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Occupancy-Based HVAC Control with Short-Term Occupancy Prediction Algorithms for Energy-Efficient Buildings

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Abstract: This study aims to develop a concrete occupancy prediction as well as an optimal occupancy-based control solution for improving the efficiency of Heating, Ventilation, and Air-Conditioning (HVAC) systems. Accurate occupancy prediction is a key enabler for demand-based HVAC control so as to ensure HVAC is not run needlessly when when a room/zone is unoccupied. In this paper, we propose simple yet effective algorithms to predict occupancy alongside an algorithm for automatically assigning temperature set-points. Utilizing past occupancy observations, we introduce three different techniques for occupancy prediction. Firstly, we propose an identification-based approach, which identifies the model via Expectation Maximization (EM) algorithm. Secondly, we study a novel finite state automata (FSA) which can be reconstructed by a general systems problem solver (GSPS). Thirdly, we introduce an alternative stochastic model based on uncertain basis functions. The results show that all the proposed occupancy prediction techniques could achieve around 70% accuracy. Then, we have proposed a scheme to adaptively adjust the temperature set-points according to a novel temperature set algorithm with customers' different discomfort tolerance indexes. By cooperating with the temperature set algorithm, our occupancy-based HVAC control shows 20% energy saving while still maintaining building comfort requirements.

Keywords: occupancy model; occupancy-based control; model predictive control; energy efficiency; building climate control

1. Introduction

1.1. Background of Research

More than 30% of building energy is consumed by HVAC systems, which usually operate on a fixed schedule predefined by building owners or operation managers. Currently, most existing building control systems still condition rooms with a set-point assuming maximum occupancy from early morning until late evening during weekdays. As a result, rooms are often needlessly over-conditioned, which may lead to a significant waste in energy consumption. Occupancy based controls, as a promising remedy for the aforementioned issue, can achieve significant energy savings by temporally matching the building energy consumption and building usage. This has the potential to reduce up to a third of HVAC energy consumption. Moreover, accurate and reliable occupancy detection is becoming a key enabler for demand-response HVAC control, which requires the capturing of occupancy changes in real time [1]. By taking advantage of occupancy information, we can reduce building energy consumption via optimized scheduling of HVAC [2], as well as shading blinds and natural ventilation to make effective use of available natural resources [3–5].

Even in the rare cases where occupancy information is integrated into the HVAC operation, only binary decisions (occupied or not) are made, with the actual number of occupants in the building is ignored. However, even under this binary case, [6] has discovered that there exists potential annual energy savings of 10–42% if actual occupancy information has been properly utilized. In actuality, the energy consumption of that building is dominated by the occupancy and related activities [7]. It follows that there exists optimal control parameters, based on the instantaneous number of humans in a building and their associated behaviors, with great energy savings potential.

In fact, occupancy in a building is stochastic both in time and space, which greatly affects actual power consumption for an individual zone or building. Consequently, this will not only affect our decisions for improving energy efficiency but also in implementing the advanced demand response (Typically peak-shaving applications in modern energy management systems [8,9]). The authors of [10,11] discovered that average occupancy level for commercial buildings is at most a third of its maximum designed-for occupancy. Thus, accurate occupancy-sensing data provides significant insight for an online adaptive HVAC control strategy utilizing to the exact number of occupants in a building over a certain time period [12–14]. Moreover, occupant behavior is well recognized as a dominant source of the discrepency between predicted and actual building performance, and developing accurate short-term occupancy prediction will greatly enhance implementation of realistic building energy modeling and control.

1.2. Literature Review

1.2.1. Occupancy Models

Despite a plethora of potential application scenarios, buildings' occupancy modeling remains a cumbersome, error-prone and expensive process [15]. A through literature review for real-time occupancy detection and modeling in commercial buildings has been delivered in [16]. For occupancy modeling, various occupant behavior models have been developed in [17–20]. Moreover, such occupancy models have been integrated with operable windows, blinds, and lighting in EnergyPlus [19]. More recently, occupancy information has also been applied in Home Energy Management System (HEMS) using Markov-chain algorithms [21] or machine learning algorithms [22].

As previously mentioned, the uncertain occupancy information plays a central role in developing demand-driven HVAC control strategy. Due to the stochastic nature of occupancy, short-term prediction of it for individual rooms remains a challenging task. Previous occupancy modeling studies have focused on representing different detailedness of occupants' behavior, such as binary data (i.e., presence and absence) [23], accurate discrete values (i.e., the number of occupants) [24], or continuous probability distributions [25]. All these models achieved a balanced trade-off between model accuracy and complexity, depending on the actual application scenario. Consequently, an appropriate modeling complexity must be chosen for any specific case.

1.2.2. Occupancy-Based Control

When provided accurate occupancy models, demand-driven control can utilize such information to coordinate real-time HVAC usage, reducing energy use and maintaining indoor thermal comfort in buildings [13,26–29]. It has been reported in [30] that a 75% energy savings can be achieved by using a robust design which is less sensitive to occupant variation. Further, when integrated with model-based control strategies, 42% energy savings have been achieved by using real-time occupancy data [1].

The main task of a traditional HVAC control system is to maintain temperature and indoor air quality within a desired comfort range while minimizing energy use. Current mainstream HVAC control practice depends on the choice of predefined dead-band values, which involves a significant amount of tedious tuning. In fact, this tuning has become increasingly challenging with the rising complexity of modern HVAC systems, particularly with regard to the uncertain characteristics of occupancy [31].

An alternative approach is to use the well-known model predictive control (MPC) approach, which takes into account weather and occupancy forecasts (as shown in Figure 1). At each sampling time, MPC minimizes the energy use by optimizing a plan for future HVAC operation based on predictions of the weather and occupancy for a future time horizon [31]. MPC has been widely applied in building climate control systems and has demonstrated promising energy savings as studies in [32,33] show.

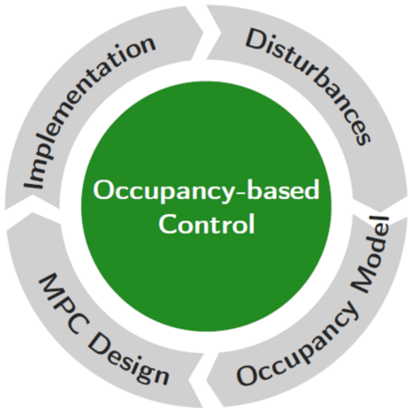


Figure 1. Scheme of occupancy-based control.

