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Transmission of Inflation and Exchange Rate Effects: The Markov Switching Vector Autoregressive Methodology

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Abstract: The aim of this study is to delve into the intricate the mechanism through which alterations in currency exchange rates give rise to shifts in inflation rates, while taking into careful consideration the country's economic cycle. In order to accomplish this objective, we used a dataset that spanned from 1 January 1999 to 1 July 2023, focusing our analytical lens on three specific geographic areas, namely the Eurozone, the United Kingdom, and Canada. In our pursuit of understanding this complex relationship, we employed the Markov Switching Vector Autoregressive model. Our research outcomes can be succinctly encapsulated as follows: in the initial stages, particularly during phases characterized by robust economic growth, the transmission of exchange rate effects onto inflation levels appeared to exhibit a partial impact across all geographic areas under examination. However, during periods marked by economic downturns, both the United Kingdom and Canada displayed a distinctly more comprehensive transmission of these effects. Moreover, the prevailing projections for the forthcoming time horizon, across all the countries encompassed by our study, strongly indicate the onset of an expansionary phase that is projected to extend over a span of 25 months. Lastly, concerning the implications of unexpected disturbances or shocks, it is noteworthy that the response of exchange rates to inflation induced shocks was neither immediate nor as pronounced as the corresponding reaction of inflation to sudden shifts in exchange rates.

Keywords: inflation; exchange rate; exchange rate pass-through; Markov Switching Vector Autoregressive

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1. Introduction

Crafting an effective monetary policy has perpetually captivated the attention of policy makers, economists, and market participants. In essence, short-term economic policy revolves around two primary aims: the mitigation of inflation, entailing upholding price stability at a level conducive to the regular operation of the economy, or the attainment of full employment, often quantified as potential output. The selection of the most fitting monetary policy hinges on the pursued objective. In this context, an array of factors must be carefully considered when establishing the monetary policy framework, with the interest rate taking center preoccupation.

In accordance with findings by Thornton (2014), the level of control that monetary authorities possess over the interest rate is significantly overemphasized. In this research, he underscored the role of money growth in managing price levels. This sentiment was echoed by Végh (2001), who not only demonstrated the potential to achieve low inflation rates through interest rate regulations but also through adhering to monetary growth guidelines.

In addition to overseeing interest rates and money growth, policymakers must react with substantial economic uncertainty when formulating monetary policies. Consequently, comprehending the repercussions of uncertain monetary policies on economic activity and macroeconomic variables becomes paramount. Nonetheless, this issue is intricate and challenging due to the inherent complexities. Uncertainty, being an underlying factor, proves arduous to quantify as an input within empirical models. Furthermore, the correlation between monetary policy uncertainty and the policy itself exists alongside other factors, such as the business cycle, which poses considerable estimation challenges. In the present scenario, matters of monetary policy contribute to consistent discoveries regarding the impacts of monetary policy uncertainty, which are universally recognized. This uncertainty is noted for its adverse influence on economic activity. A study by Creal and Wu (2017) delving into the relationship between monetary policy uncertainty, its transmission mechanisms, and economic volatility revealed a negative correlation between uncertainty and economic activity.

Another factor elucidating the transmission of uncertainty shocks to other nations is the exchange rate. Indeed, it assumes a pivotal role in the economy and exerts its influence through three primary channels. Firstly, in terms of production, currency appreciation tends to suppress inflation. This occurs because the cost of imported goods does not escalate as swiftly as the currency's value, thereby curbing overall price levels. Furthermore, this phenomenon diminishes real gross domestic product due to shifts in expenditure patterns. Secondly, fluctuations in the exchange rate impact investment and consumption, integral components of aggregate demand. The depreciation of the exchange rate diminishes net wealth, consequently leading to reduced consumption. Finally, currency depreciation can amplify the value of external financial obligations. In summary, variations in the exchange rate wield significant effects on prices, investments, and general economic stability, particularly in the context of expanding international trade. Hence, for the effective formulation of monetary policy, comprehending the degree to which the exchange rate influences prices is imperative, a phenomenon known as the exchange rate pass-through.

An extensive and growing body of literature has significantly contributed to enhancing comprehension of exchange rate pass-through, employing diverse methodologies. In fact, a consensus exists that pass-through diminishes along the pricing chain; in simpler terms, a currency increase in one country results in a price level increase in another country, albeit not in direct proportion. Nevertheless, the precise influence of exchange rate variations on inflation stays ambiguous. Numerous studies have acknowledged the intricate character of the exchange rate pass-through phenomenon. Early research attributed the incomplete transmission of exchange rate fluctuations to prices to microeconomic factors. In this regard, Krugman (1987) and Dornbusch (1987) emphasized the role of the pricing to market strategy in elucidating this phenomenon. Others have directed their attention towards macroeconomic factors like the extent of trade openness to expound on pass-through dynamics. However, a more recent line of inquiry, spearheaded by Shambaugh (2008) and Forbes et al. (2018), employs a structural Vector Autoregressive model (SVAR) to demonstrate that the exchange rate pass-through is contingent on the type of exogenous shock driving the exchange rate fluctuations.

In light of this backdrop, the present research aims to elucidate the connectedness amongst exchange rates and inflation from a distinctive perspective. It seeks to explore this relationship while factoring in the phases of an economic cycle. The focus is primarily on discerning the degree of exchange rate pass-through during the expansion phase, characterized by heightened production and inflation, and the recession phase, marked by low economic activity and inflation. To achieve this objective, a Markov Switching Vector Autoregressive model was employed.

In particular, the objective of this study is deeply rooted in a more nuanced understanding of the relationship between exchange rates and inflation, taking into account cyclical fluctuations in the economy. By focusing on expansionary and recessionary phases, the study aims to determine how exchange rates influence inflation in different economic contexts. During expansions, characterized by high output and high inflation, the exchange rate transmission mechanisms may differ from those observed during recessions, when economic activity is weak, and inflation is moderate or even falling. By examining these relationships across these different contexts, the study seeks to capture the nuances of the transmission of exchange rate shocks. To achieve this objective, the study adopted a rigorous methodological approach using a Markov regime-switching vector autoregression model. This model makes it possible to capture transitions between different economic phases and to analyze how the relationship between exchange rates and inflation evolves in these different settings. The underlying ambition of this approach is to provide new and deeper insights into how exchange rate fluctuations affect the economy as a whole, by shedding light on the underlying mechanisms that modulate this complex relationship according to economic conditions. Ultimately, this deeper analysis should contribute to a better understanding of economic and monetary policies in a dynamic and ever-changing global context.

The structure of this document is as follows: It begins with a preliminary section outlining essential study components—such as inflation, the exchange rate, and shocks. This is followed by an initial theoretical section that delves into the exchange rate pass-through phenomenon, encompassing its various facets and the proposed theoretical model, alongside alternative models within the same framework. Subsequently, a practical section is presented wherein the model is applied to the selected countries: the Eurozone, the United Kingdom, and Canada. Finally, the study concludes with summarizing remarks.

2. Literature Review

An extensive body of literature has meticulously examined the pass-through phenomenon, primarily in developed nations but also in developing countries. These analyses have employed diverse methodologies and consistently revealed that the sensitivity to exchange rate fluctuations wanes along the pricing chain. In this context, Jamar and Aimon (2021) affirmed that the transmission of exchange rate movements affects price stability through various channels. This impact is direct, involving import prices for commodities such as oil and consumer goods, as well as indirect, encompassing wage formation, profit markups, foreign direct investment, and more. The pass-through mechanism unfolds across two distinct stages. Initially, exchange rate depreciation leads to an escalation in imported prices, causing imported inputs and finished products to become more costly. This phenomenon is known as the first stage pass-through. Conversely, the indirect effects, which take more time to permeate the economy, manifest through alterations in production costs and real channels. Within the production channel, the depreciation of the exchange rate amplifies production expenses due to the increased cost of imported goods. This, in turn, prompts higher producer prices, subsequently resulting in higher consumer prices. Domestic firms adjust their pricing behavior to maintain constant mark-ups and profits, thereby generating inflation on consumer prices. This sequence is referred to as the second-stage pass-through. In terms of the real channels, exchange rate depreciation elevates import prices in the local currency and reduces export prices denominated in foreign currency. This dynamic stimulates net exports and sustains gross domestic product (GDP) growth. As the real GDP growth gains momentum, it fuels heightened labor demand and increased wages, consequently exerting pressure on consumer prices.

Furthermore, Valogo et al. (2023) asserted that the persistent devaluation of Ghana's currency had sparked concerns about its potential consequences not only for inflation but also for the broader economic landscape. Their study delved deeply into the nuanced effects associated with the threshold concept of exchange rate pass-through on patterns of inflation. Their investigation delved into the critical importance of the exchange rate threshold, particularly within the context of the Taylor principle. To uncover these dynamics, they employed the approach of autoregression as their methodological framework. The outcomes revealed a noteworthy insight: a decline in the exchange rate that surpassed the monthly threshold of 0.70% manifested a conspicuously positive and momentous impact on inflationary trends. This finding strongly substantiated the significance of this particular threshold level in the analysis. The outcomes also extended to their model of the monetary policy rule, where they uncovered a significant and advantageous influence of the exchange rate on the monetary policy rate. This underscored the intricate relationship between exchange rate movements and monetary policy dynamics. In light of these findings, they suggested that incorporating careful considerations of the exchange rate within the broader policy framework can function as a proactive measure against the exchange rate depreciation crossing the optimal threshold. By adopting this approach, the potential inflationary impacts triggered by exchange rate fluctuations can be effectively mitigated. Moreover, Anderl and Caporale (2023) embarked on an in-depth exploration into the intricate dynamics governing the transmission of exchange rates to consumer and import prices. The study focused on a comparative analysis of five countries that adhered to inflation targeting (the United Kingdom, Canada, Australia, New Zealand, and Sweden) and three countries that diverged from this approach (the United States, the Euroarea, and Switzerland). The findings pointed to a more pronounced and intricate influence of exchange rate pass-through on both consumer and import prices within the context of the nonlinear framework. In certain instances, this influence even appeared to approach full pass-through. Importantly, this effect was particularly potent during the second regime, characterized by high levels of inflation expectations within both market participants and consumers. This significant insight underscored the potential of anchoring inflation expectations as a strategy for mitigating the impacts of exchange rate pass-through. In addition, the study revealed an intriguing trend: the connection between inflation expectations and exchange rate pass-through seemed to hold greater weight in countries that had adopted measures centered around inflation targeting which underscores the potential benefits of explicit inflation targeting practices.

Adding to that, Mirza et al. (2023) explored the worldwide surge in global energy costs, which poses a significant obstacle to central banks across the globe in their pursuit of achieving stable price levels. In an effort to tackle this challenge, they assessed the viability of sustaining price stability through a focus on inflation rates, particularly when confronted with the influence of exchange rate dynamics and energy price shocks. They employed the non-linear autoregressive distributive lag (NARDL) model. The results validated that currency devaluation led to a sustained increase in domestic prices, whereas currency appreciation contributed to their alleviation. Moreover, the research established that elevated energy expenses led to heightened inflationary pressures within economies that emphasize controlling inflation.

