Journal of
Risk and Financial Management

MDPI

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Valuation of Patent-Based Collaborative Synergies under Strategic Settings with Multiple Uncertainties: Rainbow Real Options Approach

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Abstract: Recent years have seen increasing initiatives involving more applications of real options to value the strategizing process. These initiatives, referred to as Real Option Theory (ROT), imply greater inclusiveness of simple and advanced real options in strategizing processes. While substantial theoretical groundwork on ROT has been laid in corporate finance, and both qualitative and quantitative studies on ROT in business management journals are appearing on an increasing basis, there remain significant opportunities for more research on strategic synergism in patent-based acquisitions. In this vein, the current paper aims to explore a rainbow real options application (real options that are exposed to two sources of uncertainty) to measure patent-based collaborative synergies in high-tech mergers and acquisitions. Having conducted the deviant case study of ZOOX start-up's acquisition by Amazon.com in 2020, this paper justifies the proposition of the employability of rainbow real options for the valuation of network and relational synergies in highly risky patent-based acquisitions with multiple uncertainties.

Keywords: acquisition; patent-based synergy; network synergy; relational synergy; valuation; rainbow real options

Citation: Čirjevskis, Andrejs. 2024. Valuation of Patent-Based Collaborative Synergies under Strategic Settings with Multiple Uncertainties: Rainbow Real Options Approach. *Journal of Risk and Financial Management* 17: 157. https://doi.org/10.3390/jrfm17040157

Academic Editor: Cuong Nguyen

Received: 8 March 2024
Revised: 5 April 2024
Accepted: 10 April 2024
Published: 13 April 2024

1. Introduction

In recent years, patent collaboration issues have become a scholar's intriguing research topic. Patent-based collaboration has been studied for specific industries or technology, especially in the energy field such as the renewable energetic sector (Liu et al. 2021), oil and gas industry (Duch-Brown and Costa-Campi 2015), wind power technology (Tsai et al. 2016; Schleich et al. 2017), and green technology (Cho and Sohn 2018; Yin et al. 2020). However, Alvarez et al. (2023) argue that there is still a gap of knowledge in management science on the impact of patent rights on the development of firms when innovative technologies are "proprietaries" by an acquirer.

Access to the unique knowledge, patents, technology, and R&D activities of a target firm became the strategic rationale for many mergers and acquisitions with the expectation of improving the acquirer's innovation performance (Grimpe et al. 2023). Having acquired the patents' holders to monetize their inventions, the acquirers also connect and interact with a network, platform, and ecosystem of the target firms. A recent stream of research focuses on the role of acquisitions in obtaining a wider network of relations of target firms (Hernandez and Shaver 2019). However, the expected value of such obtained networks has not been empirically theorized in the literature on firm acquisitions (Grimpe et al. 2023). Teng (2019) argues that patents are one of the core values of the target firm. Currently, the valuation methods used by practitioners are mainly the cost method and income method (Teng 2019). In turn, real options application for the valuation of investment effectiveness enhances decision-makers to "capitalize on good fortune or to mitigate loss" (Brealey et al. 2003).

Recent years have seen increasing initiatives involving real options applications to value strategizing processes (Trigeorgis and Reuer 2017; Trigeorgis and Tsekrekos 2018; Chi et al. 2019; Čirjevskis 2022). In comparison with some of these domains, research on the application of advanced real options like rainbow, compound, with changing volatilities, and combination (e.g., Čirjevskis 2023a) is still nascent. While substantial theoretical groundwork on ROT has been laid in corporate finance, and both qualitative and quantitative studies on ROT in business management journals are appearing on an increasing basis (Driouchi et al. 2020; Noorizadeh et al. 2021; Araya et al. 2021), there remain significant opportunities to value strategic synergies in patent-based acquisitions.

Therefore, having strived to complement the current research gap in patent-based collaboration research and empirical application of real options analyses, the current paper asks two research questions: What types of synergies expect a high-tech acquirer by "proprietarizing" (Yu 2007, p. 749) technological patents as well as a target firm's network, platform, and ecosystem? How to value patent-based synergies when an acquirer experiences several uncertainties at the time of proprietarizing technological patents of a target company? By giving answers to research questions, the current paper aims to explore an "exotic" (Hull 2012) rainbow real options application to measure patent-based collaborative synergies in high-tech mergers and acquisitions (M&A). The rainbow option is a derivative exposed to two or more sources of uncertainty, as opposed to a simple option that is exposed to one source of uncertainty.

According to real options theory (ROT), volatility increases real options value. While the simple option exposes one source of uncertainty (e.g., price volatility of an underlying asset), the rainbow option exposes more sources of uncertainty (two-colors, three-colors, and so on). Thus, if there is more than one source of uncertainty associated with the investment, then such options are known as rainbow options (Copeland 1998; Damodaran 2002, p. 148). Hence, rainbow real options are incrementally better than other valuation approaches, especially in the context of multiple uncertainties. Damodaran (2005) exemplifies this by considering developing an oil field—one source of uncertainty for the company is the number of oil reserves and the second is the future market price of the oil.

The name rainbow financial option was coined by Rubinstein (1991), where he refers to the number of assets as the number of colors of the option (Rubinstein 1991; Boen 2020, p. 1). Rainbow options application as multifactor relative performance options (Boyle 1993) are used for global asset allocation and for ranging from valuation of foreign currency debt and bonds to risk-sharing contracts and growth opportunities involving mutually exclusive investment according to Peng and Peng (2009, p. 76). Although rainbow financial options analysis has been widely applied in the financial management literature, the rainbow real options for high-tech M&A have not been fully examined, especially within the context of patent-based collaborative synergies.

Using data from one of the strategically important Amazon patent-based acquisitions in 2020, this paper contributes to this gap by incorporating the rainbow real options application to value the types of patent-based collaborative synergies and thereby provides the theoretical and empirical novelty of this research. The nature of the deviant case study of Amazon's acquisition of ZOOX is exploratory, and the usage of this case study is confirmatory to justify the provided theoretical proposition. Moreover, Seawright and Gerring (2008) argued that one deviant case study can be used as a high-residual case or an outlier.

Specifically, the research results prove that the application of rainbow real options (the real option that is exposed to two sources of uncertainty) provides a better estimation of patent-based collaborative synergies when a high-tech acquirer experiences at least two uncertainties at the time of acquisition. In this context, this paper demonstrates that the application of rainbow real options ensures more accurate valuation results than the application of the traditional Black–Scholes option pricing model that operates single volatility.

The remaining part of the paper is organized as follows. Having considered the platform as a network of dependent complementary cooperative providers, this paper debates the economic reasons for the partners to be a node of this platform's network: network and relational synergies. Next, the paper discusses real options application as a more realistic synergies valuation model and develops a theoretical proposition. The subchapter on research methods describes data collection, treatment, and interpretation. The case study of Amazon's acquisition of Zoox in 2020 explores two real options valuation models of patent-based synergies (simple and advanced rainbow options) and justifies the proposed proposition. In the end, this paper discusses findings, contributions, limitations, and future work.

2. Key Literature Review

According to the resource-based perspective of the company (Barney 1991; Peteraf 1993), the ownership of valuable resources that are not entirely substitutable or imitable can lead to a long-term competitive advantage. Patent-protected technology is a valuable resource that frequently possesses these qualities (see, for example, Silverman 1999; Markman et al. 2004). The resource-based view posits that the firm's existing resources can exhibit complementarities to resources not in its possession, implying that their combination will lead to higher economic rents than the sum of the value that these resources could yield on their own that in the current paper are named as collaborative synergies.

