

Article

Development of Weighting Scheme for Indoor Air Quality Model Using a Multi-Attribute Decision Making Method

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Abstract: When planning the energy demand of ventilation, proper consideration should be given to the possible scenarios of indoor air quality and pollutant concentrations. The purpose of the present research is to create a practical method of prioritising indoor air pollutants, considering technical, economical and health aspects, in the Indoor Air Quality model (IAQ). In order to find the global weights for the combined IAQ_{index} model sub-elements (in practice, air pollutant concentrations), the Multi-Criteria Decision Making (MCDM) approach is used. The authors have approached the problem of a weighting scheme in a model such as the complex model of the IAQ related to making decisions with many criteria and with the Multi-Attribute Decision Making MADM approach (specifically MCDM). The basis of the MADM method is a decision matrix constructed rationally by the authors, which includes six attributes: actual indoor air carbon dioxide concentration, total volatile organic compounds (TVOCs) and formaldehyde HCHO concentration, and their anthropogenic and construction product emissions to the indoor environment. The decision model of IAQ_{index} includes five alternatives (possible situations), and the combination of pollutant concentration attributes with additional emission attributes is related to the indoor environment under specific situation. For defining the weights of criteria, the authors provide objective approaches: (i) entropy-based approach considering measuring the amount of information, and (ii) CRITIC, a statistic-based approach. The value of the presented method, i.e., the determination of global weights for IAQ components, is shown as a practical application to determine IAQ and the Indoor Environmental Quality (IEQ) index for an office building used as a case study.

Keywords: indoor air quality models; indoor air quality; indoor environmental quality; sustainable building; multi-criteria decision analysis; MCDM; MADM; user dissatisfaction; weights system; building comfort; PD; IAQ; IEQ

1. Introduction and Literature Review

1.1. Research Problem

In literature on the Multi-Attribute Decision Making (MADM) method, the combined Indoor Air Quality (Σ IAQ) model optimisation problem has not been addressed thus far to set up the weights of model sub-components (in practice due to excess indoor pollutant concentrations and risk to occupants). One of the main criteria for optimising such a decision problem should be, directly or indirectly, the cost of ventilation for the elimination of excess pollution concentrations. For the purposes of the decision problem, it is assumed that the general criterion for optimising such a weighing system is to include a range of IAQ indicators and components, i.e., types of contaminants that are most important due to health and toxicity or the amount of air to be removed by the ventilation system.

The MADM method, as a specific kind of Multi-Criteria Decision Making (MCDM) approach, has emerged as a formal methodology for assessing available information and providing values supporting decision-makers in many areas, including environmental engineering. The published results suggest that, over the past decade, there has been a significant increase in the use of MCDM in technical areas for civil engineering applications, the design of indoor environments and the improvement of outdoor environments [1]. MCDM makes it possible to assess complicated decision problems limited by numerous and divergent criteria based on expert subjective assessments (subjective weights) and decision models, the purposes of which are to calculate objective weights according to the rules of decision theory and order multiple parameters of the analysed problem. Recently, the construction industry has become one of the main stressors to the environment and society globally [2]. In the context of the continuous decrease in energy consumption by new buildings and the fundamental need to provide comfortable and healthy conditions for the users in indoor environments, there is a growing urgency to use IAQ models that meet certain criteria. IAQ models are the main components of building Indoor Environmental Quality models [3] and increasingly used for building design software (such as Building Information Modelling, BIM) or as part of building managing systems (BMSs) controlling heat, cooling and ventilation systems (HVACs).

The basic aim of this study was to develop a new approach which combines the previously developed Σ IAQ model [4], containing loosely connected components (indoor air pollution elements and their impact on user perception), with a new weighting system based on MADM that would produce a practical tool supporting choices of the best possible indoor environment and air control strategy. As a case study, a large office building was used, for which the authors had already assessed the indoor environmental parameters [4,5], and the authors recalculated the Σ IAQ and Indoor Environmental Quality (IEQ) indices using the new weighting scheme. A similar issue for the external environment was presented in [6]. This team built the MCDM decision model to study the impacts of outdoor air pollution on the economic development of selected zones in China.

How humans perceive indoor comfort is a complex phenomenon. Authors have analysed the problem from a holistic point of view following current developments described widely [3–5]. Researchers assessing indoor comfort in buildings have focused primarily on optimizing individual components of the indoor environment (thermal, light and acoustic comfort, and air quality), as these four parameters connected to the human senses are considered by occupants to be the most important in determining their comfort [3]. Our research work is aimed at identifying the relationship between the indoor parameters and the resulting reactions of building occupants (in particular, in the area of IAQ). This multi-component approach to indoor comfort focuses on the ‘human stimuli response’ and has led to well-known indoor comfort models such as the thermal comfort model developed by Fanger. In recent years, this holistic approach to the indoor environment has been presented in the European standard EN 15251 and later in EN 16798-1: 2018 [7] where, for the first time, a wider number of indoor comfort criteria were considered. These standards suggest an approach to the classification of IAQ and IEQ models and support certification of buildings taking into account specific components of indoor environment, however, they do not provide a practical guidance or an approach on how to combine them into one indicator that could be useful to classify indoor environmental conditions. The authors solved this problem in a previous publication [4]. When the predominant indoor pollutants have different characteristic values, there is a large problem regarding the choice and weighting method of IAQ sub-components. The problem is considered in this article with use of the MADM approach. Taking into consideration the characteristics and correlations of the selected pollutants, IAQ may be characterized by representative indicators. Our studies on Building Research Establishment Environmental Assessment Method (BREEAM) certified buildings [5] and Leadership in Energy and Environmental Design (LEED) system shown out that carbon dioxide, total volatile organic compounds (TVOCs) and HCHO are the main worldwide, independent and representative environmental indicators. They are being used worldwide as an evaluation index of IAQ in buildings (mainly offices with mechanical ventilation). Since each of these indicators

represents a pollutant class with comparable indoor sources and characteristic dissemination and the indexed group avoids unreliable measurements. This is based on the fact that indicators are “too small” because of decreased critical concentrations. A method of data pre-treatment is proposed by us in the procedures, which reflects concentration differences between pollutants and considers their influence on the satisfaction of the building users. Taking into consideration the existing knowledge gap, the authors proposed an objective method to determine weights in the IAQ model in this article. The article provides the procedure to calculate global weights for the IAQ model sub-components. The Σ IAQ model considers the impact of air concentrations of selected air pollutants (CO_2 , TVOCs, HCHO, etc.) on the building users’ perceptions. The weighted IAQ model was developed by the authors considering the standard EN 16798-1:2019 [7] provisions with the intention to create it as a main sub-component of the overall Indoor Environmental Quality (IEQ) model [8,9] with assumptions described in references [3,8]. The IAQ model and weight system applies to office buildings with mechanical ventilation. Quasi-static indoor environment conditions and full air mixing in the building are assumed.

1.2. Basic Information on the MADM Approach

Some analogies to the presented IAQ weighting problem can be found in the literature, and the studies described so far have led to the establishment of rankings of alternative solutions to the decision problem. The MADM method has been used as an effective tool for choosing the optimal composition of construction materials [10,11], estimating their weights [12] for the selection of the optimal air filtration method to remove pollutants [13], and choosing the optimal set of indoor environment parameters in an office building [14] when the alternatives are differently dated combinations of these parameters (attributes). Additionally, the MADM technique solved the problem of the impact of air pollution on the level of economic development of Chinese cities (for example, the city of Wuhan) [6]. This project is interesting to the authors because it concerns the impact of air pollution. The research involved determining the weights of air pollutants of various types of VOCs, and of the physical parameters affecting these pollutants (attributes of the decision problem), and establishing a ranking of the specific systems of these environmental parameters occurring in different years (alternatives to the decision problem). In these simulations, a modified Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) was used. This approach uses neural networks to determine weights.

MCDM was also proposed in choosing a strategy to design sustainable buildings [15]. The purpose of this study was to provide a method combining climate change effects, adaptive thermal satisfaction, life cycle assessment (LCA), life cycle cost (LCC) analysis and multi-criteria decision making in order to set the most valuable design strategy that would improve building object sustainability. For the assessment and selection of the most appropriate design strategies for buildings, attributes were analysed using a multi-disciplinary approach based on sustainability criteria. In order to find the best building design strategy and make decisions based on several criteria, the Complex Proportional Assessment (COPRAS) method was used [16,17]. This method [15] assumes proportional and direct dependence of the significance of the usability degree of the tested versions of buildings (alternatives) in the system of criteria (attributes) described by the parameters: (1) hours of comfort in the room, (2) primary energy demand, (3) CO_2 emissions, and (4) costs). This method was successfully used to deal with the complex selection of problems. Based on this study, by using the COPRAS method, the authors provided a ranking of the most valuable structural solutions for buildings that meet four characteristic parameters/criteria and their weights built within the decision model.

Most often, MCDM decision models have been used by researchers in the process of selecting components of construction materials by methods such as the “preference technique in classification according to the similarity to the ideal solution” (TOPSIS) [18,19], which uses the principles of analytical hierarchy process (AHP) [17] and various versions of Elimination and Choice Expressing the Reality (ELECTRE) [1]. The method using entropic weights, created on the basis of Shannon’s information theory [20] for the selection of materials [10,11], was also used. Additionally, the method

of material selection from many alternatives based on Jahan's research [12] and the analysis of correlation between Criteria Importance Through Intercriteria Correlation (CRITIC) criteria [12] was used. Alemi-Ardakani [10] analysed the possibilities of optimising the composition of composite materials using the Adjustable Mean Bars (AMB), Modified Digital Logic (MDL), Numeric Logic (NL) and CRITIC [10] methods, as well as the Peng method [21] of creating rankings of preference functions using composition enrichment technology (PROMETHEE—Preference Ranking Organization Method for Enrichment Evaluations) or a preference selection indicator [1].

MCDM methods have now become so popular for extending knowledge of environmental impacts and sustainable construction that several groups of scientists reviewed the applications and development trends of decision models in “environmental sciences” (Cegan, Linkov et al., 2011 [22] and 2017 [23]) and in sustainable construction (Navarro et al. 2019 [1]). MADM methods are a special case of MCDM methods in which decision-making criteria have been replaced by attributes, i.e., features of problems or processes that are evaluated by analysts. Each MADM technique has its own specifics, assumptions and principles. Almost all of the above-mentioned MCDM methods are characterised by the complexity of the mathematics of their moderate to extreme models. Application of these techniques is difficult, requiring advanced mathematical skills and knowledge. Therefore, the creation of undemanding MADM methods is highly desirable. Multi-purpose optimisation based on proportion analysis, named Multi-Objective Optimization by Ratio Analysis (MOORA) was proposed by Brauers and Zavadskas (2006) [24] and is used for the selection of construction materials [25], as it involves uncomplicated mathematics. Therefore, it can be used effortlessly and effectively to choose the best materials or solutions.

To solve the weight problem in the IAQ model, the authors used two methods: entropy and CRITIC. The objective weighting measure was proposed with the Shannon entropy concept provided in [20]. In MCDM, the entropy relates to the diversity degree within the attribute data set. The higher this diversity degree, the higher the attribute weight. The smaller the entropy within the associated data to the attribute, the higher the power of discrimination of the attribute in changing ranks of alternatives. The entropy weights are calculated with only one set of values dedicated to the same attribute j in all alternative levels of the decision matrix. Entropy weight is a typical attribute weight, but it is always introduced to the global weight as a factor in all decision matrix indices. In the entropy method, an attribute with a performance rating that is very different from the others has more importance in the problem because it has a greater impact on ranking outcomes. The attribute has lesser importance if all attributes have comparable performance ratings for that attribute. An objective method to weigh criteria importance through inter-criteria correlation (CRITIC) based on the SD approach was proposed by Diakoulaki and later explored by Jahan [12]. The higher the level of interdependency between attributes, the larger the ranking outcome error. The CRITIC approach is a popular objective method to calculate weights, which solves the problem of interdependencies between the attributes in all combinations by considering the correlation between the sets of variables from various alternative levels, while calculating the weights. It is one of the objective methods of weight determination that belongs to the class of correlation methods. It uses the information contained in the data matrix in the form of the degree of deviation of a variant value from a given mean value of the criteria. The CRITIC method is particularly useful in the IAQ model, as we show later, because the parameters adopted as attributes in the decision model of this problem are the concentrations of selected pollutants and the emissions of the same pollutants; therefore, in the IAQ decision matrix there are correlating attributes.

1.3. Basic Information on the Σ IAQ Model

The Σ IAQ model, in which the authors are looking for objective weight sub-elements, uses a selected number of indoor air pollutants (P_1, \dots, j) and their impact on user dissatisfaction (Percentage

Dissatisfied as $PD = f(c_j)$ in (%). The combined $\sum IAQ_{index}$ (cumulative percentage of satisfied users with indoor air quality, including selected pollutant impacts of user perception) Equation is

$$\sum IAQ_{index} = WP_1 \cdot IAQ(P_1)_{index} + WP_2 \cdot IAQ(P_2)_{index}, \dots, WP_j \cdot IAQ(P_j)_{index} \quad (1)$$

where sub-indices $\sum IAQ(P_j)$ are the percentage of users satisfied with the pollutant concentration; WP_1, \dots, P_j are weights for each IAQ sub-component for groups of air pollutants with comparable concentrations.

There is a difference in Δc_j concentration between the measured air concentration of pollutant c_j and the recommended “reference” concentration c_{ref} (reference by the European Commission “safe” values such as c_{LCI} or c_{ELV}), which may be below the actual air concentration in the contaminated rooms. Thus, the concentration excess related to the ventilation rate is

$$\Delta c_j = c_j - c_{ref} \quad (2)$$

The excess concentration weights WP_j for the IAQ model until now have been calculated based on the arithmetic mean or by adjusting (normalization) all attribute Δc_j values using Equation (3):

$$W_{P,j} = \frac{\Delta c_j}{\sum_{j1..7} \Delta c_{j..7}} \quad (3)$$

where the sum of the adjusted weights $W_{P,j}$ of all pollutants to be removed by ventilation should be in unity. This weighting scheme has been used by the authors in previous years, but in this article, we propose a better and more objective approach based on the MADM approach.

