Article
Implicit Hedging and Liquidity Costs of Structured Products

Kujtim Avdiu [1] and Stephan Unger [2,*]

[1] Department of Statstics, Oesterreichische Nationalbank (OeNB), Otto-Wagner-Platz 3, 1090 Vienna, Austria;
    kujtim.avdiu@oenb.at
[2] Department of Economics & Business, Saint Anselm College, Goffstown, NH 03102, USA
[*] Correspondence: sunger@anselm.edu; Tel.: +1-646-991-8501 or +43-681-10608892

Abstract: This article analyzes the implicit hedging and liquidity costs of structured equity products offered by various financial institutions. We replicate several payoffs of structured products, compare the calculated fair values based on the Heston model as well as geometric Brownian motion, using various optimization techniques, and compare their fair values with the historic prices traded in the market. We find that implicit hedging costs range between 0.9% and 2.9% markup on the fair value, where we find the underlying market volatility to be the relevant driver of this range for complex structures, while market liquidity can be extracted as the only driver of markups for simple structures with no hedging requirements.

Keywords: pricing; Heston model; hedging costs; structured products; optimization

JEL Classification: C15; G12; G13

1. Introduction

Structured products are customized financial products that are composed of plain vanilla financial products and derivatives. They can be categorized into leverage, participation, return optimization and capital guarantee products. Each category exhibits different markups on their fair value due to different market conditions, such as market volatility and market liquidity, and implicit risk factors, such as the payoff of the structured product. While the detection of structured product markup has widely been proven in the literature, the drivers of these markups have not been investigated extensively. Moreover, what has been neglected by the literature so far is the exploration of the drivers of the changes in this markup over time. Since there exist several reasons for issuers of structured products to charge a markup, the question remains of what determines the change in the markup over time in the secondary market.

Prior to the great financial crisis in 2009, structured products were very popular among institutional as well as retail clients. They were mainly used for hedging purposes but also for speculation. The underlying mathematical formulas that define their payoffs were broadly understood as common tradable instruments such as payoffs of options. But since payoffs of various derivatives are combined, the resulting total payoff structure is often more complex than the simple aggregation of their payoffs. These characteristics make the payoffs of structured products often look more attractive than they actually are, which is reflected (or hidden) in higher markups and the total payoff structure for the client. According to the EUSIPA, the market volume of investment and leveraged products in Belgium, Germany, Austria and Switzerland at the end of the third quarter of 2020 was EUR 275 billion. For leveraged products, the outstanding volume at the end of September 2020 was EUR 10 trillion (EUSIPA (2020)).

The purpose of this paper is to deliver an overview of the embedded costs associated with trading complex structured products that occur when clients want to trade structured payoffs. All research conducted thus far used option payoffs to replicate the fair values

of structured products. In contrast to the existing approaches, we apply financial models to replicate directly the fair value by using Monte Carlo simulations and a stochastic discount factor.

Moreover, mostly the driving factors explaining the divergence at issuance have been investigated thus far. As the fair value of a structured product fluctuates over time, meaning that the markup is not constant after issuance of the product, the drivers for these changes have not been investigated thus far. In assessing the driving factors of pricing policies, Wilkens et al. (2003) concluded that issuers orient their pricing toward the product's lifetime and the incorporated risk of a redemption by shares (given by the moneyness of the implicit options), bearing in mind the volumes of sales and repurchases to be expected from issuance until maturity.

We assess the driving factor for changes in the markup over time by stating the following hypotheses:

Hypothesis 1. *The changes in the underlying volatility explain the changes in markup.*

Hypothesis 2. *The changes in the underlying liquidity explain the changes in markup.*

Moreover, we are the first, to our knowledge, to apply the Heston model for the calculation of a fair value over the entire test period and thus for investigating the divergence between the fair value and traded price of a structured product. We replicate the payoff of the structured products on several clusters using parallel computing techniques. The replication using cluster technology represents a novelty in the pricing of structured products, as it allows more simulations to approximate the fair value of these products more closely.

Since several studies found the markup to be a nonzero value, due to incurred hedging costs, we investigate why the markup is not of a constant size and what drives the markup changes over time. We are interested in finding the reason for these fluctuations. One possible explanation might be the dependency of hedging costs on underlying market volatility or liquidity. Therefore, we regress the structured products' markup on market volatility and liquidity. Moreover, we replicate complex products and not standard products such as classic, rainbow, guarantee, turbo and barrier products but a combination of all these products by using an optimal calibrated Heston model in order to calculate a robust fair value over time.

We structure this article as follows. In Section 2, we provide a literature review of articles dealing with the hedging costs of structured products. In Section 3, we give an overview about the data and methodology we use for replication of the structured products. In Section 4, we price all these products for the past X years using the calibrated Heston model in order to be able to compare them with real traded market prices. The difference between these two prices reveals the markup and thus the implicit hedging costs. In Section 5, we conduct a regression to test if market volatility, specifically the changes in market volatility, explains the size of the markup (i.e., its changes). In Section 6, we present the results obtained and finally conclude the paper.

2. The Literature

The very fundamental model to solve stochastic differential equations used to price derivatives assuming stochastic volatility was presented by Heston (1993). Since this model is widely used nowadays for pricing structured products, we also employ the Heston model for our analysis.

