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Non-Convex Economic Dispatch of a Virtual Power Plant via a Distributed Randomized Gradient-Free Algorithm

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Abstract: The economic dispatch problem of a virtual power plant (VPP) is becoming non-convex for distributed generators' characteristics of valve-point loading effects, prohibited operating zones, and multiple fuel options. In this paper, the economic dispatch model of VPP is established and then solved by a distributed randomized gradient-free algorithm. To deal with the non-smooth objective function, its Gauss approximation is used to construct distributed randomized gradient-free oracles in optimization iterations. A projection operator is also introduced to solve the discontinuous variable space problem. An example simulation is implemented on a modified IEEE-34 bus test system, and the results demonstrate the effectiveness and applicability of the proposed algorithm.

Keywords: distributed randomized gradient-free algorithm; distributed randomized gradient-free oracles; non-convex economic dispatch; virtual power plant

1. Introduction

Distributed energy resources (DERs) include distributed generators (DGs), renewable energies (REs), and energy storage systems (ESs) [1]. Optimizing the output of DERs can greatly improve the efficiency of their energy utilization. The centralized dispatch methods adopt lambda iterative algorithms [2] and interior point lambda iterative algorithms [3], both of which need the objective functions to be smooth and derivable. Actually, the cost functions of DGs are non-convex because of their features of valve-point loading effects, prohibited operating zones, and multiple fuel options [4]. The charging and discharging of ESs also aggravates the complexity of economic dispatch problems [5]. Based on advanced management concepts and software technologies, the virtual power plant (VPP) [6,7] has been developed to be a new DERs management tool. Additionally, it is necessary to solve the non-convex economic dispatch of VPPs in order to realize the optimal scheduling of DERs. Intelligent optimization algorithms can solve non-convex optimization problems effectively, including the genetic algorithm (GA) [8], particle swarm optimization (PSO) [9], and differential evolution [10,11]. However, these algorithms rely on the dispatch center or controller to collect and process DERs' information, which may lead to a higher communication cost; they also need a centralized communication structure with high bandwidth, which has poor system reliability and is more vulnerable to a single point of failure [12].

At present, the optimization decision process of VPPs is changing from centralized ways to distributed ones [13]. According to the local communication mechanism, the operation information of the DERs is collected through communication lines built among DER units and their adjacent units, and then the real-time scheduling process can be carried out. The distributed consensus algorithm employed in the distributed scheduling of [14] can greatly reduce communication costs and communication delays. Compared with centralized solutions, the distributed gradient algorithm in [15] can not only obtain comparable optimums but also respond in a timely manner when the operation conditions of the system change. To solve the problem of the objective function of ESs being not smooth, the sub-gradient is calculated as an alternative of the gradient [16,17]. The economic dispatch of a VPP is a non-convex optimization problem for the consideration of valve-point loading effects, prohibited operating zones, and multiple fuel options. The algorithms mentioned in [14–17] cannot be applied to the non-convex optimization problem because it is difficult to estimate the gradient or sub-gradient. The distributed auction-based algorithm designed in [12] can realize the optimal power output by sharing units' bidding and then determining the auction results in the process of consensus. However, it introduces too many intermediate variables, which will make the iteration format more complex.

This paper adopts a distributed randomized gradient free algorithm (DRGF) [18] to solve a VPP's non-convex economic dispatch problem considering DGs' valve-point loading effects, prohibited operating zones, and multiple fuel options. The algorithm is established on a distributed communication structure that has a higher operation reliability and lower communication cost [12]. In addition, the DRGF approach calculates randomized gradient-free oracles, instead of gradients or sub-gradients, to implement the distributed optimization, which makes the iteration formula simple and easily solved. The modified IEEE-34 bus system is employed to verify the effectiveness of the proposed method. The simulation results show that the DRGF algorithm can formulate an economical scheme for a VPP's non-convex economic dispatch.

This paper is organized as follows. In Section 2, a VPP's non-convex economic dispatch with constraints of valve-point loading effects, prohibited operating zones, and multiple fuel options is discussed. Section 3 introduces the DRGF algorithm, and the simulation results presented in Section 4 show its effectiveness. Finally, the paper is summarized in Section 5.

