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Approximate Analysis of Multi-State Weighted *k*-Out-of-*n* Systems Applied to Transmission Lines

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Academic Editor: Rui Xiong
Received: 11 September 2017; Accepted: 24 October 2017; Published: 30 October 2017

Abstract: Multi-state weighted *k*-out-of-*n* systems are widely applied in various scenarios, such as multiple line (power/oil transmission line) transmission systems where the capability of fault tolerance is desirable. However, the complex operating environment and the dynamic features of load demands influence the evaluation of system reliability. In this paper, a stochastic multiple-valued (SMV) approach is proposed to efficiently predict the reliability of two models of systems with non-repairable components and dynamically repairable components. The weights/performances and reliabilities of multi-state components (MSCs) are represented by stochastic sequences consisting of a fixed number of multi-state values with the positions being randomly permutated. Using stochastic sequences with L multiple values, linear computational complexities with parameters n and L are required by the SMV approach to compute the reliability of different multi-state *k*-out-of-*n* systems at a reasonable accuracy, compared to the complexities of universal generating functions (UGF) and fuzzy universal generating functions (FUGF) that increase exponentially with the value of n. The analysis of two benchmarks shows that the proposed SMV approach is more efficient than the analysis using UGF or FUGF.

Keywords: multi-state weighted *k*-out-of-*n* system; transmission lines; universal generating function; fuzzy universal generating function; stochastic multi-value approach

1. Introduction

The *k*-out-of-*n* system is widely used to model various industrial systems that require the adoption of redundancy for the purpose of fault tolerance, such as multiple lines transmission systems and production systems consuming multiple resources [1]. A binary-stated *k*-out-of-*n*: G system is anticipated to be reliable unless the number of working components is no less than a pre-specified threshold k [2], and a binary-stated *k*-out-of-*n*: F system is inaccessible when the number of failed components is no less than k [3]. However, some engineering systems such as power/oil lines transmission systems perform the intended tasks at multiple performance levels, which indicates the system's capability (electric power, oil, etc.) from perfectly operational to completely out-of-order [4]. Another example is of a power generator in a power station that can work at full capacity; however, some types of failure cause the generator to completely fail and other failures lead to the generator working at a reduced capacity. The abilities of a power generating system consisting of several power generators to meet different power load demands can be regarded as different system states [5]. Hence, the binary-valued assumption is incapable of accurately reflecting the behaviors of these systems. Instead, these systems can be described as multi-state systems (MSS) [5]. A multi-state *k*-out-of-*n*: G system and a multi-state *k*-out-of-*n*: F system model were investigated in References [6,7].

The performance of a practical system is not only dependent on system operation but is also affected by the total contribution of all the components. The contribution of a component can be quantified and the total contribution of all components is supposed to be above a pre-specified performance level [8]. For example, consider a power station with three components, and each component is a generator. Each component has maybe three different states: 0, 1, 2; and the distribution probability may be different in each state. The power outputs of a component are 0 megawatts (MW), 10 MW, and 100 MW, corresponding the states 0, 1, 2. The total power output of the system is the weighted sum of each component. When the total output is greater than 100 MW, the system is in state 2; when the output is greater than 50 MW, the system is in state 1; otherwise, it is in state 0. The reliability of a system is defined by considering its structure and the weight of each component [9]. Then, the proposed models for binary and multi-state k-out-of-n systems are generalized into binary and multi-state weighted models [10–12]. For a weighted k-out-of-n system, the weight of the component i in state j is assigned to be w_{i_j}. The system is supposed to be correctly working only if the total weight of the components is no less than the pre-determined value of K.

To perform the reliability analysis of a k-out-of-n system, the method of universal generating function (UGF) is adopted. This concept was introduced in Reference [13] and it was first utilized to predict the reliability of a power system in Reference [14]. In previous research [4,15–18], the UGF technique was used to determine the reliability of several multi-state systems, which are composed of series, parallel, series-parallel, and bridge structures. The required computational complexity analysis of the UGF technique to evaluate the reliability of a multi-state weighted k-out-of-n system is performed in Reference [11]. The number of system states increases dramatically with the system scale (the adoption of more components) [19], so the UGF method incurs a large computational complexity for systems consisting of many components or multi-state components. In order to reduce the computational complexity, a fuzzy universal generating function (FUGF) method is presented in References [20,21]. It combines the fuzzy set theory [22,23], UGF, and a clustering technique [24] to obtain the reliability of a multi-state weighted k-out-of-n system. However, the clustering technique in FUGF is likely to reduce the prediction accuracy of the results considerably and usually too much time is consumed.

The UGF and FUGF methods can be used to perform exact and approximate performance assessments of a multi-state k-out-of-n system, respectively. However, the above methods are mainly focused on the steady state study of a multi-state k-out-of-n system with the assumption of non-repairable property for the component. In practice, the corresponding state probability may gradually change with time because of the aging process [25]. When considering repairable operations, the performance of a component can not only transmit to a lower level, but also return to a higher level unless the component is completely failed. A dynamic general k-out-of-n system with repairable components is investigated in Reference [26], and the discrete time Markov chain and UGF are combined to predict the system reliability. However, aiming to perform the analysis of a complex system, a large complexity is required according to the state transition matrix (STM).

In this paper, a stochastic multiple-valued (SMV) approach is presented that aims to predict the reliability of different multi-state k-out-of-n systems. The weights and reliabilities of components are encoded into randomly permuted multiple-valued sequences. The corresponding proposed stochastic architectures are composed of stochastic multi-valued logic gates. By propagating the multiple-valued sequences through the above stochastic architecture, an output sequence can be inherently obtained. Then, the corresponding reliability is predicted by analyzing the output sequence of the proposed model.

The remainder of the paper is organized as follows: In Section 2, some fundamentals related to UGF, FUGF, and the two models of multi-state weighted k-out-of-n systems are reviewed. In Section 3, corresponding stochastic models are presented for a multi-state k-out-of-n system with non-repairable components as well as for a dynamic system with repairable components. Then, the proposed stochastic multi-valued models are validated by a theoretical proof. Section 4 gives some case studies and a comparison of simulation results with different methods. Finally, Section 5 concludes the paper, as well as provides some future work.

