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Exact Performance Analysis of Amplify-and-Forward Bidirectional Relaying over Nakagami-*m* Fading Channels with Arbitrary Parameters

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Abstract: The exact performance of amplify-and-forward (AF) bidirectional relay systems is studied in generalized and versatile Nakagami-m fading channels, where the parameter m is an arbitrary positive number. We consider three relaying modes: two, three, and four time slot bidirectional relaying. Closed form expressions of the moment generating function (MGF), higher order moments of signal-to-noise ratio (SNR), ergodic capacity, and average signal error probability (SEP) are derived, which are different from previous works. The obtained expressions are very concise, easy to calculate, and evaluated instantaneously without a complex summation operation, in contrast to the nested multifold numerical integrals and truncated infinite series expansions used in previous work, which lead to computational inefficiency, especially when the fading parameter m increases. Simulation results corroborate the correctness and tightness of the theoretical analysis.

Keywords: amplify-and-forward; arbitrary fading parameter; bidirectional relaying

1. Introduction

Relay technology is a promising solution for 5G communication in the future. The Nakagami-m environment is widely studied in relay networks. When it comes to the Nakagami-m distribution, most research institutions prefer an integer parameter m. For example, an approximate formula of the outage probability was studied in Reference [1] for bidirectional relaying systems with co-channel interferes. An exact outage expression was derived in Reference [2], when m is an integer. While multi-antenna technology was introduced in Reference [3]. The exact and asymptotic outage probability formulas were derived in Reference [4] when a bidirectional relaying system suffered from asymmetric traffic, which was quantified by a traffic pattern indicator and meant that the traffic transmitted and received by the two transceivers was different. Then, the author in Reference [4] took the asymptotic outage probability as an optimization objective, in order to find the optimal power allocation and relay location. Using the expectation property of the logarithmic function, the lower bound of the average rate was obtained in Reference [5] for bidirectional amplify-and-forward (AF) relaying networks that suffer from Nakagami-m fading channels. Resorting to polynomial theory, the lower bound involved many special functions, including the incomplete gamma function, factorial, Euler psi function, and hypergeometric function. By enlarging the original integral region, an approximation of the outage probability was provided in Reference [6]. Obviously, the authors in Reference [6] failed to get an exact formula in the original integral region, so they had to change the integral region. Some common average signal error probability (SEP) expressions were determined in Reference [7] for integer m, such as the 32-cross quadrature amplitude modulation, differentially encoded quadrature phase shift keying, and $\pi/4$ quadrature phase shift keying modulation schemes. The approximate probability density function and asymptotic outage probability in high signal-to-noise ratio (SNR) region were presented in Reference [8] for AF bidirectional relaying.

The papers published about non-integer m are obviously not as many as those about integer m. The outage probability and SEP were analyzed in Reference [9] for cascading Nakagami-m fading channels in AF bidirectional relaying systems. The diversity order of the outage probability was derived for an AF bidirectional relay network with beam-forming [10]. The authors in Reference [11] claimed the asymptotic formula was a gap in bidirectional relay networks. This claim indicated the difficulty for arbitrary m values, especially for non-integer m. When the SNR tended to be infinite, the asymptotic outage probability and average SEP were derived with the aid and knowledge of limit operations [11]. An exception appeared in Reference [12], where the exact outage probability, average SEP, and average sum rate were presented for non-integer m. However, this non-integer value only defined m as an integer plus one half. Clearly, this rule was too narrow for an arbitrary parameter and many other non-integers were not included in Reference [12].

Although the Nakagami-m fading channel has been extensively studied in bidirectional AF relaying networks, there are still some disadvantages. First, most previous works prefer an integer value for parameter m in the Nakagami-m environment for bidirectional relaying systems [1–8]. Obviously, it is easier to study an integer than any real number. When m is an integer, the complex gamma channel distribution is easily expanded into a finite summation of the exponential function and the power function, and so a closed form expression is easily yielded with integer m. But integer m often limits the application of the Nakagami-m fading channel, because the actual complex environment may not be well modeled by an integer parameter m. Also, the Nakagami-m fading channel does not specify that m is an integer. This brings the second shortcoming. When parameter m takes a non-integer value, a lower or upper bound, truncated infinite series expansion, and nested multifold numerical integrals have to be utilized to evaluate approximate performance [9–12]. This is because the gamma function is only expanded into an infinite series of exponential and power functions when m takes a non-integer value. Additionally, infinite series expansions appear in References [2,3] even for integer m, let alone non-integer m. Finally, most of the previous work links performance with the probability density function (PDF) or cumulative distribution function (CDF). This method is effective for simple PDF or CDF. However, for the complex gamma channel distribution, this method often fails in finding closed-form solutions when m is an arbitrary number. To our best knowledge, no investigation on the simple exact closed form formula has been done for arbitrary parameter m, due to the inherent complexity of this problem.

