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Article

Effects of Carbon-Emission and Setup Cost Reduction in a Sustainable Electrical Energy Supply Chain Inventory System

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Received: 15 February 2019; Accepted: 28 March 2019; Published: 29 March 2019

Abstract: This article develops a sustainable electricity supply chain mathematical model that assumes linear price-dependent customer demands where the price is a decision variable under setup cost and carbon emission. The sustainable electrical supply chain system contained: (a) power generation; (b) transmission substations; (c) distribution substations; and (d) customer. The production rates depend on the demand rate, and demand for electricity by the customers is dependent on the price of electricity where the electrical energy was generated and transmitted through multiple substations to customers. Moreover, we considered that the capacities of transmission rates, power generation, and distances in between two stations are associated with the distribution costs and transmission cost. Here, we used the theory of inventory to develop a new model and suggested a procedure to deduce an optimal solution for this model. Finally, a numerical example and sensitivity analysis are employed to illustrate the present study and with managerial insights.

Keywords: sustainable electrical energy supply chain; inventory; price-dependent demand; transmission and distribution costs; carbon-emission

1. Introduction

1.1. Background of the Research

Generally, a supply chain inventory model defines the retailer-customer relationship. Setup cost plays a crucial role in today's supply chain inventory management system for shipment of items on time. The setup procedure is not evaluated as a fixed/known constraint, but needs to be considered at a time of minimizing waste, satisfying deadlines, productivity, and elaborating resource utilization. To minimize the total cost investment, the manufacturer needs to reduce setup costs. In a supply chain inventory system, the capacity of electrical energy has the same process for determining the order quantity and total profit. In an electrical supply chain inventory system, electricity generated in a power generation plant will be transmitted through a transmission line and distribution substation to the customers whose demand is influenced by the price of electricity in order to maximize the profit.

The consumption of electricity energy has rapidly increased. The total electricity consumption worldwide in 2015 was greater than in 1980. Electricity consumption for industrial, commercial, residential and transportation sectors has been enhanced since 2015. This is due to increasing climate changes upon home electricity use at a rate of more than 100 million kWh/day, due to the need for air-conditioning (mainly in summer time) in homes. The electricity price may have influenced the demand for electricity. The price of electricity is set to reduce the demand. Generally, the price is usually the maximum for commercial customers and the residential sector because of the cost to distribute electricity supply to them. Furthermore, an increase of global greenhouse gas carbon emissions (GGCE) in all countries of world is farther than ever from reaching the goals of the Paris Agreement, according to the new United Nations report, released in 2018, prior to a meeting of officials and environmental experts from all around of the world in Katowice, Poland, for climate negotiations. A few years ago at the environmental negotiations in Paris all countries agreed to bring down GGCE sufficient to maintain global warming below 2 degrees Celsius, and under 1.5 degrees. A report from the UN's top climate panel published earlier this decrease found that an additional 0.5 degrees of warming would have forceful and unfortunate effects on the environment, allowing for more empirical support for those countries pressing for more challenging targets during the negotiations at COP24. But to maintain temperature increases under 1.5 degrees, global carbon emissions are required to peak by 2020. In between 2014 to 2016, global carbon emissions stayed comparatively flat, and negotiators hope that the decade long trend of increasing carbon emissions is about to reverse. But after few years of stagnation, global carbon emissions grew again in 2017, reaching a record 53.4 gigatons of carbon emission equivalent. To meet the 1.5-degree mark, the world requires to jointly bring emissions down in the next 12 years. Therefore, governments all over the world should implement cap-and trade regulations policies to keep the emissions low. For reducing carbon emissions, production companies can monitor and enhance the emission performance of their products during their life-cycle stages. The carbon-emission assessment provides a possible mechanism to serve companies with some emission reduction. During the production process, the manufacturer should formulate a low carbon system.

1.2. Research Questions, Motivation and Contribution of the Model

In the literature review section (Section 1.3) it can be clearly seen that some works seek to determine sustainable electrical supply chain inventory models and no research tries to determine a sustainable electrical supply chain inventory model with setup cost reduction and CO2 emissions. Our research questions this model: (1) what is the electrical power distribution factor's effect on the distribution substation, and the electrical power transmission factor's impact on the transmission substation and electrical power generation factor; (2) how much is the ordering quantity?; (3) what is customer's average electricity consumption time?; and (3) what is the retailer's selling price? To answer these questions in this study, we developed and solved a sustainable electricity supply chain inventory model with setup cost reduction and CO2 emissions while considering environmental parameters. The aim of this study is to examine the effect of price-dependent demand on the sustainable electrical supply chain inventory system under setup cost reduction and carbon emissions. The energy is transmitted via distribution networks to customers whose demand is determined by the price of electricity in order to find an optimal solution. The contributions of this paper are presented in Table 1.

1.3. Literature Review

Several research papers highlighted various integrated inventory models with various key parameters. Yang [1] deduced an inventory model considering lead time and crashing cost. Hoque [2] studied an integrated inventory model considering a normal distribution lead time, setup time, batch time and the cost of transportation. Sarkar et al. [3] deduced an inventory model by adding an imperfect production concept. A non-defective product adopts a binomial distribution function and demand adopts a mixture of the normal distribution function in their model. Mishra [4] formulated a production-inventory model with price dependent demand where the production depends on the rate of demand. Multiple buyers and a single-vendor model apply in a food inventory system studied by Fauza et al. [5]. Denizel et al. [6] formulated a lot size inventory model with setup costs decreased by various amounts depending upon the raw-materials and discussed the shortest path problem. Diaby [7] demonstrated a complete model to reduce setup cost and time with determining cut setup time and minimizing the total cost. Nyea et al. [8] studied several inventory models for optimal investment of setup cost reduction or optimal setup times; and also discussed the queuing model to estimate work in the process level in their models. Later, Freimer et al. [9] demonstrated improvement in quality and setup cost reduction of the process in their model. Huang et al. [10] assumed setup cost policy reduction by an added investment cost. Sarkar and Moon [11] deduced a model by putting the policy of reorder point, quality improvement and lead time by considering that backorder rate has a great impact under a production process and is imperfect. Sarkar et al. [12] studied the concept of setup cost-reduction policy under quality improvement. Sarkar et al. [13] studied an effect of setup cost-reduction process in a two-echelon supply chain inventory model under deterioration and using the technique of quality improvement. Sarkar et al. [14] developed an integrated inventory model for a setup cost-reduction policy, carbon-emission policy and used the technique of the Stackelberg game approach to find the total cost. Little research has been done on a supply chain electrical energy inventory model. Banbury [15] was the first researcher to developed an electricity supply chain model. Thereafter, (Schneider et al. [16], Schneider et al. [17]) first developed this using a very simple inventory policy to find the electrical supply chain policy. Later, Taylor et al. [18] studied capacity and price competition in an electricity market by using a two-stage game theory model. Wu et al. [19] studied a new model to examine the formal generation with sporadic supply. Ouedraogo [20] formulated an electricity supply chain with demand in an African power system. Wangsa and Wee [21] studied an electrical supply chain inventory policy assuming the blackout cost. Recently, interesting research by Wangsa et al. [22] assumed a sustainable supply chain inventory model and the effect of price-dependent demand.

Table 1. Other authors contribution to this theme.

