A Group Decision Framework for Renewable Energy Source Selection under Interval-Valued Probabilistic linguistic Term Set

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Abstract: In recent years, the assessment of desirable renewable energy alternative has been an extremely important concern that could change the environment and economic growth. To tackle the circumstances, some authors have paid attention to selecting the desirable renewable energy option by employing the decision-making assessment and linguistic term sets. With a fast-growing interest in multi-criteria group decision-making (MCGDM) problems, researchers are tirelessly working towards new techniques for better decision-making. Decision makers (DMs) generally rate alternatives linguistically with different probabilities occurring for each term. Previous studies on linguistic decision-making have either ignored this idea or have used an only a single value for representing the weight of the linguistic term. Since expression of the complete probability distribution is hard and implicit hesitation exists, representation of weights of the linguistic terms using a single value becomes imprecise and unreasonable. To avoid this challenge, an interval-valued probabilistic linguistic term set (IVPLTS) is used, which is a generalization of (probabilistic linguistic term set) PLTS. Inspired by the usefulness of IVPLTS concept, we develop a decision framework for rational decision making. Initially, some operational laws and axioms are presented. Further, a novel aggregation operator known as interval-valued probabilistic linguistic simple weighted geometry (IVPLSWG) is developed for aggregating DMs' preferences. Also, criteria weights are determined using the newly developed interval-valued probabilistic linguistic standard variance (IVPLSV) approach and alternatives are ranked using the extended VIKOR (VlseKriterijumskaOptimizacijaKompromisnoResenje) method under IVPLTS environment. Finally, a numerical example of renewable energy assessment is demonstrated to show the practicality of the developed decision framework. Also, the strengths and weaknesses of the developed decision framework are illustrated by comparison with existing ones.

Keywords: group decision making; probabilistic linguistic term set; interval numbers; VIKOR; renewable energy policy selection

1. Introduction

During the establishment of civilization, energy resources affect the environmental, social, and economic growth of countries. Presently, various authors and practitioners have concentrated their work upon choosing desirable renewable energy sources. This form of energy has been expressed as domestic assets that have the essential capacity for providing energy with no, or only a trivial amount of, emission of greenhouse gases and other pollutants [1]. The renewable energy sources of like solar, hydropower, biomass, geothermal, and wind energy are approximately endless, at the same time offering several economic and environmental advantages in comparison with conventional sources [2]. Each renewable energy source offers a particular benefit, making it individually suitable for particular applications in appropriate disciplines [3].

Decision-making is becoming an integral part of our daily life. Human beings make decisions very often, starting from common activities like what beverages to drink to critical activities like which share to purchase. Most often, DMs prefer using linguistic terms for rating the elements. The primary reason is the ease of use, and moreover, the use of numerical terms causes a higher rate of inaccuracies due to lack of a proper method for deriving apt values [4].

So far, many scholars have proposed different methods of linguistic decision-making [5]. The very nature of decision-making encourages team contribution rather than an individual contribution, and hence, group decision making (GDM) has become an attractive area for exploration in the linguistic environment. Herrera and Verdegay [6] initiated the idea of GDM with linguistic terms. Following this, Herrera et al. [7,8] proposed different GDM models under the linguistic domain for effective decision making. Xu [8–10] further proposed methods like goal programming, linguistic order weighted geometry (LOWG), and induced LOWG operator for effective GDM under a linguistic environment.

1.1. Literature Review

In many applications, DMs often have difficulties in expressing their preferences by just using a single linguistic term. To better handle the challenge, Rodriguez et al. [11] proposed a new environment called hesitant fuzzy linguistic term set (HFLTS) by combining the concept of the hesitant fuzzy and linguistic term set. According to the notion of HFLTS, DMs enjoyed the chance to enter more than one preference value linguistically. This concept attracted many scholars to develop new methods under the HFLTS environment. Beg and Rashid [12] developed a new aggregation operator for ordering criteria preferences and also extended the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) approach for ranking under a linguistic environment. Following this, Zhu and Xu [13] developed the hesitant fuzzy linguistic preference relation and used it to solve the decision-making problem.

Further, Rodriguez et al. [14] proposed a new model for GDM (group decision-making) with comparative linguistic expression under HFLTS domain. Liu et al. [15] discussed the idea of fuzzy envelopes for HFLTS and extended the same for decision-making. With a view of minimizing computational complexity in HFLTS, Wang [16] and Wei et al. [17] came up with an idea of non-continuous linguistic terms and extended HFLTS for the same.

Generally, HFLTS allow DMs to enter different linguistic terms for evaluation, but all these terms are assumed to have equal occurring probabilities, which is irrational and unreasonable. To mitigate this concern, Zhang et al. [18] initially came up with the idea of linguistic preference relation with distribution assessment. Later, Dong et al. [19] extended the idea of linguistic distribution assessment to interval numbers with the view of handling uncertainty better. Inspired by the idea of Zhang et al. [18], Pang et al. [4] ameliorated the concept known as probabilistic linguistic term set (PLTS) and applied the same for MCGDM. The PLTS is an extension to HFLTS, where each term is associated with an occurring probability value for representing the relative importance (weight) of each term. For example, to estimate the IQ level of a student by using HFLTS, terms like good, fair, and moderate are used. These terms are assumed to have equal weights, and this assumption seems unreasonable and irrational. So, to alleviate this issue, Pang et al. [4] developed the PLTS concept, which considers different a weight value

(probability value)for each term and it is given by good (0.56), fair (0.22), and moderate (0.18) with the sum of weight values not exceeding unity. Following this, many scholars have proposed new theories and methods under the PLTS environment. Bai et al. [20] developed a comparison measure of PLTSs using the diagram method and possibility degree theory and implemented it for decision making. Gou and Xu [21] proposed new operational laws for LTS, HFLTS, and PLTS, and clarified the operations by using some three-dimensional diagrams. Zhai et al. [22] introduced a concept known as probabilistic linguistic vector term set, which has different scales as well as different degrees corresponding to each evaluator. Some operations are proposed for the same, and it is applied to hospital selection problem. Zhang et al. [23] further explored the PLTS and proposed the preference relation for PLTS. Some consistency check and repair measures were also developed, and the practicality was realized with a belt and road example. Lin et al. [24] recently proposed probabilistic uncertain linguistic term set (PULTS) where the linguistic terms are in the interval fashion, and developed aggregation, criteria weight evaluation, and ranking methods under a PULTS environment for a suitable selection of cloud storage services.

In recent years, many MCGDM (multi-criteria group decision-making) methods have been extended, proposed, and applied for solving renewable energy source (RES) assessment and selection. In addition, the literature review identified different criteria and factors that were used to evaluate and select the RESs; therefore, in this section of the study, we provide an overview to the application of proposed MCGDM methods under various uncertain environments and evaluated criteria to choose and assess the RES alternatives. Dincer and Yuksel [25] used the DEMATEL (Decision making trial and evaluation laboratory) and TOPSIS method to evaluate and select four types of renewable energy alternatives based on eight criteria and four aspects. To validate the developed method, that study compared the outcomes with other existing methods including interval type 2 hesitant fuzzy and interval type-2 fuzzy sets. Yuan et al. [26] proposed a fuzzy decision making approach using an improved Choquet integral using a linguistic hesitant fuzzy set to evaluate and select the RES alternatives in China. The results of that paper found that the best renewable energy alternative was biomass energy. Elzarka et al. [27] discussed a fuzzy decision-making technique based on a vague set to assess the onsite RESs for construction facilities. The outcomes of that study indicated that the developed approach is efficient for selecting renewable energy sources.

Çolak and Kaya [28] developed a new model to select and evaluate RES alternatives using analytical hierarchy process (AHP), TOPSIS methods under Interval type-2 fuzzy. Karunathilake et al. [29] extended the fuzzy TOPSIS method and life cycle thinking to choose and assess different RESs. The results of that paper indicated that small hydro and solar PV were the most common sources among other sources. Ghenai et al. [30] discussed and developed a new method based on SWARA (Step-wise weight assessment ratio analysis) and ARAS (Additive ratio assessment) approaches to choose and assess RESs. Boran et al. [31] used an intuitionistic fuzzy TOPSIS to select and compute RESs for electricity generation by considering four types of sources, including wind, photovoltaic, geothermal, and hydro in Turkey. The outcomes of that study found that the best RES for this country was the hydro source, and hydropower energy had the highest efficiency among other technologies. Mousavi et al. [32] developed a novel ELECTRE (Elimination and choice expressing reality) with hesitant fuzzy sets (HFSs) to select from among numerous policies established for renewable energy. Their findings showed that the wind energy policy was chosen as an important source among other sources. Mishra et al.[33] proposed a novel MCGDM method based on divergence measure to select the best RES alternative among set of RESs.

In recent study, Rani et al. [34] proposed a novel method using the VIKOR approach under Pythagorean fuzzy sets to choose and assess the criteria of RESs in India. Wang et al. [35] extended the fuzzy AHP and SWOT (Strength, weakness, opportunities and threats) model for choosing and assessing the strategic RESs in Pakistan by considering 17 sub-indicators and four indicators. The outcome of that study illustrated that socio-political and economic criteria were the influential indicators for the assessment of RESs. Wu et al. [36] introduced a new approach using the AHP

model under interval type-2 fuzzy weighted averaging set to appraise the performance of renewable energy projects based on the sustainability view. The results of that proposed method found that the GHG (greenhouse gas) emission reduction had the best rank among other criteria. Campisi et al. [1] proposed multi criteria decision making(MCDM) methods for RES and economic efficiency evaluation residential buildings in Italy.

1.2. Motivation of the Study

Though PLTS better handles the issue of HFLTS, it does suffer from the issue of proper and precise determination of weight values for each linguistic term. Motivated by this challenge and to circumvent the same, Bai et al. [37], extended the idea of PLTS with interval numbers and presented the interval-valued probabilistic linguistic term set (IVPLTS) concept. Attracted by the power of IVPLTS and with the view of extending the same to a decision-making framework, some crucial challenges are identified, and efforts are made to address the same in our proposed work.

- The operational laws presented in [37] use transformation function, which complicates the process of evaluation. So, presenting simple and straightforward operational laws as a direct extension to [1] is of interest.
- The aggregation operator in [37] aggregates preferences with some information loss and uses the operational law for aggregation, which eventually complicates the process. Proposing an aggregation operator that aggregates preferences without much loss of information and the virtual set is an open challenge.
- Criteria weights are directly provided by the DMs, which causes inaccuracies in the MCGDM process. Hence, proposing a systematic way for weight evaluation under IVPLTS context is an interesting challenge.
- Based on the comparative study conducted by Opricovic et al. [38], TOPSIS method performs weakly when compared to its close counterpart VIKOR. Hence, extending a better compromised ranking method for a suitable selection of object under IVPLTS context is an open challenge.

Now, consider the same example of IQ level of a student, and the use of PLTS yields importance values (as a probability) for each term, which are single-valued. Most often in practical cases, determination of the importance of each term by using a single value is unreasonable and imprecise. Thus, to address this challenge effectively, we use interval values for representing the weights of the linguistic terms, and this provides a range of values with possible importance for each term. This helps DMs by offering a specific range of values as occurrence probability for each term rather than specifying occurrence probability as a single-valued entity. So, in the example, IQ level of a student is represented as good [0.5, 0.6], fair [0.2, 0.3], and moderate [0.1, 0.2]. This gives the DMs a range of values for each term, and hence, imprecision and uncertainty is handled in a much more sensible manner.