1.3. Main Idea and Outline

We note here that most of the aforementioned techniques require an off-line training process using a large amount of collected measurement. However, the focus of our paper is to provide an alternative on-line strategy for short-term occupancy prediction. The proposed occupancy-based control framework aims to minimize the total HVAC energy consumption while maintaining a comfortable indoor environment in buildings by utilizing a non-fixed temperature setpoint for the HVAC controller. To accomplish this, we recall one efficient algorithm [34] for optimally assigning temperature set-point based on both real-time forecasts of occupancy information of the building. To determine the temperature set-points for the planning horizon, a novel temperature setpoint algorithm is introduced, where a discomfort tolerance index is also included. After determining the optimal future temperature set-points, it is further integrated with an MPC framework to complete the occupancy-based control strategy.

Statement of contributions:

Our contribution in this paper is threefold: Firstly, we design a suitable utility function alongside a temperature set algorithm to capture the trade-off between occupancy comfort and energy consumption. Secondly, we propose three different occupancy estimation algorithms that enable short-term stochastic modeling of occupancy in buildings. Finally, we analyze and validate energy-saving performance of the proposed techniques. Detailed comparisons are provided for energy consumptions both with various occupancy estimation algorithms, and without any occupancy information. As mentioned in [35], very few implementations of occupancy models in building simulation are reported. To the best knowledge of the authors, there exist few available guidelines or analysis utilizing predicted occupancy information for occupancy-based HVAC control techniques. This paper bridges the gap between reliable stochastic occupancy modeling and energy efficiency building simulation.

It is an extension of authors' previous work [34], where a more powerful prediction algorithm—Uncertain Basis has been brought into the picture. Most importantly, we present novel quantified analysis for true energy savings with demand-based HVAC control strategy in this paper. Finally, some discussions are provided to better illustrate true energy-saving benefit by using the proposed demand-based control strategy.

The paper is structured as follows: Section 2 defines the mathematical problem under consideration and HVAC model we use. We also set up a temperature setpoint algorithm which could adaptively tune the temperature setpoint of the room based on the real occupancy information. Section 3 contains details about the occupancy prediction algorithms. Three different prediction techniques, especially the uncertain basis technique is introduced to predict the occupancy for the first time. Section 4 presents the simulation results regarding the estimation performance of all the aforementioned estimation algorithms. Finally, Section 5 draws the conclusions and ideas for future directions of development.

2. Problem Formulation

In order to use occupancy for demand-based HVAC control, we first must illustrate our HVAC model settings and control strategy. MPC is a class of algorithms designed to exploit building models and forecasts of interior and exterior disturbance signals. An MPC algorithm then computes open-loop optimal control actions by optimizing a cost function over a finite time horizon.

The reference temperature set point will serve as a bridge to relate occupancy prediction with MPC control strategy via the temperature set algorithm proposed in [34].

2.1. Building Thermal Model

In this section, we describe the typical one-dimensional resistance-capacitance (RC) model used in MPC design. The model stems from a physics-based continuous-time model, which captures the dynamics of indoor temperature, interior-wall surface temperature, as well as exterior-wall core temperature. This building thermal model has been widely applied in dozens of researches [32,36,37] for simulating residential and commercial buildings. It is described by

$$\begin{aligned}
\dot{x}_1 &= \frac{1}{C_1}\left[(K_1+K_2)(x_2-x_1)+K_5(x_3-x_1)+K_3(\delta_1-x_1)+u_1+u_2+\delta_2+\delta_3\right] \\
\dot{x}_2 &= \frac{1}{C_2}\left[(K_1+K_2)(x_1-x_2)+\delta_2\right] \\
\dot{x}_3 &= \frac{1}{C_3}\left[K_5(x_1-x_3)+K_4(\delta_1-x_3)\right]
\end{aligned}$$

where the variables are defined in Table 1, and the parameter values are provided in Table 2.

Table 1. Building parameter definition.

Variables	Definition
x_1	Indoor air temperature (°C)
x_2	Interior-wall temperature (°C)
x_3	Exterior-wall core temperature (°C)
u_1	Cooling power (≤ 0) (kW)
u_2	Heating power (≥ 0) (kW)
δ_1	Ambient temperature (°C)
δ_2	Solar radiation (kW/m^2)
δ_3	Internal heat gain (kW)

Table 2. Building parameter values.

Building Parameter Values	Unit
C_1 = 9.356 ×10 ^{5}	kJ/°C
C_2 = 2.970 × 10 ^{6}	kJ/°C
C_w = 6.695 × 10 ^{5}	kJ/°C
K_1 = 16.48	kJ/°C
K_2 = 108.5	kJ/°C
K_3 = 5	kJ/°C
K_4 = 30.5	kJ/°C
K_5 = 23.04	kJ/°C

The system states are x_1, x_2, and x_3. The model inputs consist of control decision variables and exterior disturbance signals. The control decision variables are the demands sent to HVAC systems (with u_1 represents the cooling power, while u_2 corresponds to the heating power). The disturbances are δ_1, δ_2, and δ_3.

To translate the model into an MPC-friendly model, we must define the state vector x, the control signal vector u, and the environment stochastic disturbance vector ω as:

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad u := \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad \omega := \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}.$$

The continuous-time state-space model can then be described compactly as:

$$\dot{x} = A_c x + B_c u + C_c \omega \tag{1}$$

where

$$A_c := \begin{bmatrix} -\frac{1}{C_1}(K_1 + K_2 + K_3 + K_5) & \frac{1}{C_1}(K_1 + K_2) & \frac{K_5}{C_1} \\ \frac{K_1+K_2}{C_2} & -\frac{(K_1+K_2)}{C_2} & 0 \\ \frac{K_5}{C_3} & 0 & -\frac{(K_4+K_5)}{C_3} \end{bmatrix}$$

$$B_c := \begin{bmatrix} \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_c := \begin{bmatrix} \frac{K_3}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & 0 \\ \frac{K_4}{C_3} & 0 & 0 \end{bmatrix}. \tag{2}$$

We then consider the discrete-time (sampled) version of Equation (1) described by

$$x_{k+1} = A_d x_k + B_d u_k + C_d \omega_k \tag{3}$$

where k is the discrete-time index, $x_k = [x_{1,k}\ x_{2,k}\ x_{3,k}]^T$ and the parameters $[A_d, B_d, C_d]$ are computed from the continuous-time model parameters in Equation (2).