Author	Inventory Model	Electrical Supply Chain System	Setup Cost Reduction	Carbon Emission
Yang [1]	✓	×	×	×
Hoque [2]	✓	×	×	×
Sarkar et al. [3]	✓	×	×	×
Fauza et al. [5]	✓	×	×	×
Denizel et al. [6]	×	×	✓	×
Diaby [7]	×	×	✓	×
Nyea et al. [8]	×	×	✓	×
Freimer et al. [9]	×	×	✓	×
Huang et al. [10]	✓	×	✓	×
Sarkar and Moon [11]	×	×	✓	×
Sarkar et al. [12]	×	×	✓	×
Sarkar et al. [14]	✓	×	✓	✓
Banbury [15]	×	✓	×	×
Schneider et al. [16]	✓	✓	×	×
Schneider et al. [17]	✓	✓	×	×
Taylor et al. [18]	×	✓	×	×
Wu et al. [19]	×	✓	×	×
Ouedraogo [20]	×	✓	×	×
Wangsa and Wee [21]	✓	✓	×	×
Wangsa et al. [22]	✓	✓	×	×
Hammami et al. [23]	✓	×	×	✓
Tang et al. [24]	×	×	×	✓
Tang et al. [25]	×	×	×	✓
Ouyang et al. [26]	✓	×	×	×
This paper	✓	✓	✓	✓

Finally, in this study we investigate many research articles involving supply chain inventory model-related carbon emissions, setup cost reduction and electricity energy. Research paper related to the above are the following: Hammami et al. [23] developed a multi-echelon supply chain model with reducing carbon emission, several manufacturing facilities, different outside suppliers, and distinct distribution centers. Tang et al. [24] studied a carbon-emission policy with minimal frequency of shipments for a periodic inventory review system. Thereafter, Tang et al. [25] developed a sustainable supply chain network for consumers, and environmental manners are added by inventory, routing, and location.

1.4. Methodology

In this section, we consider a sustainable electricity supply chain power system with price-dependent demand electricity demand. In reality, it is shown that determining the capacity of a sustainable electrical supply chain system has the same methodology as finding the order quantity q in the supply chain inventory system. The supply chain inventory system involves a vendor–buyer coordination and freight forwarding. The buyer sells items to the customers and orders items from the vendor. The vendor produces the items and sends in batch to the buyer. The buyer will then sell the items to the customers. But in sustainable electrical supply chain system case, the electricity generated from a power generation will be transmitted through a transmission line and distribution substation to the customers whose demands are influenced by the price of electricity where the electricity is continuously supplied to consumers without any interruptions. The electricity demand is considered as $D(p)$ kWh/year. The power generation produces the electricity in a batch size of $qT\zeta\eta\rho$ kWh where ζ (positive integer) is the distribution factor's effect on the distribution substation, η (positive integer) is the transmission factor's impact on the transmission substation and ρ (positive integer) is a power generation factor. The finite power supply rate is $P = \lambda D(p)$ kWh/year, $[\lambda > 1]$ and a setup cost. The electricity energy of $qT\zeta\eta$ kWh is supplied by the power generator to the transmission substation, then $qT\zeta$ kWh of electricity is supplied to the distribution substation and $E = qT$ kWh of electricity is consumed by the customers. Hence, to maximize the profit of a sustainable electrical supply chain system, we consider the sales revenue, production cost, setup cost reduction of the power generation, ordering cost of customers and transmission/distribution costs of substations. The transmission and distribution costs are functions of the power plant, the transmission substation and the distribution substation with maximum capacities of in z_p^x kWA, z_t^x kWA, z_d^x kWA, respectively. The comparison in supply chain inventory system and sustainable electrical supply chain system are shown in Figure 1.

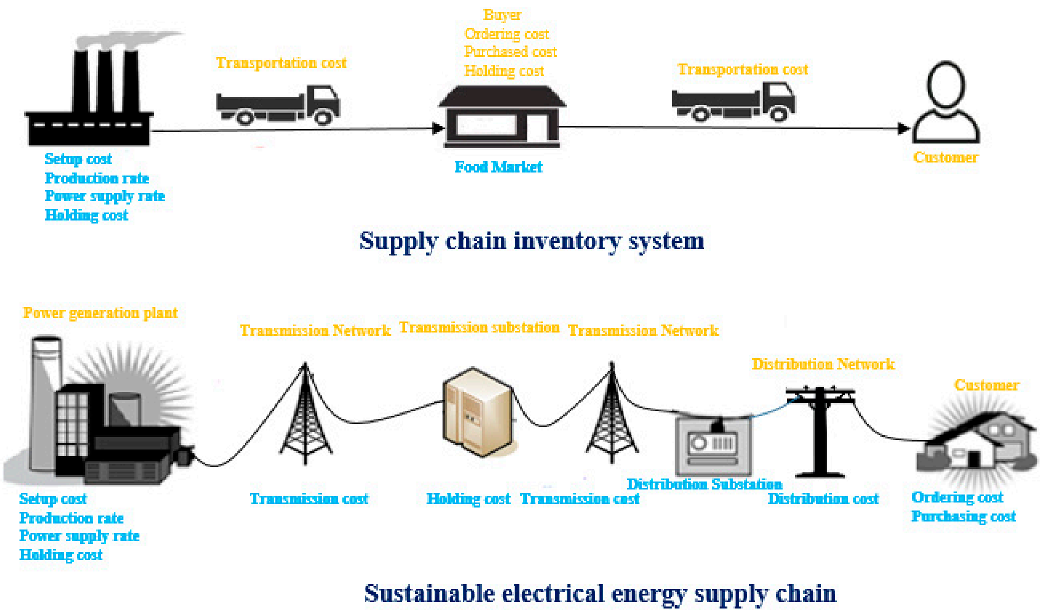

Figure 1. Supply chain inventory and sustainable electrical supply chain.

The rest of the paper is prepared as follows: in Section 2, the assumption and notation for model formulation of the electrical supply chain inventory system are given; in Section 3, we describe the model formulation of the electrical supply chain inventory system. In Section 4, a numerical example is presented for validation of this model. A discussion of the managerial implications is presented in Section 5. Section 6 concludes the work by offering directions for future research work.

2. Assumptions and Notations

In this section, some notation can be used for development of the mathematical model (see Section 3) using the following assumptions:

2.1. Assumptions

- An integrated inventory model is considered.
- A single-buyer for single-types of items are considered.
- Power supply blackouts are not considered.
- The finite power supply rate is greater than demand rate and power supply and demand relation is $P = \lambda D(p)$, where $\lambda > 1$.
- Demand is a linear function of the selling price, which is a more realistic representation of the real world than is the assumption that the demand is a fixed parameter. The linear demand function is considered to be $D(p) = a - bp$ (as shown in Mishra et al. [27]; Mishra et al. [28]) and the electricity demand rate of the customers depends on selling price; where $a > 0$ is scaling factors, $b > 1$ is the elasticity coefficient and satisfied the condition $p < \frac{a}{b}$.
- To reduce setup cost, investment cost I is considered. The expression of setup cost $S(I) = v_0 e^{-\tau I}$, where $v_0(> 0)$ setup cost at the initial stage and $\tau(> 0)$ is a constant parameter. $dS(I)/dI = -\tau v_0 e^{-\tau I}, d^2S(I)/dI^2 = \tau^2 v_0 e^{-\tau I} > 0$, means that if the investment will be a higher value than the setup cost it will be smaller value. Therefore, the investment function, setup cost for every production run can be lower value. The investment I decisions will consequence on the setup cost and the setup cost can be a major function of the production system. For example, see Sarkar et al. (2016).
- The total power consumption is q in kW.
- $E = qT$ kWh of electricity is consumed by the customers within a particular time T.
- $qT\zeta$ kWh of electricity is supplied to the distribution substation; where ζ (positive integer) is the distribution factor's effect on the distribution substation.
- $qT\zeta\eta$ kWh is the electricity energy supplied by the power generator to the transmission substation; where ζ (positive integer) is the distribution factor's effect on the distribution substation and η (positive integer) is the transmission factor's impact on the transmission substation.
- $qT\zeta\eta\rho$ kWh of power generation produces the electricity in a batch size; where ζ (positive integer) is the distribution factor's effect on the distribution substation,η (positive integer) is the transmission factor's impact on the transmission substation and ρ (positive integer) is a power generation's factor.
- Power generation, transmission and distribution are following functions; Maximum capacity power generation plant is z_p^x in kWA. Maximum capacity the transmission substation is z_t^x in kWA. Maximum capacity the distribution substation is z_d^x in kWA.
- The relations of the maximum capacity of power generation, transmission substation and distribution substation is $z_p^x > z_t^x > z_d^x$.
- The total power generation process should include the transmission and distribution costs.

2.2. Notation

Notations are used in the model are shown in Table 2 as follows:

Table 2. Notations.

Parameter/Decision Variable/Function	Notations	Descriptions	Units
Parameter	A	Ordering cost per order.	\$/unit/order
	a	Scaling factor.	units
	b	Price elasticity coefficient.	units
	k	Production cost.	\$/order/year
	v_0	Setup cost at the initial stage.	\$/setup
	τ	Known parameter of setup cost $S(I)$.	unit
	r_1	Annual percent of energy holding cost of the transmission substation.	\$/unit/year
	r_2	Annual percent of energy holding cost of power generation.	\$/unit/year
	Ω	Power factor correction.	kVA/kWh
	c_1	The per mile transmission and distribution rates.	\$/kVA/mile
	m_1	The per mile transportation network from power generation to the transmission substation.	miles
	m_2	The per mile transportation network from the transmission substation to the distribution substation.	miles
	m_3	The per mile transportation network from the distribution substation to the customers.	miles
	z_p^x	Maximum capacity of power generation.	kVA
	z_t^x	Maximum capacity of transmission substation.	kVA
	z_d^x	Maximum capacity of distribution substation.	kVA
	β	The power supply loss factor $(0 \leq \beta \leq 1)$.	units
Emission parameter	$\hat{r}_1$	Per unit carbon emission of energy holding of the transmission substation.	ton/year
	$\hat{r}_2$	Per unit carbon emission of energy holding of power generation.	ton/year
	$\hat{c}_1$	Per unit carbon emission of transmission and carbon emissions distribution rates.	ton/kVA/mile
	ξ	Carbon tax per unit (money units for each unit of carbon emitted as tax)	\$/ton/year
Decision variable	I	Investment cost for setup of power generation.	\$/setup
	ζ	The electrical power distribution factor's effect on the distribution substation (positive integer).	units
	η	The electrical power transmission factor's impact on the transmission substation (positive integer).	units
	ρ	The electrical power generation's factor (positive integer).	units
	T	Customer's average electricity (positive integer).	Consumption in hour.
	p	Selling price of electricity (positive integer).	\$/kWh.
	q	The customer's power	kW
Function	Π	Total profit.	\$/year
	$D(p)$	Demand of customer.	kWh/year
	z_d^y	Actual capacity of the distribution substation.	KVA
	z_t^y	Actual capacity of the transmission substation.	KVA
	z_p^y	Actual capacity of the power generation.	KVA

3. Model Formulation

In this section, we explain how to find the total cost function with regard to customers, the distribution and transmission substations, carbon emission cost, and the total profit function for power generation. The total cost functions are given by the following components:

3.1. Ordering Cost

Ordering cost is defined as the customer's total cost per unit time:

$$TC_O = \frac{AD(p)}{qT} \tag{1}$$

3.2. Distribution Cost

The same methodology as (Wangsa et al. [22]) to determine the distribution cost is obtained from the distribution substation. The cost of distribution for partial load G can be written as $G = \frac{c_1 z_d^x}{z_d^y}$. The transmission function, increase in rate per kVA/mile when z_d^y increases. The distribution cost per kVA/mile is $G_Z = \beta G + (1-\beta)c_1$, where $0 \le \beta \le 1$ is the coefficient of the adjusted inverse function. Therefore $G_Z = \beta c_1 \left[\frac{z_d^x - z_d^y}{z_d^y}\right] + c_1$. The estimated total cost for the distribution substation as a function of demand, corrections factor and distance yields: $G_D = \left[\beta c_1 \left[\frac{z_d^x - z_d^y}{z_d^y}\right] + c_1\right] D(p) m_3 \Omega$. The actual power supply capacity is given by $z_d^y = qT\zeta\,\Omega$; therefore, the cost for distribution can be written as in the following equation:

$$TC_D = \frac{D(p)\beta c_1 z_d^x m_3}{qT\zeta} + D(p) m_3 \Omega (1-\beta) c_1 \tag{2}$$

3.3. Carbon Emission Distribution Cost

The carbon emission distribution cost has one component. This carbon emission component is related to total distribution rates. The component is summated and briefly represented by:

$$CE_D = \hat{c}_1 \left[\frac{D(p)\xi\beta z_d^x m_3}{qT\zeta} + D(p)\xi m_3 \Omega (1-\beta)\right] \tag{3}$$

3.4. Transmission Substation Cost

This total cost comprises of the energy holding cost and transmission cost. The cost of energy holding at the transmission substation is presented by $\frac{r_1 p\zeta\eta E}{2} = \frac{r_1 p\zeta\eta qT}{2}$. The transmission cost can be calculated as: $G_t = \frac{D(p)\beta c_1 z_t^x m_2}{qT\zeta\eta} + D(p) m_2 \Omega (1-\beta) c_1$. Therefore, the total cost obtained at the transmission substation is the sum of energy holding cost and transmission cost. Therefore, the total transmission substations cost;

$$TC_t = \frac{r_1 p\zeta\eta qT}{2} + \frac{D(p)\beta c_1 z_t^x m_2}{qT\zeta\eta} + D(p) m_2 \Omega (1-\beta) c_1 \tag{4}$$

3.5. Carbon Emission Transmission Substation Cost

The carbon emission transmission substation cost has three components. The first component is related to total energy holding rates at the transmission substation, the second and third components are related to the total transmission rates. These components are summated and briefly represented by

$$CE_t = \frac{\hat{r}_1 \xi p\zeta\eta qT}{2} + \frac{D(p)\beta\xi \hat{c}_1 z_t^x m_2}{qT\zeta\eta} + D(p) m_2 \Omega (1-\beta)\xi \hat{c}_1 \tag{5}$$