Cappelli et al. (2023) argued that firms may acquire resources (i.e., patented inventions) and obtain access to externally developed technologies and associated rights. In this sense, a patent-based acquisition allows firms to connect the acquirer's technological resources with the technological resources of the target that form the basis of the firm's sustainable competitive advantage, if the patents are a core competence of the target firm. The extent to which an invention offers a competitive advantage may determine the relative worth of obtaining patents. Since patents provide a certain degree of protection against rival inventions, patents offer an evident competitive advantage (Klein 2022).

The contract theory on patents (Denicolò and Franzoni 2004) suggests a social benefit of patent protection since it encourages inventors to bear the costs associated with a patent's information disclosure requirements. In contrast, the reward theory on the patent system (Klein 2022) suggests that patent protection engenders rent-seeking behavior. In terms of patent-based mergers and acquisitions, economic rent-seeking can be interpreted as an expectancy of patent-based synergies.

Although the patent system was established to provide certainty of returns to innovators, the patenting process itself and its monetization contains elements of uncertainty (Dohse et al. 2023) Therefore, there is growing concern regarding uncertainties about the ultimate market value added of the patented innovation (e.g., Boldrin and Levine 2013). In this sense, the electric vehicle industry patenting breakthrough technology and strategizing with multiple uncertainties is direct evidence of this phenomenon. The next sub-section discusses the specifics of the electric vehicle industry as a patent-based platform business.

2.1. Electric Vehicle Industry as a Patent-Based Platform Business

Battery electric vehicles (BEVs), hybrid electric vehicles (HEVs), and fuel cell electric vehicles (FCEVs) have gained relevance as viable solutions to mitigate the climate change effects and to open up new business opportunities (Christensen 2011; Manzetti and Mariasiu 2015; Aaldering et al. 2019, p. 236). Electric vehicles (EVs) business is a "platform goods as well because value comes from the vehicle plus complementary providers" (Anderson et al. 2022, p. 135). The price volatility of strategically important commodities needed for EV production, such as nickel and lithium, makes the incorporation of platform strategies crucially important for EV firms to survive and sustain advantages (Anderson et al. 2022).

Anderson defined a "platform" industry as an industry that depends on a network of complementors (Anderson et al. 2022, p. 136). Once a platform company becomes dominant in its sector of economics, it starts growing via acquisitions in other sectors (Pagani et al. 2021). Magney (2020) argued that tech giants such as Amazon, Apple, and Alphabet are well-positioned to participate in the business of automated self-driving vehicles and they have an anchor position in the greater technology ecosystem (Magney 2020, p. 1).

Thus, the EV industry competes not only on the traditional dimensions of automotive quality but also on its network of complementary providers, such as charging stations. Platforms and their associated ecosystems are an active area of study in economics and management literature. Jacobides et al. (2018) defined the ecosystems as "(1) interacting organizations, (2) enabled by modularity, (3) not hierarchically managed, (4) bound together by the non-deplorability of their collective investment elsewhere" (Jacobides et al. 2018; Anderson et al. 2022, p. 136).

Aaldering et al. (2019) argued that alternative powertrain systems are pushing the underlying technological knowledge ecosystem to adopt a collaborative patent-based growth strategy. Having extended an EV platform, the innovator can acquire a new powerful technological competency (Teece et al. 2022) that is robustly protected with technological patents. Therefore, the first research question has arisen: What types of synergies expect a high-tech acquirer by proprietarizing technological patents as well as a target firm's network, platform, and ecosystem?

Hence, the next subchapter explores the types of collaborative synergies in patent-based M&A deals.

2.2. Exploring Types of Synergies and Valuation of Synergies in M&A Deals

Synergy originates from a compound of assets that are more valuable cooperatively than when isolated (Shaver 2006). Reciprocal collaboration still appears when the acquired firm is no longer independent (i.e., it can no longer decide whether to share its intellectual property (IP) or not). There is no collaborative synergy, or it becomes quite problematic in a hostile takeover, where a target's shares are being bought up by an acquiring company against the target's management's wishes. The other path, denoted by a strategic partnership, leads businesses toward a collaborative synergy when both parties consent to unite (Chelniciuc 2013).

Merger and Acquisition (M&A) studies have accentuated two well-known types (internal and market power) that jointly lower costs and enhance revenue, also known as operational synergies (Feldman and Hernandez 2022). What is more, Feldman and Hernandez (2022) unpacked novel synergy sources (relational and network) arising from collaborative strategy-induced changes in external cooperative environments of firms.

If a platform is a network of dependent complementary cooperative providers (Anderson et al. 2022), then there should be economic reasons for the partners to be a node of this platform's network. In this sense, recently Hernandez and Shaver have also discovered a novel source of synergy by coining created network synergy (Hernandez and Shaver 2019, p. 171). In the context of merger and acquisition (M&A) deals, Hernandez and Shaver have defined network synergy as "the extent to which combining an acquirer's and a target's networks through node collapse results in a more favorable structural position for the combined firm as the acquirer gains control of the target's existing ties" (Hernandez and Shaver 2019, p. 171). Thus, one can hypothesize that acquirers prefer targets that generate greater expected network synergies, and among them, by "proprietarizing" the protected assets (patents) of the targets, obtaining property power rights (PPR). Moreover, as patents are embedded in networks, M&As should have PPR consequences that generate network synergies.

Moreover, recently Feldman and Hernandez (2022) have added valuable insights to acquisitions that had already been obtained from transaction cost economics (TCE) (Williamson 1991). Even though from the TCE perspective all acquisitions involve a decision to govern activities internally, "the acquirer also inherits and recombines other valuable interactions located outside the boundaries of the acquirer or the target, some of which will be governed by "hybrid" modes (e.g., alliances) and others through "market" modes" (Feldman and Hernandez 2022, p. 552).

According to Feldman and Hernandez (2022), network synergies are based on acquiring a target whose platform, network, and ecosystem, when combined with that of the acquirer, puts the combined entity in an improved competitive position. When a firm acquires, it gains legal control over the assets of the target (Hernandez and Shaver 2019). Thus, relational synergies are based on partner-specific assets, such as patents, and technological capabilities that allow collaborative parties to create and appropriate more value. Moreover, Hernandez and Shaver (2019) argued that the likelihood of selecting a target increases when the expected network synergy is greater.

Therefore, the second intriguing research question has arisen: How to value patent-based synergies when an acquirer experiences several uncertainties at the time of proprietarizing technological patents of a target firm? Next, this paper discusses an option-based valuation model approach to measure the expected relational and network patent-based synergies in M&A deals.

2.3. Developing a More Realistic Synergies Valuation Model: Enhancing Rainbow Real Options Application

Finding appropriate valuations for target firms on rational grounds is a desirable goal of scholars and practitioners. Although quantitative models underlying the valuation process are helpful, they may not be deterministic by nature and can be intrinsically subject to non-quantifiable elements (Bogdan and Villiger 2007) such as acquiring a target's patents, platform, network, and ecosystem. Traditionally, a project's Net Present Value (NPV) is calculated via discounted free cash flow (DFCF) techniques (without any options), and it is the most frequently used technique to estimate capital budgeting effectiveness in corporate finance.