2. Multi-State *k*-Out-of-*n* System and UGF, FUGF Methods

In this section, two types of multi-state weighted *k*-out-of-*n* systems [11], using universal generating function (UGF) and fuzzy universal generating function (FUGF), are presented.

2.1. Multi-State Weighted k-Out-of-n System

The definition of model I of multi-state weighted *k*-out-of-*n* systems is given as follows [11]:

Definition 1. *In a system with n components, each component can be in M + 1 states: 0, 1, 2, ..., M. Component i ($1 \le i \le n$) in state j carries a weight of w_{i_j} with a probability of p_{i_j} ($0 \le j \le M$). The system is in state j or a higher state if the total weights of all the components is greater than or equal to K, a pre-defined value. Then, we have $P_r\{\phi \ge j\} = P_r\{W \ge K\}$, where ϕ is the structure function of a system and W describes the total weight of all components.*

Because in some systems the components whose states are below *j* do not make any contribution for the system to be in state *j* or a higher state, then the model II is described below.

Definition 2. *The system is in state j or a higher state if the sum of the weights of the components in state j or a higher state is greater than or equal to K. We can have $P_r\{\phi \ge j\} = P_r\{W \ge K\}$, where ϕ is the structure function of a system and W denotes the total weight of the components in state j or a higher state.*

2.2. UGF and FUGF

For UGF, the output performance distribution (OPD) is described by a polynomial function of $U_S(z)$. The function relates to the weight w_j, and the corresponding probability (reliability) P_j of the system in the state *j* is given by:

$$U_s(z) = \sum_{j=0}^{M} P_j \cdot z^{w_j}, \tag{1}$$

where *M* is the highest state of the system. The probability distribution (PD) of component *i* can be obtained by:

$$u_i(z) = \sum_{j=0}^{M} p_{i_j} \cdot z^{\alpha(w_{i_j} - l)}, \left(w_{i_j} - l\right) = \begin{cases} w_{i_j}, w_{i_j} \ge l \\ 0, w_{i_j} < l \end{cases}. \tag{2}$$

where *l* is zero and a pre-defined non-zero value in model I and model II of the multi-state weighted system, respectively. p_{i_j} represents the corresponding probability if component *i* is supposed to be in state *j* ($0 \le j \le M$), while w_{i_j} denotes the corresponding weight of the above scenario, and $w_{i_j} \ge 0$.

In order to obtain the PD of a multi-state weighted *k*-out-of-*n* system with an arbitrary structure function ϕ, a composition operator Ω_ϕ is represented by the UGF of *n* system components as follows [11]:

$$\begin{aligned} U_s(z) &= \Omega_\phi\{u_1(z), \ldots, u_n(z)\} \\ &= \Omega_\phi\{\textstyle\sum_{j=0}^{M} \mathrm{p}_{1_j} \cdot z^{\alpha(w_{1_j} - l)}, \ldots, \sum_{j=0}^{M} p_{n_j} \cdot z^{\alpha(w_{n_j} - l)}\}, \\ &= \textstyle\sum_{j=0}^{M} \cdots \sum_{j=0}^{M} p_{1_j} \ldots p_{n_j} \cdot z^{\alpha(w_{1_j} - l) + \cdots + \alpha(w_{n_j} - l)}, \end{aligned} \tag{3}$$

where $U_s(z)$ represents the OPD of the system.

Then, we can obtain the reliability of a system for any required weight (*K*) using the operator δ_A in Equation (4), when the UGF of a multi-state weight *k*-out-of-*n* system is obtained:

$$R_s(K) = \delta_A(U_s(z), K) = \delta_A\left(\sum_{j=0}^{M} P_j \cdot z^{w_j}, K\right) = \sum_{j=0}^{M} P_j \cdot \alpha(w_j - K), \tag{4}$$

where $\alpha(x) = \begin{cases} 1\,, x \geq 0. \\ 0\,, x < 0. \end{cases}$

In some systems, the dimension of the weight vector/state vector of subsystems is too large. To reduce the computational overhead, a fuzzy universal generating function (FUGF) is proposed [19,21]. The weight vector and state vector of the subsystem are divided into some clusters [24], and these clusters are represented by the fuzzy values. Hence, a multi-state weight of k-out-of-n system with n components is transformed into a fuzzy multi-state weight of k-out-of-n system with n^* subsystems, and each subsystem i^* consists of n_{i^*} components. The PD of subsystem i^* can be determined by adopting the UGF method [21].

3. The Stochastic Multiple-Valued Models

3.1. Stochastic Multiple-Valued Logic

For stochastic computation, signal probability is indicated by a random binary bit stream [27]. In the binary steam, a proportional number of bits is set to be 1, aiming to denote the signal probability. Stochastic computation can also perform a probabilistic analysis of multiple-valued signals by using logic gates. For certain signals with a discretization level of m, the probability vector is usually described as $P = [p_{m-1} \ldots p_0]$, with $\sum_{h=0}^{m-1} p_h = 1$. In Figure 1, a probability vector of a ternary-valued signal is indicated by a multi-valued sequence [28].

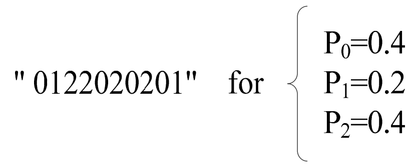

Figure 1. An illustration of the encoding process of a ternary-valued probability vector with a 10 digit sequence of 0, 1, 2.

A multiple-valued smaller (MVS) operator and a multi-valued equal or larger (MVEL) operator perform the functions:

$$MVS(A < a) = \begin{cases} A, A < a \\ 0\,, A \geq a \end{cases}. \tag{5}$$

$$MVEL(A \geq a) = \begin{cases} A, A \geq a \\ 0\,, A < a \end{cases}. \tag{6}$$

The ternary-valued gates such as an inverter, an MVS operator, an MVEL operator, and a 4-to-1 multiplexer are introduced in Figure 2.