Three levels of complexity of the IAQ model were proposed, and in each, pollutants have been defined that were included in the IAQ model. Figure 1 shows the extended $IAQ_{quality}$ model with its sub-indices and also with the sub-components of the $IAQ_{comfort/health}$ model type, i.e., indoor air pollutants important to health with an impact on the energy balance of a building using a mechanical ventilation system.

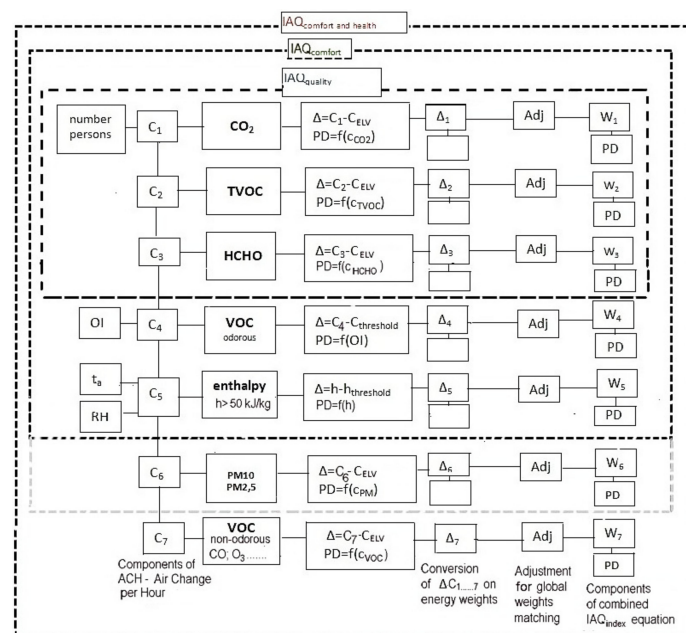


Figure 1. Combined Indoor Air Quality ($\sum IAQ$) model with weighting scheme; horizontally, the model is bound by the weighing processes of individual sub-models; vertically, the model is bound by a chain of input components (pollutant concentrations in ventilated indoor air) and a chain of output components (elements of the combined IAQ_{index} equation).

The experimental dependencies of the percentage of persons dissatisfied (%PD, where %PD = 1-IAQ) and the concentrations values of air pollutants, c_j , perceived in the adequate ranges are of fundamental significance to the sub-components relevant to the IAQ and IEQ models [9]. In the Σ IAQ model proposed by the authors, the accepted common approach is engaged to transform individual pollutant concentrations into sub-components before these are combined into a single index. The aggregated summary of sub-components, however, may lead to a situation in which all components are below the individual health threshold, but the final index shows that the health threshold is exceeded. Conversely, the averaging of partial sub-components may lead to an outcome whereby the overall index shows an acceptable IAQ value, but one or more partial indicators are higher than their individual health threshold. The solution proposed is to use all sub-component maximum values to provide a final form of the Σ IAQ. Considering this, we proposed the Σ IAQ model with three possible levels of model complication due to the application potential, as we show in Figure 2:

- i. building sustainable assessment schemes (like BREEAM assessment)—three sub-components, named quality, Σ IAQ_{quality};
- ii. design purpose, considering the perceptible pollutants affecting indoor satisfaction and comfort, with the intention to use this index for calculating the IEQ—five sub-components, called comfort, Σ IAQ_{comfort}; and
- iii. complex design purpose, where Σ IAQ_{index} represents health and comfort (seven sub-components, or more if necessary), called comfort and health, Σ IAQ_{comfort/health}.

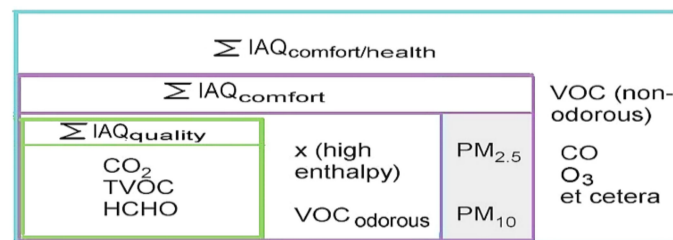


Figure 2. The IAQ model with three complexity levels reflecting the application potential.

The simplest IAQ index level can be used with the main purpose of supporting a sustainable building assessment and certification by using only three sub-components: CO₂, HCHO and TVOC. Σ IAQ_{quality} model is used in this article to assess the office building (case study).

From the dependencies, expressed as the curves for the percentage of satisfied users with pollutant IAQ(CO₂) or IAQ(VOC_{odorous}), the following Equations (4)–(6) of the models are provided:

$$\Sigma \text{IAQ}_{\text{quality}} = W_1 \cdot \text{IAQ}(\text{CO}_2) + W_2 \cdot \text{IAQ}(\text{TVOC}) + W_3 \cdot \text{IAQ}(\text{HCHO}) \quad (4)$$

$$\Sigma \text{IAQ}_{\text{comfort}} = W_1 \cdot \text{IAQ}(\text{CO}_2) + W_2 \cdot \text{IAQ}(\text{TVOC}) + W_3 \cdot \text{IAQ}(\text{HCHO}) + W_4 \cdot \text{IAQ}(\text{VOC}_{\text{odorous}}) + W_{4a} \cdot \text{IAQ}(\text{VOC}_{\text{odorous}}) + W_5 \cdot \text{IAQ}(\text{h}) \quad (5)$$

$$\Sigma \text{IAQ}_{\text{comfort/health}} = W_1 \cdot \text{IAQ}(\text{CO}_2) + W_2 \cdot \text{IAQ}(\text{TVOC}) + W_3 \cdot \text{IAQ}(\text{HCHO}) + W_4 \cdot \text{IAQ}(\text{VOC}_{\text{odorous}}) + W_{4a} \cdot \text{IAQ}(\text{VOC}_{\text{odorous}}) + W_5 \cdot \text{IAQ}(\text{h}) + W_6 \cdot \text{IAQ}(\text{PM}_{2.5}, \text{PM}_{10}) + W_7 \cdot \text{IAQ}(\text{VOC}_{\text{non-odorous}}) + W_{7a} \cdot \text{IAQ}(\text{VOC}_{\text{non-odorous}}) + W_8 \cdot \text{IAQ}(\text{CO}) + W_9 \cdot \text{IAQ}(\text{NO}_2) + W_{10} \cdot \text{IAQ}(\text{O}_3) \quad (6)$$

The structure of Σ IAQ_{comfort/health} is made of seven (or possibly more) components or IAQ sub-models, which are occupants satisfaction functions for the various types of indoor air pollutants: IAQ(TVOC), IAQ(HCHO), IAQ(CO₂), IAQ(VOC_{odorous}), IAQ(PM_{2.5}, PM₁₀), IAQ(enthalpy, h) and the selected IAQ(VOC_{non-odorous}). The IAQ(VOC_{non-odorous}) and IAQ(VOC_{odorous}) models should be multiplied, depending on the number of dominant VOC substances; hence, the Σ IAQ_{comfort/health} index may have more than 11 sub-components in practice.

Combined $\Sigma\text{IAQ}_{\text{index}}$ is an element of the IEQ index in Equation (7) [8], where the authors adopted a crude weighting system in which all elements are weighted identically (0.25 for weights W_1 – W_4):

$$\text{IEQ}_{\text{index}} = 0.25 \cdot \text{TC}_{\text{index}} + 0.25 \cdot \Sigma\text{IAQ}_{\text{index}} + 0.25 \cdot \text{ACC}_{\text{index}} + 0.25 \cdot \text{L}_{\text{index}} \quad (7)$$

where TC_{index} is the thermal comfort index, expressed as the number of users satisfied with an indoor thermal comfort (in %); $\text{ACC}_{\text{index}}$ is the acoustic comfort index, i.e., the number of users satisfied with sound level (in %); and L_{index} is Daylight Quality, i.e., the number of users satisfied with daylight in the building (in %).

2. Materials and Methods

2.1. Research Procedure Diagram

Figure 3 shows the necessary research phases to calculate the global weights of the combined ΣIAQ model with MADM and further usage of these weights to calculate the occupant satisfaction $\text{IAQ}_{\text{index}}$ and $\text{IEQ}_{\text{index}}$ for the case study building.

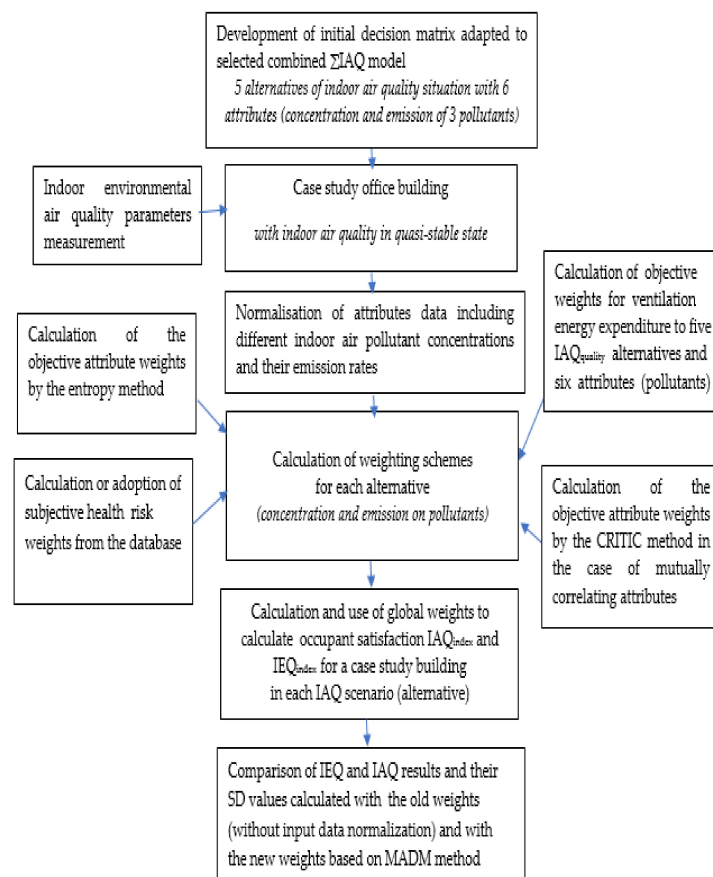


Figure 3. The research phases to calculate the global weights for the ΣIAQ model (based on the case study building).

2.2. MADM Method Applied to the ΣIAQ Model

The basis for applying the MADM method is a decision matrix, as shown in Table 1. The issue of a weighting scheme is a problem related to making decisions with several criteria, which can be analysed using the model [26]. For every type of ΣIAQ model with Equations (4), (5) or (6), the decision matrix should be created separately.

Table 1. Decision matrix scheme in the MADM model for the concordance of economic and human comfort criteria for each Σ IAQ scheme described by Equations (4)–(6) separately with alternatives (e.g., Σ IAQ_{comfort} scheme configurations) and with attributes (air pollution components in quasi-stable states and emissions).

Alternatives	Attributes $X_{1..m}(j = 1 \dots m)$									
$a_{i..n}(i = 1 \dots n)$	X_1	X_2	X_3	X_4		$X_5(e)$	$X_6(e)$	$X_7(e)$		$X_m(e)$
a_1	x_{11}	x_{12}	x_{13}	x_{14}	X_{15}	X_{16}	X_{17}	x_{1m}
a_2	x_{21}	x_{22}	X_{23}			X_{25}	X_{26}			x_{2m}
a_3	x_{31}	X_{32}				X_{35}				x_{3m}
....
a_n	x_{n1}	X_{n2}	X_{n3}	X_{n4}		X_{n5}	X_{n6}	X_{n7}	x_{nm}

In Table 1, alternatives $a_1, a_2 \dots a(n)$ for Σ IAQ_{comfort} configurations are defined further (in Methods). Attributes X_1, X_2 and X_j are the indoor air pollution concentrations of CO₂, HCHO, TVOC and VOC_{odorous} in a quasi-stable state of the indoor environment. $X_5(e), X_6(e)$ and $X_7(e)$ are indoor emission processes and their rates per emission surface of CO₂ (e), HCH(e), introduced by anthropogenic emissions or directly emitted from construction products. In the decision matrix of the IAQ weighting scheme according to Table 1, the flow chart presented in Figure 3 provides the necessary steps and, therefore, includes MADM elements:

(1) attributes in the number $j = 1 \dots 3 \dots$, which are indoor air pollution concentrations covered by the models Σ IAQ_{quality}, Σ IAQ_{comfort} and Σ IAQ_{comfort/health} described by Equations (4)–(6) and Figure 2; however, the number of attributes beyond $j = 7$ (e.g., for Σ IAQ_{comfort/health}) is not limited because model components like VOC_{odorous} or VOC_{non-odorous} and other pollutants can be multiplied.

(2) alternatives $i = 1 \dots n$, which are the Σ IAQ models described by Equations (4)–(6) and Figure 2 assigned to the quasi-stable state, or defined by emissions. The selected decision model alternatives correspond to three combined indoor air quality models in various attribute set configurations (i.e., changes in attribute sets, IAQ sub-models). Alternative models, i.e., IAQ_{comfort} and IAQ_{comfort/health} models, have more attributes than the IAQ_{quality} model; moreover, the number of these attributes is variable in both the pollutant and emission groups. In the group of alternatives, IAQ model systems not disturbed by emissions, e.g., IAQ_{quality}, and models distorted by emission attributes, (e) for example, were considered. The number of alternatives (Table 1) cannot be too small because the standard deviations for the set of alternatives assigned to a given attribute are calculated (i.e., for the attribute column in the decision matrix).

The assumptions in the decision model are adequate for the task of measuring the indoor air ventilation rate to obtain the assumed air quality, setting IAQ model alternatives so they meet the criteria and taking into account their importance ranking and possibility for ongoing diagnosis of alternatives to the assumed overall IEQ assessment. The model should provide a set of attributes that are important for maintaining hygienic and efficient ventilation, both of which are connected to variable air pollutants. This is important because the decisions relate to the scope of complex indoor air quality models, while planning the ventilation system depends on the amount of “excess air pollution” including VOC and SVOC compounds.