Some research has been carried out on the topic of structured product markup. However, the investigation of the drivers for these markups is lacking in quality as well as quantity. The most recent branch of research dealing with this topic found some evidence for the existence of a deliberate exploitation of asymmetric information in the structured products retail market.

The fundamental existence of different markups has been commonly agreed upon in the literature. Celerier and Vallee (2015) found that relatively more complex products had higher markups. One could argue that the more complex a product is, the more difficult it is for the retail clients to obtain accurate pricing information. Kronlid and Bengtsson (2017) analyzed whether higher complexity gave lower returns in structured products and discovered that higher complexity could be used to hide risks and fees. Arnold et al. (2021) investigated the compensation of counter-party exposure in the prices of structured products. They found a difference in compensation in the retail prices for structured products for the retail clients pre- and post-Lehman default. After the Lehman default, counter-party exposure was compensated more when attention was higher, which was reflected in the markup of the structured products.

Bertrand and Prigent (2014) examined French retail structured products by computing the initial values of these products and comparing them to the actual traded prices, and found an average mispricing of 2% to 7%. However, it was not investigated if this mispricing was due to the market conditions or exploitation of asymmetrical information bias.

Henderson and Pearson (2011) analyzed offering prices of 64 issues of a popular retail structured equity product and determined that offered prices were almost 8% greater than the estimates of the products' fair market values obtained using option pricing methods. Henderson et al. (2020) found evidence for market manipulation by broker-dealers in the structured products market by detecting abnormal returns on structured equity products. They showed that pretrade hedging altered the prices at which derivative trades occurred. This version of front-running could be one explanation for the drivers of markups. Nervertheless, their results do not explain the variation in occurring markups.

Ammann et al. (2023) identified specific sources of asymmetric information between the issuers and investors in this market. They showed that issuers exploited this information friction to offer products to investors that appeared more profitable for the issuer. This incentivized issuers to design products with higher information asymmetry. Thus, information asymmetry can be seen as another driver for the markups. However, the remaining question is whether these markups diminish when institutional counterparts enter the market. This is addressed by our approach, which takes into account the realized volatility and liquidity of the prevailing market conditions.

Burth et al. (2001) distinguished between distinguish convex strategies, which conveyed a payoff resembling a long position in a stock portfolio together with a protective put, and concave strategies, which replicated covered call payoffs. They studied 275 concave products sold in the Swiss market in the late 1990s, and compared their prices to an estimate of the cost of creating the payoffs using options traded on Eurex. Not surprisingly, they discovered that prices of the concave products seemed to be rather favorable for the banks. They also noted considerable pricing dispersion, with distinct differences across various issuing institutions, between the products that paid a coupon versus those which did not, and between the instruments issued by a single bank versus those with the co-lead managers.

Stoimenov and Wilkens (2005) examined the pricing of equity-linked structured products in the German market. They compared the closing prices of 2566 equity-linked structured products on the German stock index DAX (Deutscher Aktienindex) with theoretical values derived from the prices of options traded on Eurex (European Exchange) and found that at issuance, structured products on DAX stocks sold at an average of 3.89% above their theoretical values based on Eurex options.

Bergstresser (2008) presented evidence on the abnormal returns of a broad sample of SEPs that is consistent with the findings of other researchers, such as Rogalski and Seward (1991) as well as Jarrow and O'Hara (1989).

Wilkens et al. (2003) compared the daily closing quotes of roughly 170 reverse convertibles and 740 discount certificates to values based on duplication strategies in November 2001, using call options traded on Eurex (European Exchange). They investigated the average price differences depending on product type, issuer, and underlying. They put a special focus on the possible influence of order flow, i.e., they analyzed whether the price

quotes depend on the expected volume of purchases and sales using product life cycles and moneyness as proxies. The study revealed significant differences in the pricing of structured products, which can mostly be interpreted as being favorable for the issuing institution.

3. Data and Methodology

The structured products are based on the historic price development of the most important American and European indices such as EURO STOXX, DAX, SMI, S&P and Dow Jones. We used historical end-of-day closing prices of 94 products with maturities between 2 and 5 years from Interactive Brokers. Our methodology comprises the daily closing prices of the above-mentioned structured products for the regression analysis, the daily closing prices of the indices for the calculation of the fair value and the daily option closing prices for the calibration of Heston parameters. The payoffs were calculated by using the descriptions, i.e., the formulas given in the respective term sheet of the corresponding product.

Wallmeier and Diethelm (2009) developed a clever valuation technique based on a multinomial tree and used it to examine how multiple barrier reverse convertibles (MBRCs) are priced in the market. They showed that prices exceed the model values on average, with greater overpricing when the stocks are less commonly used in MBRCs and are denominated in low-interest-rate currencies.

Since some of the products include multiple assets, we used a multivariate geometric Brownian motion in order to price and reproduce the dependency of the underlyings' dependencies on each other:

$$dS_t^i = X_t^i dt + \sigma_i S_t^i dW_t^i, \tag{1}$$

Here, the Wiener processes are correlated such that $\mathbb{E}(dW_t^i, dW_t^j) = \rho_{i,j}, dt$, where $\rho_{i,i} = 1$. This enabled us to calculate a more precise fair value. To calculate the co-variance matrix and μ of the multivariate geometric Brownian motion, we used historic data of the past year. Our total data availability ranges from 2 to 5 years of historic data using daily closing prices.

