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Development of Algorithms for Effective Resource Allocation among Highway–Rail Grade Crossings: A Case Study for the State of Florida

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Abstract: Smart cities directly rely on a variety of elements, including water, gas, electricity, buildings, services, transportation networks, and others. Lack of properly designed transportation networks may cause different economic and safety concerns. Highway–rail grade crossings are known to be a hazardous point in the transportation network, considering a remarkable number of accidents recorded annually between highway users and trains, and even solely between highway users at highway–rail grade crossings. Hence, safety improvement at highway–rail grade crossings is a challenging issue for smart city authorities, given limitations in monetary resources. In this study, two optimization models are developed for resource allocation among highway–rail grade crossings to minimize the overall hazard and the overall hazard severity, taking into account the available budget limitations. The optimization models are solved by CPLEX to the global optimality. Moreover, some heuristic algorithms are proposed as well. A case study focusing on the public highway–rail grade crossings in the State of Florida is performed to evaluate the effectiveness of the developed optimization models and the solution methodologies. In terms of the computational time, all the solution approaches are found to be effective decision support tools from the practical standpoint. Moreover, the results demonstrate that some of the developed heuristic algorithms can provide near-optimal solutions. Therefore, the smart city authorities can utilize the proposed heuristics as decision support tools for effective resource allocation among highway–rail grade crossings.

Keywords: smart cities; highway–rail grade crossings; resource allocation; optimization; heuristics; crossing hazard

1. Background

Smart city operations are significantly influenced by a variety of elements, including water, gas, electricity, buildings, services, transportation networks, and others [1–9]. Transportation networks play a critical role for the populations inhabiting smart cities. The railroad transportation concept was introduced for the first time in the United States (U.S.) in the early 18th century [10]. There were only a few trains by the middle of the 18th century moving at a fairly low speed. Hence, safety at highway–rail grade crossings did not receive a lot of attention in the U.S. By the end of 1960s, the number of accidents at highway–rail grade crossings started increasing due to rapid industrialization, rail network expansion, and increasing train speed. Therefore, safety at highway–rail grade crossings became a major concern [11]. Although the maximum number of fatalities was recorded in 1928, the

rate of fatalities to rail traffic kept increasing. The rate of highway user fatalities per million train miles comprised 1.13 and 1.95 in 1950 and 1966, respectively. The latter tendencies can be explained due to an increase in the railway and highway traffic along with limited monetary resources for installation of traffic safety devices at highway–rail grade crossings. Hence, the Federal Aid Rail–Highway Crossing Program, as a part of the Federal Highway Act of 1973 (commonly referred to as "the Section 130 Program"), was issued to provide financial resources for safety improvement at highway–rail grade crossings [11].

In 2010, it was reported that there were a total of approximately 255,000 highway–rail grade crossings all around the U.S., with 52% of them open for public use [12]. Each highway–rail grade crossing is a potential location for an accident between a highway user and a train. Furthermore, existence of a highway–rail grade crossing can increase the possibility of accidents that do not involve trains, including the following: (1) rear-end accidents between two vehicles, one of which is stopped at a highway–rail grade crossing, with the other one stopped on the highway; (2) accidents with traffic control devices at highway–rail grade crossings (e.g., signal equipment or signs); and (3) non-collision accidents, where a driver loses control of the vehicle. The analysis of the accident data collected for the highway–rail grade crossings in the U.S. showcases that the minimum number of accidents, injuries, and fatalities was recorded for 2009, while the maximum number was recorded for 2007 [13]. In general, the number of accidents, injuries, and fatalities fluctuated between 2007 and 2017, which means that the highway–rail grade crossing upgrades have not been implemented in the optimal way to produce a decreasing pattern for the number of accidents, injuries, and fatalities from 2007 to 2017.

Based on the information provided by the Federal Railroad Administration (FRA), 841 injuries and 308 fatalities among 2117 accidents were reported at highway–rail grade crossings across the country in 2017, while 54 injuries and 23 fatalities were the consequences of 106 accidents occurred in the State of Florida [13]. The frequent accident occurrence at highway–rail grade crossings in the State of Florida can be explained by the following factors [14]: (1) the State of Florida is the third most populous state in the U.S.; (2) about 201,040 million vehicle miles of travel were recorded in the State of Florida in 2015; (3) the majority of the population travel to work by automobiles; (4) the State of Florida is largely urbanized; and (5) the metropolitan areas in the State of Florida record a high volume of freight and passenger rail traffic. This study aims to improve the safety of railway and highway travelers in smart cities and proposes two optimization models that can assist with resource allocation among highway–rail grade crossings. The objective of the first optimization model is to minimize the overall hazard, while the second optimization model aims to minimize the overall hazard severity.

In order to solve the developed optimization models and to achieve the global optimality, two exact optimization approaches are adopted. Moreover, a group of heuristic algorithms that prioritize highway–rail grade crossings for upgrading based on certain rules are developed to solve the proposed optimization models within a reasonable computational time for the realistic problem instances. A case study, considering the public highway–rail grade crossings in the State of Florida, is used to evaluate the performance of the solution approaches. In particular, the performance of the solution approaches is assessed from two points of view: (1) the solution quality (i.e., the overall hazard and the overall hazard severity at the highway–rail grade crossings after implementation of the suggested countermeasures); and (2) the computational time (so that decisions regarding selection of the highway–rail grade crossings for upgrading and identification of the appropriate upgrading type can be made effectively and in a timely manner). The input data required to conduct the numerical experiments (e.g., the set of highway–rail grade crossings, the set of countermeasures considered for implementation, and the hazard severity categories) will be further described in the manuscript.

The remainder of this paper is structured as follows. In the Section 2, the related literature is reviewed. The resource allocation problem addressed herein is discussed in the Section 3. The developed optimization models and their formulations are described in the Section 4. The adopted exact optimization approaches and the developed heuristic algorithms are explained in the Section 5.

The Section 6 covers the input data description, evaluation of the solution methodologies, and the managerial insights. Conclusions and future research extensions are highlighted in the Section 7.

2. Literature Review

Under this section of the manuscript, two groups of previously conducted studies are comprehensively described: (1) previous safety efforts at highway–rail grade crossings; and (2) resource allocation studies for highway–rail grade crossings. Towards the end of the literature review section, the contributions of this study are clearly stated as well.

2.1. Previous Efforts on Safety at Highway–Rail Grade Crossings

Some of the state Departments of Transportation (DOTs) in the U.S. have conducted a number of studies, aiming to improve safety at highway–rail grade crossings. A total of 13 accident prediction models and hazard prediction models were investigated by the Virginia Highway and Transportation Research Council [15]. In that study, the models were evaluated in terms of the accident prediction ability, and the adoption of models among different states was investigated as well. Moreover, the main accident and hazard model predictors were identified in the study, where the number of vehicles per day and the number of trains per day were recognized as the most commonly used factors. The study classified the identified formulae as follows: (1) relative formulae that rely on the hazard index value; and (2) absolute formulae that rely on the number of accidents. The results from the conducted analysis demonstrated that the U.S. DOT accident prediction formula outperformed the other formulae in terms of ranking the most hazardous highway–rail grade crossings in the State of Virginia.

The Tennessee DOT in collaboration with the University of Memphis developed a set of models to allocate the available monetary resources for safety improvement projects at the highway–rail grade crossings in the State of Tennessee. The objective was to reduce the expected number of accidents and the expected accident severity [16]. The study performed a survey on the nationally recognized methods and the state-specific approaches used for prediction of accidents and for assessing a potential hazard of highway–rail grade crossings. Moreover, a detailed review was conducted on the Tennessee Roadway Information Management System (TRIMS) database, which included some important information about the highway–rail grade crossings in the State of Tennessee. Several heuristic approaches were presented in order to solve the developed mathematical models for resource allocation. Comprehensive numerical experiments were conducted to show the applicability of the proposed methodology.

Weissmann et al. [17] proposed a methodology for prioritizing highway–rail grade crossings in the State of Texas. Some of the previously developed accident and hazard prediction models were reviewed. The study presented the revised Texas priority index to address the limitations, which were reported for the existing Texas priority index. The revised Texas priority index value for a given highway–rail grade crossing was calculated by considering the predicted number of accidents per year and the number of accidents observed over the last 5 years. A comprehensive set of experiments were conducted over 9108 highway–rail grade crossings, with the results demonstrating the superiority of the revised Texas priority index formula in identification of the most hazardous highway–rail grade crossings. Moreover, the methodologies that were used to issue warrants for passive highway–rail grade crossings were reviewed in the study. The results showed that the Florida DOT (FDOT) methodology was stricter than the other methods. It was concluded that active highway–rail grade crossings should be prioritized considering the revised Texas priority index, while passive highway–rail grade crossings should be prioritized based on the revised Texas priority index and the Texas passive crossing index.

The Iowa DOT in collaboration with the Institute for Transportation at Iowa State University conducted a project for the development of a methodology to prioritize the highway–rail grade crossings for safety improvement projects in the State of Iowa [18,19]. The Iowa DOT [18] developed a safety action plan, aiming to reduce the number of accidents at the highway–rail grade crossings in the State of Iowa, to identify the most hazardous highway–rail grade crossings, and to determine specific

engineering solutions to improve safety at the highway–rail grade crossings. The benefit–cost analysis was adopted by the Iowa DOT to allocate the available monetary resources among the highway–rail grade crossings that required safety improvement projects. Hans et al. [19] showed that the following factors should be considered throughout the selection of highway–rail grade crossings for safety improvement projects: (1) demand factor; (2) alternate route factor; and (3) certain other railway- and roadway-related factors. In conclusion, a weighted-index method and an accompanying tool for Microsoft Excel spreadsheets were developed for prioritization of the highway–rail grade crossings in the State of Iowa based on the aforementioned factors.

The Ohio DOT in collaboration with Ohio University and Texas A&M Transportation Institute developed a decision support methodology to assist with selection of highway–rail grade crossings for safety improvement projects [20]. The existing highway–rail grade crossing improvement program in the State of Ohio can be summarized in the following three steps: (1) development of the list of candidate grade crossing locations; (2) diagnostic review of the candidate grade crossing locations; and (3) project implementation. A review of the projects, conducted by other state DOTs and focusing on the adopted accident and hazard prediction formulae, was performed. The review showed that there were a number of factors that other states considered in their models but they were not included in the model by the State of Ohio. These factors included: (1) stopping sight distance; (2) school bus or special vehicle volume; (3) highway traffic speed; (4) proximity of a highway–rail grade crossing to a nearby intersection; and (5) "close call" data. In conclusion, it was recommended to use the U.S. DOT accident prediction formula for resource allocation among the highway–rail grade crossings in the State of Ohio. Moreover, the study suggested deploying the Missouri Exposure Index and the North Carolina Investigative Index in order to rank passive highway–rail grade crossings upon completion of the initial prioritization.

In addition to the aforementioned state DOT efforts, other state DOTs have conducted some research works on safety at highway–rail grade crossings as well. A detailed description of the other state DOT efforts, as well as the accident and hazard prediction models that have been used for highway–rail grade crossings, can be found in Elzohairy and Benekohal [21] and Dulebenets et al. [22]. A significant number of studies aimed to improve safety at highway–rail grade crossings in other countries as well. For example, Hu et al. [23] developed a logit model to analyze the severity of accidents at the highway–rail grade crossings in Taiwan. The data were collected for a total of 592 highway–rail grade crossings. The findings indicated that the annual daily train traffic, the annual daily roadway traffic, highway separation, approaching crossing markings, and obstacle detection devices could substantially influence the accident severity.

Evans [24] investigated fatal accidents that occurred at the highway–rail grade crossings in Great Britain between 1946 and 2009. The considered highway–rail grade crossings were classified into three groups, including: (1) railway-controlled; (2) automatic; and (3) passive groups. The highest fatality rate was observed for the automatic highway–rail grade crossings (52% of fatalities), while the best safety performance was recorded for the railway-controlled highway–rail grade crossings. Boros et al. [25] focused on improving safety at the highway–rail grade crossings in Hungary. The required data were collected from approximately 1700 highway–rail grade crossings. The analysis results indicated that the annual daily train traffic and the annual daily roadway traffic were significant predictors that influenced the safety level the most. On the other hand, crossing angle, number of tracks, track alignment, and sight distances were found to be non-significant.