However, according to the above discussions and literature review, IVPLTSs and decision-making methods are powerful and effective tools and methods to select, rank, and assess the RESs problems because since the data in the RES selection problem are often imprecise, therefore IVPLTSs theory successfully handles this problem; also, IVPLTS has the qualification to deal with vague and imprecise information. Furthermore, it provides a consistent basis for information processing and a neat, mathematically well-known illustration of the linguistic data.

1.3. Contribution of the Study

Owing to the powerfulness of IVPLTS and with the view of circumventing the challenges mentioned above, some concrete contributions of this paper are listed below:

- Some simple and straightforward operational laws are presented under IVPLTS context, which is readily usable and is a direct extension to [4].

- A new aggregation operator called interval-valued probabilistic linguistic simple weighted geometry (IVPLSWG) is developed for sensible aggregation of preference information without much loss of information and formation of the virtual set.
- An organized process for criteria weight evaluation is also introduced under IVPLTS context by extending statistical variance (SV) approach.
- Further, the conventional VIKOR ranking approach is extended under IVPLTSs by gaining motivation from [38]. This helps DMs to evaluate an option object from the set of options.
- Apart from these contributions, the implementation of the proposed decision framework is also discussed by solving RES selection problem. Also, the strengths and weaknesses of the framework are mentioned under theoretic and numeric sense by comparison with the existing methods.

1.4. Organization of the Paper

The rest of the paper is constructed as follows: Section 2 reviews the basics, Section 3 discusses the proposed new operational laws, operators, properties, and decision framework under IVPLTS context. Next, Section 4 presents an illustrative problem for RES selection to demonstrate the practicality and usefulness of the proposed approach. Further, in Section 5, a comparative investigation is put forward to realize the strengths and weaknesses of the developed method from theoretic as well as numeric perspectives, and finally, in Section 6, a concluding remark is presented.

2. Preliminaries

In the current section, we discuss some basics of linguistic term set (LTS) and PLTS.

Definition 1 [39]. *Consider a term set S, given by $\{s_\beta | \forall \beta = 0, 1, \ldots, \tau\}$, with τ being a positive integer, s_0 and s_τ are the lower and upper bounds of the linguistic term (LT), and s_β is a linguistic term with the following postulates:*

(a) *Consider two LTs s_m and s_n, the relation $s_m > s_n$ is true if $m > n$.*
(b) *The negation of a linguistic term s_m is giving by $neg(s_m) = s_n$, such that, $m + n = \tau$.*

Definition 2 [4]. *Consider a LTS $S = \{s_0, s_1, \ldots, s_\tau\}$, then the PLTS is given by,*

$$L(p) = \left\{ L^t\left(p^t\right) | L^t \in S, p^t \geq 0, t = 0, 1, \ldots, \#L(p), \sum_{t=1}^{\#L(p)} p^t \leq 1 \right\} \tag{1}$$

where $L^t\left(p^t\right)$ is the linguistic term L^t with a probability value p^t and $\#L(p)$ is the number of linguistic terms (instance).

Remark 1. *For brevity, $\alpha_i = \{r_i^t(p_i^t)\}$ is known as probabilistic linguistic element (PLE). The term r_i^t is called the subscript of the PLE, and p_i^t is the occurrence probability associated with each term. Ordering of PLEs is done by arranging the terms in descending order based on the value obtained from $\left(r^t \times p^t\right)$*

Definition 3 [4]. *Consider the PLTS $L(p)$ with $\sum_{t=1}^{\#L(p)} p^t \leq 1$ then the normalized PLTS $L(p^*)$ is given by,*

$$L(p^*) = \left\{ L^t\left(p^{*t}\right) | L^t \in S, p^{*t} \geq 0, t = 0, 1, \ldots, \#L(p), \sum_{t=1}^{\#L(p^*)} p^{*t} = 1 \right\} \tag{2}$$

where $p^{*t} = \frac{p^t}{\sum_{t=1}^{\#L(p)} p^t}$ with $t = 0, 1, \ldots, \#L(p)$

Definition 4 [4]. *Consider a PLE as defined before, then the score and deviation degree is defined by,*

$$E(\alpha_i) = s_{\overline{\beta}} \tag{3}$$

$$\sigma(\alpha_i) = \sqrt{\frac{\sum_{t=1}^{\#L(p)} \left(p^t\left(r^t - \overline{\beta}\right)\right)^2}{\sum_{t=1}^{\#L(p)} p^t}} \tag{4}$$

where $\overline{\beta} = \frac{\sum_{t=1}^{\#L(p)} r^t p^t}{\sum_{t=1}^{\#L(p)} p^t}$.

Remark 2. *Any two PLEs α_1 and α_2 can be compared in the following manner using score and deviation:*

(a)   *If $E(\alpha_1) < E(\alpha_2)$ then $\alpha_1 < \alpha_2$.*

(b)   *If $E(\alpha_1) = E(\alpha_2)$ then calculate deviation using Equation (4). If $\sigma(\alpha_1) < \sigma(\alpha_2)$ then $\alpha_1 > \alpha_2$. If $\sigma(\alpha_1) = \sigma(\alpha_2)$ then $\alpha_1 = \alpha_2$.*

Definition 5 [4]. *Consider two PLEs α_1 and α_2, which are ordered, then these PLEs follow some operational laws, which are depicted as*

$$\alpha_1 \oplus \alpha_2 = \cup_{L_1^t \in \alpha_1, L_2^t \in \alpha_2} \left(p_1^t L_1^t \oplus p_2^t L_2^t\right) \tag{5}$$

$$\alpha_1 \otimes \alpha_2 = \cup_{L_1^t \in \alpha_1, L_2^t \in \alpha_2} \left(p_1^t L_1^t \otimes p_2^t L_2^t\right) \tag{6}$$

$$\alpha_1 = \cup_{L_1^t \in \alpha_1} \lambda p_1^t L_1^t , \ \lambda \geq 0 \tag{7}$$

$$(\alpha_1)^\lambda = \cup_{L_1^t \in \alpha_1} \left(L_1^t\right)^{p_1^t \lambda}, \ \lambda \geq 0 \tag{8}$$

Definition 6 [4]. *Consider two PLEs α_1 and α_2, then the distance measure between these two PLEs is given by,*

$$d(\alpha_1, \alpha_2) = \sqrt{\frac{\sum_{t=1}^{\#L(p)} \left(p_1^t r_1^t - p_2^t r_2^t\right)^2}{\#L(p)}} \tag{9}$$

where the length of α_1 and α_2 must be equal and, hence, we denote the length as $\#L(p)$. If the lengths are unequal, then the procedure defined in [4] is adopted to make the lengths equal.

3. Proposed Decision Framework under IVPLTSs

3.1. IVPLTS and Some Operational Laws

In the current section, we present some simple and straightforward operational laws, which are a direct extension to [4].

Definition 7 [37]: *Consider a LTSS $= \{s_0, s_1, \ldots, s_\tau\}$, then the IVPLTS is given by,*

$$LI(p_l, p_u) = \left\{ L^t\left(\left[p_l^t, p_u^t\right]\right) \middle| L^t \in S, 0 \leq p_l^t \leq 1, 0 \leq p_u^t \leq 1, p_l^t \leq p_u^t \right\} \tag{10}$$

where $L^t\left(\left[p_l^t, p_u^t\right]\right)$ is the linguistic term L^t with probability in the range $[p_l^t, p_u^t]$, $t = 0, 1, \ldots, \#LI(p)$ and $\#LI(p)$ refers to the number of terms/instances.

Note 1. The IVPLTS is named so because it is a direct extension of the PLTS concept [4] (the genesis of the idea is from [18]). The IVPLTS associates interval values with occurrence probability for each

LT. Though the genesis of this idea was from Dong et al. [19], in this paper, efforts are discussed to ameliorate the issues present in the previous concept. To avoid confusion and to emphasize the direct extension of PLTS, throughout this paper, we will use the term IVPLTS.

Definition 8. *Consider an IVPLTS as defined before, then the normalized IVPLTS is defined as,*

$$LI^*(p_l, p_u) = \left\{ L^{*t}\left([p_l^{*t}, p_u^{*t}]\right) \middle| L^{*t} \in S, 0 \le p_l^{*t} \le 1, 0 \le p_u^{*t} \le 1, p_l^{*t} \le p_u^{*t} \right\} \tag{11}$$

where $L^{*t}([p_l^{*t}, p_u^{*t}])$ is the normalized linguistic term L^{*t} with normalized probability in the range $[p_l^{*t}, p_u^{*t}]$ and normalized probability p_l^{*t} and p_u^{*t} is calculated using Equations (12)–(13).

$$p_l^{*t}(i, j) = \frac{p_l^t(i, j)}{\sqrt{0.5\left(\sum_{t=1}^m \left(\left(p_l^t(i, j)\right)^2 + (p_u^t(i, j))^2\right)\right)}} \forall m \text{ alternatives} \tag{12}$$

$$p_u^{*t}(i, j) = \frac{p_u^t(i, j)}{\sqrt{0.5\left(\sum_{t=1}^m \left(\left(p_l^t(i, j)\right)^2 + (p_u^t(i, j))^2\right)\right)}} \forall m \text{alternatives} \tag{13}$$

where i represents the ith object (row) and j shows the jth criterion (column).

Remark 3. *For the ease of use, from now on, we represent the interval-valued probabilistic linguistic element (IVPLE) in the form $la = \{r^t, [p_l^t, p_u^t]\}$, where r^t is the subscript associated with each LT and $[p_l^t, p_u^t]$ is the interval probability value related with each LT.*

Definition 9. *Consider an LTSS $= \{s_0, s_1, \ldots, s_\tau\}$, then the empty IVPLTS and full IVPLTS is defined by,*

(a)   *Empty IVPLTS $LI(p_l, p_u) = \{\varnothing\}$;*
(b)   *Full IVPLTS $LI(p_l, p_u) = S|with respective probabilities.$*

Definition 10. *The lower and upper bounds of IVPLEs are defined by (a) lower bound $LI^-(p_l, p_u) = \underset{t}{min} \left(r^t \times \left(\frac{p_l^t + p_u^t}{2}\right)\right)$; (b) upper bound $LI^+(p_l, p_u) = \underset{t}{max} \left(r^t \times \left(\frac{p_l^t + p_u^t}{2}\right)\right)$ with $t = 0, 1, \ldots, \#L(p)$.*

Definition 11. *The union and intersection of two IVPLTS $LI_1(p_l, p_u)$ and $LI_2(p_l, p_u)$ are given by*

$$\begin{aligned} &LI_1(p_l, p_u) \cup LI_2(p_l, p_u) \\ &= \left\{ L^t\left([p_l^t, p_u^t]\right) \middle| L^t\left([p_l^t, p_u^t]\right) \in LI_1(p_l, p_u) orL^t\left([p_l^t, p_u^t]\right) \in LI_2(p_l, p_u) \right\} \end{aligned} \tag{14}$$

$$\begin{aligned} &LI_1(p_l, p_u) \cap LI_2(p_l, p_u) \\ &= \left\{ L^t\left([p_l^t, p_u^t]\right) \middle| L^t\left([p_l^t, p_u^t]\right) \in LI_1(p_l, p_u) andL^t\left([p_l^t, p_u^t]\right) \in LI_2(p_l, p_u) \right\} \end{aligned} \tag{15}$$