For products consisting of only one underlying S_t, we used the Heston model for pricing, calibrated with call option data

$$dS_t = \mu S_t dt + \sqrt{v_t} t S_t dW_t^S, \tag{2}$$

where v_t, the instantaneous variance, is a CIR process

$$dv_t = \kappa(\theta - v_t) dt + \xi \sqrt{v_t}, dW_t^v, \tag{3}$$

and W_t^S, W_t^v are Wiener processes (i.e., random walks) with correlation ρ, or, equivalently, with covariance ρ dt, and where μ is the the asset's rate of return, θ is the long-run variance, κ is the rate at which v_t reverts to θ and ξ is the volatility of the volatility and determines the variance in v_t.

We simulated, for each underlying one, 500,000 random paths. After that, we applied the payoffs of all ten products to these simulated paths. Then, we used the average of all outcomes to calculate the fair value.

4. Structured Products

We use the following abbreviations, applicable to all products: T: maturity; IC: invested notional; CP: capital protection; C_t: coupon at time t; L_t: level at time t; B_t: barrier at time t; r_t: risk-free rate at time t; $P_i(t)$: price of underlying i at time t; and t_i i-th observation with coupon payment. The subject of the investigation is equity products due to better data availability of historic equity option prices for calibration of the Heston model (Appendix A). The structured products we replicated are the following:

4.1. Dual Index Kick-Out, Capital Risk

The dual index kick-out incorporates two indices. The performance of this product depends on the closing prices of both indices at maturity. If both indices are at maturity at or above a chosen strike level then the client receives a predetermined coupon and the invested capital. If this is not the case, the capital is not protected and the client receives the worse return of both indices. The dual index kick-out offers a predetermined coupon at a specific observation date in the future, if both indices close at or above a predetermined level. In this case, the payoff at time τ is calculated as follows:

$$\text{Payoff}(\tau) = \min_{t > \tau}\{C_t * e^{-r_\tau(t-\tau)}\} + IC * e^{-r_\tau*(T-\tau)}, t : P_1(t) \geq L_1(t) \wedge P_2(t) \geq L_2(t) \quad (4)$$

Otherwise, if this is not the case, the investor bears the following capital risk:

$$\text{Payoff}(\tau) = IC * \min\left\{\frac{P_1(T)}{P_1(t_0)}, \frac{P_2(T)}{P_2(t_0)}\right\} * e^{-r_\tau(T-\tau)} \quad (5)$$

4.2. Dual Index + Coupon, Capital Risk

The dual index + coupon consists of two indices. The client receives a coupon at maturity. If the daily closing prices of both indices remain above a certain predetermined strike over the course of the product's whole lifespan, the investor receives the invested capital. If this is not the case, he receives the worse return of both indices. If both indices trade during the whole lifetime T of the product always above the corresponding, predetermined levels during the product's whole lifespan T, the investor obtains the following payoff at time τ:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{T} C_t * e^{-r_\tau*(t-\tau)} + IC * e^{-r_\tau*(T-\tau)} \quad (6)$$

If one or both indices reach or drop below the corresponding level, then the investor bears the following capital risk:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{T} C_t * e^{-r_\tau*(t-\tau)} + IC * \min\left\{\frac{P_1(T)}{P_1(t_0)}, \frac{P_2(T)}{P_2(t_0)}\right\} * e^{-r_\tau*(T-\tau)} \quad (7)$$

4.3. Triple Index + Coupon, Capital Risk

This product is based on three indices. If all indices quote at or above a certain level at a certain observation time, then the investor receives a coupon until early redemption. If this is not the case, the product expires and the investor receives the invested capital.

$$C_t = IC * (N + 1), \quad (8)$$

where N defines the amount of unpaid coupons until the prearranged observation date. If all indices trade on a predetermined observation date T_l at or above a predetermined barrier, then an early redemption happens and the product is closed. The payoff at this time τ is then

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{t_n} C_t * e^{-r_\tau*(t-\tau)} + IC * e^{-r_\tau*(T_l-\tau)} \quad (9)$$

Otherwise, if this is not the case, the investor bears the following capital risk:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{t_n} C_t * e^{-r_\tau*(t-\tau)} + IC * \min\left\{\frac{P_1(T)}{P_1(t_0)}, \frac{P_2(T)}{P_2(t_0)}, \frac{P_3(T)}{P_3(t_0)}\right\} * e^{-r_\tau*(T-\tau)}, \quad (10)$$

where n is the amount of paid-out coupons until maturity.