Laapotti [26] compared the accident occurrence at the passive and active highway–rail grade crossings in Finland. The passive highway–rail grade crossings were found to be riskier. The study highlighted that human error was one of the main factors causing the accident occurrence. Larue and Wullems [27] and Larue et al. [28] also pointed out that human errors could negatively affect safety at highway–rail grade crossings. However, both studies were performed for the highway–rail grade crossings in Australia.

Beanland et al. [29] studied the driver compliance at the rural highway–rail grade crossings with passive protection in Australia. A total of 22 drivers participated in the experiments. It was observed that 14% of drivers violated stop controls, while 27% made rolling stops (<10 km/h). The study highlighted that the compliant and non-compliant drivers had the same approach speed, but the non-compliant drivers spent less time visually checking for trains. Beanland et al. [30] evaluated three new treatments for the rural highway–rail grade crossings in Australia using a driving simulator, including the following: (1) GPS average speed interface; (2) ecological interface design crossing; and (3) simple but strong crossing. The results indicated that the proposed new treatments had some advantages and disadvantages, which should be further studied in the future. A number of other studies also focused on reducing safety risks at the highway–rail grade crossings in Australia [31–34].

2.2. Resource Allocation Studies for Highway–Rail Grade Crossings

Berg [35] highlighted that many research efforts relied on the effectiveness ratios for various safety improvement actions and incorporated the accident history into accident prediction models and hazard prediction models, as well as resource allocation procedures. It was shown that the existing methods could introduce some bias in the analysis, which could further cause inaccurate resource allocation among highway–rail grade crossings. The study recommended ignoring the adjustment of the predicted accident rate based on the recent accident history. Forgionne [36] proposed a high-speed rail grade crossing analysis tool, which could assist the FRA in evaluating the costs and benefits of potential infrastructure projects. In the conducted study, the FRA objectives and the role of a cost–benefit analysis in achieving these objectives were discussed. The study also demonstrated the applicability of the developed decision support system using real-life data.

Konur et al. [37] developed an optimization model to assist the Tennessee DOT with resource allocation among the highway–rail grade crossings for safety improvement projects. The developed optimization model aimed to maximize the safety benefits subjected to the operational constraints. A set of sorting heuristics were developed to solve the optimization models, which were based on the safety benefit-to-cost ratios. The computational experiments demonstrated that the proposed mathematical modeling approach was more accurate than the developed sorting heuristics. Rezvani et al. [38] proposed a benefit–cost methodology for highway–rail grade crossing safety protocols to be utilized for prioritization of safety improvement projects. The proposed methodology included five major steps: (1) measuring the accident cost; (2) cost-based screening; (3) benefit–cost analysis; (4) project prioritization; and (5) funding. It was concluded that the proposed framework could be effectively used by decision makers to allocate the available funds among highway–rail grade crossings.

2.3. Discussion and Contributions

A review of the relevant literature revealed a significant number of research efforts aiming to improve safety at highway–rail grade crossings. Some studies presented methodologies that can be used for resource allocation among highway–rail grade crossings. Considering a substantial number of accidents at the highway–rail grade crossings in the State of Florida, this study presents two optimization models that can assist the relevant stakeholders, including the smart city authorities, with identification of the highway–rail grade crossings that require upgrading, as well as selection of the appropriate upgrading type, taking into account the available budget limitations. The objective of the first optimization model is to minimize the overall hazard at the highway–rail grade crossings, while the second optimization model minimizes the overall hazard severity at the highway–rail grade crossings. A set of solution algorithms, including exact and heuristic methods, are proposed to solve each one of the optimization models. A case study is further conducted for the public highway–rail grade crossings in the State of Florida to evaluate the performance of the developed solution algorithms, as well as to showcase some important managerial insights that would be of interest to the smart city authorities.

3. Problem Description

A detailed description of the resource allocation optimization problems investigated herein is presented in this section of the manuscript. Each state in the U.S. is allocated a limited budget to upgrade the highway–rail grade crossings, with the aim of improving the level of safety. The relevant stakeholders, including the smart city authorities, are expected to effectively distribute the available budget among the considered highway–rail grade crossings to implement the countermeasures.

Let $X = \{1, \ldots, n\}$ denote a set of highway–rail grade crossings considered for upgrades. Some specific countermeasures, which were determined based on the relevant procedures, can be assigned to the highway–rail grade crossings throughout the resource allocation process. Here, $C = \{1, \ldots, m\}$ is a set of countermeasures, which can be implemented at highway–rail grade crossings. Two optimization models are developed in this study, where one of them minimizes the overall hazard at the highway–rail grade crossings and the other one minimizes the overall hazard severity at the highway–rail grade crossings. The overall hazard ($OH_x \in \mathbb{R}^+$ $\forall x \in X$) will be estimated based on the Florida index priority formula (see Section 6.1 for more details). In order to calculate the overall hazard severity for the second optimization model, the overall hazard should be divided into different severity categories (denoted using set $S = \{1, \ldots, k\}$) based on the methodology provided by the GradeDec.NET reference manual [39]. Note that the hazard of severity category s for highway rail grade crossing x will be referred to as $HS_{xs} \in \mathbb{R}^+$ $\forall x \in X, s \in S$. A weight (i.e., $W_s \in \mathbb{R}^+$ $\forall s \in S$) is associated with each severity category, and the weight value increases with the severity level (i.e., the fatality hazard should have a higher weight value as compared to the injury hazard for a given highway–rail grade crossing).

The current features of a given highway–rail grade crossing (i.e., the existing traffic control devices installed at the highway–rail grade crossing) determine the eligibility of the highway–rail grade crossing for implementation of different types of countermeasures. This concept is applied within the developed resource allocation optimization models using the parameter $p_{xc} \in \{0, 1\}$ $\forall x \in X, c \in C$. In particular, if $p_{xc} = 1$, then countermeasure $c \in C$ can be implemented at highway–rail grade crossing $x \in X$. On the other hand, if $p_{xc} = 0$, countermeasure $c \in C$ cannot be implemented at highway–rail grade crossing $x \in X$. Moreover, the influence of implementing a given countermeasure at the highway–rail grade crossings is designated by a specific effectiveness factor, which represents the percentage reduction of a potential hazard or hazard severity at a given highway–rail grade crossing after the countermeasure implementation. The effectiveness factor is applied within the developed resource allocation optimization models using the parameter $EF_{xc} \in \mathbb{R}^+$ $\forall x \in X, c \in C$. The cost of implementing countermeasure $c \in C$ at highway–rail grade crossing $x \in X$ is designated using the parameter $CA_{xc} \in \mathbb{R}^+$ $\forall x \in X, c \in C$. The total cost of implementing countermeasures assigned to the highway–rail grade crossings throughout the resource allocation process should not exceed the total available budget ($TAB \in \mathbb{R}^+$).

4. Model Formulation

This section of the manuscript provides a detailed explanation of the two developed optimization models, along with all of the components and notations, which will be used within the mathematical models. Moreover, two integer programming mathematical formulations are presented for the two developed optimization models, where the first model aims to minimize the overall hazard at the highway–rail grade crossings, and the second model aims to minimize the overall hazard severity at the highway–rail grade crossings. The constraint sets, which are used within the developed mathematical models, will be further described in detail as well.

4.1. Nomenclature

The nomenclature used throughout the mathematical model development is explained in this section of the manuscript. Table 1 provides a detailed description of all the components and notations

adopted in the integer programming mathematical models, which were developed for resource allocation among the respective highway–rail grade crossings.

Table 1. Description of the mathematical model components and notations.

Model Component		Description
Type	**Nomenclature**	
Sets	$X = \{1, \ldots, n\}$	set of highway–rail grade crossings (highway–rail grade crossings)
	$C = \{1, \ldots, m\}$	set of countermeasures (countermeasures)
	$S = \{1, \ldots, k\}$	set of severity categories (severity categories)
Decision Variables	$z_{xc} \in \{0, 1\} \ \forall x \in X, c \in C$	=1 if countermeasure c is applied at highway–rail grade crossing x (=0 otherwise)
Parameters	$n \in N$	number of highway–rail grade crossings (highway–rail grade crossings)
	$m \in N$	number of considered countermeasures (countermeasures)
	$k \in N$	number of severity categories (severity categories)
	$OH_x \in \mathbb{R}^+ \ \forall x \in X$	overall hazard at highway–rail grade crossing x (no units)
	$HS_{xs} \in \mathbb{R}^+ \ \forall x \in X, s \in S$	hazard of severity s at highway–rail grade crossing x (no units)
	$W_s \in \mathbb{R}^+ \ \forall s \in S$	weight associated with severity s (varies from 0.0 to 1.0)
	$p_{xc} \in \{0, 1\} \ \forall x \in X, c \in C$	=1 if countermeasure c can be potentially applied at highway–rail grade crossing x (=0 otherwise)
	$EF_{xc} \in \mathbb{R}^+ \ \forall x \in X, c \in C$	effectiveness factor for countermeasure c when applied at highway–rail grade crossing x
	$CA_{xc} \in \mathbb{R}^+ \ \forall x \in X, c \in C$	cost of applying countermeasure c at highway–rail grade crossing x (USD)
	$TAB \in \mathbb{R}^+$	total available budget (USD)

4.2. Minimizing the Overall Hazard

The integer programming model, developed for the first resource allocation problem (**RAP-1**) among the existing highway–rail grade crossings, is presented under this section of the manuscript. The **RAP-1** mathematical model aims to minimize the overall hazard at the highway–rail grade crossings. The **RAP-1** mathematical model can be formulated as follows.

RAP-1:

$$min \sum_{x \in X} \left[1 - \sum_{c \in C} (EF_{xc} \cdot z_{xc}) \right] \cdot OH_x \tag{1}$$

Subject to:

$$\sum_{c \in C} z_{xc} \leq 1 \ \forall x \in X \tag{2}$$

$$z_{xc} \leq p_{xc} \ \forall x \in X, c \in C \tag{3}$$

$$\sum_{x \in X} \sum_{c \in C} CA_{xc} \cdot z_{xc} \leq TAB \tag{4}$$

$$z_{xc}, p_{xc} \in \mathbb{B} \ \forall x \in X, c \in C \tag{5}$$

$$OH_x, EF_{xc}, CA_{xc}, TAB \in \mathbb{R}^+ \ \forall x \in X, c \in C \tag{6}$$

The objective function (1) minimizes the overall hazard at the highway–rail grade crossings. Constraint set (2) ensures that no more than one countermeasure can be implemented at each one of the considered highway–rail grade crossings. Constraint set (3) indicates that a given countermeasure can be implemented only at the highway–rail grade crossings that are eligible for that countermeasure. Constraint set (4) guarantees that the total cost of upgrading the highway–rail grade crossings will not be greater than the total available budget. The nature of decision variables and parameters of the **RAP-1** mathematical model is defined within constraint sets (5) and (6) (note that "$\mathbb{B}$" refers to a set of binary numbers and "$\mathbb{R}^+$" refers to a set of positive real numbers).

4.3. Minimizing the Overall Hazard Severity

The integer programming model, developed for the second resource allocation problem (**RAP-2**) among the existing highway–rail grade crossings, is presented under this section of the manuscript. The **RAP-2** mathematical model aims to minimize the overall hazard severity at the highway–rail grade crossings. The **RAP-2** mathematical model can be formulated as follows.