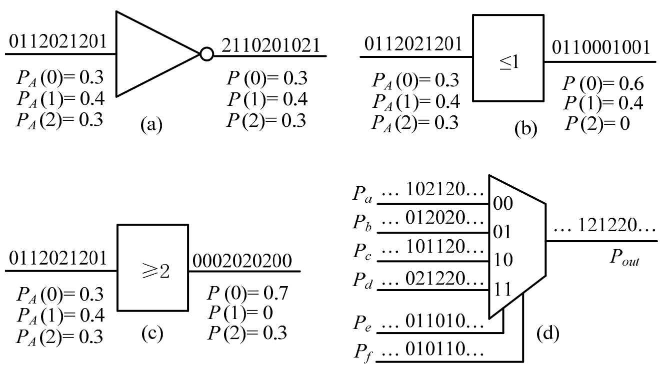

Figure 2. Stochastic logic gates: (**a**) a ternary-valued inverter; (**b**) an MVS logic; (**c**) an MVEL logic; (**d**) a 4-to-1 multiplexer.

As discussed in Reference [27], the result of stochastic computational analysis is usually a probabilistic value because of the inevitable fluctuations of stochastic computation. For a Bernoulli sequence S_a, which is adopted to encode probability P_a, the final result P_{a} is usually different from P_a due to random fluctuations. If the length of S_a is L, the fluctuation error e_f is assumed to be the difference between P_a and P_{a}. It can be approximated as [29]:

$$e_f = \sqrt{\frac{P_a(1 - P\_{a})}{L}}. \quad (7)$$

Equation (7) indicates that the fluctuation error can be reduced by adopting a longer sequence length. It has been shown that stochastic fluctuation can be significantly attenuated if the initial inputs are non-Bernoulli sequences; here, the corresponding sequence is composed of fixed numbers of 1s and 0s while the positions are randomly permutated [30]. Furthermore, the application of non-Bernoulli sequences is capable of reducing the result inaccuracy. Hence, the stochastic multi-valued sequence in Reference [28] is adopted to deal with multi-valued scenarios and utilized as the input of stochastic multi-valued models proposed in this paper.

3.2. SMV Models for a Multi-State k-Out-of-n System with Non-Repairable Components

For the stochastic multi-valued model, the weights of components are encoded in the multi-values stochastic sequences. As shown in Figure 3, the weights and reliabilities of component i at different states are simultaneously encoded into stochastic multi-value sequences.

State	0	1	2	3
Reliabilities	0.1	0.1	0.5	0.3
weight	1	1.5	2	3

↓ encoded in a stochastic sequence

1, 2, 1.5, 2, 2, 2, 3, 2, 3 ,3

Figure 3. The stochastic encoding of the weights and reliabilities of component i using a multi-valued sequence with sequence length L = 10.

The weights and PD of multi-state weighted k-out-of-n systems consisting of non-repairable components can be obtained by the output sequence of an add operation, as shown in Figure 4.

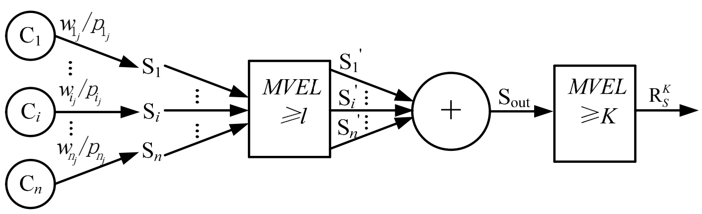

Figure 4. The stochastic multi-valued model for a multi-state weighted k-out-of-n system consisting of non-repairable components.

In Figure 4, w_{i_j} and p_{i_j} are the weight and probability of component C_i in state j, the sequence S_i encodes the weights and reliabilities of C_i in all states, and the total number of values in sequence S_i is L, $i \in [1, n]$, $j \in [0, M]$. The sequence $S_i{}'$ is the output of an MVEL gate. As defined in Equation (2), the value of l can be zero or a pre-defined non-zero value—it is determined by model I or model II of the multi-state weighted k-out-of-n system. The sequence S_{out} represents the sum of sequence $S_i{}'$, and the weights and the PD of the system can be obtained from S_{out}. K is the required weight of the system. R_S^K is a proportional number of non-zero values in S_{out}; it denotes the reliability of the system for the required weight K.

3.3. SMV Models for a Multi-State k-Out-of-n System with Repairable Components

For some systems, the corresponding state of a component may gradually change with time due to aging. Hence, a multi-state weighted k-out-of-n system that considers the time factor was proposed in Reference [26] with the incorporation of dynamic behavior. A state of each component may be changed to a lower state. In addition, the component can return to a higher state when the component is repairable. Assuming that a time period T is divided into a number of time intervals Δ_T, the representation of the states for a repairable component i within the given period T are presented in Figure 5.

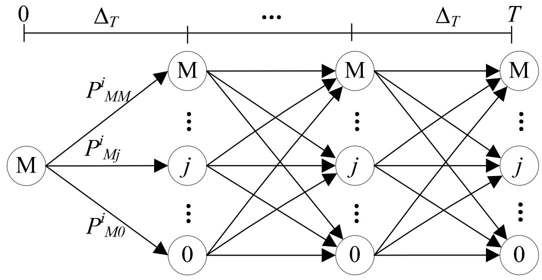

Figure 5. Different states of a component i during the time period T.

Let component i be in state M (which is being regarded as perfectly operational) at the start of the mission time. P^i_{Mj} denotes the probability transforming from the state M to another state j of component i. Hence, component i has a state transition matrix (STM) as:

$$STM^i = \begin{bmatrix} P^i_{00} & \cdots & P^i_{0M} \\ \vdots & \ddots & \vdots \\ P^i_{M0} & \cdots & P^i_{MM} \end{bmatrix} \begin{matrix} S^i_0 \\ \vdots \\ S^i_M \end{matrix} . \quad (8)$$

In Equation (8), $\forall i$, $\sum_{j=0}^{M} P^i_{mj} = 1$, $m \in [0, M]$. Let $P^i_j(t)$ and $P^i_m(t-1)$ indicate the probabilities of component i in state j at time t and in state m at time $t-1$, respectively. At time $t-1$, any state of component i can be changed to state j, and the transition probability is P^i_{mj}. Based on STM^i and Figure 5, $P^i_j(t)$ is given by:

$$P^i_j(t) = \sum_{m=0}^{M} P^i_m(t-1) \cdot P^i_{mj},\ i\ [1,\ n],\ t\ \ [0,\ \frac{T}{\Delta_T}], \quad (9)$$

where $P^i_M(0) = 1$, $P^i_h(0) = 0$, $h \in [0, M-1]$.