4.4. Single Index + Coupon, Capital Risk

　　This product includes only one index. If the index closes on a predetermined observation date above a predetermined level, then the investor receives the following coupon until early redemption of the product:

$$C_t = IC * (N + 1), \tag{11}$$

There, N is the amount of unpaid coupons until the predetermined observation date. If the index closes at a predetermined observation date T_l above the initial level, then the product expires early and the investor is returned his invested money:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{t_n} C_t * e^{-r_\tau*(t-\tau)} + IC * e^{-r_\tau*(T_l-\tau)} \tag{12}$$

Otherwise, if this is not the case, the investor bears the following capital risk:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{t_n} C_t * e^{-r_\tau*(t-\tau)} + IC * \frac{P_1(T)}{P_1(t_0)} * e^{-r_\tau*(T-\tau)} \tag{13}$$

4.5. Single Index + Maturity Coupon, Capital Protection

　　This product offers an X % capital protection with following payoff:

$$\text{Payoff}(\tau) = (KP + IC * max((P_1(T) - L_T), 0)) * e^{-r_\tau*(T-\tau)} \tag{14}$$

Single Index + Payoff, Strike Capital Protection

　　The investor receives a predetermined coupon, C_T, at the end of the term if the index reaches or falls below the predetermined level at least once. Additionally, he receives a reduced return of the index. Also, the invested capital is 100% protected.

$$\text{Payoff}(\tau) = C_T * e^{-r_\tau*(T-\tau)} + IC * (1 + max(\frac{P_1(T) - B_T}{P_1(t_0)}, 0)) * e^{-r_\tau*(T-\tau)} \tag{15}$$

4.6. Single Index + Barrier Payoff, Capital Protection

　　This 100% capital-protected product contains an index and generates a coupon if the price of the index remains at or above the strike during the entire term of the product. If this is not the case, the investor receives a reduced return. This product generates the following payoff if the price of the index trades at or above the strike during the product's whole lifespan:

$$\text{Payoff}(\tau) = (C_T + IC) * e^{-r_\tau*(T-\tau)} \tag{16}$$

Otherwise, if this is not the case, the investor obtains the following payoff:

$$\text{Payoff}(\tau) = IC * (max(\frac{P_1(T) - B_T}{P_1(t_0)}, 1)) * e^{-r_\tau*(T-\tau)} \tag{17}$$

4.7. Single Index + Ongoing Coupon, Capital Protection

　　The payoff of this product offers a 100% capital guarantee. If the index closes on a predetermined observation date above a predetermined level, then the investor receives the following coupon until early redemption of the product:

$$C_t = IC * (N + 1), \tag{18}$$

There, N is the amount of unpaid coupons until the predetermined observation date. If the index closes at a predetermined observation date T_l above the initial level, then the product expires early and the investor is returned his invested money:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{t_n} C_t * e^{-r_\tau*(t-\tau)} + IC * e^{-r_\tau*(T_l-\tau)} \tag{19}$$

Otherwise, the investor receives the following capital protect payoff:

$$\text{Payoff}(\tau) = \sum_{t=t_1}^{t_n} C_t * e^{-r_\tau*(t-\tau)} + IC * \left(max\left(\frac{P_1(T) - B_T}{P_1(t_0)}, 1\right)\right) * e^{-r_\tau*(T-\tau)} \tag{20}$$

4.8. Single Index + Payoff, Capital Protection

In addition to the capital invested, the investor in this single index product receives the return of the index if the index return is positive. With this product, the investor's capital is 100% protected. Additionally, the investor obtains the following payoff:

$$\text{Payoff}(\tau) = IC * \left(max\left(\frac{P_1(T)}{P_1(t_0)}, 1\right)\right) * e^{-r_\tau*(T-\tau)} \tag{21}$$

4.9. Single Index, Capital Risk

This product offers a participation in the index performance without capital protection. This product offers no capital protection, but offers a participation in the EuroStoxx performance:

$$\text{Payoff}(\tau) = IC * \frac{P_1(T)}{P_1(t_0)} * e^{-r_\tau*(T-\tau)} \tag{22}$$

5. Driver Analysis

In order to analyze the explanatory power of volatility and liquidity on the markup, we first regress on a weekly basis each structured product markup on the corresponding yearly volatility. Then we regress on a weekly basis each structured product markup on the liquidity measured in seconds. We opted for the regression on a weekly basis due to computation time restrictions. This allows us to extract the drivers of the markup size. Thus, we use volatility and liquidity as a proxy for market trust and regress $Markup_{i,t}$ for structured product i at time t on the volatility $\sigma_{i,t}$, respectively, and on the liquidity $\lambda_{i,t}$, considering time lags.

5.1. Single Driver Analysis

For single products with just one underlying, we conduct a standard regression based on the volatility $\sigma_{i,t}$ of product i:

$$Markup_{i,t} = \alpha + \gamma_p \sigma_{i,t} + \epsilon_{i,t}. \tag{23}$$

To check for liquidity as the driver of the dynamics in the markup of single underlying structured products, we also need to regress $Markup_{i,t}$ for structured product i at time t on the liquidity $\lambda_{i,t}$ over time:

$$Markup_{i,t} = \alpha + \gamma_p \lambda_{i,t} + \epsilon_{i,t}. \tag{24}$$

5.2. Multivariate Driver Analysis

For structured products with multiple underlyings, we conduct a multivariate regression for $Markup_{i,t}$ for product i at time t, on the volatility $\sigma_{j,t}$ of underlying j:

$$Markup_{i,t} = \alpha + \sum_{j=1}^{N} \gamma_j \sigma_{j,t} + \epsilon_{i,t}. \tag{25}$$

Then, we run a regression on the liquidity $\lambda_{j,t}$ of product i of underlying j:

$$Markup_{i,t} = \alpha + \sum_{j=1}^{N} \gamma_j \lambda_{j,t} + \epsilon_{i,t}. \tag{26}$$

We provide the results for the single and multivariate driver analysis in Tables 2 and 3.