In the Materials and Methods and Results sections, an example of the implemented IAQ_{quality} model is provided, as well as a shortened Table 5 decision matrix with a set of attributes of type (e) (emission), which includes additional sources of bio-emissions (CO₂ emissions), emission sources from additional equipment and finishing products (HCHO emissions), and emissions from building materials (TVOC). The choice of the attributes was facilitated by the fact that the effects of air pollution, in another open environment and system of conditions and tasks, on the economy were provided by Wang et al., 2017 [6]. This team built a Smart MCDM decision model to study the impact of outdoor air pollution on the economic development of selected zones in China. In addition, looking at the goal of the present paper to rank the importance of selection parameters for human comfort, insights could be

derived from already established research procedures for the selection of construction materials [12] or waste disposal (Savic [27]). Therefore, the attributes of the decision matrix (Tables 1 and 2) include the most typical and expected air pollutants belonging to successively developed Σ IAQ systems. The selection of attributes, i.e., types of pollution, consistent with Equations (4)–(6) also corresponds to the choice of Air Quality Index (AQI) [28].

2.3. Flow Chart of Weighting Scheme Determination

In the analysed IAQ model, the authors considered excess concentrations that can be measured in a quasi-steady state, and emissions from additional and sometimes temporary sources that periodically increase in the Σ IAQ system of excess concentrations. The number of pollution type attributes is as strictly defined as the Σ IAQ specified composition. The number of emission type attributes is variable, adapted to the selected set of decision pollutants, and does not usually exceed the number of pollution type attributes.

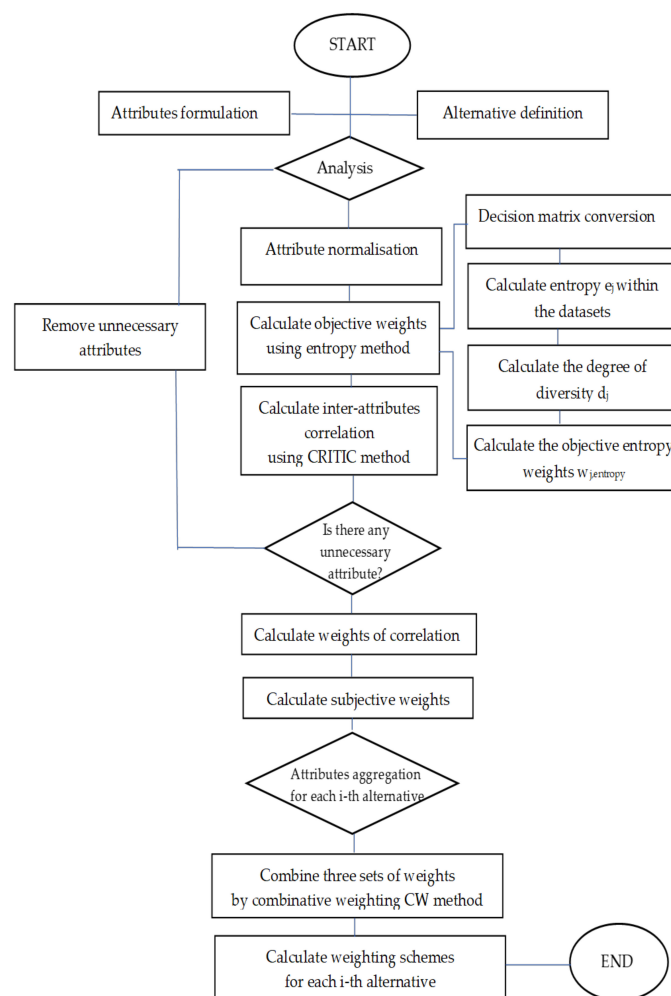


Figure 4. Chart for subjective and objective weight determinations for the combined Σ IAQ model.

The process of determining the weight for the IAQ_{index} model, seen in a number of variants of the composition of its sub-indexes, i.e., in various alternatives, involves weighting scheme calculations adequate to the level of a particular alternative. Each weighting scheme contains relative values of global weights. These weights represent the energy expenditure of the ventilation system on the elimination of excess concentration of a given pollutant (excess concentration (2) [4]) in ventilated air. The process for determining the weighting scheme for the Σ IAQ model is shown in the flow chart in Figure 4. It should be noted that the authors considered it appropriate, after specifying the

technical conditions and physics of the building for which the decision model is used, to add the costs of ventilating the anticipated excess pollutant concentrations. Based on the literature of this multi-parameter problem, however, the authors found it advisable not to insert this attribute at the current level of the generalized decision model for selecting the Σ IAQ weighting scheme.

The idea of the correlation effect on weight $w_{j,CRITIC}$ stems from the fact that when the criteria correlation with other attributes is significant, it should have lower importance due to the roles of other criteria. An increase in the value of the concentration of a given indoor air pollutant, with a sudden (dramatic) increase in emissions, is accompanied by an increase in global weight because it must include an additional component, which is the correlation weight.

2.4. General Principles of Estimating IAQ Model Weighting Schemes

In this section, the authors provide a method with a step-by-step procedure for setting priorities in the group of dominant indoor air pollutants, considering technical, economic and health aspects, by global weights for each component of the IAQ model. Developed for the Σ IAQ_{quality} model, the MADM decision matrix includes six attributes: carbon dioxide, total VOC (TVOC) and formaldehyde concentrations (HCHO) in the indoor environment as well as additional carbon dioxide bio-emissions and total VOCs and formaldehyde emissions. All six attributes are associated with cost criteria, and in future the decision model can be developed along this trend. The MADM decision matrix of the IAQ_{quality} model includes five alternatives (combination of various IAQ_{quality} sub-components) for defining indoor air quality in a building. The information contained in every attribute is related to the contrast of each criterion. Standard deviation (SD) and entropy are the measures of intensity and ways to present objective criteria weights. In order to calculate the weights of criteria, the authors use objective methods: entropy and CRITIC, presented in Figures 3 and 4.

Adaptation of the IAQ model with the “new” weighting procedure based on the MDMA approach to our office building case study (see Results and Discussion) was performed mainly for illustrative purposes in the context of the presented MDMA method for IAQ model development. The authors did not focus in depth on any technical issues of the building. Other Indoor Environmental Quality (IEQ) sub-components indexes, such as thermal, acoustic and visual satisfaction (in %), are presented in a paper [5] in order to provide background to the actual research problem.

The authors present the order of steps and mathematical operations leading to the determination of weighting schemes of Σ IAQ models (4), (5) and (6) separately. All model decision matrix variants are covered in an overview (Table 2), which presents decision matrices for the three types of IAQ models: Σ IAQ_{quality}, Σ IAQ_{comfort} and Σ IAQ_{comfort/health}, covering various types of contaminants.

Table 2. Decision matrices for three Σ IAQ models with attribute data.

Alternatives a = 1 ... n	Attributes $X_1 \dots m (j = 1 \dots m)$								
	CO ₂	TVOC	HCHO	VOC Odorous	x High Enthalpy	PM2.5	Emissions		
							CO ₂ (e)	TVOC (e)	HCHO (e)
IAQ _{quality} in quasi-stable state with load by emissions									
IAQ _{quality}	x ₁₁	x ₁₂	x ₁₃						
IAQ _{quality} (e _{min})	x ₂₁	x ₂₂	x ₂₃				x ₂₇	x ₂₈	x ₂₉
IAQ _{quality} +(e _{25%})	x ₃₁	x ₃₂	x ₃₃				x ₃₇	x ₃₈	x ₃₉
IAQ _{comfort} in quasi-stable state with load by emissions									
IAQ _{comfort}	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	x ₁₆	x ₁₇	x ₁₈	x ₁₉
IAQ _{comfort} (e _{min})	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	x ₂₆	x ₂₇	x ₂₈	x ₂₉
IAQ _{comfort} +(e _{25%})	x ₃₁	x ₃₂	x ₃₃	x ₃₄	x ₃₅	x ₃₆	x ₃₇	x ₃₈	x ₃₉
IAQ _{comfort/health} in quasi-stable state with load by emissions									
IAQ _{comfort/health}	x ₁₁	x ₁₂	x ₁₃	x ₁₄	x ₁₅	x ₁₇	x ₁₈	x ₁₉
IAQ _{comfort/health} (e _{min})	x ₂₁	x ₂₂	x ₂₃	x ₂₄	x ₂₅	x ₂₇	x ₂₈	x ₂₉
IAQ _{comfort/health} (+e _{25%})	x ₃₁	x ₃₂	x ₃₃	x ₃₄	x ₃₅	x ₃₇	x ₃₈	x ₃₉

The first step for estimating the MADM framework is normalisation of data (henceforth, the decision matrix indicators), which belong to different types and have different excess concentrations of the most important air pollutants and VOC emissions affecting IAQ indoor environments. In an indoor quasi stable state, attribute parameters include the concentration of these pollutants and additional increases in their concentration when the level is disturbed by additional sources of their emission or bio-pollutant emissions (1). Normalisation of attributes in various units (values of pollutant concentration and emission rate) in large ranges (attribute) is performed to obtain dimensionless quantities. Attribute normalisation is possible in several ways; however, for the research problem discussed here, the most common methods are as follows:

(a) Linear normalisation of attributes belonging to the decision matrix carried out according to the pattern of elements in the normalised decision matrix, belonging to the set of “cost attributes” according to Equation (8):

$$n_{ij} = \min(x_{ij})/x_{ij} \quad (8)$$

where $j \in N_c$ and N_c represents a set of criteria or attributes. The minimum value of the indicator x_{ij} of the decision matrix, i.e., the normalisation of attributes in the form of the levels of air pollutants or their emissions in the decision matrix of a complex IAQ model, can be more conveniently determined using Equation (9) provided by Körth [29]:

$$n_{ij} = 1 - \frac{x_{ij}}{x_j^{\max}} \quad (9)$$

if air pollution is a negative value indicator.

(b) Normalisation of attributes according to Wang’s scheme [6] already applied in relation to the description of air pollution. In addition, positive and negative indicator values represent different meanings (10) and (11). Air pollution is a negative value indicator (negative values), while profit from the absence of pollutants would be a positive indicator. Similarly, the values of the emission rate of individual pollutants are also negative indicators, which can be read from the emission curves, both $\max(x_{ij})$ and $\min(x_{ij})$ values. Therefore, in order to normalise the attributes, the authors chose negative values according to Equation (10). Positive values (type benefit) are

$$n_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \quad (10)$$

Negative values (type costs) are

$$n_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \quad (11)$$

where x_{ij} represents the original value and n_{ij} represents the value after normalisation. However, this way of normalising the IAQ decision matrix in the actual system of its application, in which some configurations (alternatives) have some attributes values equal to 0, proved to be impracticable. The same group also uses the transformer version of Equation (10) provided by Zavadskas [30] and Vujcic [31]:

$$r_{ij} = \frac{x_{ij} - \max x_{ij}}{\min x_{ij} - \max x_{ij}} \quad (12)$$

The stage of data normalisation of the decision matrix is carried out after it is built, and at the same time, the $\max(x_{ij})$ and $\min(x_{ij})$ values are determined in the attribute columns j . These \max and \min values of the x_{ij} indicators are inserted into the formula for normalisation of the input data. Therefore, prior to data normalisation, the maximum and minimum values x_{ij} adopted for the attributes in the studied conditions of the IAQ model should be compiled from the literature. These values will be used to test the normalisation of the input dataset adopted in the decision matrix,

while the test normalisation will also address the question of whether a given normalisation method (formulas) normalises the input dataset correctly (the normalisation result cannot be negative) [31] and whether the set of alternatives (combinations of attributes) in the decision matrix has been determined correctly. Literature [2,5,7,32–40] on the minimum pollutant values (min x_{ij}) include the long-term concentrations [32] or c_{LCI} concentration values. The maximum values [2,7,36–38] of the max (x_{ij}) pollutant concentrations are either short-term values or allowable concentration values c_{ELV} [32] according to EN 16798-1: 2019, World Health Organization WHO guidelines [33] and EU recommendations [34]. The results of maximum and minimum concentration tests in representative groups of UK, Danish and Polish building tests published by Shrubsole [2] and Johnston [35] were used. Attributes are initially normalised assuming that, for attributes $X_1 \dots m$, the values from Table 3 can be inserted into Equations (9) and (12) to obtain normalised attributes n_{ij} .

Table 3. Example data for the Σ IAQ model decision matrix. Attribute indices x_{ij} and normalisation results n_{ij} with two methods calculated by Equations (9) and (12), as example alternatives.

Parameters	Attributes, $X_1 \dots m$								
	CO ₂ ppm	TVOC $\mu\text{g}/\text{m}^3$	HCHO $\mu\text{g}/\text{m}^3$	VOC _{odorous} $\mu\text{g}/\text{m}^3$ e.g., NO ₂	^x Enthalpy kJ/kg	PM2.5 $\mu\text{g}/\text{m}^3$	Emission Bio-CO ₂ e mg/m^3 h.pers.	Emission TVOC e mg/m^3 h	Emission HCHO e $\mu\text{g}/\text{m}^2$ h
max(x_{ij})	1000	1000	160	660	80	100	64.84 persons	57.1	68.2
min(x_{ij})	200	154	5.8	20	34	10	16.2 one person	13.6	4.26
$\frac{\min x_{ij} - \max x_{ij}}$	-800	-846	-194.2	-640	-46	-90	-48.6	-43.5	-63.94
x_{ij} lit.value	450	787	18	40	45	35	32.4 mean value	28.6 mean value	36
Normalised attribute values n_{ij} calculated according to Equation (9)									
	0.55	0.213	0.88	0.94	0.44	0.65	0.5	0.5	0.47
Normalised attribute values n_{ij} calculated according to Equation (12)									
	0.68	0.25	0.73	0.97	0.76	0.72	0.67	0.65	0.50

2.5. Calculation of the Objective Attribute Weights by the Entropy Method

Step 1. Normalisation of indicators

Since the analysed indicators of indoor air contamination may be provided in different units, authors normalised them as a first step. Normalised decisions, however, depend on the attribute’s nature. In the entropy weights procedure, all attributes must be beneficial (positive values), but in the decision matrix all attributes are of a non-beneficial type. Therefore, the conversion of normalised indicators of “cost” type into indicators of “profit” requires building a matrix of “profit type”. As such, the higher their indicator values are, the higher the assessment of the decision variant will be. The indicators for which the criteria are based on the assessment of current air pollutants, and the assessment of the emission rate belong to the cost type indicators, are designated by non-beneficial or cost sets. In order to determine weights using the Shannon entropy method, a specific data normalisation method is used [20]. We built a matrix $Y = (y_{ij})$, in which all criteria will be of “profit type”. Then, the elements of the matrix are determined from Equation (13):

$$y_{ij} = 1/x_{ij} \tag{13}$$

The normalised decision matrix $Z = [z_{ij}]_{n \times m}$ with the elements z_{ij} for determining entropy weights is determined with Equation (14):

$$z_{ij} = \frac{y_{ij}}{\sum_{i=1}^n y_{ij}} = \frac{1/x_{ij}}{\sum_{i=1}^n 1/x_{ij}} \quad (14)$$

for various alternatives (more or less complex indoor air quality models) and their respective attribute sets (selected IAQ sub-models to assess various pollutants and emission rates).