2. VPPs' Non-Convex Economic Dispatch

2.1. The Operation Models of the DER's Units

In traditional economic dispatch, the cost function of a DG is a standard quadratic function [19]. If valve-point loading effects, simulated by sinusoidal terms, and multiple fuel options are taken into account, the cost function can be described as [4,12]:

$$C(P_i) = \begin{cases} a_i^1 P_i^2 + b_i^1 P_i + c_i^1 + \left| d_i^1 \sin(e_i^1 P_i - e_i^1 P_i^{\min}) \right|, & P_i^{\min} \le P_i \le P_{1i} \\ \vdots \quad\quad \vdots \quad\quad \vdots & i = 1, \cdots, n_g. \\ a_i^q P_i^2 + b_i^q P_i + c_i^q + \left| d_i^q \sin(e_i^q P_i - e_i^q P_i^{\min}) \right|, & P_{(q-1),i} \le P_i \le P_i^{\max} \end{cases}, \tag{1}$$

where n_g is the total number of DGs and the power output of DGi is P_i, yielding to the upper limit $P_i^{\max}$ and lower limit $P_i^{\min}$. When P_i exceeds the value of $P_{(q\text{-}1),i}$, unit i chooses the fuel q and its cost function $C(P_i)$ uses coefficients of $a_i^q, b_i^q, c_i^q, d_i^q, e_i^q$.

Because the power output is usually concentrated in some areas, the operational efficiency can be greatly promoted by prohibiting units from running in low productivity areas. The power output constraints considering DGs' prohibited operating zones can be expressed as [4,12]:

$$\begin{cases} P_i^{\min} \le P_i \le L_{mi}, & mi = 1 \\ U_{(m-1)i} \le P_i \le L_{mi}, & mi = 2, \cdots, Mi. \\ U_{mi} \le P_i \le P_i^{\max}, & mi = Mi \end{cases}, \tag{2}$$

where unit i has Mi prohibited operating zones, the mi^{th} of which subjects to the upper boundary U_{mi} and lower boundary L_{mi}.

ESs can work in charging or discharging modes, and their cost functions and operation constraints are as follows [20,21]:

$$C(P_{ei}) = 0.5 \cdot E_i \cdot |P_{ei}| \ , \ ei = 1, \cdots, n_e., \tag{3}$$

$$-P_{ch}^{\max} \le P_{ei} \le P_{dch}^{\max}, \tag{4}$$

$$\begin{cases} S_{up} < S_{OC} \le S_{\max}, & dch \\ S_{down} \le S_{OC} \le S_{up}, & dch \ or \ ch \\ S_{\min} \le S_{OC} < S_{down}, & ch \end{cases}, \tag{5}$$

where n_e is the total number of ESs, and the power output of ESi is P_{ei}. C (P_{ei}) represents the charging (ch)/discharging (dch) cost, and E_i is the charging/discharging efficiency. The maximum charging and discharging power of ESi is $P_{ch}^{\max}$, $P_{dch}^{\max}$. Additionally, the minimum of both is 0. For the value of the state-of-charge (S_{OC}), $S_{\max}$, $S_{\min}$, S_{up}, S_{down} are the maximum, minimum, upper, and lower values, respectively [20].

Compared with operating at maximum power, REs will be more flexible in a schedulable mode. However, this will also cause some profit losses, that is, the schedulable cost C (P_{ri}) [21]:

$$C(P_{ri}) = \rho_V \ (or \ \rho_W) \ \cdot [P_{ri}^{\max} - P_{ri}] \ , \ ri = 1, \cdots, n_r. \tag{6}$$

$$s.t. \ 0 \le P_{ri} \le P_{ri}^{\max} \tag{7}$$

where n_r is the total number of REs, and the power output of REi is P_{ri}. The ρ_V (photovoltaic systems) and ρ_W (wind turbine) are the grid-connected prices, including electricity prices and generation compensations. The maximum available power of REi is $P_{ri}^{\max}$, and it can be either the maximum photovoltaic system tracking power P_{V} [22] or the maximum wind power P_W [23]:

$$P_{\mathrm{V}} = P_{\mathrm{Vmax}} \frac{G_C}{G_{\mathrm{Cmax}}} [1 + K(T_c - T_r)], \tag{8}$$

$$P_W = \begin{cases} 0 & v < v_{ci}, v \ge v_{co} \\ \frac{v - v_{ci}}{v_r - v_{ci}} \cdot P_r & v_{ci} < v < v_r \\ P_r & v_r \le v \le v_{co} \end{cases}, \tag{9}$$

where P_{Vmax} represents the maximum output under standard test conditions. G_C means the actual light intensity; G_{Cmax} is the reference intensity under standard test conditions. The conversion coefficient of temperature to power is depicted by K. T_c and T_r are the environment temperature and the reference temperature under standard test conditions, respectively. P_r is the rated power output of the wind generators (WGs). v, v_{ci}, v_{co}, and v_r are the wind speeds, cut-in wind speeds, cut-out wind speeds, and the rated wind speeds, respectively.