Regarding this aspect, a study conducted by Chroufa and Chtourou (2023) engaged in a thorough exploration of the intricate and asymmetric interplay between currency exchange rates and inflation in Tunisia. They used the multi-threshold non-linear autoregressive distributed lag model (MTNARDL). Additionally, a quantile Granger causality analysis was employed to delve into the nuanced causal relationships between fluctuations in exchange rates and variations in inflation. The empirical findings unveiled a multifaceted influence of currency exchange rates on inflation, which exhibited distinct characteristics over both extended and shorter timeframes. Notably, the estimated coefficients of exchange rate changes displayed significant variations across different quantiles. They affirmed that this indicated that alterations in inflation respond in varying ways depending on the magnitude of fluctuations observed in the currency exchange rate. Moreover, the examination of causality across different quantiles offered compelling insights. It revealed a unidirectional causal link from shifts in inflation to movements in the currency exchange rate. This confirmation of a reciprocal effect at higher quantiles underscored the intricate dynamics at play, where changes in inflation can exert a notable influence on the behavior of currency exchange rates. In the work by Gereziher and Nuru (2023), the objective was to explore the imbalanced repercussions stemming from shifts in the exchange rate, impacting inflation within the context of a small open economy, specifically South Africa. The methodology adopted encompassed a threshold Vector Autoregressive model, which accommodated the alteration of parameters based on whether a designated threshold variable surpassed an established threshold value. The determination of the threshold value was intrinsically governed by the Hansen (1996) test. To scrutinize the implications of exchange rate shocks on inflation contingent upon their magnitude, direction, and synchronization with the inflation cycle, the authors made use of generalized impulse responses, as introduced by

Koop et al. (1996). Additionally, they employed a Cholesky decomposition identification scheme within the framework of the non-linear model to identify and characterize exchange rate shocks. The obtained findings underlined the existence of a nonlinear influence attributed to exchange rate shocks on inflation. Specifically, the enduring consequences of exchange rate shocks, whether positive (appreciation) or negative (depreciation), spanning one or two standard deviations, manifest as relatively minor over the long term. However, these effects were somewhat more pronounced within the sphere of elevated inflation compared to instances of low inflation.

In the context of the Southern African Development Community (SADC), Olamide et al. (2022) affirmed that the dynamics among exchange rate volatility, inflation, and economic growth have sustained their prominence in economic discourse due to historical background and the close grouping of economies within the region. Nevertheless, a significant knowledge gap exists concerning the intricate interplay, synergy, or interchangeability of exchange rate instability and inflation's impacts on the economic growth trajectory within SADC nations. To achieve their objective, Pooled Mean Group (PMG), Generalised Moments (GM), Dynamic Fixed Effect (DFE), and the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model were employed. The research outcomes brought to light that a negative correlation exists between exchange rate instability, inflation, and the economic growth of the region. Moreover, the results affirmed that heightened levels of exchange rate instability amplified the detrimental relationship between inflation and growth in the region. This corroborated that as inflation rates escalated, the swifter the impact of exchange rate pass-through manifested. Based on these findings, it was strongly advocated that member nations prioritize policies geared towards fostering the appreciation of local currencies to better address these issues. Also, El Aboudi et al. (2023) delved into the intricate relationship between exchange rate and inflation metrics and their ramifications on overall macroeconomic performance. The objective revolved around assessing the repercussions stemming from the interplay between the exchange rate and inflation on the trajectory of economic growth within the context of Morocco. To achieve it, they adopted an autoregressive distributed lag model (ARDL). The conclusions confirmed that in the short term, the interaction between the exchange rate and inflation, commonly referred to as the pass-through effect, exerts a notable and significant influence on the trajectory of economic growth, particularly when considering the lagged impact in the realm of first differences. However, the long-term perspective presents a distinct pattern, suggesting that sustainable economic growth might not necessarily hinge upon the confluence of both exchange rate dynamics and inflationary forces.

Similarly, Citci and Kaya (2023) examined the relationship between the volatility of exchange rates and the presence of inflationary pressures. They introduced a risk premium that was passed on to consumers through amplified price levels. This pricing strategy established a conduit through which fluctuations in exchange rate uncertainty reverberate throughout the broader realm of price dynamics. The empirical estimations showed that the degree of uncertainty associated with exchange rates exerted a statistically significant and positive impact on inflationary tendencies, but this effect was characterized by nonlinearity. As the level of exchange rate uncertainty escalated, the magnitude of its influence on inflation gradually diminished. Furthermore, Sari et al. (2023) utilized the Vector Autoregressive (VAR) impulse response model to gauge the extent of. Within the framework of pandemic-induced conditions, this research employed consumer price index (CPI) and the wholesale trade price index (WPI) as the dependent variables, juxtaposed with the exchange rate serving as the independent variable. Results implied that the exchange rate pass-through's impact on both the CPI and WPI exhibited incompleteness, implying that the escalation in the price level or inflation was not as pronounced or directly proportional to fluctuations in the exchange rate.

Moreover, Gereziher and Nuru (2023) conducted a thorough investigation on the asymmetric repercussions of exchange rate shocks on inflation dynamics utilizing a threshold Vector Autoregressive model. The analysis employed generalized impulse responses and Cholesky decomposition to examine the magnitude, direction, and temporal alignment of exchange rate shocks with the inflationary cycle. Results indicated a nuanced, nonlinear impact of exchange rate shocks on inflation, with lasting effects being relatively muted over extended periods, particularly within lower inflationary environments. Notably, Peer and Baig (2023) studied the interplay between the pursuit of inflation targeting (IT) and the transmission of fluctuations in exchange rates onto consumer prices, commonly denoted as exchange rate pass-through (ERPT). The central emphasis resided in extracting invaluable insights to illuminate the strategies of nations that are in the process of adopting IT, coupled with a concerted effort to quantify ERPT in the specific context of India. Results approved the necessity for monetary policy to shift towards future trajectories of inflation, accentuating the significance of harmonizing monetary and fiscal policy actions. The results additionally uncovered a noticeable downtrend in the magnitude of ERPT during the period characterized by the enforcement of IT measures.

In connection with this, Masoudi et al. (2023) delved into a pivotal intersection between exchange rates and inflation that exerted a dual-fold impact. The study took on the responsibility of dissecting the causal interconnection between inflation and exchange rates, delving deep into the intricacies across different frequencies and temporal scales. They employed the discrete wavelet transform. The outcomes provided thought-provoking revelations, uncovering that over the long term, the causal linkages stemmed from inflation to the exchange rate. This significant finding underscored that relying solely on exchange rate stabilization is an inadequate strategy for taming the pressures of inflation. Moreover, the study conducted by Sheferaw and Sitotaw (2023) had, as an objective, the exploration of the extent to which exchange rate pass-through (ERPT) influences local price levels. To undertake this, a structural Vector Autoregressive (SVAR) analysis was employed. The findings uncovered that a unitary alteration in the nominal effective exchange rate, indicating appreciation, corresponded to a decline of 0.059 in consumer prices within a one-year timeframe. This underscored the significance of ERPT's effect on domestic inflation in Ethiopia. Furthermore, the study delved into the intricate dynamics through the lens of impulse response and variance decomposition analyses. These explorations shed light on the fact that government expenditure plays a role, and the impact of international energy prices further amplifies the influence on domestic prices.

In addition, Jambaldorj (2023) provided tangible empirical insights into the complex interaction between inflation targeting and the pass-through mechanism stemming from fluctuations in exchange rates that influence consumer prices. The primary aim of the study was to estimate a vector-autoregressive model while examining the impulse responses related to consumer prices in response to exchange rate shocks. As a result of empirical investigation, in the pre-inflation targeting period, a distinct pass-through effect was observable, wherein changes in the exchange rate directly impacted consumer prices. However, a significant shift occurred during the post-inflation targeting era. The empirical scrutiny revealed a dampening of the pass-through effect during this phase. The author confirmed that attenuation could be attributed to the inherent mechanics of the forward-looking monetary policy framework adopted within inflation targeting regime. This framework acts as a mechanism for shaping the expectations of domestic economic agents. He explained that these agents are less prone to altering prices in response to exchange rate shocks.

In the current post-COVID-19 and geopolitical turmoil, central banks face the task of addressing heightened inflation. In this context, Alexius and Holmberg (2023) employed Bayesian VAR modeling to scrutinize pass-through effects for understanding inflation dynamics. Their analysis unveiled that those pass-through effects amplified with exchange rate volatility. The connection between pass-through and domestic price shocks followed an expected pattern, strengthening with shock scale and persistence. Results showed that exchange rate shock persistence correlates with heightened pass-through effects, particularly in low inflation scenarios. Notably, the study identified an asymmetry in the impact of inflation persistence on pass-through, with appreciations having a stronger link than depreciations. Equally important, Cakir and Kaya (2023) explored the evolution of exchange rate pass-through effects on consumer prices in Turkey. Using the time-varying Granger causality test, they analyzed the relationships between the U.S. dollar, Euro currency basket, and the consumer price index (CPI). The findings revealed a distinct downward trajectory in the impact of exchange rate pass-through effects on both the CPI and consumer spending categories. They explained that this shift was attributed to Turkey's economic structure, featuring a floating exchange rate and an inflation-targeting regime.

In conjunction with, Solórzano (2023) studied the exchange rate pass-through dynamics in Mexico, focusing on regional and product-specific attributes. Using CPI micro-data, the research revealed notable variations in pass-through rates across regions and industries. Specifically, low pass-through regions exhibited one-quarter of the elasticity compared to high pass-through counterparts over 12 months, persisting over extended periods. The findings attributed these differences to factors like demand dynamics, economic development, proximity to the U.S. border, import intensity, price fluctuation dispersion, and expenditure allocation. Market density emerged as a tempering factor. Additionally, Iliyasu and Sanusi's (2022) study investigated the impact of announced exchange rate policies on pass-through effects in Nigeria. Using a structural VAR model, the research revealed a significant reduction in the pass-through effect when exchange rate interventions were pre-announced and anticipated. The pass-through rate transitions indicated an incomplete and gradual nature of pass-through in Nigeria. The study suggested that well-communicated exchange rate depreciation can strategically mitigate inflationary costs compared to sudden depreciation. The authors concluded that an effective communication strategy by central banks have the potential to reduce the cascading impact of exchange rate fluctuations on consumer prices, providing nuanced insights for policy actions in Nigeria.

In addition to this, Ozdogan's (2022) investigated exchange rate pass-through (ERPT) in Turkey, focusing on the completeness of ERPT and its evolution post the 2001 flexible exchange rate regime. Utilizing Vector Auto Regression, this study unveiled two key findings: incomplete ERPT in Turkey with distinct patterns, and a notable decrease in pass through post-2001, followed by a surge after 2017 due to factors like high inflation and exchange rate volatility. Further, Nguyen Hong et al. (2022) focused on Vietnam's pursuit of price stability. They employed ARDL and NARDL methodologies to scrutinize exchange rate pass-through dynamics. Their study unveiled the substantial impact of exchange rates and money supply on the CPI, emphasizing a successful shift to a central-rate mechanism. Delving deeper, asymmetries in exchange rate dynamics were revealed through the NARDL model, adding complexity to Vietnam's economic landscape. The study concluded by offering actionable policy implications, providing policymakers with informed strategies for managing exchange rates, inflation, and economic stability in Vietnam.

Moreover, Ndou's (2022) study explored the impact of South Africa's adoption of a flexible inflation-targeting framework in 2000, accompanied by an official inflation target band of 3–6%. The analysis, conducted using a regime-dependent VAR modeling framework, revealed a transformative effect on exchange rate pass-through (ERPT) dynamics. Within the 3–6% target band, the ERPT coefficient was halved compared to scenarios exceeding the 6% threshold, emphasizing a significant structural shift induced by the inflation target band. This insight has far-reaching implications for inflation forecasting and policy decisions, offering a nuanced understanding to enhance the effectiveness of monetary policymaking. In essence, Ndou's study contributed valuable insights that resonate in the realm of policy formulation, informed by a nuanced comprehension of ERPT dynamics within the South African economic landscape. In a similar fashion, Fandamu et al. (2023) explored asymmetric exchange rate pass-through (ERPT) in Zambia, focusing on consumer price inflation. Using a structural Vector Autoregressive model, the study spanned from 1985 to 2017, revealing incomplete and asymmetric ERPT dynamics. Notably, kwacha depreciation had a more pronounced impact on consumer prices than appreciation. Temporal analyses showed that depreciation shocks persisted longer and contributed more to variance in inflation than appreciation shocks. It has confirmed that these findings hold even when considering external factors like commodity price booms, emphasizing the robust nature of the asymmetries.