It should be mentioned that such state-space matrices A, B, G can be easily generated for any given buildings through either physics-based or data-driven modeling techniques.

2.2. Baseline Control Strategy (or RBC)

The performance of the proposed adaptive control scheme will be compared with a baseline rule-based on/off control (RBC) scheme commonly used by thermostats in residential homes. These RBC algorithms represent the core logic behind the most popular mechanical and digital controls of thermostats in residential homes. Figure 2a,b describe the overall schemes for summer and winter cases, respectively. Basically, the RBC rules compare the indoor temperature T with a given reference temperature T_{ref}, which is allowed to drift by a cooling/heating dead band ΔT_c or ΔT_h, respectively.

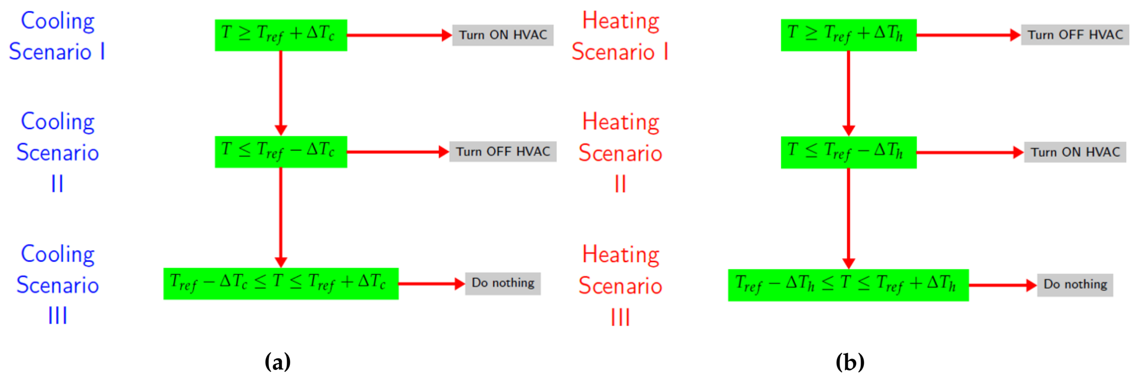


Figure 2. Overall scheme of baseline control (or RBC): (**a**) Overall scheme of baseline line control (or RBC) in summer cooling case; (**b**) Overall scheme of baseline line control (or RBC) in winter heating case.

2.3. System Model

The system states are the room air temperature t_1, interior wall surface temperature t_2, and exterior-wall core temperature t_3. u represents the control input, which is heating power S_h in this scenario (S_h comes as electrical power kW, since power conversion coefficients from heating load are absorbed in the model), and v is the outside disturbance including: the outdoor dry-bulb temperature $T_{outdoor}$°C, the approximated sky temperature T_{sky}°C, the internal load of space $Q_{internal}$ [W], and the solar radiation on the nodes Q_{solar} [W]. All variables with subscripts H correspond to the HVAC.

The thermal model for any given building can then be described as:

$$\dot{X} = AX + BU + GV \tag{4}$$

where

- *States* $X = [t_1, t_2, t_3]$, *Inputs* $U = [S_h]$,
- *Disturbance* $V = \left[T_{outdoor}, T_{sky}, Q_{internal}, Q_{solar}\right]$.

We assume the following constraints are imposed on the temperature and control inputs (for winter heating):

$$23.3\ ^\circ\mathrm{C}(74\ ^\circ\mathrm{F}) \leq t_{1,k} \leq 25.5\ ^\circ\mathrm{C}(78\ ^\circ\mathrm{F}), \quad 0 \leq U_k \leq 1\ \mathrm{kW}, \tag{5}$$

where $U_k \leq 0$ means cooling (we can consider similar heating case when U_k positive). It should be mentioned that sub-index k represents the kth time step. Additionally, we consider a variable speed HVAC system, where 0 represents HVAC totally OFF, and 1 (-1) means working at the maximum heating (cooling) power. It should be mentioned that while the temperature comfort interval has been chosen by ASHRAE, it can be adjusted to any other values based on user's preference.

2.4. Cost Function

In this MPC problem, we desire to minimize both the temperature deviation from the reference setpoints and the energy consumption while simultaneously enforcing a performance guarantee that ensures the indoor temperature always falls in a pre-defined comfort zone. We can assign set-points for all three temperature states, where x_r represents the indoor temperature set-points (dominated by current and future occupancy information). Ultimately, our objective is to find the finite-horizon control sequences which minimize the following finite horizon objective function:

$$V_N(e_0, U, V) := \frac{1}{2}\left[(x_N - x_r)^T P(x_N - x_r) + \sum_{k=1}^{N-1} e_k^T Q e_k + \sum_{k=0}^{N-1} u_k{}^T R u_k\right], \quad (6)$$

where $P \geq 0$, $Q \geq 0$ (i.e., semi-definite positive matrices), $R > 0$ (i.e., positive definite matrix), N is the prediction horizon, and

$$X := [x_0^T, ..., x_N^T]^T, \quad U := [u_0^T, ..., u_{N-1}^T]^T, \quad V := [\omega_0^T, ..., \omega_{N-1}^T]^T.$$

Thus, once we have accurate occupancy predictions available provided in Section 3, we can adaptively adjust temperature set-points according to the novel temperature set algorithm proposed in [34]. From this, we will be able to achieve occupancy-based optimal control (as shown in Figure 3) to improve energy efficiency of the buildings.

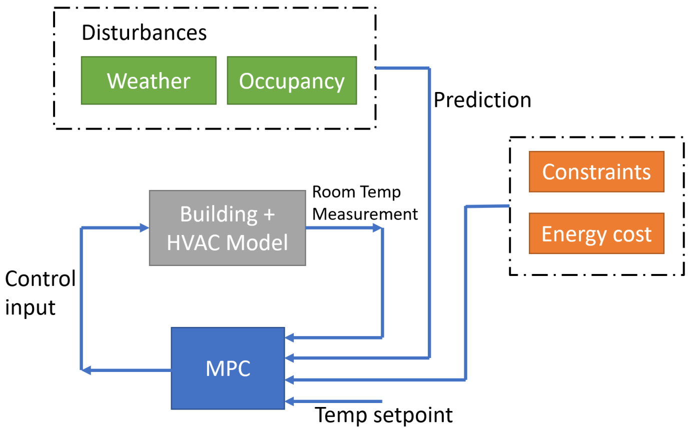


Figure 3. The proposed occupancy-based control setup for a building HVAC system.

