum_{t=1}^{n_j} X_t X_t^\top \right)^{-1} \left(\sum_{t=1}^{n_j} X_t y_t^\top \right), \tag{6}$$

and

$$\hat{\sigma}_i^2(\gamma, d) = \frac{1}{n_i - p_i} \sum_{t=1}^{n_j} (y_t - X_t^\top \hat{\Phi}_i(\gamma, d))(y_t - X_t^\top \hat{\Phi}_i(\gamma, d))^\top. \tag{7}$$

The sum of the residual square is

$$S(\gamma, d) = (n_1 - p_1)\hat{\sigma}_1^2(\gamma, d) + (n_2 - p_2)\hat{\sigma}_2^2(\gamma, d). \tag{8}$$

Then, the conditional OLS of γ and d is

$$(\hat{\gamma}, \hat{d}) = \arg\min_{\gamma, d} S(\gamma, d), \tag{9}$$

where $1 \leq d \leq p$ and $\gamma \in \mathbb{R}_0$.

The asymptotic properties of the OLS estimators were carried out by the consistency theorem given by Petruccelli (1986), who proved that the estimator for a known γ is consistent and asymptotically normal distributed. If γ is unknown, the estimator consistency under certain regularity conditions is proved.

3.3. TGARCH Model

Let y_t be a TGARCH(p, q) process (Zakoian 1994), defined as

$$\begin{aligned} y_t &= \sigma_t \epsilon_t, & (10)\\ \sigma_t &= \alpha_0 + \sum_{i=1}^{q} \left\{ \alpha_i^+ y_{t-i}^+ - \alpha_i^- y_{t-i}^- \right\} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}, & (11) \end{aligned}$$

where (α_i^+), (α_i^-), $i = 1, \ldots, q$, and (β_j), $j = 1, \ldots, p$, is a sequence of real scalars. In addition, ϵ_t is i.i.d., and independently of y_{t-1}, for all t, with $\mathbb{E}[\epsilon_t] = 0$ and $Var[\epsilon_t] = 1$. σ_t is based on the GARCH model developed by Bollerslev (1986).

The TGARCH approach is closely related to the TAR one of Tong (1993) on conditional mean modeling. As an advantage, TGARCH considered a model for the σ_t scalar instead of conditional variance. Thus, some restrictions about positivity are not needed because the conditional variance of σ_t is non-negative by construction. Therefore, we obtain a simpler specification for the inferential procedure. However, if σ_t is non-positive, the inference is hard to develop, so Zakoian (1994) provides the following certain positivity conditions:

$$\alpha_0 > 0, \quad \alpha_i^+ \geq 0, \quad \alpha_i^- \geq 0, \quad \beta_j \geq 0, \quad \forall i, j. \tag{12}$$

The simplest case is the TGARCH$(1, 1)$ model defined by

$$\sigma_t = \alpha_0 + \alpha_1^+ y_{t-1}^+ - \alpha_1^- y_t^- + \beta_{\sigma_{t-1}}, \tag{13}$$

under conditions (12). Model (19) allows to rewrite (19) to obtain an $AR(1)$ process for σ_t with a random component that depends on Z, say $\sigma_t = \alpha_0 + B(z_{t-1}\sigma_{t-1})$, with $B(z_{t-1}) = \alpha_1^+ z_{t-1}^+ - \alpha_1^- z_{t-1}^- \beta$. This relationship is used to solve problems related to strong stationarity. Nelson (1990) proved the existence of a strong stationary solution for (19) that depends on the sign of σ_t.

3.3.1. Quasi-Maximum Likelihood Estimation

Let $\Omega = [\varphi, \omega^\top]$ be a parametric subspace in $\mathbb{R}^{2q+p+1}$, with $\omega^\top = \{\alpha_0, \alpha_1^+, \ldots, \alpha_q^+, \alpha_1^-, \ldots, \alpha_q^-, \beta_1, \ldots, \beta_p\}$. The log-likelihood function for a sample with T observations (ignoring constants) is

$$\ell(\theta) = -\sum_{t=q+1}^{T} \log \sigma_t - \frac{1}{2} \sum_{t=q+1}^{T} \left(\frac{y_t}{\sigma_t} \right)^2. \tag{14}$$