In addition, Tiamiyu (2022) investigated exchange rate pass-through and its impact on inflation in Nigeria. Using various analytical tools, including ADF and unit root tests, cointegration tests, and NARDL methodologies, the research revealed short-term relationships and incorporates asymmetric effects in the exchange rate dynamics. The findings highlighted declining but substantial estimates of exchange rate pass-through, emphasizing the nuanced impact of positive and negative changes. The study identified the industrial production index as a key factor influencing inflation, suggesting a need for government focus on productive sectors. Furthermore, Hüseyin and Kutlu's (2022) investigated the pass-through mechanism of exchange rate fluctuations onto Turkey's domestic prices. Using the SVAR model, the study explored the complex interplay between exchange rate fluctuations, producer price index, and consumer price index, focusing on cumulative exchange rate pass-through (ERPT) elasticities. The findings revealed an incomplete ERPT within Turkey, emphasizing the nuanced translation of exchange rate fluctuations into domestic prices. Importantly, the study uncovered variations in ERPT across economic variables, with a higher impact on producer prices compared to consumer prices. Additionally, the research challenged conventional notions about the role of the central bank's policy interest rate in influencing exchange rates and prices, emphasizing the multifaceted nature of these interactions. The study concluded with pragmatic policy recommendations, advocating for central bank policies characterized by stability and reliability to foster a more resilient economic environment.

Notably, Pham et al. (2022) conducted a comprehensive study on exchange rate pass-through effects (ERPT) onto consumer prices in Vietnam. Using a structural Vector Autoregressive (SVAR) approach, the research unveiled Vietnam's unique ERPT dynamics, surpassing those of both emerging and developed economies. The results affirmed that high and volatile inflation, coupled with an open economic landscape and the exchange rate regime, contributed to this heightened ERPT degree. The study delved into the interplay between ERPT, inflation levels, and volatility across different sectors, emphasizing the nuanced dynamics shaped by sectoral attributes. Namely, the focus on a single country revealed that observed differences were more likely industry-specific, enriching our understanding of ERPT dynamics and highlighting the importance of sectoral nuances.

3. Hypothesis Development

The relationship between the exchange rate and the inflation rate is essential in economic and monetary analysis. A common assumption is that exchange rate fluctuations can have a significant impact on a country's inflation rate. This assumption is based on the concept of the transmission of exchange rate variations to the prices of goods and services, often referred to as exchange rate pass-through.

When the exchange rate of a currency depreciates, imported products become more expensive in domestic currency, resulting in an increase in the price of imported goods. This increase in the price of imported goods can spread throughout the economy, in turn affecting the prices of locally produced goods and services. This first stage of exchange rate pass-through, where changes in the exchange rate translate directly into rapid adjustments in the prices of imported goods, contributes to rising inflation.

Then, there is the second stage pass-through, which occurs more indirectly. The depreciation of the exchange rate can increase the production costs of local companies, due to the higher prices of imported inputs. These higher production costs can encourage companies to increase their selling prices to maintain their profit margins, which leads to a further increase in the price of goods and services on the domestic market. This rise in prices, in turn, can contribute to inflation in the longer term.

Consequently, there is a close link between exchange rate fluctuations and changes in the inflation rate. Exchange rate movements can directly influence the prices of imported goods and indirectly influence local production costs, both of which contribute to the evolution of inflation in a given economy. An accurate understanding of this relationship is crucial to the formulation of effective monetary policies aimed at maintaining price stability and promoting economic growth.

4. Methodology

With the aim of comprehend the intricate exchange rate pass-through (ERPH) phenomenon, researchers have engaged a diverse array of models. Among these, Özyurt (2016) and Goldberg and Campa (2010) have chosen to employ single equation time series models. Conversely, Comunale and Kunovac (2017) and Forbes et al. (2018) have embraced the vector auto regression (VAR) methodology. Ben Cheikh and Rault (2017) adopted a dynamic panel approach, utilizing the generalized method of moments estimation.

In our study, our aim was to illuminate the repercussions of exchange rate pass-through on inflation, with a specific emphasis on the fluctuations of a country's business cycle. However, to obtain a more comprehensive perspective and consider the complexity of global economic interconnections, we also wished to introduce the concept of connectedness or interconnection. To accomplish this, we opted to employ a Markov Switching Vector Autoregressive (MS-VAR) model for several compelling reasons.

Indeed, the Markov switching Vector Autoregressive (MS-VAR) model introduces a nonlinear multivariate framework wherein certain parameters are allowed to dynamically shift over time and in accordance with specific states, while other parameters remain steadfastly regime-invariant. This unique property bestows upon the model the capability to perform efficient coefficient estimation even amid periods of regime transitions. Nevertheless, traditional VAR models often grapple with challenges rooted in macroeconomic condition uncertainties. For instance, when a severe financial or debt crisis emerges while an economic system is still recovering from a prior crisis, levels of uncertainty can escalate to a degree where conventional VAR models falter in their effectiveness. Here, the Markov model emerges as a potent solution, addressing the intricacies of stochastic factors and skillfully pinpointing the probabilities associated with transitioning the system from one regime to another. Appreciating these distinctive attributes, the MS-VAR models have found versatile application across a spectrum of domains. In this context, Bessac et al. (2016) leveraged the MS-VAR model to characterize wind regimes within meteorological time series, while Monbet and Ailliot (2017) employed it to delineate fluctuations in daily mean temperature patterns.

Moreover, these models have assumed significant roles within econometric time series analyses. For example, Papadamou and Markopoulos (2018) harnessed the MS-VAR to unveil the intricate transmission mechanism of a positive shock within the EONIA index to Greek banks' retail rates, a process that necessitated an evaluation of its stochastic factors. In a different context, Aimer and Lusta (2021) utilized the MS-VAR model to dissect the repercussions of the U.S. economic policy uncertainty index and oil price fluctuations on the dollar exchange rate, utilizing monthly data from January 2006 to August 2020. Adding to that, Shahrestani and Rafei (2020) adopted the MS-VAR model with dual regimes to explore the influence of global oil price shocks on the Tehran Stock Exchange. His investigation revealed varying intercepts, coefficients, and variances within each regime. The matrices that depict transition probabilities offer insight into the persistence of both states, indicating that shocks evoke both positive and negative effects on the Tehran Stock Exchange within the respective first and second regimes.

In this context, the Markov Switching Vector Autoregressive (MS-VAR) model can be elucidated as an amalgamation of concepts. Essentially, it is a type of Hidden Markov model where the structure is composed of two distinct components: the observable variables modeled through Vector Auto Regressive (VAR) processes, and the hidden component representing the number of states, which conforms to a first-order Markov chain. In this context, Hamilton (2010) introduced the concept of the Markov switching method. This theory posits that economic systems display diverse characteristics across different states, and these distinctions are often reflected in the parameters of economic models. Consequently, the relationships between variables may alter in response to various shocks.

Therefore, Krolzig (1998) combined the traditional Vector Autoregressive (VAR) model with the Markov switching approach, giving rise to the Markov Switching-Vector Autoregressive model. Unlike the conventional VAR model, this novel iteration effectively captures the repercussions of changes in macroeconomic factors induced by distinct shocks. The fundamental premise based on the notion that the vector of observable time series relies on an unobservable state. To simplify, within each regime denoted as S_t, the time series vector Y_t is engendered by a VAR process involving a certain number of lags (p).

Consequently, the MS-VAR model enables the inclusion of dynamic shifts within an economy's behavior by acknowledging multiple states, each marked by a different set of parameters in the VAR process. This innovation affords the model the capability to accurately capture the intricate interplay between variables even as the underlying economic conditions undergo shifts. This is especially relevant in cases where economic systems experience changes in response to exogenous shocks, which can start a transition between different states characterized by distinct relationships between variables. The MS-VAR model, with its Hidden Markov structure, empowers researchers to distinguish the complex interdependencies that unfold within dynamic economic systems, making it a powerful tool for unraveling the multifaceted dynamics of economic phenomena.

The MS-VAR model is a Hidden Markov model where the observable variables' model is the vector auto regressive per sate. We distinguish the two components of the MS-VAR model: The first component Y_t describes the evaluation of the observable variables. The second component S_t describes the number of states. It is a hidden component that takes a value between {1, …., M} and models a first-order Markov chain. The model is given as follows:

$$Y_t = \begin{cases} \beta_{01} + \beta_{11} Y_{t-1} + \cdots + \beta_{p1} Y_{t-p} + A_1 \varepsilon_t & \text{if } S_t = 1 \\ \vdots & \\ \beta_{0M} + \beta_{1M} Y_{t-1} + \cdots + \beta_{pM} Y_{t-p} + A_M \varepsilon_t & \text{if } S_t = M \end{cases} \tag{1}$$

where A is a regime dependent matrix.

As mentioned earlier, a defining attribute of the MS-VAR framework lies in its adaptability across variables. This flexibility arises from the fact that certain parameters are subject to regime-specific variations while others remain consistent. Specifically, within the VAR framework, which encompasses three key parameters—the intercept (or mean), the variance, and the autoregressive coefficient—we discern several notable extensions within the MS-VAR model.

Introduced by Krolzig (1998), this concept involves a distinct approach. The essentials of this idea reside in the fact that solely the intercept of each regression undergoes alterations across the various states, while the autoregressive coefficients and variance remain constant across all states. To elucidate further, the intercept, representing the starting point or baseline level of the relationship between variables, can exhibit shifts in response to changes in economic regimes. This acknowledges that different states or economic conditions might yield different starting points for the relationships between variables. However, the autoregressive coefficients, which capture the relationship between a variable's current value and its past values, as well as the variance, reflecting the dispersion of data points around the mean, remain stable and unaffected by changes in states. In particular, for this idea, only the intercept of each regression varies across the M sates, while the autoregressive coefficients and the variance are constant above all states.

As a matter of fact, this structure offers a nuanced understanding of how different facets of economic relationships are influenced by shifts in regimes. By permitting regime specific intercept adjustments while keeping the autoregressive coefficients and variance constant, the MS-VAR model effectively captures the intricate dynamics of variables under changing economic circumstances. This concept finds resonance in economic reality, where certain relationships between variables might be more sensitive to regime shifts, while others remain relatively stable. Ultimately, this adaptability within the MS-VAR structure enhances its capacity to unravel the complex interplay of variables across diverse economic states. The model is described as follows:

$$Y_t = \begin{cases} \beta_{01} + \sum_{i=1}^{p} \beta_i Y_{t-i} + A\varepsilon_t & \text{if } S_t = 1 \\ \vdots & \\ \beta_{0M} + \sum_{i=1}^{p} \beta_i Y_{t-i} + A\varepsilon_t & \text{if } S_t = M \end{cases} \quad (2)$$

Illustrating the applicability of the Markov Switching Intercept Vector Autoregressive model (MSI-VAR) framework, Wai et al. (2013) employed it to discern the seamless progression of stock index fluctuations, shifting from recessionary phases to periods of growth. Using the MSI-VAR model, they delved into the dynamic shifts within stock indices, capturing the underlying mechanisms that facilitate the transition between economic states marked by downturns and those characterized by expansion.

In a more contemporary context, Lebari and Didi (2021) embarked on a study that delved into the intricate nexus between international trade and macroeconomic stability in Nigeria, adopting the MSI-VAR model as a robust analytical tool. He endeavored to unravel the interplay between international trade dynamics and the overall macroeconomic equilibrium of Nigeria. Through the application of the MSI-VAR models, Lebari and Didi (2021) scrutinized the transitions between various states of economic stability and trade interdependence, offering insights into the nuanced relationship between these two critical elements.