2.2. Dispatch Objectives and Constraints

According to the operation mode of a VPP, the economic dispatch objective function and the system constraints can be defined as [16,21]:

$$\max F_{VPP} = \rho_d P_D + \rho_s P_S - C_{VPP}, \tag{10}$$

$$C_{VPP} = \sum_{i=1}^{ng} C(P_i) + \sum_{i=1}^{ne} C(P_{ei}) + \sum_{i=1}^{nr} C(P_{ri}), \tag{11}$$

$$\sum_{i=1}^{ng} P_i + \sum_{i=1}^{ne} P_{ei} + \sum_{i=1}^{nr} P_{ri} = P_D + P_S, \tag{12}$$

$$P_{VPP}^{\max} - P_D \ge \gamma_1 \cdot P_D + \gamma_2 \cdot P_V + \gamma_3 \cdot P_W, \tag{13}$$

where F_{VPP} denotes the total income of the VPP, and C_{VPP} represents its total generation cost. The maximum generation capacity of the VPP is $P_{VPP}^{\max}$, that is, the sum of the maximum power output of each DER unit. The total load demand is P_D and P_S is the interface power at the point of common coupling (PCC). If P_S is negative, the power will flow from the VPP into the main grid. ρ_d is the bidding of the VPP and ρ_S is the electricity price of the main grid. γ_1, γ_2, and γ_3 are the reserved capacity coefficients of the load demand, photovoltaic systems, and wind turbines, respectively.

3. Distributed Randomized Gradient-Free (DRGF) Algorithm

The DERs' power output vector $P = [P_1, P_2, \cdots P_{ng}, P_{e1}, P_{e2}, \cdots P_{ne}, P_{r1}, P_{r2}, \cdots P_{nr}]^T$ can be denoted as a variable vector $x = [x_1, x_2, \cdots x_n]^T$, and there exists $n = n_g + n_e + n_r$. Accordingly, the upper and lower bounds of the variables are represented by $x_i^{\max}, x_i^{\min}$. Thus, the active power balance constraint of the original VPP economic dispatch problem can be denoted as:

$$P_S = \sum_{i=1}^{n} x_i - P_D, \tag{14}$$

Substituting formula (14) into formula (10), one can obtain:

$$\max F_{VPP} = (\rho_d - \rho_s) \cdot P_D - \sum_{i=1}^{n} [C(x_i) - \rho_s \cdot x_i], \tag{15}$$

It can be seen that the F_{VPP} is only dependent on variables x_i when the values of ρ_d, ρ_S, and P_D are constant. If we set $f_i(x_i) = C(x_i) - \rho_s \cdot x_i$, the original objective function can be equivalent to a minimization problem:

$$\min_{x \in X} f(x) = \sum_{i=1}^{n} f_i(x_i), \tag{16}$$

where $f(x)$ denotes the objective function, and X is the feasible space of x, that is, $x \in X$ may represent the power output constraints of DERs.

Based on a distributed optimization framework, the DERs can collect neighboring units' information and the information at the PCC, and then this information will be calculated by weighted mean values [14,18]

$$\overline{x_i}[k] = \sum_{j=1}^{n} W_{ij}[k] x_j[k] + \varepsilon_i \cdot P_S, \tag{17}$$

where ε_i is the power regulation factor, and $\overline{x_i}$ is the weighted mean value of x_i. Unit j is connected to unit i with the communication weight W_{ij} $[k]$, and its calculation format is shown in [12].

This paper employs a DRGF algorithm [18] to solve the problems of a discontinuous solution space and the non-convex objective function. Although the objective function of a VPP is not smooth, it is Lipschitz continuous in the variable space [24], and its smooth form can be written as:

$$\min_{x \in X} f^{\mu}(x) = \sum_{i=1}^{n} f_i^{\mu i}(x_i), \tag{18}$$

where μ_i is the smoothing coefficient of the objective function, and $f_i^{\mu i}(x_i)$ is its smooth form, calculated by:

$$f_i^{\mu i}(x_i) = \frac{1}{\gamma} \int_X f_i(x_i + \mu_i \tau_i) e^{-0.5\tau^2} d\tau_i, \tag{19}$$

where the conversion coefficient denotes as $\gamma = (\sqrt{2\pi})^n$, and the random sequence τ_i meets the Gaussian distribution. The theory of Gauss approximation is shown in the Appendix A.