Step 2. Calculation of the entropy measure

The entropy vector e_j for a given attribute j from the sum of components z_{ij} and $\ln(z_{ij})$ calculated for all alternatives $i = 1 \dots n$ is determined as

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^n z_{ij} \ln z_{ij} \quad (15)$$

where $z_{ij} \cdot \ln z_{ij}$ is equal to 0 for $z_{ij} = 0$ and the variation (divergence) level vector d_j calculated for each attribute as

$$d_j = 1 - e_j \quad (16)$$

Step 3. Defining the objective weight of each j as follows, based on the entropy concept

$$w_{jentropy} = \frac{d_j}{\sum_{j=1}^m d_j} \quad (17)$$

2.6. Calculation of Subjective Attribute Weights Depending on the Toxicity of Air Pollution to Humans

The weights of IAQ_{index} models must also contain, in addition to the objective entropy weights, the subjective attribute weights that describe factors related to health human toxicity from indoor air pollution.

Variant 1. S_j^{health} subjective weights from databases for criteria of “air pollution specified chemical compound” with tested chemical properties, toxicity, and “health level” and “health scores” assigned to air concentrations of this compound are treated as reference data taken directly from the literature [36] and [40–44]. For example, Yuan [40] adopted the following weights for selected pollutants as part of the research program: “French permanent survey on Indoor Air Quality” [43], US-EPA and WHO databases (Table 4). In some cases when detecting a larger number of indoor air contamination compounds with air concentration $>5 \mu\text{g}/\text{m}^3$, additive health effects are assumed, and the subjective weight S_j^{health} can be created with the EU-LCI approach.

Table 4. Subjective weights of ten pollutants (plus hot and humid air enthalpy).

Different Types of Air Pollutants and Subjective Weights									
CO	CO ₂	NH ₃	O ₃	HCHO *	PM10	TVOC	Rn	microbes	benzene
0.21	0.01556	0.01556	0.158	0.21	0.0525	0.21	0.105	0.0233	0.19
The levels of indoor air enthalpy and related user dissatisfaction (%)									
Enthalpy kJ/kg		34.5		44.5		59.5		70.5	
Dissatisf. %		10		20		40		60	
								81.9	

* Based on the recommendation of the European Collaborative Action expert group. This group focuses on the harmonisation of indoor emission assessments by means of developing Lowest Concentration Interest (LCI) values (since 2011). This expert group developed a list of relevant substances and described the procedure of LCI calculation. For assessing combination effects of substances, the R-value as the health hazard index was proposed. The R_i value was established as $R_i = C_i/LCI_i$, where C_i is the concentration of compound in a test chamber i . For $R_i < 1$, it is assumed that there will be no health effects. $R = \text{sum of all } R_i = \text{sum of all ratios } (C_i/LCI_i) \leq 1$.

Variant 2. The subjective s_j^{health} weights determined by health experts depend on the toxicity of pollutants based on a number of expert opinions r greater than the number of experts k . In the case of not listing compounds in databases, the relative value (signification) of the s_j^{health} weight for the j th criterion (with a total number of m criteria) can be calculated by the SWARA method presented by Kersulienė et al. [45] and Zavadskas [46] using weight estimation procedures in several steps as follows.

Step 1. The relative importance of the criterion is determined by reference to the weight of s_j and the weight of the previous criterion s_{j-1} . Then, the weights of s_j^{health} based on the opinions of all experts are calculated. An example of a simple algorithm for determining the average subjective weight of attributes s_j is based on the average of opinions (weight proposals) of attributes collected from k experts, i.e., from the formulas given by [46]. The average value of expert attribute reviews is calculated using Equation(18):

$$\bar{t}_j = \frac{\sum_{k=1}^r t_{jk}}{r} \quad (18)$$

where t_{jk} is the ranking of attribute j according to respondent k , and r is the number of respondents.

Step 2. The subjective weights can be adjusted by dividing the average values of each attribute by the sum of the average values of the attributes (19):

$$s_j^{\text{health}} = \frac{\bar{t}_j}{\sum_{j=1}^m \bar{t}_j} \quad (19)$$

Step 3. Having determined the average values of attribute reviews from Equation (18) and knowing the established ranking of these opinions, you can calculate the standard deviation of attribute j in the ranking of opinions (“dispersion of expert ranking values”) by Equation (20):

$$\sigma_j^2 = \frac{1}{r-1} \cdot \sum_{k=1}^r (t_{jk} - \bar{t}_j)^2 \quad (20)$$

Value σ_j^2 is useful for calculating the weights of the effects of attribute correlation $w_{j,\text{CRITIC}}$ formula.

2.7. Calculation of the Objective Weights of Correlation between Attributes by the CRITIC Method

Use of the CRITIC (Criteria Importance Through Inter-criteria Correlation) method in the case of mutually correlating attributes (e.g., attributes constituting “quasi-fixed” impurities and attributes constituting their emissions) to estimate the correlation weights between the criteria is justified. Additionally to the intensity contrast of dataset attributes in the decision matrix, there was another concept more recently taken into consideration by researchers using MCDM. Diakoulaki’s team [47] found that the greater the level of inter-dependency between attributes, the higher the error ranking outcome. CRITIC was analysed in [47] as an objective and new weighting system that took into consideration criteria correlations. The approach included contrast intensities (by means of SD criteria) and combined them with correlations weights. All material selection studies assume that the criteria are independent of each other, while they are also likely to influence each other. Some approaches proposed in recent years for weighting criteria when choosing material not only bypassed this approach, but they did not take into account the interdependence of criteria and proposed methods of objective weighting, which have relevant shortcomings. When correlation between criteria is suspected, the validity of the criteria can be adjusted by entering correlation weights. Calculating the weights of correlations between the criteria of a decision model using Pearson’s correlation coefficients is recommended [9,11]. The CRITIC weights marked $w_{j,\text{CRITIC}}$ specify a correlation between the criteria, which can include all criteria (or attributes) or only criteria that are influenced. The CRITIC method by Diakoulaki [47], Jahan et al. [12] and Ardakani et al. [10] includes estimations of differences in the “intensity” of criteria values (expressed by the value of their standard deviations) and combines them with weights based on correlation coefficients (step 4). Alternatively, the modified model in Jahan [12] uses only Pearson’s correlation coefficients and does not include standard deviation criteria in the equations. As justification, it was stated that the differences resulting from changes in the “intensity” of the criteria were already taken into account by the entropy method, and the correlation weights were calculated accordingly. Therefore, the values of the correlation weights between the criteria are calculated by [10,12] using the

values of Pearson coefficients R_{jk} , which represent the correlation between the two selected criteria j and k (correlated criteria) and which can be calculated easily, even in MS Excel. R_{jk} values are read from the symmetrical linear criteria correlation matrix or calculated from Equation (21):

$$R_{jk} = \frac{\sum_{i=1}^n (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sqrt{\sum_{i=1}^n (x_{ij} - \bar{x}_j)^2 \cdot \sum_{i=1}^n (x_{ik} - \bar{x}_k)^2}} \quad j \text{ and } k = 1 \dots m \quad (21)$$

where x_{ij} and x_{ik} are n total number of alternatives (alternative IAQ models) and matrix elements (Table 1) x_{ij} (with a peak) of average values of the j criterion and x_{ik} of average values of k criteria (emission criteria $k_{1..3}$ correlating with other criteria, e.g., emission rate of pollutants, which include selected criteria pollutant concentrations). If R_{jk} is close to +1 or -1, this means that the criterion is correlated highly, while R_{jk} close to 0 means there is no correlation. Then, the correlation weights are calculated by the CRITIC method using R_{jk} determined by Equation (21). The effects of the correlation impact of all criteria on other criteria can be calculated, or the impact of selected k criteria (e.g., emissions) on related criteria of the decision model j (e.g., polluting emissions), and only then are selected correlation coefficients used. A common form of the formula for correlation weights using the CRITIC method is Equation (22) [10]:

$$w_{j,CRITIC} = \frac{\sum_{k=1}^m (1 - \beta \cdot R_{jk})}{\sum_{j=1}^m (\sum_{k=1}^m (1 - \beta \cdot R_{jk}))} \quad j \text{ and } k = 1 \dots m \quad (22)$$

where β is equal to +1 if two objective criteria are of the same type (either greater is better) or $\beta = -1$ otherwise. In Equation (22), Equation (23) denotes a conflict of measure created by criterion k in relation to the decision:

$$Conflict = \sum_{k=1}^m (1 - R_{jk}) \quad (23)$$

According to Zawadskas [30], the sum (23) ought to be multiplied by σ_j^- , – the standard deviation (SD) of the j th criterion. The inclusion of SD follows a similar approach to the entropy approach, assigning a weight to the attribute if it has a corresponding attribute value across alternatives. A “symmetrical matrix” is developed with dimensions $n \times m$ and a generic element R_{jk} . The higher the divergence in the scores of alternatives in criteria j and k , the lower the value of R_{jk} . This step determines the conflict and then the weight of each decision criterion. The sum provided below denotes the measure of conflict created by criterion j with respect to the decision.

The modified and simplified CRITIC method, which has not been used in this study, calculates the correlation weights between criteria without calculating the symmetric linear correlation matrix R_{ij} . The method determines the objective weights $w_{j,CRITIC}$ on a different principle. It determines the objective weights of the attributes by using contrast intensities for each measure, considered as SD, conflict and correlation coefficient between criteria by Zawadsas et al. [30], Vujicic et al. [31] and Adali et al. [48].

Step 1. Calculation of the normalised decision matrix for $w_{j,CRITIC}$

For each criterion x_{ij} , function r_{ij} which transforms all the values of criteria into the interval [0, 1], is provided:

$$r_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \quad (24)$$

The basis of the transformation uses the concept of an ideal point. The initial matrix is converted into generic element matrix r_{ij} . Here, it should be noted that normalisation does not take account of the type of criteria.

Step 2. Calculation of standard deviations in attribute column j for all alternatives $1 \dots n$.

Step 3. Calculation of C_j specifying the amount of information contained in the given attribute j .

In Equation (25), SD represents the deviation of various values for a given criterion of a mean value. The information of C_j contained in criterion j is calculated by Equation (25):

$$C_j = \sigma_j \sum_{j=1}^m (1 - R_{jk}) \quad (25)$$

where σ_j is the SD of the j th criterion or alternative, and the expression r_{jk} is considered equivalent to the correlation coefficient between the two attributes: selected attribute j and correlation attribute k .

Step 4. Calculation of the weight of the correlation of the criteria correlating k with a given criterion j is carried out using Equation(26):

$$w_{j,CRITIC} = C_j / \sum_{k=1}^m C_k \quad j = 1, \dots, m \quad (26)$$

Calculation of correlation coefficients should not be ignored; therefore, this study adopted a modified method of data normalisation and calculation of correlation weights between attributes based on the symmetric linear correlation matrix using the full CRITIC method. Calculation of correlation weights between criteria based on the symmetric linear correlation matrix by the CRITIC method uses a modified data normalisation method. In summary, the calculations were based on the determined correlation coefficients for attribute pairs related to the same impurities $w_{j,CRITIC}$ by using the following steps.

Step 1. Normalisation of initial decision matrix (Table 1)

The recommended [6] equation for normalisation of the original decision matrix for cost criteria is Equation (27)

$$r_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \quad (27)$$

but the authors used the transformer version [32,33] in the form:

$$r_{ij} = \frac{x_{ij} - \max x_{ij}}{\min x_{ij} - \max x_{ij}} \quad (28)$$

Step 2. Calculation of standard deviations in data sets assigned to attributes j from the value r_{ij} for all alternatives $1 \dots n$.

Step 3. Calculation of symmetric linear correlation matrix R_{ij} .

The primary matrix (Table 1) is converted to the normalized decision matrix and later to symmetric linear correlation matrix of generic R_{ij} elements, which are Pearson's correlation coefficients (21) determined for all pairs of normalised sets on data r_{ij} assigned to selected attributes, at all levels of alternatives n . Then, the most likely correlations corresponding to attribute pairs (e.g., concentrations and emissions of the same compound) were selected from the matrix, and weights were calculated for them $w_{j,CRITIC}$. A coefficient of linear correlation between each pair of measures is estimated using the following formula that allows to quantify the conflict occurring among various criteria. It is obvious that more discordant the scores of the alternatives in both matrix vertical i and horizontal j sections, the lower the value R_{ij} .

Step 4. Calculation of objective weights using the CRITIC method. The weight of the j th criterion is obtained using Equation (29) presented as:

$$w_{j,CRITIC} = C_j / \sum_{k=1}^m C_k \quad (29)$$

where

$$C_i = \sigma_j \cdot \sum_{k=1}^m (1 - R_{jk}) \quad (30)$$

A higher value of C_j indicates a more considerable amount of information contained in a particular criterion. Therefore, its weight is higher. The CRITIC method assigns greater weight to criteria that have high standard deviations and low correlations with other criteria. A higher C_j value means that more information can be obtained from this criterion, which increases the relative importance of the criterion for the decision problem. The study of correlation effects by the CRITIC method is carried out by calculating the weight $w_{j,CRITIC}$ for the j attribute that requires it due to the anticipated impact of the k attribute, e.g., when it is known that the validity of the attribute “TVOC pollution” in a “quasi-stable state” will increase under the influence of the attribute correlating with “TVOC emission”.