This is so because the discount rate used in traditional DFCF analysis represents the rate of return investors would require for the investment if it were a traded asset. In this vein, Bogdan and Villiger (2007) recommended quantitative valuation, which is based on considering volatility by using a tree structure and the concept of real options (Bogdan and Villiger 2007). At the outset of ROT applications, research focused on single uncertainty in many cases (Dockendorf 2010). The concept of the rainbow option was first introduced by Rubinstein (1991). Hull (2012) defined rainbow options as options on two or more risky assets. Trigeorgis and Mason (1987) discussed the market-added value of an investment as a sum of static Net Present Value (NPV) plus the option's premium. Particularly, higher uncertainty decreases the static NPV but increases the value of options.

The logic underlying ROT is based on financial options where uncertainty can be considered as an opportunity, and not necessarily as a negative risk, and its impact on the value of an underlying asset can become positive (Iazzolino and Migliano 2015). In this sense, we are on the opposite side of orthodox financial management's statement "that higher risk reduces the value of an asset" (Dockendorf 2010, p. 17).

In this context, the real options application is not only ensuring a numerical exercise but enhancing a strategic decision-making roadmap in the form of decision tree analysis (as recombining and not-recombining lattices) as scholars argued (Trigeorgis and Mason 1987; Dockendorf 2010). Non-recombining binomial lattices are more robust, but with a higher number of periods, they are also more difficult to calculate. Therefore, while such lattices conventionally are not preferred, they are needed for rainbow options when volatility changes over time (Mun 2002, pp. 232–39; Brach 2003, p. 61).

The practical challenge in valuing rainbow options is the mathematical complexity introduced by multiple uncertainties in the form of stochastic processes (Dockendorf 2010). The calculation complexity of trinomial and multinomial lattices can become staggering as the number of periods increases; this considerably diminishes their appeal in real options analysis (Mun 2003, pp. 90–91; Kodukula and Papudesu 2006, pp. 72–73). Such complex lattices are seldom used in practice, and most commonly, analysts resort to binomial lattices instead (Schwarth and Trigeorgis 2001).

When two uncertainties are analyzed (as a two-color rainbow option), the options value estimation involves a quadrinomial tree instead of a binomial. This is because the underlying values as well as the values of real options obtain one of four values when an analyst moves from one node to the next steps' nodes in the quadrinomial lattice (Kodukula and Papudesu 2006, p. 163).

Therefore, an acquirer firm investing in a target firm is not only acquiring the present value of future free cash flows (FCF), but it is also creating several real options obtaining collaborative synergies: abandon options, sequential compound options, options with changing volatility, and rainbow options with multiple volatilities.

The most typical valuation model (FCF approach) assumes that managers decide whether to pursue an investment or not at time zero based on the information and the best forecast available at the moment. However, in practice, investment in other firms is a dynamic process. The acquirer resolves uncertainty through learning after first purchasing a block of shares of a target firm. In this vein, real options (growth options) provide a more realistic framework to value these synergies.

Initiating a strategic alliance can be viewed as exercising the initial option to generate synergies which in turn leads to the option to merge or abandon. Carrefour and Tesco initiated a strategic alliance in 2018; however, the two companies split in 2021 (did not exercise the option to expand) (Čirjevskis 2022). Brazilian Natura Cosméticos S.A. bought a 65% stake in the Aesop brand in 2012. In 2016, Brazilian Natura Cosméticos S.A. completed a full takeover (exercised the option) (Čirjevskis 2023a).

Based on this reasoning, stock market and commodity price uncertainties, real options, and the value of collaborative synergies can be linked within a theoretical framework, which is represented in Figure 1.

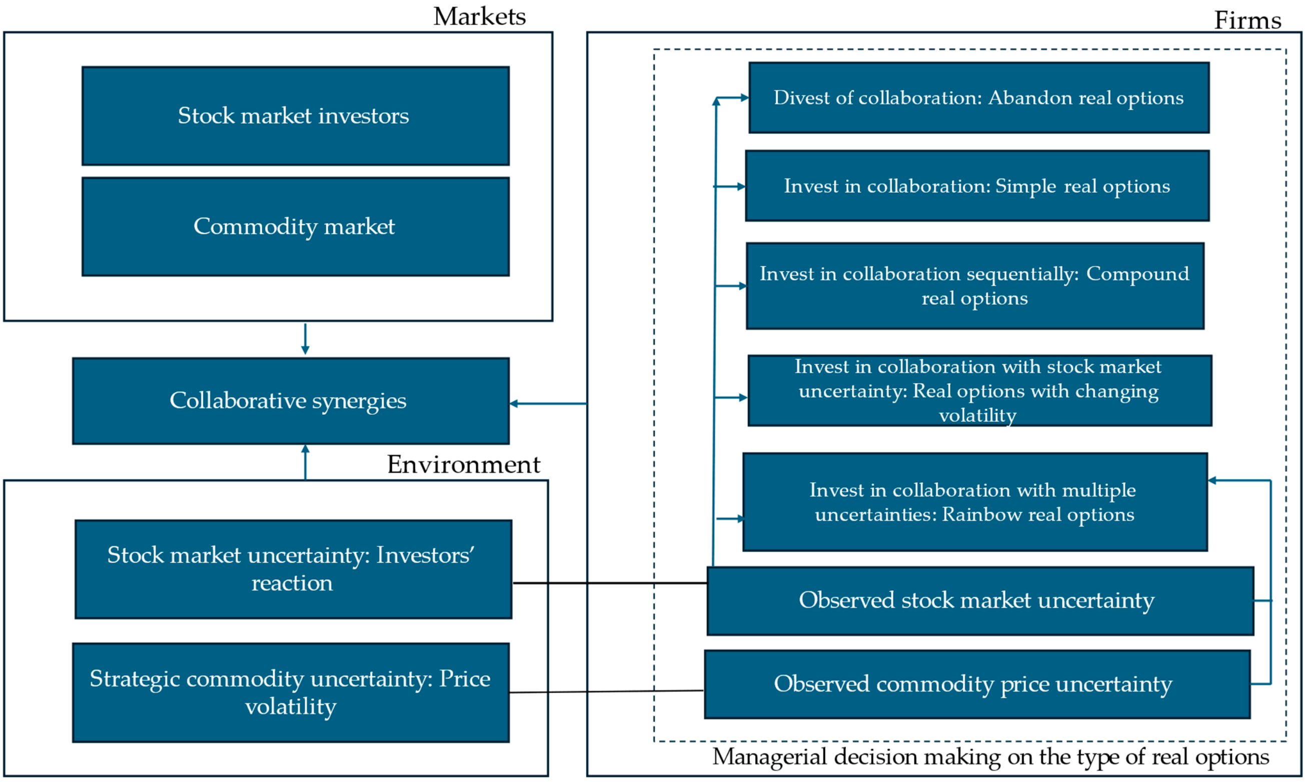

Figure 1. Theoretical framework of the research: environmental uncertainty, investment decisions, real options, and collaborative synergies. Source: Adapted from Orlani and Sobrero (2008) and extended by the author.

Thus, other things being equal:

Proposition 1. *Patent-based synergies are, in fact, network and relational synergies in high-tech M&As and they can be measured with rainbow real options application when an acquirer experiences two uncertainties, for instance, its stock prices' volatility and price volatility of their strategic feedstock needed for its production.*

In the next chapter, the paper will move from this framework to build an overall research model.