RAP-2:

$$min \sum_{x \in X} \sum_{s \in S} [1 - \sum_{c \in C} (EF_{xc} \cdot z_{xc})] \cdot W_s \cdot HS_{xs} \tag{7}$$

Subject to:

$$\sum_{c \in C} z_{xc} \leq 1 \ \forall x \in X \tag{8}$$

$$z_{xc} \leq p_{xc} \ \forall x \in X, c \in C \tag{9}$$

$$\sum_{x \in X} \sum_{c \in C} CA_{xc} \cdot z_{xc} \leq TAB \tag{10}$$

$$z_{xc}, p_{xc} \in \mathbb{B} \ \forall x \in X, \ c \in C \tag{11}$$

$$HS_{xs}, W_s, EF_{xc}, CA_{xc}, TAB \in \mathbb{R}^+ \ \forall x \in X, c \in C, s \in S \tag{12}$$

The objective function (7) minimizes the overall hazard severity at the highway–rail grade crossings. Constraint set (8) ensures that no more than one countermeasure can be implemented at each one of the considered highway–rail grade crossings. Constraint set (9) indicates that a given countermeasure can be implemented only at the highway–rail grade crossings that are eligible for that countermeasure. Constraint set (10) guarantees that the total cost of upgrading the highway–rail grade crossings will not be greater than the total available budget. The nature of decision variables and parameters of the **RAP-2** mathematical model is defined within constraint sets (11) and (12) (note that "$\mathbb{B}$" refers to a set of binary numbers and "$\mathbb{R}^+$" refers to a set of positive real numbers).

5. Solution Methods

In general, resource allocation problems that distribute a limited budget among certain areas can be reduced to a typical Knapsack problem, which has non-deterministic polynomial time complete (NP-complete) complexity [22]. Two groups of solution methodologies were developed to solve the proposed **RAP-1** and **RAP-2** mathematical models: (1) exact optimization approaches; and (2) heuristic algorithms. Exact optimization methodologies provide the global optimum for the developed optimization models (i.e., **RAP-1** and **RAP-2**). However, exact optimization methodologies may require significant computational time to solve a given mathematical model to the global optimality. Therefore, a group of heuristic algorithms were developed to solve **RAP-1** and **RAP-2** and to provide high-quality solutions within a reasonable computational time. The adopted exact optimization approaches are presented inSection 5.1 of the manuscript, while the developed heuristic algorithms are described in Section 5.2 of the manuscript.

5.1. Exact Optimization Approaches

Two exact optimization approaches, including the MATLAB function "intlinprog" and CPLEX, were adopted to solve the **RAP-1** and **RAP-2** mathematical models to the global optimality. The function "intlinprog" is a solver embedded in the MATLAB optimization toolbox, which is widely used for mixed-integer linear programming optimization models [40]. On the other hand, CPLEX executes a set of heuristics to facilitate exploration of various domains of the search space. The heuristics, deployed by CPLEX, can be categorized into the following two groups: (1) node heuristics; and (2) neighborhood exploration heuristics. In this study, CPLEX was executed using the general algebraic modeling system (GAMS) [41], where the target optimality gap was set to 0.01%, and the computational time was restricted to 7200 sec. GAMS was called via the MATLAB environment throughout the numerical experiments.

5.2. Heuristic Algorithms

A total of four heuristic algorithms were developed to solve the **RAP-1** mathematical model: (a) the most profitable hazard reduction (MPHR) heuristic; (b) the most effective hazard reduction (MEHR) heuristic; (c) the profitable hazard reduction (PHR) heuristic; and (d) the effective hazard reduction (EHR) heuristic. Additionally, four heuristic algorithms were developed to solve the **RAP-2** mathematical model: (a) the most profitable severity reduction (MPSR) heuristic; (b) the most effective severity reduction (MESR) heuristic; (c) the profitable severity reduction (PSR) heuristic; and (d) the effective severity reduction (ESR) heuristic. The developed heuristic algorithms create a priority list for highway–rail grade crossings by sorting them based on certain attributes. The following sections of the manuscript describe each heuristic in detail.

5.2.1. The Most Profitable Hazard Reduction and Severity Reduction Heuristics (MPHR/MPSR)

The first heuristic for the **RAP-1** mathematical model was named the most profitable hazard reduction (MPHR) heuristic. Figure 1 provides the main steps of the MPHR heuristic. Note that two abbreviations, "*CM*" and "*HRCR*", stand for the terms "countermeasure" and "hazard reduction-to-cost ratio", respectively. Other notations are adopted from Table 1. The data structures required throughout the algorithm execution are initialized in step 0 (i.e., the countermeasure-to-crossing assignment decision variable—z; the highway–rail grade crossing priority list—*List*; the hazard reduction-to-cost ratios—*HRCR*; and the remaining budget—*RB*). The *HRCR* values are estimated for the available highway–rail grade crossing and countermeasure pairs in step 1. The highest *HRCR* ($\overline{HRCR}$) and feasible countermeasure with the highest *HRCR* ($\overline{CM}$) are determined for each highway–rail grade crossing in step 2. The priority list is developed by sorting the highway–rail grade crossings based on the $\overline{HRCR}$ values following the descending order in step 3. Also, all the highway–rail grade crossings that are not eligible for any of the considered countermeasures are eliminated in step 3.

Then, the first loop of the MPHR heuristic (steps 4–10) is executed. Steps 5 and 6 are allocated to select the next highway–rail grade crossing in the priority list and the corresponding countermeasure, respectively. The considered highway–rail grade crossing is assigned the respective countermeasure in step 7. Step 8 is used to update the remaining budget. In step 9, the highway–rail grade crossing that was selected for upgrading is eliminated from the priority list. The first MPHR loop is terminated when either the priority list is empty or the remaining budget is not sufficient to implement the countermeasure with $\overline{HRCR}$ for the next highway–rail grade crossing belonging to the priority list. After that, the second loop of MPHR (steps 11–21) is executed. The next highway–rail grade crossing belonging to the priority list is selected in step 12. Then, MPHR searchers for the first countermeasure that a given highway–rail grade crossing is eligible for, considering the remaining budget (steps 13–19). In step 20, the highway–rail grade crossing that was selected is eliminated from the priority list. The second loop continues until the priority list is empty.

Algorithm 1: The Most Profitable Hazard Reduction (MPHR) Heuristic

$\mathbf{MPHR}(X, C, OH, p, EF, CA, TAB)$

in: $X = \{1, \ldots, n\}$ - set of crossings; $C = \{1, \ldots, m\}$ - set of countermeasures; OH - overall hazard at each crossing; p - crossing eligibility for upgrading; EF - effectiveness factors for countermeasures; CA - cost of applying each countermeasure; TAB - total available budget

out: $\mathbf{z}$ - selection of countermeasures at the considered crossings

 0: $|\mathbf{z}| \leftarrow n \cdot m; List \leftarrow \oslash; |HRCR| \leftarrow n \cdot m; RB \leftarrow TAB$ ◁ Initialization

 1: $HRCR \leftarrow (OH \cdot EF)/CA$ ◁ Estimate the hazard reduction-to-cost ratio

 2: $[\overline{HRCR}, \overline{CM}] \leftarrow \mathbf{argmax}(p, HRCR)$ ◁ Determine the highest $HRCR$ and feasible CM with the highest $HRCR$

 3: $[List, \overline{CM}] \leftarrow \mathbf{sort}(p, \overline{CM}, \overline{HRCR})$ ◁ Sort the eligible crossings and associated $\overline{CM}$ based on $\overline{HRCR}$

 4: **while** $List \neq \oslash$ **and** $RB \geq CA_{(List_1)(\overline{CM}_1)}$ **do**

 5: $\quad x \leftarrow List_1$ ◁ Select the next crossing in the list

 6: $\quad c \leftarrow \overline{CM}_1$ ◁ Select the CM with $\overline{HRCR}$ for that crossing

 7: $\quad \mathbf{z}_{xc} \leftarrow 1$ ◁ Assign the CM with $\overline{HRCR}$ for the selected crossing

 8: $\quad RB \leftarrow RB - CA_{xc}$ ◁ Update the remaining budget

 9: $\quad List \leftarrow List - \{x\}$ ◁ Remove the selected crossing from the list

10: **end while**

11: **while** $List \neq \oslash$ **do**

12: $\quad x \leftarrow List_1$ ◁ Select the next crossing in the list

13: $\quad$ **for all** $c \in C$ **do**

14: $\qquad$ **if** $CA_{xc} \leq RB$ **and** $p_{xc} = 1$ **do** ◁ Check the remaining budget and upgrade eligibility

15: $\qquad\quad \mathbf{z}_{xc} \leftarrow 1$ ◁ Assign the CM for the selected crossing

16: $\qquad\quad RB \leftarrow RB - CA_{xc}$ ◁ Update the remaining budget

17: $\qquad\quad$ **break** ◁ Break "for" loop (starting at line 13)

18: $\qquad$ **end if**

19: $\quad$ **end for**

20: $\quad List \leftarrow List - \{x\}$ ◁ Remove the selected crossing from the list

21: **end while**

22: **return** $\mathbf{z}$

Figure 1. The most profitable hazard reduction (MPHR) heuristic.

The MPHR heuristic developed for **RAP-1** is similar to the MPSR heuristic developed for **RAP-2**. The only difference between the MPHR and MPSR algorithms is that MPSR relies on the hazard severity reduction-to-cost ratios (denoted as "$SRCR$") rather than the hazard reduction-to-cost ratios throughout resource allocation. The hazard severity reduction-to-cost ratio is estimated by MPSR in step 1 for each crossing–countermeasure pair as follows: $SRCR \leftarrow (HS \cdot W \cdot EF)/CA$.

5.2.2. The Most Effective Hazard Reduction and Severity Reduction Heuristics (MEHR/MESR)

The second heuristic for the **RAP-1** mathematical model was named the most effective hazard reduction (MEHR) heuristic. Figure 2 provides the main steps of the MEHR heuristic. The notations used for description of steps in Algorithm 2 are adopted from Table 1. The data structures, required throughout the algorithm execution, are initialized in step 0 (i.e., the countermeasure-to-crossing assignment decision variable—z; the highway–rail grade crossing priority list—$List$; the hazard reduction-to-cost ratios—$HRCR$; and the remaining budget—RB). The effectiveness factor for the most effective and feasible countermeasure is identified for each highway–rail grade crossing in step 1. The $HRCR$ value is estimated for each highway–rail grade crossing and corresponding most effective countermeasure in step 2. The highest $HRCR$ ($\overline{HRCR}$) and feasible countermeasure with the highest $HRCR$ ($\overline{CM}$) are determined for each highway–rail grade crossing in step 3. The priority list is developed by sorting the highway–rail grade crossings based on the $\overline{HRCR}$ values following the

descending order in step 4. Also, all the highway–rail grade crossings that are not eligible for any of the considered countermeasures are eliminated in step 4.

Algorithm 2: The Most Effective Hazard Reduction (MEHR) Heuristic

$MEHR(X, C, OH, p, EF, CA, TAB)$

in: $X = \{1, \ldots, n\}$ - set of crossings; $C = \{1, \ldots, m\}$ - set of countermeasures; OH - overall hazard at each crossing; p - crossing eligibility for upgrading; EF - effectiveness factors for countermeasures; CA - cost of applying each countermeasure; TAB - total available budget

out: z - selection of countermeasures at the considered crossings

0: $|z| \leftarrow n \cdot m; List \leftarrow \oslash; |HRCR| \leftarrow n \cdot m; RB \leftarrow TAB$ ◁ Initialization

1: $\overline{EF} \leftarrow \boldsymbol{argmax}(p, EF)$ ◁ Determine the effectiveness factor for the most effective and feasible CM

2: $HRCR \leftarrow (OH \cdot \overline{EF})/CA$ ◁ Estimate the hazard reduction-to-cost ratio

3: $[\overline{HRCR}, \overline{CM}] \leftarrow \boldsymbol{argmax}(p, HRCR)$ ◁ Determine the highest $HRCR$ and feasible CM with the highest $HRCR$

4: $[List, \overline{CM}] \leftarrow \boldsymbol{sort}(p, \overline{CM}, \overline{HRCR})$ ◁ Sort the eligible crossings and associated $\overline{CM}$ based on $\overline{HRCR}$

5: **while** $List \neq \oslash$ **and** $RB \geq CA_{(List_1)(\overline{CM}_1)}$ **do**

6:    $x \leftarrow List_1$ ◁ Select the next crossing in the list

7:    $c \leftarrow \overline{CM}_1$ ◁ Select the CM with $\overline{HRCR}$ for that crossing

8:    $z_{xc} \leftarrow 1$ ◁ Assign the CM with $\overline{HRCR}$ for the selected crossing

9:    $RB \leftarrow RB - CA_{xc}$ ◁ Update the remaining budget

10:    $List \leftarrow List - \{x\}$ ◁ Remove the selected crossing from the list

11: **end while**

12: **while** $List \neq \oslash$ **do**

13:    $x \leftarrow List_1$ ◁ Select the next crossing in the list

14:    **for all** $c \in C$ **do**

15:        **if** $CA_{xc} \leq RB$ **and** $p_{xc} = 1$ **do** ◁ Check the remaining budget and upgrade eligibility

16:            $z_{xc} \leftarrow 1$ ◁ Assign the CM for the selected crossing

17:            $RB \leftarrow RB - CA_{xc}$ ◁ Update the remaining budget

18:            **break** ◁ Break "for" loop (starting at line 14)

19:        **end if**

20:    **end for**

21:    $List \leftarrow List - \{x\}$ ◁ Remove the selected crossing from the list

22: **end while**

23: **return** z

Figure 2. The most effective hazard reduction (MEHR) heuristic.