3.6. Profit of Power Generation

The total profit of power generation can be presented by:

$$\Pi_{PG} = SR - PC - SC - INVC - HC - TRC - CE_{TR} - CE_{PG} \tag{6}$$

Sales revenue

$$SR = D(p)p \tag{7}$$

Production cost

$$PC = D(p)k \tag{8}$$

Power generation creates electrical energy in ($qT\zeta\eta\rho$) kWh in one production run. Therefore, the setup cost for power generation can be found by using the equation below. Setup cost is:

$$SC = \frac{D(p)v_0e^{-\tau I}}{qT\zeta\eta\rho} \tag{9}$$

Total setup investment cost is:

$$INVC = \frac{D(p)I}{qT\zeta\eta\rho} \tag{10}$$

Energy holding cost is:

$$HC = r_2k\frac{\left[E\zeta\eta\rho\left(\frac{E\zeta\eta}{P} + (\rho-1) - \frac{E}{D(p)}\right) - \frac{\rho^2[E\zeta\eta]^2}{2P}\right] - \left[\frac{E\zeta\eta}{D(p)}(1+2+3\ldots\ldots+(\rho-1))E\right]}{\frac{E\zeta\eta\rho}{D(p)}}$$

HC can be written as rewritten as $HC = r_2k\frac{E\zeta\eta}{2}\left[\rho\left(1-\frac{D(p)}{P}\right) - 1 + \frac{2D(p)}{P}\right]$.
Putting $P = \lambda D(p)$ in HC. Therefore, the holding cost for is

$$HC = r_2k\frac{qT\zeta\eta}{2\lambda}[\rho(\lambda-1) - \lambda + 2] \tag{11}$$

Transmission cost can be calculated as:

$$G_p = \frac{D(p)\beta c_1 z_p^x m_1}{qT\zeta\eta} + D(p)m_1\Omega(1-\beta)c_1 \tag{12}$$

3.7. Total Carbon Emission of Transmission Cost

The carbon emission transmission cost has one component. This transmission emission component is related to total transmission rates. The component is summated and briefly represented by:

$$CE_{TR} = \hat{c}_1\left[\frac{D(p)\xi\beta z_p^x m_1}{qT\zeta\eta} + D(p)\xi m_1\Omega(1-\beta)\right] \tag{13}$$

3.8. Total Carbon Emission of Energy Holding Cost for Power Generation

The carbon emission energy holding cost has one component. The holding emission component is related to total energy hold in a power generation plant. The component is summated and briefly represented by:

$$CE_{PG} = \hat{r}_2\left[k\frac{qT\xi\zeta\eta}{2\lambda}[\rho(\lambda-1) - \lambda + 2]\right] \tag{14}$$

Therefore, the total profit for power generation Π_{PG} can be defined as:

$$\begin{aligned}\Pi_{PG} = D(p)(p-k) - \frac{D(p)v_0e^{-\tau I}}{qT\zeta\eta\rho} - \frac{D(p)I}{qT\zeta\eta\rho} - (r_2 + \hat{r}_2\xi)k\frac{qT\zeta\eta}{2\lambda}[\rho(\lambda-1) - \lambda + 2] \\ - \left[\frac{D(p)\beta(c_1+\hat{c}_1\xi)z_p^x m_1}{qT\zeta\eta} + D(p)m_1\Omega(1-\beta)(c_1 + \hat{c}_1\xi)\right]\end{aligned} \tag{15}$$

Therefore, the total profit is:

$$\begin{aligned} \Pi &= \Pi_{PG} - TC_O - TC_D - CE_D - TC_t - CE_t \\ &= D(p)\left[p - k - (m_1 + m_2 + m_3)\Omega(1-\beta)(c_1 + \overset{\wedge}{c_1}\xi)\right] \\ -\frac{D(p)}{qT\zeta\eta\rho}&\left[A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] \\ &-\frac{qT\zeta\eta}{2\lambda}\left[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda-1) - \lambda + 2)\right] \end{aligned} \quad (16)$$

Next, we analyze the consequence of I, ζ, η, ρ, T and p on $\prod$ fixed q by using the second order partial derivatives of Equation (16) with respect to I, ζ, η, ρ, T and p:

$$\begin{aligned} \frac{\partial\Pi}{\partial\zeta} = \frac{D(p)}{qT\zeta^2\eta\rho}&\left[\beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] \\ &-\frac{qT\eta}{2\lambda}\left[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda-1) - \lambda + 2)\right] \end{aligned} \quad (17)$$

$$\frac{\partial^2\Pi}{\partial\zeta^2} = -\frac{2D(p)}{qT\zeta^3\eta\rho}\left[\beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] < 0 \quad (18)$$

$$\begin{aligned} \frac{\partial\Pi}{\partial\eta} = \frac{D(p)}{qT\zeta\eta^2\rho}&\left[\beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] \\ &-\frac{qT\zeta}{2\lambda}\left[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda-1) - \lambda + 2)\right] \end{aligned} \quad (19)$$

$$\frac{\partial^2\Pi}{\partial\eta^2} = -\frac{2D(p)}{qT\zeta\eta^3\rho}\left[\beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] < 0 \quad (20)$$

$$\frac{\partial\Pi}{\partial\rho} = \frac{D(p)}{qT\zeta\eta\rho^2}\left[v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] - \frac{qT\zeta\eta(r_2 + \overset{\wedge}{r_2}\xi)k(\lambda-1)}{2\lambda} \quad (21)$$

$$\frac{\partial^2\Pi}{\partial\rho^2} = -\frac{2D(p)}{qT\zeta\eta\rho^3}\left[v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] < 0 \quad (22)$$

$$\begin{aligned} \frac{\partial\Pi}{\partial T} = \frac{D(p)}{qT^2\zeta\eta\rho}&\left[A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] \\ &-\frac{q\zeta\eta}{2\lambda}\left[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda-1) - \lambda + 2)\right] \end{aligned} \quad (23)$$

$$\frac{\partial^2\Pi}{\partial T^2} = -\frac{2D(p)}{qT^3\zeta\eta\rho}\begin{bmatrix} A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi) \\ \times z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1 \end{bmatrix} < 0 \quad (24)$$

$$\begin{aligned} \frac{\partial\Pi}{\partial p} &= a - bp - b\left[p - k + (\beta-1)\Omega(c_1 + \overset{\wedge}{c_1}\xi)(m_1 + m_2 + m_3)\right] - \frac{qT\zeta\eta(r_1 + \overset{\wedge}{r_1}\xi)}{2} \\ +\frac{b}{qT\zeta\eta\rho}&\left[A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\right] \end{aligned} \quad (25)$$

and

$$\frac{\partial\Pi}{\partial p} = -2b < 0 \quad (26)$$

From Equations (18), (20), (22), (24) and (26) it can be concluded that, for any feasible solution of q the total profit function $\prod$ (Equation (16) is a concave function of ζ, η, ρ, T and p.