Now, distributed randomized gradient-free oracles can be constructed to implement the optimal iteration:

$$g^{\mu i}(x_i[k]) = \frac{f_i^{\mu i}(x_i[k] + \mu_i \tau_i[k]) - f_i^{\mu i}(x_i[k])}{\mu_i} \tau_i[k], \tag{20}$$

where $g^{\mu i}(x_i[k])$ represents the distributed randomized gradient-free oracle of x_i at the kth iteration.

By the above steps, the iterative form of the optimization variables can be derived as:

$$x_i[k+1] = P_X\left[\overline{x_i}[k] - \alpha[k] g^{\mu i}(x_i[k])\right], \tag{21}$$

where the projection operator P_X is defined as in [25]. The iteration step-size satisfies the following conditions:

$$\alpha[k] > 0, \sum_{k=0}^{\infty} \alpha[k] = \infty, \sum_{k=0}^{\infty} (\alpha[k])^2 < \infty, \tag{22}$$

The implementation process of a VPP's non-convex economic dispatch via the DRGF algorithm is shown in Figure 1, and the specific procedures are as follows:

1. The input data includes the coefficients of cost functions, various limits of the DERs' power output, the total load demand, etc. The maximum available power output of the REs is calculated by formula (8) and (9).
2. The optimization variable x_i [0] is initialized according to references [8,10,15]. Then, set up the smoothing coefficient of the objective function and generate the random sequence.
3. According to formula (14), calculate the initial P_S at PCC.
4. Correct the iteration step by $k = k + 1$, where the initial number of iteration steps is $k = 1$.
5. According to formula (17), calculate the weighted mean values; and according to formula (19), calculate the Gauss approximation. When DGs have multiple fuel options, as shown in equation (2), select the cost function curves on the basis of the DG's actual power output.
6. According to formula (20), calculate distributed randomized gradient-free oracles; according to formula (22), calculate the current iteration step by $\alpha[k] = 1/\sqrt{k+1}$.
7. According to formula (21), implement the optimal iteration of the power output variables.
8. Determine whether the current variables are within the available space. If they satisfy, proceed to the next step; otherwise, the variables take the upper ($x_i[k+1] \geq x_i^{\max}$) or lower ($x_i[k+1] \geq x_i^{\min}$) limits of the constraints. When variable x_i falls into prohibited zone mi during the decreasing process, such as x_i [k] > x_i [k + 1], its value will be set at the upper boundary U_{mi}. Additionally, when x_i falls into prohibited zone mi during the increasing process, such as x_i [k] < x_i [k + 1], the value will be set at the lower boundary L_m.
9. According to formula (14), update the initial P_S at PCC.
10. Determine whether the current power imbalance satisfies the allowable value. If it satisfies, proceed to the next step; otherwise, return to step (5) to recalculate the weighted mean values.
11. Calculate the iteration error.
12. Determine whether the iteration error satisfies the allowable value. If it satisfies, proceed to the next step; otherwise, return to step (4) for the next iteration.
13. Output the optimal solution vector.