Then, the first loop of MEHR (steps 5–11) is executed. Steps 6 and 7 are allocated to select the next highway–rail grade crossing in the priority list and the corresponding countermeasure, respectively. The considered highway–rail grade crossing is assigned the respective countermeasure in step 8. Step 9 is used to update the remaining budget. In step 10, the highway–rail grade crossing that was selected for upgrading is eliminated from the priority list. The first MEHR loop is terminated when either the priority list is empty or the remaining budget is not sufficient to implement the countermeasure with $\overline{HRCR}$ for the next highway–rail grade crossing belonging to the priority list. After that, the second loop of MEHR (steps 12–22) is executed. The next highway–rail grade crossing belonging to the priority list is selected in step 13. Then, MEHR searchers for the first countermeasure that a given highway–rail grade crossing is eligible for, considering the remaining budget (steps 14-20). In step 21, the highway–rail grade crossing that was selected is eliminated from the priority list. The second loop continues until the priority list is empty.

The MEHR heuristic developed for **RAP-1** is similar to the MESR heuristic developed for **RAP-2**. The only difference between the MEHR and MESR algorithms is that MESR relies on the hazard

severity reduction-to-cost ratios (denoted as "*SRCR*") rather than the hazard reduction-to-cost ratios throughout resource allocation. The hazard severity reduction-to-cost ratio is estimated by MESR in step 2 for each crossing–countermeasure pair as follows: $SRCR \leftarrow \left(HS \cdot W \cdot \overline{EF}\right)/CA$.

5.2.3. The Profitable Hazard Reduction and Severity Reduction Heuristics (PHR/PSR)

The third heuristic for the **RAP-1** mathematical model was named the profitable hazard reduction (PHR) heuristic. Figure 3 provides the main steps of the PHR heuristic. The notations used for description of steps in Algorithm 3 are adopted from Table 1. The data structures required throughout the algorithm execution are initialized in step 0 (i.e., the countermeasure-to-crossing assignment decision variable—z; the highway–rail grade crossing priority list—*List*; the hazard reduction-to-cost ratios—*HRCR*; and the remaining budget—*RB*). The *HRCR* values are estimated in the first loop of Algorithm 3 (steps 1-6) to establish the priority list, which includes all the highway–rail grade crossing–countermeasure pairs (unlike the MPHR heuristic, which creates the priority list based on the highway–rail grade crossings only). The generated priority list is sorted based on the *HRCR* values following the descending order in step 7. Additionally, all the highway–rail grade crossings that are not eligible for any of the considered countermeasures are eliminated in step 7.

Algorithm 3: The Profitable Hazard Reduction (PHR) Heuristic

$PHR(X, C, OH, p, EF, CA, TAB)$

in: $X = \{1, \ldots, n\}$ - set of crossings; $C = \{1, \ldots, m\}$ - set of countermeasures; OH - overall hazard at each crossing; p - crossing eligibility for upgrading; EF - effectiveness factors for countermeasures; CA - cost of applying each countermeasure; TAB - total available budget

out: z - selection of countermeasures at the considered crossings

0: $|z| \leftarrow n \cdot m; List \leftarrow \oslash; |HRCR| \leftarrow n \cdot m; RB \leftarrow TAB$　　　　　　◁ Initialization

1: **for all** $x \in X$ **do**

2:　　**for all** $c \in C$ **do**

3:　　　　$HRCR_{xc} \leftarrow (OH_x \cdot EF_{xc})/CA_{xc}$　　　　　　◁ Estimate the hazard reduction-to-cost ratio

4:　　　　$List \leftarrow List \cup \{x, c\}$　　　　　　◁ Add a crossing-countermeasure pair to the list

5:　　**end for**

6: **end for**

7: $List \leftarrow sort(List, p, HRCR)$　　　　　　◁ Sort the list based on *HRCR*

8: **while** $List \neq \oslash$ **and** $RB \geq min(CA)$ **do**

9:　　$x \leftarrow List_1^x$　　　　　　◁ Select the next crossing in the list

10:　　$c \leftarrow List_1^c$　　　　　　◁ Select the next *CM* in the list

11:　　**if** $RB \geq CA_{xc}$ **do**

12:　　　　$z_{xc} \leftarrow 1$　　　　　　◁ Assign the *CM* for the selected crossing

13:　　　　$RB \leftarrow RB - CA_{xc}$　　　　　　◁ Update the remaining budget

14:　　　　$List \leftarrow List - \{x, :\}$　　　　　　◁ Remove all the crossing-countermeasure pairs from the list

15:　　**else**

16:　　　　$List \leftarrow List - \{x, c\}$　　　　　　◁ Remove the selected crossing-countermeasure pair from the list

17:　　**end if**

18: **end while**

19: **return** z

Figure 3. The profitable hazard reduction (PHR) heuristic.

Then, the second loop of PHR (steps 8–18) is executed. Steps 9 and 10 are allocated to select the next highway–rail grade crossing in the priority list and the corresponding countermeasure, respectively. In step 11, the budget available for assignment of a given countermeasure to the considered highway–rail grade crossing is checked. If the remaining budget is sufficient, the considered highway–rail grade crossing is assigned the respective countermeasure in step 12. Step 13 is used to update the remaining budget. In step 14, all the crossing–countermeasure pairs associated with the highway–rail grade crossing that was selected for upgrading are eliminated from the priority list. If the remaining budget for assignment of a given countermeasure to the considered highway–rail grade crossing (checked in

step 11) is insufficient, the PHR heuristic removes the selected crossing–countermeasure pair from the priority list in step 16. However, PHR will be able to investigate the other countermeasures that have lower hazard reduction-to-cost ratios and lower installation costs due to the fact that the other crossing–countermeasure pairs associated with that highway–rail grade crossing may still be present in the priority list. The loop continues until the priority list is empty or the remaining budget is not sufficient to implement any of the available countermeasures.

The PHR heuristic developed for **RAP-1** is similar to the PSR heuristic developed for **RAP-2**. The only difference between the PHR and PSR algorithms consists of the fact that PSR relies on the hazard severity reduction-to-cost ratios (denoted as "*SRCR*") rather than the hazard reduction-to-cost ratios throughout resource allocation. The hazard severity reduction-to-cost ratio is estimated by PSR in step 3 for each crossing–countermeasure pair as follows: $SRCR_{xc} \leftarrow ([\sum_{s \in S} HS_{xs} \cdot W_s] \cdot EF_{xc})/CA_{xc} \ \forall x \in X, c \in C$.

5.2.4. The Effective Hazard Reduction and Severity Reduction Heuristics (EHR/ESR)

The fourth heuristic for the **RAP-1** mathematical model was named the effective hazard reduction (EHR) heuristic. Figure 4 provides the main steps of the EHR heuristic. The notations used for description of steps in Algorithm 4 are adopted from Table 1. The additional abbreviation "*HR*" stands for the term "hazard reduction". The data structures required throughout the algorithm execution are initialized in step 0 (i.e., the countermeasure-to-crossing assignment decision variable—*z*; the highway–rail grade crossing priority list—*List*; the hazard reduction values—*HR*; and the remaining budget—*RB*). The *HR* values are estimated in the first loop of Algorithm 4 (steps 1–6) to establish the priority list, which includes all the highway–rail grade crossing countermeasure pairs (unlike the MEHR heuristic, which creates the priority list based on the highway–rail grade crossings only). The generated priority list is sorted based on the *HR* values following the descending order in step 7. Also, all the highway–rail grade crossings that are not eligible for any of the considered countermeasures are eliminated in step 7.

Then, the second loop of EHR (steps 8–18) is executed. Steps 9 and 10 are allocated to select the next highway–rail grade crossing in the priority list and the corresponding countermeasure, respectively. In step 11, the budget available for assignment of a given countermeasure to the considered highway–rail grade crossing is checked. If the remaining budget is sufficient, the considered highway–rail grade crossing is assigned the respective countermeasure in step 12. Step 13 is used to update the remaining budget. In step 14, all the crossing–countermeasure pairs associated with the highway–rail grade crossing that was selected for upgrading are eliminated from the priority list. If the remaining budget for assignment of a given countermeasure to the considered highway–rail grade crossing (checked in step 11) is insufficient, the EHR heuristic removes the selected crossing–countermeasure pair from the priority list in step 16. However, EHR will be able to investigate the other countermeasures that have lower hazard reduction values and lower installation costs due to the fact that the other crossing–countermeasure pairs associated with that highway–rail grade crossing may be still present in the priority list. The loop continues until the priority list is empty or the remaining budget is not sufficient to implement any of the available untermeasures.

The EHR heuristic developed for **RAP-1** is similar to the ESR heuristic developed for **RAP-2**. The only difference between the EHR and ESR algorithms consists in the fact that ESR relies on the hazard severity reduction values (denoted as "*SR*") rather than the hazard reduction values throughout resource allocation. The hazard severity reduction value is estimated by ESR in step 3 for each crossing–countermeasure pair as follows: $SR_{xc} \leftarrow [\sum_{s \in S} HS_{xs} \cdot W_s] \cdot EF_{xc} \ \forall x \in X, c \in C$.

Algorithm 4: The Effective Hazard Reduction (EHR) Heuristic

$EHR(X, C, OH, p, EF, CA, TAB)$

in: $X = \{1, \ldots, n\}$ - set of crossings; $C = \{1, \ldots, m\}$ - set of countermeasures; OH - overall hazard at each crossing; p - crossing eligibility for upgrading; EF - effectiveness factors for countermeasures; CA - cost of applying each countermeasure; TAB - total available budget

out: z - selection of countermeasures at the considered crossings

0: $|z| \leftarrow n \cdot m; List \leftarrow \oslash; |HR| \leftarrow n \cdot m; RB \leftarrow TAB$ ◁ Initialization

1: **for all** $x \in X$ **do**

2:     **for all** $c \in C$ **do**

3:        $HR_{xc} \leftarrow (OH_x \cdot EF_{xc})$ ◁ Estimate the hazard reduction

4:        $List \leftarrow List \cup \{x, c\}$ ◁ Add a crossing-countermeasure pair to the list

5:     **end for**

6: **end for**

7: $List \leftarrow sort(List, p, HR)$ ◁ Sort the list based on HR

8: **while** $List \neq \oslash$ **and** $RB \geq min(CA)$ **do**

9:     $x \leftarrow List_1^x$ ◁ Select the next crossing in the list

10:    $c \leftarrow List_1^c$ ◁ Select the next CM in the list

11:    **if** $RB \geq CA_{xc}$ **do**

12:       $z_{xc} \leftarrow 1$ ◁ Assign the CM for the selected crossing

13:       $RB \leftarrow RB - CA_{xc}$ ◁ Update the remaining budget

14:       $List \leftarrow List - \{x, :\}$ ◁ Remove all the crossing-countermeasure pairs from the list

15:    **else**

16:       $List \leftarrow List - \{x, c\}$ ◁ Remove the selected crossing-countermeasure pair from the list

17:    **end if**

18: **end while**

19: **return** z

Figure 4. The effective hazard reduction (EHR) heuristic.