Definition 12. *The complement of an IVPLTS is denoted by $LI^c(p_l, p_u)$ and it is defined by $LI^c(p_l, p_u) = S - LI(p_l, p_u)$ with $p_l^c = 1 - p_u$, and $p_l^c p_u^c$.*

Proposition 1. *The complement of an IVPLTS is involutive.*

Proof. The complement of an IVPLTS is given by Definition 12. To prove the involutive property, we consider $(LI^c(p_l, p_u))^c = S - (S - LI(p_l, p_u)) = = LI(p_l, p_u)$ with $(p_l^c)^c = 1 - p_u^c = p_l$ and $(p_u^c)^c = 1 - p_l^c = p_u$. Hence, it is proved. □

Definition 13. *Consider two IVPLEsla$_1$ and la$_2$ as given before, then the operational laws are defined by*

$$la_1 \oplus la_2 = \left\{ s_{r_1^t + r_2^t} \left[p_{1l}^t + p_{2l}^t, p_{1u}^t + p_{2u}^t \right] \right\} \tag{16}$$

$$la_1 \otimes la_2 = \left\{ s_{r_1^t \times r_2^t} \left[p_{1l}^t \times p_{2l}^t, p_{1u}^t \times p_{2u}^t \right] \right\} \tag{17}$$

$$\lambda la_1(p_l, p_u) = \left\{ s_{(\lambda \times r_1^t)} \left[\lambda p_{1l}^t, \lambda p_{1u}^t \right] \right\}, \ 0 \le \lambda \le 1 \tag{18}$$

Remark 4. *The operational laws given in Definition 13 produce IVPLEs that sometimes exceed the bounds. To transform the linguistic part of IVPLE within the bounds, we apply the procedure given in [40]. Further, to transform the occurring probabilities within the bounds, we apply Equations (19) given below:*

$$[p_l, p_u] = \begin{cases} [1, \ p_u] \ \ if \ \ p_l > 1 \ and \ 0 \ \le p_u \le 1 \\ [p_l, \ 1] \ \ if \ \ p_u > 1 \ and \ 0 \ \le p_l \le 1 \end{cases} \tag{19}$$

　　Also, when the result of the operational laws yields the same linguistic terms, redundancy is removed by considering the linguistic term along with the mean of the probability values (for both lower and upper bounds). Readers are encouraged to see example 1 for clarification.

Property 1. Commutative

$$LI_1(p_l, p_u) \cup LI_2(p_l, p_u) = LI_2(p_l, p_u) \cup LI_1(p_l, p_u)$$

$$LI_1(p_l, p_u) \cap LI_2(p_l, p_u) = LI_1(p_l, p_u) \cap LI_2(p_l, p_u)$$

Property 2. Associative

$$LI_1(p_l, p_u) \cup (LI_2(p_l, p_u) \cup LI_3(p_l, p_u)) = (LI_1(p_l, p_u) \cup LI_2(p_l, p_u)) \cup LI_3(p_l, p_u)$$

$$LI_1(p_l, p_u) \cap (LI_2(p_l, p_u) \cap LI_3(p_l, p_u)) = (LI_1(p_l, p_u) \cap LI_2(p_l, p_u)) \cap LI_3(p_l, p_u)$$

Property 3. Distributive

$$LI_1(p_l, p_u) \cup (LI_2(p_l, p_u) \cap LI_3(p_l, p_u)) = (LI_1(p_l, p_u) \cup LI_2(p_l, p_u)) \cap (LI_1(p_l, p_u) \cup LI_3(p_l, p_u))$$

$$LI_1(p_l, p_u) \cap (LI_2(p_l, p_u) \cup LI_3(p_l, p_u)) = (LI_1(p_l, p_u) \cap LI_2(p_l, p_u)) \cup (LI_1(p_l, p_u) \cap LI_3(p_l, p_u))$$

Theorem 1. *Consider two IVPLEs la$_1$ and la$_2$ as defined before. Then for real λ, λ_1 and λ_2*

(1)　$\lambda(la_1 \oplus la_2) = \lambda la_1(p_l, p_u) \oplus \lambda la_2(p_l, p_u)$
(2)　$(\lambda_1 + \lambda_2)la_1 = \lambda_1 la_1 \oplus \lambda_2 la_1$
(3)　$\lambda(LI_1(p_l, p_u) \otimes LI_2(p_l, p_u)) = \lambda LI_1(p_l, p_u) \otimes \lambda LI_2(p_l, p_u)$
(4)　$(\lambda_1 \times \lambda_2)la_1 = \lambda_1 la_1 \otimes \lambda_2 la_1(p_l, p_u)$.

Proof. The proof for Theorem 1 is obvious from Definition 13.　□

Example 1. *Let S be a LTS of the form $S = \{s_0 = nothing, s_1 = verybad, s_2 = bad, s_3 = moderate, s_4 = good, s_5 = verygood, s_6 = perfectoderate, theform8888888888888888888888888888888888\}$. Consider*

*two IVPLEs la_1 and la_2 of the form la_1 = $\{3, [0.2, 0.3], 4, [0.33, 0.42]\}$ and $la_2(p_l, p_u)$ =
$\{4, [0.33, 0.42], 2, [0.23, 0.36]\}$, with $\lambda = 0.3$ then*

$$la_1 \oplus la_2 = \{3 + 4, [0.53, 0.72], 4 + 2, [0.56, 0.78]\} = \{s_7, [0.53, 0.72], s_6, [0.56, 0.78]\}$$
$$\approx \{s_6, [0.53, 0.72], s_6, [0.56, 0.78]\} \approx \{s_6, [0.55, 0.75]\}$$

$$LI_1(p_l, p_u) \oplus LI_2(p_l, p_u) = \{3 \times 4, [0.066, 0.126], 4 \times 2, [0.076, 0.151]\}$$
$$= \{s_{12}, [0.066, 0.126], s_8, [0.076, 0.151]\} \approx \{s_6, [0.066, 0.126], s_6, [0.076, 0.151]\}$$
$$= \{s_6, [0.071, 0.139]\}$$

$$\lambda LI_1(p_l, p_u) = \{s_{0.3 \times 3}[0.3 \times 0.2, 0.3 \times 0.3], s_{0.3 \times 4}[0.3 \times 0.33, 0.3 \times 0.42]\}$$
$$= \{s_1, [0.06, 0.09], s_1, [0.099, 0.126]\} = \{s_1, [0.08, 0.11]\}.$$

3.2. Construction of the Proposed Decision Framework

In this section, we depict the construction of the proposed framework in Figure 1. The architecture is self-contained and simple. The architecture illustrates a new decision-making framework where the first stage aggregates the DMs' IVPLTS information rationally. Afterwards, in the second stage, weights of each criterion are determined by the proposed method under IVPLTS context, and finally, ranking is done by the extension of VIKOR approach to IVPLTS context. A detailed discussion of the developed decision-making framework is demonstrated in the following sub-sections.
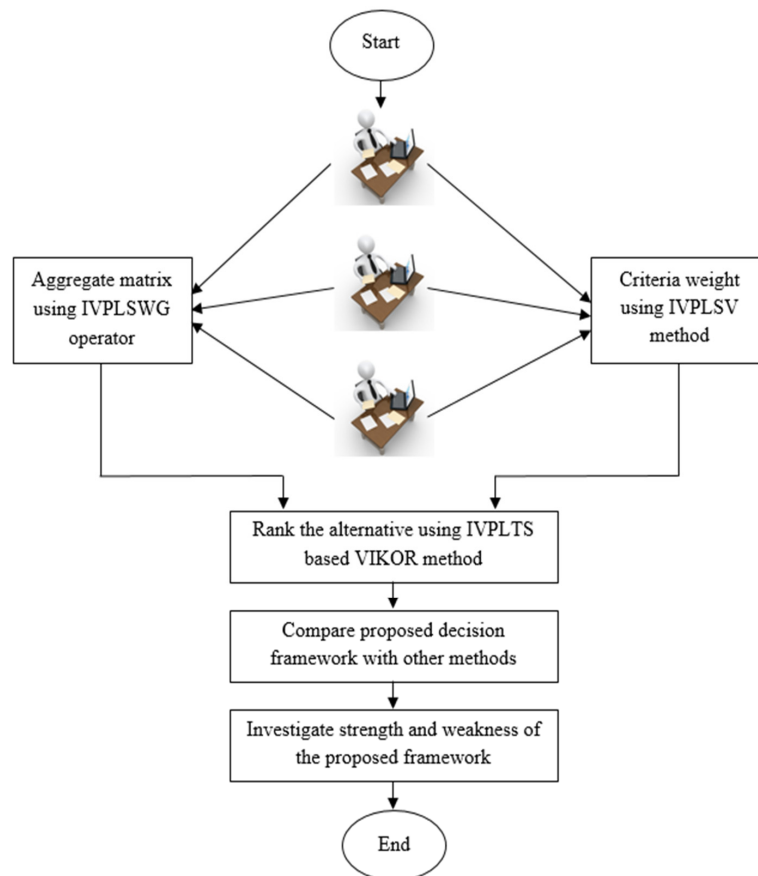

Figure 1. Structural design of developed decision framework under interval-valued probabilistic linguistic term set (IVPLTS).

3.3. Proposed IVPLSWG Aggregation Operator

Here, we propose a new aggregation operator known as IVPLSWG for fusing DMs' preferences. The operator does the aggregation in two folds by initially aggregating the linguistic terms and then aggregating the associated occurring probabilities.

Definition 14. *Consider IVPLTSs* $LI_i(p_l, p_u)$ *in the form* $\{L_i^t([p_{il}^t, p_{iu}^t]) | L_i^t \in S, 0 \leq p_{il}^t \leq 1, 0 \leq p_{iu}^t \leq 1, p_{il}^t \leq p_{iu}^t, p_{il}^t + p_{iu}^t \geq 0\}$ *for each of the i objects, then the aggregation operator IVPLSWG is a function given as* $U^n \to U$ *such that,*

$$IVPLSWG(r_i^t) = \begin{cases} Scheme\ 1 & if\ all\ linguistic\ terms\ are\ unique \\ Scheme\ 2 & otherwise \end{cases} \tag{20}$$

$$IVPLSWG([p_{il}^t, p_{iu}^t]) = \prod_{j=1}^n (p_{il}^t)^{\lambda_j}, \prod_{j=1}^n (p_{iu}^t)^{\lambda_j} \tag{21}$$

where n is number of DMs, λ *is the weight or relative importance of each DM, such that* $\lambda \epsilon [0, 1]$ *and* $\sum \lambda = 1$.

Scheme 1. If all the linguistic terms are unique, then take the mean of the subscripts (r_i^t) and apply round-off principle for producing non-virtual subscripts.
Scheme 2. If the linguistic terms are not unique, then choose that subscript that repeats a maximum number of times as the aggregated value.