The stochastic multi-valued architecture for a dynamic multi-state weighted k-out-of-n system with repairable components is presented as in Figure 6. Here, the transition probability $P^i{}_{mj}$ ($m, j \in [0, M]$; $i \in [1, n]$) and corresponding states are encoded in sequence $S^i{}_m$ with length L (as shown in Equation (8)). A proportional number of 0s in sequence $S^i{}_m(t-1)$ denotes the corresponding probability of component i being in state m at time point $t-1$ (i.e., $P^i{}_m(t-1)$). In sequence $S^i{}_m(t-1)$ with length L, $P^i{}_m(t-1) \cdot L$ digits are valid, $L \times (1 - P^i{}_m(t-1))$ digits are filled with an invalid number V (i.e., elements in sequence $S^i{}_m(t-1)$ are only '0' and V), V is used to compensate sequence $S^i{}_m(t-1)$, and '0' ensures that the maximum state of component i is still M at time point t. If sequences $S^i{}_m$ and $S^i{}_m(t-1)$ are the inputs of an add operation, the proportional number of valid values in the output sequence indicates the probability of component i being in the state of m at time point t. Sequence $S^i(t)$ denotes the PD of components i in different states at time t. The states in the sequence $S^i(t)$ are replaced by the corresponding weights to be sequence $S^{i\prime}(t)$ by the block STW (state to weight). The sequences $S^{i\prime}(t)$ are the inputs of the model for a multi-state weighted k-out-of-n system with non-repairable components (referred to as MMWN) in Figure 4. $R^K{}_S$ (t) denotes the corresponding system reliability meeting the requirement of a pre-specified value of K at time t.

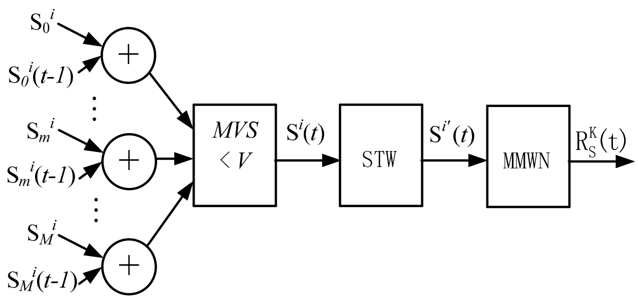

Figure 6. The stochastic multi-value model for a dynamic multi-state weighted *k*-out-of-*n* system with repairable components.

3.4. Validation of the Stochastic Multi-Valued Models

To validate the proposed stochastic multi-valued model for a multi-state weighted *k*-out-of-*n* system with non-repairable components, a theoretical proof is first presented.

Theorem 1. *The product of polynomials* $u_1(z) = \left(\sum_{j=0}^{M} P_j \cdot z^{w_j}\right)$ *and* $u_2(z) = \left(\sum_{i=0}^{M} P_i \cdot z^{w_i}\right)$ *, where* P_j *and* w_j *are encoded in a stochastic multi-valued sequence* s_1 *, and* P_i *and* w_i *are encoded in a stochastic multi-valued sequence* s_2 *as in Figure 3, is encoded in the sum of* s_1 *and* s_2*.*

Proof of Theorem 1. Assuming $n = 2$ in Equation (3), the probability mass functions (PMFs) of two components (u_1 and u_2) indicate the reliability and weight of each state for a component. As per Equation (2), u_1 and u_2 can be represented by polynomials $u_1(z)$ and $u_2(z)$. The reliabilities and the weights of all states for component i are encoded in the stochastic multi-valued sequence s_i, $i \in [1, n]$.

As shown in Reference [29], the product of polynomials $u_1(z)$ and $u_2(z)$ can be expressed as a convolution of u_1 and u_2. Equation (3) performs a polynomial multiplication. Hence, Equation (3) can be described by:

$$U_s(z) \; = \Omega_\phi\{u_1(z),\; u_2(z)\} \; = u_1 \otimes u_2, \tag{10}$$

where $u_1 \otimes u_2$ denotes the convolution of u_1 and u_2.

The PD of the sum of two or more independent random variables is the convolution of their individual distributions [31], so we have:

$$u_1 \otimes u_2 = u_{temp}, \tag{11}$$

where u_{temp} is the PMF of a temporary variable; the weights of the variable are obtained by summing the two components' weights.

In the stochastic multi-valued models, the probabilities and weights of the two components are encoded in the stochastic multi-value sequences s_1 and s_2, and the number of the multi-values in these sequences is L. In other words, sequences s_1 and s_2 encode u_1 and u_2, respectively. u_{temp} is then encoded by summing the sequence s_1 and s_2. This completes the proof.

Hence, the proposed model for a multi-state weighted *k*-out-of-*n* system with non-repairable components shown in Figure 4 is validated.

The validation of the proposed stochastic multi-value model for a dynamic multi-state weighted *k*-out-of-*n* system with repairable components is presented as follows.

As defined in Equation (1), the transition probabilities P_{mj}^i of component i and its states are given by:

$$u_m^i = \sum_{j=0}^{M} P_{mj}^i \cdot z^j, \tag{12}$$

where $M + 1$ is the number of states. Let $u_m^i(t-1)$ denote the probability of component i at time $t-1$ (i.e., $P_m^i(t-1)$); then, we have:

$$u_m^i(t-1) = P_m^i(t-1) \cdot z^0 + \left(1 - P_m^i(t-1)\right) \cdot z^v, \tag{13}$$

where v is an invalid number. Let $U_m^i(t)$ denote the performance distribution of component i in state m at time t; thus, we obtain:

$$\begin{gathered} U_m^i(t) = \Omega_\phi\{u_m^i, u_m^i(t-1)\} \\ = \sum_{j=0}^{M} P_{mj}^i P_m^i(t-1) \cdot z^{j+0} + \sum_{j=0}^{M} P_{mj}^i (1 - P_m^i(t-1)) \cdot z^{j+v} \\ = \sum_{j=0}^{M} P_{mj}^i P_m^i(t-1) \cdot z^j. \end{gathered} \tag{14}$$

assuming that the transition probabilities P_{mj}^i and the states of component i are encoded in a stochastic multi-value sequence s_m^i. In the sequence $S_m^i(t-1)$, a probability of component i at time $t-1$ in state m (i.e., $P_m^i(t-1)$) is encoded as a proportional number of 0s, and the other values in the sequence are invalid numbers v ($m, j \in [0, M]$; $i \in [1, n]$).