6. Case Study

This section of the manuscript presents a case study that was used to evaluate the effectiveness of the developed optimization models and solution approaches for the existing highway–rail grade crossings in the State of Florida. Note that only public highway–rail grade crossings will be considered throughout the analysis.

6.1. Input Data

Both **RAP-1** and **RAP-2** optimization models require certain input data in order to perform resource allocation among the highway–rail grade crossings in the State of Florida. The input data related to the set of highway–rail grade crossings ($X = \{1, \ldots, n\}$) were extracted from the publicly available crossing inventory database, which is maintained by the FRA [42]. The inventory database includes detailed information regarding basic crossing characteristics (e.g., average daily traffic volume, average daily train volume, protection type, train speed, crossing surface). The overall hazard at highway–rail grade crossings (OH_x, $x \in X$) for the **RAP-1** mathematical model was calculated using the Florida priority index formula [22]:

$$FPI_x = V_x \cdot T_x \cdot (0.1 \cdot S_x) \cdot PF_x \cdot \left(0.01 \cdot A_x^{1.15}\right) \tag{13}$$

where FPI_x = the Florida priority index at highway–rail grade crossing x (no units); V_x = average daily traffic volume at highway–rail grade crossing x (vehicles per day); T_x = average daily train volume at highway–rail grade crossing x (trains per day); S_x = train speed at highway–rail grade crossing x (mph); PF_x = protection factor at highway–rail grade crossing x (PF = 1.00 for passive; PF = 0.70 for

flashing lights; PF = 0.10 for gates); A_x = accident history parameter at highway–rail grade crossing x (accidents), representing the total number of accidents in the last 5 years or since the year of last improvement (if there was an upgrade).

Note that many factors may influence the overall hazard of a given highway–rail grade crossing (i.e., average daily train volume, number of cars in a train, train type, train speed, number of tracks, average daily traffic volume, sight distance, number of traffic lanes, highway vehicular speed, approach gradient, crossing angle, location, existing protection, etc.). Dulebenets et al. [22] evaluated a wide range of different accident and hazard prediction models for the highway–rail grade crossings in the State of Florida, including the Coleman–Stewart model, NCHRP Report 50 accident prediction formula, Peabody–Dimmick formula, U.S. DOT accident prediction formula, New Hampshire formula, California hazard rating formula, Connecticut hazard rating formula, Illinois hazard index formula, Michigan hazard index formula, Texas priority index formula, and the Florida priority index formula. The considered accident and hazard prediction models directly accounted for many of the aforementioned factors that influence the overall hazard of highway–rail grade crossings. The actual 2007–2016 accident data provided by the FRA were used to estimate the predicted number of accidents and the predicted overall hazard by applying the candidate models, while the actual 2017 accident data were used to validate the candidate models. It was found that over-represented accident and hazard prediction models (i.e., the ones that consider many factors at the same time) have low accident and hazard prediction accuracy for the highway–rail grade crossings in the State of Florida. On the other hand, the Florida priority index formula demonstrated the best performance; therefore, it was used throughout this study.

The overall hazard of a given highway–rail grade crossing was set equal to the Florida priority index (i.e., $FPI_x = OH_x \ \forall x \in X$). The FRA highway–rail grade crossing accident database [13] was used for estimation of the accident history parameter ($A_x, x \in X$). Both **RAP-1** and **RAP-2** mathematical models relied on a set of countermeasures, denoted as $C = \{1, \ldots, m\}$, to improve the level of safety at the highway–rail grade crossings. Each countermeasure has a specific effectiveness factor, which represents the percentage reduction of a potential hazard at a given highway–rail grade crossing [39,43]. However, not all the countermeasures can be implemented at a given highway–rail grade crossing, considering its technical characteristics. In this study, a total of 11 countermeasures were adopted for the **RAP-1** and **RAP-2** mathematical models from the GradeDec.NET reference manual [39]. The basic information regarding the countermeasures was retrieved from the GradeDec.NET reference manual (see Table 2), including the (1) effectiveness factors ($EF_{xc}, x \in X, c \in C$) and (2) installation costs ($CA_{xc}, x \in X, c \in C$). Note that the **RAP-1** and **RAP-2** mathematical models can be applied for other types of countermeasures as well (i.e., not just the ones that are suggested by the GradeDec.NET reference manual). In order to introduce new countermeasures in the developed mathematical models, the user will need to add new elements to the set of countermeasures (C) and specify the effectiveness factor ($EF_{xc}, x \in X, c \in C$), as well as the installation cost ($CA_{xc}, x \in X, c \in C$), for each new countermeasure.

In order to estimate the overall hazard severity at each highway–rail grade crossing for the **RAP-2** mathematical model, the overall hazard estimated by the Florida priority index should be divided into different severity categories. In this study, the hazard severity was categorized based on the GradeDec.NET reference manual as follows [39]: (1) fatal accidents—accidents with at least one fatality; (2) injury accidents—accidents with at least one injury, but no fatality; and (3) property damage only accidents. The severity weights attributed to different severity categories in the **RAP-2** mathematical model were adopted based on the project conducted by the Iowa DOT [18]. Specifically, the base values for the weights of fatality hazard (W_{FH}), injury hazard (W_{IH}), and property damage hazard (W_{PH}) were set to $W_{FH} = 0.60$, $W_{IH} = 0.30$, and $W_{PH} = 0.10$, respectively.

Table 2. Basic information for the considered countermeasures.

a/a	Countermeasure	Effectiveness	Installation Cost
1	passive to flashing lights	0.57	$74,800
2	passive to flashing lights and gates	0.78	$180,900
3	flashing lights to gates	0.63	$106,100
4	4 quadrant (no detection) for gated crossings	0.82	$244,000
5	4 quadrant (with detection) for gated crossings	0.77	$260,000
6	4 quadrant (with 60' medians) for gated crossings	0.92	$255,000
7	mountable curbs (with channelized devices) for gated crossings	0.75	$15,000
8	barrier curbs (with or without channelized devices) for gated crossings	0.80	$15,000
9	one-way street with gate for gated crossings	0.82	$5000
10	photo enforcement for gated crossings	0.78	$65,000
11	grade separation	1.00	$1,500,000

6.2. Evaluation of the Solution Algorithms

All the solution algorithms developed for the **RAP-1** and **RAP-2** mathematical models were evaluated for all the public highway–rail grade crossings in the State of Florida. There were a total of 6089 public highway–rail grade crossings in the State of Florida based on the FRA crossing inventory database as of November 2018. A total of 121 scenarios were developed to evaluate the performance of the exact optimization algorithms and the heuristic algorithms. The scenarios were developed by varying the number of highway–rail grade crossings and the number of available countermeasures. The following values were considered for the number of highway–rail grade crossings: $|X| = \{600, 1200, 1800, 2400, 3000, 3600, 4200, 4800, 5400, 6000, 6089\}$. The number of countermeasures was altered from 1 to 11 by an increment of 1. A total of 12 problem instances were developed by changing the values for the total available budget as follows: TAB ($\$M$) = $\{7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0\}$. Note that the adopted values for the total available budget are in line with the ones reported by the FDOT [14].

All the developed heuristic algorithms (see Section 5.2 of the manuscript) were encoded in the MATLAB environment. Moreover, the GAMS environment was used to encode the **RAP-1** and **RAP-2** mathematical models and to solve them to the global optimality with CPLEX. A CPU with Dell Intel(R) Core™ i7 Processor, 32 GB of RAM, and Windows 10 operating system was utilized to perform all the numerical experiments throughout this study. Note that the **RAP-1** and **RAP-2** mathematical models were solved using the function "intlinprog" available within the MATLAB optimization toolbox. However, the initial numerical experiments showed that "intlinprog" required a significant computational time as compared to the other solution methodologies adopted in this study. Furthermore, in some cases, "intlinprog" violated certain constraint sets of the **RAP-1** and **RAP-2** mathematical models. Therefore, the MATLAB "intlinprog" function had to be withdrawn from the analysis. Sections 6.2.1 and 6.2.2 provide more details regarding evaluation of the candidate solution algorithms for the **RAP-1** and **RAP-2** mathematical models, respectively.

6.2.1. Solution Quality and Computational Efforts for **RAP-1**

The **RAP-1** mathematical model was solved utilizing CPLEX, MPHR, MEHR, PHR, and EHR solution algorithms for all the developed scenarios of each problem instance considered. Note that the same objective function values were returned by each algorithm after each iteration, as they are deterministic in nature. However, in order to estimate the average computational time, a total of 5 replications were performed for each scenario and each problem instance. Table 3 and Figure 5 present the average over the developed scenarios overall hazard values returned by CPLEX, MPHR, MEHR, PHR, and EHR for each of the considered problem instances of the **RAP-1** mathematical model. Furthermore, the average over the considered problem instances overall hazard values returned by CPLEX, MPHR, MEHR, PHR, and EHR for each of the developed scenarios of the **RAP-1** mathematical model are presented in Figure 6.

Table 3. The average overall hazard values returned by the candidate solution algorithms for the considered problem instances of the **RAP-1** mathematical model.

| Instance	CPLEX	MPHR	MEHR	PHR	EHR
1	1,802,758.3	1,815,438.4	2,121,317.9	1,815,406.3	2,101,462.2
2	1,791,588.0	1,804,746.6	2,112,177.9	1,804,708.2	2,072,767.2
3	1,781,509.6	1,795,053.4	2,120,578.3	1,794,991.0	2,064,098.6
4	1,772,367.5	1,786,104.7	2,071,113.3	1,786,081.1	2,056,414.6
5	1,763,982.3	1,778,066.2	2,064,504.8	1,778,038.9	2,047,866.9
6	1,756,185.5	1,770,446.2	2,060,952.6	1,770,434.4	2,040,753.4
7	1,748,741.0	1,763,316.8	2,050,754.1	1,763,305.9	2,033,795.5
8	1,741,493.6	1,756,434.4	2,040,743.6	1,756,369.4	2,025,694.2
9	1,734,805.1	1,749,871.5	2,036,525.8	1,749,860.9	2,003,522.8
10	1,728,510.2	1,743,901.9	2,033,874.6	1,743,871.3	1,992,358.2
11	1,722,629.4	1,738,196.8	2,016,882.2	1,738,184.9	1,988,633.1
12	1,716,912.7	1,732,753.7	2,013,398.7	1,732,688.4	1,983,089.9
Average:	**1,755,123.6**	**1,769,527.6**	**2,061,902.0**	**1,769,495.1**	**2,034,204.7**
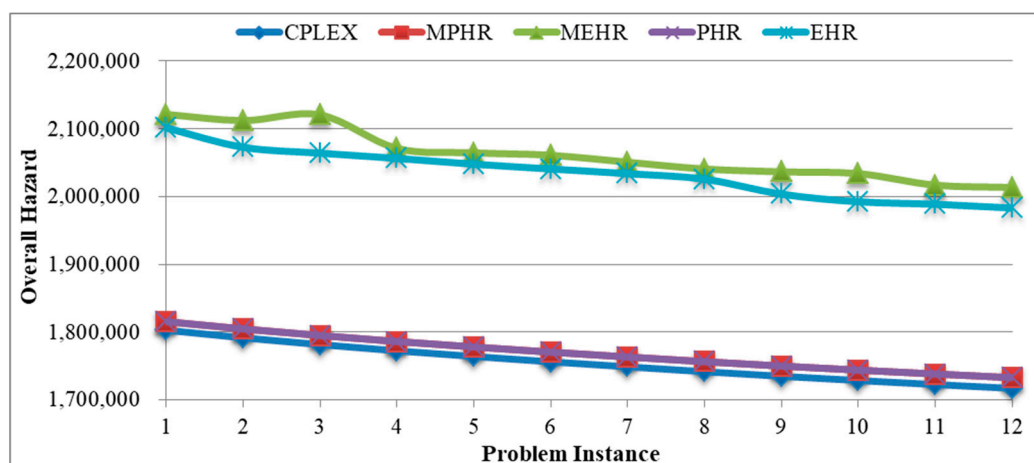
Figure 5. The average overall hazard values identified by the candidate solution algorithms for the considered problem instances of the **RAP-1** mathematical model.
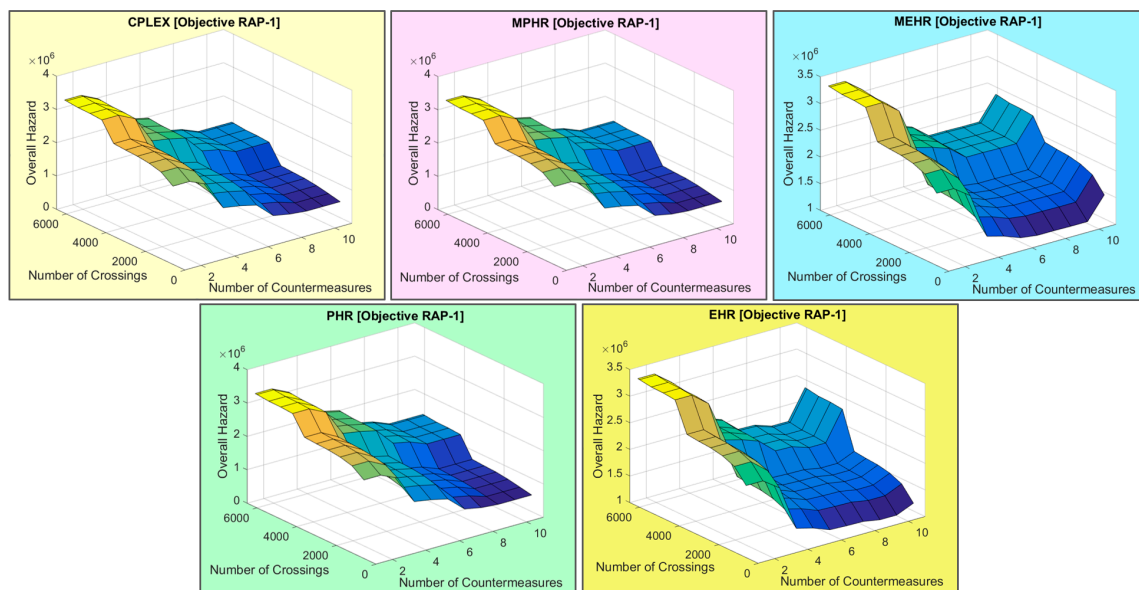
Figure 6. The average overall hazard values returned by the candidate solution algorithms for the developed scenarios of the **RAP-1** mathematical model.