Comparing with the GARCH model, function $\ell(\theta)$ is continuous in θ and differentiable with respect to ω but not always with respect to φ, given the threshold presence. The maximum likelihood $\hat{\theta}$ of θ is thus a particular case of the M-estimator based on the continuous function and differentiable by the right. Under certain regularity conditions, $\hat{\theta}$ is consistent and asymptotically normal,

$$\sqrt{T}(\hat{\theta}-\theta) \xrightarrow{D} N(0, J^{-1}),$$

where
$$J = \mathbb{E}_0\left[\frac{\partial^+\ell(\theta_0;Y)}{\partial\theta}\frac{\partial^+\ell(\theta_0;Y)}{\partial\theta^\top}\right] = \left[\frac{\partial}{\partial\theta}\mathbb{E}_0\left[\frac{\partial^+\ell(\theta_0;Y)}{\partial\theta}\right]\right]_{\theta=\theta_0},$$

and $\partial^+/\partial\theta$ denotes the right derivation, component by component, $\ell(\theta;Y)$ is the log-likelihood of Y_t conditional to Y_{t-1}, and "$\xrightarrow{D}$" means convergence in distribution.

3.3.2. ARMA-TGARCH Model with Skew-*t* Innovations

Another model is the so-called ARMA-TGARCH one, which models $\{y_t\}$ as an ARMA (1), and residuals $\{\epsilon_t\}$ are modeled through a TGARCH (10) and (11) to obtain

$$y_t = \sum_{i=1}^{p}\phi_i Y_{t-i} + \sum_{j=1}^{q}\theta_j\epsilon_{t-j} + \epsilon_t, \quad \epsilon_t \sim^{i.i.d.} N(0,1), \tag{15}$$

$$\epsilon_t = \sigma_t\nu_t, \tag{16}$$

$$\sigma_t = \alpha_0 + \sum_{i=1}^{q_g}\{\alpha_i^+\epsilon_{t-i}^+ - \alpha_i^-\epsilon_{t-i}^-\} + \sum_{j=1}^{p_g}\beta_j\sigma_{t-j}, \tag{17}$$

which is denoted ARMA(p,q)-TGARCH(p_g,q_g), and innovations $\{\nu_t\}$ could be normal distributed. However, in high-volatility data (outliers), innovations could be heavy-tailed and asymmetrically distributed. Thus, we considered here the skew-*t* distribution (Abid et al. 2021) with mean 0, scale 1, asymmetry parameter $\lambda \in \mathbb{R}$, and $m > 2$ degrees of freedom, denoted as $ST(0,1,\lambda,m)$, with probability density function (pdf) given by

$$t_\lambda(x;\nu) = 2t(x;m)T\left(\lambda x\sqrt{\frac{m+1}{m+x^2}};m+1\right),$$

where
$$t(x;m) = \frac{\Gamma(\frac{m+1}{2})}{\sqrt{m\pi}\Gamma(\frac{m}{2})}\left(1+\frac{x^2}{m}\right)^{-(m+1)/2},$$

is the Student-*t* pdf and $T(\cdot;m+1)$ their respective cumulative density function. Without loss of generality, the quasi-log-likelihood function (14) is based on normal innovations; thus, this function could change when the skew-*t* distribution is selected for the innovations.

3.4. Model Selection Criteria

Akaike Information Criterion (AIC) is considered as selection model. Let $\{M_k\}$, $k = 1,2,\ldots,K$, be a set of competing models indexed by k. AIC defined over $\{M_k\}$ is

$$AIC(M_k) = -2\,\ell(\hat{\theta}_k) + 2r, \tag{18}$$

where r is the number of parameters of the M_k model with estimated set of parameters $\hat{\theta}_k$. We selected as best model among $\{M_k\}$, the model with smallest AIC value.

3.5. Computational Implementation

Models were implemented with `R` software (R Core Team 2022). Augmented Dickey–Fuller (ADF) test (Dickey and Fuller 1979) was carried out with `adf.test` function of `aTSA` package. ADF test considered a null hypothesis of a unit root in the time series. ARIMA models were carried out with `auto.arima` function of `forecast` package. This function considered the AIC and Bayesian Information Criterion (BIC) in an automatized algorithm to select the best ARIMA model.

Non-linearity test of Section 3.2.1 was carried out with `thr.test`, and `uTAR.est` function computed TAR model estimates with multiple regimes. Both functions were implemented in the `NTS` package. The `ugarchfit` function of `rugarch` package (Ghalanos 2022) allows to fit, among other models, the TGARCH and ARMA-TGARCH ones and with skew-t innovations.