As per *Theorem* 1, Equation (14) can be obtained by summing the stochastic multi-value sequences S_m^i and $S_m^i(t-1)$, and Equation (14) is equivalent to Equation (9). Hence, the reliability of component i at time t in state m can be obtained by summing the stochastic multi-value sequences S_m^i and $S_m^i(t-1)$.

In a stochastic multi-value model for a dynamic multi-state weighted k-out-of-n system with repairable components (as shown in Figure 6), when the probability of component i in state m at time t, and $P_m^i(\text{t})$ is used for the inputs of the MMWN, then the reliability of the system can be obtained accordingly.

4. Analysis of Illustrative Examples

In this section, consider the multiple line transmission system proposed in Reference [1] to investigate the efficiency and the accuracy of the proposed SMV approach. As shown in Figure 7, power/oil is delivered from the source to five stations through six transmission lines. Any lines with weight/performance that characterizes the flow available at its different points connected to the stations can be considered as a multi-state component [1]. The system depicted in Figure 7 can be regarded as a multi-state weighted k-out-of-n system with $n = 6$, $M = 5$, where n is the number of multi-state components (lines), and M is the highest state of the lines. Each transmission line has six states. Different states of each transmission line i are described by different random weight vector w_i consisting of w_{i_j}, where w_{i_j} is the weight/performance of line i in state j, $i \in [1, n]$ and $j \in [1, M]$. Different demands are required by different stations.

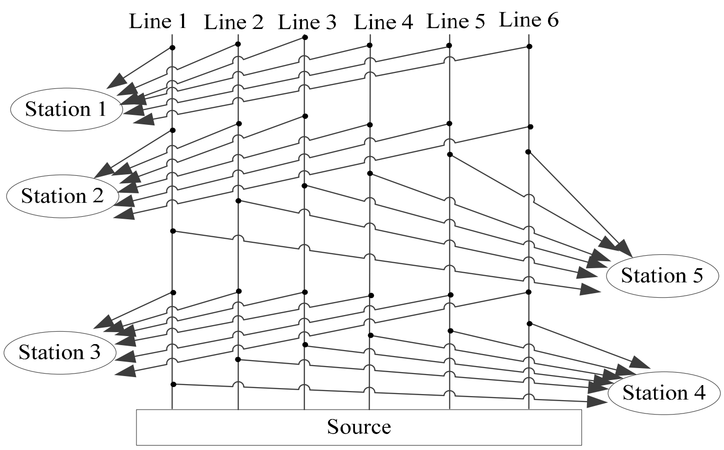

Figure 7. Multiple line transmission system [1].

In order to investigate more complex systems, for the same structure as in Figure 7, larger values of n and M may be selected. The results are compared with those obtained by adopting the UGF [11,26] and FUGF methods [19,21]. The computer programs for all approaches are developed in MATLAB, and all simulations are run on a computer with a 3.4-GHz Intel microprocessor with 16 GB memory.

4.1. Analysis of a Multiple Line Transmission System with Non-Repairable Components

For a reliability evaluation of a multiple line transmission system using model I, the computational complexity of the UGF method is $O\left((M+1)^n\right)$ [21]. For model II, the computational complexity of the UGF method for state j or a higher state is $O\left((M+2-j)^n\right)$ [19]. For a multiple line transmission system divided into n^* subsystems, the computational complexities of the FUGF method are $O\left((M+1)^{n^*}\right)$ and $O\left((M_*+1)^{n^*}\right)$ for model I and II, respectively [19], when n^* is large enough (M_* is the maximum possible state in the subsystem). However, when n^* is a small value, FUGF will incur a large computational overhead due to the clustering operation [19].

The computational complexity of the SMV approach for a multiple line transmission system is presented in this section. For reliability evaluation using model I, $(n-1)\cdot L$ multiple-valued logic add operations and L MVEL operations are required; hence, the computational complexity of the SMV approach with sequences is $O(n\cdot L)$ for an accuracy indicated by Equation (7), where L is the number of multi-values in the sequence. For model II, $(n-1)\cdot L$ multiple-valued logic add operations, L MVEL operations, and L MVS operations are required; hence, the computational complexity of the SMV approach is $O((n+1)\cdot L)$ for an accuracy indicated by Equation (7).

Several examples are used to compare the efficiency and accuracy of the SMV approach with the UGF [11,26] and FUGF [19,21] methods. The central processing unit (CPU) time is obtained for n components, and the highest state value of component M. The weights of components are real values randomly selected from (0, 10), and '0' indicates the weight of the component in state 0. The probability of component i in state j is randomly selected from (0, 1), and meets the requirement of $\sum_{j=0}^{M} p_{ij} = 1$. The reliabilities obtained by different methods are compared in order to evaluate the accuracy of the SMV approach.

4.1.1. Model I

The run time of different methods are presented in Tables 2 and 3, for a system with n = 6 and 12 components and a pre-defined weight threshold value $K = 0.5\sum_{i=0}^{n} w_i M$. As shown in Tables 1 and 2, the CPU time required by the UGF and FUGF methods increases with the number of component states $M + 1$. However, the CPU time using the SMV approach is not influenced by the number of component states, but increases with the length of stochastic sequences. When the system has six components, the UGF method is more efficient than the SMV approach which L (the sequence length) is 10 k (k =1000), only when the value of M is 10 or less. When the system has 12 components, the SMV approach which L is 1 k is faster than the UGF and FUGF methods for all M values.

Table 1. Efficiency comparison for a system of model I with six components.