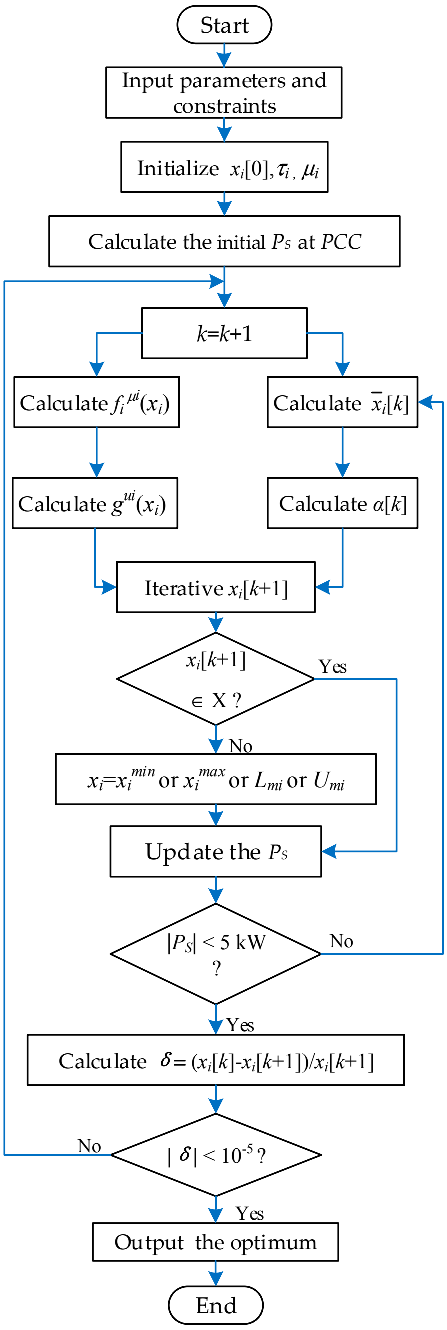

Figure 1. The flowchart for a virtual power plant's (VPP's) non-convex economic dispatch via the distributed randomized gradient free algorithm (DRGF) algorithm.

4. Numerical Examples

Based on a modified IEEE 34 bus system, a VPP system is built to verify the effectiveness of the proposed algorithm. It mainly investigates DGs' valve-point loading effects, prohibited operating zones, and multiple fuel options. The reference [18] shows that the convergence coefficient has little effect on the convergence of the algorithm, so the smoothing coefficient of the cost function is set to 0.0005 in this example. The communication topology is shown in Figure 2. The operation parameters are listed in Tables 1 and 2. The total load demand is 650 kW, and the initial power outputs of the DGs, REs, and ESs are 75, 75, and 0 kW, respectively. For solving non-convex economic dispatch problems, a PSO solution used in [9] has achieved the lowest cost among numerous centralized algorithms.

As contrast, we will also adopt the PSO [9] (one of the centralized dispatch method) to deal with the VPP's economic dispatch model. Table 3 provides the optimization results when one of the centralized dispatch method (PSO) is adopted, and Table 4 shows the VPP's average profits made by PSO and DRGF. This section sets up three simulation scenarios as follows: (A) the VPP's distributed economic dispatch with valve-point loading effects; (B) the VPP's distributed economic dispatch with prohibited operating zones; (C) the VPP's distributed economic dispatch with multiple fuel options.

Table 1. Coefficients of the distributed generators' (DGs') production cost. DER, Distributed Energy Resources.

DERs Units	Fuel Type	$a_i^q P_i{}^2 + b_i^q P_i + c_i^q + \left\| d_i^q \sin(e_i^q P_i - e_i^q P_i^{\min}) \right\|$ (\$/kWh)					Operation Range (kW)
		a_i	b_i	c_i	d_i	e_i	
DG 1	1	0.0751	25.734	996.57	310	0.053	40–55
	2	0.0578	21.462	996.57	295	0.048	55–80
DG 2	1	0.0814	21.215	1002.1	225	0.062	40–55
	2	0.0581	19.893	1002.1	218	0.061	55–80
DG 3	1	0.0857	22.188	1058.3	436	0.042	40–55
	2	0.0596	22.095	1058.3	402	0.041	55–80
DG 4	1	0.0704	28.026	978.52	289	0.046	40–55
	2	0.0672	27.684	978.52	275	0.039	55–80

Table 2. The virtual power plant's (VPP's) other operation parameters.

Operation Parameters	Values
$\gamma_1, \gamma_2, \gamma_3$	0.05, 0.2, 0.15
$SOC_{(min,\ down,\ up,\ max)}$	0.05, 0.20, 0.80, 0.95
$[L_1, U_1]$, $[L_2, U_2]$	[45, 50], [55, 65] (kW)
$\rho_{rV}, \rho_{rW}, \rho_d, \rho_S$	0.0839, 0.0721, 0.0780, 0.0736 (\$/kWh)
E_i; E_C; $P^{\max}(ch, dch)$	85%; 100 (kWh); 20, 20 (kW)
P_r; v, v_{ci}, v_{co}, v_r	120 (kW); 12, 3.0, 25, 15 (m/s)
$P_{V\max}$; $G_{C\max}$, G_C; K; T_r, T_c	180 (kW); 1, 0.9 (kw/m^2); −0.45%; 25, 18 (°C)

Table 3. Optimization results under one of the centralized dispatch method (PSO).