Theorem 2. *The aggregation of IVPLEs by using developed IVPLSWG operator yields an IVPLE.*

Proof. The fusion of IVPLTS information by using IVPLSWG operator can be achieved two-fold viz., aggregation of LTs and aggregation of associated occurrence probability values in the interval format. To prove that the aggregation of IVPLTS information also yields an IVPLTS information, it is enough to prove that both the LT and the probability value associated with the LT are interval-valued probabilistic linguistic (i.e., they obey Definition 7).

The linguistic term that is obtained by aggregation using IVPLSWG is non-virtual, and since the operator uses count and score measure for aggregation of the linguistic term, the term is non-virtual. Now, the associated occurring probability can be proved by showing that $p_{il}^t \leq p_{iu}^t$ with $0 \leq p_{il}^t \leq p_{iu}^t \leq 1$. For this, motivation is gained from the lemma discussed in [41], which states that $\prod_{k=1}^{\#DM} (x_{ij})^{\lambda_k} \leq \sum_{k=1}^{\#DM} \lambda_k x_{ij}$ where x_{ij} is some sequence of values, $0 \leq \lambda_k \leq 1$ and $\sum_k \lambda_k = 1$. Now, this lemma is extended to the occurring probability values and hence, $0 \leq \prod_{k=1}^{\#DM} (p_{il}^t)^{\lambda_k} \leq \prod_{k=1}^{\#DM} (p_{iu}^t)^{\lambda_k} \leq \sum_{k=1}^{\#DM} \lambda_k p_{il}^t \leq \sum_{k=1}^{\#DM} \lambda_k p_{iu}^t \leq \sum_k \lambda_k = 1$. Thus, the aggregation of IVPLEs using IVPLSWG operator yields an IVPLE.
□

Example 2. *Consider three DMs with the relative importance* $(\lambda_1, \lambda_2, \lambda_3) = (0.3, 0.4, 0.3)$ *rating an alternative concerning a criterion; the IVPLTS rating of each DM is given as* $LI_1(p)$, $LI_2(p)$, and $LI_3(p)$.
Now, $LI_1(p_l, p_u) = \{(2, [0.3, 0.4]), (3, [0.24, 0.36])\}$, $LI_2(p_l, p_u) = \{(2, [0.34, 0.45]), (4, [0.3, 0.4])\}$ *and* $LI_3(p_l, p_u) = \{(3, [0.35, 0.46]), (2, [0.4, 0.5])\}$. *Thus, the aggregated value* $LI_{123}(p_l, p_u)$ *is calculated by using Equations (20)–(21).*
By using Equation (20), we obtain the subscript of $LI_{123}(p)$. *Since there are two instances in each term, the resultant* $LI_{123}(p)$ *also has two terms. In the first instance, all terms are not unique, and hence, we adopt Scheme 2, which gives* $r^1 = 2$. *Correspondingly, the aggregated expectation values are calculated for the first instance by*

using Equation (21), which gives $\left[p_l^1, p_u^1\right] = [0.33, 0.44]$. *Similarly, instance two is calculated. Here, r^2 follows Scheme 1 as all elements are unique. So, $r^2 = 3$ and $\left[p_l^2, p_u^2\right] = [0.32, 0.43]$. Thus, the aggregated value is*

$$LI_{123}(p) = \{(2, [0.33, 0.44]), (3, [0.31, 0.41])\} = \{(s_2, [0.33, 0.44]), (s_3, [0.31, 0.41])\}$$

.

Remark 5. *The proposed IVPLSWG operator satisfies properties like commutative, idempotent, bounded, and monotone.*

3.4. Proposed Interval-Valued Probabilistic Linguistic Standard Variance Method

In this section, we make efforts to propose a new process for criteria weight evaluation. This method is a generalization to the classical statistical variance (SV) method under the interval-valued probabilistic linguistic environment. The main motivation for proposing IVPLSV method is three-fold: (a) Liu et al. [42] argued that common weight estimation methods like entropy measures, analytical hierarchy process (AHP), and optimization models yield unrealistic and irrational weights for the criteria. (b) Rao et al. [43] argued that the standard variance (SV) method, not like other statistical methods, concentrates on all data points for evaluating the distribution and hence, offers much more reasonable and rational criteria weight estimation. (c) Rao et al. [43] presented a geometric proof with the help of frontiers and projection to support the claim that the SV method provides high importance to those criteria that have high confusion and uncertainty during the process of preference elicitation. (d) Also, the SV method is an easy and straightforward method for implementation, which is successfully extended over PLTS environment for estimating criteria weights.

The procedure of estimation of criteria weights using IVPLSV approach is given by

Step 1. Create a weight assessment matrix of order $(m \times n)$, where m and n are the total number of DMs and criteria, respectively. The DMs rate each criterion using IVPLTS information.

Step 2. Transform the IVPLTS values into a single term by applying Equations (22)–(23).

$$LI(p_l, p_u) = \left[r^t p_l^t, r^t p_u^t\right] \tag{22}$$

$$LI_{ij}(p_l, p_u) = \frac{\left[r^t p_l^t + r^t p_u^t\right]}{2} \tag{23}$$

where $t = 0, 1, \ldots, \#LI(p)$, r^t is the subscript of the linguistic term, (i, j) denotes the row and column instance of the evaluation matrix, and $\left[p_l^t, p_u^t\right]$ is the probability value in the interval fashion.

Step 3. Compute the mean for each criterion and apply Equation (24) to evaluate the variance value for each criterion.

$$var_j = \frac{\sum_{i=1}^m \left(LI_{ij}(p_l, p_u) - \overline{LI_{ij}(p_l, p_u)}\right)^2}{m - 1} \tag{24}$$

where $\overline{LI_{ij}(p_l, p_u)}$ and m are the mean each criterion and total number of DMs, respectively.

Step 4. Evaluate the criteria weight (relative importance) by normalizing the variance obtained for each criterion from Step 3.

$$var_j^* = \frac{var_j}{\sum_{j=1}^n var_j} \tag{25}$$

where var_j^* is the normalized weight value of the jth criterion, n is the total number of criteria, and $\sum_{j=1}^n var_j^* = 1$.

3.5. Proposed IVPLTS-based VIKOR Ranking Method

In this section, we discuss the framework for ranking the alternatives utilizing the developed IVPLTS-based classical VIKOR approach. It is a compromised ranking method based on the L_p metric. Sayadi et al. [44] developed the VIKOR under interval numbers. The interval VIKOR [44] can handle DMs' choices better by allowing them to enter a range of values as preferences rather than forcing them to give a single value. Motivated by the strength of interval VIKOR, we set our proposal to extend VIKOR to IVPLTS environment. Here, the relative significance of the linguistic term is interval-valued, and hence, the framework for IVPLTS-based VIKOR approach is given by

Step 1. Estimate the positive and negative ideal solutions (PIS and NIS) for each criterion by Equations (26)–(27).

$$LI^*(p_l, p_u) = max_{j \in benefit} \left(\frac{\left(r_{ij}^t p_{ijl}^t + r_{ij}^t p_{iju}^t \right)}{2} \right) (or) \ min_{j \in cost} \left(\frac{\left(r_{ij}^t p_{ijl}^t + r_{ij}^t p_{iju}^t \right)}{2} \right) \tag{26}$$

$$LI^-(p_l, p_u) = min_{j \in benefit} \left(\frac{\left(r_{ij}^t p_{ijl}^t + r_{ij}^t p_{iju}^t \right)}{2} \right) (or) max_{j \in cost} \left(\frac{\left(r_{ij}^t p_{ijl}^t + r_{ij}^t p_{iju}^t \right)}{2} \right) \tag{27}$$

where $LI^*(p_l, p_u)$ is the PIS, $LI^-(p_l, p_u)$ is the NIS, and i, j, and t are the number of alternatives, criteria, and instances, respectively; r is the subscript of the linguistic term and $[p_l, p_u]$ is the probability value in the interval form.

Note 2. The Equations (26)–(27) are used to calculate the PIS and NIS values. They are single-valued terms and evaluated for each criterion. The IVPLTS information corresponding to these terms is presented for further calculation.

Step 2. Evaluate the group utility (S) and individual regret (R) for each alternative under the interval domain by using Equations (28)–(31).

$$S^l = \left(\sum_{j \in benefit} \omega_j \left(\frac{d\left(LI_{ij}(p_u), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right) + \sum_{j \in cost} \omega_j \left(\frac{d\left(LI_{ij}(p_l), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right) \right) \tag{28}$$

$$S^u = \left(\sum_{j \in benefit} \omega_j \left(\frac{d\left(LI_{ij}(p_l), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right) + \sum_{j \in cost} \omega_j \left(\frac{d\left(LI_{ij}(p_u), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right) \right) \tag{29}$$

$$R^l = max \left(\omega_j \left(\frac{d\left(LI_{ij}(p_u), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right), \omega_j \left(\frac{d\left(LI_{ij}(p_l), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right) \right) \tag{30}$$

$$R^u = max \left(\omega_j \left(\frac{d\left(LI_{ij}(p_l), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right), \omega_j \left(\frac{d\left(LI_{ij}(p_u), LI^*(p_l, p_u) \right)}{d(LI^*(p_l, p_u), LI^-(p_l, p_u))} \right) \right) \tag{31}$$

where $d(a, b)$ is the distance between linguistic terms a and b, which is defined in Equation (9), $\left[S^l, S^u \right]$ is the group utility in the interval form, and $\left[R^l, R^u \right]$ is the individual regret in the interval form with $S^l \leq S^u$ and $R^l \leq R^u$.

Step 3. Compute the merit function (Q) for each alternative under the interval domain using Step 2, and it is given by Equations (32)–(33).

$$Q^l = v \left(\frac{S^l - S^*}{S^- - S^*} \right) + (1 - v) \left(\frac{R^l - R^*}{R^- - R^*} \right) \tag{32}$$

$$Q^u = v\left(\frac{S^u - S^*}{S^- - S^*}\right) + (1 - v)\left(\frac{R^u - R^*}{R^- - R^*}\right) \tag{33}$$

where $\left[Q^l, Q^u\right]$ is the merit function in the interval range, v is the strategy of the DM such that $v \,\epsilon\, [0, 1]$, and $S^* = \min\left(S^l\right)$, $R^* = \min\left(R^l\right)$, $S^- = \max(S^u)$, $R^- = \max(R^u)$.

Step 4. The final merit function (Q) is calculated by calculating the mean of Q^l and Q^u. The preference order is evaluated by determining the values of Q in ascending order.

Before signifying the practicality of developed framework under IVPLTS environment, it is worth discussing the procedure here.

(a) Initially, the DMs' preferences are aggregated into a single decision matrix without much loss of information using the newly proposed IVPLSWG operator. The operator has two folds viz., (a) aggregating linguistic terms and (b) aggregating the interval-based probability terms. Since the details of this operator can be found in Section 3.2, we confine our discussion here.

(b) Secondly, the criteria weights are determined utilizing a proposed IVPLSV approach, as an extension to the SV method under IVPLTS environment. The result of this method is a weight vector, whose length is equal to the number of criteria and whose sum is equal to unity. Since the procedure for weight estimation is clarified in Section 3.3, we confine our discussion here.