Fundamentally, the MSI-VAR model's distinctive feature lies in its capacity to accommodate regime-specific fluctuations in the variance-covariance matrix, while maintaining other parameters as constants across all regimes. In essence, this means that while the magnitudes and relationships between variables might change under different economic states, certain structural aspects remain unaltered. This mechanism ensures that the model can effectively capture shifts in the dispersion of data points around the mean, reflecting changes in the underlying volatility of the system.

By allowing the variance-covariance matrix to vary according to different regimes, the MSI-VAR model incorporates the changing nature of data distributions and relationships under varying economic conditions. This modeling approach acknowledges that the dynamics of certain variables might display distinct patterns during economic transitions, necessitating a nuanced treatment of the underlying covariance structure. Such flexibility enhances the MSI-VAR's capability to uncover the complex interactions and transitional dynamics within economic systems, shedding light on how variables respond to shifts from one state to another. The model is described as follows:

$$Y_t = \begin{cases} \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + A_1\varepsilon_t & \text{if } S_t = 1 \\ \vdots & \\ \beta_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + A_M\varepsilon_t & \text{if } S_t = M \end{cases} \quad (3)$$
$$\varepsilon_t \sim N\left(0, \sigma^2\right)$$

Within the framework of the model, a distinctive feature emerges: the variance within each state exhibits differentiation from the variances present in other states. Each state's variance, denoted as ${\sigma_i}^2$, is capable of undergoing variations across different economic regimes. However, two key parameters, namely the intercept β_0 and the autoregressive coefficient β_i, remain consistent across all these regimes. This characteristic maintains the structural foundation of the model while allowing for dynamic changes that are reflective of real-world economic transitions.

Lanne et al. (2010) illustrated the model's utility. By leveraging this modeling approach, they endeavored to uncover and identify the sources of structural shocks within the economy. In essence, the model functions as a powerful tool for untangling the intricate web of underlying factors that lead to shifts in economic conditions, offering insights into the fundamental drivers behind fluctuations and transitions.

A unique variant of this model is one where all parameters are permitted to undergo changes across states. In simpler terms, each state is characterized by distinct autoregressive coefficients, resulting in differing means and variances. This parameter variation across states captures the dynamic nature of economic transitions, where different states are associated with varying relationships between variables and fluctuations in magnitudes. This adaptable modeling structure enables researchers to delve into the intricate details of economic shifts, effectively painting a comprehensive picture of how different facets of an economy evolve across changing conditions. The model is described as follows:

$$Y_t = \begin{cases} \beta_{01} + \sum_{i=1}^{p} \beta_i Y_{t-i} + A\varepsilon_t & \text{if } S_t = 1 \\ \vdots \\ \beta_{0M} + \sum_{i=1}^{p} \beta_i Y_{t-i} + A\varepsilon_t & \text{if } S_t = M \end{cases} \quad (4)$$

Commencing our analysis, we started by introducing the vector Y_t. This vector covers the observed variables up to time t, providing a comprehensive overview of the empirical data at that point in time. Essentially, it serves as a composite representation of the various metrics and indicators that have been monitored and recorded up until the specified moment. In conjunction with, we introduced the vector θ, a pivotal element in our procedure. This vector serves as a repository for the parameters that are subject to estimation. These parameters collectively encompass a spectrum of vital components within our modeling framework. Specifically, they include the intercept, which represents the baseline or starting point of relationships between variables. The autoregressive variables are also integral, capturing the lagged influence of a variable on its own future values, thereby incorporating the temporal dependencies present in the data. Additionally, the variance parameter is a crucial piece of the puzzle, signifying the extent of dispersion of data points around the mean and providing insights into the overall data distribution. Lastly, the vector θ incorporates the transition probabilities, which play a pivotal role in delineating the likelihood of transitions between different states within the model.

By regrouping these parameters within the vector θ, we aimed to effectively capture and quantify the underlying dynamics and relationships present within the data. The process of estimating these parameters was at the core of our analytical journey, allowing us to unlock insights, uncover patterns, and model the intricate interplay between variables and states. Through rigorous estimation, we tried to discover the hidden mechanisms that govern the observed phenomena, thereby enriching our understanding of the complex interactions that drive economic and statistical processes. $\theta = \left(\beta_0^{S_t}, \sum \beta_i^{S_t}, \sigma^2, P_{ij}\right)$. It is important to recognize that the MS-VAR model can be estimated using two methods, which are the expectation-maximization (EM) algorithm and the Bayesian method.

According to Krolzig (1998), for a given regime S_t and lagged endogenous variables $Y_{t-1} = \left(Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}\right)'$, the conditional probability density function is given by $f(y_t|S_t; Y_{t-1})$. Where ε_t is assumed to be Gaussian $\varepsilon_t \sim N(0, \sigma^2)$. Then, the error term ε_t is presupposed to follow a Gaussian distribution $\varepsilon_t \sim N(0, \sigma^2)$. In such a case,

$$f(y_t|S_t; Y_{t-1}) = (2\pi)^{-\left(\frac{1}{2}\right)} \sigma^{-1} \exp\left\{-\frac{1}{2}\left(y_t - \overline{y_s}\right)^2 \sigma^{-2}\right\} \quad (5)$$

In which case $\overline{y_s}$ signifies the conditional mean of y_t within the context of regime S_t; $\overline{y_s} = E(y_t|S_t, Y_{t-1})$.

We made the presumption that the data accessible as of time $t-1$ encompass the recorded instances from the dataset gathered at Y_{t-1} derived from the states of the Markov chain up to S_{t-1}; the conditional distribution of y_t takes the form of a combination of Gaussian distributions.

$$f(y_t|S_{t-1}=i;Y_{t-1}) = \sum_{m}^{M} f(y_t|S_{t-1},Y_{t-1})P(S_t=m|S_{(t-1)}=i) \quad (6)$$

$$f(y_t|S_{t-1}=i;Y_{t-1}) = \sum_{m=1}^{M}\sum_{i=1}^{M}\left((2\pi)^{-\left(\frac{1}{2}\right)}\sigma^{-1}\exp\left\{\frac{1}{2}(y_t-\overline{y_s})^2\sigma^{-2}\right\}\right)P_{im} \quad (7)$$

Two factors hinder the application of the maximum likelihood (ML) function for estimating the MS-VAR model. These factors involve the latent variables such as the regime S_t and the conditional distribution of y_t. Hence, we resorted to employing the EM (expectation-maximization) algorithm. This algorithm was introduced by Dempster et al. (1977) and is an iterative procedure. It serves as a parametric estimation technique that enables the identification of the maximum likelihood parameters of a probabilistic model in scenarios where unobservable variables are present. The EM algorithm derives its name from the fact that each iteration encompasses two separate stages, namely:

→ The expectation step (E step): During this stage, we compute the anticipated value relying on the observed variables and the parameter values established in the preceding iteration. This computation can be expressed as follows:

$$\varphi\left(\theta\middle|\theta^{(0)}\right) = E_{\left(s|y,\theta^{(0)}\right)}[\ln P(y,s|\theta)] \quad (8)$$

- The maximization step (M step): This phase employs the approximation of the unfamiliar information derived from the earlier stage to optimize the likelihood function and revise the parameters for the forthcoming iteration. Consequently, the shift from one iteration (m) to the subsequent iteration (m + 1) of the algorithm involves ascertaining:

$$\widehat{\theta^{(m+1)}} = \mathrm{argmax}_{\theta}\left\{E_{\left(s|y,\theta^{(0)}\right)}[\ln P(y,s|\theta)]\right\} \quad (9)$$

These two steps are repeated until the difference between the likelihood function of the iteration (m + 1) and that of the iteration (m) does not change.

The EM algorithm stands as the conventional method for deducing the optimal MS-VAR. Nevertheless, an alternative statistical approach is available, known as Bayesian statistics. It encompasses the domain of statistics through a perspective grounded in Bayesian interpretation, where probability incorporates the level of confidence in an occurrence. To elaborate further, consider the following instance: When presented with two events, A and B, there exists an initial belief in the occurrence of event A, denoted as $P(A)$. Given the occurrence of event A, the impact on event B is comprehended, represented as $P(B|A)$. The initial probability $P(A)$ can be modified by taking event B into account, utilizing Bayes' theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (10)$$

$P(A|B)$ is referred to as the posterior probability, whereas $P(A)$ is denoted as the prior probability, incorporating anticipations concerning event A prior to the unfolding of event B. $P(B|A)$ serves as the likelihood function, signifying the probability of event B being true given that event A is already true. $P(B)$ embodies the likelihood of event B being true; its calculation is as follows:

$$P(B) = \sum_{i} P(B|A_i)P(A_i) \quad (11)$$

This statistical principle concerning causality centers on the concept of anticipation. In fact, if a variable y_1 "Granger-causes" another variable y_2, it signifies that the historical values of y_1 should encompass information that contribute to the prediction of y_2, augmenting the knowledge already embedded within the past values of y_2. The testing procedure is outlined as follows. The null hypothesis postulates the absence of causality in the context of, namely, the conveyed values of the variable y_1 do not trigger the presence of y_2 in the current context. In simpler terms, y_1 is not the cause of y_2 according to the Granger causality criterion. In mathematical terms, the test can be formulated as:

$$C_{y_1 \to y_2} = \log \frac{\det V_{\varepsilon}[y_2|y_{2,t-1}]}{\det V_{\varepsilon}[y_2|y_{2t-1}, y_{1t-1}]} \tag{12}$$

And the test statistic is as follows: $\delta = TC_{y_1 \to y_2}$; if $\delta < p_{value}$, we accept the null hypothesis.

The reaction to an external alteration, often referred to as an exogenous shock, can be elucidated through the impulse response function (IRF). This concerns to the array of equations interconnecting multivariate autoregressive variables. Essentially, IRFs provide insights into the dynamic patterns exhibited by the assorted variables within the system when confronted with brusque changes. In simpler terms, IRFs facilitate comprehension of how the system's various variables evolve over time due to unexpected influences.

5. Empirical Analysis

5.1. Data Description

In this segment, we provide an overview of the datasets employed in our empirical examination. The nations under consideration for this study encompass the Eurozone, Canada, and the United Kingdom. These specific selections were guided by a triad of principal rationales. Firstly, we considered the accessibility and availability of comprehensive data. Secondly, we considered the robust political stability within these nations, which, in turn, minimizes the likelihood of inflation stemming from governance issues. Lastly, we considered the distinctive aspect of these geographic areas operating with their own distinct currencies: the euro (EUR), Canadian dollars (CAD), and the pound (GBP). These currencies are processed in alignment with the U.S. dollar as the base currency, all within the context of their well-established histories characterized by floating market-driven exchange rate frameworks. Our analytical framework hinges on the utilization of monthly data spanning the contiguous period, specifically commencing from 1 January 1999 to 1 July 2023.

The primary aim of this research was to gain comprehensive insights into the intricate interactions between inflation dynamics and the inherent fluctuations witnessed within the exchange rate arena. In this vein, the pivotal variables under scrutiny encompassed inflation and the exchange rate. We sought to unravel the nuanced ways in which variations in the exchange rate can influence and shape the trajectory of inflation trends.

In order to assess the stationarity characteristics of the time series data under investigation, we employed two distinct testing methodologies: the augmented Dickey–Fuller test (ADF test) and the Kwiatkowski–Phillips–Schmidt–Shin test (KPSS test) (see Table 1).

Table 1. Basic statistics of monthly log returns.