Distributed Energy Resources(DERs) Units	The Optimization Results (kW)		
	Scenarios A	Scenarios B	Scenarios C
Distributed Generator (DG) 1	44.6364	44.4128	47.6625
Distributed Generator (DG) 2	47.8850	50.0000	44.4135
Distributed Generator (DG) 3	41.3855	41.1637	41.1637
Distributed Generator (DG) 4	51.1345	50.9133	50.9136
Renewable Energy (RE) 1	123.7492	123.5162	123.7500
Renewable Energy (RE) 2	109.0612	108.8276	109.0614
Renewable Energy (RE) 3	118.8540	118.6203	118.8541
Renewable Energy (RE) 4	113.9585	113.7249	113.9587
Energy Storage (ES) 1	2.6756	1.8496	2.0834
Energy Storage (ES) 2	4.2515	6.0170	6.2511
Energy Storage (ES) 3	−3.4227	−2.3173	−2.0838
Energy Storage (ES) 4	−4.4134	−6.4850	−6.2517

Table 4. The VPP's average profits obtained by two dispatch strategies (PSO and DRGF).

Algorithm	The PSO [8]	The DRGF
Scenarios a	0.0642 (\$/kWh)	0.0642 (\$/kWh)
Scenarios b	0.0645 (\$/kWh)	0.0645 (\$/kWh)
Scenarios c	0.0649 (\$/kWh)	0.0649 (\$/kWh)

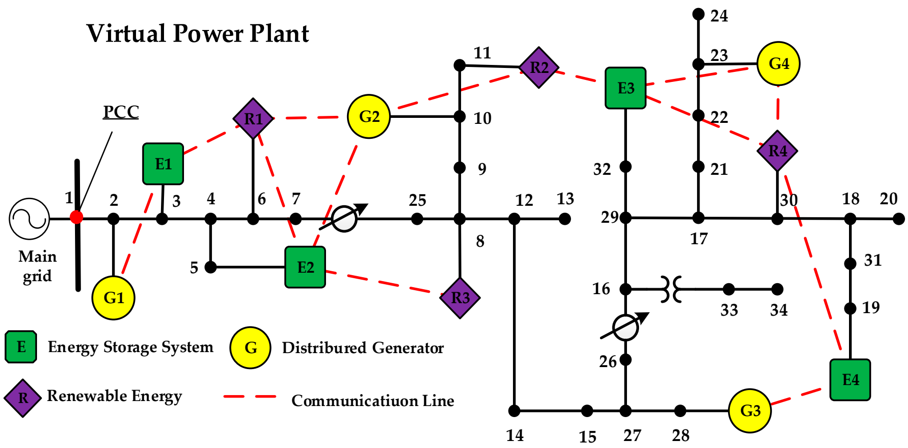

Figure 2. The communication topology based on a modified IEEE 34 bus system.

4.1. Scenario A: The VPP's Distributed Economic Dispatch with Valve-Point Loading Effects

The characteristic of valve-point loading effects makes the DGs' cost function have many non-differentiable points. The operation coefficients of DGs are shown in Tables 1 and 2. Figures 3 and 4 provide the optimal scheduling process of this scenario. It can be seen that the DGs' power output will have a great fluctuation during the initial stage of optimization, and then the ESs will change their power output to reduce the impact of valve-point loading effects on the system's active power balance. In addition, the VPP also exchanges power with the main grid through the PCC to stabilize the supply-demand balance. After a certain number of iterations, the DERs' optimization curves tend to be stable, and the DRGF algorithm finally achieves the same results as centralized algorithms (shown in Tables 3 and 4). However, if considering the communication cost and operability, the distributed dispatch will be more economical and practical.