Stimulated by the above observations, this paper studies the accurate performance of AF bidirectional relay systems in the Nakagami-m fading channel with arbitrary parameter m. In our analysis, closed form expressions for the moment generating function (MGF), higher order moments of the SNR, ergodic capacitym and average SEP are derived, which differ from previous work. In addition, our formulas without summation operations are very easy and instantaneous to calculate, in sharp contrast to previous work, which had to resort to the evaluation of truncated infinite series or multifold numerical integrals. Finally, some mathematical notations are summarized in Table 1.

Table 1. Mathematical notations.

Notation	Description
α_1	Scaling factor of the total relay power assigned to S_1
α_2	Scaling factor of the total relay power assigned to S_2
$B(\cdot,\cdot)$	Beta function
$erfc(\cdot)$	Complementary error function
${}_2F_1(\cdot)$	Gaussian hypergeometric function
$\Gamma(\cdot)$	Gamma function
$G(\cdot)$	Meijer's G function
$H(\cdot)$	Fox's function
$K_\nu(\cdot)$	The νth order modified Bessel function of the second kind

2. System Model

Consider an AF relaying system, where two transceivers S_1 and S_2 exchange messages via one relay R. We consider three bidirectional modes: two, three, and four time slot bidirectional relaying [13]. The transmit powers of the two transceivers and the relay are denoted as p_1, p_2, and p_r, respectively. In three time slot bidirectional relaying, α_1 and α_2 denote the scaling factors of the total relay power p_r assigned to S_1 and S_2, respectively. Let h_1 and h_2 be the channel coefficients pertaining to the links $S_1 \leftrightarrow R$ and $S_2 \leftrightarrow R$, respectively. N_0 represents the noise variance. All channel links suffer from independent Nakagami-m fading, where m is an arbitrary positive number. The equivalent SNR between two transceivers can be expressed by [13]:

$$\gamma = \frac{a\gamma_1\gamma_2}{\gamma_1 + \gamma_2}, \tag{1}$$

where a, γ_1 ,and γ_2 depend on the different relaying modes and are given in Table 2.

Table 2. Parameter values for the three relay modes.

Relay Mode	Transmission Direction	a	γ_1	γ_2
Two time slot bidirectional relay	$S_2 \rightarrow R \rightarrow S_1$	$\frac{p_r}{p_1+p_r}$	$\frac{p_2\|h_2\|^2}{N_0}$	$\frac{(p_r+p_1)\|h_1\|^2}{N_0}$
	$S_1 \rightarrow R \rightarrow S_2$	$\frac{p_r}{p_r+p_2}$	$\frac{p_1\|h_1\|^2}{N_0}$	$\frac{(p_r+p_2)\|h_2\|^2}{N_0}$
Three time slot bidirectional relay	$S_2 \rightarrow R \rightarrow S_1$	$\frac{p_r}{p_r+\alpha_1 p_1}$	$\frac{\alpha_2 p_2\|h_2\|^2}{N_0}$	$\frac{(p_r+\alpha_1 p_1)\|h_1\|^2}{N_0}$
	$S_1 \rightarrow R \rightarrow S_2$	$\frac{p_r}{p_r+\alpha_2 p_2}$	$\frac{\alpha_1 p_1\|h_1\|^2}{N_0}$	$\frac{(p_r+\alpha_2 p_2)\|h_2\|^2}{N_0}$
Four time slot bidirectional relay	$S_1 \rightarrow R \rightarrow S_2$	1	$\frac{p_1\|h_1\|^2}{N_0}$	$\frac{p_r\|h_2\|^2}{N_0}$
	$S_2 \rightarrow R \rightarrow S_1$	1	$\frac{p_2\|h_2\|^2}{N_0}$	$\frac{p_r\|h_1\|^2}{N_0}$

3. Performance Analysis

The PDF and CDF are often used to obtain the performance expression; most researchers write the performance expression as an integrand of the PDF or CDF. However, for an arbitrary parameter m, this method can only obtain approximate solutions by means of a lower or upper bound, nested multifold numerical integrals, or truncated infinite series. Even when the parameter m is an integer value, the performance expression is often the summation of multiple special functions. Obviously, the previous formulas are computationally inefficient and time consuming, especially when the parameter m increases [1,2,5,7,11]. To avoid this dilemma, we make use of the MGF for the inverse SNR. As will be seen later, our derived expressions allow us to evaluate and quantify performance at a low complexity.