(c) The aggregated matrix from Section 3.2 and criteria weights from Section 3.3 are taken as input to the newly proposed ranking method called IVPLTS-based VIKOR. Some details of the method are clarified below:

- Step 1 of the ranking method is used to find the positive ideal solution (PIS) and negative ideal solution (NIS). Since benefit criteria correspond to a maximum in PIS and cost criteria correspond to a minimum in NIS (vice versa under cost criteria), we set our formulation in such a manner. Suppose, we consider, three alternatives with two benefits (b) and one cost (c) criteria, the PIS and NIS will have three values (2b + 1c). When Equation (26) and Equation (27) are applied for PIS and NIS, we obtain a single-valued term, and the corresponding IVPLTS information for each criterion is chosen. Such selection of PIS and NIS values yield sensible ranking as every instance of the rating is concentrated separately.

- In Step 2, the parameters like S and R are determined for both lower and upper bounds. We adopt the distance measure to assess the closeness to PIS. The PIS and NIS values are transformed into single-valued terms by using $\left(\frac{rp_l + rp_u}{2}\right)$. Then, the distance measure is applied. For simplicity, we consider the values produced by each IVPLTS information for PIS and NIS as $LI^*(p_l, p_u)$, and $LI^-(p_l, p_u)$ itself (all these values are single-valued). The S and R values are determined for each alternative.

- Step 3 is used to estimate the merit function under lower and upper bounds. The merit function is also determined for each alternative.

- The average of Q^l and Q^u is calculated using Step 4 for each alternative to determine the ranking order.

- The parameter v is considerably varied from [0, 1] to realize the effect of fuzziness on the proposed method.

4. Numerical Example

In the current section, a case study for evaluating the optimal RES is considered in India to illustrate the feasibility and the procedural acceptance of VIKOR model under IVPLTSs. To cope with the issue, three alternatives $M = (M_1, M_2, M_3)$ over four criteria $C = (C_1, C_2, C_3, C_4)$ are taken. The group decision framework is executed by three decision makers $E = (e_1, e_2, e_3)$. In the RES selection problem, the decision makers' risk preferences are considered in the form of linguistic values for given alternatives over the assessed criteria. The feasible alternatives and criteria are given by

M_1 : Hydropower;

M_2 : Solar energy;

M_3 : Wind energy.

And

C_1 : Technology

C_2 : Environmental;

C_3 : Social;

C_4 : Economical.

The steps implicated in the proposed framework are as follows:

- We signify the set of all RES as $M = (M_1, M_2, M_3)$, over set of criteria as $C = (C_1, C_2, C_3, C_4)$ and set of all experts as $E = (e_1, e_2, e_3)$. These three experts rate the mangers using IVPLTS information, and three decision matrices are formed with the order (3×4). The LTS used for the analysis is given by $S = \{s_0 = nothing, s_1 = verybad, s_2 = bad, s_3 = moderate, s_4 = good, s_5 = verygood, s_6 = perfectmoderate, the form$1414141414141414141414141414141414141414 1414141414141414141414141414141414141414}

- We aggregate the given three matrices into a decision matrix by using proposed IVPLSWG operator from Section 3.2. This aggregated matrix is further normalized using Equations (12) and (13).

Table 1 shows the evaluation matrix given by all three experts and also the resultant aggregated matrix formed using the newly proposed IVPLSWG operator. The rating is done using IVPLTS information and the aggregated matrix that yield probability values as [0, 0], are converted to [0.1, 0.1] for better implementation of the ranking procedure. By making this conversion, *divide by zero* issues can be avoided. Though some readers argue about the reliability of the data, such conversion helps in the implementation context and preserves the integrity of the data.

The criteria weights are calculated using the proposed IVPLSV procedure from Section 3.4.

Table 2 depicts the evaluation matrix by the three DMs. The criteria are rated using IVPLTS information. The mean value of each criterion is estimated using Equations (22)–(23), and the value is given by (0.64, 1.25, 1.07, 2.23). The variance of given criteria are computed by Equations (24)–(25), and obtained as follows: (0.068, 0.5, 0.084, 0.16). Finally, Equation (19) is applied to evaluate the criteria weight as (0.1, 0.6, 0.1, 0.2).

Next, the renewable energy source are ranked using the developed IVPLTS-based VIKOR approach from Section 3.4.

The PIS and NIS values are computed utilizing Equations (20)–(21). The criteria C_2 and C_4 are benefit type, while C_1 and C_3 are cost type. Table 3 represents the PIS and NIS values for RES selection.

Table 4 shows the values of S, R, and Q parameters for the rank estimation. These parameters are estimated using Equations (28)–(34).

For estimating the merit function (Q), the parameters viz., S^*, R^*, S^-, and R^- are needed, which are given by 0.4683, 0.1356, 0.9664, and 0.6293. The final merit function is estimated by taking mean of Q^l and Q^u.

Next, sensitivity analysis is also performed by using different values for v in the range 0 to 1 to realize the effect of uncertainty and fuzziness on the method.

Table 5 shows the sensitivity analysis of the proposed IVPLTS-based VIKOR method. From the analysis, we infer that the final preference order is $M_3 > M_2 > M_1$ and M_3 is the compromise solution, which means wind energy is the optimal renewable energy source. The proposed IVPLTS-based VIKOR method is unaffected by vagueness and imprecision and is highly robust.

Table 1. Decision matrix with IVPLTS for renewable energy source (RES) selection.

DMs	Managers	Assessment criteria			
		C_1	C_2	C_3	C_4
e₁	M_1	{1, [0.2, 0.4]} {2, [0.3, 0.4]} {3, [0.32, 0.44]}	{4, [0.4, 0.5]} {2, [0.3, 0.4]} {3, [0.33, 0.42]}	{3, [0.4, 0.5]} {2, [0.3, 0.4]} {4, [0.44, 0.52]}	{2, [0.4, 0.5]} {3, [0.37, 0.45]} {4, [0.33, 0.43]}
	M_2	{2, [0.35, 0.42]} {3, [0.42, 0.52]} {4, [0.36, 0.46]}	{4, [0.42, 0.52]} {5, [0.36, 0.44]} {3, [0.38, 0.45]}	{4, [0.36, 0.44]} {3, [0.48, 0.55]} {5, [0.3, 0.4]}	{4, [0.5, 0.6]} {3, [0.38, 0.42]} {5, [0.42, 0.54]}
	M_3	{4, [0.37, 0.42]} {3, [0.33, 0.4]} {2, [0.28, 0.38]}	{5, [0.54, 0.62]} {4, [0.34, 0.42]} {2, [0.44, 0.5]}	{4, [0.48, 0.54]} {3, [0.38, 0.42]} {2, [0.29, 0.38]}	{3, [0.47, 0.54]} {4, [0.4, 0.5]} {5, [0.52, 0.64]}
e_2	M_1	{4, [0.5, 0.7]} {3, [0.3, 0.5]} {3, [0, 0]}	{4, [0.7, 0.9]} {2, [0.1, 0.3]} {2, [0, 0]}	{4, [0.7, 0.9]} {3, [0.1, 0.3]} {3, [0, 0]}	{5, [0.5, 0.7]} {3, [0.3, 0.5]} {3, [0, 0]}
	M_2	{3, [0.7, 0.9]} {5, [0.1, 0.3]} {3, [0, 0]}	{3, [0.4, 0.6]} {4, [0.15, 0.35]} {2, [0.15, 0.35]}	{2, [0.4, 0.6]} {3, [0.15, 0.35]} {1, [0.15, 0.35]}	{3, [0.7, 0.9]} {4, [0.1, 0.3]} {3, [0, 0]}
	M_3	{3, [0.5, 0.7]} {4, [0.3, 0.5]} {3, [0, 0]}	{3, [0.65, 0.85]} {4, [0.15, 0.35]} {3, [0, 0]}	{5, [0.23, 0.43]} {4, [0.23, 0.43]} {3, [0.23, 0.43]}	{4, [0.7, 0.9]} {6, [0.1, 0.3]} {4, [0, 0]}
e_3	M_1	{2, [0.4, 0.5]} {3, [0.38, 0.44]} {1, [0.48, 0.58]}	{3, [0.4, 0.5]} {2, [0.35, 0.44]} {4, [0.48, 0.54]}	{3, [0.4, 0.52]} {4, [0.38, 0.44]} {5, [0.52, 0.62]}	{4, [0.44, 0.51]} {5, [0.4, 0.5]} {3, [0.38, 0.44]}
	M_2	{2, [0.44, 0.54]} {3, [0.36, 0.44]} {4, [0.4, 0.5]}	{4, [0.4, 0.5]} {3, [0.36, 0.42]} {2, [0.28, 0.35]}	{4, [0.36, 0.42]} {3, [0.38, 0.48]} {2, [0.42, 0.52]}	{5, [0.42, 0.54]} {4, [0.4, 0.5]} {3, [0.36, 0.46]}
	M_3	{3, [0.37, 0.48]} {4, [0.44, 0.52]} {5, [0.45, 0.54]}	{4, [0.42, 0.52]} {5, [0.44, 0.54]} {3, [0.36, 0.44]}	{3, [0.38, 0.44]} {4, [0.4, 0.5]} {5, [0.46, 0.54]}	{4, [0.46, 0.58]} {2, [0.53, 0.62]} {1, [0.26, 0.35]}
e_{123}	M_1	{2, [0.35, 0.52]} {3, [0.33, 0.44]} {3, [0.1, 0.1]}	{4, [0.47, 0.60]} {2, [0.16, 0.23]} {3, [0.1, 0.1]}	{3, [0.47, 0.61]} {3, [0.24, 0.35]} {4, [0.1, 0.1]}	{4, [0.44, 0.56]} {3, [0.36, 0.48]} {3, [0.1, 0.1]}
	M_2	{2, [0.47, 0.58]} {3, [0.26, 0.41]} {4, [0.1, 0.1]}	{4, [0.41, 0.53]} {4, [0.28, 0.40]} {2, [0.23, 0.38]}	{4, [0.37, 0.47]} {3, [0.31, 0.45]} {3, [0.36, 0.43]}	{4, [0.52, 0.65]} {4, [0.26, 0.41]} {4, [0.1, 0.1]}
	M_3	{3, [0.40, 0.52]} {4, [0.36, 0.48]} {3, [0.1, 0.1]}	{4, [0.52, 0.64]} {4, [0.3, 0.44]} {3, [0.1, 0.1]}	{4, [0.35, 0.46]} {4, [0.33, 0.45]} {3, [0.33, 0.45]}	{4, [0.53, 0.65]} {4, [0.29, 0.47]} {3, [0.1, 0.1]}
e_{123}^*	M_1	{2, [0.42, 0.62]} {3, [0.31, 0.42]} {3, [0.09, 0.09]}	{4, [0.51, 0.65]} {2, [0.15, 0.21]} {3, [0.09, 0.89]}	{3, [0.35, 0.45]} {3, [0.16, 0.23]} {4, [0.06, 0.06]}	{4, [0.30, 0.38]} {3, [0.22, 0.30]} {3, [0.06, 0.06]}
	M_2	{2, [0.57, 0.70]} {3, [0.24, 0.39]} {4, [0.09, 0.09]}	{4, [0.44, 0.58]} {4, [0.26, 0.38]} {2, [0.21, 0.34]}	{4, [0.28, 0.35]} {3, [0.21, 0.31]} {3, [0.23, 0.27]}	{4, [0.35, 0.43]} {4, [0.52, 0.25]} {4, [0.06, 0.06]}
	M_3	{3, [0.49, 0.62]} {4, [0.34, 0.44]} {3, [0.09, 0.09]}	{4, [0.56, 0.69]} {4, [0.28, 0.41]} {3, [0.09, 0.09]}	{4, [0.26, 0.34]} {4, [0.22, 0.30]} {3, [0.20, 0.29]}	{4, [0.35, 0.44]} {4, [0.18, 0.29]} {3, [0.06, 0.06]}

Note. The IVPLTS information presented in the table follows Remark 3 for representation. When Equations (12)–(13) are applied to e_{123}^*, the associated probability values become unity.