	R-EAI	R-UKI	R-CAI	R-EUR	R-GBP	R-CAD
Mean	0.0081	0.0052	0.0084	0.0004	0.0011	−0.0005
Minimum	−6.9077	−0.8782	−2.1972	−0.0618	−0.0598	−0.0600
Maximum	8.0063	0.6951	2.3025	0.0779	0.0954	0.1129
Student	0.1430	0.5030	0.3424	0.3490	0.9112	−0.5148
ADF	−6.2521	−5.0730	−6.2835	6.0782	−6.6741	−6.3452
KPSS	0.0394	0.1351	0.0411	0.1361	0.0989	0.2309

According to the Table 1, we can affirm that the series of the returns were quite stationary. We note that log returns European Annual Inflation (R-EAI), log returns United

Kingdom Inflation (R-UKI), log returns Canadian Inflation (R-CAI), log returns euro (R-EUR), log returns Great Britain pound (R-GBP), and log returns Canadian dollar (R-CAD).

The initial data density graph, as depicted in Figure 1, and the subsequent examination of Quantile-Quantile plots (QQ plots), as presented in Figure 2, collectively serve to offer a comprehensive and global understanding of the underlying distributions inherent within the dataset.

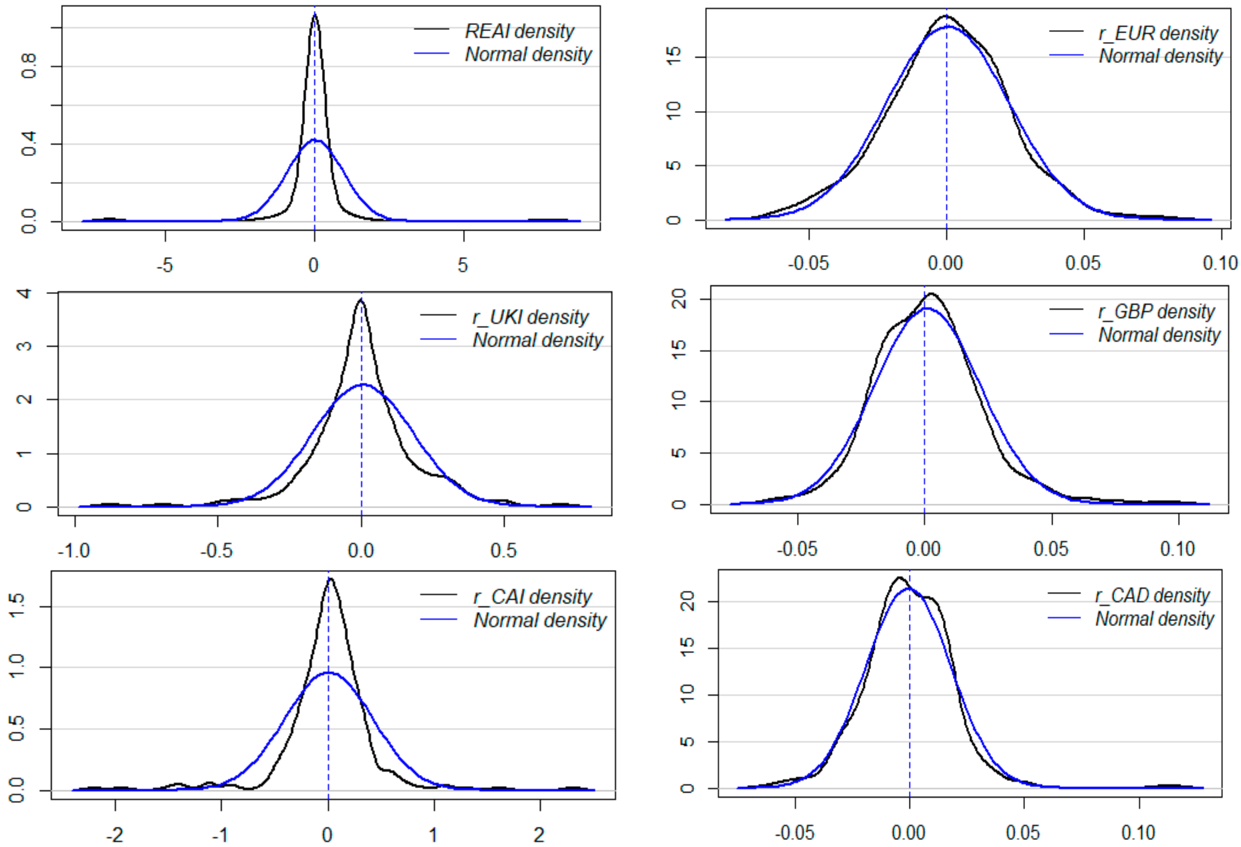

Figure 1. Unconditional density estimation.

Upon careful analysis of these representations, it became evident that the distribution exhibited by the inflation dataset deviated from the typical normal distribution. Indeed, the distribution displayed characteristics that were notably more acute and peaked in comparison to the conventional normal distribution. Similarly, the distribution observed within the exchange rate dataset approximated a normal distribution to a certain extent. However, distinct asymmetrical traits were clearly observable.

By examining the QQ plots, we embark on a comparative exploration of two probability distributions by juxtaposing their respective quantiles. This enabled us to discern how the dataset's distribution contrasts with that of a normal distribution. Evidently, these QQ plots affirm that the data's distribution, both within the context of inflation rates and exchange rates, significantly deviated from the norm. Notably, the quantiles of the samples deviated distinctly from the quantiles characterizing a standard normal distribution. This empirical evidence highlights the non-normal nature of the datasets under scrutiny and further illuminates the distinctive distribution characteristics that set them apart.

In order to examine the normality characteristics inherent within the series, our focus turned towards a set of pertinent statistical measures. Specifically, we considered and assessed the skewness and the kurtosis and applied the Jarque–Bera test (see Table 2). These indicators collectively serve as valuable tools in lighting on the distributional properties of the series under scrutiny.

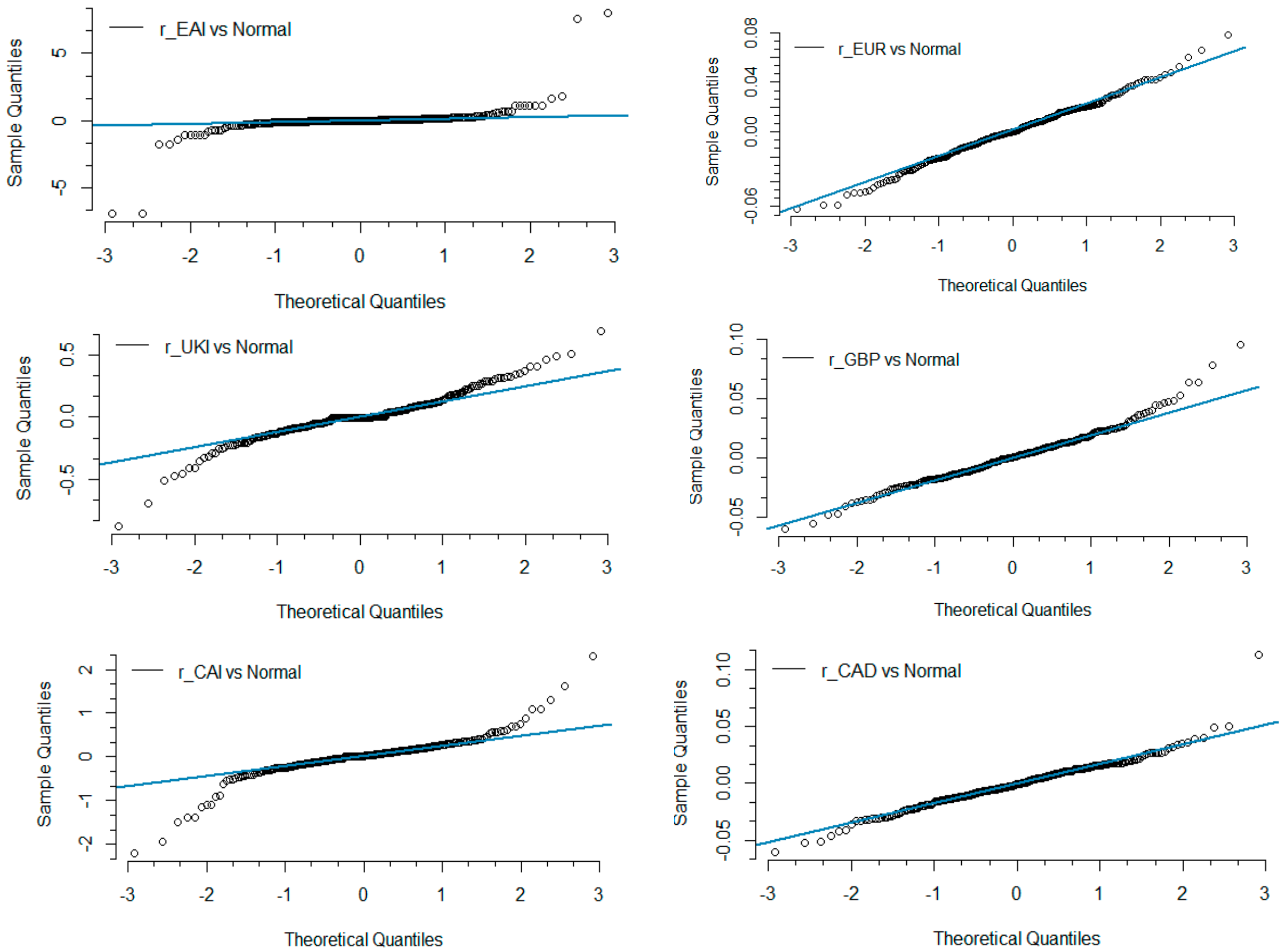

Figure 2. QQ-plots for the returns.

Table 2. Normality tests for the return series.

	R-EAI	R-UKI	R-CAI	R-EUR	R-GBP	R-CAD
skewness	1.1646	−0.3301	−0.4926	0.0065	0.5788	0.5849
kurtosis	51.9555	6.8660	12.1508	3.4763	4.9235	7.6080
J.B test (P_{value})	$<2.2 \times 10^{-16}$	$<2.2 \times 10^{-16}$	$<2.2 \times 10^{-16}$	0.2633	1.38×10^{-13}	$<2.2 \times 10^{-16}$

To analyze the autocorrelation patterns inherent within the series, a valuable tool at our disposal is the Ljung–Box test (see Table 3). This statistical test is specifically designed to assess the presence of serial autocorrelation within the observed data. The null hypothesis posits the absence of any significant autocorrelations within the errors of the model, while the alternative hypothesis operates under the assumption of the contrary—a scenario where autocorrelations do indeed exist.

Table 3. Ljung–Box test for the return series.

	R-EAI	R-UKI	R-CAI	R-EUR	R-GBP	R-CAD
Q(20)	132.04	44.925	43.732	48.776	37.966	43.732
P_{value}	$<2.2 \times 10^{-16}$	0.0011	0.0016	0.0003	0.0089	0.0016

As we observe the contents of the table, the *p*-values associated with all variables were notably lower than the conventional significance threshold of 5%. This signifies a significant departure from the null hypothesis, ultimately leading to its rejection. In essence, these findings underscore the presence of autocorrelation within the residuals of the series under consideration. This empirical evidence points towards a temporal dependence within the dataset's residual elements.

In order to assess the presence of heteroscedasticity within the dataset, we employed a dual-pronged testing approach. Specifically, we applied two distinct tests to shed light on this crucial aspect: the Ljung–Box test, focusing on the squared returns (see Table 4), and the Lagrange multiplier test (see Table 5). These tests collectively enabled us to comprehensively explore the variance patterns within the data, and by extension, gauge the presence of heteroscedastic tendencies.

Table 4. Ljung–Box test.