Based on the information presented in Table 3, the averages of the overall hazard values for the developed scenarios were 1,755,123.6, 1,769,527.6, 2,061,902.0, 1,769,495.1, and 2,034,204.7 for CPLEX, MPHR, MEHR, PHR, and EHR, respectively. Hence, the MPHR and PHR heuristics returned the solutions that were close to the optimal solutions identified by CPLEX for all the developed scenarios of each problem instance of the **RAP-1** mathematical model. However, PHR was found to be superior to MPHR for certain scenarios, as it consistently assigns countermeasures to the highway–rail grade crossings from the priority list using the hazard reduction-to-cost ratios (unlike MPHR, which assigns countermeasures with the highest hazard reduction-to-cost ratios to the highway–rail grade crossings from the priority list, while the remaining budget is allocated for implementation of eligible countermeasures at certain highway–rail grade crossings that were not selected for upgrading). The numerical experiments show that the MEHR and EHR heuristics were outperformed by CPLEX in terms of the overall hazard values on average by 17.48% and 15.90%, respectively, over the considered problem instances. Hence, prioritization of the highway–rail grade crossings based on the hazard reduction-to-cost ratios (implemented within MPHR and PHR) was found to be a more effective strategy throughout resource allocation as compared to prioritization of the highway–rail grade crossings based on either combination of the hazard reduction-to-cost ratios and the hazard reduction (implemented within MEHR) or based on the hazard reduction only (implemented within EHR).

Table 4 presents the average over the developed scenarios computational time recorded for CPLEX, MPHR, MEHR, PHR, and EHR for each of the considered problem instances of the **RAP-1** mathematical model. The averages of computational time values for the developed scenarios were 81.19 sec, 29.89 sec, 10.25 sec, 16.78 sec, and 11.68 sec for CPLEX, MPHR, MEHR, PHR, and EHR, respectively. Although all the candidate solution algorithms were able to solve all the developed scenarios of each problem instance of the **RAP-1** mathematical model within a reasonable computational time, it is expected that the CPLEX computational time may significantly increase for certain problem instances due to the computational complexity of the **RAP-1** mathematical model. Therefore, based on the results from the conducted analysis, the PHR heuristic algorithm is recommended for resource allocation among the highway–rail grade crossings in the State of Florida, aiming to minimize the overall crossing hazard.

Table 4. The average computational time recorded for the candidate solution algorithms for the considered problem instances of the **RAP-1** mathematical model.

Instance	CPLEX	MPHR	MEHR	PHR	EHR
1	81.2341	22.9996	2.8711	9.6014	4.3790
2	81.1790	23.4100	4.3074	10.7004	5.7838
3	81.1772	24.7488	5.7418	12.9317	7.2130
4	81.1997	26.0894	7.1103	14.2918	8.6722
5	81.1887	27.4256	8.0442	14.8401	10.0820
6	81.1955	28.8453	9.4199	15.8962	10.9678
7	81.1694	30.1043	10.7273	17.2545	11.7212
8	81.1568	33.0137	12.0542	18.5729	13.1138
9	81.1526	34.5303	13.3985	19.8464	15.3944
10	81.1760	34.0772	14.7706	21.1538	16.7427
11	81.2046	35.6770	16.1121	22.4718	17.3051
12	81.1855	37.7855	18.4550	23.8091	18.7206
Average:	**81.1849**	**29.8922**	**10.2510**	**16.7808**	**11.6746**

6.2.2. Solution Quality and Computational Efforts for **RAP-2**

The **RAP-2** mathematical model was solved utilizing CPLEX, MPSR, MESR, PSR, and ESR solution algorithms for all the developed scenarios of each problem instance considered. Note that the same objective function values were returned by each algorithm after each iteration, as they are deterministic in nature. However, in order to estimate the average computational time, a total of 5 replications were performed for each scenario and each problem instance. Table 5 and Figure 7 present the average over the developed scenarios overall hazard severity values returned by CPLEX, MPSR, MESR, PSR, and ESR for each of the considered problem instances of the **RAP-2** mathematical model. Furthermore, the average over the considered problem instances overall hazard severity values returned by CPLEX, MPSR, MESR, PSR, and ESR for each of the developed scenarios of the **RAP-2** mathematical model are presented in Figure 8.

Table 5. The average overall hazard severity values returned by the candidate solution algorithms for the considered problem instances of the **RAP-2** mathematical model.

| Instance	CPLEX	MPSR	MESR	PSR	ESR
1	348,227.2	350,835.6	414,025.1	350,820.8	409,626.2
2	345,960.7	348,706.3	413,755.8	348,694.1	407,723.8
3	343,946.4	346,773.6	412,041.1	346,768.7	403,869.5
4	342,080.2	345,002.4	404,133.4	344,992.4	402,275.5
5	340,414.2	343,390.1	405,017.3	343,376.2	400,441.6
6	338,779.6	341,825.1	402,815.9	341,815.0	398,961.1
7	337,344.6	340,412.2	400,018.3	340,402.6	397,559.1
8	335,944.2	339,064.2	397,008.1	339,053.5	396,298.2
9	334,608.9	337,801.3	397,052.9	337,796.5	394,716.8
10	333,545.3	336,630.6	393,861.8	336,623.7	389,561.8
11	332,224.7	335,490.5	390,915.7	335,486.0	388,406.8
12	331,327.9	334,426.2	390,775.0	334,419.4	387,516.3
Average:	**338,700.3**	**341,696.5**	**401,785.0**	**341,687.4**	**398,079.7**
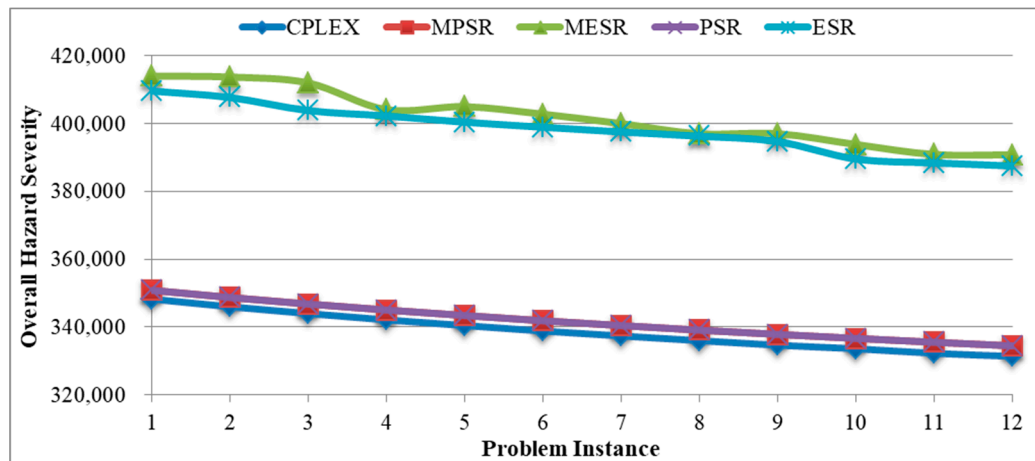
Figure 7. The average overall hazard severity values identified by the candidate solution algorithms for the considered problem instances of the **RAP-2** mathematical model.
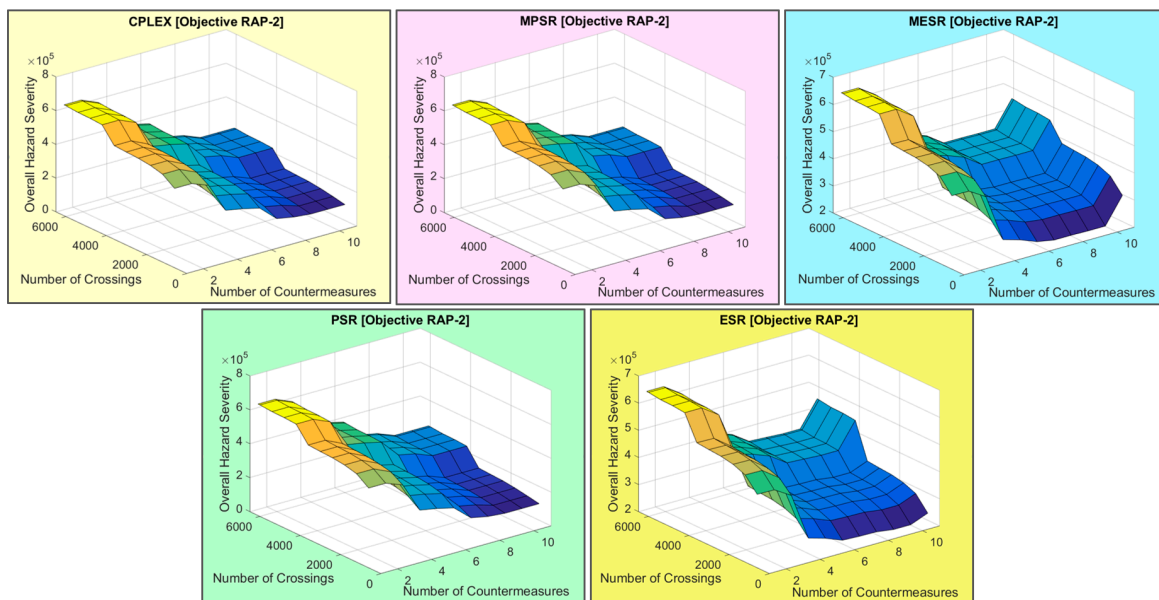

Figure 8. The average overall hazard severity values returned by the candidate solution algorithms for the developed scenarios of the **RAP-2** mathematical model.