Table 2. Assessment of criteria weight matrix for RES selection.

DMs	Assessment Criteria			
	C_1	C_2	C_3	C_4
e_1	$\{2, [0.2, 0.3]\}$	$\{2, [0.4, 0.5]\}$	$\{2, [0.3, 0.4]\}$	$\{4, [0.42, 0.52]\}$
e_2	$\{1, [0.42, 0.54]\}$	$\{2, [0.36, 0.42]\}$	$\{3, [0.34, 0.4]\}$	$\{4, [0.46, 0.55]\}$
e_3	$\{2, [0.44, 0.5]\}$	$\{4, [0.46, 0.58]\}$	$\{3, [0.42, 0.52]\}$	$\{5, [0.52, 0.6]\}$
e_1	$[0.4, 0.6]$	$[0.8, 1]$	$[0.6, 0.8]$	$[1.68, 2.08]$
e_2	$[0.42, 0.54]$	$[0.72, 0.84]$	$[1.02, 1.2]$	$[1.84, 2.20]$
e_3	$[0.88, 1]$	$[1.84, 2.32]$	$[1.26, 1.56]$	$[2.60, 3]$

Note. IVPLTS information by each DM follows Remark 3 for representation.

Table 3. Estimation of ideal solution.

Ideal Solution	Assessment Criteria			
	C_1	C_2	C_3	C_4
PIS	$\{2, [0.42, 0.62]\}$ $\{3, [0.24, 0.39]\}$ $\{3, [0.09, 0.09]\}$	$\{4, [0.56, 0.691]\}$ $\{4, [0.28, 0.41]\}$ $\{2, [0.21, 0.33]\}$	$\{3, [0.35, 0.45]\}$ $\{3, [0.16, 0.23]\}$ $\{4, [0.06, 0.06]\}$	$\{4, [0.35, 0.44]\}$ $\{4, [0.18, 0.29]\}$ $\{4, [0.06, 0.06]\}$
NIS	$\{3, [0.49, 0.62]\}$ $\{4, [0.34, 0.44]\}$ $\{4, [0.09, 0.09]\}$	$\{4, [0.44, 0.58]\}$ $\{2, [0.15, 0.21]\}$ $\{3, [0.09, 0.09]\}$	$\{4, [0.28, 0.35]\}$ $\{4, [0.22, 0.3]\}$ $\{3, [0.23, 0.27]\}$	$\{4, [0.3, 0.4]\}$ $\{3, [0.22, 0.3]\}$ $\{3, [0.06, 0.06]\}$

Note. The PIS and NIS produce a single value as result, the IVPLTS information that is corresponding to this single value is presented here. Calculation is made for each instance to better handle uncertainty and imprecision in preference information.

Table 4. Estimation of parameters S, R, and Q under interval context.

Parameters	Renewable Energy Sources		
	M^1	M^2	M^3
S^l	0.5881	0.4683	0.5620
S^u	0.9664	0.8685	0.5620
R^l	0.5171	0.1356	0.2439
R_u	0.6293	0.4302	0.2439
Q	0.7533	0.3500	0.1789

Table 5. Sensitivity analysis for proposed IVPLTS-VIKOR method.

v Parameter	Merit Function (Q)			Ranking Order
0.1	**0.8598**	0.3087	0.2112	$M_3 \succ M_2 \succ M_1$
0.2	0.8332	0.3190	0.2031	$M_3 \succ M_2 \succ M_1$
0.3	0.8066	0.3293	0.1950	$M_3 \succ M_2 \succ M_1$
0.4	0.7799	0.3397	0.1869	$M_3 \succ M_2 \succ M_1$
0.5	0.7533	0.3500	0.1789	$M_3 \succ M_2 \succ M_1$
0.6	0.7267	0.3604	0.1708	$M_3 \succ M_2 \succ M_1$
0.7	0.7001	0.3707	0.1627	$M_3 \succ M_2 \succ M_1$
0.8	0.6735	0.3810	0.1546	$M_3 \succ M_2 \succ M_1$
0.9	0.6469	0.3914	0.1465	$M_3 \succ M_2 \succ M_1$

Finally, the developed approach is compared with existing methods to validate the strength and weakness. We refer to Section 5 for discussion.

5. Comparative Analysis

In this section, we further compare the developed framework with existing ranking methods to realize the strengths and weaknesses. In order to make the comparison sensible, we concentrate on the environment as well as on the method and hence, IVPLTS-based TOPSIS [37], PLTS-based TOPSIS method [4], HFLTS-based VIKOR method [45], and HFLTS-based TOPSIS methods [12] are taken for comparison with the proposed decision framework.

Before getting into the further investigation, let us review some differences between VIKOR and TOPSIS method:

- The VIKOR method uses linear normalization, which is independent of the measuring unit of the criteria. TOPSIS, on the other hand, uses vector normalization, and the measuring unit of the criteria is essential for the study.
- The VIKOR method adopts aggregation, which aggregates all criteria, their relative importance, and balances the satisfaction rates, while theTOPSIS method concentrates on the distance between positive and negative ideal solution without paying much attention to relative importance.
- In the VIKOR method, the alternative, which gains the first rank, is the one that is closest to the ideal solution. While in the TOPSIS method, the alternative, which is ranked first, is better in terms of the rank index but is generally not the closest to the ideal solution.

With a view of finding alternatives close to an ideal solution, we extend the VIKOR method in our proposal. Table 6 presents the different ranking outcome produced by other decision-making methods.

Table 6. Investigation of a ranking order by different ranking methods.

Methods	Renewable Energy Sources for Evaluation			Ranking Order	Compromise Solution
	M_1	M_2	M_3		
IVPLTS-based VIKOR	**3**	2	1	$M_3 > M_2 > M_1$	M_3
IVPLTS-based TOPSIS [37]	3	2	1	$M_3 > M_2 > M_1$	M_3
PLTS-based TOPSIS [4]	2	3	1	$M_3 > M_1 > M_2$	M_1, M_3
PLTS-based aggregation [4]	2	3	1	$M_3 > M_1 > M_2$	M_1, M_3
HFLTS-based VIKOR [45]	2	1	3	$M_2 > M_1 > M_3$	M_2
HFLTS-based TOPSIS [12]	2	1	3	$M_2 > M_1 > M_3$	M_2

Note: For PLTS-based ranking, the average probability values are taken. For HFLTS-based ranking, the linguistic terms are alone considered, and the relative importance of each term is neglected. As mentioned in [18], it is lucid that diverse linguistic models use different preference structures, and hence, comparison of such models is difficult.

Figure 2 depicts the correlation between the proposed IVPLTS-based VIKOR and various existing methods using Spearman rank correlation method [46]. In the inference, we gain that the developed IVPLTS-based VIKOR approach is moderately consistent with different existing methods. The methods proposed by Pang et al. [4] produces a ranking order, which causes confusion between M_1 and M_3 due to some loss of information in the occurring probability values. Further, the methods discussed in [42] and [12] produce a different ranking order and cause negative correlation values with the proposed method due to the ignorance of occurring probability values.
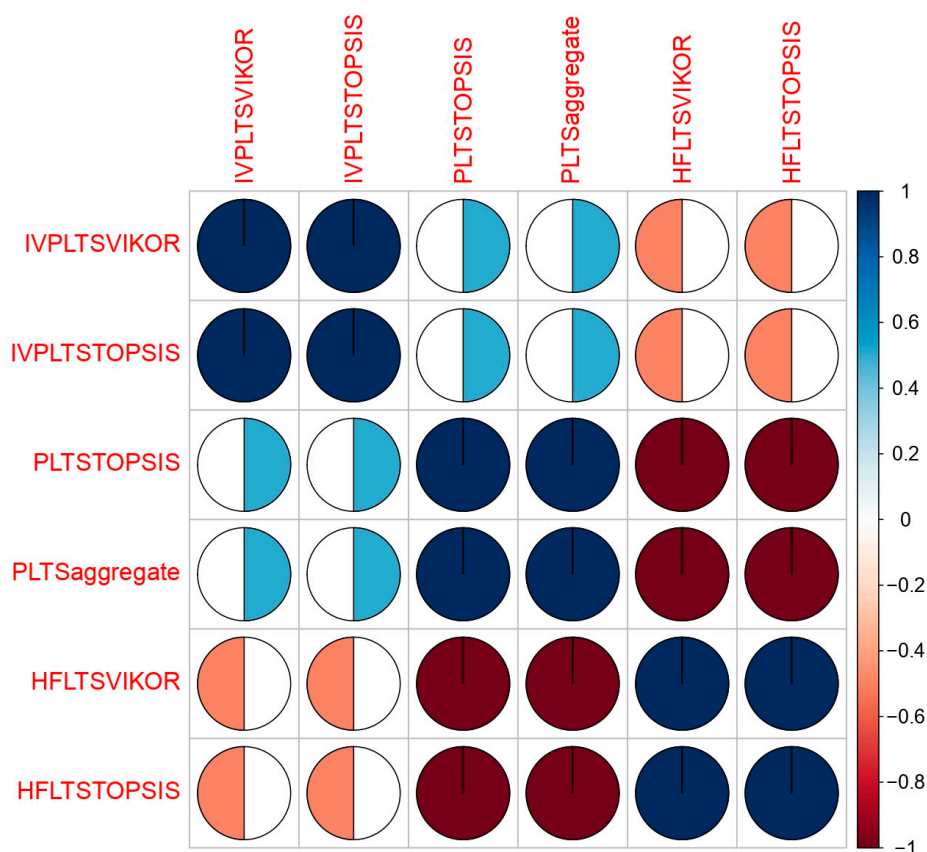
Figure 2. Spearman correlation for different ranking methods.

From Table 7, we investigate the innovation of the proposed IVPLTS-based decision framework, and some details are listed below:

- The proposed IVPLTS concept is a direct extension to the PLTS concept [4], which associates interval values for the occurring probability of each linguistic term. Hence, we name the concept IVPLTS. Though Dong et al. [19] initiated the idea of interval value-based distribution assessment, it suffers from significant weaknesses, as mentioned earlier (see Section 1). On the other hand, in this paper, the proposed IVPLTS concept overcomes these weaknesses by (a) retaining the linguistic terms as it is, without adopting any conversion procedure. This mitigates the potential loss of information and originality. (b) Also, a new scientific decision framework is developed on IVPLTS context, which performs decision making in a much more sensible and rational manner than the arithmetic (expectation measure) based ranking used in Dong et al.'s [19] framework. (c) As mentioned by Dong et al. [19], another weakness of their proposal is the inability to compare the linguistic models because of the lack of criteria for evaluation. However, in this paper, we have surveyed the literature and identified suitable factors from both theoretic and numeric perspective for comparing the developed framework with existing methods.
- Further, the work in [37] is a close counterpart to the proposed decision framework, and it suffers from the following weaknesses: (a) Direct elicitation of criteria weights causes error and inaccuracy in decision-making process; (b) operational laws adopt transformation measures, which increase the computational overhead; (c) the ranking approach is affected by the rank reversal issue; and (d) the rank value set is narrow and it causes trouble in backup management.
- The newly proposed IVPLSWG aggregation operator aggregates DMs' IVPLTS preferences in two folds viz., (a) LTs and (b) the relative importance of the LTs (as probability values in interval fashion), which produces a rational and sensible aggregated matrix without loss of originality. The other methods directly use this aggregated matrix without using any new aggregation method.