6. Results

We calculated the results for all 10 structured products. However, in this chapter, we are presenting only the results of the first product as an example.

6.1. Example 1: Dual Index Kick-Out, Capital Risk

For the dual index kick-out, capital risk, we calculated over the past 4 years its historic fair value, using a calibrated Heston model, and compared it to the historic traded prices of the structured product. We can see in Figure 1 that the traded market prices exhibit a significant markup in price, which reflects the implicit hedging costs for this product, as well as the banks' premiums.
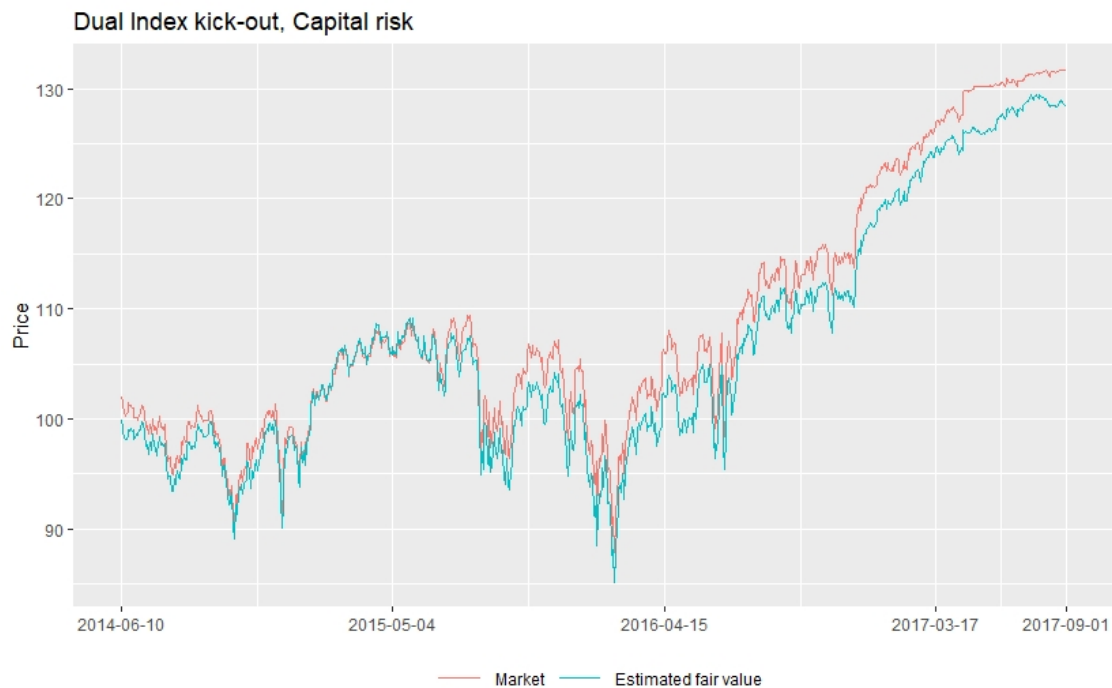

Figure 1. Calculated fair value vs. historic traded price.

When looking at the volatility of the underlyings of the dual index kick-out, capital risk, we can see, first of all, a positive correlation between both indices. In Figure 2, we plot the difference (blue) between the market value (red) and the estimated fair value (green) of Figure 1. The difference in both volatilities is the hedging costs associated with this product. The hedging costs are the input for our regressions in Equations (25) and (26) as they depict

the markup. When plotting the corresponding hedging costs over time in Figure 2, we can see an increase in hedging costs with an increase in the volatility of the underlying constituents of the product.
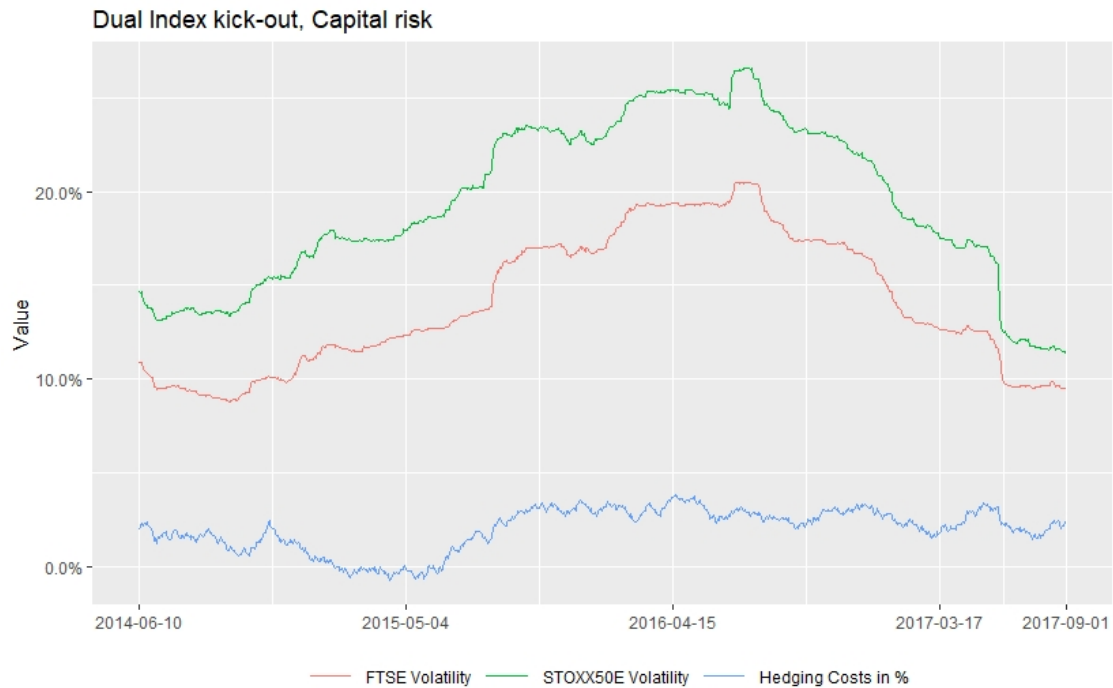