3.1. Moment Generating Function

The MGFs of inverse γ_1 and γ_2 are found in (Equations (32) and (33), Reference [12]). Due to the statistical independence of γ_1 and γ_2, the MGF of inverse γ is achieved via:

$$\begin{aligned} M_{\frac{1}{\gamma}}(s) =& M_{\frac{1}{\gamma_1}}\left(\frac{s}{a}\right) M_{\frac{1}{\gamma_2}}\left(\frac{s}{a}\right) \\ =& \frac{4\beta_s^{\frac{m_s}{2}}\beta_r^{\frac{m_r}{2}}}{\Gamma(m_s)\Gamma(m_r)}\left(\frac{s}{a}\right)^{\frac{m_s+m_r}{2}} K_{m_s}\left(2\sqrt{\frac{\beta_s s}{a}}\right) K_{m_r}\left(2\sqrt{\frac{\beta_r s}{a}}\right), \end{aligned} \tag{2}$$

where $m_s\,(m_r)$ is the fading parameter between $S_1\,(S_2)$ and R. β_s and β_r are scale parameters on the corresponding links. $\Gamma(\cdot)$ is the gamma function (Equation (8.311), [14]) and $K_\nu(\cdot)$ is the νth order modified Bessel function of the second kind (Equation (8.432), [14]). According to the relationship between the MGF of a random variable and its inverse (Equation (42), [15]), the MGF of the SNR γ is given by:

$$\begin{aligned} M_\gamma(s) =& 1 - 2\sqrt{s}\int_0^\infty J_1\left(2\sqrt{s}\xi\right) M_{\frac{1}{\gamma}}\left(\xi^2\right) d\xi \\ =& 1 - 2\sqrt{s}\int_0^\infty J_1\left(2\sqrt{s}\xi\right) H_{0,2}^{2,0}\left[\frac{\beta_s\xi^2}{a}\middle| (0,1),(m_s,1)\right] H_{0,2}^{2,0}\left[\frac{\beta_r\xi^2}{a}\middle| (0,1),(m_r,1)\right] d\xi \\ =& 1 - \frac{1}{\Gamma(m_s)\Gamma(m_r)} H_{2,(0:0),0,(2:2)}^{1,0,0,2,2}\left[\begin{array}{c|c} \frac{\beta_s}{as} & (1,1),(0,1) \\ & -;- \\ \frac{\beta_r}{as} & - \\ & (0,1),(m_s,1);(0,1),(m_r,1) \end{array}\right], \end{aligned} \tag{3}$$

where $J_\nu(\cdot)$ is the νth order Bessel function of the first kind (Equation (8.402), [14]), $H(x)$ and $H(x,y)$ are the univariable and bivariable Fox's functions (Equations (1.1.1) and (2.2.1), [16]), respectively. The second equality in Equation (3) is obtained by rewriting the Bessel function in terms of Fox's function (Equation (2.9.19), [17]), i.e.,:

$$2\left(\frac{\beta s}{a}\right)^{\frac{m}{2}} K_m\left(2\sqrt{\frac{\beta s}{a}}\right) = H_{0,2}^{2,0}\left[\frac{\beta s}{a}\middle| (0,1),(m,1)\right]. \tag{4}$$

From $M_\gamma(s)$, the average SEP of some simple modulations can be immediately obtained. For example, the SEP of the differential phase shift keying (DPSK) modulation is given by $P_e = \frac{1}{2}M_\gamma(1)$.