4. Results

The time series were first tested through an ACF test on the unit root presence. For the CPI, the p-value was 0.116, indicating the null hypothesis of stationarity was rejected; thus, the CPI is a non-stationary time series. The BCI p-value was 0.514, and hence the null hypothesis was rejected, indicating that the BCI is a non-stationary time series. This result led us to consider integrated processes such as the ARIMA or SARIMA, as follows.

4.1. ARIMA and SARIMA Models

Considering the smallest AIC value (Section 3.4), the best models for the BCI and CPI time series were the SARIMA$(1,0,1) \times (0,0,2)_{12}$ and ARIMA$(1,0,1)$, respectively. Table 2 shows the estimated parameters where, for both indexes, the estimated autoregressive parameters were close to 1, indicating invertibility problems. Hence, this kind of model is not the most adequate for the indexes. The next section covers the TAR models that consider threshold variables.

Table 2. Estimates and standard errors (in parentheses) for SARIMA and ARIMA models for BCI and CPI, respectively.

Parameter	BCI Estimates	CPI Estimates
ϕ_1	0.926 (0.030)	0.965 (0.019)
θ_1	−0.189 (0.075)	−0.400 (0.067)
θ_1^s	0.097 (0.074)	–
θ_2^s	−0.202 (0.077)	–

4.2. TAR Model

The first step was the non-linearity test for the BCI and CPI time series, where the p-values 0 (statistic = 7.787) and 0.002 (statistic = 6.223) were obtained, respectively. The null hypothesis related to the TAR(1) was rejected for both time series; hence, the BCI and CPI can be represented by a TAR(r) model with $r > 1$. The second step was the definition of an adequate number of partitions for the time series. A useful technique is the change-point detection in the time series, where a generic approach was considered. Here, the Breaks For Additive Seasonal and Trend (BFAST) technique proposed by Verbesselt et al. (2010) was used. This method detects structural changes in the seasonal series and the trend, and the other time-series components.

For the BCI time series, two breakpoints, and hence partitions, were detected. As in the ARIMA models, the number of autoregressive parameters in each partition was determined using the AIC for a maximum of five, indicating that the best BCI model is the TAR$(2;1,5)$ (Table 3) with a third partition (p_3). However, for the CPI, two partitions were detected using the BFAST technique. As is observed in Table 3, the CPI was fitted for a maximum of five autoregressive parameters for the second partition. The best TAR model also required five parameters; thus, the best CPI one was a TAR$(1;5)$. This means the BCI and CPI were modeled with a high number of autoregressive parameters and could be higher, indicating that the data volatility makes it difficult to estimate and determine the best model. A heteroscedastic model that involves threshold variables is considered in the next section to summarize the information of the time series.

Table 3. AIC for TAR models fitted to BCI and CPI time series. The bold entries highlight the smallest AIC values for each model.

BCI								CPI		
p_1	p_2	p_3	AIC	p_1	p_2	p_3	AIC	p_1	p_2	AIC
1	1	1	736.6063	2	1	1	735.6555	1	1	849.3376
1	1	2	738.6053	2	1	2	737.6545	1	2	842.3831
1	1	3	736.4068	2	1	3	735.4560	1	3	831.2180
1	1	4	731.8181	2	1	4	730.8673	1	4	828.2918
1	1	5	729.6451	2	1	5	728.6943	1	5	826.5194
1	2	1	737.2708	2	2	1	736.3200	2	1	845.1154
1	2	2	739.2698	2	2	2	738.3190	2	2	844.0723
1	2	3	737.0713	2	2	3	736.1205	2	3	832.9071
1	2	4	732.4827	2	2	4	731.5319	2	4	829.9810
1	2	5	730.3096	2	2	5	**729.3588**	2	5	**828.1809**

4.3. ARMA-TGARCH Models

Comparing the AIC values in Table 4, it was determined that the ARMA$(1,1)$-TGARCH$(1,1)$ and ARMA$(1,1)$-TGARCH$(2,2)$ were the optimal models for the BCI and CPI, respectively. They considered the ARMA$(1,1)$ and normal innovations as the base model and distribution for the BCI time series, respectively, because this combination generated the smallest AIC values. However, for the CPI time series, the skew-t distribution was considered for the innovation because it produced a smaller AIC (7.2523) than the normal one (7.2869).