2.5. Temperature Set Algorithm

In conventional practice, the HVAC operates under a fixed dead-band (chosen by the users) for indoor temperature. Currently, most temperature set-points are predefined by the building owner or administrator, and do not change frequently during the regular operation periods.

We recall our own simple yet effective algorithm (shown in Algorithm 1) to assign reference temperature set-points for each half hour of the day based on the real-time occupancy [34]. Inside this temperature set algorithm, we need first define the maximum (max(O_h)) and minimum(min(O_h)) occupancy values (based on previous field measurement or survey) during normal operation. Similarly, we also must define the comfort band chosen by the customers. Obviously, the larger the band is, the more energy savings will be achieved. The beauty of Algorithm 1 is its ability to identify a temperature set-point depending on the occupancy information. The temperature set-points are then assigned to each half hour of the day based on the range in which the occupancy number of corresponding half hours fall in.

Following [38], a discomfort tolerance index α is defined to characterize building users' different choices/tolerance on thermal comfort (discomfort). Discomfort tolerance is considered high when $\alpha > 0$, and low when $\alpha < 0$.

Algorithm 1 Temperature Setting Algorithm [34]

Step 1:
    Initialize α
Step 2:
    $n \leftarrow \frac{T_{max} - T_{min}}{k} + 1$
for all hour $h = 1$ to 48 **do**
    $Range \leftarrow max(O_h) - min(O_h)$
    $r_0 \leftarrow min(O_h)$
end for
if $\alpha = 0$ **then**
    Go to step 3
else
    Go to step 4
end if
Step 3:
for all set-point j ($j = 1$ to n) **do**
    $r_j \leftarrow r_{j-1} + \frac{Range}{n}$
    Go to Step 5
end for
Step 4:
for all set-point j ($j = 1$ to n) **do**
    $r_j \leftarrow r_{j-1} + Range * \frac{2^{\alpha(j-1)}(1-2^{\alpha})}{(1-2^{\alpha n})}$
end for
Step 5:
for all hour $h = 1$ to 48 **do**
    $T_h^{set} \leftarrow k[argmin\{j : O_h \leq r_j\} - 1] + T_{min}$
end for

3. Occupancy Prediction Algorithms

In this section, we introduce our algorithm for the detailed estimation of future occupancy, and we show how we could predict the number of occupants in the room. An overview of all the developed algorithms is summarized in Figure 4.

In order to study a practical occupancy model, we collected real occupancy data from an occupancy sensor for a room. We randomly pick a segment of data dated "13 October 2010–5 April 2011", i.e., 174 days. The sampling interval is 30 min, so each individual sensor collects 48 occupancy samples each day. i.e., we have 8352 samples for each sensor. For simplicity, we consider the 8352 samples from a single sensor.

Natural questions which arise are: what is the probability for this room to be occupied; and how many people will be in the room?

To answer the first question, we could compute the probability for the room to be occupied by observing historic data. This has been reported in our earlier paper [34], so we skip this model here due to space constraints.

For the second problem, we need to apply more intelligent techniques (proposed in Sections 3.1–3.5) to predict the number of people in the room.

Due to occupancy's stochastic characteristic, it is not realistic to expect the real occupancy of the room to exactly follow the given schedule. Therefore, we should desire to accurately predict the occupancy information based on the most recent measurement. Moreover, detailed occupancy estimation considers not only whether the building is occupied or not, but also takes into account the number of occupants in the building. Here we will introduce some background and several alternative techniques that may be adequate for occupancy estimation.

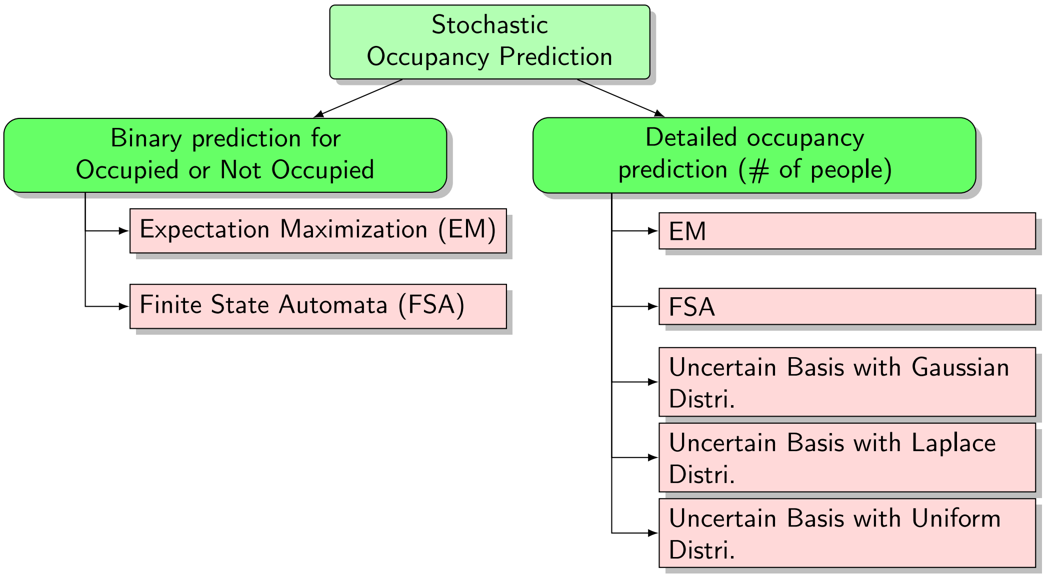


Figure 4. Overview of the occupancy prediction algorithms.

3.1. Expectation Maximization (EM)

The first occupancy modeling algorithm relies on the state-space model, which has been very popular in both societies of control systems and signal processing due to its advantage of on-line recursive implementation. The EM algorithm is a data-driven approach that builds and updates the estimated occupancy relying purely on collected measurements. Its core mechanism consists of two simple equations , i.e., a state x_k Equation (7) and a measurement y_k Equation (8).