Based on the information presented in Table 5, the average overall hazard severity values for the developed scenarios were 338,700.3, 341,696.5, 401,785.0, 341,687.4, and 398,079.7 for CPLEX, MPSR, MESR, PSR, and ESR, respectively. Hence, the MPSR and PSR heuristics returned solutions that were close to the optimal solutions identified by CPLEX for all of the developed scenarios of each problem instance of the **RAP-2** mathematical model. However, PSR was found to be superior to MPSR for certain scenarios, as it consistently assigns countermeasures to the highway–rail grade crossings from the priority list using the hazard severity reduction-to-cost ratios (unlike MPSR, which assigns countermeasures with the highest hazard severity reduction-to-cost ratios to the highway–rail grade crossings from the priority list, while the remaining budget is allocated for implementation of eligible countermeasures at certain highway–rail grade crossings that were not selected for upgrading). The numerical experiments show that the MESR and ESR heuristics were outperformed by CPLEX in terms of the overall hazard severity values on average by 18.63% and 17.53%, respectively, over the considered problem instances. Hence, prioritization of the highway–rail grade crossings based on the hazard severity reduction-to-cost ratios (implemented within MPSR and PSR) was found to be a more

effective strategy throughout resource allocation as compared to prioritization of the highway–rail grade crossings based on either combination of the hazard severity reduction-to-cost ratios and the hazard severity reduction (implemented within MESR) or based on the hazard severity reduction only (implemented within ESR).

Table 6 presents the average over the developed scenarios computational time recorded for CPLEX, MPSR, MESR, PSR, and ESR for each of the considered problem instances of the **RAP-2** mathematical model. The average computational time values for the developed scenarios were 79.64 sec, 14.52 sec, 13.94 sec, 15.01 sec, and 15.24 sec for CPLEX, MPSR, MESR, PSR, and ESR, respectively. Although all the candidate solution algorithms were able to solve all the developed scenarios of each problem instance of the **RAP-2** mathematical model within a reasonable computational time, it is expected that the CPLEX computational time may significantly increase for certain problem instances due to the computational complexity of the **RAP-2** mathematical model. Therefore, based on the results from the conducted analysis, the PSR heuristic algorithm is recommended for resource allocation among the highway–rail grade crossings in the State of Florida, aiming to minimize the overall crossing hazard severity.

Table 6. The average computational times recorded for the candidate solution algorithms for the considered problem instances of the **RAP-2** mathematical model.

Instance	CPLEX	MPSR	MESR	PSR	ESR
1	69.9911	3.9560	3.3479	4.6314	4.4462
2	70.9460	6.0069	5.2160	6.4073	6.2969
3	73.0394	8.0679	7.1945	8.2830	8.5575
4	72.2399	10.1245	9.6549	10.7905	10.6734
5	75.9433	12.2229	11.0833	12.8405	12.5062
6	81.3307	13.7113	12.7959	14.1511	14.6446
7	79.6609	15.5128	14.6753	16.0397	16.3512
8	82.1378	17.2391	16.5429	17.9721	18.0089
9	82.4937	19.4961	18.4784	19.4171	19.4612
10	89.0162	20.8236	20.3190	21.3125	21.3299
11	87.8737	22.5603	22.2184	23.2041	24.2457
12	90.9694	24.5690	25.7563	25.0734	26.3221
Average:	**79.6368**	**14.5242**	**13.9402**	**15.0102**	**15.2370**

6.3. Managerial Insights

This section of the manuscript provides the results of the sensitivity analyses, which were conducted to reveal how the developed optimization models and solution methodologies can assist the relevant stakeholders, including the smart city authorities, with important managerial insights. In particular, the sensitivity of the resource allocation decisions to the total available budget and the number of available countermeasures will be further investigated.

6.3.1. Sensitivity Analysis for the Total Available Budget

A sensitivity analysis was conducted to determine how the total available budget may impact the resource allocation decisions among the highway–rail grade crossings in the State of Florida. A total of 12 budget availability scenarios were developed by altering the total available budget from $7.5 million to $13.0 million, with increments of $500,000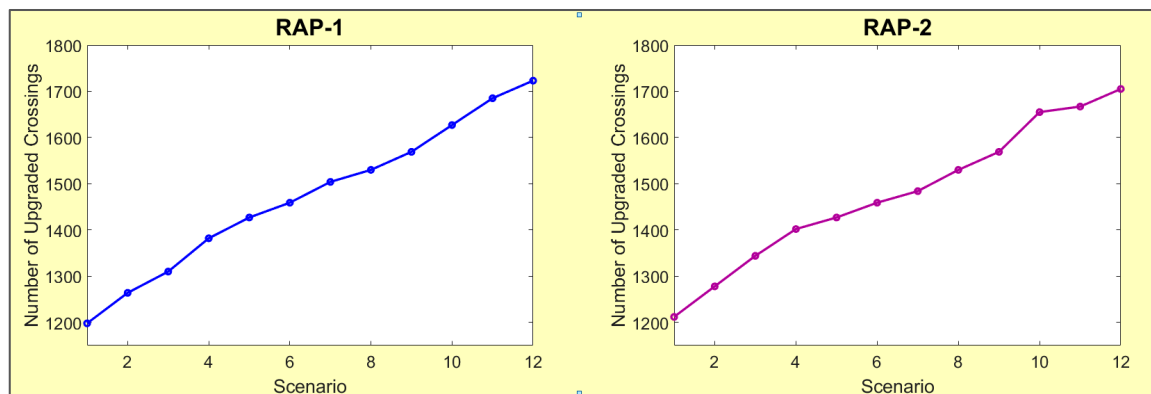. The impact of the total available budget on the number of upgraded highway–rail grade crossings by **RAP-1** and **RAP-2** is illustrated in Figure 9. Note that PHR and PSR were used as solution approaches for **RAP-1** and **RAP-2**, respectively. As expected, the total

number of highway–rail grade crossings upgraded by **RAP-1** and **RAP-2** increased after increasing the total available budget. Specifically, a total of 1198 and 1723 highway–rail grade crossings were selected for upgrading by **RAP-1** for budget availability scenarios "1" and "12", respectively. On the other hand, a total of 1212 and 1705 highway–rail grade crossings were selected for upgrading by **RAP-2** for budget availability scenarios "1" and "12", respectively. Therefore, the total available budget may substantially influence the resource allocation decisions and directly affect the number of upgraded highway–rail grade crossings.

Figure 9. The impact of the total available budget on the number of highway–rail grade crossings upgraded by **RAP-1** and **RAP-2**.

Figure 10 illustrates the overall hazard before and after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-1** for all 12 developed budget availability scenarios. It can be noticed that neither the overall hazard before nor the overall hazard after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-1** substantially changed from increasing the total available budget. Such a finding can be explained by the fact that **RAP-1** selected for upgrading the most hazardous highway–rail grade crossings, even with the total available budget of \$7.5 million (i.e., scenario "1"). Increasing the total available budget allowed upgrading of additional highway–rail grade crossings, however their overall hazard was not as significant as the one recorded for the highway–rail grade crossings upgraded under budget availability scenario "1". Nevertheless, implementation of the countermeasures that were suggested by **RAP-1** significantly reduced the overall hazard at the highway–rail grade crossings. Specifically, the overall hazard was reduced by $\left(3.69 \cdot 10^6 - 1.01 \cdot 10^6\right)/\left(3.69 \cdot 10^6\right) = 72.63\%$ for budget availability scenario "1". Furthermore, the overall hazard was reduced by $\left(3.79 \cdot 10^6 - 1.04 \cdot 10^6\right)/\left(3.79 \cdot 10^6\right) = 72.56\%$ for budget availability scenario "12". Therefore, the developed **RAP-1** mathematical model can serve as an efficient methodology for reducing the overall hazard at the highway–rail grade crossings under various budget availability scenarios.
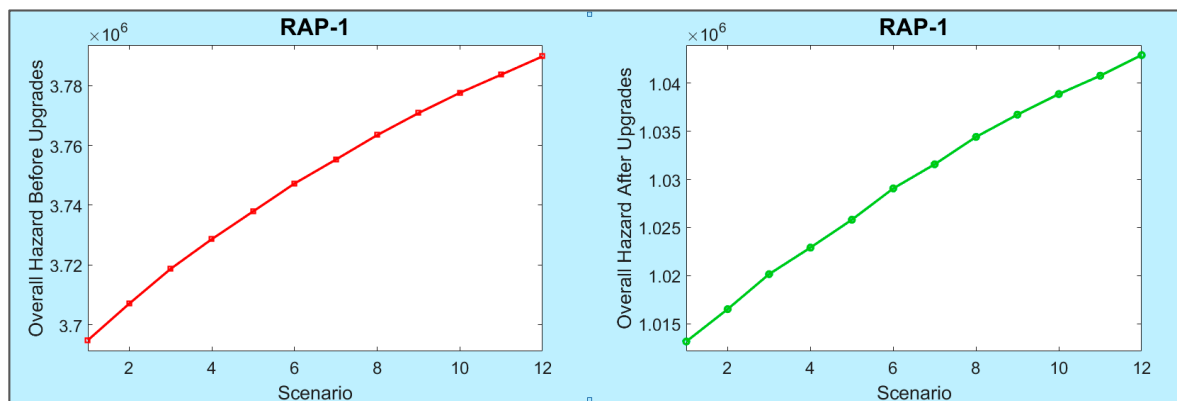
Figure 11 illustrates the overall hazard before and after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-2** for all 12 developed budget availability scenarios. Similar to the resource allocation results that were observed for **RAP-1**, neither the overall hazard before nor the overall hazard after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-2** substantially changed by increasing the total available budget. Nevertheless, implementation of the countermeasures that were suggested by **RAP-2** significantly reduced the overall hazard at the highway–rail grade crossings. Specifically, the overall hazard was reduced by $\left(3.69 \cdot 10^6 - 1.01 \cdot 10^6\right)/\left(3.69 \cdot 10^6\right) = 72.63\%$ for budget availability scenario "1". Furthermore, the overall hazard was reduced by $\left(3.79 \cdot 10^6 - 1.04 \cdot 10^6\right)/\left(3.79 \cdot 10^6\right) = 72.56\%$ for budget availability scenario "12". Therefore, the developed **RAP-2** mathematical model can serve as an efficient methodology for reducing the overall hazard at the highway–rail grade crossings under various budget availability scenarios.

Figure 10. The impact of the total available budget on the overall hazard before and after implementing countermeasures at the highway–rail grade crossings upgraded by **RAP-1**.
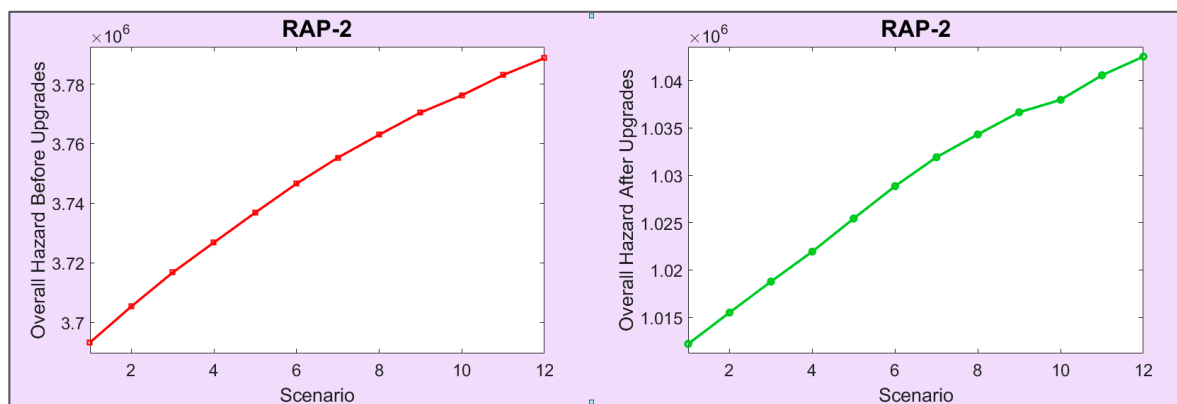

Figure 11. The impact of the total available budget on the overall hazard before and after implementing countermeasures at the highway–rail grade crossings upgraded by **RAP-2**.