3.2. Higher Order Moments

Dependent on (Equation (34), Reference [18]) and (Equation (6.576.4), [14]), the nth order moment of the SNR γ is given by:

$$\begin{aligned} E(\gamma^n) =& -\frac{1}{n!}\int_0^\infty s^n \frac{\partial M_{\frac{1}{\gamma}}(s)}{\partial s} ds \\ =& \frac{1}{(n-1)!}\int_0^\infty s^{n-1} M_{\frac{1}{\gamma}}(s)\, ds \\ =& \frac{a^n\Gamma(m_s+n)\Gamma(m_r+n)\Gamma(m_s+m_r+n)}{\beta_r^n\Gamma(m_s)\Gamma(m_r)\Gamma(m_s+m_r+2n)}\,{}_2F_1\left(n, m_r+n; m_s+m_r+2n; 1-\frac{\beta_s}{\beta_r}\right) \\ =& (n-1)!\left(\frac{a}{\beta_r}\right)^n \frac{B(m_s+m_r+n,n)}{B(m_s,n)B(m_r,n)}\,{}_2F_1\left(n, m_r+n; m_s+m_r+2n; 1-\frac{\beta_s}{\beta_r}\right), \end{aligned} \tag{5}$$

where $E(\cdot)$ is the expectation operation, $B(\cdot,\cdot)$ is the beta function (Equation (8.380.1), [14]), and ${}_2F_1(\cdot)$ is the Gaussian hypergeometric function (Equation (9.111), [14]).

3.3. Ergodic Capacity

Following a similar procedure as for higher order moments, according to the relationship between the ergodic capacity and the MGF of the inverse SNR (Equation (58), [18]), the ergodic capacity is given by:

$$\begin{aligned} E\left[\eta \log_2\left(1+\gamma\right)\right] =& \frac{\eta}{\ln 2}\int_0^\infty \frac{1-e^{-s}}{s} M_{\frac{1}{\gamma}}\left(s\right) ds \\ =& \frac{\eta}{\ln 2}\int_0^\infty H_{1,2}^{1,1}\left[s \middle| \begin{matrix} (0,1) \\ (0,1),(-1,1) \end{matrix} \right] M_{\frac{1}{\gamma}}\left(s\right) ds \\ =& \frac{\eta a}{\ln 2\beta_s \Gamma\left(m_s\right)\Gamma\left(m_r\right)} H_{2,(1:0),0,(2:2)}^{2,1,0,1,2}\left[\begin{matrix} \frac{a}{\beta_s} \\ \frac{\beta_r}{\beta_s} \end{matrix} \middle| \begin{matrix} (1,1),(m_s+1,1) \\ (0,1);- \\ - \\ (0,1),(-1,1);(0,1),(m_r,1) \end{matrix} \right], \end{aligned} \tag{6}$$

where $\eta = \frac{1}{2}, \frac{1}{3}$, and $\frac{1}{4}$ correspond to two, three, and four time slot bidirectional relaying, respectively. The second equality in Equation (6) follows from (Equation (37), [19]):

$$\begin{aligned} \frac{1-e^{-s}}{s} =& \frac{H_{1,2}^{1,1}\left[s \middle| \begin{matrix} (1,1) \\ (1,1),(0,1) \end{matrix} \right]}{s} \\ =& H_{1,2}^{1,1}\left[s \middle| \begin{matrix} (0,1) \\ (0,1),(-1,1) \end{matrix} \right]. \end{aligned} \tag{7}$$

3.4. Average SEP

Similarly, using (References [20], Equation (21)) and ([16], Equation (2.5.1)), the average SEP is given by:

$$\begin{aligned} E\left[b erfc\left(\sqrt{c\gamma}\right)\right] =& -\frac{b}{\sqrt{\pi}}\int_0^\infty G_{1,3}^{2,0}\left[cs \middle| \begin{matrix} 1 \\ \frac{1}{2},0,0 \end{matrix} \right] \frac{\partial M_{\frac{1}{\gamma}}\left(s\right)}{\partial s} ds = b - \int_0^\infty \frac{b\sin\left(2\sqrt{cs}\right)}{\pi s} M_{\frac{1}{\gamma}}\left(s\right) ds \\ =& b - \frac{b}{\sqrt{\pi}\Gamma\left(m_s\right)\Gamma\left(m_r\right)} H_{2,(0:0),0,(2:2)}^{1,0,0,2,2}\left[\begin{matrix} \frac{\beta_s}{ac} \\ \frac{\beta_r}{ac} \end{matrix} \middle| \begin{matrix} \left(\frac{1}{2},1\right),(0,1) \\ -;- \\ - \\ (0,1),(m_s,1);(0,1),(m_r,1) \end{matrix} \right], \end{aligned} \tag{8}$$