A standard EM model in discrete time can be written as:

$$x_{k+1} = A_k x_k + B_k w_k \quad (7)$$
$$y_k = C_k x_k + D_k v_k \quad (8)$$

where $x_{k+1} \in \mathbb{R}^{n\times 1}$ ($\mathbb{R}^{n\times 1}$ denotes the space of real vectors with dimension $n \times 1$) is the state that characterizes the occupancy; it is a variable of the time series $\{x_k\}$ determined by the previous state x_k and the noise term $w_k \in \mathbb{R}^{m\times 1}$ introduced at each k. $A_k \in \mathbb{R}^{n\times n}$ and $B_k \in \mathbb{R}^{n\times m}$ are corresponding coefficients.

The beauty of the EM algorithm is the time varying nature inherited in the state-space model, which enables the model to adapt dynamically to the most recent occupancy model. Moreover, it takes into account the noise terms w_k and v_k that capture small perturbations or uncertainties introduced at each time k. This greatly alleviates the challenges associated with occupancy uncertainties in the model.

We can estimate the unknown system parameters $\beta_k = \{A_k, B_k, C_k, D_k\}$ and states $\{x_k\}$ through a finite set of received signal measurement data $Y = \{y_1, y_2, ...\}$ following [39]. Finally, we can achieve a one-step prediction of the occupancy by [40]:

$$\hat{y}_{k+1} = \hat{C}_k(\hat{A}_k \hat{x}_{k/k} + \hat{K}_k(y_k - \hat{C}_k \hat{x}_{k/k})) \quad (9)$$

where $\hat{y}_{k+1}$ denotes the predicted occupancy at $k+1$ and $\hat{K}_k$ is the Kalman gain.

3.2. Finite State Automata (FSA)

Probabilistic FSA [41] have previously been introduced to describe distributions over strings. FSA has been used quite successfully to address several complex sequential pattern recognition problems, such as continuous speech recognition, computational biology [42] and linguistics [43]. The general Systems Problem Solver (GSPS) proposed in [44], provides a novel and highly effective method for reconstructing the input/output behavior of FSA. The detailed algorithm and system formulation can be found in [45].

Given a system and a sequence of length n generated by that system, we may posit a relation between its variables. This relation takes the form of a function f that maps observations of some variables at times $n, n-1, \ldots, k$ with $k \geq 1$ to observations of other variables at time n.

In this occupancy problem, we want to know the exact number of people in the room. So we assume different numbers of occupants as different states in the FSA. As long as finite states are involved, the general form of the rule and the methods for constructing and forecasting are the same.

Such a relation is called a *rule*. Typically this purpose is forecasting; i.e., to predict future observations from past and current observations. This relation takes the form of a function f that maps observations of some variables at times $t, t-1, \ldots, t-n$ to observations of other variables at time t. Rules are posited by the observer for some particular purpose.

Continuing the example above, consider strings comprised of the letters a, b, c. Take these strings to be our system and the variable v_1 to be a single character in a string. Observations of this variable are indexed by position in the string ordered from left to right so that the first character in the string is $v_1(1)$, the second is $v_1(2)$, and so forth. One example of a sequence for this system is *abcabcabc*.

A possible rule that predicts the next letter in this string is $v_1(n) = f(v_1(n-1))$. This rule relates the characters in subsequences of length two such that the leftmost character predicts the rightmost character. Interpreting the sequence *abcabcabc* with this rule yields the following function: $f(a) = b$ which occurs three times, $f(b) = c$ which occurs three times, and $f(c) = a$ which occurs two times.

3.3. Simplified Binary States FSA

In general, the length of the "look back depth" used is decided by actual problem. For our specific occupancy prediction problem, the system of interest has a single variable with two possible values: occupied (b) or not occupied (a). After tail and error check, we posit a rule that looks back three steps and also considers the time of day, i.e., $v_1(n) = f(v_1(n-1), v_1(n-2), v_1(n-3), t(n))$. The time of day $t(n) = t_n$ is used to characterize the different rules for different time period during a day; for the available data use there are 48 times that can be considered, as the sampling interval of the sensor is 30 min. Given the data and a rule, we can build a model for forecasting with the simple procedure described by Klir [44]. More details about working mechanisms for FSA can be found in [34].

To illustrate this procedure, consider the mask and sequence in Table 3. The system that generated this sequence has time variable t_n and a single variable v that can take the value b or a. The rule for this mask built with 3000 data points is shown in Table 4. It should be noted that in order to simplify the explaination, we decouple the time variable from the rule. To be specific, Table 4 represents the rule for 3:00 p.m. using 3000 data points. Considering the sampling interval is every 30 min, we could generate 48 such rule tables for the whole day (24 h).

Table 3. A sequence and mask for a system with uncertain output. Circles are input data for the function f and the square is the output data.

Round	Time (v_1)	Occupancy (v_2)
11	t_{11}	$\boxed{a}$
10	t_{10}	b
9	t_9	a
8	t_8	b
7	t_7	a
6	t_6	a
5	t_5	b
4	t_4	a
3	t_3	b
2	t_2	b
1	t_1	a

Table 4. Rule for the sequence and mask using a subset of 3000 points restricted to 3:00 p.m. (*a*-unoccupied, *b*-occupied).

Input	Output	Count	Likelihood
aaa	a	47	0.959
	b	2	0.041
aab	a	0	0
	b	1	1
aba	a	1	1
	b	0	0
abb	a	0	0
	b	1	1
baa	a	1	0.5
	b	1	0.5
bab	a	1	0.33
	b	2	0.67
bba	a	0	0
	b	1	1
bbb	a	0	0
	b	4	1

To illustrate how this model is applied for forecasting, suppose we begin with the latest observation sequence as *bab* at 3:00 p.m. The next output is *a* with probability 0.33 or *b* with probability 0.67. If we were at a different time step other than 3:00 p.m., then the output is selected according to the corresponding rule table at that time step. Once the time step is fixed, then the second output is *a* with some probability p or *b* with probability $1 - p$. Continuing in this fashion, we can construct a tree of possible futures and use these possible futures to inform the control system.

3.4. Estimating Number of Occupants

We consider three methods for anticipating the actual number of occupants within a room.

FSA with 3 or More Input/Output Values

Though the method described in Section 3.3 only involves two states, it is readily extended to estimate the exact number of occupancy by using the number of occupants as variable rather than the binary occupied/unoccupied.