Figure 2. Volatility of FTSE and ESTX50 future, with corresponding implicit hedging costs.

When looking at the market liquidity of the underlying of the dual index kick-out, capital risk, in Figure 3, we do not see any correlation or direct relationship between the liquidity of both indices and the corresponding hedging costs.
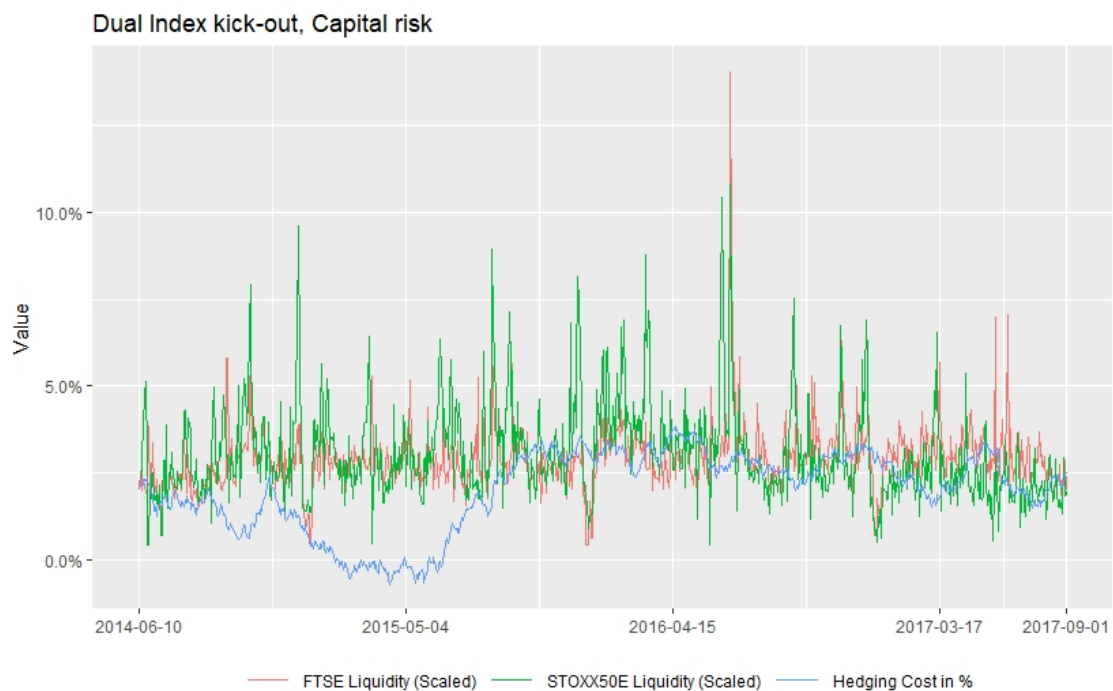

Figure 3. Liquidity of FTSE and ESTX50 future, with corresponding implicit hedging costs.

6.2. Hedging and Liquidity Costs

The 10 structured products were replicated on the basis of their payoffs and priced with multivariate geometric Brownian motion or the optimal Heston model (fair values).

The difference between fair values and historical prices is defined as hedging and liquidity costs (markup).

Table 1 shows the range and average markup on each structured product. We can see that the average markup on a structured product ranges from 0.9% to 2.9%. We can see that hedging costs depend heavily on market volatility. Thus, products that track more volatile markets exhibit a higher markup. As most of the products incorporate barriers, we find that the distance between the barrier and the spot level influences the average markup. The more difficult it is to reach these barriers, i.e., the further away they are from the current spot price, the higher the hedging costs for the bank, since the moneyness levels are of course highly dependent on the corresponding forward rates. Therefore, the client has to expect higher markup costs as banks incur these hedging costs. Also, the coupon payments play a significant role in determining the markup, since the options which are used to hedge these payoffs increase in price with distance to the strike price.

Table 1. Hedging and liquidity costs.