Note that similar percentages were observed in terms of the overall hazard reduction at the highway–rail grade crossings that were suggested for upgrading for both **RAP-1** and **RAP-2**. However, **RAP-1** suggested a total of 1198 and 1723 highway–rail grade crossings to be upgraded for budget availability scenarios "1" and "12", respectively, while **RAP-2** suggested a total of 1212 and 1705 highway–rail grade crossings to be upgraded for budget availability scenarios "1" and "12". Such a variation can be explained by differences in the resource allocation objectives (i.e., **RAP-1** aims to minimize the overall crossing hazard, while **RAP-2** aims to minimize the overall crossing hazard severity).

6.3.2. Sensitivity Analysis for the Number of Available Countermeasures

A sensitivity analysis was conducted to determine how the number of available countermeasures may impact the resource allocation decisions among the highway–rail grade crossings in the State of Florida. A total of 11 countermeasure availability scenarios were developed by altering the number of available countermeasures from 1 to 11 by an increment of 1 countermeasure. The impact of the number of available countermeasures on the number of highway–rail grade crossings upgraded by **RAP-1** and **RAP-2** is illustrated in Figure 12. Note that PHR and PSR were used as solution approaches for **RAP-1** and **RAP-2**, respectively. As expected, the total number of highway–rail grade crossings upgraded by **RAP-1** and **RAP-2** increased after increasing the number of available countermeasures. Specifically, a total of 100 and 1198 highway–rail grade crossings were selected for upgrading by **RAP-1** for countermeasure availability scenarios "1" and "11", respectively. On the other hand, a total of 100 and 1212 highway–rail grade crossings were selected for upgrading by **RAP-2** for countermeasure availability scenarios "1" and "11", respectively. Such a significant increase in the number of upgraded

highway–rail grade crossings can be explained by the introduction of low-cost countermeasures (i.e., countermeasures "7", "8", and "9"—see Table 2). Therefore, the number of available countermeasures may substantially influence the resource allocation decisions and directly affect the number of upgraded highway–rail grade crossings.
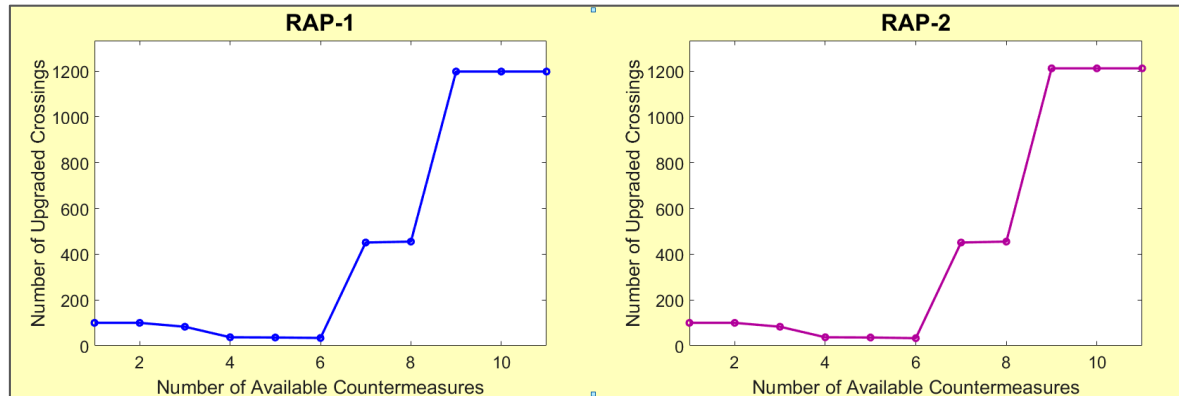

Figure 12. The impact of the number of countermeasures on the number of highway–rail grade crossings upgraded by **RAP-1** and **RAP-2**.

Figure 13 illustrates the overall hazard before and after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-1** for all 11 developed countermeasure availability scenarios. It can be noticed that the overall hazard before and after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-1** substantially increased after increasing the number of available countermeasures. This finding can be explained by the fact that the number of upgraded highway–rail grade crossings significantly increased after introducing additional countermeasures. Moreover, implementation of the countermeasures that were suggested by **RAP-1** significantly reduced the overall hazard at the highway–rail grade crossings. Specifically, the overall hazard was reduced by $\left(1.19 \cdot 10^6 - 5.14 \cdot 10^5\right) / \left(1.19 \cdot 10^6\right) = 56.81\%$ for countermeasure availability scenario "1". Furthermore, the overall hazard was reduced by $\left(3.69 \cdot 10^6 - 1.01 \cdot 10^6\right) / \left(3.69 \cdot 10^6\right) = 72.63\%$ for countermeasure availability scenario "11". Therefore, the developed **RAP-1** mathematical model can serve as an efficient methodology for reducing the overall hazard at the highway–rail grade crossings under various countermeasure availability scenarios.
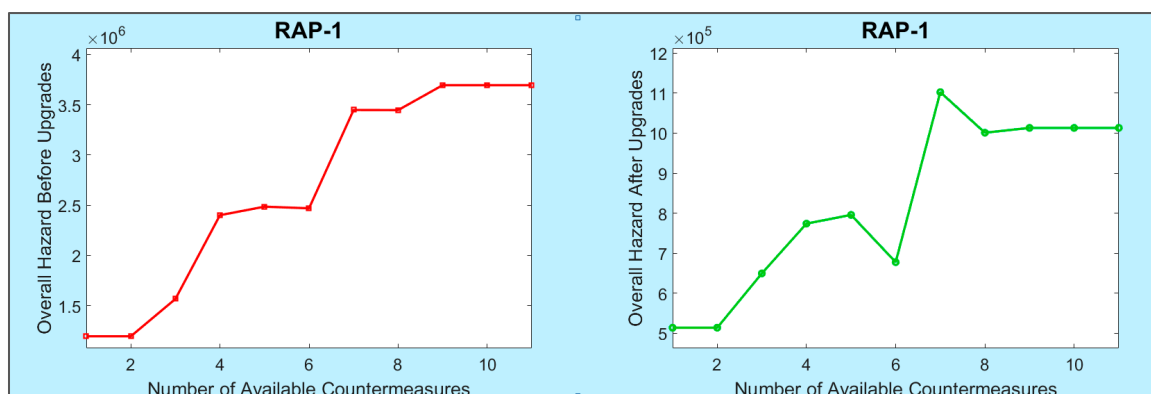

Figure 13. The impact of the number of countermeasures on the overall hazard before and after implementing countermeasures at the highway–rail grade crossings upgraded by **RAP-1**.

Figure 14 illustrates the overall hazard before and after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-2** for all 11 developed

countermeasure availability scenarios. Similar to the resource allocation results that were observed for **RAP-1**, the overall hazard before and after implementation of countermeasures at the highway–rail grade crossings that were suggested for upgrading by **RAP-2** substantially increased after increasing the number of available countermeasures. Moreover, implementation of the countermeasures that were suggested by **RAP-2** significantly reduced the overall hazard at the highway–rail grade crossings. Specifically, the overall hazard was reduced by $(1.19{\cdot}10^6 - 5.14{\cdot}10^5)/(1.19{\cdot}10^6) = 56.81\%$ for countermeasure availability scenario "1". Furthermore, the overall hazard was reduced by $(3.69{\cdot}10^6 - 1.01{\cdot}10^6)/(3.69{\cdot}10^6) = 72.63\%$ for countermeasure availability scenario "11". Therefore, the developed **RAP-2** mathematical model can serve as an efficient methodology for reducing the overall hazard at the highway–rail grade crossings under various countermeasure availability scenarios.
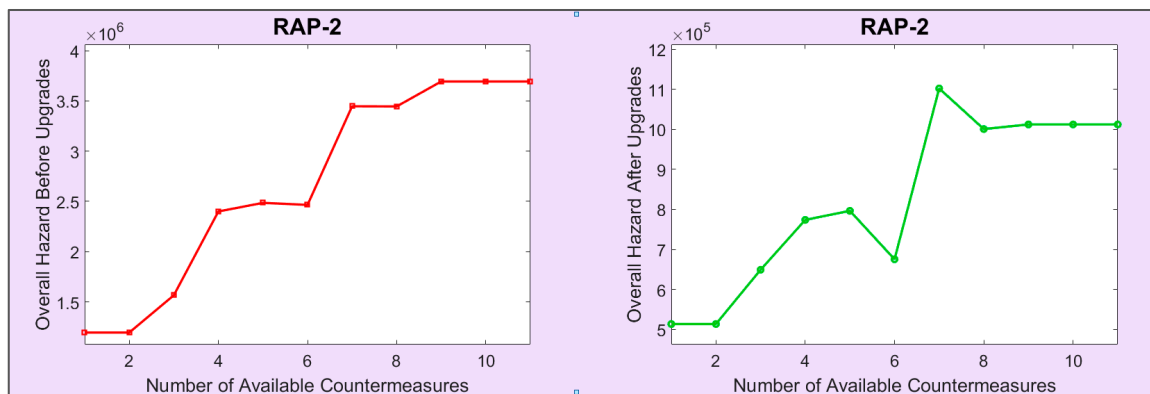

Figure 14. The impact of the number of countermeasures on the overall hazard before and after implementing countermeasures at the highway–rail grade crossings upgraded by **RAP-2**.

Note that similar percentages were observed in terms of the overall hazard reduction at the highway–rail grade crossings that were suggested for upgrading for both **RAP-1** and **RAP-2**. However, **RAP-1** suggested a total of 100 and 1198 highway–rail grade crossings to be upgraded for countermeasure availability scenarios "1" and "11", respectively, while **RAP-2** suggested a total of 100 and 1212 highway–rail grade crossings to be upgraded for countermeasure availability scenarios "1" and "11". Such a variation can be explained by differences in the resource allocation objectives (i.e., **RAP-1** aims to minimize the overall crossing hazard, while **RAP-2** aims to minimize the overall crossing hazard severity).

7. Conclusions and Future Research Directions

Smart cities directly rely on a variety of elements, including water, gas, electricity, buildings, services, transportation networks, and others. Lack of properly designed transportation networks may cause different economic and safety concerns. Considering a significant number of accidents at the highway–rail grade crossings across the United States, safety improvement projects have to be conducted on a regular basis. Given the limited budget available for highway–rail grade crossing safety improvement projects, effective resource allocation has been a challenging issue for the state Departments of Transportation (DOTs). This study presented two optimization models for resource allocation among highway–rail grade crossings. The objective function of the first optimization model aimed to minimize the overall hazard at the highway–rail grade crossings. On the other hand, the objective function of the second optimization model aimed to minimize the overall hazard severity at the highway–rail grade crossings. The developed optimization models were solved to the global optimality using CPLEX. Also, several heuristic algorithms were developed to solve each one of the optimization models. The heuristic algorithms constructed the priority list for the available highway–rail grade crossings based on certain attributes (i.e., hazard reduction-to-cost ratios, hazard severity reduction-to-cost ratios, hazard reduction, and hazard severity reduction).

A case study was performed for the public highway–rail grade crossings in the State of Florida, aiming to evaluate the performance of the developed solution methodologies. It was found that the developed heuristic algorithms were promising in terms of the solution quality and the computational time. Furthermore, prioritization of highway–rail grade crossings for safety improvement projects based on the hazard and hazard severity reduction-to-cost ratios was found to be a more promising strategy as compared to prioritization based on the hazard and hazard severity reduction values. The conducted case study showed that allocation of additional funds for safety improvement projects and introduction of additional countermeasures increased the number of upgraded highway–rail grade crossings and reduced the overall hazard at the highway–rail grade crossings. The proposed methodology can be further used by the relevant stakeholders, including the smart city authorities, for effective resource allocation among the highway–rail grade crossings, not only in Florida but across the nation.

The future research extensions include: (1) conducting additional sensitivity analyses for the model parameters (e.g., sensitivity of the models for different groups of highway–rail grade crossings—private, public, or both private and public); (2) applying the proposed methodology to the highway–rail grade crossings located in other states; (3) developing metaheuristic algorithms to solve the optimization models (e.g., evolutionary algorithms, ant colony optimization, particle swarm optimization, artificial bee colony, tabu search, simulated annealing, variable neighborhood search, and others [44–48]); (4) considering the application of multiple countermeasures at a given highway–rail grade crossing through resource allocation; and (5) performing resource allocation for both private and public highway–rail grade crossings in the State of Florida.

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