In the following simulation results, we show that the discussion for the binary model applies directly to the model with multiple states.

3.5. Basis Function

Finally, we introduce the third powerful algorithm, i.e., the uncertain basis functions. If we consider y_k as the current occupancy measurements detected by occupancy sensor, it can be well represented by the following basis functions:

$$y_k = \sum_{i=1}^{p} A_i \phi_{i,k} \quad k = 1, 2, ..., N, \tag{10}$$

where $\{\phi_{1,k}, ..., \phi_{p,k}\}$ are the basis functions, and $p < N$, and A_i is the corresponding coefficient of each basis function [46,47].

However, this trivial representation may not be able to capture all the dynamics of occupancy due to the limitation of fixed basis functions, which eliminates the uncertainty nature. An alternative way to cast this formulation is to assume that each basic function depends on some unknown parameter vectors $\boldsymbol{\theta}_i$, where only some statistics of the distribution of $\boldsymbol{\theta}_i$ are known. Then, the corresponding coefficients can be estimated by minimizing an expected cost function following the technique developed in [48].

Following [48], we assume that each $\boldsymbol{\theta}_i$ is independent. The basic functions are further represented by $\phi_{i,k}(\boldsymbol{\theta}_i)$. The main objective here is to find the best coefficients to minimize the expected cost function $\hat{J}(\boldsymbol{A})$ defined as:

$$\hat{J}(\boldsymbol{A}) = \mathbb{E}_{\boldsymbol{\theta}} \left[\sum_{k=1}^{N} |y_k - \sum_{i=1}^{p} A_i \phi_{i,k}(\boldsymbol{\theta}_i)|^2 \right] \tag{11}$$

where $\mathbb{E}_{\boldsymbol{\theta}}$ is the expectation with respect to $\boldsymbol{\theta}_i$. The measured values are real, so we could estimate the coefficients $\boldsymbol{A}$ following steps presented in [39].

We focus on predicting future occupancy using the basis function:

$$\hat{y}_k = \sum_{i=1}^{p} \hat{A}_i \mathbb{E}_{\boldsymbol{\theta}}[\phi_{i,k}(\boldsymbol{\theta}_i)] \tag{12}$$

This means that we can predict occupancy values with p random basis functions. Similarly as in [39], we assume the basis functions to be governed by three different distributions, i.e., *Gaussian, Laplace and Uniform*. Consequently, we are able to compute occupancy predictions under each distribution.

4. Case Studies

To illustrate the effectiveness of the occupancy prediction techniques proposed in the last section, we assess their performance using the aforementioned occupancy measurement (at the beginning of Section 3). In the first part, performance of three occupancy prediction techniques are examined, and corresponding accuracy comparison are provided. In the second part, different temperature set trajectories s are obtained using our temperature set algorithm. The algorithm is employed to assign reference temperature set-points for each hour of the day. Lastly, these reference temperature set-points are utilized in the standard HVAC control strategies. In essence, different energy saving benefits are studied for traditional ON/OFF control and advanced MPC control, respectively.

4.1. Definition of the Performance Indexes

We define the estimation accuracy as the total number of correct predictions divided by the total number of predictions. The Root Mean Squared Error (RMSE), which characterizes the absolute estimation errors.

More formally, if we let ACC represent the accuracy, then using indicator functions, we obtain the accuracy of the estimator:

$$ACC(\hat{O}) := \frac{N - \sum_{k=1}^{N} \mathbb{1}(|O(k) - \hat{O}(k)|)}{N}. \quad (13)$$

where O and $\hat{O}$ are true and estimated occupancy, respectively. Let us denote the characteristic function of estimation error $\mathbb{1}(EO(k))$ as:

$$\mathbb{1}(EO(k)) := \mathbb{1}(|O(k) - \hat{O}(k)|) = \begin{cases} 1 & \text{if } O(k) > 0 \\ 0 & \text{otherwise,} \end{cases} \quad (14)$$

Here, the RMSE is defined as the square root of the mean square error:

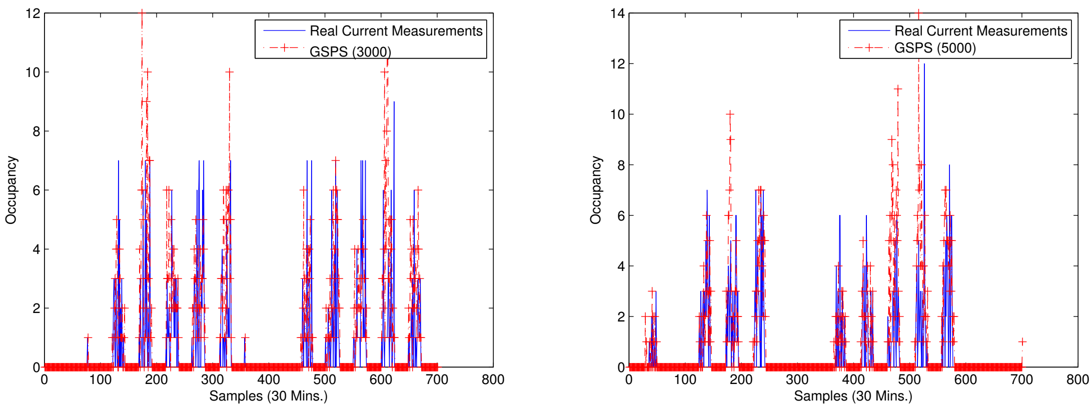
$$MSE(\hat{O}) := \frac{1}{N} \sum_{k=1}^{N} (\hat{O}(k) - O(k))^2. \quad (15)$$

Therefore, RMSE will be the square root of (15).

4.2. Occupancy Prediction Performance

4.2.1. GSPS Model

In this GSPS-based scenario, we assume we have access to a large enough historical data set. We trained the model using the last 3000 and 5000 data points, respectively. The prediction results are shown in Figure 5a,b. As expected, the more data involved in training the FSA, the better the prediction results. This is also confirmed by the estimation error comparison shown in Table 5.



(a) (b)

Figure 5. Occupancy estimation using GSPS model: (**a**) Occupancy estimation using GSPS model 3000 points; (**b**) Occupancy estimation using GSPS model 5000 points.