Theorem 1. *For the any positive integer $(\zeta, \eta, \rho, T, p)$, the required objective function $\prod$ (Equation (16) is concave function of I and q. Therefore, the maximum value $\prod$ (Equation (16) is settled at the point I and q which satisfies $\frac{\partial\prod}{\partial I} = 0$ and $\frac{\partial\prod}{\partial q} = 0$.*

Proof. To calculate the optimal solution for fixed-integers $(\zeta, \eta, \rho, T, p)$, used the partial derivatives with respect to I and q, as shown in the following equations:

$$\frac{\partial \Pi}{\partial I} = -\frac{D(p)\left(1 - \tau v_0 e^{-\tau I}\right)}{qT\zeta\eta\rho} \tag{27}$$

$$\begin{aligned}\frac{\partial \Pi}{\partial q} = \frac{D(Ap)}{q^2 T\zeta\eta\rho}\Big[A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + c_2)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\Big] \\ -\frac{T\zeta\eta}{2\lambda}\Big[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda - 1) - \lambda + 2)\Big]\end{aligned} \tag{28}$$

Now, set Equations (27) and (28) equal to zero and solve for I and q:

$$-\frac{D(p)\left(1 - \tau v_0 e^{-\tau I}\right)}{qT\zeta\eta\rho} = 0 \Rightarrow I = \frac{1}{\tau}\log[\tau v_0] \tag{29}$$

Then:

$$\begin{aligned}\frac{D(Ap)}{q^2 T\zeta\eta\rho}\Big[A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + c_2\xi)z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1\Big] \\ -\frac{T\zeta\eta}{2\lambda}\Big[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda - 1) - \lambda + 2)\Big] = 0 \\ \Rightarrow q = \frac{1}{T\zeta\eta}\sqrt{\frac{2\lambda\left[D(p)\left[\begin{array}{c} A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + c_2) \\ \times z_t^x m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1 \end{array}\right]\right]}{\rho\left[\lambda(r_1 + \overset{\wedge}{r_1}\xi)p + (r_2 + \overset{\wedge}{r_2}\xi)k(\rho(\lambda - 1) - \lambda + 2)\right]}}\end{aligned} \tag{30}$$

In order to prove the concavity of the required objective profit function $\prod$; can show that the following conditions:

$$\frac{\partial^2 \Pi}{\partial I^2} = -\frac{D(p)\left(1 + \tau^2 v_0 e^{-\tau I}\right)}{qT\zeta\eta\rho} < 0 \tag{31}$$

$$\frac{\partial^2 \Pi}{\partial q^2} = -\frac{2D(p)}{q^3 T\zeta\eta\rho}\left[\begin{array}{c} A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho + \beta(c_1 + c_2)z_t^x \\ \times m_2\rho + v_0 e^{-\tau I} + I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1 \end{array}\right] < 0 \tag{32}$$

$$\frac{\partial \Pi}{\partial I \partial q} = \frac{\partial \Pi}{\partial q \partial I} = \frac{D(p)\left(1 - e^{-\tau I}\tau v_0\right)}{q^2 T\zeta\eta\rho} \tag{33}$$

The Hessian matrix can be found as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \Pi}{\partial I^2} & \frac{\partial^2 \Pi}{\partial I \partial q} \\ \frac{\partial^2 \Pi}{\partial q \partial I} & \frac{\partial^2 \Pi}{\partial q^2} \end{bmatrix} = \begin{bmatrix} -\frac{D(p)\left(1 + \tau^2 v_0 e^{-\tau I}\right)}{qT\zeta\eta\rho} & \frac{D(p)\left(1 - e^{-\tau I}\tau v_0\right)}{q^2 T\zeta\eta\rho} \\ \frac{D(p)\left(1 - e^{-\tau I}\tau v_0\right)}{q^2 T\zeta\eta\rho} & -\frac{2D(p)}{q^3 T\zeta\eta\rho}\left[\begin{array}{c} A\zeta\eta\rho + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_d^x m_3\eta\rho \\ +\beta(c_1 + c_2)z_t^x m_2\rho + v_0 e^{-\tau I} \\ +I + \beta(c_1 + \overset{\wedge}{c_1}\xi)z_p^x m_1 \end{array}\right] \end{bmatrix}.$$

The determinant of $|H|$ is followed:

$$\begin{aligned}
|H| = & -\frac{[D(p)]^2}{q^4T^2\zeta^2\eta^2\rho^2} + \frac{2[D(p)]^2e^{-\tau I}\tau v_0}{q^4T^2\zeta^2\eta^2\rho^2} + \frac{2[D(p)]^2e^{-\tau I}I\tau^2 v_0}{q^4T^2\zeta^2\eta^2\rho^2} + \frac{2A[D(p)]^2e^{-\tau I}\tau^2 v_0}{q^4T^2\zeta\eta\rho} \\
& + \frac{[D(p)]^2e^{-2\tau I}\tau^2 v_0^2}{q^4T^2\zeta^2\eta^2\rho^2} + \frac{2[D(p)]^2e^{-\tau I}\beta\tau^2 c_1 m_3 v_0 z_d^x}{q^4T^2\zeta^2\eta\rho} + \frac{2[D(p)]^2e^{-\tau I}\beta\tau^2 \hat{c}_1\xi m_3 v_0 z_d^x}{q^4T^2\zeta^2\eta\rho} \\
& + \frac{2[D(p)]^2e^{-\tau I}\beta\tau^2 c_1 m_2 v_0 z_t^x}{q^4T^2\zeta^2\eta^2\rho} + \frac{2[D(p)]^2e^{-\tau I}\beta\tau^2 \hat{c}_1\xi m_2 v_0 z_t^x}{q^4T^2\zeta^2\eta^2\rho} + \frac{2[D(p)]^2e^{-\tau I}\beta\tau^2 c_1\xi m_1 v_0 z_p^x}{q^4T^2\zeta^2\eta^2\rho^2} \\
& + \frac{2[D(p)]^2e^{-\tau I}\beta\tau^2 \hat{c}_1\xi m_1 v_0 z_p^x}{q^4T^2\zeta^2\eta^2\rho^2} \\
= & \frac{[D(p)]^2\left[\tau^2 v_0^2 e^{-2\tau I} + 2e^{-\tau I}\tau v_0 \left[\begin{array}{c} 1+\tau I + A\tau\zeta\eta\rho + \tau\beta\hat{c}_1\xi\left(\eta\rho m_3 z_d^x + \rho m_2 z_t^x + m_1 z_p^x\right) \\ +\tau\beta\hat{c}_1\xi\left(\eta\rho m_3 z_d^x + \rho m_2 z_t^x + m_1 z_p^x\right)\end{array}\right] - 1\right]}{q^4T^2\zeta^2\eta^2\rho^2} \\
= & \frac{[D(p)]^2\left[\tau^2 v_0^2 e^{-2\tau I} + 2B - 1\right]}{q^4T^2\zeta^2\eta^2\rho^2} > 0
\end{aligned}$$

where $B = e^{-\tau I}\tau v_0 \left[\begin{array}{c} 1+\tau I + A\tau\zeta\eta\rho + \tau\beta\hat{c}_1\xi\left(\eta\rho m_3 z_d^x + \rho m_2 z_t^x + m_1 z_p^x\right) \\ +\tau\beta\hat{c}_1\xi\left(\eta\rho m_3 z_d^x + \rho m_2 z_t^x + m_1 z_p^x\right)\end{array}\right] > 1.$

The behavior of the concavity for objective function $\prod$ with respect to our decision variables $\zeta, \eta, \rho, T,\ p,\ I$ and q, the algorithm is similar to Wangsa et al. [22], and was developed to draw the global maximum feasible solutions for $I^*,\ \zeta^*, \eta^*, \rho^*, T^*,\ p^*,\ q^*$ and $\prod^*$. □