3. Methods

"Qualitative case study methodology enables researchers to conduct an in-depth exploration of intricate phenomena within some specific context" (Rashid et al. 2019, p. 1). The research phenomena are patient-based collaborative synergies in the context of high-tech M&A. Seawright and Gerring (2008) argued that case selection in case study research desires (a) a representative sample and (b) useful variation in the dimensions of theoretical interest. When it comes to sampling, according to Eisenhardt and Graebner (2007), it is appropriate to use a single case if a phenomenon-driven research question "how" is subject to investigation. Ultimately, each case can be viewed as a discrete experiment that could be repeated (Yin 2009). Regarding research investigating a single case, Siggelkow (2007, p. 20) notes that it "can be a very powerful example".

Because the main theoretical interest of the current paper is a valuation of a collaborative in patent-based acquisition and its valuation, Amazon as a high-tech giant with its ecosystem throughout the world is an appropriate object of current research. When it comes to a unit of research, namely, patent-based synergies, Amazon introduced unmanned vehicles in its logistics system, and this is why it bought ZOOX—an autonomous vehicle company (Hankookandcompany.com 2023) is headquartered in Foster City, California and has offices of operations in the San Francisco Bay Area and Seattle, Washington. This concrete case study is instrumental to answering research questions and obtaining the research aim for to following reasons.

According to the cross-case methods of case selection and analysis (Seawright and Gerring 2008), the case study of Zoox startup can be characterized as a deviant case study that deviates from some cross-case relationships seen in recent acquisitions by Amazon.com like the acquisition of Souq.com in Dubai in 2017, Whole Food in the US in 2017, or the more recent acquisition of One Medical in 2022.

The rationales behind acquiring Dubai-based start-up Souq.com were to enter a new geographic market and to acquire Souq.com's core competencies, capabilities, and logistic system to "navigate a complicated region" (MAGNiTT 2017, p. 1). Thus, consumers in the Middle East can buy Amazon.com products using the Souq.com platform (Banerjee 2021). Regarding the highly strategic and not standard Amazon's acquisition of Whole Foods in 2017, with limited knowledge and experience in the offline retail environment, Amazon needed to acquire more expertise in perishable grocery procurement, more knowledge of the retail market, improve the management of its supply chain for the offline retail store, and continue investing in R&D for the grocery retail business (Čirjevskis 2023b). As recently announced in July 2022, Amazon's decision to acquire One Medical has pursued to strengthen the physical presence of Amazon in the US healthcare market and expand its online pharmacy business as well as diagnostic business. These acquisitions deviate from the existing explanation of patent-based collaborative synergies.

This deviant case study provides insights into why Amazon chose to acquire ZOOX over the many other alternatives available (Columbus 2020; Frąckiewiczin 2023) by obtaining patent-based collaborative synergies. ZOOX establishes the ecosystem that is required to bring a fully autonomous vehicle fleet to the market. Even if this acquisition would have aimed at neutralizing a potential competitor, or buying technology before some other competitor in the autonomous vehicle space would do so, the case study using rainbow real options application in the context of ZOOX and Amazon is a worthwhile illustration to justify the proposed theoretical proposition and answer research questions. In this vein, Amazon acquired or "proprietaried" 154 patent families of ZOOX that are reinforced by the intersection of robotics, machine learning, and Artificial Intelligence (AI) technology development.

The nature of the deviant case study of Amazon's acquisition of ZOOX is exploratory, and the usage of this case study is confirmatory to justify the provided theoretical proposition. Moreover, Seawright and Gerring (2008) argued that one deviant case study can be used as a high-residual case or an outlier.

The primary quantitative research method is simple and advanced real options analyses, which is the most known and widely approved method in the financial management community and is quite new and in demand in strategic management. Recently, Lambrecht argued that "although the academic literature on real options has grown enormously over the past three decades, the adoption of formal real option valuation models by practitioners appears to be lagging" (Lambrecht 2017, p. 166).

The data for real option analyses were collected from V-Lab (the Volatility Laboratory), Finbox (an online platform for securities and trading data for the stock market), Macrotrends (a research platform for long-term investors), and the US Department of Treasury databases, which are all official data sources with verified content also used by other researchers and practitioners. Finally, regarding the number of EV car companies' acquisitions, the author chose a spectacular single acquisition: Amazon's acquisition of ZOOX, the startup that develops driverless innovation named "robo-taxis".

The proposed research model consists of the following analysis steps and is outlined as follows (Figure 2). In the following subsections, a detailed description of each analysis step is provided.

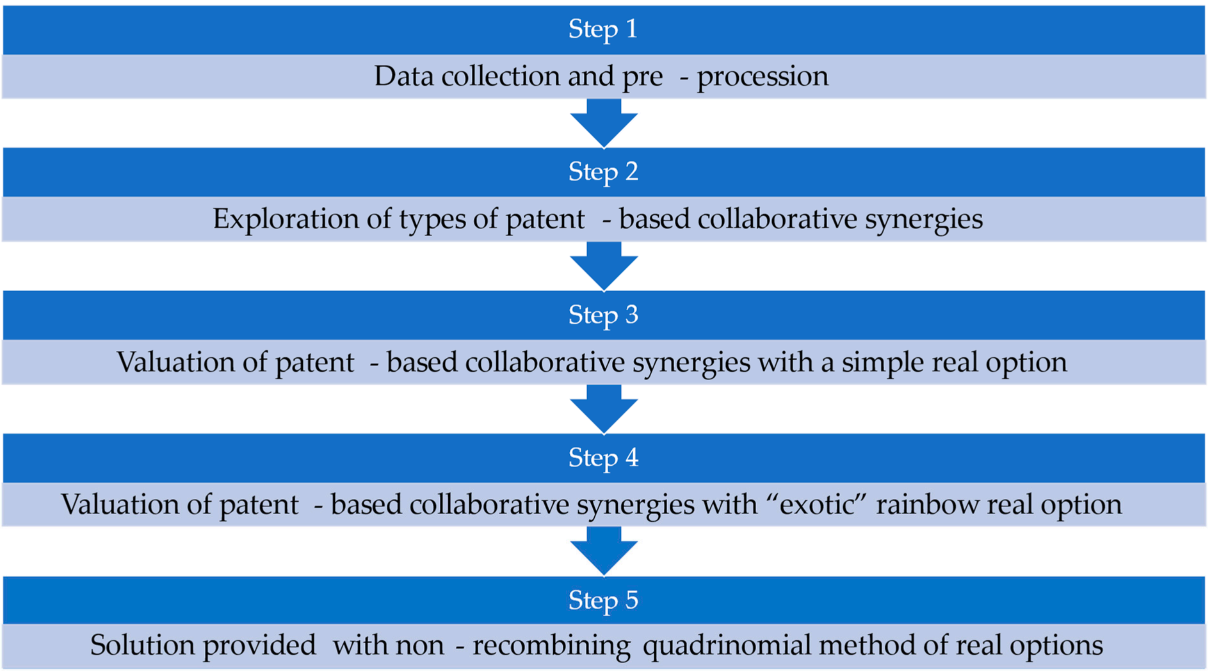

Figure 2. Overall research model. Source: Adopted from Aaldering et al. (2019) and extended by the author.

Having adopted the acknowledged recommendations of Dunis and Klein (2005) on the correspondence of parameters or real options with parameters of financial options when valuing synergy, the parameters of simple and rainbow (two-colors) real options are summarized in Table 1.

The results of the simple and advanced (rainbow) real options analysis to measure collaborative synergies are self-explanatory with a clear quantitative output and delivered in the next chapter.