Table 4. AIC for ARMA-TGARCH models fitted to BCI and CPI time series. The bold entries highlight the smallest AIC values for each model.

BCI						CPI					
p_g	q_g	AIC	p_g	q_g	AIC	p_g	q_g	AIC	p_g	q_g	AIC
1	1	**6.6832**	3	5	6.7260	1	1	7.3196	3	5	7.3439
1	2	6.6842	4	1	6.7270	1	2	7.3305	4	1	7.3415
1	3	6.6956	4	2	6.7265	1	3	7.3405	4	2	7.3180
1	4	6.7073	4	3	6.7323	1	4	7.3501	4	3	7.3621
1	5	6.7126	4	4	6.7426	1	5	7.3163	4	4	7.3396
2	1	6.7034	4	5	6.7466	2	1	7.3018	4	5	7.3646
2	2	6.7048	5	1	6.7467	2	2	**7.2869**	5	1	7.3607
2	3	6.7257	5	2	6.7475	2	3	7.3249	5	2	7.3389
2	4	6.7279	5	3	6.7513	2	4	7.2992	5	3	7.3743
2	5	6.7328	5	4	6.7616	2	5	7.3103	5	4	7.3609
3	1	6.7051	5	5	6.7611	3	1	7.3258	5	5	7.3698
3	2	6.7039				3	2	7.3046			
3	3	6.7095				3	3	7.3345			
3	4	6.7220				3	4	7.3190			

Ghalanos (2022) proposed a reparameterized version of the TGARCH model for σ_t in Equation (11), given by

$$\sigma_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j \sigma_{t-j}|u_{t-j}| + \sum_{j=1}^{p} \beta_j \sigma_{t-j}, \tag{19}$$

where $|u_{t-j}|$ is related to the threshold values of the TAR representation (4), i.e., $\gamma_{k-1} \leq u_{t-d} < \gamma_k$. Thus, we have the ordered values $\gamma_0 < \gamma_1 < \ldots < \gamma_d$, where d regimes of the model are defined. We considered representation (19) of σ_t for the ARMA-TGARCH model results in Table 5.

Table 5 gives the estimated parameters, where for the BCI time series the significant parameters were ϕ_1, θ_1, α_1, and β_1. This means autoregressive, heteroscedastic, and threshold components were present in the model and suitable to model the BCI time series. For the CPI time series, the parameters ϕ_1, θ_1, β_1, λ, and ν were significant for a 95% confidence level. This indicated that asymmetry and heavy-tails were present in the innovations and autoregressive and heteroscedastic components in the model. In addition, $\nu \approx 5$ indicated the presence of atypical innovations, so a normal distribution was not suitable for the residuals. Note that α_1 and α_2 were not significant for a 95% confidence level, indicating that the CPI time series do not require a threshold component. Therefore, the CPI time series could be fitted by the simplest model, such as the ARMA$(1,1)$-GARCH$(0,2)$ one (Ramírez-Parietti et al. 2021).

Table 5. Estimated parameters of ARMA$(1,1)$-TGARCH$(1,1)$ and ARMA$(1,1)$-TGARCH$(2,2)$ models for BCI and CPI time series, respectively.

	Parameter	Estimation	Std. Error	t-Value	*p*-Value
BCI	ϕ_1	0.940	0.034	28.014	0.001
	θ_1	−0.162	0.084	−1.924	0.054
	α_0	0.497	0.332	1.499	0.134
	α_1	0.081	0.033	2.422	0.016
	β_1	0.857	0.066	12.939	0.001
CPI	ϕ_1	0.994	0.009	106.930	0.001
	θ_1	−0.411	0.059	−6.920	0.001
	α_0	0.005	0.061	0.075	0.940
	α_1	0.001	0.001	0.007	0.994
	α_2	0.031	0.021	1.437	0.151
	β_1	0.931	0.001	3972.814	0.001
	β_2	0.001	0.012	0.001	0.999
	λ	0.888	0.076	11.645	0.001
	m	4.951	1.505	3.290	0.001

The Ljung–Box test (Ljung and Box 1978) was considered for several lags to check if the residuals of the fitted models are white noise. Table 6 shows that at a 95% confidence level, both the standardized and standardized squared residuals of the ARMA$(1,1)$-TGARCH$(1,1)$ and ARMA$(1,1)$-TGARCH$(2,2)$ models for the BCI and CPI time series were white noise, respectively. In addition, Figures 2 and 3 indicate that the autocorrelation functions were within Bartlett's bands (Contreras-Reyes and Palma 2013), indicating that the residuals of the models fitted to the BCI and CPI time series were not correlated from 1 to 25 lags. The histograms illustrate that well-fitted normal and skew-t distributions form the empirical distribution of the innovations related to the BCI and CPI, respectively. Therefore, these diagnostics indicated a good performance of the fitted models.