2.8. Calculation of the Objective Weights for Ventilation Energy Expenditure

The objective weights for ventilation energy refer to the combined Σ IAQ model described by polynomial (1), whose words have been assigned weights described by Equation (3), based on the concept of “excess concentration”, creating the energy expenditure of ventilation. These are the relative weights calculated for each alternative of the IAQ model. The values Δc_j for pollutants adopted in the decision matrix differ by several orders. In the IAQ index model decision matrix (Table 1), it is necessary to normalise the set of values of “excess concentrations” for the first three attributes in the Σ IAQ_{quality} system described by Equation (4). The basic problem in data normalisation for determining weights $w_{ij,energy}$ based on the concept of “excess concentration” is the selection of formulas that can be useful for this purpose. Referring to Körth et al. [29] and Vojcica [31], it seems that the following equation can be used to normalise datasets of “type cost”, which after conversion to attributes of “type profit” (or beneficial) adjust for IAQ systems corresponding to subsequent alternatives a1 to a5. Normalised values Δc_j will correspond to “energy load” weights $w_{ij,energy} = n_{ij}$:

$$n_{ij} = \left[1 - \frac{x_{ij}^{\min}}{x_{ij}} \right]^{-1} \quad (31)$$

Taking into account the input data of the decision model summarised in Tables 1 and 5, a dimensionless set of values corresponding to excess concentrations according to (2), normalised for weights, is calculated for the alternatives of this model $w_{ij,energy}$. for non-zero attributes $j = 1, 2, 3$ in the form of CO₂, TVOC and HCHO pollutant concentrations.

2.9. Combining the Weights by Calculation of Global Weights

The global weights $w_{ij,global}$ contain specified weight multiplication products of various types, adjusted to the sum of these products according to the widely used procedure for building weighting development. In the flow chart (Figure 3), the $w_{j,global}$ weights include basic entropy weights $w_{j,entropy}$, subjective weights related to health human toxicity determined using “health-related” databases s_j^{health} and correlation weights $w_{j,CRITIC}$, and ventilation energy weights $w_{ij,energy}$. At the same time, correlation weights occur only in those alternatives (Σ IAQ systems) in which decision matrices contain correlation criteria (this criterion is “pollutant emissions” in Table 1). The weights are assigned to individual attributes $j = 1 \dots 6$ of the decision matrix, and the weighting schemes for five alternative IAQ levels $i = 1 \dots n$ are composed of weights assigned to specific attributes at a given level of alternatives from a1 to a5, which are adjusted to the sum of weights at a given alternative level i . A modified integrated weighting method, presented by Deepa et al. [49] and [15], proposed an approach to combine all weights

calculated using different weight assignment approaches into a single set of weights. The equation to calculate the weights of attributes in alternative levels is

$$w_{ij,global} = \frac{\prod_{g=1}^n w_g}{\sum_{g=1}^n (\prod_{g=1}^n w_g)} \quad (32)$$

where $w_{ij,global}$ represents the combined weights of the criteria, w_g denotes the criteria weights calculated by using one of the weight calculation methods, and $g = 1, 2, \dots, n$, where n is the number of methods used.

If a decision is made to assign a given attribute to the objective weight $w_{j,entropy}$ and the subjective weight associated with the risk to s_j^{health} health, but with adjustment to the sum of weights at a given alternative level i , then the global weight of toxic pollution is

$$w_{ij,global} = \frac{[s_j^{health} \cdot w_{j,entropy}]}{\sum_{j=1}^m [s_j^{health} \cdot w_{j,entropy}]} \quad (33)$$

When the alternative and the attribute are assigned the relative weight $w_{ij,energy}$ and when the alternative indicators are compared to the attributes of type k correlating with, e.g., "pollution emission", then the composite weight of the attribute correlating with the attribute "emission" also includes the weight of the correlation effect $w_{j,CRITIC}$ and weight $w_{j,energy}$ for the given attribute j at the level of the i -th alternative:

$$w_{ij,global} = \frac{[s_j^{health} \cdot w_{j,entropy} \cdot w_{j,CRITIC} \cdot w_{ij,energy}]}{\sum_{j=1}^m [s_j^{health} \cdot w_{j,entropy} \cdot w_{j,CRITIC} \cdot w_{ij,energy}]} \quad (34)$$

The denominators of the formulas for global attribute weights $j = 1 \dots m$ at alternative levels $i = 1 \dots n$ are, therefore, calculated as the sum of the products of entropy weights $w_{j,entropy}$ multiplied by s_j^{health} and $w_{j,CRITIC}$ and other weights, e.g., $w_{j,energy}$ representing the importance of the attribute for the energy load.

It should be noted that the use of the Σ IAQ model (for calculating the total energy expenditure for ventilation and using the global weight value in the IEQ in Equation (7)) requires to determine the global weight values only for the attributes of "pollution concentrations" (in the case of the IAQ_{quality} model, only for $j = 1, 2, 3$) in the form of relative global weights adjusted at all levels. The measures of the significance of the other attributes (emissions) are determined by the CRITIC method and are given by correlation weights that additionally impact the global weights of the "concentration" attributes. Therefore, although entropic weights are calculated just like CRITIC weights for all attributes, taking advantage of the fact that they are weights characteristic for given attributes (they are weights calculated in columns j , not relative weights), they can be "selected" and adjusted in the range of attributes to calculate relative global weights for all levels of alternatives.

2.10. Characteristics of the Case Study Building

In order to determine the weights of the IAQ model, the authors present a case study building and use the real scenarios in indoor environment quality. The case study building is a newly built high tower office with 49 floors and a net area of 59,000 m² made of steel and concrete materials with a heavy glass facade. The building is located in Warsaw. The experimental part (measurement in situ) included analysis of CO₂, HCHO and VOCs indoor concentrations in the building interiors. Provided in [5], datasets on the pollutants supported the authors' decision matrix development including scenarios/alternatives. At the time of the test, the building was in the pre-occupancy phase and had empty spaces with no furniture inside. The indoor walls were plaster covered and freshly painted. Suspended ceilings were mounted, and the floors were covered with a synthetic carpet. The building installations were active during tests. The building was tested three days after the internal

work was finished. Measurements were made on the 47th floor, covering area of almost 3000 m². The main focus was on the IAQ_{index} for selected open spaces in which a large number of users have to reside, and which represent the largest occupied floor space. ISO and CEN international standard based analytical methods were engaged to assess “in situ” HCHO, TVOC and CO₂ concentration values. Their samples for VOCs tests were collected via active sampling with an electronic mass flow controller that controlled the air flow via test probes (10 dm³/h for VOCs and 30 dm³/h for HCHO). Indoor air samples were taken from selected representative office locations and tested later in an accredited laboratory in accordance with ISO 16000-3:2011 and ISO 16000-6:2011. VOCs were sampled on the probe tubes filled with Tenax adsorbent. Later, they were thermally desorbed using a TD-20 device (Shimadzu, Kyoto, Japan). The process of separation and analysis of VOCs was achieved with a GC/MS gas chromatograph equipped with a mass spectrometer (model GCMS-QP2010, Shimadzu, Japan). VOCs were identified by comparing the retention times of chromatographic peaks with the retention times of reference compounds, by searching the NIST mass spectral database (USA). Identified VOC compounds were quantified using a relative identification factor from standard solution calibration curves. TVOC concentration was calculated by summing identified and unidentified VOCs eluting between n-hexadecane and n-hexane.

2.11. Building-Related Research Limitations

Building-related research limitations were also recognized because the Σ IAQ model and provided weighting scheme applied mainly to office buildings with mechanical ventilation. Quasi-static indoor environment conditions and full air mixing in the building were assumed by the authors (perfect mix of indoor air). The IAQ model is based on sensory curves that were created by examining people’s response to a stimulus with an almost neutral influence of other indoor environment factors. This means that the applicability of the presented IAQ model is limited to indoor thermal conditions in the range of 20–26 °C and relative humidity of 30–70%. The valid clothing level of occupants was 0.5–0.8 clo, and metabolic rate accepted was 1–1.2 m. The number of air pollutants as sub-components in the IAQ model was limited to the sub-models presented in [4]. In the example case study building, we analysed building pollution situations based on six attributes from which we built alternative situations (a1–a5) for the building. These attributes are most commonly used in commercial environmental building assessment systems like BREEAM or LEED. The range of applicability of the developed weighing system was valid up to the maximum pollution values: 1800 mg/m³ for Δ CO₂, 1000 µg/m³ for TVOC and 160 µg/m³ for HCHO.

Health risk in the context of human exposure to a given pollutant is an issue usually analysed in occupational medicine research. In this context, we consciously limited our research scope and did not focus directly on human health risk.

3. Development of Σ IAQ_{quality} Model Weighting Schemes for the Case Study Building

3.1. Development of the Initial Decision Matrix Adapted to the Combined Σ IAQ_{quality} Model

This section provides a step-by-step approach for a numerical calculation of combined Σ IAQ_{quality} model sub-element weights presented in the example case study building.

The results of the case study building identified pollutants [5] and were the basis in building a decision matrix. Table 5 represents the initial decision matrix (the input data table) for the defined scenario (fitting of the weighting schemes to various IAQ_{quality} quasi-stable states). The matrix consisted of six attributes (concentrations and emissions of three indoor air pollutants) and five alternatives, where:

a1—alternative based on the case study specifics presented in a previous report in [4,5]. In the building under investigation, with quasi-stable air quality, for the indoor pollutant concentrations of CO₂, TVOC and HCHO, the assumption was made that the additional emissions of these three air pollutants were not measurable;

a2—alternative based on the assumption of minimum air pollution concentration levels changing very little because the emissions rate at the minimum level does not affect the quasi-stable state of IAQ; for the CO₂ bio-pollution rate and the CO₂ concentration minimum level value, the bio-effluents rate from a single person during one hour was accepted [42];

a3—alternative predicting the measured air pollution quasi-stable concentration at a minimum level, but under the influence of CO₂ (e), TVOC (e) and HCHO (e) emission rates, it did so at a higher level (addition under minimum level was within the 0.25 measuring range); the TVOC or HCHO minimum levels of emission rates were calculated as 0.25 of the (max–min) range estimated by the literature on the pollution emission rate;

a4—alternative is similar to a3, but the higher influence of the emission rate of CO₂ (e) does not exist (there is no addition under the minimum level of CO₂ emission rate);

a5—alternative is similar to a3, but the higher influence of emission rates of TVOC and HCHO does not exist (there is no addition under the minimum level of TVOC and HCHO emission rates).

Table 5. Initial decision matrix with the data assigned to various IAQ_{quality} component sets and action scenarios presented by individual alternatives.

Alternatives	Attributes					
	Δ CO ₂ ^{(1),(2)}	TVOC	HCHO	CO ₂ e ⁽²⁾	TVOC e ⁽³⁾	HCHO e ⁽⁴⁾
	ppm mg/m ³	µg/m ³	µg/m ³	mg/m ³ h	µg/m ³ h	µg/m ² h
a1 test study [5] (e) emission rate = 0	450 ppm 810 mg/m ³	787	18	0	0	0
a2 minimum [3] (e) minimum TVOC and HCHO; (e) minimum CO ₂ one person	360 + 16.2 = 376.2 mg/m ³	154 + 13.6 = 167.6	5.8 + 4.26 = 10.6	16.2 one person	13.6	4.26
a2-2 test study [5] (e) minimum TVOC and HCHO; (e) minimum CO ₂ one person	450 ppm 810 mg/m ³	787	18	16.2 one person	13.6	4.26
a3 minimum + 1h (e) minimum + 25% concentration (e) CO ₂ two persons ²⁾	200ppm 360 + 32.4 = 392.4 mg/m ³	154 + 24.3 = 178.3	5.8 + 20.2 = 26.0	32.4 two persons	10.8 + 13.6 = 24.3	15.9 + 4.26 = 20.2
a5 minimum + 1h (e) minimum TVOC, HCHO; (e) CO ₂ minimum + 25% concentration two persons	200ppm 360 + 32.4 = 392.4 mg/m ³	167.6	10.6	32.4 two persons	13.6	4.26
Maximum x _{ij} level of ventilation load (lit.)	1000 ppm 1800 mg/m ³	1000	160	64.8 mg/m ³ h four persons	57.1	68.2
Maximum x _{ij} value in attribute j column	810	787	26.0	32.4	24.3	20.2
Minimum x _{ij} level of ventilation load (lit.)	200 ppm 360 mg/m ³	154	5.8 10.6	16.2 mg/m ³ hone person	13.6	4.26
Minimum x _{ij} value in attribute j column	376.2	167.6	10.6	0	0	0

⁽¹⁾ General conversion of units, e.g., CO₂ from ppm to mg/m³ in accordance with [50]. ⁽²⁾ Assumptions for the attribute of CO₂ concentration—(1) minimum 1 person max. 4 people; (2) in a room of 20 m³ per person; (3) the CO₂ concentration caused by occupant exhalation is approximately 400–1700 ppmv, while the average volume is 20–150 m³·person⁻¹ and the air change ACH is 0.45–0.70 h⁻¹ [42]. The time for indoor concentration of CO₂ to reach a quasi-steady state is between 6 and 8 h approximately; (4) the mass balance method [42] can be used to link indoor CO₂ emission rate concentration, as shown in equation $m = p \cdot c \cdot Q$, where m is the emission rate of the target compound, µg s⁻¹, p is occupant number, c is the CO₂ concentration in occupant exhaled air (400 ppmv [42]), µg m⁻³, and Q is the respiratory rate of humans, m³ s⁻¹, and is assumed as 7.5 l min⁻¹ (1.25×10^{-4} m³ s⁻¹). ⁽³⁾ TVOC—Hori [38] study ACH is ca 0.5 h⁻¹. ⁽⁴⁾ HCHO—[35] ACH is set to 0.5 h⁻¹.

The set of alternatives is used for checking how the weighting scheme changes when the emission rate of a particular air pollutant is absent or increasing.