Next, the deviant case study of Amazon's acquisition of Zoox in 2020 provides the solution to a valuation of collaborative synergies with two sources of uncertainty.

Table 1. The correspondence of parameters of financial options with real options with the parameters of simple and rainbow (two-colors) real options.

Financial Options Variables	Real Option Variables	Sources of Data
Share price	The cumulated market value of collaborative business partners before the announced deal terms, excluding the week of an announcement (four-week average).	CompaniesMarketcap.com (accessed on 10 August 2023) Forbes.com (accessed on 27 August 2023)
Strike price	The hypothetical future market value of the separated entities forecast by the DCF or EV-based multiples.	Finbox.com (accessed on 10 August 2023) Macrotrends.net (accessed on 10 August 2023 Finerva.com (accessed 10 August 2023)
Standard deviation	The annualized standard deviation of stock within one week after the announcement.	https://vlab.stern.nyu.edu/docs/volatility/GARCH (accessed on 10 August 2023), own calculation
Risk-free rate	Domestic three-month rate with the leading collaborated partner.	US Department of the Treasury, https://home.treasury.gov (accessed on 10 August 2023)
Time to maturity	One year or based on the expectation of the management to obtain collaborative synergy.	The synergy life cycle.

Source: Adopted from Dunis and Klein (2005, p. 8) and extended by the author.

4. Case Study: Amazon's Acquisition of ZOOX—Data Analysis and Interpretation

Zoox started in 2014 with the vision of purpose-built, zero-emission vehicles designed for autonomous ride-hailing, along with an end-to-end autonomy software stack. "Zoox is working to imagine, invent, and design a world-class autonomous ride-hailing experience", said Jeff Wilke, Amazon's CEO. Amazon has signed an agreement to acquire Zoox in 2020, a California-based company working to design autonomous ride-hailing vehicles from the ground up (Amazon.com 2020, p. 1)

The COVID-19 pandemic influenced Zoox's financial standing, necessitating significant cost-cutting measures and selling the business (Stone 2020). Early in March 2020, Zoox said that it would cease testing its cars on public roads in San Francisco and Las Vegas as localities started to implement shelter-in-place measures in response to the COVID-19 outbreak (Porter 2020). As the COVID-19 outbreak "threw some curveballs" at the startup in 2020, Zoox Chief Technology Officer and co-founder Jesse Levinson told Business Insider that the company had been discussing with Amazon on and off for a few years (Matousek 2020).

According to VentureBeat (2020), by leveraging the technological capabilities and patents of Zoox, Amazon, which holds more than 210 transportation-related patents, including a 2017 patent to provide on-demand transportation services through a network of autonomous vehicles, could convert ZOOX's driverless innovation named "robo-taxis" into automated delivery vans further down the road (VentureBeat 2020). An exploration of the antecedents of network and relational patent-based synergies generated by Amazon's acquisition of Zoox is given in Table 2.

In this sense, Table 2 provides an answer to the first research question: What types of synergies can a high-tech acquirer expect by proprietarizing technological patents as well as a target firm's network, platform, and ecosystem? First, a relational synergy was generated based on the partner's patents and technological competencies. Indeed, the acquisition encompassed not only its patents but also its talented employees and their technological expertise. Second, a network synergy was created based on a target's platform, network, and ecosystem.

Table 2. Exploration of antecedents of patent-based (relational and network) synergies: Amazon's acquisition of Zoox: Types, definitions, analyses, and results.

Patent-Based Synergies	Definition of Synergies	Antecedents of Synergies: Amazon's Acquisition of Zoox	Duration of Synergy Gains	Timing of Initial Synergy Realization	Results
Relational synergies	Synergies are based on partners' patents and technological competencies.	Zoox's patent portfolio is particularly strong in the areas of driving assistance, signaling, and cruise control, three areas Amazon needed to strengthen along with reinforcement-based machine learning techniques to support autonomous vehicles (Columbus 2020).	Medium, requiring continued investment	Medium to long	Autonomous ride-hailing services and logistics are two markets Amazon looks to expand into quickly with Zoox's expertise. (Columbus 2020). By acquiring Zoox, Amazon will significantly strengthen its position in autonomous driving technology patents obtaining patents-based relational synergies.
Network synergies	Synergies are based on acquiring a target's platform, network, and ecosystem.	Zoox has 154 patent families, with many of them reflecting the startup's expertise with reinforcement machine learning (Columbus 2020) and the development of its EVs platform.	Short	Immediate	As a result of network synergies because of the acquisition, Amazon's competitive impact on autonomous vehicles nearly tripled (Columbus 2020).

Source: adapted from Feldman and Hernandez (2022), Columbus (2020, p. 1), and Čirjevskis (2021a) and extended by the author.

Tarsalewska (2021) argues that acquisitions of firms with patents can pose significant risks to the acquirer's firm. In this vein, developing and navigating the autonomous vehicles' ecosystem is a challenging deal, particularly due to uncertain and unpredictable external factors (Kasly and Bhagyalakshmi 2020), among which are the price volatility of the acquirer and uncertainty and price volatility of strategically important exchange commodities such as nickel. Thus, this is a call for rainbow (two-color) real options valuation.

Now, the solution to a rainbow option problem requires a quadrinomial method because of multiple sources of uncertainty (stock volatility of an acquirer −38% and nickel commodity price volatility −25%), and the calculations are cumbersome compared to a recombining binomial tree. In the case of Zoox's acquisition by Amazon, a two-year option life cycle was used to represent the duration for obtaining synergies as an option to keep the illustration simple due to the dramatic escalation of the volatility of the nickel commodity market after Russia's invasion of Ukraine on 24 February 2022, which reached 35.8% (V-Lab 2023a).

The collaborative synergies payoff is influenced by two types of uncertainty: market demand for the strategically important commodity (nickel), the annual volatility of which is estimated to be 25% (Sharypin et al. 2020), and stock volatility of Amazon within the first week after the announcement of the acquisition of Zoox. What is the value of the synergies that collaborative partners expect to obtain at the end of the option life of two years, assuming a continuous annual risk-free rate of 0.14%?

The real options valuation parameters are provided in Table 3 below.

The application of the Black–Scholes option pricing model with single volatility (stock volatility of an acquirer) provides the result of synergism valuation in Table 4. To measure a simple call option premium (a value of collaborative synergy), the Black–Sholes model is usually applied with the following Formula (1) (Black and Scholes 1973):

$$C = S\,N(d_1) - K\,e^{-rT}\,N(d_2) \quad (1)$$

where S is the cumulated market values of Amazon and Zoox as the separated entities (four-week average) before the announcement of the acquisition; K is a strike price or the sum of hypothetical future market values of companies as separated entities; T is the exercise date or duration of obtaining synergies; r is the continuously compounded risk-free interest rate (% p.a.); σ is the volatility. Regarding real options reasoning, N (d_1) is the factor by which the combined values of the collaborating firms exceed the current values of these firms without collaboration. The value of firms without collaboration is discounted by the factor of risk-adjusted probability N (d_2). The result has evidenced that collaborative synergies equal USD 404.1 bn, as shown in Table 4.

Table 3. The parameters of real options pricing model's input variables: Amazon.com's acquisition of Zoox.com.