- The proposed IVPLSV procedure is an extension to the SV procedure that is used for evaluating the criteria weights. The other methods directly use these weight values for evaluation.

- The proposed IVPLTS-based VIKOR method constructs broad and sensible rank value set, which promotes easy and effective backup management while the other methods generate narrow rank value set and sometimes yield negative values, which affects the process of backup maintenance of alternatives.

- Motivated by the numerical factors introduced by Rodriguez et al. [48], we investigate the effectiveness of the developed method in terms of adequacy changes to alternatives, criteria, computational complexity, and scalability. The other factors considered for investigation are inspired by intuition.

- The results indicate that the proposed IVPLTS-based VIKOR method remains unaffected by rank reversal concern, by staying unaffected to adequate changes on alternatives (three new test cases formed by repeating each of the three alternatives) and criteria (four new test cases formed by repeating each of the criteria). The ranking order of $M_3 > M_2 > M_1$ is maintained even after the repetition of alternatives and criteria.This also ensures the stability of the proposed ranking method. The proposed decision framework also satisfies the scalability test by adhering to the Miller principle [47,49] of preference information to objects.

- Though the developed method enjoys such elegant strengths, it does suffer from some weakness, as mentioned in Table 7. The key weakness of the proposal is the computational complexity, which comes to $O(nmk(i + i))$, where n, m, and k are the number of alternatives, criteria, and instances, respectively, for each pair of alternative criteria, and i is the corresponding probability value for each term.

To further realize the power of the developed method, a simulation study is conducted with 300 decision matrices of order 4×4 that follow IVPLTS information. The constraints are adapted from Definition 7. *Each matrix is fed as input to the developed framework, and the criteria weights are considered from the above-mentioned example.* The rank value set is obtained for all 300 matrices, and the standard deviation is determined for each rank value set. These 300 matrices, along with criteria weight values, are also given as input to [37], and the standard deviation is computed for each matrix. These values are depicted in Figure 3, and this clearly shows that the developed method produces broad and sensible rank value set, which helps DMs for the rational decision-making procedure. Also, the adequacy test for alternatives and criteria and partial adequacy test for alternatives and criteria are determined for all 300 matrices, and the outcomes are tabulated in Table 8. From Table 8, it can be inferred that the developed method is less affected by rank reversal issue than its counterpart (method [37]).

Table 7. Investigation of salient features and innovations: Proposed IVPLTS decision framework vs. others.

Context	IVPLTS-VIKOR (Proposed)	IVPLTS-TOPSIS [37]	PLTS-TOPSIS [4]	PLTS-Aggregation [4]	HFLTS-VIKOR [45]	HFLTS-TOPSIS [12]
Input	IVPLTS information	IVPLTS information	PLTS information	PLTS information	HFLTS information	HFLTS information
Aggregation	Using the proposed IVPLSWG operator	IVPLTS-based weighted arithmetic and geometry operator	Average operator		No	
Probability values	Considered as interval numbers		Considered as single values		Not considered	
Criteria weights	Estimated using the proposed IVPLSV method.	Use the same weight values as of IVPLTS framework.				
Rank value set	Broad and sensible.	Narrow	Narrow with some negative values.		Narrow	
Backup management	Done sensibly using the rank value set.	Since the rank value set is narrow, the alternative backup becomes a little tough.	Because of the negative value set, backup management becomes irrational.		Since the rank value set is narrow, the alternative backup becomes a little tough.	
Adequacy test(alternatives)	The method is affected when adequate amendments are applied to the options.		The method is affected by adequate changes to alternatives.		The method is unaffected by adequate changes to alternatives.	The method is affected by adequate changes to alternatives.
Adequacy test (criteria)	The method remains unaffected by adequate changes to criteria.	The method is affected when adequate changes are made to the criteria.	The method is affected when adequate modifications are used to the criteria.		The method is unaffected by adequate changes to criteria.	The method is affected by adequate changes to criteria.
Scalability	The method is scalable and follows the principle of Saaty et al. [47].					
Computation complexity	$O(mnk(i+i))$		$O(mnk(i))$		$O(mnk)$	

Table 7. *Cont.*

Context	IVPLTS-VIKOR (Proposed)	IVPLTS-TOPSIS [37]	PLTS-TOPSIS [4]	PLTS-Aggregation [4]	HFLTS-VIKOR [45]	HFLTS-TOPSIS [12]
Strengths	• IVPLTS is a generalization of the PLTS concept. • Partial ignorance is accepted. • Preference information is aggregated sensibly using IVPLSWG operator. • Criteria weights are estimated objectively. • Handles uncertainty better by offering DMs a range of values to rate each linguistic term. • The proposed approach is: (a) *robust* in nature (sensitivity analysis), (b) *moderately consistent* with existing methods and (c) *stable* (adequacy checks) to rank reversal issue.	• IVPLTS information is used, which is a generalization of PLTS. • Partial ignorance is allowed. • Possibility degree method is used for the comparison of IVPLEs.	• PLTS is a generalization of computing with words concept. • Partial ignorance is accepted.		• HFLTS is computationally feasible. • Partial ignorance is accepted.	
Weakness	• Computationally complex, because of the probability concept. • Probability values for each LTs must be collected.	• Computationally complex. • Probability value must be collected for each term. • DMs directly give their weight values for each criterion, and this causes inaccuracies in the MCDM process	• Computationally complex, as the information regarding the importance of linguistic terms must be presented. • Uncertainty is not handled properly by offering single value as a probability for each LT.		• The relative significance of each LT is ignored. • Balancing of partial ignorance without weighting of linguistic terms causes some loss of originality.	
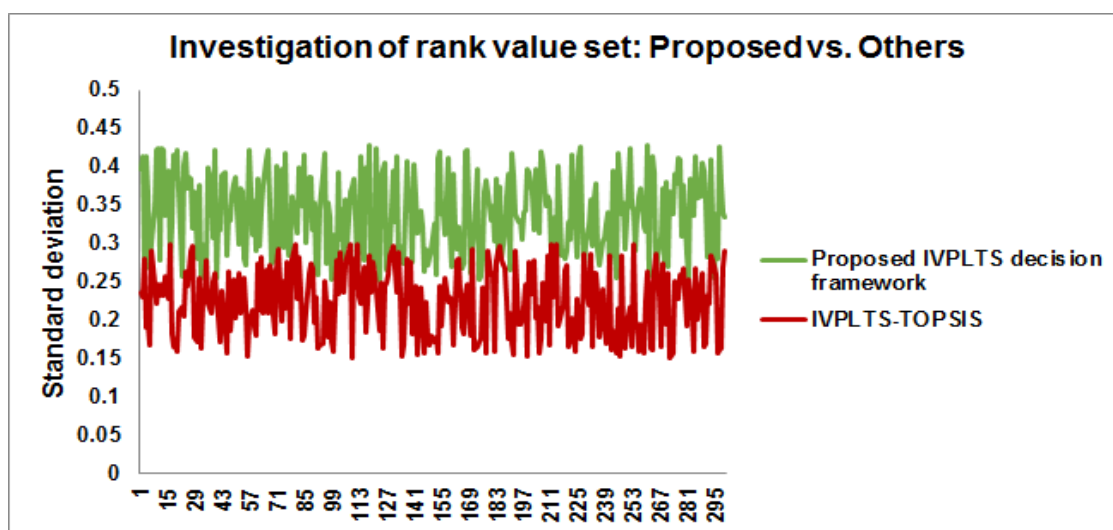
Figure 3. Analysis of the rank value set using standard deviation: Proposed vs. others.

Table 8. Investigation of adequacy test: Proposed vs. others.

Context	Method(s)	
	Proposed framework (%)	**IVPLTS-TOPSIS [37] (%)**
Adequacy test (alternatives)	69.00	68.00
Adequacy test (criteria)	92.33	87.67
Partial adequacy test (alternatives)	92.33	85.67
Partial adequacy test (criteria)	100	93.33

Note. These percentage values are determined by calculating the number of matrices that satisfy the test condition out of 300 decision matrices. Adequacy test is inspired by [48] and partial adequacy test checks for the change of highly preferred alternative (first place) in the ranking order.

6. Conclusions

Prioritizing and evaluating optimal RES is considered a complex MCDM process in the energy region. For this concern, various informative and conflicting criteria may be measured to choose the desirable RES alternative in uncertain circumstances. In this paper, we propose a new concept called interval-valued probabilistic linguistic term set (IVPLTS) to handle the weakness of PLTS. The IVPLTS concept is further extended to GDM process by proposing (a) a new aggregation operator called IVPLSWG to fuse DMs' IVPLTS information, (b) a new criteria weight evaluation procedure called IVPLSV method for determining the weights of the criteria, and (c) a new ranking method called IVPLTS-based VIKOR method under IVPLTS context for choosing the best renewable energy source alternative. Finally, the developed framework is utilized to solve the renewable energy source selection problem for realizing the practicality of the framework.

Some managerial implications that can be derived are as follows:

➢ The proposed decision framework under the new IVPLTS environment is a "ready-made" framework for rational MCGDM under uncertain situations.
➢ The IVPLTS is a new data structure for providing preference information, which handles uncertainty and vagueness in a much sensible and rational manner. Managers, with little training, can provide effective preference information for sensible decision making.
➢ Very often, managerial decision making cannot be accurate by using simple ad-hoc methods, and hence, there is a substantial need for systematic scientific methods.

➢　The developed framework is flexible and ready-to-use, which helps DMs to make better decisions under uncertain situations. Also, the researchers can use such frameworks as an aid for proper planning and decision making.

As a part of the plan for future research, we address the weakness of the proposed framework and also take efforts to propose a new aggregation operator, weight estimation operation under IVPLTS domain. Also, imputation of missing values and checking of consistency are planned under IVPLTS context. Finally, the proposed concept could be integrated with theories like prospect theory [50,51], possibilistic theory [52], evidence theory [53], regret theory [54], etc., for solving practical issues like sensor fault detection, medical diagnosis, weather forecasting, business analytics, etc.