Simultaneously, the authors calculated all weights for the additional alternative version a2-2, which differs from alternative a2. This is based on the assumption of air pollution concentration levels, taken from the case study, practically not changing the quasi-stable state because the emissions rate is at a minimum level, like in alternative a2. A minimum level for the CO₂ bio-pollution rate, and TVOC (e) and HCHO (e) minimum level emission rates were established. It can, therefore, be argued that the weighting scheme for the IEQ model with the a2-2 alternative will have a screen emission (more probable than for the a2 alternative). The decision model and its weighting schemes with alternative a2-2 and the same set of attributes have different structures than alternative a2. However, as the values of indoor pollution concentrations are the same in this study (with the assumption of minimum levels of emissions alone), version a2-2 can be used for comparing the impact of the two alternatives a2 and a2-2. Maximum x_{ij} levels of ventilation load are based on the research [2,7,32,35,38,42] and Minimum x_{ij} levels by [2,35,38,39,42].

3.2. Attribute Weights Obtained by the Entropy Method

Determination of objective attribute weights (only for attributes 1, 2 and 3, named “pollution concentration” sub-indices in IAQ_{quality}) from Equation (4), calculated in attribute columns by all levels of alternatives belonging to the decision matrix in accordance to the entropy approach, is based on uncertain information measurements contained in the decision matrix and directly provides a set of weights for criteria based on mutual individual criteria values contrasted with variants for each criterion alone and then for all criteria at the same moment. The following steps are suggested in this direction.

Step 1. Normalisation of indicators for the entropy method—since the indicators of air pollutants are in different units, and since the attributes are of a non-beneficial type, the most recommended method to normalise them is used first. In particular, the transformation method according to Equation (34) solves this issue:

$$z_j = \frac{y_{ij}}{\sum_{i=1}^n y_{ij}} = \frac{1/x_{ij}}{\sum_{i=1}^n 1/x_{ij}} \quad (35)$$

Table A1 (in Appendix A) presents the normalised decision matrix (according to Equation (35)) with dimensionless numbers z_j representing the normalised response of alternative i on attribute j . The structure of the decision matrix contains the input parameters of attribute $j = 1, 2, 3$ determined on the assumption of an indoor quasi-stable state environment with minimum level of air pollution concentration and minimum emission rates.

Table 6a presents the shortage of normalised decision matrices (according to Equation (35)). The structure of alternative a2-2 (presented in Table 6b) contains the input parameters of attribute $j = 1, 2, 3$ based on the current experiment [5] with the assumption of an indoor quasi-stable state environment with the experimentally determined level of air pollution concentration and with minimum emission rates.

Step 2. The entropy measure calculation. The measure of the j th attribute under indicator i (i ranges from 1 to n) is calculated using the following Equation (36):

$$e_j = -k \cdot \sum_{i=1}^n z_{ij} \ln z_{ij} = -\frac{1}{\ln n} \sum_{i=1}^n z_{ij} \ln z_{ij} \quad (36)$$

where constant $k = 1/\ln n$ guarantees that e_j ($j = 1, 2, \dots, n$) belongs to the interval $[0;1]$. A constant k in the matrix with five alternatives can be calculated by Equation (37):

$$k = \frac{1}{\ln(n)} = \frac{1}{\ln(5)} = 0.621335 \quad (37)$$

Table 6. Normalised decision matrices with dimensionless numbers z_j for (a) alternative a2 based on the minimum level of air pollution in a quasi-stable state with minimum emission rates for (b) alternative a2-2 based on air pollution values from the case study in a quasi-stable state with emissions rates at a minimum.

Alternatives	Attribute X = 1 ... m					
	ΔCO_2	TVOC	HCHO	CO_2 (e)	TVOC (e)	HCHO (e)
(a)						
a1	0.1043	0.0531	0.175	0	0	0
a2	0.2304	0.2449	0.2937	0.3333	0.3201	0.4134
a3	0.2174	0.2286	0.1187	0.1666	0.1798	0.0866
a4	0.2304	0.2286	0.1187	0.3333	0.1798	0.0866
a5	0.2174	0.2449	0.2937	0.1666	0.3201	0.4134
(b)						
a1	0.1188	0.0656	0.1986	0	0	0
a2	0.1188	0.0656	0.1986	0.3333	0.3201	0.4134
a3	0.2475	0.2828	0.1347	0.1666	0.1798	0.0866
a4	0.2624	0.2828	0.1347	0.3333	0.1798	0.0866
a5	0.2475	0.3030	0.3333	0.1666	0.3201	0.4134

Therefore, it is possible to calculate the value of k for all attributes $j = 1 \dots 6$ and alternatives $n = 5$. With the help of Table 6a,b, it was possible to calculate e_j , d_j and entropy weights of criteria values $w_{j,entropy}$, which are presented for Variant 1 with alternative a2 and Variant 2 with a2-2 in Table 7. The degree of divergence (d_j), where d_j ($j = 1, 2 \dots, m$), is the inherent intensity of criteria contrast C_j . The value (d_j) of the average intrinsic information contained in each criterion is calculated as Equation (38):

$$d_j = 1 - e_j \quad (38)$$

Step 3. Calculation of the entropy weights of attributes

Next, the entropy weights of each j are calculated as follows. The final criteria weight, in the third step of the approach, may be calculated by an additive normalisation (39):

$$w_{j,entropy} = (d_j) / \sum (d_j) \quad (39)$$

Table 7 presents the results of calculations using Equations (38) and (39).

Table 7. Entropy, degree of divergence and the relative weights of attribute j .

Attributes	ΔCO_2	TVOC	HCHO
Decision matrix with alternative a2			
Entropy e_j	0.9790	0.9444	0.9509
Divergence d_j	0.0210	0.0556	0.0491
Relative weight $w_{j,entropy}$	0.16706	0.44232	0.39061
Decision matrix with alternative a2-2			
Entropy e_j	0.9620	0.8907	0.9619
Divergence d_j	0.0381	0.1093	0.0380
Relative weight $w_{j,entropy}$	0.2055	0.5895	0.20496

3.3. Attribute Weights by the CRITIC Method

Step 1. Normalisation of indicators for the CRITIC method

For each criterion j and indicator x_{ij} , membership function r_{ij} , which transforms the values of attributes into the interval $[0,1]$, is provided(40):

$$r_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \tag{40}$$

and the transformer version of formulas is [30,31]:

$$r_{ij} = \frac{x_{ij} - \max x_{ij}}{\min x_{ij} - \max x_{ij}} \tag{41}$$

The transformation considers the concept of an ideal point. With Equation (41), the initial matrix is converted into the normalised decision matrix with the generic elements r_{ij} (Table 8a,b).

Table 8. Normalised decision matrices. (a) (with alternative a2) based on minimum air pollution levels in a quasi-stable state with minimum emission rates; with zero-dimensional generic elements r_{ij} . (b) (with alternative a2-2) based on air pollution test study levels in a quasi-stable state with minimum emission rates and zero-dimensional generic elements r_{ij} .

Alternatives	ΔCO_2	TVOC	HCHO	CO_2 e	TVOC e	HCHO e
(a)						
σ_j	0.393	0.394	0.447	0.374	0.369	0.428
a1	0	0	0.519	1	1	1
a2	1	1	1	0.5	0.440	0.789
a3	0.963	0.967	0	0	0	0
a4	1	0.967	0	0.5	0	0
a5	0.963	1	1	0	0.440	0.789
(b)						
σ_j	0.478	0.479	0.376	0.374	0.369	0.428
a1	0	0	0.519	1	1	1
a2	0	0	0.519	0.5	0.440	0.789
a3	0.963	0.967	0	0	0	0
a4	1	0.967	0	0.5	0	0
a5	0.963	1	1	0	0.440	0.789

Table A2 (located in Appendix A) presents the normalised decision matrix Equation (35) with dimensionless indicators r_{ij} representing the normalised response of alternative i on attribute j .

Step 2. Calculation of symmetric linear correlation matrix R_{jk} based on all indicators of normalised decision matrices (Table 9) and shortages of symmetric linear correlation matrices R_{jk} with alternatives a2 (Table 9a) and a2-2 (Table 9b).

Step 3. Determining criteria weights for “pollutant concentration” attributes using the CRITIC method.

The amount of information necessary to calculate C_j (42) includes measures of conflict C_j determined with Equations (30) or (42). Attribute (criteria) weights $w_{j,CRITIC}$ were obtained from Equation (43) (Table 10).

Table 9. Symmetric linear correlation matrix R_{ij} (with alternative a2) calculated for all correlation possibilities. (a) Shortage of symmetric linear correlation matrix R_{ij} (with alternative a2) calculated for all correlation possibilities (in this shape of the matrix, it can be assumed that only six attributes are burdened with additional correlation weights, but in the recognised attribute set only the first three will be used). (b) Shortage of symmetric linear correlation matrix R_{ij} (with alternative a2-2) calculated for all correlation possibilities.

Attributes	ΔCO_2	TVOC	HCHO	CO_2 e	TVOC e	HCHO e
ΔCO_2	1.0000	0.9984	-0.0170	-0.7759	-0.8451	-0.5654
TVOC	0.9984	1.0000	0.0205	-0.8012	-0.8252	-0.5346
HCHO	-0.0170	0.0205	1.0000	0.0136	0.5477	0.8340
CO_2 e	-0.7759	0.0136	0.0136	1.0000	0.6782	0.4537
TVOC e	-0.8451	-0.8252	0.5477	0.6782	1.0000	0.9185
HCHO e	-0.5654	-0.5346	0.8340	0.4537	0.9185	1.0000
(a)						
ΔCO_2	1			-0.775907		
TVOC		1			-0.8252373	
HCHO			1			0.83395897
CO_2 e	-0.775907			1		
TVOC e		-0.825237			1	
HCHO e			0.833959			1
(b)						
ΔCO_2	1			-0.7496659		
TVOC		1			-0.750258	
HCHO			1			0.8279349
CO_2 e	-0.7496659			1		
TVOC e		-0.750258			1	
HCHO e			0.8279349			1

Table 10. Results of the CRITIC method application for decision matrix with alternatives a2 and a2-2.

Attributes	$(1 - R_{jk})$		
	CO_2	TVOC	HCHO
Standard deviation	0.393	0.394	0.447
CO_2	0		
TVOC		0	
HCHO			0
CO_2 e	$1 + 0.7759 = 1.7759$		
TVOC e		$1 + 0.8252 = 1.8252$	
HCHO e			$1 - 0.83396 = 0.16604$
$\text{SUM}_{\text{attribute}}$	1.7759	1.8252	0.16604
C_j —with Equation (42)	0.6979	0.7191	0.0742
$\text{SUM } C_j$	1.4912 (for three attributes) (100%)		
$w_{j,\text{CRITIC}}$ —with Equation (43)	and with alternative a2 (Table 9a)		
	0.46801	0.48223	0.04976
	and with alternative a2-2 (Table 9b)		
$w_{j,\text{CRITIC}}$ —with Equation (43)	0.48083	0.48198	0.0372

A conflict measure created by criterion j due to the decision defined by the rest of the criteria can be calculated, which determines a quantity of the information in relation to each criterion:

$$C_j = \sigma_j \sum_{k=1}^m (1 - R_{jk}) \tag{42}$$

Objective CRITIC criteria weights for a chosen attribute are obtained by normalising (adjusting) the values of C_j and, in particular, alternative levels (43):

$$w_{j,CRITIC} = C_j / \sum_{k=1}^m C_k \quad j = 1, \dots, m \tag{43}$$

3.4. Relative Weights Obtained by Normalisation of the “Excess Concentration”

Taking into account the input data of the decision model compiled in the input decision matrix with alternatives (Table 5), alternatives a1 to a5 (with values of excess concentrations Δc_j according to (2) [4]) were calculated with weights $w_{ij,energy}$ for non-zero attributes $j = 1, 2, 3$. In order to determine the “energy load” weight, the following steps are recommended.

Step 1. Assuming that the minimum concentration value c_{ref} in the definition of “excess concentration” given in Equation (2) may be the minimum value $\min(x_{ij})$, then the formula for normalisation (43) can be determined, based on the data for the alternative a1 obtained in real case studies [5] and data from the literature [2,51,52] regarding the minimum values of pollutants (TVOC and HCHO) occurring in a building. Justification for the manner of assuming the minimum and maximum values of pollution in a building with CO₂ bio-pollution is discussed in the notes of Table 5.

Step 2. Normalisation of weights (energy load) $w_{ij,energy}$ using Equation (9) transformed to (44) for attributes of type “cost”. According to [29,31], it seems that the equation can be used to normalise non-beneficial datasets, which after conversion to profit (or beneficial) attributes can be adjusted for IAQ systems corresponding to subsequent alternatives from a1 to a5. Transformation of the excess concentration values, which correspond to the indicators x_{ij} at the levels of subsequent alternatives a1 to a5 taken from Table 5, gives the weight of “energy load” $w_{ij,energy}$:

$$w_{ij,energy} = \left[1 - \frac{x_{ij}^{min}}{x_{ij}} \right]^{-1} \tag{44}$$

Normalisation of weights based on the concept of “excess concentration”—correct application of weights of Equation (44) is possible on the assumption that the $\min(x_{ij})$ value of indoor pollution determined on the basis of insitu measurements and given in the input decision matrix (Table 5) corresponds to the standard value c_{ref} from Equations (2) and (3).

Step 3. Calculation of the relative values of “energy load” weights assigned to attributes of CO₂, TVOC and HCHO pollutant indoor concentrations (Table 11).

Table 11. Normalisation of the energy load relative weights calculated with Equation (44).

Alternatives	ΔCO_2	TVOC	HCHO
alternative a1			
x_{ij}	810 mg/m ³	787 µg/m ³	18 µg/m ³
$\min(x_{ij})$	360 mg/m ³	154 µg/m ³	5.8 µg/m ³
$w_{ij,energy}$	0.3984	0.2744	0.3265
alternative a2			
x_{ij}	376.2 mg/m ³	167.6 µg/m ³	10.6 µg/m ³
$\min(x_{ij})$	360 mg/m ³	154 µg/m ³	5.8 µg/m ³
$w_{ij,energy}$	0.6148	0.3357	0.0585

Table 11. Cont.