Parameters of Financial Options	The Parameters of Simple and Rainbow (Two-Colors) Real Options, Data
Stock price (So)	The cumulated market values of Amazon and Zoox as the separated entities (four-week average) before the announcement of the acquisition. Amazon's market capitalization on 29 May 2023 was USD 1220B, and on 26 June 2023, it was 1340B, thus the average market value of Amazon was USD 1280B (CompaniesMarketcap.com 2023). According to the Financial Times, Amazon paid USD 1.2B for Zoox (Columbus 2020). Thus, the stock price was USD 1281.2 bn.
The strike price (K)	The hypothetical future market value of Amazon as a separate entity is forecast by the EV/EBITDA multiples. The ev/EBITDA multiple for Amazon.com in the twelve months of 2019 was 27.7×, (Finbox 2023). EBITDA in 2019 was USD 36.330B (Macrotrends 2023); therefore, Amazon's EV was USD 1006.34 bn. The hypothetical future market value of the separate entities is forecast by the EV/EBITDA multiples for Amazon and EV/Revenues for ZOOX in 2020. The median revenue multiple for self-driving and smart vehicle companies in the second quarter of 2020 was 2.0× (Finerva 2022). The annual revenue of Zoox varies between USD 100 M and USD 500 M. Thus, the hypothetical future market value of Zoox equals USD 500 M × 2.0 = 1.000 M or 1.0 bn. Thereby, the sum of hypothetical future market values as separated entities (strike price) was USD 1006.34 bn plus USD 1.0 bn equals USD 1007.34 bn.
Stock volatility of Amazon within the first week after the announcement of the acquisition of Zoox (σ1)	Amazon's historical volatilities within the first week after the announcement of the partial acquisition of Zoox on 26 July 2020 was 38% (V-Lab 2023b).
Price volatility of a strategically important commodity for self-driving car production—the nickel spot (σ2)	The volatility of nickel price has remained at 25%, and this unprecedented turbulence has continued to reshape the nickel market throughout 2020 (Sharypin et al. 2020, p. 2).
Risk-free rate (r1)	Domestic US three-month rate in 2020 was 0.14% on 26 June 2020 (US Department of the Treasury 2023).
Time to maturity (T)	Duration (T) was the period from 2020 to 2020 (two years) when the volatility of nickel spiked upward by 35.8% (V-Lab 2023b), resulting in volatilities of Nickel of 35.8% in 2023 due to the Russian invasion of Ukraine.
Time increment (δt)	One-year time intervals for two years to account for the change in the up and down factors of the quadruple lattice-based real options method.

Source: Developed by the author.

To answer the second research question—how to value patent-based synergies when an acquirer experiences several uncertainties at the time of proprietarizing technological patents of a target firm—the application of the rainbow real options is employed. The input parameters for rainbow real options have been identified in Table 1, which are used to calculate the quadrinomial non-recombining lattice parameters, as shown in Table 5.

The quadrinomial tree was built by employing one-year time intervals for two years of the options life cycle and calculated as the joint market values of collaborative partners at each node of the tree. The lattice in this case study is a quadrinomial instead of a binomial (e.g., Čirjevskis 2022) because the two volatility factors yield two up and down factors, as given in Table 3. At any given node of the tree, therefore, there are four possible asset values in the next period (Kodukula and Papudesu 2006, p. 164). The factors used to determine these values are u1u2, uld2, dlu2, and dld2. Having started with the node "A", the four up and down factors were calculated separately to obtain "B"u1u2, "C"u1d2, "D"d1u2, and "E"d1d2 for the first step, according to Kodukula and Papudesu's (2006) recommendations.

Table 4. Real options valuation with Black–Scholes option pricing model (BSOPM): Amazon's acquisition of Zoox.

Simple Real Options Valuation with the Black–Scholes Option Pricing Model (BSOPM)	
Stock price (So): Cumulative average capitalization of Amazon and Zoox before announcement on 29th of May till 26th of June 2020 (in USD bn)	1281.20
Strike price (K): Cumulative theoretical market value of two separated entities without merger after one year in 2021 (in USD bn)	1007.34
The risk-free rate of return (Rf) on 26 June 2020 in the USA (%)	0.14
Assumption on the time to expiration (T) (in years)	2.0
The volatility of the stock price of Amazon (σ) within one week after the announcement of a deal during the period from 26th of June to 3rd of July of 2020 (%)	38.0
d_1 = [(ln (So/X) + risk-free rate + variance/2) × T]/[(square root of variance) × (square root of T)]	0.721
$d_2 = d_1 -$ (square root of variance) × (square root of T)	0.184
Value of the call option (C) = Synergies (USD bn)	404.1

Source: Developed by the author by adopting Dunis and Klein's (2005) recommendations.

Table 5. Rainbow real options parameters: Amazon's acquisition of Zoox.

Rainbow Real Options Parameters and Data	
Time increment: ΔT (years)	1.00
First Up factor: $u_1 = e^{\sigma 1 \sqrt{\Delta T}}$	1.462
First Down factor: $d_1 = \frac{1}{u_1}$	0.684
First risk-neutral probability: $p_1 = \frac{e^{r\Delta T} - d_1}{u_1 - d_1}$	0.408
Second Up factor: $u_2 = e^{\sigma 2 \sqrt{\Delta T}}$	1.284
Second Down factor: $d_2 = \frac{1}{u_2}$	0.779
Second risk-neutral probability: $p_2 = \frac{e^{r\Delta T} - d_2}{u_2 - d_2}$	0.441

Where: p_1 and p_2 are risk-neutral probabilities; u_1 and u_2 are values of up nodes; d_1 and d_2 are values of down nodes; e = mathematical constant of exponential function; rf = risk-free rate; ΔT = stepping time. Source: Developed by the author.

Therefore, to continue in a similar mode for every node of the quadrinomial tree in the next step, the top values at each node were calculated using the Excel Microsoft program, as shown in Figure 2. Then, the option values (synergies) across the quadrinomial tree were estimated by pursuing backward induction. The option values are shown as bottom italicized numbers and calculated with the application of two RNPs as the discounted at-risk-free rates with a weighted average of the potential future option value using risk-neutral probabilities. For instance, the value of node "B" is $((p_1 \times p_2 \times F + p_1 \times (1 - p_2) \times G + p_2 \times (1 - p_1) \times H + (1 - p_1) \times (1 - p_2) \times I)) \times \exp(-r\delta t)$ (Kodukula and Papudesu 2006).

Therefore, the theoretical proposition has been justified in Figure 3. The valuation of patent-based collaborative synergies (relational and network) by rainbow (two-colors) real options explains to what extent (in terms of market value added) the acquiring platform company would capture the full value of the exercised option by fully integrating the target firm.