	REAI	RUKI	RCAI	REUR	RGBP	RCAD
$Q^2(20)$	174.96	154.85	88.77	49.723	42.362	88.77
P_{value}	$<2.2 \times 10^{-16}$	$<2.2 \times 10^{-16}$	1.215×10^{-10}	0.0002	0.0024	1.215×10^{-10}

The results derived from the application of the Ljung–Box test reveal that the *p*-values corresponding to all the series were consistently below the established significance level of 5%. This collective outcome strongly indicates the presence of an Autoregressive Conditional Heteroskedasticity (ARCH) effect within the data. In essence, the Ljung–Box test outcomes substantiate the notion that the variances of the series are not uniformly distributed, signifying the presence of conditional heteroscedasticity. This empirical insight offers a clear indication that the level of variability within the data is not constant, which has important implications for understanding the potential patterns of volatility inherent within the series.

Table 5. Lagrange multiplier test.

	REAI	RUKI	RCAI	REUR	RGBP	RCAD
LM statistic	88.4793	14.9185	44.0564	3.8587	16.1984	0.9107
P_{value}	0	0.0001	1.6577×10^{-10}	0.0502	7.3536×10^{-5}	0.3407

The outcomes of the Lagrange test yielded nuanced insights. Specifically, by examining the *p*-values associated with the returns of the EUR and CAD series, it was evident that they surpassed the conventional threshold of 5%. This validation aligns with the acceptance of hypothesis H_0, indicating that the data for these series do not manifest an ARCH effect. This divergence becomes especially pronounced when compared to the remaining series, where the *p*-values registered below 5%. This observation signifies the presence of an ARCH effect within these series, underlining the presence of conditional heteroscedasticity.

To address the variance-autocorrelation within these series, we turned to the application of an ARCH (p) model. The selection of the appropriate model was based on the selection criteria, namely the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). This methodology lies in opting for the model that displays the weakest information criterion, thereby providing a robust foundation for model selection. The determination of the lag parameter p draws on the squared returns series. This process involves identifying the number of lags that extend beyond the bounds of the confidence interval.

5.2. Empirical Results and Interpretations

Within this section, we undertook the estimation of distinct models for each of geographic areas, namely the Eurozone, the United Kingdom, and Canada. For each individual estimation, we progressed through a sequence of methodological phases. In the first phase, we determined of the optimal number of lags. This process was guided by the application of two criteria, namely AIC and BIC. In the second phase, with the optimal lag count, we pivoted towards the presentation of the appropriate model that aligns with the distinctive regimes evident within the data. This strategic alignment ensures that the models accurately capture the complex dynamics unfolding within each regime, fostering a more accurate depiction of the real-world intricacies. The subsequent step concerns the transition matrix, which effectively outlines the transitions between the different identified regimes. This matrix offers valuable insights into the pathways through which economic conditions shifted from one regime to another. In the fourth phase, we proceeded to apply the Granger causality test, a pivotal analytical tool that enabled us to probe the causal interplays within the data. This examination allowed us to unveil the temporal cause-and-effect relationships between variables, unraveling hidden dynamics that might be otherwise obscured. In the fifth phase and following the completion of these methodological steps, we engaged in a comprehensive interpretation of the results. To conclude, in the final step, we delved into the discussion of impulse response functions. This allowed us to explore the dynamics of how the system responds to external shocks, uncovering the trajectories of adjustments and reactions that ripple through the data.

Model 1: Eurozone inflation and exchange rate EUR/USD

We began by delimiting two distinct regimes that incorporate the divergent states of the economic landscape. The initial regime pertained to the phase of economic expansion, characterized by growth and vigor. The subsequent regime, in contrast, corresponded to the phase of economic recession, marked by contraction and retrenchment. By segmenting the data into these two pivotal regimes, we laid the foundation for a comprehensive exploration of the differential behaviors and interactions that manifested across these contrasting economic states.

Our analysis identified that the optimal lag count is precisely 2 (see Table 6). With this critical insight in hand, we proceeded to the estimation of a Vector Autoregressive (VAR) model that impeccably aligns with the two identified regimes, each encompassing its unique dynamics and behaviors. This model configuration, denoted as MS (2) VAR (2), effectively captures the intricate interplay within the dataset under the lens of both two regimes and two delays. This approach ensures a robust depiction of the underlying relationships and dependencies, allowing us to comprehensively analyze the data within the context of the identified economic phases and time lags.

Table 6. Information criteria for estimated model—EAI.

	AIC	BIC
P = 1	−7.9382	−7.8595
P = 2	−8.2331	−8.1019
P = 3	−8.2254	−8.0417
P = 4	−8.2191	−7.9829
P = 5	−8.2327	−7.9451
P = 6	−8.2118	−7.8707

Covariance matrix:

$$\begin{pmatrix} 0.03626 & 0.000 \\ 0.000 & 0.00044 \end{pmatrix}$$

Covariance matrix:

$$\begin{pmatrix} 0.91951 & 0.000 \\ 0.000 & 0.00050 \end{pmatrix}$$

Transition matrix:

Subsequent to the estimation of the model for each distinct regime, we computed the Markov matrix probability. It summarizes the transition probabilities between the different identified economic regimes. Its calculation and interpretation serve as a crucial bridge that connects the observed data to the nuanced dynamics of economic shifts. By quantifying the likelihood of transitioning between these regimes, we enhance our ability to comprehend the underlying patterns that govern economic behavior across different states.

$$A = \begin{matrix} \text{expansion} \\ \text{recession} \end{matrix} \begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix}$$

$$A = \begin{bmatrix} 0.96 & 0.04 \\ 0.09 & 0.91 \end{bmatrix} \text{ where } P_{1,1} + P_{1,2} = 1 \text{ and } P_{2,1} + P_{2,2} = 1$$

Beginning with the inflation equation, our analytical findings unveil noteworthy insights. During a phase of economic expansion, the observed variations in the inflation rate demonstrated a robust negative dependency on its historical values, specifically those two periods prior (at time −2). However, this linkage became relatively subdued, oscillating within the range from 0.07 to 0.1 within the context of a recessionary period (see Tables 7 and 8).

Table 7. Estimation results for regime 1: expansion—Eurozone.

	$DL(EAI_t)$ (P_{Value})	$DL(EUR_t)$ (P_{Value})
Const	0.02 (0.00)	0.0007 (0.62)
$DL(EAI_{t-1})$	−0.96 (0.00)	−0.0005 (0.5)
$DL(EAI_{t-2})$	−0.96 (0.00)	−0.0006 (0.7)
$DL(EUR_{t-1})$	0.47 (0.4)	0.34 (0.00)
$DL(EUR_{t-2})$	0.41 (0.5)	−0.11 (0.1)

Table 8. Estimation results for regime 2: recession—Eurozone.

	$DL(EAI_t)$ (P_{Value})	$DL(EUR_t)$ (P_{Value})
Const	0.02 (0.8)	0.00 (0.8)
$DL(EAI_{t-1})$	0.07 (0.57)	−0.004 (0.7)
$DL(EAI_{t-2})$	0.10 (0.33)	−0.002 (0.9)
$DL(EUR_{t-1})$	−0.35 (0.9)	0.26 (0.03)
$DL(EUR_{t-2})$	−0.66 (0.8)	0.14 (0.2)

The outcomes show that the transmission of exchange rate variations, whether in an expansive or recessionary regime, is characterized by a partial influence. Specifically, in times of expansion, the degree of transmission registered at approximately 0.47, whereas in a recessionary phase, it took a negative stance of around −0.35, both corresponding to time −1. This implies that a 1% fluctuation in the exchange rate yields a proportional effect of approximately 0.47% on the inflation rate during expansion and an inverse influence of around −0.35% during recession. This confirms the interaction between exchange rates and inflation within the two regimes. Notably, the observed outcomes reflect that the transmission of exchange rate dynamics into inflation trends remained incomplete across both expansive and recessionary phases. This pivotal insight underscores the intricate nature of these relationships and sheds light on the nuanced dynamics governing economic shifts across these distinct states.

Analyzing the equation governing exchange rate changes reveals intriguing insights, suggesting that the dynamics of exchange rate fluctuations were influenced by their historical values and, to a lesser extent, by the levels of inflation within both expansion and recession states for the Eurozone. This underscores the intricate web of interactions that contributed to the shifts in exchange rates within these economic phases.

The examination of the transition matrix indicates that when the current economic state is characterized by expansion, there exists a 96% probability of transitioning into another expansion phase in the subsequent period, and a 4% probability of transitioning into a recession phase. Conversely, in the context of a current recession state, there is a 9% likelihood of transitioning into expansion and a dominant 91% probability of moving into another recession phase. Utilizing the transition matrix, the expected duration of a state can be calculated as d = 1/(1 − P_1,1). This calculation provides insights into the anticipated duration of a given state, further enhancing our comprehension of regime dynamics. Drawing upon the transition matrix and the context of the Eurozone's current state being expansionary due to ongoing geopolitical factors, such as the Russian War, it can be inferred that there is a 96% likelihood of remaining in this regime, with an expected duration of 25 months (1/0.04).

Lastly, examining the Granger causality table, we observe that the *p*-values associated with the variables exceeded the conventional threshold of 5%. Consequently, we reject the null hypothesis that posits the absence of Granger causality between the Eurozone Inflation and the exchange rate. Instead, we conclude that Granger causality indeed exists between these two variables (see Table 9).

Table 9. Granger causality test—Eurozone.

	$F_{Statistic}$	P_{Value}
REUR → REAI	0.1496	0.8611
REAI → REUR	0.8042	0.4484

As depicted in Figure 3, within the first regime, there was a distinct observation: the smoothing process assumed a prominent and sustained stature. This high level of smoothing persisted resolutely until it reached a pivotal juncture at period 120 (equivalent to the year 2009). At this juncture, a noteworthy shift occurred as the smoothing process began to experience fluctuations in a downward trajectory, indicative of an impending period of recession. Notably, this pivotal phase aligns with the aftermath of the subprime crisis—a period marked by economic turbulence and contraction.

Subsequently, a second notable recessionary phase emerged in 2015, further underscoring the inherent volatility within economic cycles. Despite these fluctuations, the graph unequivocally depicts a decisive trajectory: the prevailing regime within the Eurozone was unmistakably characterized by economic expansion.

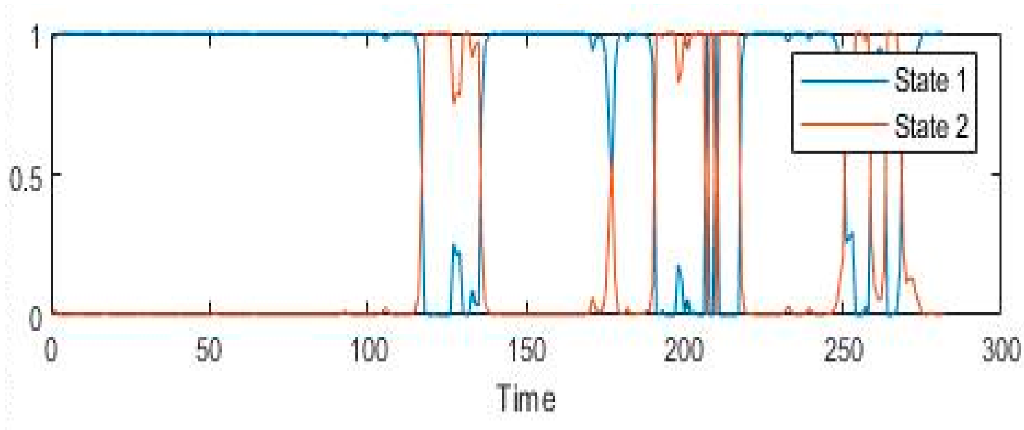

Figure 3. Smoothed states' probabilities for the Eurozone.