Alternatives	ΔCO_2	TVOC	HCHO
alternative a3			
x_{ij}	392.4 mg/m ³	178.3 $\mu\text{g}/\text{m}^3$	26.0 $\mu\text{g}/\text{m}^3$
$\min(x_{ij})$	360 mg/m ³	154 $\mu\text{g}/\text{m}^3$	5.8 $\mu\text{g}/\text{m}^3$
$w_{ij,energy}$	0.5841	0.3538	0.0621
alternative a4			
x_{ij}	376.2 mg/m ³	178.3 $\mu\text{g}/\text{m}^3$	26.0 $\mu\text{g}/\text{m}^3$
$\min(x_{ij})$	360 mg/m ³	154 $\mu\text{g}/\text{m}^3$	5.8 $\mu\text{g}/\text{m}^3$
$w_{ij,energy}$	0.7290	0.2305	0.0404
alternative a5			
x_{ij}	392.4 mg/m ³	167.6 $\mu\text{g}/\text{m}^3$	10.6 $\mu\text{g}/\text{m}^3$
$\min(x_{ij})$	360 mg/m ³	154 $\mu\text{g}/\text{m}^3$	5.8 $\mu\text{g}/\text{m}^3$
$w_{ij,energy}$	0.454567	0.462546	0.082887

4. Results

4.1. Global Weights Calculated for the IAQ_{quality} Model Alternatives

The final step of the weight development method in Section 3 is to aggregate the calculated set of “component” weights and obtain the resultant global weights. Table 12 shows the determination of global weights calculated according to Equation (33), when the attributes do not take part in the emission process (attribute type j), and Equation (34) when attributes do take part in the emission process (attribute type k). Weight values adequate for the numerical example are taken for the set of alternatives a2 and a2-2.

Table 12. Relative global weights in the weighting schemes calculated for five alternatives of the IAQ model (a1 to a5); global weights obtained with Equation (32).

	Attributes (Pollution Concentration Levels)					
	ΔCO_2	TVOC	HCHO	ΔCO_2	TVOC	HCHO
	(alternative a1) test study [5] concentration level without emission			(alternative a2) minimum concentration level with (e) minimum emission rate levels		
$w_{j,entropy}$	0.16706	0.44232	0.39061	0.16706	0.44232	0.39061
s_j^{health}	0.01556	0.21	0.21	0.01556	0.21	0.21
$w_{j,CRITIC}$	–	–	–	0.46801	0.48223	0.04976
$w_{ij,energy}$	0.3984	0.2744	0.3265	0.6148	0.3357	0.0585
$w_{j,entropy} \times s_j^{health} \times w_{j,CRITIC} \times w_{ij,energy}$	0.00103562	0.025488	0.026782	0.000747946	0.0150370	0.00023878
$w_{ij,global}$	0.0194279	0.478148	0.5024236	0.0466775	0.938422	0.0149017
	(alternative a2-2) test study [5,8] concentration level with minimum emission rate levels			(alternative a3) minimum concentration level; (e) minimum +25% range of emission rate		
$w_{j,entropy}$	0.2055	0.5895	0.20496	0.16706	0.44232	0.39061
s_j^{health}	0.01556	0.21	0.21	0.01556	0.21	0.21
$w_{j,CRITIC}$	0.48083	0.48198	0.0372	0.46801	0.48223	0.04976
$w_{ij,energy}$	0.3984	0.2744	0.3265	0.5841	0.3538	0.0621
$w_{j,entropy} \times s_j^{health} \times w_{j,CRITIC} \times w_{ij,energy}$	0.000612536	0.0163725	0.0005227	0.0007106	0.0158478	0.00025347
$w_{ij,global}$	0.0349865	0.935155	0.0298593	0.0422678	0.9426554	0.0150769
	(alternative a4) minimum concentration level; (e) minimum +25% range of TVOC and HCHO emission rate			(alternative a5) minimum concentration level; (e) minimum +25% range of CO ₂ emission rate		
$w_{j,entropy}$	0.16706	0.44232	0.39061	0.16706	0.44232	0.39061
s_j^{health}	0.01556	0.21	0.21	0.01556	0.21	0.21
$w_{j,CRITIC}$	0.46801	0.48223	0.04976	0.46801	0.48223	0.04976
$w_{ij,energy}$	0.7290	0.2305	0.0404	0.5841	0.3538	0.0621
$w_{j,entropy} \times s_j^{health} \times w_{j,CRITIC} \times w_{ij,energy}$	0.000886878	0.010324785	0.000164901	0.000710597	0.01584780	0.000251997
$w_{ij,global}$	0.07795658	0.9075486	0.014494798	0.0422712976	0.942738165	0.014990546

4.1.1. The Results of the New Weighting Method Applied on IAQ_{index} and IEQ_{index} Results for the Case Study Building

A numerical example of the combined \sum IAQ_{quality} model with the new weighting scheme, developed according to Figure 3, is presented in Table 13 in the context of the overall Indoor Environmental Quality (IEQ) assessment of the case study office building (IEQ is the predicted percentage of users satisfied with all components of indoor environmental quality). The input indoor parameters for the case study (in order to assess IEQ sub-components) are taken from previous papers [4,5], where the measured pollutant concentrations were $c_{CO_2} = 450$ ppm, $c_{TVOC} = 787$ $\mu\text{g}/\text{m}^3$ and $c_{HCHO} = 18$ $\mu\text{g}/\text{m}^3$ in the case study building a week after completion of the finishing works.

Table 13. The physical parameters representing the indoor environment of the case study and the IEQ_{index} results calculated from Equation (7) with sub-component \sum IAQ calculations (old approach and new); assuming realistic uncertainty of measurements for a case study (old and new weight systems applied and compared).

Sub-Index	IEQ Sub-Component Models	Input Values	Sub-Index (Satisfied Users) \pm SD
Thermal comfort TC _{index}	PMV (Fanger and ISO 7730) PMV = f(M, I _{cl,dyn} , t _a , t _r , v _a , P _{a,r}) PD _{TC} = f(PMV)	I _{cl} = 0.55 clo t _a = 24 °C t _r = 24.5 °C v _a = 0.15 m/s RH = 45% M = 1.1 met	90.0% \pm 3.2%
\sum IAQ _{index} sub-indices	PD _{IAQ(CO2)} = 395·exp(−15.15·C _{CO2} ^{−0.25}) PD _{IAQ(TVOC)} = 405·exp(−11.3·C _{TVOC} ^{−0.25}) PMV _{HCHO} = 2·log(C _{HCHO} /0.01) PD _{HCHO} = 100−95·exp(−0.03353·PMV ⁴ − −0.2179·PMV ²)	c = 450 ppm c = 787 $\mu\text{g}/\text{m}^3$ c = 18 $\mu\text{g}/\text{m}^3$	85.2% \pm 0.6% 52.0% \pm 18.0% 65.8% \pm 10.7%
\sum IAQ _{index} (new) without emission	\sum IAQ _{index} = W ₁ ·(CO ₂) + W ₂ ·(TVOC) + W ₃ ·(HCHO) = 0.0194279·(85.2) + 0.478148·(52.0) + 0.5024236·(65.8) = = 1.65525708 + 24.863696 + 33.05947288 = 59.58		59.58% \pm 10.2%
\sum IAQ _{index} (new) with emission on minimum level	\sum IAQ _{index} = W ₁ ·(CO ₂) + W ₂ ·TVOC) + W ₃ ·(HCHO) = = 0.0349865·(85.2) + 0.935155·(52.0) + 0.0298593·(65.8) = = 2.980885 + 48.62806 + 1.96474194 = 53.57		53.57% \pm 16.8%
\sum IAQ _{index} (old)	IAQ _{VOC} = 0.96·IAQ(TVOC)+0.04·IAQ(HCHO) \sum IAQ _{index} = 0.5·IAQ(CO ₂)+0.5·IAQ(VOC)		53.0% \pm 17.3% 69.1% \pm 9.0%
Acoustic comfort ACC _{index}	PD _{ACC} = 2·(Actual _{Sound_Pressure_Level} (dB(A)− Design _{Sound_Pressure_Level} (dB(A)) Measured actual sound level Designed sound level	55 dB(A) 45 dB(A)	80.0% \pm 6.7%
Daylight L _{index}	PD _L = −0.0175 + 1.0361/(1 + exp(+4.0835·(log ₁₀ (E _{min})−1.8223)))	455 lux	98.4% \pm 9.0%
\sum IEQ _{index} \pm SD (new)	W ₁ ·TC _{index} + W ₂ · \sum IAQ _{index} + W ₃ ·ACC _{index} + W ₄ ·L _{index}		
no emission	0.25·90.0 + 0.25·59.6 + 0.25·80.0 + 0.25·98.4 = 82.0%		82.0% \pm 3.9%
with emission	0.25·90.0 + 0.25·53.6 + 0.25·80.0 + 0.25·98.4 = 80.5%		80.0% \pm 5.12%
\sum IEQ _{index} \pm SD (old)	IEQ _{index} = W ₁ ·TC _{index} + W ₂ · \sum IAQ _{index} + W ₃ ·ACC _{index} + W ₄ ·L _{index}		84.4% \pm 3.7%

The uncertainty of user dissatisfaction with pollutant concentration is determined by a combination of two groups of components. The first measurement uncertainty is related to the real uncertainty of parameters biases when there are measured in indoor environments. The second group concerns the uncertainty of votes in the sensory tests (reliability of responses to a given stimulus of the indoor environment are discussed in literature). In practice, a metrological analysis of the IEQ model (where \sum IAQ index is the sub-model) was provided in our previous paper [8] considering provisions of the

international metrological guideline: ISO BIPM JCGM 100:2009 Guide to the expression of uncertainty in measurement. For all other possible scenarios, the uncertainty for determining the Σ IAQ index would be up to $\pm 10\%$ to $\pm 30\%$ (depending on each pollution model uncertainty assessment and the number of IAQ model components taken into account).

5. Discussion

5.1. New Weighting Method Applied on IAQ_{index} and IEQ_{index} Estimation for a Case Study Building and Comparison of Its Results with Old Crude Weighting System Results

The previous approach by their weighting scheme (Table 13) was based on the treatment of all main pollutants in a uniform way. The weights were previously set as equal (for pollutants with disproportionately different concentrations) or as dependent only on the energy expenditure on ventilation (for pollutants with similar concentrations). The new IAQ and IEQ index results based on MDMA-developed weights were compared to the old method. The system of objectified weights used for a numerical example of IAQ and IEQ building assessment leads to important conclusions. As expected, the use of objectified weights means that the results of the IAQ and IEQ indices were different than those with subjective weights. For example, at low CO_2 indoor concentrations, the role of this parameter in the IAQ_{index} assessment was overstated. This is the case for most BMS systems controlling building ventilation, where currently the number of air exchanges (ACH) depends mainly on CO_2 concentration measurements. At relatively low CO_2 air concentrations < 1500 ppm, the CO_2 priority compared to HCHO or TVOC attributes is much lower. The cumulative weight of the HCHO and TVOC arguments compared to the “old” IAQ calculation method increased by 48%. This is important from the point of view of planning and managing ventilation systems. According to calculations, the number of users satisfied with the level of CO_2 (85% satisfied) was a good result. Nevertheless, at the same time, the theoretical number of users satisfied with TVOC concentration was only 52%. With novel objective weights for the TVOC argument, the number of users satisfied with IAQ will decrease. In the case of the present study and the new weight system, the value of IAQ_{index} decreased about 10% compared to previous calculations using the old method. Further, if one considers the existence of additional emissions from the building, finishing materials and furniture, the value of satisfied users will decrease by another 5.5%.

The authors strongly believe that the new approach to weights gives objective and more adequate results for the IAQ and IEQ models. Wider use of the new weighting method can be effective upon supplementing databases with the subjective weights of health risks for various volatile substances in the air.

5.2. The Weighting Problem in Indoor Air Quality Component Sets Framework

The weights in any decision-making model should not be arbitrary [53]. Some researchers still use weights in modelling without justification. A solution to the weight problem in indoor air quality modelling is proposed. Global weights are not only measures of the importance of attributes in the human–indoor building environment relationship, health risks and possible disturbances of the quasi-stable state due to correlations between attributes, but they are also measures of the potential load of ventilation impacted by emissions and excess concentrations of additional costs for energy use. In this context, the adoption of objectified weights in the IAQ model (also any other model, where the proposed method can be transferred) has significant practical contribution. It is suggested, knowing the specific air pollution values (attributes) for the analyzed building, to calculate the weights for a new alternative created by the user in accordance with the method proposed in article. For less insightful research, the authors suggest using a set of weights developed by us (Table 12) for scenarios/alternatives that are most similar to the scenario under consideration. According to the adopted MADM method for each analyzed alternative (a1–a5 scenarios for the indoor environment described by concentration/emission levels for attributes or types of pollutants adopted in the IAQ

model), global weights for each attribute (CO₂ or TVOC or HCHO concentration) are calculated separately as the numerical coefficients of Equations (4) or (6). This is because the component weights (e.g., CRITIC correlation weight) of global weights depend on the values of the attributes (air pollution concentrations and emissions). Table 12 shows the calculated global weights $w_{ij,global}$ for each attribute for each alternative a1–a5. For alternative a1 (measured concentrations in the case study building with an assumption of no additional emissions from construction products and anthropogenic CO₂), global weights of the IAQ_{index} model are 0.0194279 for CO₂, 0.478148 for TVOC and 0.5024236 for HCHO. For alternative a2 (minimum concentration levels with minimum emission rates), global weights are 0.0466775 for CO₂, 0.938422 for TVOC and 0.0149017 for HCHO. For alternative a2-2 (case study result concentrations with minimum emission rates), global weights are 0.0349865 for CO₂, 0.935155 for TVOC and 0.0298593 for HCHO. For alternative a3 (minimum concentration level; (e) minimum + 25% range of emission rate), global weights are 0.0422678 for CO₂, 0.9426554 for TVOC and 0.0150769 for HCHO. For alternative a4 (minimum concentration level; (e) minimum +25% range of TVOC and HCHO emission rates), global weights are 0.07795658 for CO₂, 0.9075486 for TVOC and 0.014494798 for HCHO. For alternative a5 (minimum concentration level; emission minimum +25% range of CO₂ emission rate), global weights are 0.0422712976 for CO₂, 0.942738165 for TVOC and 0.014990546 for HCHO. The obtained global weight results clearly indicate which pollutant attributes have priority. In ranges up to about 1500 ppm CO₂ in indoor air, the weight of this attribute in the IAQ model is very small. This means that the ventilation system should take into account, in this respect, the removal of excess TVOC concentration first and, in the second place, excess HCHO. For all alternatives, the weight of the TVOC attribute is significant (for a2–a5, weight is over 90%). The weights for TVOC and HCHO are significantly higher than for CO₂ (one order of magnitude). A 25% increase in CO₂ emissions will not increase its weight in the model.