The M Value	CPU Time (s) by UGF (T_1)	CPU Time (s) by FUGF (T_2)	CPU Time (s) by SMV Approach (T_3)	
			L = 1 k	L = 10 k
5	0.00383	0.80163	0.02026	0.21170
10	0.10356	1.92038	0.02049	0.21593
15	0.40486	3.74018	0.02128	0.21545
20	2.32911	4.60791	0.02023	0.22064
25	10.9098	5.40370	0.02018	0.21960
30	31.4016	6.97095	0.02104	0.22146

Table 2. Efficiency comparison for a system of model I with 12 components.

The M Value	CPU Time (s) by UGF (T_1)	CPU Time (s) by FUGF (T_2)	CPU Time (s) by SMV Approach (T_3)	
			L = 1 k	L = 10 k
5	6.4141	1.4204	0.05372	0.54842
8	726.37	4.7341	0.05447	0.55142
10	6173.62	16.2819	0.05381	0.55647
12	/	53.3925	0.05523	0.56025
15	/	201.8218	0.05695	0.56324

When a system has six components with 15 states for each component, Table 3 illustrates the accuracy of the comparison between the FUGF and SMV approaches. The results reveal that absolute and relative errors calculated by the SMV approach which L is 10 k are smaller than that calculated by FUGF in all cases. In short, the accuracy of the SMV approach is higher than that of FUGF.

Table 3. Accuracy of the SMV approach and FUGF [19,21], compared with UGF [11,26], when a system of model I has six components with 15 states.

The k_j Value	Reliability by UGF	Reliability by FUGF	Reliability by SMV Approach with L = 10 k	Absolute/Relative Error by FUGF	Absolute/Relative Error by SMV Approach with L = 10 k
0	1.0000	1.0000	1.0000	0/0%	0/0%
10	0.9920	0.9913	0.9920	0.0007/0.071%	0/0%
20	0.7727	0.7468	0.7716	0.0059/0.763%	0.0011/0.142%
30	0.2461	0.2414	0.2440	0.0047/1.909%	0.0021/0.853%
40	0.0902	0.0955	0.0930	0.0053/5.875%	0.0028/3.104%
50	0.0033	0.0038	0.0030	0.0005/15.15%	0.0003/9.090%
60	0.000001	0	0	0.000001/0%	0.000001/0%

4.1.2. Model II

For this model, when a system has n = 6 and 12 components, the run time of different methods are shown in Tables 4 and 5. The pre-defined weight K is $0.5\sum_{i=0}^{n} w_i M$, the value of l defined in Equation (2) is 2.

Table 4. Efficiency comparison for a system of model II with six components.

The M Value	CPU Time (s) by UGF (T_1)	CPU Time (s) by FUGF (T_2)	CPU Time (s) by SMV Approach (T_3)	
			L = 1 k	L = 10 k
5	0.00261	0.21352	0.02717	0.28382
10	0.09704	0.52451	0.02802	0.28211
15	0.37363	1.3736	0.02829	0.28575
20	1.8075	2.1417	0.02874	0.28298
25	6.1865	3.5425	0.02939	0.28638
30	19.2374	4.7721	0.02985	0.28673

Table 5. Efficiency comparison for a system of model II with 12 components.

The M Value	CPU Time (s) by UGF (T_1)	CPU Time (s) by FUGF (T_2)	CPU Time (s) by SMV Approach (T_3)	
			L = 1 k	L = 10 k
5	0.08263	0.96846	0.06119	0.65196
8	1.5327	1.2458	0.06161	0.65514
10	2.1682	2.6382	0.06793	0.65930
12	18.9435	3.7936	0.06534	0.66064
15	22.3949	5.8395	0.06759	0.66199

As revealed in Table 4, UGF and FUGF are faster than the SMV approach which L is 1 k or 10 k, when M is 5 or less. As shown in Table 5, when the system has 12 components, the SMV approach which L is 1 k or 10 k is more efficient than the UGF and FUGF methods for all M values. The results in Tables 4 and 5 show that the run time of the UGF and FUGF methods increases with the number of components n and states $M + 1$, and the run time of the SMV approach increases with the number of components n and the length of the sequence. As shown in Table 6, when the system has six components with 15 states, the system reliability obtained by the SMV approach which L is 10 k is more accurate than that obtained by FUGF.

Table 6. Accuracy of the SMV approach and FUGF [19,21], compared with UGF [11,26], when a system of model II has six components with 15 states.

The k_j Value	Reliability by UGF	Reliability by FUGF	Reliability by SMV Approach with L = 10 k	Absolute/Relative Error by FUGF	Absolute/Relative Error by SMV Approach with L = 10 k
0	1.0000	1.0000	1.0000	0/0%	0/0%
10	0.9897	0.9986	0.9894	0.0011/0.111%	0.0003/0.030%
20	0.6946	0.6903	0.6924	0.0043/0.619%	0.0022/0.316%
30	0.2275	0.2204	0.2246	0.0071/3.120%	0.0029/1.274%
40	0.0774	0.0731	0.0769	0.0043/5.555%	0.0025/3.229%
50	0.0027	0.0020	0.0024	0.0007/25.92%	0.0003/11.11%
60	0.000001	0	0	0.000001/0%	0.000001/0%

4.2. Analysis of a Multiple Line Transmission System with Repairable Components

An example is introduced to validate the effectiveness of the proposed stochastic multi-value model in Figure 6. Similar to the system structure shown in Figure 7, a dynamic multi-state line transmission system is considered here with six repairable components/transmission lines and five stations. Each component has six possible states. It is assumed that five time periods have been considered for the working period of the system. The weights of each component in each state are provided in Table 7.

Table 7. The weights/performance of each component in each state.