Table 6. Ljung–Box test for standardized and standardized squared residuals of ARMA$(1,1)$-TGARCH$(1,1)$ and ARMA$(1,1)$-TGARCH$(2,2)$ models for BCI and CPI time series.

	Standardized Residuals			Standardized Squared Residuals		
	Lag	Statistic	*p*-Value	Lag	Statistic	*p*-Value
BCI	1	0.170	0.681	1	1.674	0.196
	5	2.260	0.887	5	2.571	0.491
	9	3.834	0.729	9	3.978	0.593
CPI	1	0.351	0.553	1	0.817	0.366
	5	1.468	0.998	11	6.085	0.424
	9	2.592	0.943	19	9.299	0.528

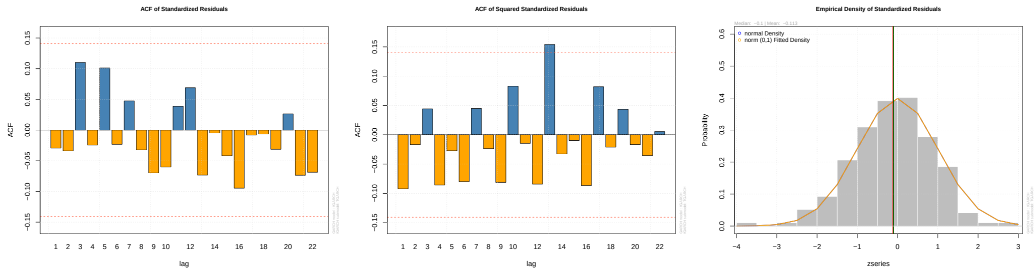

Figure 2. Histogram and ACF of standardized residuals of the ARMA-TGARCH model for BCI time series. Red lines correspond to Bartlett's bands related to 95% confidence level.

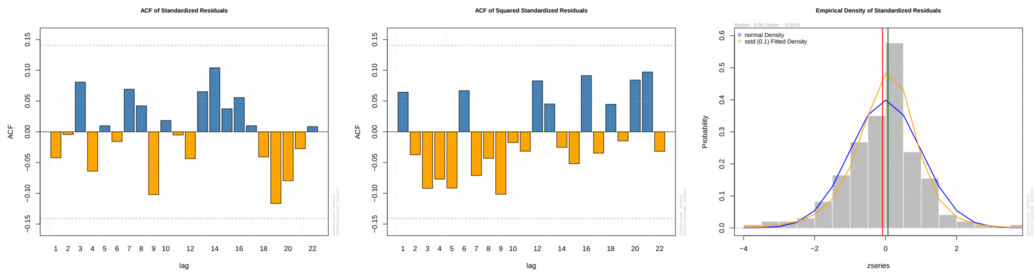

Figure 3. Histogram and ACF of standardized residuals of the ARMA-TGARCH model for CPI time series. Red lines correspond to Bartlett's bands related to 95% confidence level.

Finally, a cross-validation study analyzed the predictions of the fitted models. Specifically, the first 75% of the observations were taken to predict one observation with one prediction horizon (one step ahead). The prediction performance (an observation–prediction comparison) was summarized using the root square mean error (RMSE). The results appear in Figure 4, where in general the last 25% of the observations were well predicted. Only for the observations related to the months of pessimism (the COVID-19 crisis) did a small difference emerge between the original observations and the predictions, where the observations increased the variability and produced volatility in the time series. The BCI and CPI RMSE was 13.092 and 31.224, respectively.