Product	Minimum	Maximum	Average
Dual index kick-out, capital risk	−0.7%	3.8%	2.0%
Dual index + coupon, capital risk	0.1%	1.4%	0.9%
Trippel index + coupon, capital risk	1.3%	3.1%	2.9%
Single index + coupon, capital risk	−0.2%	1.6%	1.4%
Single index + maturity coupon, capital protection	−0.3%	1.8%	1.3%
Single index + payoff, strike capital protection	−0.7%	3.5%	2.1%
Single index + barrier payoff, capital protection	0.3%	2.9%	1.4%
Single index + ongoing payoff, capital protection	0.5%	2.9%	1.9%
Single index + payoff, capital protection	0.6%	4.1%	2.2%
Single index, capital risk	−0.1%	0.5%	0.2%

Table 2 shows the descriptive statistics (median, standard deviation (sd), the 25% quartile (Q1) and 75% quartile (Q3)) of the returns of the 10 structured products on average.

Table 2. Descriptive statistics.

Product	Median	Sd	Q1	Q3
Dual index kick-out, capital risk	0.17%	0.97%	−0.53%	0.8%
Dual index + coupon, capital risk	0.13%	0.63%	−0.23%	0.48%
Trippel index + coupon, capital risk	0.23%	1.12%	−0.38%	0.93%
Single index + coupon, capital risk	0.04%	0.78%	−0.32%	0.56%
Single index + maturity coupon, capital protection	0.12%	0.84%	−0.38%	0.68%
Single index + payoff, strike capital protection	0.15%	1.03%	−0.41%	0.86%
Single index + barrier payoff, capital protection	0.09%	0.8%	−0.38%	0.66%
Single index + ongoing payoff, capital protection	0.22%	0.93%	−0.36%	0.74%
Single index + payoff, capital protection	0.3%	1.03%	−0.41%	0.81%
Single index, capital risk	0.31%	1.82%	−1.53%	2.4%

Table 3 shows the results of the driver analysis for volatility. We can see for most of the products a significant explanatory power of volatility on the markup.

Table 3. Volatility.

Product Group	Nr. of Products	Correlation	*p*-Value	R-Squared
Dual index kick-out, capital risk	15	0.4–0.78	0.02–0.06	0.42–0.68
Dual index + coupon, capital risk	12	0.42–0.62	0.03–0.04	0.55–0.68
Trippel index + coupon, capital risk	7	0.55–0.84	0.06–0.09	0.53–0.78
Single index + coupon, capital risk	13	0.42–0.65	0.008–0.02	0.55–0.88
Single index + maturity coupon, capital protection	7	0.52–0.68	0.03–0.06	0.45–0.38
Single index + payoff, strike capital protection	5	0.61–0.89	0.007–0.02	0.61–0.91
Single index + barrier payoff, capital protection	4	0.55–0.81	0.01–0.03	0.48–0.62
Single index + ongoing coupon, capital protection	9	0.61–0.78	0.01–0.04	0.53–0.78
Single index + payoff, capital protection	13	0.62–0.9	0.008–0.02	0.57–0.81
Single index, capital risk	9	0.21–0.32	0.08–0.2	0.15–0.26

Table 4 shows the results of the driver analysis for liquidity. Interestingly, we can see for only one product a significant explanatory power of liquidity on the markup. Nevertheless, for most of the products, liquidity does not seem to play an important role in the markup determination.

Table 4. Liquidity.

Product Group	Nr. of Products	Correlation	*p*-Value	R-Squared
Dual index kick-out, capital risk	15	−0.13–0.24	>0.05	0.16–0.32
Dual Index + coupon, capital risk	12	0.15–0.28	>0.05	0.03–0.32
Trippel index + coupon, capital risk	7	−0.38–0.14	>0.05	0.15–0.36
Single index + coupon, capital risk	13	−0.12–0.15	>0.05	0.12–0.28
Single index + maturity coupon, capital protection	7	0.12–0.38	>0.05	0.01–0.15
Single index + payoff, strike capital protection	5	−0.25–0.32	>0.05	0.22–0.52
Single index + barrier payoff, capital protection	4	0.18–0.34	>0.05	0.01–0.46
Single index + ongoing coupon, capital protection	9	0.25–0.52	>0.05	0.25–0.58
Single index + payoff, capital protection	13	0.25–0.62	>0.05	0.25–0.38
Single index, capital risk	9	0.29–0.58	0.03–0.9	0.12–0.38

7. Conclusions

We replicated the payoff of 10 different structured products and compared their calculated fair values with the real traded prices in order to determine the markup of the products. We found significant markups, ranging from 0.9% to 2.9%, on the fair values, which indicates implicit hedging costs related to the corresponding product. The markup also included all premiums earned by the issuing bank, but it is not possible to infer the share how much of that markup can be assigned towards the hedging costs and how much towards the bank premium.

Overall, we found that the better the product's payoff for the investor, the higher the hedging costs for the bank. Products with reduced diversification of risk exhibited higher markups as banks incurred higher hedging costs. Also, the higher the capital protection, the higher the markup. Additionally, products with barriers further away from the spot price incurred more hedging costs and thus higher markups. Investors who had been rewarded with better coupons were facing higher net present values when purchasing the products. Moreover, products which were exposed to more volatile markets also incurred more hedging costs and thus caused higher markups.