State / Component (Line)	0	1	2	3	4	5
1	0	2	3	4	6	7
2	0	1	2	4	5	6
3	0	1	2	3	5	6
4	0	1	3	4	6	7
5	0	1	2	3	4	5
6	0	3	4	5	6	7

The state transition matrix for component i (STM^i) denotes the transition probabilities of component i.

$$\mathrm{STM}^{1} = \begin{bmatrix} 0.2 & 0.15 & 0.25 & 0.15 & 0.1 & 0.15 \\ 0.1 & 0.5 & 0.1 & 0.15 & 0.05 & 0.1 \\ 0.05 & 0.05 & 0.55 & 0.15 & 0.1 & 0.1 \\ 0.03 & 0.1 & 0.17 & 0.6 & 0.05 & 0.05 \\ 0.03 & 0.07 & 0.2 & 0.15 & 0.45 & 0.1 \\ 0.05 & 0.05 & 0.1 & 0.2 & 0.2 & 0.4 \end{bmatrix}, \mathrm{STM}^{2} = \begin{bmatrix} 0.4 & 0.2 & 0.1 & 0.15 & 0.1 & 0.05 \\ 0.1 & 0.45 & 0.15 & 0.15 & 0.1 & 0.05 \\ 0.05 & 0.05 & 0.5 & 0.25 & 0.1 & 0.05 \\ 0.06 & 0.14 & 0.2 & 0.5 & 0.05 & 0.05 \\ 0.08 & 0.1 & 0.12 & 0.2 & 0.4 & 0.1 \\ 0.05 & 0.1 & 0.15 & 0.2 & 0.1 & 0.4 \end{bmatrix}$$

$$\text{STM}^{3} = \begin{bmatrix} 0.4 & 0.2 & 0.15 & 0.1 & 0.1 & 0.05 \\ 0.1 & 0.5 & 0.15 & 0.15 & 0.05 & 0.05 \\ 0.1 & 0.2 & 0.4 & 0.15 & 0.05 & 0.1 \\ 0.05 & 0.1 & 0.15 & 0.5 & 0.15 & 0.05 \\ 0.05 & 0.05 & 0.1 & 0.15 & 0.5 & 0.15 \\ 0.05 & 0.05 & 0.1 & 0.1 & 0.15 & 0.55 \end{bmatrix}, \text{STM}^{4} = \begin{bmatrix} 0.45 & 0.2 & 0.15 & 0.1 & 0.05 & 0.05 \\ 0.15 & 0.5 & 0.15 & 0.1 & 0.05 & 0.05 \\ 0.05 & 0.1 & 0.6 & 0.1 & 0.1 & 0.05 \\ 0.05 & 0.15 & 0.1 & 0.55 & 0.1 & 0.05 \\ 0.1 & 0.1 & 0.05 & 0.15 & 0.5 & 0.1 \\ 0.05 & 0.1 & 0.15 & 0.1 & 0.05 & 0.55 \end{bmatrix}$$

$$\text{STM}^{5} = \begin{bmatrix} 0.35 & 0.25 & 0.15 & 0.1 & 0.1 & 0.05 \\ 0.1 & 0.5 & 0.1 & 0.15 & 0.05 & 0.1 \\ 0.05 & 0.15 & 0.5 & 0.15 & 0.1 & 0.05 \\ 0.2 & 0.1 & 0.05 & 0.55 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.2 & 0.2 & 0.45 & 0.05 \\ 0.1 & 0.1 & 0.05 & 0.05 & 0.2 & 0.5 \end{bmatrix}, \text{STM}^{6} = \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.1 & 0.05 & 0.05 \\ 0.15 & 0.55 & 0.1 & 0.1 & 0.05 & 0.05 \\ 0.1 & 0.1 & 0.5 & 0.15 & 0.1 & 0.05 \\ 0.06 & 0.13 & 0.21 & 0.5 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.1 & 0.5 & 0.4 & 0.1 \\ 0.05 & 0.05 & 0.1 & 0.1 & 0.15 & 0.55 \end{bmatrix}$$

The reliability and accuracy of model I and model II systems in each period using the UGF, FUGF, and SMV methods are provided in Tables 8 and 9. Accuracy is defined as the ratio of the disparity between an approximate reliability and the accurate value over the accurate result. The pre-defined weight K is 20 in the system of model I and model II. In model II, a component does not make a contribution to the system when the states of the components are below j, and the weight threshold in component state j is set to be 2. From Tables 8 and 9, the results indicate that the SMV approach which L is 1 k or 10 k is faster than the UGF method for the model I and model II systems. The SMV approach which L is 1 k or 10 k is more efficient and more accurate than the FUGF method for the systems of model I and model II.

Table 8. The reliability and accuracy of a dynamic system of mode I in each period using UGF, FUGF, and SMV methods; L: The sequence length for the SMV approach.

	Reliability/Accuracy in Each Period						CPU Time (s)
	0	1	2	3	4	5	
UGF [11,26]	1	0.9557	0.8074	0.6717	0.5870	0.5401	0.4097
FUGF [19,21]	1/0%	0.9551/ 0.063%	0.8091/ 0.211%	0.6729/ 0.179%	0.5852/ 0.29%	0.5424/ 0.424%	1.3267
SMV approach with L = 1 k	1/0%	0.9549/ 0.083%	0.8069/ 0.062%	0.6720/ 0.045%	0.5864/ 0.102%	0.5409/ 0.123%	0.0912
SMV approach with L = 10 k	1/0%	0.9557/ 0.0%	0.8075/ 0.012%	0.6715/ 0.029%	0.5868/ 0.034%	0.5403/ 0.037%	0.3571

Table 9. The reliability and accuracy of a dynamic system of mode II in each period using UGF, FUGF, and SMV methods; L: The sequence length for the SMV approach.

	Reliability/Accuracy in Each Period						CPU Time (s)
	0	1	2	3	4	5	
UGF [11,26]	1	0.9407	0.7656	0.6185	0.5310	0.4837	0.4372
FUGF [19,21]	1/0%	0.9423/ 0.17%	0.7639/ 0.222%	0.6162/ 0.372%	0.5329/ 0.358%	0.4811/ 0.536%	1.3321
SMV approach with L = 1 k	1/0%	0.94/ 0.074%	0.7648/ 0.1045	0.6189/ 0.065%	0.5316/ 0.113%	0.4829/ 0.165%	0.0936
SMV approach with L = 10 k	1/0%	0.9405/ 0.021%	0.7657/ 0.013%	0.6182/ 0.049%	0.5313/ 0.056%	0.4833/ 0.083%	0.3794

For a multi-state weighted k-out-of-n system with repairable components, a $(M + 1) \times (M + 1)$ state transition matrix (STM) is provided for each component (M is the maximum possible state of each component), resulting in a complexity of $O\left(T(M+1)^{n}\right)$ and $O\left(T(M+2-j)^{n}\right)$ by the UGF method to obtain the system reliability of model I and model II, respectively. The complexity of the FUGF method can be expressed as $O\left(T(M+1)^{n^*}\right)$ and $O\left(T(M_*+1)^{n^*}\right)$ when the system model I and model II have n^* subsystems, where T is the number of the time period, and m_* is the maximum possible state in the subsystem. However, a computational complexity of $O(nTL)$ is required to calculate the PD of the system at each time period using the SMV approach. Then the reliability of the system can be obtained by the comparison of the PD of the system and the pre-defined value K (the PD of six components is presented in the Appendix A, where L is the length of the randomly permuted multiple-valued sequence used in the SMV approach).