where b and c are modulation specific constants, $erfc\left(\cdot\right)$ is the complementary error function, and $G\left(\cdot\right)$ is Meijer's G function (Equation (9.301), [14]). The third equality in Equation (8) follows from (Equation (2.9.7), [17]):

$$\begin{aligned} \frac{\sin\left[2\sqrt{cs}\right]}{\sqrt{\pi}s} =& \frac{H_{0,2}^{1,0}\left[cs \middle| \begin{matrix} - \\ \left(\frac{1}{2},1\right),(0,1) \end{matrix} \right]}{s} \\ =& cH_{0,2}^{1,0}\left[cs \middle| \begin{matrix} - \\ \left(-\frac{1}{2},1\right),(-1,1) \end{matrix} \right] \end{aligned} \tag{9}$$

Now, closed form expressions are found for the MGF, higher order moments, ergodic capacity, and average SEP in the AF bidirectional relaying system over a Nakagami-m fading environment, when m is an arbitrary positive number. These formulas are very compact and easy to calculate without complex numerical integration and summation operations. As far as we know, the formulas we provide are brand new and have not been publicly reported. However, in previous work, finding the closed form solution is a task far from non-intricate, and involved truncated infinite series expansions and nested multifold numerical integrals, which become verbose and slow even for small m values. Obviously, our derived formulas lower computational complexity.

4. Simulation Results

Simulation results are shown to examine the impacts of our compact formulas. The channel coefficients of all links are normalized to unity. For simplicity, we assume equal transmit power at the two transceivers and the relay node. The average SNR per hop is defined as p_1/N_0. As an example, we show the performance of the two time slot bidirectional relay system, while the three and four time slot bidirectional patterns can be presented similarly.

Figures 1 and 2 show the average SEPs of DPSK and 4-ary pulse amplitude modulation (4PAM), respectively. Six sets of random non-integer fading parameters are selected. Other arbitrary parameters m can be simulated in a similar way. The reason for two figures here is that the error probability expressions of different modulation methods are different. The error probabilities of DPSK and 4PAM are derived from the MGF in Equation (3) and the complementary error function in Equation (8), respectively. Figures 1 and 2 demonstrate the universality and correctness of the formulas. It is further observed that the two figures highlight the same decline speed, which implies the same slope and diversity order.

Figure 3 plots the first order moment $E(\gamma)$ while other higher order moments can be drawn similarly using Equation (5). The theoretical values accurately coincide with the simulation curves. Notice that because of the symmetry of the SNR γ, Equation (5) still holds when the positions of β_s and β_r are swapped.

Finally, the ergodic capacity is shown in Figure 4, where the theoretical and simulated curves are indistinguishable from each other. The ergodic capacity increases as the fading parameters increase due to the improvement of the channel quality. It is observed that the capacity curves nearly keep the same slope in the high SNR region but shift to the right as m increases. This implies that m has no impact on the multiplexing gain, which always stays constant.

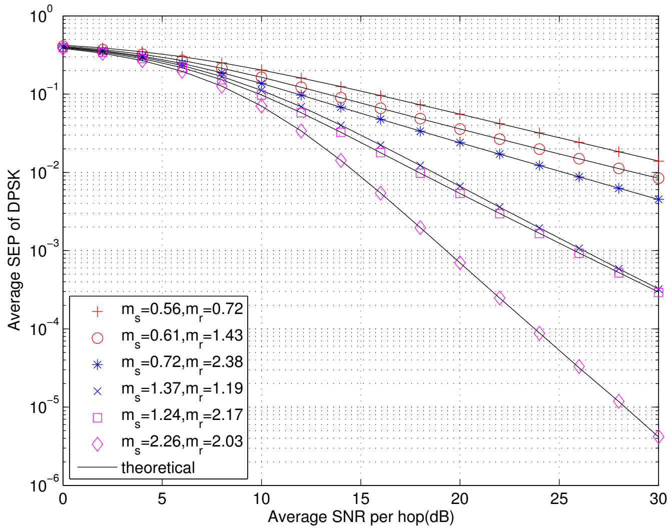

Figure 1. Average signal error probability (SEP) of the differential phase shift keying (DPSK) modulation. SNR: signal-to-noise ratio.