4.2.2. EM Method

Next, the elegant EM algorithm is applied to occupancy prediction. The performance is depicted in Figure 6, where only the last 20 sample points are used to predict the occupancy information at the very next time step.

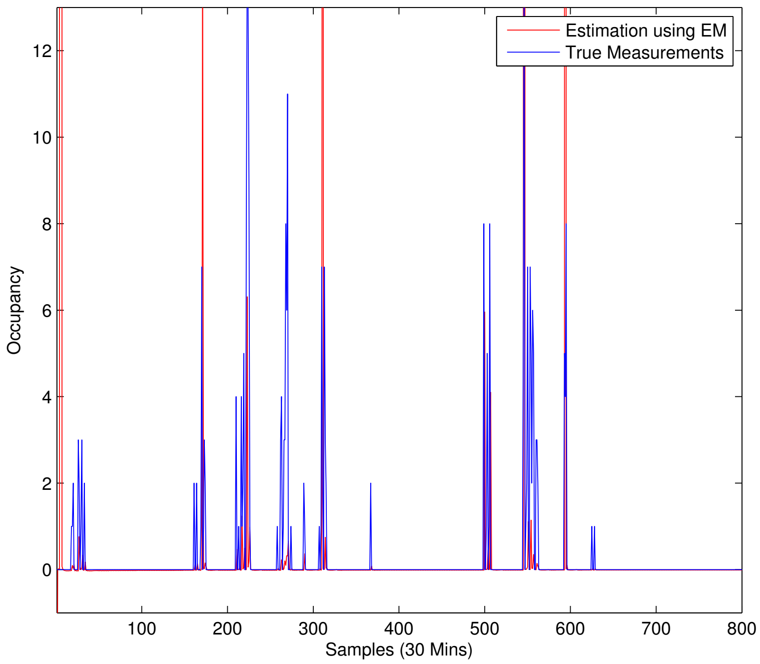


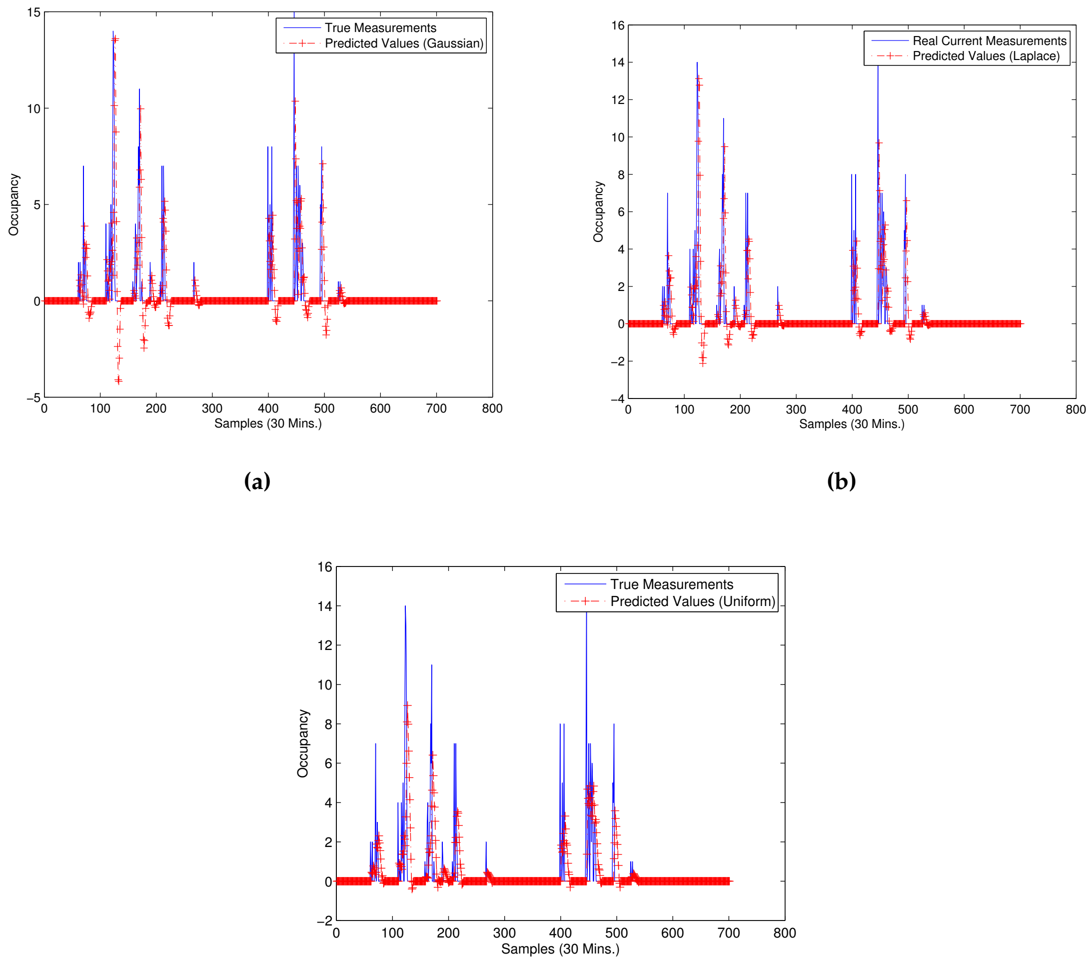
Figure 6. Occupancy estimation using EM algorithm.

4.2.3. Uncertain Basis Functions

In order to the show robustness of the proposed uncertain basis technique, we tested it with three different distributions for θ_i. It should be mentioned that only 10 last sampling points are used to build the optimal basis for each distribution. Figure 7 shows prediction comparison results using three different distributions.

Table 5. Performance comparison of occupancy prediction algorithms.

Methods	Estimation RMSE	Accuracy
GSPS (3000)	3.078	70.0%
GSPS (5000)	2.646	71.5%
EM	3.715	61.5%
Basis-Gaussian	3.211	68.4%
Basis-Laplace	2.946	70.9%
Basis-Uniform	2.571	72.6%



(a) **(b)** **(c)**

Figure 7. Occupancy estimation using three different basis functions: (**a**) Gaussian basis functions; (**b**) Laplace basis functions; (**c**) Uniform basis functions.

4.3. Temperature Set Points

Through the above simulation results, we achieve the desired occupancy prediction. However, our goal is to design temperature set-points based on these occupancy sequences. In order to further determine the effectiveness of these occupancy prediction results, we integrate the occupancy prediction results into the temperature set algorithms. The one corresponding to basic functions using *uniform distribution* is presented in Figure 8.