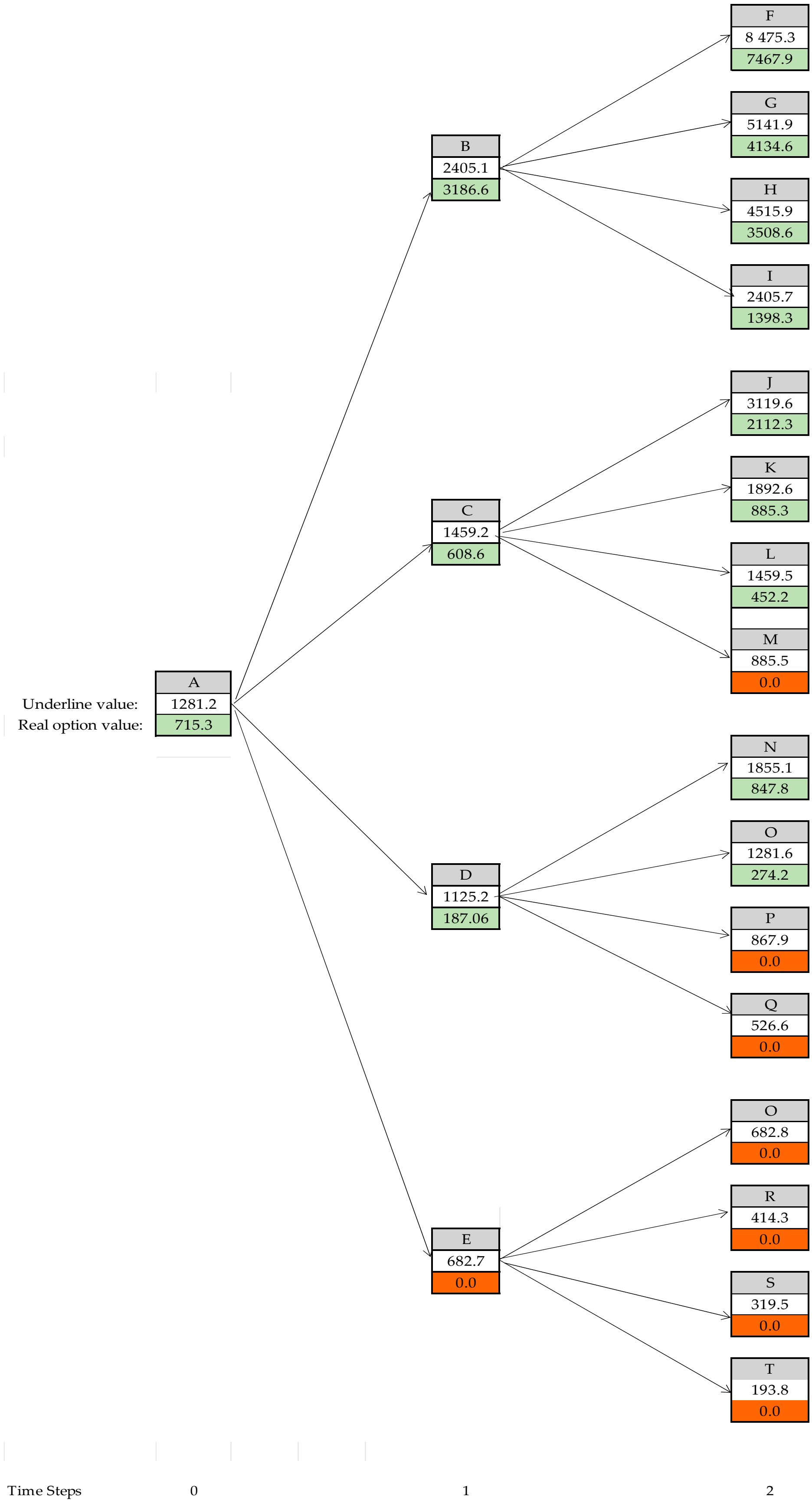

Figure 3. Valuing patent-based (relational and network) collaborative synergies with rainbow (two colors) real options: Amazon's acquisition of Zoox in 2020.

5. Discussion

Alvarez et al. (2023) have called for research that enhances our understanding of property rights in business models, platforms, and ecosystems. This paper contributes to these scientific requests and provides empirical evidence on the measurement of patent-based collaborative synergies by rainbow real options with a quadrinomial tree application. This occurs when a high-tech acquirer experiences two uncertainties at the time of an acquisition of a target with a portfolio of patents; specifically, the volatility of stock prices and the price volatility of the strategic feedstock are needed for future production.

This paper provides an answer to the first research question: what types of synergies expect a high-tech acquirer by proprietarizing technological patents as well as a target firm's network, platform, and ecosystem? Two types of patent-based synergies can be generated when the acquirer is "proprietarizing" the patents on advanced innovation of a target. The first is the relational synergy that is based on the target's patents and technological capabilities. Indeed, by acquiring Zoox, Amazon has significantly strengthened its position in autonomous driving technology patents, thereby obtaining patents-based relational synergies. The second is a network synergy that is based on a target's platform, network, and ecosystem. Columbus argued that because of Zoox's acquisition, Amazon's competitive impact on autonomous vehicles nearly tripled (Columbus 2020) due to network synergies.

When it comes to the second research question—how to value patent-based synergies when an acquirer experiences several uncertainties at the time of proprietarizing technological patents of a target firm—the application of the rainbow real options is employed to provide an answer. Because of multiple sources of uncertainty, a rainbow option was required, and the non-recombining quadrinomial method was employed following the acknowledged recommendations by Kodukula and Papudesu (2006, p. 167). In the case study of the Zoox acquisition by Amazon, the duration of obtaining synergies as an option life cycle of two years and two volatilities were used to keep the illustration simple. The results indicate that the longer the duration, the higher the value of the option.

Moreover, more volatility (color) can be added to the rainbow real options. For instance, experts anticipate long-term volatility in the lithium market and lithium is a crucial component in electric vehicles (EVs). China's majority control of the lithium commodity as well as the imposition of sanctions on major lithium suppliers from Russia because of the Ukraine invasion are instrumental reasons for uncertainty (volatility) (Darabshaw 2023, p. 1). In this sense, more than two uncertainties magnify the value of an option, but, at the same time, the valuation becomes even more complex due to the growing quadrinomial tree and,, therefore it goes beyond the current research. Regarding the calculation technique, although the calculations are complex with this approach, the underlying theme in computing the option value remains the same as with the binomial method.

What is more, research shows that the standard Black–Scholes equation cannot accommodate the multiple sources of uncertainty and is not useful for rainbow options. The result of a simple real option (one color) with an application of BSOPM result (USD 404.1 bn) is lower than the result of two-color rainbow real options (USD 715.3 bn) with an application quadrinomial method.

Therefore, the deviant case study of the acquisition of ZOOX company by Amazon.com justifies the proposition of the paper: patent-based collaborative synergies in high-tech M&As are network and relational synergies; when an acquirer experiences multiple uncertainties, those synergies can be measured by rainbow real options with quadrinomial tree application.

Indeed, the more volatilities that are added in the process of the valuation, the higher the value of the real option would be. Having generalized research results, this paper argues that when an acquirer experiences multiple uncertainties at the time of acquisition, collaborative synergies can be valued with an application of multicolor rainbow real options.

The rainbow option result is USD 715.3 bn; therefore, it can be forecasted that the theoretical market value of Amazon and Zoox after two years of cooperation can be USD

1007.34 bn plus USD 715.3 bn, which is equal to USD 1722.3 bn. Having explored the real market capitalization of Amazon on 30 November 2021 (before the Russian invasion of Ukraine in February 2022), the result shows that Amazon fully realized collaborative synergies by acquiring Zoox in June 2020, and the market capitalization of Amazon was 1780 B on 30 November of 2021 (CompaniesMarketcap.com 2023).

However, it should be noted that Amazon is a large company with several divisions and factors affecting its stock price; thus, the assumptions that link Amazon stock prices in November 2021 with the acquisition of ZOOX in 2020 may need further investigation. Platform companies have many divisions with concurrent strategies. Attributing outcomes two years down the road to a single decision within a platform does not acknowledge the complexity of platform companies and the ecosystems they operate. This will be the case for any platform company, making it difficult to attribute value to one single strategic decision by comparing valuation results with real market capitalization. Nevertheless, a rainbow real options approach allows for better ex-ante valuation, as it captures more sources of uncertainty compared to DCF valuation methods and the Black–Scholes option pricing model.