We were the first to use a calibrated Heston model to replicate the payoffs of single-asset structured products. This approach yielded better results than pricing methods based on standard Brownian motions using long-term data. We used cluster calculation to replicate the payoffs. This cluster calculation is a new technology that uses parallel computing.

Based on these results, we were able to identify possible drivers for the markup, especially if these drivers explained the change in the markup over time. We tested two possible drivers: volatility and market liquidity of the underlying. We found that volatility is a significant driver for the markup of 8 of 10 products, while liquidity turned out to be a driver for just one product, whose payoff did not require any hedging. Our results indicated that most of the markup could be assigned to hedging costs, while hedging costs due to market liquidity could not be extracted in complex payoffs, as the underlying's volatility seemed to dominate the cost of hedging in complex structures. In the presence of hedging, liquidity costs were embedded in the underlying's hedging costs. We found that volatility is the main driver of the markup of complexly structured products. The liquidity costs could only be extracted from noncomplex structures as we found them to be the single driver of product markups. Only in the absence of hedging necessity did market liquidity seem to drive the dynamics of the markup changes.

We can conclude that products with hedging requirements are exposed to volatility, as it drives the hedging costs up. The markup of products with no hedging requirements can be explained by market liquidity. Our results bear economic significance and are important due to the enormous leverage caused by derivatives holdings. Since most of the exposure of the financial market is sitting in derivatives, identifying the drivers of hedging costs as part of the charged markup is very important to understand the underlying risk of banks' exposure to volatility and liquidity. This, in turn, carries important implications for policymakers, as a lower offer of structured product offerings is the only way to reduce the exposure caused by derivatives. The introduction of the Basel accords in the European Union and the Dodd–Frank act in the USA were the first steps in this direction. However, retail clients all around the globe have felt the consequences, as structured products have since only been available for financial institutions.

Our analysis was limited by the restricted calculations of the fair values of the products due to the chosen pricing models. We used the most common pricing models, i.e., the Heston model and the geometric Brownian motion. However, other pricing models, such as Levy models or mean-reverting models, might lead to other conclusions.

Future research could look into the feasibility for retail clients to invest in structured products. Since banks have to hedge their exposure, it would be very important to analyze the profitability of structured products in comparison to other investment possibilities, such as regular ETFs, which could be achieved via calculation of the Sharpe ratio.

Appendix A

Appendix A.1. Calibration of the Heston Model

To calibrate the Heston parameters, we performed Levenberg–Marquardt algorithms in combination with genetic algorithms, since this is the best method in order to minimize

the root mean square error between the estimated plain vanilla Heston option price and the realized option price of option *i* on the estimation day:

$$arg\ min_\Omega \sqrt{\frac{\sum_{i=1}^N (C_0(i,r,M_i,S,K_i) - C_i)^2}{N}}, \tag{A1}$$

where Ω is the set of Heston parameters to be estimated, N is the number of options on the estimation day, C_0 is the Heston call function, which denotes the dollar adjusted call plain vanilla option price, r is the interest rate, M_i is the maturity of option *i*, S is the closing price of the underlying and K_i is the strike of option *i*.

We showed in Avdiu (2021) that options with shorter maturities, at-the-money options and out-of-the money options yield the most accurate fair values. Thus, we took the best 23 options of the last 5 days in order to obtain the best calibration results.

Appendix A.2. Market Liquidity Estimation

To calculate the market liquidity, we used a measure which generated an estimated traded volume per time unit. The Heston model assumed a stochastic volatility development of the bid (*i*) and ask (*j*) prices ($S_{i,j}$) with the parameters $\mu_{i,j}$ (bid/ask price drift), $V(t)_{i,j}$ (bid/ask price variance), κ (rate of mean reversion), $\omega_{i,j}$ (long run variance), $\sigma_{i,j}$ (volatility of variance) and $W_{1,2}^{i,j}$ (standard Brownian movements). Thus, we took, for the bid-ask dynamics, the following:

$$\frac{dS(t)_{ij}}{S(t)_{ij}} = \mu_{ij}dt + \sqrt{V(t)_{ij}}dW_1^{ij}, \tag{A2}$$

$$dV_{ij} = \kappa_{ij}(\omega_{ij} - \sigma_{ij})dt + \sigma(t)_{ij}\sqrt{V(t)_{ij}}dW_2^{ij}, \tag{A3}$$

where W_1 and W_2 are correlated by $dW_1 \cdot dW_2 = \rho dt$ due to the leverage effect between the asset price and instantaneous volatility. To estimate the traded bid/ask volumes, we used inverse transform sampling applied to the historic data:

$$P(Q_{i/j}(t) = x_{i/j}^k) = \sum_{t=1}^k p_{i/j}^t - \sum_{t=1}^{k-1} p_{i/j}^t = p_{i/j}^k \tag{A4}$$

The resulting compound volume process $Y(t)$ at time *t* characterizes the volume generating process induced by trading.

$$Y(t) = \begin{cases} min\{Q_i(t), Q_j(t)\} & \text{if } S_i(t) \geq S_j(t) \\ 0 & o.w. \end{cases}. \tag{A5}$$

By matching bid and ask prices and taking the average of each possible generated volume, we obtained the average traded volume (liquidity) over a certain time period *n*:

$$\lambda = \frac{\sum_{t=1}^n Y(t)}{n} \tag{A6}$$

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