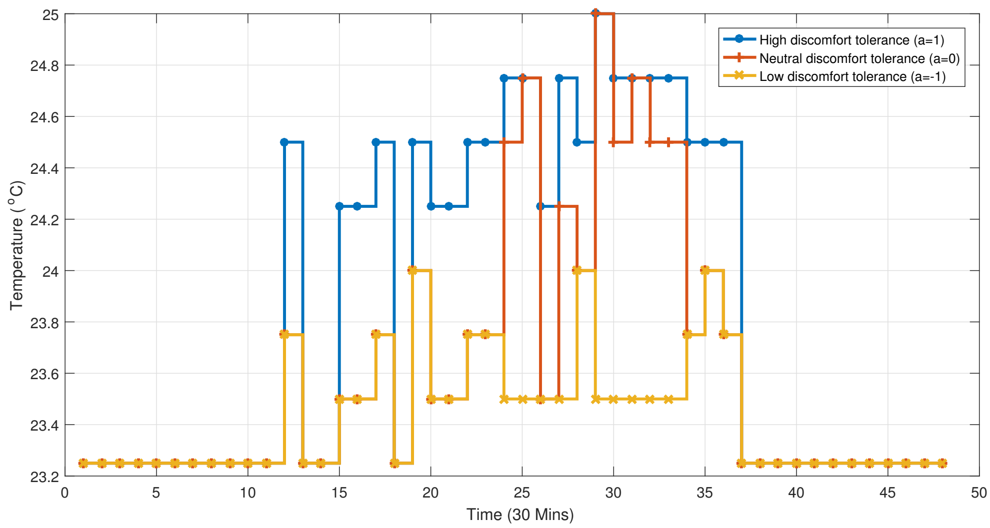


Figure 8. Temperature set point for uncertain basis method.

4.4. Occupancy-Based Control

To show the energy-saving performance using the proposed stochastic models for occupancy and temperature set algorithm, we insert the obtained temperature reference into simple ON/OFF switching control framework. A fixed reference temperature 23 °C is chosen for the baseline scenario. An occupancy-dependent reference temperature generated via our temperature set algorithm replaces original fixed schedule. This simple step will adaptively tune the set-point of HVAC systems according to current occupancy information, avoiding wasting energy for empty rooms.

In order to quantify the performance using different algorithms, we utilize the energy cost using traditional ON/OFF control strategy without any occupancy information as a benchmark. It should be mentioned that energy cost is defined as 2-norm, i.e., $U_{total} = \sqrt{\sum_{k=1}^{T} U_k^2}$.

Next, a comparison between different algorithms is made by changing only the occupancy information. Detailed numerical result is given in the right end column of Table 5. We are expected to save approximately 13% and 20% energy consumptions for traditional ON/OFF and MPC control strategies, respectively.

Table 6. Comparison of energy saving.

Methods	Control Cost (U_{total})	Energy Saving
Basic Control (No Occupancy info)	5.43	0%
Basis-Gaussian (Basic Control)	4.77	13%
Basis-Gaussian (MPC)	4.38	20%

4.5. Summary of the Results

In this section we compare four occupancy prediction algorithms, all trained using the same training set described at the beginning of Section 3. Figure 8 shows the realizations of occupancy predictions can be applied to the corresponding test set. Ideally, we can increase temperature set point when less occupants will be present in the room. As expected, we notice higher temperature set points are achieved corresponding to a larger tolerance index, as we are studying a summer cooling case in this paper. Table 5 summarizes the achieved numerical performance and accuracy comparison for the three algorithms. Generally speaking, both EM and Uncertain basis methods can provide reliable predictions with just a few historical data points. The GSPS method meanwhile, requires many more points to build a reliable model. Additionally, GSPS and Uncertain basis methods achieve a higher accuracy, while the EM method provides a degraded prediction result. Each method may find its own suitable application scenarios depending on the accuracy requirement and data structure.

It should be remarked that, although some mismatches exist for non-zero jumps in Figures 5a–7c, all prediction algorithms track the 0 base line (non-occupied status) perfectly. Therefore, all prediction techniques are effective for identifying empty rooms, with an over 90 percent accuracy rate. Moreover, the accuracy conditions we set are extremely restrictive. In other words, the accuracy is said to be satisfied only when the estimated number of occupants is exactly the same as the real measurement. So in this case, the accuracy is void if the estimated number is 13 while actual number is 12.

Furthermore, the obtained occupancy status is successfully applied into the temperature set algorithm which dominates energy consumption in building climate systems. In this final experiment, we applied the designed temperature setpoints into two different algorithms - basic control and MPC (designed in Section 2), and compare their energy consumption. The building thermal zone model we picked has also been introduced in Section 2. The detailed energy consumption data of the simulation has been given in Table 6. These control tests complete our occupancy-based control framework, which showcases up to 20% energy saving benefits via the proposed corresponding control framework.

5. Conclusions and Future Work

In this paper, we propose three different occupancy prediction methods for demand-based HVAC control. All three proposed short-term stochastic modeling methods, GSPS, EM and uncertain basis, achieved more than 70% accuracy in the experimental studies. Furthermore, we have designed a novel temperature set algorithm to correctly assign temperature set points based on the instantaneous occupancy information. To complement the occupancy-based framework, we have integrated the temperature set point naturally into the MPC algorithm. Finally, detailed comparisons are provided for energy consumptions with various occupancy estimation algorithms and without any occupancy information. This paper delivered a novel end-to-end solution, which connects a reliable stochastic occupancy modeling study with the occupancy-based control design. Consequently, we have seen up to 20% energy saving by integrating the proposed technique into two standard HVAC control strategies.

A great number of increasingly complex models are being developed for HVAC systems. However, the limited number of implementations of such models in demand-based control and the lack of occupants' effects limits their ability to improve efficiency while guaranteeing a comfortable temperature environment in buildings. Our near future work will involve detailed internal heat gain subject to different occupancy situations and various application scenarios, particularly the hot topic of building-to-grid integration. Another interested direction is to perform the sensitivity analysis for changing the set point. Basically, to answer the question, when is the best time to change the set-point; and how long/much will the electricity consumption reflect the change. This knowledge is critical for doing demand-response using buildings.

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