3.9. Algorithm

Step 1. First compute I from Equation (29).
Step 2. a. set $\zeta = 1$.
b. set $\eta = 1$.
c. set $\rho = 1$.
d. set $T = 1$.
e. set $p = 20$.
Step 3. Compute the optimal q^* by using Equation (30).
Step 4. After completed Step 3, compute the actual electrical power capacities.
a. Power generation
The actual capacity of the power generation $z_p^y = qT\zeta\eta\rho\,\Omega$.
If $z_p^y \le z_p^x$ then find $q = \frac{z_p^y}{T\zeta\eta\rho\,\Omega}$ and go to Step 5.
b. Transmission substation
The actual capacity of the transmission substation $z_t^y = qT\zeta\eta\,\Omega$.
If $z_t^y \le z_t^x$ then find $q = \frac{z_t^y}{T\zeta\eta\,\Omega}$ and go to Step 5.
c. Distribution substation
The actual capacity of the distribution substation $z_d^y = qT\zeta\,\Omega$.
If $z_t^y \le z_t^x$ then find $q = \frac{z_d^y}{T\zeta\,\Omega}$ and go to Step 5.
Step 5. Computed $\prod$ from Equation (16).
Step 6. Set $\zeta = 1 + 1$ and repeated Step 3 to Step 5.
Step 7. If $\prod(q_\zeta^*, \zeta, \eta_\zeta, \rho_\zeta, T_\zeta, p_\zeta) \ge \prod(q_{\zeta-1}^*, \zeta-1, \eta_{\zeta-1}, \rho_{\zeta-1}, T_{\zeta-1}, p_{\zeta-1})$ then go to Step 8.
Otherwise go to Step 6.
Step 8. Set $\eta = 1 + 1$ and repeated Step 2b to Step 7.
Step 9. If $\prod(q_\eta^*, {\zeta_\eta}^*, \eta, \rho_\eta, T_\eta, p_\eta) \ge \prod(q_{\eta-1}^*, {\zeta_{\eta-1}}^*, \eta-1, \rho_{\eta-1}, T_{\eta-1}, p_{\eta-1})$ then go to Step 9.
Otherwise go to Step 8.
Step 10. Set $\rho = 1 + 1$ and repeated Step 2c to Step 9.
Step 11. If $\prod(q_\rho^*, {\zeta_\rho}^*, {\eta_\rho}^*, \rho, T_\rho, p_\rho) \ge \prod(q_{\rho-1}^*, {\zeta_{\rho-1}}^*, \eta_{\rho-1}, \rho-1, T_{\eta-1}, p_{\eta-1})$ then go to Step 12.
Otherwise go to Step 10.

Step 12. Set $T = 1 + 1$ and repeated Step 2d to Step 11.

Step 13. If $\prod(q_T^*, \zeta_T{}^*, \eta_T{}^*, \rho_T^*, T, p_T) \geq \prod(q_{T-1}^*, \zeta_{T-1}{}^*, \eta_{T-1}{}^*, \rho_{T-1}^*, T-1, p_{T-1})$ then go to Step 14. Otherwise go to Step 12.

Step 14. Set $p = 1 + 1$ and repeated Step 2e to Step 13.

Step 15. If $\prod(q_p^*, \zeta_p{}^*, \eta_p{}^*, \rho_p^*, T_p^*, p) \geq \prod(q_{p-1}^*, \zeta_{p-1}{}^*, \eta_{p-1}{}^*, \rho_{p-1}^*, T_{p-1}^*, p-1)$ then go to Step 16. Otherwise go to Step 14.

Step 15. If $\prod(q_p^*, \zeta_p{}^*, \eta_p{}^*, \rho_p^*, T_p^*, p^*) \geq \prod(q_{p-1}^*, \zeta_{p-1}{}^*, \eta_{p-1}{}^*, \rho_{p-1}^*, T_{p-1}^*, p-1)$ then find I^*, $\zeta^*, \eta^*, \rho^*, T^*, p^*, q^*$ and go to Step 16.

Step 16. Stop.

4. Numerical Example

In this section, used data to demonstrate the application of the model. This study considers the sustainable electrical energy supply chain inventory system in the Taiwanese electrical production industry to determine the optimal ordering quantity and total profit. The parameters in this section are assumed from a previous published paper. The data values are:

4.1. Customer Data

Let $a = 80$ units, $b = 2$ units, $A = \$100/unit/order$, $r_1 = \$\ 0.004/unit/year$, $\hat{r}_1 = 0.002\,ton/year$, $r_2 = \$\ 0.002/unit/year$, $\hat{r}_2 = 0.003\,ton/year$, $\beta = 0.01\,unit$, $\Omega = 1.2\ kVA/kWh$, $v_0 = \$7/setup$, $\xi = \$1/ton/year$ and $\tau = 0.2\,unit$.

4.2. Transmission Substation, Distribution Substation and Power Generation Data

Let $k = \$4/kWh$, $\lambda = 2\,units$, $c_1 = \$0.00011/kVA/mile$, $z_p^x = 6\,kVA$, $z_t^x = 5\,kVA$, $z_d^x = 2\,kVA$, $m_1 = 0.2\,mile$, $m_2 = 0.15\,mile$ and $m_3 = 0.1\,mile$.

Based on input data, by using Algorithm and Mathematica 9.0, the optimal solutions (see Tables 3 and 4); $\zeta^* = 2$, $\eta^* = 1$, $\rho^* = 1$, $T^* = 1$, $p^* = 22$, $D(p)^* = 36$, $I^* = 1.68236$, $q^* = 183.875$, $z_p^{y*} = 441.30$, $z_t^{y*} = 441.30$, $z_d^{y*} = 441.30$ and $\Pi^* = 601.653$. Furthermore, Figures 2 and 3 for graphical representations of the total profit function is a concave with respect to feasible optimal values p^* and q^*.

Table 3. Details of the procedures for the solution.

Iteration	ζ^*	η^*	ρ^*	T^*	p^*	$D(p)^*$	I^*	q^*	z_p^{y*}	z_t^{y*}	z_d^{y*}	Π^*
1	1	1	1	1	20	40	1.68236	66.6182	79.94184	79.94184	79.94184	571.609
2	2	1	1	1	20	40	1.68236	185.451	445.0824	445.0824	445.0824	593.597
3	3	1	1	1	20	40	1.68236	338.854	1219.874	1219.874	1219.874	561.851
4	4	1	1	1	20	40	1.68236	520.277	2497.329	2497.329	2497.329	496.906
5	5	1	1	1	20	40	1.68236	725.913	4355.478	4355.478	4355.478	398.489
6	6	1	1	1	20	40	1.68236	953.188	6862.953	6862.953	6862.953	264.009
7	2	2	1	1	20	40	1.68236	520.277	2497.329	2497.329	1248.664	496.906
8	2	3	1	1	20	40	1.68236	953.188	6862.953	6862.953	2287.651	264.009
9	2	1	2	1	20	40	1.68236	381.779	1832.539	916.2696	916.2696	575.894
10	2	1	1	2	20	40	1.68236	370.901	1780.324	1780.324	1780.324	537.988
11	2	1	1	1	21	38	1.68236	184.879	443.7096	443.7096	443.7096	599.611
12	2	1	1	1	22	36	1.68236	183.875	441.3000	441.3000	441.3000	601.653←
13	2	1	1	1	23	34	1.68236	182.430	437.8320	437.8320	437.8320	599.736

* The local maximum solution; ← the optimal solution.

Table 4. The results.