6. Conclusions, Contributions, Limitations, and Future Work

In this paper, the author brings up the notion that patent-based synergies are, in fact, network and relational synergies in high-tech M&As and they can be measured with rainbow real options with quadrinomial tree application when an acquirer experiences two market uncertainties, for instance, its stock prices' volatility and the price volatility of their strategic feedstock needed for its production.

6.1. The Theoretical Contribution

Having conducted the case study of the ZOOX start-up's acquisition by Amazon.com in 2020, this paper contributes to this scientific discourse and explores the rainbow real options application for the valuation of network and relational synergies as a market value added in patent-based acquisitions of the EVs platform industry and its impact on Amazon's development of global transportation logistics network. At the intersection of the financial management and strategic management disciplines, this paper shows that the sum of values of different types of synergies in M&A deals is the market value added of M&A deals, which can be valued with real options application.

Garcia-Castro and Aguilera (2015) argued that value is a core concept in strategic management research. "There exists a great interest in the strategic management field to advance in the study of the dynamics of value creation..." (Garcia-Castro and Aguilera 2015, p. 145). What is more, Lieberman et al. (2018) argued that 'Value creation' is central to strategy ... and achieving economic gain is a necessary condition to increasing competitive advantage" (Lieberman et al. 2018, pp. 1546–50).

Lieberman et al. (2018) are convinced that "measures such as accounting profits, economic value added (EVA) are static in that they focus on the value created in a single period" and argued that "economic gain can arise through innovation or when a superior firm displaces competitors". Lieberman et al. (2018) also argued that firms create value not only for shareholders, but also for other stakeholders, including employees, customers, and suppliers. However, the aspect of patent-based acquisition as a value (synergies) creation strategy was missed in their research.

This research contributes to the current discussion in three ways. First, the current paper contributed to this discussion by providing the practically applicable rainbow real options valuation approach that allows a broader notion of value creation and appropriation that can be assessed and quantified in many M&A cases even beyond the ICT industry.

Second, the main scope of the paper is not only to support the application of real options with multiple uncertainties (rainbow options) in the context of patent acquisitions but also to generalize the research results on real options valuation application. In this vein, the paper contributes to Lieberman et al.'s "Static and Dynamic Measures of Value Creation by a Firm" framework (Lieberman et al. 2018), where a real options analysis (ROA) can be added to the quadrant with the ordinate 'Dynamic' Measures (change between periods) and the abscissa is "Total Economic Value", as shown in Figure 4.

	Shareholder value	**Total economic value**
«Static» measures (within a given time period)	*Current period shareholders' return:* • **Economic Value Added (EVA)** • **Return on Equity (ROE,)** • **Earning per Share (EPS)** *Current and anticipated shareholders' return:* • **Total Shareholders'Return (TSR)** • **Market capitalisation** • **Tobin Q**	*Current period return to «all» shareholders:* • **Total surplus** **= Profit + Consumer surplus** **= Williness to pay – Opportunities cost** **But here we do not**
«Dynamic» measures (change between time period)	• **Change in ROE, EVA, TSR** • **Change in market capitalisation** • **Change in Tobin Q** **Here we have good measures**	**This quadrant we tipically don't even think about** • **Economic gain: Real option analyses (ROA): Abandon, simple, compound, with changing volatility, with multiple volatilities.**

Figure 4. Dynamic measures of economic value creation by ROA. Source: Adopted from (Lieberman et al. 2018, p. 1548; Lieberman et al. 2020) and extended by the author.

Therefore, real option valuation explains to what extent the acquiring company would dynamically capture the full value of the exercised option by fully integrating the target firm's innovation.

6.2. The Managerial and Policy Implications

The current paper also provides insights into the sensitivities of real option values by contrasting synergies valuation with simple (one uncertainty) and advanced rainbow (two uncertainty) real options. Kodukula and Papudesu argued that, typically, practitioners simplify their business life by aggregating uncertainties that drive the asset value and employ one combined volatility for the valuation (Kodukula and Papudesu 2006, p. 167). Monto Carlo simulation is one the most common methods for this (Kodukula and Papudesu 2006; Čirjevskis 2021b). Thus, on the empirical side of the research, the paper enhances practitioners' acceptance of advanced real options application as the extra valuation technique under multiple uncertainties, bridging the gap between theory and practice, and thereby contributing to the awareness and application of ROT.

From the perspectives of regulators, it is critical to create approaches that can clarify how patent-based M&A transactions and different aspects of the patent system impact technological platforms and ecosystems of merging partners. Regarding the role of public policy in supporting and enabling patent-based M&A transactions, many unanswered concerns remain. What roles, if any, should governments play in such types of transactions that open nascent businesses such as the automated delivery of electric vans driverless industry? The answers to these questions require further investigation (OECD 2004; Anton et al. 2006).

6.3. Limitations and Future Work

When it comes to limitations, the two sources of volatility (the price of the raw material and the value of underlining assets) might be related; however, it was assumed that the correlation between the acquirer's stock prices and strategic commodity prices is insignificant for illustration. In fact, in high market uncertainty, different sources of uncertainty may not be completely independent. In such circumstances, an analyst can analyze the uncertainties separately which would provide deeper insight into what type of uncertainty has the strongest impact on the value of the option (Kodukula and Papudesu 2006). Despite these limitations, the proposed rainbow real options valuation of collaborative synergies offers greater precision to "the fundamental question of how value is created" (Garcia-Castro and Aguilera 2015, p. 146)

Further, there are assumptions about projecting future market values of partners without collaboration based on EBITDA multiples. Peeyush and Werner (2023) argue that multiples can be misleading when industries face significant one-off events or when a company's business mix is in a transitioning phase (Peeyush and Werner 2023, p. 7). Accordingly, it was assumed that both limitations are absent in the case study. What is more, Bianconi and Tan employed the ratio of EV/EBITDA as the fair metric for firm value, when they explored the impact of mergers and acquisitions on firm values with a large sample of 65,521 M&A deals globally from the Communications, Technology, Energy, and Utilities sectors, including Facebook, Google, Microsoft Corporation, and other giants of the high-tech sector (Bianconi and Tan 2019, p. 73).

The author hopes that this research will help to connect M&A studies with other current streams of financial management and strategic management research and encourage future research that better accounts for the implications of M&A for the high-tech sector. The effects of network and relational synergies in patent-based acquisitions may be different in various institutional and international contexts.

Even though the paper investigates the applicability of advanced rainbow real options with two uncertainties in the EV business, using multi-factor options with three or more uncertainties can provide a more accurate valuation result for industry settings beyond the current focus. To enhance generalizability, rainbow real options valuation can be applicable for industries with different technology landscapes that are affected by three types of uncertainties: market (demand and price uncertainty), technical (uncertainty regarding the performance of new technology), and technological uncertainty (uncertainty about more efficient competing technology that arrives in the future) (Azevedo and Paxson 2007).

Among such industries are the pharmaceutical industry (Mohanty 2022), Civil engineering (Siripongvakin and Athigakunagorn 2020), the petrochemical industry (Dockendorf 2010), and other industries with multi-factor uncertainties. Therefore, future studies may also test the provided theoretical proposition by conducting comparative studies across different platform industries and different ecosystems.

Funding: This research received no external funding.

Data Availability Statement: Publicly available datasets were analyzed in this study. These data can be found in the reference list.

Conflicts of Interest: The author declares no conflict of interest.

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