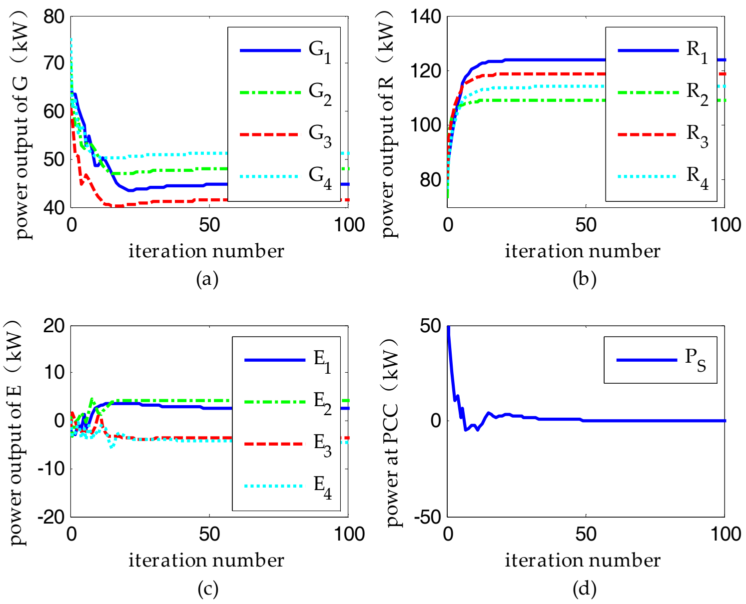

Figure 3. The optimized scheduling scheme of scenario A: (**a**) Optimization results of distributed generators (DGs); (**b**) Optimization result of renewable energies (REs); (**c**) Optimization results of energy storage systems (ESs); (**d**) The power P_S at the point of common coupling (PCC).

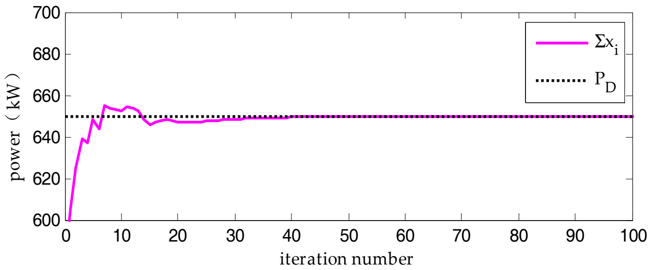

Figure 4. The active power balance of scenario A.

4.2. Scenario B: The VPP's Distributed Economic Dispatch with Prohibited Operating Zones

The DGs' power output is discontinuous for some prohibited operating zones. Figures 5 and 6 show the simulation results under this scenario. In Figure 5a, the platforms in the curve indicate the situation where DGs fall into prohibited operating zones. When DGs jump out of these areas, they will resume normal operation. ESs can flexibly charge and discharge, greatly reducing the impact of prohibited operating zones on the active power balance. As can be seen from Table 3, the DRGF algorithm can get the same results as the centralized algorithms, which demonstrates that the DRGF algorithm can effectively deal with DGs' prohibited operating zones.

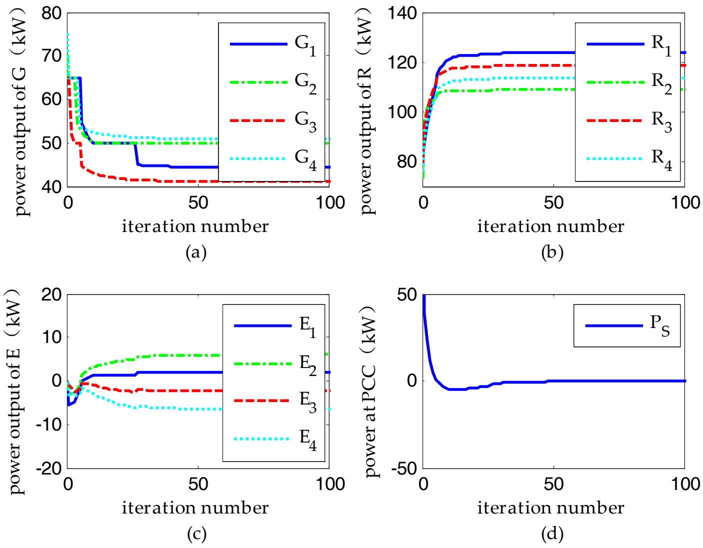

Figure 5. The optimized scheduling scheme of scenario B: (**a**) Optimization results of DGs; (**b**) Optimization result of REs; (**c**) Optimization results of ESs; (**d**) The power P_S at PCC.

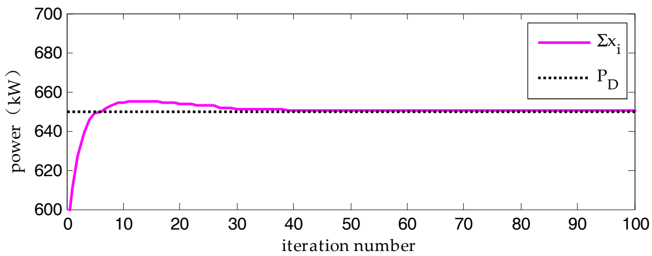

Figure 6. The active power balance of scenario B.