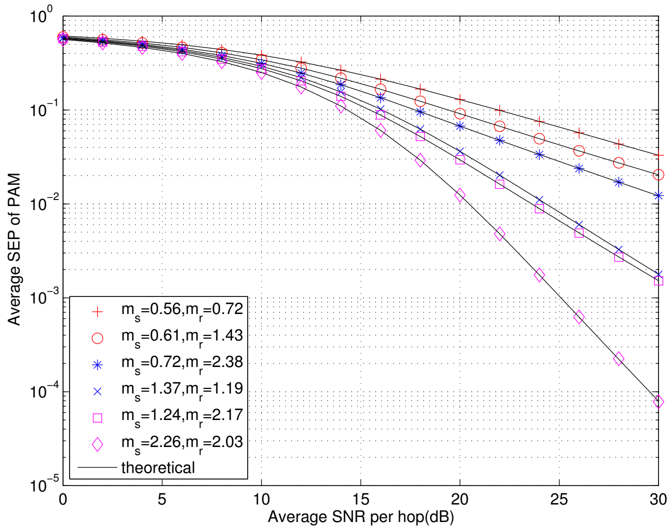

Figure 2. Average SEP of 4-ary pulse amplitude modulation (4PAM).

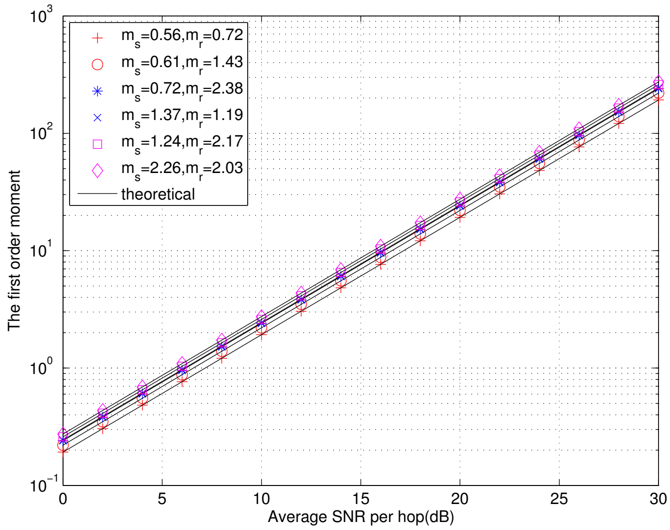

Figure 3. Comparison of the average SNR.

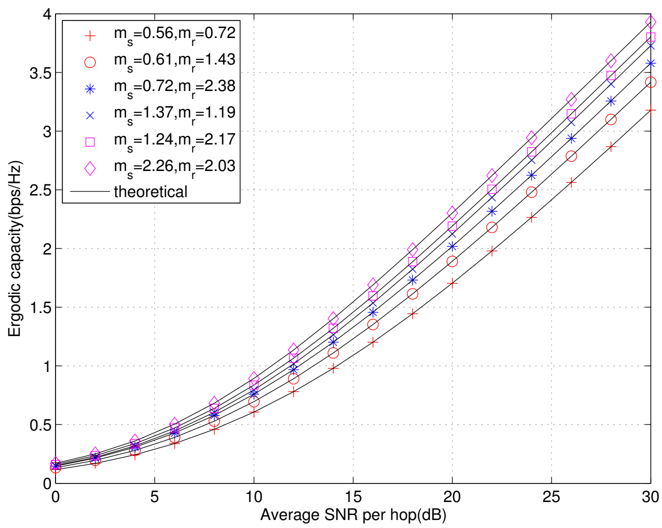

Figure 4. Comparison of the ergodic capacity.

5. Conclusions

In this paper, we consider three types of bidirectional relay modes: two, three, and four time slot bidirectional relaying. In the Nakagami-m fading channel with arbitrary m value, we derive new, highly exact expressions for the MGF, higher order moments, ergodic capacity, and average SEP. All expressions are closed. In particular, the expressions are very concise and easy to calculate, as opposed to complex truncated infinite series expansions or nested multifold numerical integrals. Unlike most researchers who are used to starting a performance analysis with the PDF or CDF, we explore another effective tool, the MGF of the inverse SNR, which is often overlooked. Moreover, our analysis is based on an arbitrary value of m, in line with actual complex environments.

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