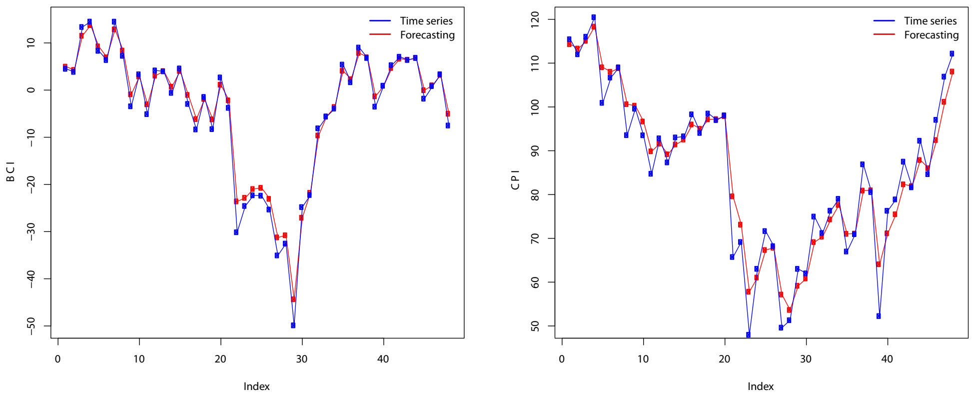

Figure 4. Predictions for ARMA-TGARCH models for BCI (**left**) and CPI (**right**) time series.

5. Discussion and Conclusions

In this paper, we modeled Chilean economic perception time series using three kinds of models. The first was the simplest linear model, the ARIMA (or SARIMA) one, which presented stationarity problems given that the autoregressive polynomial was not invertible. The second model, the TAR one, considered an autoregressive model in each partition separated by regimes, whose results produced the problem of finding a suitable number of autoregressive parameters. The third model, the ARMA-TGARCH (or ARMA-GARCH), considered an ARMA process for the economic index and a TGARCH for the residuals.

This model performed best in terms of the AIC values, i.e., for the determination of a small number of parameters with respect to the ARIMA (or SARIMA) and TAR models. In addition, the diagnostics showed good model performance in terms of the white noise and autocorrelated residuals. The predictions of both indexes were close to the original observations.

The results indicate that the volatility and structural changes in the time series were better modeled by an ARMA-TGARCH (or ARMA-GARCH) model, assuming normal (or skew-t) innovations. This was mainly produced by the volatility of the time series related to the political, economic, and sanitary crises affecting Chile. These features produced asymmetry and heavy-tails on innovations (Lyu et al. 2017; Maleki et al. 2020; Shum 2020), leading to considerations to use new models with skew-t errors. In addition, a cross-validation analysis for the forecasting performance was performed, showing that the pessimism periods produced by the 2019 social uprising and the 2020-2022 COVID-19 crisis increased the bias between the original observations and the predictions. However, the RMSE is relatively small because the proposed heteroscedastic model leads with observations that produced volatility in the uncertainty indexes. In addition, the proposed models could be useful for decision making related to public policies based on microeconomic indicators, such as construction (Idrovo-Aguirre and Contreras-Reyes 2019; Idrovo-Aguirre et al. 2021) and natural resources (Idrovo-Aguirre and Contreras-Reyes 2021a) figures. It is expected that the estimation and prediction performance of the proposed heteroscedastic model detects uncertainty in the construction sector, specifically for the periods when a lot of business confidence measured by the BCI was lost, and thus of a lower sector investment (CChC 2022; Idrovo-Aguirre and Contreras-Reyes 2019).

Further research could involve implementing a cross-sample entropy for the synchronization of the BCI and CPI indexes (Ramírez-Parietti et al. 2021). With respect to the proposed models, further studies might focus on other indexes, such as a daily S&P500 index, the Hang–Seng index (Shum 2020), foreign exchange rates (Ramírez-Parietti et al. 2021), or another time series. Moreover, economic uncertainty could be modeled by a multivariate approach (Arnold and Günther 2001; Contreras-Reyes 2022). The proposed models may be applied to similar economies of other Latin American countries by adapting the relevant variants, such as the external factors. We encourage researchers to consider the proposed models for the study of economic uncertainty in other countries, especially those affected by similar crises (Kliestik et al. 2020).

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Notes

1 A methodological note about building this index is available at https://ceen.udd.cl/estudios-y-publicaciones/ice/ (accessed on 12 November 2022).

2 A methodological note about building this index is available at https://ceen.udd.cl/estudios-y-publicaciones/ipeco/ (accessed on 12 November 2022).

3 Constructed through the percentage of participation with respect to each sector's GDP.

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