4.3. Scenario C: the VPP's Distributed Economic Dispatch with Multiple Fuel Options

According to actual power output, DGs will choose the most economical fuel selection with different cost coefficients, leading to some non-differentiable points. Figures 7 and 8 provide this scenario's simulation results. In Figure 7a and Table 1, when the power output is within 40–55 kW, DGs select the No. 1 fuel; if the power output is within 55–80 kW, DGs will select the No. 2 fuel. Then, the allocation of DERs should be re-optimized. The VPP's total power output may have a large fluctuation when the fuel changes, and the system will recover the active power balance quickly by absorbing some power from the main grid. The distributed dispatch can get the same result as the centralized one, which shows the DRGF algorithm's effectiveness in solving DGs' multiple fuel options.

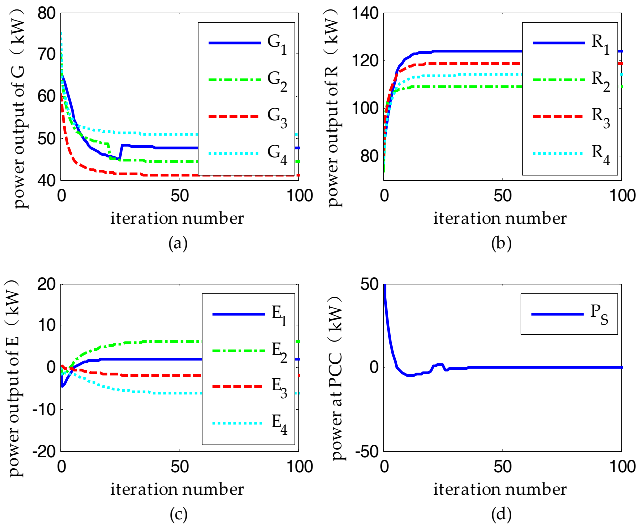

Figure 7. The optimized scheduling scheme of scenario C: (**a**) Optimization results of DGs; (**b**) Optimization result of REs; (**c**) Optimization results of ESs; (**d**) The power P_S at *PCC*.

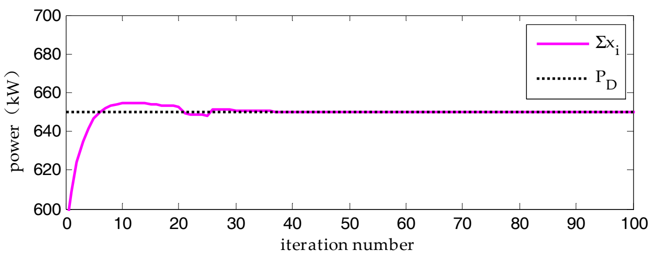

Figure 8. The active power balance of scenario C.

5. Summary

A technology of a VPP is adopted to manage DERs by modeling its non-convex economic dispatch considering DGs' characteristics of valve-point loading effects, prohibited operating zones, and multiple fuel options. A DRGF algorithm is introduced to solve this nonlinear and non-differentiable optimization problem. The objective function is converted to its Gauss approximation, and then used to construct distributed randomized gradient-free oracles instead of gradients or sub-gradients. A projection operator is also employed to deal with the discontinuous variable space. Three typical simulation scenarios are implemented on a modified IEEE 34 bus system. The results indicate that the proposed DRGF algorithm can effectively cope with a VPP's non-convex economic dispatch, and shows a good applicability.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

According to [24] (theorem 1 and 4, Formula (11) and (22)), the following lemma is introduced to provide some important properties of the function $f_i^{\mu i}(x_i)$ and the random gradient-free oracle $g^{\mu i}(x_i[k])$.

Lemma A1. *For each i, there has:*

(a) $f_i^{\mu i}(x_i)$ *is convex and differentiable, and it satisfies:*

$$f_i(x_i) \le f_i^{\mu i}(x_i) \le f_i(x_i) + \sqrt{n}\mu_i G_0(f_i), \quad \text{(A1)}$$

(b) *The gradient* $\nabla f_i^{\mu i}(x_i)$ *satisfies:*

$$\left[g^{\mu i}(x_i[k])\right] = \nabla f_i^{\mu i}(x_i), \quad \text{(A2)}$$

(c) *The random gradient-free oracle satisfies:*

$$\left[\|g^{\mu i}(x_i[k])\|^2\right] \le (n+4)^2 G_0(f_i)^2, \quad \text{(A3)}$$

where $G_0(f_i)$ is Lipschitz constant; E[x] denotes the expected value of a random variable x. The further principle description and proof can be found in [18,24].

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