In this article, the authors have described in detail a procedure for the subjective and objective weight determination for a combined Σ IAQ model, which is composed of the following basic steps:

- i. Development of an initial decision matrix adapted to the type of the combined Σ IAQ model;
- ii. Normalisation of attribute data including different indoor air pollutant concentrations and their emission rates;
- iii. Calculation of the objective attribute weights by the entropy method;
- iv. Calculation or adoption of subjective health-connected weights from the database;
- v. Use of the CRITIC (Criteria Importance Through Inter-criteria Correlation) method in the case of mutually correlating attributes;
- vi. Calculation of objective weights for ventilation energy expenditure to various IAQ sub-components (pollutants);
- vii. Calculation of relative global weights for all significant air pollutants of the Σ IAQ model, considered in various schemes (various alternatives in the framework of decision model).

In our case study building (Table 13), the IAQ model determines global weight values for the attributes of “pollution concentrations” in the form of relative global weights adjusted at all alternative levels. The measures of the significance of the other attributes (additional emissions) are determined by the CRITIC method and are given by correlation weights that additionally impact the global weights of the “concentration” attributes. Therefore, although entropy weights are calculated just like CRITIC weights for all attributes, taking advantage of the fact that they are weights characteristic for given attributes, they can be “selected” and adjusted in the range of the first “concentration” attributes in order to calculate relative global weights for all levels of alternatives.

Thus, the determined global weights are valid within the IAQ_{quality} model (4) treated as a sub-expression in the IEQ equation model described by Equation (7). It seems that the approach presented in this study may easily be replicated by other similar IAQ models.

In our future research, the authors will try to unambiguously link the proposed IAQ model and weight system with the demand for ventilation air for various alternatives and the ventilation systems used in case study buildings.

6. Conclusions

Currently, the issue of comfort assessment, based on user satisfaction models, is discussed in numerous research works, but in most of them indoor comfort is analyzed in terms of thermal comfort only [54–56]. Fortunately, the recently published European standard EN 16798-1: 2018 [7] included a wider number of indoor comfort criteria. This standard provides an approach to the classification of IAQ and IEQ models and supports certification of buildings taking into account specific components of the environmental comfort (IEQ sub-models) including Σ IAQ (which is also composed of sub-components in functions of pollutant concentrations). This standard and any related research does not provide practical guidance on how to combine Σ IAQ model components into one indicator that could be used to classify indoor air conditions and does not provide information of how to aggregate components by an objective weighting system. With the research challenge in mind, the authors presented a relevant IAQ model in [4]. In this paper, the authors provided a step-by-step procedure for determining the combined weighting scheme for the IAQ index equation, using the MADM decision model including calculation of the objective attribute weights by the entropy method, calculation or adoption of subjective health-connected weights from the reference database, use of the CRITIC method in the case of mutually correlating attributes and calculation of objective weights for ventilation energy expenditure to various IAQ sub-components (pollutants). The assumption of the decision model is adequate for the task of rating indoor air ventilation to determine the assumed air quality (our actual work), and setting IAQ model alternatives so that they can meet the criteria taking into account their importance ranking and possibility of ongoing diagnostics of alternatives connected to the perceived actual indoor air situation. The presented weighting scheme approach for the IAQ model may be regarded as objective, and it generates weighted criteria values (for each analysed attribute and indoor scenario) directly from the variations in criteria values. The provided method eliminates the problems of IAQ model weights with using the subjective point of view as well as the incompetence or absence of a responsible decision-maker. The MADM approach, as shown in this case study, is based on simple mathematics and can, therefore, be used effectively for choosing the best IEQ and ventilation strategies.

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Appendix A

Table A1. Normalised decision matrix for the entropy method based on Equation (35).

Alternatives	Attribute X = 1 ... m					
	ΔCO_2	TVOC	HCHO	CO_2 (e)	TVOC (e)	HCHO (e)
	mg/m^3	$\mu\text{g/m}^3$	$\mu\text{g/m}^3$	$\text{mg/m}^3\cdot\text{h}$	$\mu\text{g/m}^3\text{h}$	$\mu\text{g/m}^2\text{h}$
a1 test study [5] +(e) emission rate = 0	810 mg/m^3 $1/x_i = 0.0012$ $z_{ij} = 0.1043$	787 $\mu\text{g/m}^3$ $1/x = 0.0013$ $z_{ij} = 0.0531$	18 $\mu\text{g/m}^3$ $1/x_{ij} = 0.056$ $z_{ij} = 0.175$	0	0	0
a2 minimum; (e) minimum TVOC and HCHO emission +1h; (e) minimum CO_2 one person	376.2 mg/m^3 $1/x_{ij} = 0.00265$ $z_{ij} = 0.2304$	167.6 $\mu\text{g/m}^3$ $1/x = 0.0060$ $z_{ij} = 0.2449$	10.6 g/m^3 $1/x_{ij} = 0.094$ $z_{ij} = 0.2937$	16.2 $\text{mg/m}^3\text{h}$ $1/x_{ij} = 0.062$ $z_{ij} = 0.3333$	13.6 $\mu\text{g/m}^3\text{h}$ $1/x_{ij} = 0.073$ $z_{ij} = 0.3201$	4.26 $\mu\text{g/m}^2\text{h}$ $1/x_{ij} = 0.234$ $z_{ij} = 0.4134$
a3 minimum+1h; (e) minimum TVOC HCHO +25% concentration range (e) minimum CO_2 +25% concentration two persons ²⁾	392.4 mg/m^3 $1/x = 0.0025$ $z_{ij} = 0.2174$	178.3 $\mu\text{g/m}^3$ $1/x = 0.0056$ $z_{ij} = 0.2286$	26.0 $\mu\text{g/m}^3$ $1/x = 0.038$ $z_{ij} = 0.1187$	32.4 mg/m^3 $1/x_{ij} = 0.031$ $z_{ij} = 0.1666$	$\Sigma 24.3 \mu\text{g/m}^3\text{h}$ $1/x_{ij} = 0.041$ $Z_{ij} = 0.1798$	$\Sigma 20.2 \mu\text{g/m}^2 \text{h}$ $1/x_{ij} = 0.049$ $z_{ij} = 0.0866$
a4 minimum+1h; (e) minimum TVOC, HCHO +25% concentration range; (e) CO_2 minimum one person	376.2 mg/m^3 $1/x = 0.00265$ $z_{ij} = 0.2304$	178.3 $\mu\text{g/m}^3$ $1/x = 0.0056$ $z_{ij} = 0.2286$	26.0 $\mu\text{g/m}^3$ $1/x_{ij} = 0.038$ $z_{ij} = 0.1187$	16.2 $\text{mg/m}^3\text{h}$ $1/x_{ij} = 0.062$ $z_{ij} = 0.3333$	$\Sigma 24.3 \mu\text{g/m}^3\text{h}$ $1/x_{ij} = 0.041$ $z_{ij} = 0.1798$	$\Sigma 20.2 \mu\text{g/m}^2 \text{h}$ $1/x_{ij} = 0.049$ $z_{ij} = 0.0866$
a5 minimum+1h; (e) minimum TVOC, HCHO; (e) CO_2 minimum + 25% concentration range; two persons	392.4 mg/m^3 $1/x = 0.0025$ $z_{ij} = 0.2174$	167.6 $\mu\text{g/m}^3$ $1/x = 0.0060$ $z_{ij} = 0.2449$	10.6 g/m^3 $1/x = 0.094$ $z_{ij} = 0.2937$	32.4 $\text{mg/m}^3\text{h}$ $1/x_{ij} = 0.031$ $z_{ij} = 0.1666$	13.6 $\mu\text{g/m}^3\text{h}$ $1/x_{ij} = 0.073$ $z_{ij} = 0.3201$	4.26 $\mu\text{g/m}^2\text{h}$ $1/x_{ij} = 0.234$ $z_{ij} = 0.4134$
$\Sigma 1/x_{ij}$	0.0115	0.0245	0.32	0.186	0.228	0.566

Table A2. Normalisation of decision matrix with alternative a2 (Table 5) for the CRITIC method.

Alternatives	Attribute X = 1 ... m					
	ΔCO_2	TVOC	HCHO	CO_2 (e)	TVOC (e)	HCHO (e)
	mg/m^3	$\mu\text{g/m}^3$	$\mu\text{g/m}^3$	$\text{mg/m}^3\cdot\text{h}$	$\mu\text{g/m}^3\text{h}$	$\mu\text{g/m}^2\text{h}$
a1 Test report [5] ; +(e) emission rate = 0	810 mg/m^3 810–810/ –433.8 = 0	787 $\mu\text{g/m}^3$ 787–787/ –619.4 = 0	18 $\mu\text{g/m}^3$ 18–26/ –15.4 = 0.519	0 0–32.4/ –32.4 = 1	0 0–24.3/ –24.3 = 1	0 0–20.2/ –20.2 = 1
a2 minimum; (e) minimum TVOC, HCHO +1h; (e) minimum CO_2 one person	376.2 mg/m^3 376.2–810/ –433.8 = 1	167.6 $\mu\text{g/m}^3$ 167.6–787/ –629.4 = 1	10.6 $\mu\text{g/m}^3$ 10.6–26/ –15.4 = 1	16.2 $\text{mg/m}^3\text{h}$ 16.2–32.4/ –32.4 = 0.5	13.6 $\mu\text{g/m}^3\text{h}$ 13.6–24.3/ –24.3 = 0.440	4.26 $\mu\text{g/m}^2\text{h}$ 4.26–20.2/ –20.2 = 0.789
a2-2 Test study [5] ; CO_2 TVOC, HCHO; (e) minimum emission rate; (e) CO_2 one person	810 mg/m^3 810–810/ –433.8 = 0	787 $\mu\text{g/m}^3$ 787–787/ –619.4 = 0	18 $\mu\text{g/m}^3$ 18–26/ –15.4 = 0.519	16.2 $\text{mg/m}^3\text{h}$ 16.2–32.4/ –32.4 = 0.5	13.6 $\mu\text{g/m}^3\text{h}$ 13.6–24.3/ –24.3 = 0.440	4.26 $\mu\text{g/m}^2\text{h}$ 4.26–20.2/ –20.2 = 0.789
a3 minimum +1h; (e) minimum +25% TVOC, HCHO concentration range; (e) CO_2 two persons	392.4 mg/m^3 392.4–810/ –433.8 = 0.963	178.3 $\mu\text{g/m}^3$ 178.3–787/ –629.4 = 0.967	26.0 $\mu\text{g/m}^3$ 26.0–26.0/ –15.4 = 0	32.4 $\text{mg/m}^3\text{h}$ two persons 32.4–32.4/ –32.4 = 0	24.3 $\mu\text{g/m}^3\text{h}$ 24.3–24.3/ –24.3 = 0	20.2 $\mu\text{g/m}^2\text{h}$ 20.2–20.2/ –20.2 = 0
a4 minimum+1h; (e) minimum +25% TVOC, HCHO concentration range (e) minimum CO_2 one person	376.2 mg/m^3 376.2–810/ –433.8 = 1	178.3 $\mu\text{g/m}^3$ 178.3–787/ –629.4 = 0.967	26.0 $\mu\text{g/m}^3$ 26.0–26.0/ –15.4 = 0	16.2 $\text{mg/m}^3\text{h}$ 16.2–32.4/ –32.4 = 0.5	24.3 $\mu\text{g/m}^3\text{h}$ 24.3–24.3/ –24.3 = 0	20.2 $\mu\text{g/m}^2\text{h}$ 20.2–20.2/ –20.2 = 0
a5 minimum+1h; (e) minimum +25% CO_2 2 persons; (e) minimum TVOC and HCHO	392.44 mg/m^3 392.4–810/ –433.8 = 0.963	167.6 $\mu\text{g/m}^3$ 167.6–787/ –629.4 = 1	10.6 $\mu\text{g/m}^3$ 10.6–26/ –15.4 = 1	32.4] $\text{mg/m}^3\text{h}$ 32.4–32.4/ –32.4 = 0	13.6 $\mu\text{g/m}^3\text{h}$ 13.6–24.3/ –24.3 = 0.440	4.26 $\mu\text{g/m}^2\text{h}$ 4.26–20.2/ –20.2 = 0.789
Maximum x_{ij} value of concentration range lit.	1800 mg/m^3	1000 $\mu\text{g/m}^3$	160 $\mu\text{g/m}^3$	64.8 $\text{mg/m}^3\cdot\text{h}$ four persons	57.1 $\mu\text{g/m}^3\text{h}$	68.2 $\mu\text{g/m}^2\text{h}$
Maximum x_{ij} value in j attribute column	810 mg/m^3	787 $\mu\text{g/m}^3$	26.0 $\mu\text{g/m}^3$	32.4 $\mu\text{g/m}^3$	24.3 $\mu\text{g/m}^3\text{h}$	20.2 $\mu\text{g/m}^2\text{h}$
Minimum x_{ij} level of ventilation load (lit.)	360 mg/m^3	154 $\mu\text{g/m}^3$	5.8 $\mu\text{g/m}^3$	16.2 $\text{mg/m}^3\text{h}$	13.6 $\mu\text{g/m}^3\text{h}$	4.26 $\mu\text{g/m}^2\text{h}$
Minimum x_{ij} value in attribute j column	376.2	167.6	10.6	0	0	0
Minimum x_{ij} –maximum x_{ij}	–433.8	–619.4	–15.4	–32.4	–24.3	–20.2

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