Model 2: United Kingdom inflation and exchange rate GBP/USD

Upon evaluation of the Information Criteria AIC, the optimal count of delays was unequivocally established as 2 (see Table 10). Our analytical pursuit leads us to the estimation of a Vector Autoregressive (VAR) model that impeccably aligns with the identified dual regimes. Each of these regimes signifies a distinctive phase in the economic landscape, contributing to a comprehensive understanding of the inherent dynamics. This specialized model configuration, denoted as MS (2) VAR (3), encompasses the interplay of these economic states, harmonizing seamlessly with the three delays integral to the analytical framework. This approach ensures that the model captures the temporal dependencies and interactions within the data.

Table 10. Information criteria for estimated model—UKI.

	AIC	BIC
P = 1	−11.2228	−11.1441
P = 2	−11.2199	−11.0887
P = 3	−11.2450	−11.0614
P = 4	−11.2282	−10.9921
P = 5	−11.2285	−10.9399
P = 6	−11.2152	−10.8742

Covariance matrix:

$$\begin{pmatrix} 0.00760 & 0.000 \\ 0.000 & 0.00042 \end{pmatrix}$$

Covariance matrix:

$$\begin{pmatrix} 0.07301 & 0.000 \\ 0.000 & 0.00030 \end{pmatrix}$$

Transition matrix:

$$\mathrm{A} = \begin{bmatrix} 0.96 & 0.4 \\ 0.7 & 0.93 \end{bmatrix}$$

The estimation tables furnish information regarding the transmission dynamics of exchange rate fluctuations in distinct economic contexts. In the expansion regime, the observed degree of transmission registers at −0.61 (see Table 11). This signifies that within this phase, the relationship between exchange rate variations and UK inflation was characterized by a partial transmission mechanism. In more concrete terms, a 1% exchange rate fluctuation induced a corresponding −0.61% change in UK inflation. Conversely, within a recession regime, the transmission phenomenon took on a different facet, showcasing a complete transmission degree of 1.09 (see Table 12). This implies that a 1% exchange rate variation generated a corresponding 1.09% inflation change in the UK within a recessionary context. This dichotomy underscores the nuanced nature of exchange rate pass-through (ERPT) in the United Kingdom, reflecting a partial ERPT within expansion states and a complete ERPT within recession states.

Shifting our focus to the exchange rate equation, an insightful pattern emerges. The findings indicate that the variation in exchange rates is influenced by their historical values and, to a lesser extent, the inflation levels of the country within both economic states. Further delving into the transition matrix, its estimations closely parallel those of the Eurozone, thereby revealing comparable regime transition patterns. When the current state is expansionary, the probability of transitioning into another expansion phase is 96%, with an expected duration of 25 months (1/0.04). Conversely, when the current state is recessionary, there is a notable likelihood of transitioning into another recession phase (93%), with an anticipated duration of 14 months. In light of these insights and given the prevailing geopolitical context such as the Russian war, it is reasonable to infer a high probability of 96% for the United Kingdom to persist in an expansionary state for a duration of 25 months. Finally, the Granger causality table lends compelling evidence. The *p*-value, significantly exceeding the threshold of 5%, leads us to reject the null hypothesis of no Granger causality between UK inflation and its exchange rate. This discernment reinforces the presence of Granger causality, affirming a significant interaction between the two variables in a manner that aligns coherently with both empirical reality and the results derived from the model estimation (see Table 13).

Table 11. Estimation results for regime 1: expansion—United Kingdom.

	$DL(UKI_t)$ (P_{Value})	$DL(GBP_t)$ (P_{Value})
Const	0.0045 (0.5)	0.0011 (0.5)
$DL(UKI_{t-1})$	−0.03 (0.7)	0.0031 (0.8)
$DL(UKI_{t-2})$	0.15 (0.06)	0.002 (0.8)
$DL(UKI_{t-3})$	0.24 (0.0)	0.01 (0.7)
$DL(GBP_{t\_1})$	−0.61 (0.07)	0.26 (0.0)
$DL(GBP_{t\_2})$	0.50 (0.17)	−0.01 (0.8)
$DL(GBP_{t\_3})$	−0.61 (0.08)	0.2 (0.02)

Table 12. Estimation results for regime 2: recession—United Kingdom.

	$DL(UKI_t)$ (P_{Value})	$DL(GBP_t)$ (P_{Value})
Const	−0.01 (0.6)	−0.002 (0.4)
$DL(UKI_{t-1})$	0.08 (0.4)	−0.0016 (0.8)
$DL(UKI_{t-2})$	−0.10 (0.3)	0.02 (0.01)
$DL(UKI_{t-3})$	0.15 (0.19)	0.01 (0.3)
$DL(GBP_{t\_1})$	1.09 (0.5)	0.27 (0.03)
$DL(GBP_{t\_2})$	−0.25 (0.8)	−0.03 (0.7)
$DL(GBP_{t\_3})$	0.51 (0.7)	−0.17 (0.1)

Table 13. Granger causality test—United Kingdom.

	F-Statistic	*p*-Value
RGBP → RUKI	0.521	0.594
RUKI → RGBP	2.455	0.088

The smoothed probabilities provide us with a representation that incorporates the alternation between periods of expansion and recession within the country's economic landscape. Figure 4 illustrates cyclic nature of economic states as they varied between phases of growth and contraction.

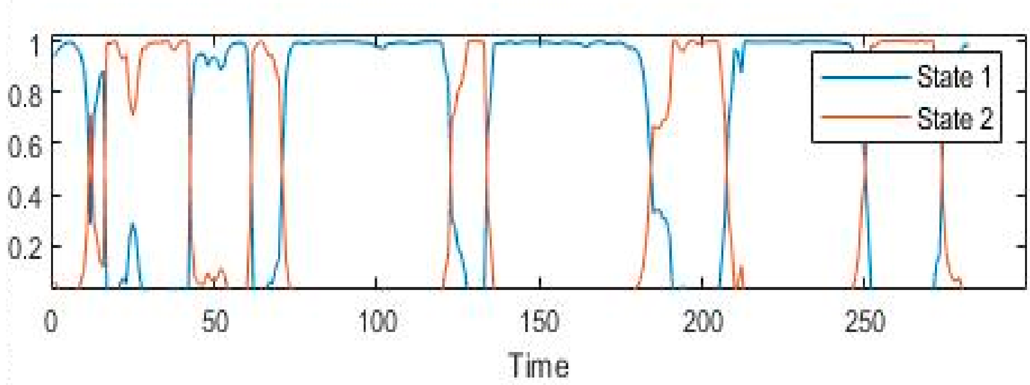

Figure 4. Smoothed states' probabilities for the United Kingdom.

Model 3: Canada inflation and exchange rate CAD/USD

Utilizing the Information Criteria, the ideal number of delays was unequivocally identified as 2 (see Table 14). Our analytical pursuit led us to the estimation of a Vector Autoregressive model that aligns with the two discerned economic regimes. Each of these regimes encapsulates a distinctive economic phase, contributing to a comprehensive understanding of the intricate dynamics at play. This model denoted as MS (2) VAR (2) captures the interplay of these economic states while accommodating the two delays inherent to the analytical framework.

Table 14. Information criteria for estimated model (CAI).

	AIC	BIC
P = 1	−9.7956	−9.7159
P = 2	−9.8467	−9.7166
P = 3	−9.8378	−9.6542
P = 4	−9.8280	−9.5919
P = 5	−9.8256	−9.5371
P = 6	−9.8076	−9.4665

Covariance matrix:

$$\begin{pmatrix} 0.04190 & 0.000 \\ 0.000 & 0.00025 \end{pmatrix}$$

Covariance matrix:

$$\begin{pmatrix} 0.67048 & 0.000 \\ 0.000 & 0.00057 \end{pmatrix}$$

Transition matrix:

$$A = \begin{bmatrix} 0.96 & 0.04 \\ 0.18 & 0.82 \end{bmatrix}$$

For the expansionary regime, the observed degree of transmission was recorded at −0.29 (see Table 15). This points to a partial degree of exchange rate pass-through where a 1% variation in the exchange rate induced a corresponding −0.29% change in Canadian inflation within an expansion phase. However, the dynamics took on a distinct nature during recession periods. The degree of exchange rate pass-through became complete and notably elevated, reaching a value of 3.9. This signifies that in times of recession, a 1% exchange rate fluctuation led to a substantial 3.9% alteration in Canadian inflation. This duality of exchange rate pass-through degrees accentuates the nuanced interplay between exchange rate variations and inflation trends, underscored by the contrasting dynamics observed within these two economic states. Moving on to the equation governing exchange rate changes, the findings indicate that these fluctuations were influenced by their historical values and, to a lesser extent, the levels of inflation within Canada during both expansion and recession states (see Table 15).

Table 15. Estimation results for regime 1 (expansion): Canada.

	$DL(CAI_t)$ (P_{Value})	$DL(CAD_t)$ (P_{Value})
Const	0.02 (0.1)	−0.0009 (0.4)
$DL(CAI_{t-1})$	−0.11 (0.1)	0.00086 (0.8)
$DL(CAI_{t-2})$	−0.17 (0.01)	−0.0036 (0.3)
$DL(CAD_{t-1})$	−0.29 (0.7)	0.34 (0.00)
$DL(CAD_{t-2})$	0.82 (0.3)	−0.2 (0.0)

By examining the transition matrix and when the current regime is expansionary, there is a robust 96% likelihood of transitioning into another expansion phase, while the probability of transitioning to a recessionary phase is notably lower at 4%. Conversely, when the current state is recessionary, the probability of transitioning into another recessionary phase is registered at 82%. The consistent agreement across the three geographic areas in the sample regarding the anticipated expansionary state in the subsequent period, extending for 25 months, reinforces the robustness and reliability of the analytical approach (see Table 16).

Table 16. Estimation results for regime 2 (recession): Canada.

	$DL(CAI_t)$ (P_{Value})	$DL(CAD_t)$ (P_{Value})
Const	0.0014 (−)	0.0001 (0.9)
$DL(CAI_{t-1})$	−0.23 (0.13)	−0.0002 (−)
$DL(CAI_{t-2})$	−0.33 (0.03)	0.002 (0.6)
$DL(CAD_{t-1})$	−3.90 (0.4)	0.27 (0.06)
$DL(CAD_{t-2})$	−5.37 (0.3)	0.18 (0.13)

Lastly, the Granger causality table offers compelling evidence. The *p*-value, exceeding the 5% threshold, leads us to reject the null hypothesis of no Granger causality between Canadian inflation (CAI) and its exchange rate (see Table 17).

Table 17. Granger causality test (Canada).

	F-Statistic	*p*-Value
RCAD → RCAI	1.7544	0.1749
RCAI → RCAD	0.2551	0.7750

The smoothed probabilities provide us with a representation that takes into account the alternation between periods of expansion and periods of recession within the economic landscape of the country. Figure 5 illustrates the cyclical nature of economic conditions as they fluctuated between periods of growth and contraction.

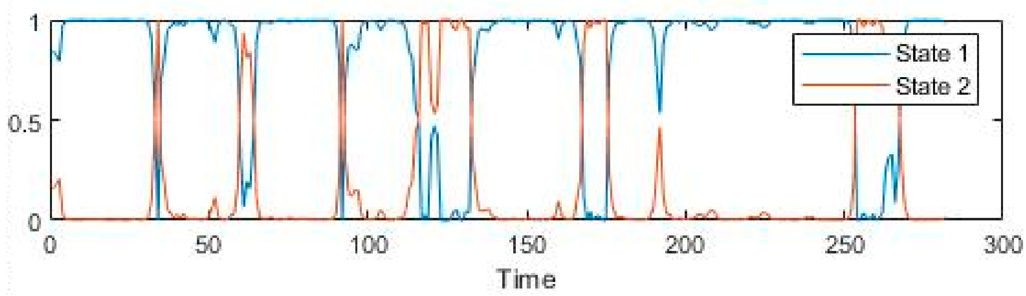

Figure 5. Smoothed states' probabilities for Canada.