Decision Variables	Values
Electrical power distribution factor	2 Times
Electrical power transmission factor	1 Times
Electrical power generation	1 Times
Customer's average electricity consumption	1 Year
Retailer's price of electricity	$22/kWh.
Demand of customer	36 kWh/year
Electrical power consumption	183.875 kW
Investment for setup cost reduction	$1.68236/ production run
Energy transmitted by power generation	8.55471 kWh
Energy transmitted by transmission substation	4.277 kWh
Energy transmitted by distribution substation	6.41593 kWh
Energy consumed by customer	183.875 kWh
Actual capacity of power generation	441.3000 kVA
Actual capacity of transmission substation	441.3000 kVA
Actual capacity of distribution substation	441.3000 kVA
Power supply rate to power generation	72 kWh/year

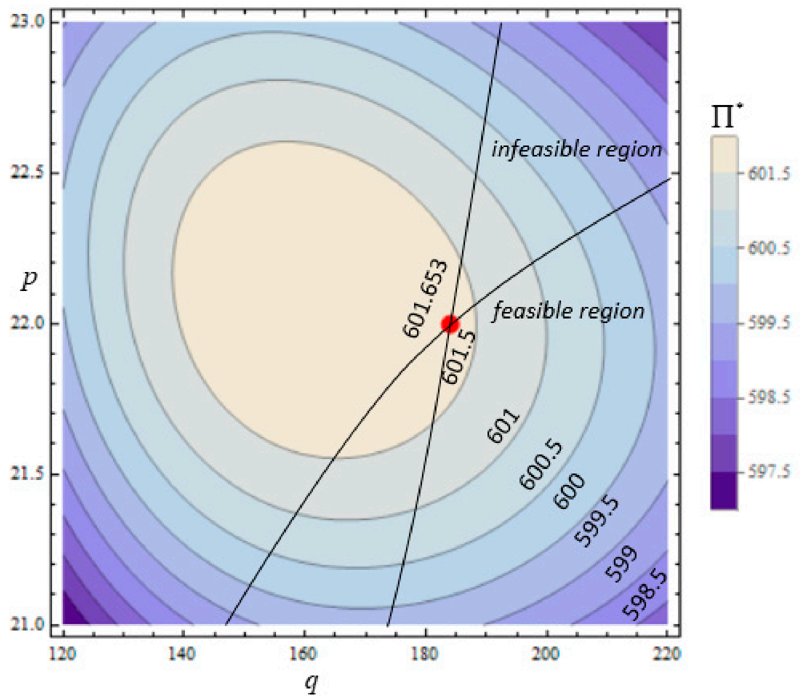

Figure 2. The contour of the objective function $\prod^*$ with respect to $\zeta = 2$, $\eta = 1$, $\rho = 1$, $I = 1.68236$ and $T = 1$.

From Table 3, we obtain the $\rho = 1$ batch. It is intended that the electricity produced by a power generator is 8.55471 kWh. But not all the electricity energy (8.55471 kWh) induced is transmitted at once, but in periods with 8.55471 kWh each. As the generator has a device to minimize the total energy holding cost of the electricity, for each batch, the transmission substation obtains 4.277 kWh of electricity; it then transmits in 2 batches to the distribution station at 6.41593 kWh each. This is done to minimize the transmission cost and distribution cost. Based on those results, the total electrical power consumption of customer 183.875 kW. The demand of customer is 36 kWh/year.

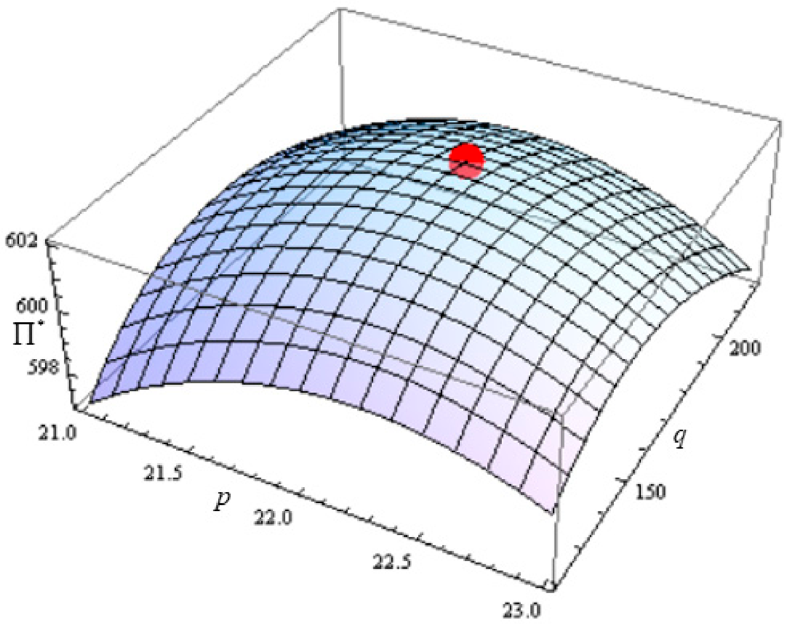

Figure 3. Concavity of $\prod^*$ with respect to p and q at $\zeta = 2$, $\eta = 1$, $\rho = 1$, $I = 1.68236$ and $T = 1$.

5. Discussion and Managerial Implication

In this section, we consider the effect of changes in the main parameters as well as summarize the results of the sensitivity analysis, which shows (see Table 5) the impact of each of the parameters which are a, b, k, λ, v_0, τ, A, $\hat{c}_1$, $\hat{r}_1$, $\hat{r}_2$ and β, respectively, on the total profit.

Table 5. Sensitivity analysis of key parameters of the model.

Parameter	Changes	ζ^*	η^*	ρ^*	T^*	p^*	$D^*(p)$	q^*	I^*	$z_p^{y^*}$	$z_t^{y^*}$	$z_d^{y^*}$	Π^*
	60	2	1	1	1	20	20	131.133	1.68236	314.719	314.719	314.719	287.189
	70	2	1	1	1	20	30	160.605	1.68236	385.452	385.452	385.452	439.814
a	80	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	90	2	1	1	1	24	42	206.829	1.68236	496.389	496.389	496.389	787.868
	100	2	1	1	1	27	46	228.775	1.68236	549.060	549.060	549.060	997.886
	1.8	3	1	1	1	24	36.8	353.749	1.68236	1273.49	1273.49	1273.49	643.645
	1.9	2	1	1	1	23	36.3	188.50	1.68236	452.400	452.400	452.400	641.897
b	2	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	2.1	2	1	1	1	21	35.9	179.698	1.68236	431.275	431.275	431.275	565.211
	2.2	2	1	1	1	20	36	175.934	1.68236	422.241	422.241	422.241	531.978
	2	2	1	1	1	21	38	181.449	1.68236	435.477	435.477	435.477	676.583
	3	2	1	1	1	22	36	182.249	1.68236	437.397	437.397	437.397	638.158
k	4	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	5	2	1	1	1	23	34	183.965	1.68236	441.516	441.516	441.516	565.209
	6	2	1	1	1	23	34	185.486	1.68236	445.166	445.166	445.166	530.673
	1.8	2	1	1	1	22	36	220.455	1.68236	529.092	529.092	529.092	575.565
	1.9	2	1	1	1	22	36	204.23	1.68236	490.152	490.152	490.152	590.137
λ	2	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	2.1	2	1	1	1	22	36	157.798	1.68236	378.715	378.715	378.715	609.451
	2.2	2	1	1	1	22	36	122.397	1.68236	293.752	293.752	293.752	611.235