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Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Campisi, D.; Gitto, S.; Morea, D. An evaluation of energy and economic efficiency inresidential buildings sector:Amulti-criteria analisys on an Italian case study. *Int. J. Energy Econ. Policy* **2018**, *8*, 185–196.
2. Büyüközkan, G.; Güleryüz, S. An integrated DEMATEL-ANP approach for renewable energy resources selection in Turkey. *Int. J. Prod. Econ.* **2016**, *182*, 435–448.
3. Poudyal, R.; Loskot, P.; Nepal, R.; Parajuli, R.; Khadka, S.K. Mitigating the current energy crisis in Nepal with renewable energy sources. *Renew. Sustain. Energy Rev.* **2019**, *116*, 109388. [CrossRef]
4. Pang, Q.; Wang, H.; Xu, Z. Probabilistic linguistic term sets in multi-attribute group decision making. *Inf. Sci.* **2016**, *369*, 128–143. [CrossRef]
5. Xu, Z. *LinguisticDecisionMaking: Theory and Methods*; Science Press: Beijing, China, 2012. [CrossRef]
6. Herrera, F.; Herrera-Viedma, E.; Verdegay, J.L. Linguistic measures based on fuzzy coincidence for reaching consensusin group decision making. *Int. J. Approx. Reason.* **1997**, *16*, 309–334. [CrossRef]
7. Herrera, F.; Herrera-Viedma, E.; Verdegay, J.L. A rational consensus model in group decision making using linguistic assessments. *Fuzzy Sets Syst.* **1997**, *88*, 31–49. [CrossRef]
8. Herrera, F.; Herrera-Viedma, E.; Verdegay, J.L. A model of consensus in group decision making under linguistic assessments. *Fuzzy Sets Syst.* **1996**, *78*, 73–87. [CrossRef]
9. Xu, Y.; Da, Q. A method for multiple attribute decision making within complete weightin formation underun certain linguistic environment. *Knowl. Based Syst.* **2008**, *21*, 837–841. [CrossRef]
10. Xu, Z. An approach based on the uncertain LOWG and induced uncertain LOWG operators to group decision making with uncertain multiplicative linguistic preference relations. *Decis. Support Syst.* **2006**, *41*, 488–499. [CrossRef]
11. Rodriguez, R.M.; Martinez, L.; Herrera, F. Hesitant fuzzy linguistic term sets for decision making. *IEEE Trans. Fuzzy Syst.* **2012**, *20*, 109–119. [CrossRef]
12. Beg, I.; Rashid, T. TOPSIS for hesitant fuzzy linguistic term sets. *Int. J. Intell. Syst.* **2014**, *29*, 495–524. [CrossRef]
13. Zhu, B.; Xu, Z. Consistency measures for hesitant fuzzy linguistic preference relations. *IEEE Trans. Fuzzy Syst.* **2014**, *22*, 35–45. [CrossRef]

14. Rodríguez, R.M.; Martínez, L.; Herrera, F. A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Inf. Sci.* **2013**, *241*, 28–42. [CrossRef]
15. Liu, H.; Rodríguez, R.M. A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. *Inf. Sci.* **2014**, *258*, 220–238. [CrossRef]
16. Wang, H. Extended hesitant fuzzy linguistic term sets and their aggregation in group decision making. *Int. J. Comput. Intell. Syst.* **2015**, *8*, 14–33. [CrossRef]
17. Wei, G.; Alsaadi, F.E.; Hayat, T.; Alsaedi, A. Hesitant fuzzy linguistic arithmetic aggregation operators in multiple attribute decision making. *Iran. J. Fuzzy Syst.* **2015**, *13*, 1–16.
18. Zhang, G.; Dong, Y.; Xu, Y. Consistency and consensus measures for linguistic preference relations based on distribution assessments. *Inf. Fusion.* **2014**, *17*, 46–55. [CrossRef]
19. Dong, Y.; Wu, Y.; Zhang, H.; Zhang, G. Multi-granular unbalanced linguistic distribution assessments with interval symbolic proportions. *Knowl. Based Syst.* **2015**, *82*, 139–151. [CrossRef]
20. Bai, C.; Zhang, R.; Qian, L.; Wu, Y. Comparisons of probabilistic linguistic term sets for multi-criteria decision making. *Knowl. Based Syst.* **2016**, *119*, 284–291. [CrossRef]
21. Gou, X.; Xu, Z. Novel basic operational laws for linguistic terms, hesitant fuzzy linguistic term sets and probabilistic linguistic term sets. *Inf. Sci.* **2016**, *372*, 407–427. [CrossRef]
22. Zhai, Y.; Xu, Z.; Liao, H. Probabilistic linguistic vector-term set and its application in group decision making with multi-granular linguistic information. *Appl. Soft Comput.* **2016**, *49*, 801–816. [CrossRef]
23. Zhang, Y.; Xu, Z.; Wang, H.; Liao, H. Consistency-based risk assessment with probabilistic linguistic preference relation. *Appl. Soft Comput.* **2016**, *49*, 817–833. [CrossRef]
24. Lin, M.; Xu, Z.; Zhai, Y.; Yao, Z. Multi-attribute group decision-making under probabilistic uncertain linguistic environment. *J. Oper. Res. Soc.* **2018**, *69*, 1–14. [CrossRef]
25. Dinçer, H.; Yüksel, S. Multidimensional evaluation of global investments on the renewable energy with the integrated fuzzy decision-making model under the hesitancy. *Int. J. Energy Res.* **2019**, *43*, 1775–1784. [CrossRef]
26. Yuan, J.; Li, C.; Li, W.; Liu, D.; Li, X. Linguistic hesitant fuzzy multi-criterion decision-making for renewable energy:Acase study in Jilin. *J. Clean. Prod.* **2018**, *172*, 3201–3214. [CrossRef]
27. Elzarka, H.M.; Yan, H.; Chakraborty, D. Avague set fuzzy multi-attribute group decision-making model for selecting on site renewable energy technologies for institutional owners of constructed facilities. *Sustain. Cities Soc.* **2017**, *35*, 430–439. [CrossRef]
28. Çolak, M.; Kaya, İ. Prioritization of renewable energy alternatives by using an integrated fuzzy MCD Mmodel: A real case application for Turkey. *Renew. Sustain. Energy Rev.* **2017**, *80*, 840–853. [CrossRef]
29. Karunathilake, H.; Hewage, K.; Mérida, W.; Sadiq, R. Renewable energy selection for net-zero energy communities: Life cycle based decision making under uncertainty. *Renew. Energy* **2019**, *130*, 558–573. [CrossRef]
30. Ghenai, C.; Albawab, M.; Bettayeb, M. Sustainability indicators for renewable energy systems using multi-criteria decision-making model and extended SWARA/ARAS hybrid method. *Renew. Energy* **2020**, *146*, 580–597. [CrossRef]
31. Boran, F.E.; Boran, K.; Menlik, T. The Evaluation of Renewable Energy Technologies for Electricity Generation in Turkey Using Intuitionistic Fuzzy TOPSIS. *Energy Sources Part B Econ. Plan. Policy* **2012**, *7*, 81–90. [CrossRef]
32. Mousavi, M.; Gitinavard, H.; Mousavi, S. A soft computing based-modified ELECTRE model for renewable energy policy selection with unknown information. *Renew. Sustain. Energy Rev.* **2017**, *68*, 774–787. [CrossRef]
33. Mishra, A.R.; Kumari, R.; Sharma, D.K. Intuitionistic fuzzy divergence measure-based multi-criteria decision-making method. *Neural Comput Appl.* **2019**, *31*, 2279–2294. [CrossRef]
34. Rani, P.; Mishra, A.R.; Pardasani, K.R.; Mardani, A.; Liao, H.; Streimikiene, D. A novel VIKOR approach based on entropy and divergence measures of Pythagorean fuzzy sets to evaluate renewable energy technologies in India. *J. Clean. Prod.* **2019**, *238*, 117936. [CrossRef]
35. Wang, Y.; Xu, L.; Solangi, Y.A. Strategic renewable energy resources selection for Pakistan:Basedon SWOT-Fuzzy AHP approach. *Sustain. Cities Soc.* **2020**, *52*, 101861. [CrossRef]
36. Wu, Y.; Xu, C.; Ke, Y.; Tao, Y.; Li, X. Portfolio optimization of renewable energy projects undertype-2 fuzzy environment with sustainability perspective. *Comput. Ind. Eng.* **2019**, *133*, 69–82. [CrossRef]
37. Bai, C.; Zhang, R.; Shen, S.; Huang, C.; Fan, X. Interval-valued probabilistic linguistic term sets in multi-criteria group decision making. *Int. J. Intell. Syst.* **2018**, *33*, 1301–1321. [CrossRef]

38. Opricovic, S.; Tzeng, G.H. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *Eur. J. Oper. Res.* **2004**, *156*, 445–455. [CrossRef]

39. Herrera, F.; Herrera-Viedma, E.; Verdegay, J.L. A sequential selection processin group decision making with a linguistic assessment approach. *Inf. Sci.* **1995**, *239*, 223–239. [CrossRef]

40. Wang, H.; Xu, Z. Interactive algorithms for improving incomplete linguistic preference relations based on consistency measures. *Appl. Soft Comput.* **2016**, *42*, 66–79. [CrossRef]

41. Xia, M.; Xu, Z. Entropy/crossentropy-based group decision making under intuitionistic fuzzy environment. *Inf. Fusion.* **2012**, *13*, 31–47. [CrossRef]

42. Liu, S.; Chan, F.T.S.; Ran, W. Decision making for the selection of cloud vendor:An improved approach under group decision-making with integrated weights and objective/subjective attributes. *Expert Syst. Appl.* **2016**, *55*, 37–47. [CrossRef]

43. Rao, R.V.; Patel, B.K.; Parnichkun, M. Industrial robot selection using a novel decision making method considering objective and subjective preferences. *Rob. Auton. Syst.* **2011**, *59*, 367–375. [CrossRef]

44. Sayadi, M.K.; Heydari, M.; Shahanaghi, K. Extension of VIKOR method for decision making problem with interval numbers. *Appl. Math. Model.* **2009**, *33*, 2257–2262. [CrossRef]

45. Liao, H.; Xu, Z.; Zeng, X.J. Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making. *IEEE Trans. Fuzzy Syst.* **2014**, *23*, 1–14. [CrossRef]

46. Spearman, C. TheProof and Measurement of Association between Two Things. *Am. J. Psychol.* **1904**, *15*, 72–101. [CrossRef]

47. Saaty, T.L.; Ozdemir, M.S. Why the magic number seven plus or minus two. *Math. Comput. Model.* **2003**, *38*, 233–244. [CrossRef]

48. LimaJunior, F.R.; Osiro, L.; Carpinetti, L.C.R. A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection. *Appl. Soft Comput.* **2014**, *21*, 194–209. [CrossRef]

49. Miller, G. The magical numbe rseven, plus or minus two:Some limits on our capacity for processing information. *Psychol. Rev.* **1956**, *101*, 343–352. [CrossRef]

50. Tversky, A.; Kahneman, D. Advances in prospect-theory—Cumulative representation of uncertainty. *J. Risk Uncertain.* **1992**, *5*, 297–323. [CrossRef]

51. Kahneman, D.; Tversky, A. Prospecttheory:Ananalysis of decision under risk. *Econometrica* **1979**, *47*, 263–292. [CrossRef]

52. Cremona, C.; Gao, Y. The possibilistic reliability theory:The oretical aspects and applications. *Struct. Saf.* **1997**, *19*, 173–201. [CrossRef]

53. Hummel, R.A.; Landy, M.S. A statistical view point on the theory of evidence. *IEEE Trans. Pattern Anal. Mach. Intell.* **1988**, *10*, 235–247. [CrossRef]

54. Loomes, G.; Sugden, R. Regret theory: An alternative theory of rational choice under uncertainty. *Econ. J.* **1982**, *92*, 805–824. [CrossRef]