Given our primary focus on understanding the impact of shocks in this study, our analytical naturally steers us towards a comprehensive examination of how our variables responded to these disturbances. To this end, we employed impulse response functions. Through their application, we gained valuable insights into the behavior of our variables when subjected to shocks. These enabled us to decipher the intricate patterns that unfolded as our variables reacted to these disruptive forces. By delving into this analysis, we unearthed a wealth of information that shed light on the nuanced dynamics that govern the interplay between variables and shocks. The culmination of this analytical exploration is succinctly encapsulated within the subsequent findings.

The representations provided above offer a comprehensive illustration of the impulse response functions (IRFs) across various lag intervals. The blue line denotes inflation, while the red line signifies the exchange rate. The graphics in the initial column elucidate the reactions of both variables to shocks in inflation, whereas the second column articulates the responses to shocks in the exchange rate.

Upon an examination of the first column, a notable convergence in responses becomes apparent. At the onset of the shock in inflation, an instantaneous and perceptibly substantial surge in inflation was witnessed across all the three geographic areas in the sample. This initial reaction was swiftly succeeded by a decline in the second period, followed by a subsequent, though milder, increase. Notably, the impact of the shock on inflation gradually dissipated, signifying the presence of a decay in its effects.

Shifting our focus to the exchange rate's response, a distinct pattern emerged. The reaction of the exchange rate to an inflation shock was not immediate, manifesting as a minor dip in exchange rates, particularly evident in GBP/USD and CAD/USD. Over time, the influence of the inflation shock waved, consistent with the observed decay effect. Transitioning to the second column of images, which depict the response of variables to a shock in the exchange rate, the response of exchange rates following an exchange rate shock was immediate and robust in the initial period. However, this sharp response rapidly diminished in the subsequent period, effectively nullifying the impact by the third period.

Turning to the inflation response the immediate inflation response is nearly negligible. This is succeeded by a modest fluctuation that attenuates and converges to baseline levels. This pattern is evident within different timeframes, with the attenuation occurring in the third period for Canada, the fourth period for the Eurozone, and the fifth period for the United Kingdom.

In sum, the IRFs illustrated unveil the intricate interplay between variables and shocks, showcasing the dynamics of immediate and subsequent responses. The contrast in behavior between inflation and exchange rate shocks offers valuable insights into the complex interactions that underlie these economic dynamics across different timeframes and regions (see Figure 6).

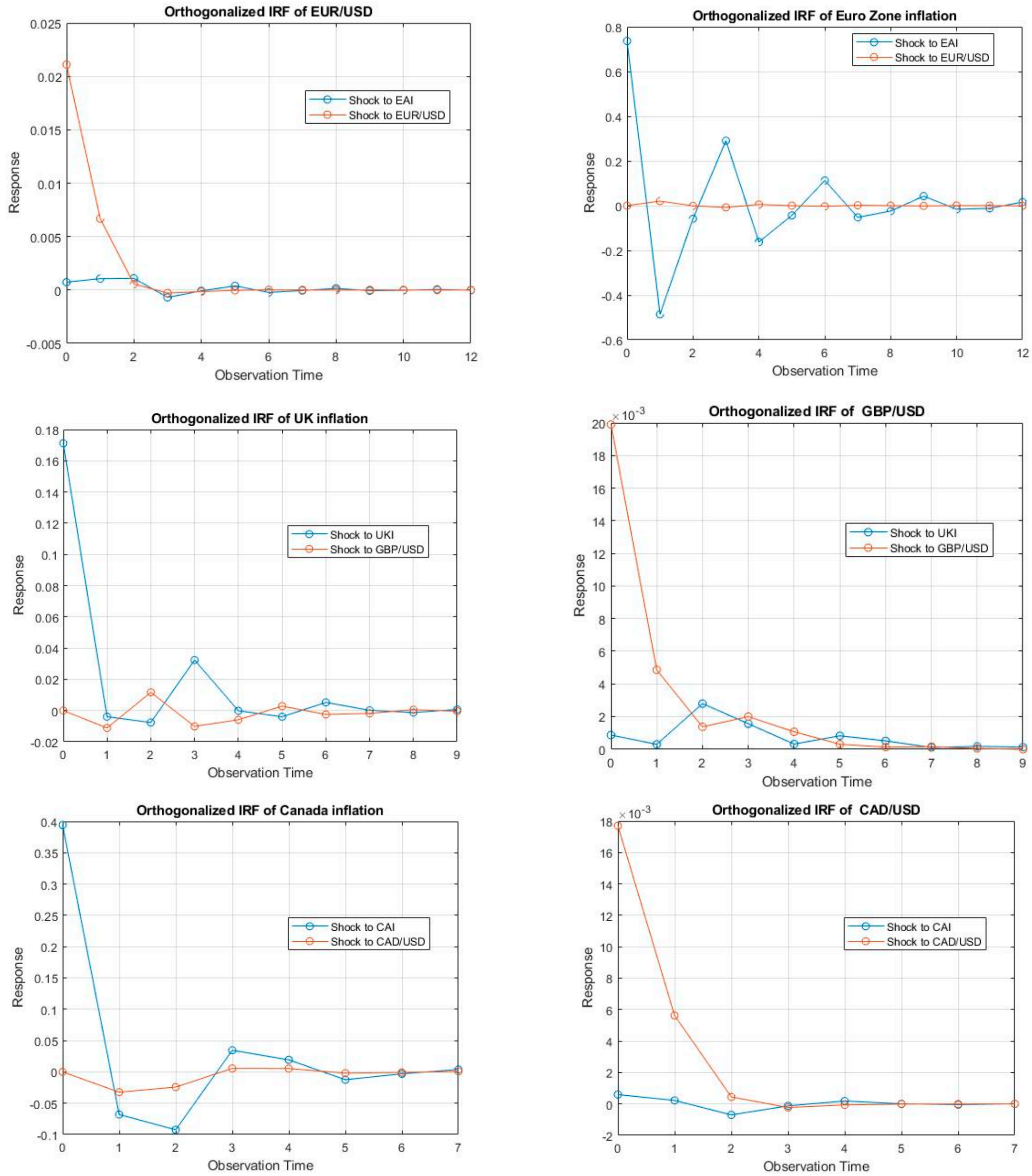

Figure 6. Orthogonalized IRF to inflation rates and exchange rates.

By analyzing the link between the hypotheses described in Section 3 and the results obtained, we can say that the initial hypotheses suggested that exchange rate fluctuations could have a significant impact on a country's inflation rate, due to the concept of transmission of exchange rate variations to the prices of goods and services, also known as exchange rate pass-through. The econometric results seem to confirm these hypotheses, in particular by showing significant relationships between the exchange rate and the inflation rate in the various countries examined.

In this context, the hypotheses initially formulated suggest that exchange rate fluctuations exert a significant influence on inflation, operating through mechanisms such as pass-through and the transmission of the exchange rate to prices. The results obtained from various econometric methods, including impulse response functions, smoothed state probabilities, and Granger causality tests, enrich our understanding of these hypotheses. Firstly, impulse response functions provide insights into the reaction of variables to shocks. They confirm that fluctuations in the exchange rate can induce variations in inflation, thus supporting the idea of a dynamic relationship between these two variables. Secondly, smoothed state probabilities, representing transitions between economic regimes, provide indications of the persistence of different economic states. Their analysis shows that expansionary regimes are characterized by distinct dynamics from recessionary ones, which supports the idea that exchange rate fluctuations can have different implications depending on the economic context.

The results of the Granger causality tests confirm the existence of causality between the inflation rate and the exchange rate for the Eurozone, the United Kingdom, and Canada. The fact that the results of these tests confirm the existence of causal relationships between the exchange rate and inflation reinforces the validity of the initial hypotheses.

6. Conclusions

The exchange rate pass-through holds a central position within the models that central banks rely upon when shaping their monetary policy decisions. Understanding the scope of ERPT's influence on final consumer prices stands as a pivotal endeavor for comprehending the interplay between exchange rates and inflation. This insight not only aids in inflation prediction and the formulation of monetary policy but also casts light on the consequential expenditure shifts prompted by fluctuations in exchange rates, thereby impacting real economic activity.

In this study, our interest laid in discerning the extent of ERPT during both expansionary and recessionary phases. We used the Markov Switching Vector Autoregressive model. Contrary to prevailing scholarly literature, our findings depart from convention. We unveiled a nuanced reality where the degree of exchange rate transmission exhibited partial characteristics during economic expansion. However, when the lens shifted to periods of recession, both the United Kingdom and Canada demonstrated a complete transmission of the exchange rate's influence. An observation emerged; all sampled geographic areas are poised for an upcoming expansion regime, forecasted to endure for a span of 25 months.

Employing impulse response functions, we traced the ripple effects of these shocks on both inflation and exchange rates. Results showed that an inflationary shock triggered an immediate and pronounced response, gradually tapering off over time until equilibrium was regained. The response of exchange rates to inflation shocks, on the other hand, unfurled differently. It was not instantaneous but rather subtle, fading over time. Similarly, for the exchange rate shocks, the initial effect was rapid but diminished over time. However, the impact on inflation was not immediate, and its magnitude was comparatively modest.

To enhance our findings, we propose delving into the causes behind complete transmission during recessions. Adding additional variables, such as uncertainty metrics, to the model could offer a more nuanced understanding. Moreover, extending the analysis to annual data could offer insights into the duration of ERPT, bolstering the depth of comprehension in this intricate economic relationship.

This study has shed light on the significant transmission of exchange rate movements to inflation across the diverse set of three geographic areas studied. However, it is important to note that while our findings provide valuable insights, they are subject to inherent limitations. One critical aspect to consider is the potential omission of crucial variables from our analysis. For example, fiscal policy plays a key role in influencing inflationary pressures within an economy. Government spending, tax policy, and the level of public debt can all affect inflation dynamics. Failure to include these variables can lead to an incomplete understanding of the relationship between exchange rates and inflation. In addition, external shocks, such as geopolitical events or changes in global commodity prices, can exert significant pressure on inflation rates. Ignoring these external factors may obscure the true nature of the relationship between exchange rate movements and inflation. Moreover, the implications of our study underscore the need for central banks to tailor monetary policy to national specificities. Economic conditions, institutional frameworks, and policy preferences differ across countries, affecting the effectiveness of monetary policy measures.

Continuous monitoring of the impact of exchange rate movements on inflation is crucial for effective policymaking. Exchange rate movements can have both direct and indirect effects on inflationary pressures. For example, a depreciation of the domestic currency may lead to higher import prices, thereby fuelling inflation. However, the extent to which exchange rate movements translate into inflationary pressures may vary depending on factors such as the degree of import penetration. Therefore, central banks need to remain vigilant in assessing the evolving relationship between exchange rates and inflation and adjust their policy stance accordingly.

Moreover, diversification of monetary policy instruments is essential to mitigate the impact of exchange rate volatility on inflation. While interest rate adjustments remain a primary tool for influencing inflation trends, central banks should explore alternative measures, such as foreign exchange interventions or macroprudential policies, to complement traditional monetary policy instruments. By diversifying their toolkit, central banks can enhance their ability to manage exchange rate fluctuations and maintain price stability in the face of evolving economic conditions.

To sum up, while our study provides valuable insights into the transmission of exchange rate movements to inflation, it is important to acknowledge its limitations and consider the broader economic context. By addressing these limitations and adopting a nuanced approach to policymaking, central banks can effectively navigate the complexities of the exchange rate–inflation relationship and promote macroeconomic stability.

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