5. Conclusions

Computational overhead is a key challenge in the evaluation of the reliability of multi-state k-out-of-n systems with large numbers of components and states that can describe the behavior of multiple line transmission systems (e.g., power/oil transmission systems). In addition, the exact values of system reliability are not always necessary. This work therefore proposes an SMV approach to approximately evaluate the reliability of multi-state k-out-of-n systems. Two stochastic multi-value models are presented for steady state and dynamic multi-state k-out-of-n systems, respectively. The efficiency and accuracy of the proposed models are compared with the UGF and FUGF methods.

The results indicate that the run time of the SMV approach increases with the length of the sequences and the number of components, while the run time required by UGF and FUGF increases with the number of components and the number of components' states. Furthermore, as shown in the analysis of computational complexities of SMV, UGF, and FUGF methods, the number of component states is not a crucial factor that affects the run time in the reliability evaluation using the SMV approach compared to the use of the UGF and FUGF methods. The SMV approach is more efficient than the UGF and FUGF methods for a complex system with a larger number of component states. Due to the influence of component states, an SMV approach with L = 10 k is more accurate than the FUGF method for evaluating the reliability of complex multi-state k-out-of-n systems.

Acknowledgments: This research is partly supported by the Project of the National Natural Science Foundation of China (Grant No. 61371024 and No. 61601371), the Aerospace Science Technology Fund, the Fundamental Research Funds for the Central University under Grant 3102016QD042, Shaanxi Science and Technology Co-ordination and Innovation Project under Grant 2016KZTDGY02-01 and the Natural Sciences and Engineering Research Council (NSERC) of Canada.

Author Contributions: Xiaogang Song designed the SMV models, drafted the manuscript; Zhengjun Zhai carried out the data acquisition, performed the experiments; Yangming Guo analyzed the data; Peican Zhu and Jie Han reviewed and refined the paper.

Conflicts of Interest: The authors declared no conflicts of interest with respect to the research, authorship, and/or publication of this article.

Appendix A

For a dynamic multi-state weighted k-out-of-n system with repairable components, as shown in Figure 7, with six components and six states of each component, the PD of components at each time period obtained by the SMV approach are presented in Tables A1–A6 (sequence length L = 10 k).

Table A1. Probability distribution of component 1 using the SMV approach.

State \ Period	0	1	2	3	4	5
0	0	0.05	0.052	0.0532	0.0546	0.0556
1	0	0.05	0.0915	0.1133	0.1237	0.1284
2	0	0.1	0.1865	0.2297	0.2481	0.2552
3	0	0.2	0.26	0.2776	0.2826	0.2839
4	0	0.2	0.1975	0.1728	0.1563	0.1479
5	1	0.4	0.2125	0.1534	0.1347	0.1290

Table A2. Probability distribution of component 2 using the SMV approach.

State \ Period	0	1	2	3	4	5
0	0	0.05	0.0775	0.0904	0.0959	0.0983
1	0	0.1	0.1405	0.157	0.1637	0.1664
2	0	0.15	0.207	0.228	0.2355	0.238
3	0	0.2	0.26	0.2774	0.2823	0.2835
4	0	0.1	0.12	0.123	0.123	0.1228
5	1	0.4	0.195	0.1242	0.0996	0.091

Table A3. Probability distribution of component 3 using the SMV approach.

State \ Period	0	1	2	3	4	5
0	0	0.05	0.075	0.0883	0.0956	0.0997
1	0	0.05	0.1	0.135	0.1569	0.17
2	0	0.1	0.14	0.1585	0.1679	0.1729
3	0	0.1	0.155	0.1832	0.1973	0.2043
4	0	0.15	0.185	0.187	0.1817	0.1763
5	1	0.55	0.345	0.248	0.2006	0.1768

Table A4. Probability distribution of component 4 using the SMV approach.

State \ Period	0	1	2	3	4	5
0	0	0.05	0.0825	0.1023	0.1138	0.1203
1	0	0.1	0.15	0.1756	0.1890	0.1961
2	0	0.15	0.2075	0.2275	0.2332	0.2341
3	0	0.1	0.1475	0.1706	0.1821	0.1878
4	0	0.05	0.085	0.106	0.1176	0.1237
5	1	0.55	0.3275	0.218	0.1643	0.1380

Table A5. Probability distribution of component 5 using the SMV approach.

State \ Period	0	1	2	3	4	5
0	0	0.1	0.1175	0.1254	0.1319	0.1367
1	0	0.1	0.1475	0.1719	0.1858	0.1942
2	0	0.05	0.1175	0.1539	0.1687	0.1734
3	0	0.05	0.125	0.1767	0.2055	0.2199
4	0	0.2	0.2125	0.1887	0.167	0.153
5	1	0.5	0.28	0.1834	0.1411	0.1228

Table A6. Probability distribution of component 6 using the SMV approach.

State \ Period	0	1	2	3	4	5
0	0	0.05	0.0835	0.1063	0.1217	0.1319
1	0	0.05	0.0955	0.1306	0.1553	0.1716
2	0	0.1	0.151	0.178	0.192	0.199
3	0	0.1	0.16	0.1893	0.2017	0.2061
4	0	0.15	0.1775	0.1707	0.1582	0.1479
5	1	0.55	0.3325	0.2251	0.1711	0.1435

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