Table 5. *Cont.*

Parameter	Changes	ζ^*	η^*	ρ^*	T^*	p^*	$D^*(p)$	q^*	I^*	z_p^{y*}	z_t^{y*}	z_d^{y*}	Π^*
	5	2	1	1	1	22	36	183.81	1.53742	441.144	441.144	441.144	601.668
	6	2	1	1	1	22	36	183.81	1.61042	441.144	441.144	441.144	601.600
v_0	7	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	8	2	1	1	1	22	36	183.906	1.75328	441.374	441.374	441.374	601.644
	9	2	1	1	1	22	36	183.937	1.82322	441.448	441.448	441.448	601.636
	0.18	2	1	1	1	22	36	183.945	1.28395	441.468	441.468	441.468	601.635
	0.19	2	1	1	1	22	36	183.911	1.50094	441.386	441.386	441.386	601.643
τ	0.2	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	0.21	2	1	1	1	22	36	183.837	1.83458	441.208	441.208	441.208	601.662
	0.22	2	1	1	1	22	36	183.797	1.96265	441.112	441.112	441.112	601.672
	80	2	1	1	1	22	36	165.126	1.68236	396.302	396.302	396.302	606.378
	90	2	1	1	1	22	36	174.752	1.68236	419.404	419.404	419.404	603.952
A	100	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	110	2	1	1	1	22	36	192.566	1.68236	462.158	462.158	462.158	599.462
	120	2	1	1	1	22	36	200.881	1.68236	482.114	482.114	482.114	597.366
	0.00012	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	0.0002	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.651
$\hat{c}_1$	0.0003	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.649
	0.0004	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.647
	0.0005	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.645
	0.002	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	0.003	2	1	1	1	22	36	197.606	1.68236	474.254	474.254	474.254	596.761
$\hat{r}_1$	0.004	2	1	1	1	22	36	210.443	1.68236	505.063	505.063	505.063	591.175
	0.005	2	1	1	1	22	36	222.541	1.68236	534.099	534.099	534.099	584.989
	0.006	2	1	1	1	22	36	234.014	1.68236	561.634	561.634	561.634	578.275
	0.003	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.653
	0.004	2	1	1	1	22	36	185.165	1.68236	444.396	444.396	444.396	601.240
$\hat{r}_2$	0.005	2	1	1	1	22	36	186.447	1.68236	447.473	447.473	447.473	600.821
	0.006	2	1	1	1	22	36	187.719	1.68236	450.526	450.526	450.526	600.395
	0.007	2	1	1	1	22	36	188.983	1.68236	453.559	453.559	453.559	599.962
	0.008	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.652
	0.009	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.652
β	0.01	2	1	1	1	22	36	186.447	1.68236	441.300	441.300	441.300	601.652
	0.011	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.652
	0.012	2	1	1	1	22	36	183.875	1.68236	441.300	441.300	441.300	601.652

Based on the computational results (Table 5), the following managerial insights can be obtained:

5.1. Impact on Demand Parameters

An increase in the value of a results in an increase in demand, which forces the retailer to increase the selling price p, fixed ζ, η, ρ, T, I, increases the actual capacity of the distribution, transmission, power generation substation and the customer's power consumption in order to increase the electrical supply chain profit. On the other hand, an increase in the value of b could reduce the demand. So, the retailer then reduces the selling price p, fixed η, ρ, T, I, reduces the actual capacity of the distribution, transmission, power generation substation and the customer's power consumption. In this case, although the demand rate could be maintained on the higher side with the high value of b. Therefore, the profit of the electrical supply chain system would be decreased.

5.2. Impact on Production Parameters

An increase in the value of λ results in an increase in production with fix at demand rates, which forces the retailer to fix the selling price p, ζ, η, ρ, T, I, decreases the actual capacity of the distribution, transmission, power generation substation and the customer's power consumption in order to increase the electrical supply chain profit. An increase in the value of k could reduce the demand. So, the retailer then increases the selling price p with fixed ζ, η, ρ, T, I, and increases an actual capacity of the distribution, transmission, power generation substation and the customer's power consumption. In this case, although the demand rate could be maintained in the less with the high value of k, customer's power consumption is higher, and actual capacity higher. Therefore, the profit of the electrical supply chain system is decreased.

5.3. Impact on Setup Cost Reduction Parameters

An increase in the value of v_0 results in an increase in setup cost with fixed at demand rates, which forces the retailer to fix the p, ζ, η, ρ, T, increases the actual capacity of the distribution, transmission, power generation substation and the customer's power consumption, increases the setup investment cost, and decreases the electrical supply chain profit. In this case, the profit of the electrical supply chain system decreased because all actual capacity and customer's power consumption with fixed selling price increase. On the other hand, an increase in the value of τ results in a decrease in setup cost with a fix at demand rates, which forces the retailer to fix the p, ζ, η, ρ, T, decreases the actual capacity of the distribution, transmission, power generation substation and the customer's power consumption, increases the setup investment cost and increases the electrical supply chain profit. In this case, the profit of the electrical supply chain system is increased because the demand rate maintained in the fixed with the high value of τ, customer's power consumption is lower, and all actual capacities are lower.

5.4. Impact on Ordering Cost

An increase in the value of A results in an increases the customer's power consumption rate with fixed demand rates; selling price increases the actual capacity of the distribution, transmission, and power generation substation. This result indicates that profit of the electrical supply chain system decreased because of higher of all actual capacity and customer's power consumption rates with fixed selling price.

5.5. Impact on Loss Factor

An increase in the value of all carbon emission parameters β, results in an no change of the customer's power consumption rate; demand rates; selling price, increases the actual capacity of the distribution, transmission, and power generation substation. This result indicates that, total profit can be unchanged because of all actual capacity and customer power consumption rates with fixed selling price are unchanged when small changes of β. Therefore, small changes of loss factor result in unchanged profit.

5.6. Impact on Carbon Emission Parameters

An increase in the value of all carbon emission parameters $\hat{c}_1$, $\hat{r}_1$ and $\hat{r}_2$ results in an increase the customer's power consumption rate with fixed demand rates; selling price increases the actual capacity of the distribution, transmission, and power generation substation. This result indicates that profit of the electrical supply chain system decreased because of higher actual capacity and customer power consumption rates at a fixed selling price.

6. Conclusions and Future Research

In this paper, a sustainable electricity supply chain mathematical model that assumes linear price-dependent customer demands where the price is a decision variable with reduction of setup cost under carbon emission, is considered. This model has been developed based on the inventory management theory, and examined how the all optimal decision variables and the total profits for sustainable electrical supply chain are affected by key parameters. Based on our computational results, it supplies managerial insights to the production system and marketing managers to help in planning a successful and sustainable electrical energy supply chain. For a future study, researchers can extend the present model to include green technology investment under carbon emissions regulation. Researchers can also study incorporating price discount strategies as well as the effect of green technology investment under carbon emissions with a multi-transmission and distribution substation.

Author Contributions: Data Curation, Software U.M. and J.-Z.W.; Formal Analysis, Investigation, Writing Original Draft Preparation, Writing-Review, Visualization and Editing U.M.; Conceptualization, Methodology, Validation A.C and J.-Z.W.; Funding Acquisition, Supervision, Project Administration J.-Z.W.

Funding: This research was funded by Ministry of Science and Technology, Taiwan, grant number MOST106-2218-E-007-024 & MOST108-2911-I-031-502; and The APC was funded by MOST106-2218-E-007-024 & MOST108-2911-I-031-502.

Acknowledgments: The authors would like to thank the editor and anonymous reviewers for their valuable and constructive comments, which have led to a significant improvement in the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

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