

# Elliptic Array Luminescent Solar Concentrators for Combined Power Generation and Microalgae Growth

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## Supplementary Material

### A. Description of an ellipse for the Elliptic Array Solar Spectrum Splitter

An ellipse can be defined as a curved line that forms a closed loop around two focal points,  $f_1$  and  $f_2$ , with the property that for any point  $S$  on the curve, the sum of the distances from  $f_1$  to  $S$  and  $f_2$  to  $S$  is constant. Moreover, the normal to the curve at any point  $S$  on the ellipse bisects the angle between the lines connecting  $f_1$  to  $S$  and  $f_2$  to  $S$ . As shown in Figure S1a, from a ray-optics perspective, this implies that all light emitted from  $f_1$  ( $f_2$ ) that is specularly reflected from the internal surface of the ellipse will be directed towards  $f_2$  ( $f_1$ ) because the angle of incidence  $\theta_1$  ( $\theta_2$ ) is equal to the angle of reflection  $\theta_2$  ( $\theta_1$ ). In this work we utilize this internal refocusing property of an ellipse to design a LSC panel with minimal optical losses.

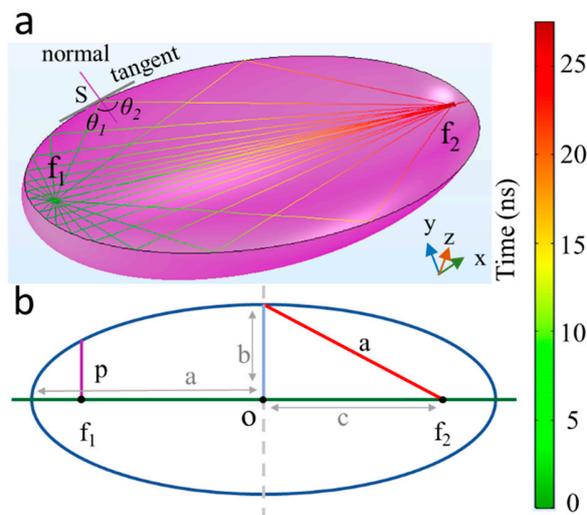


Figure S1. a) Cross-section of an ellipsoid as a closed loop curve and; b) the geometrical parameters for an ellipse. A light ray is emitted from  $f_1$  at  $t = 0$  ns. The trajectory of this emitted ray changes color in accordance with the colored vertical bar on the right, and after 25 ns the light ray has reached  $f_2$ .

The equation defining an ellipse using Cartesian coordinates is provided in Equation S1. Where the center of the ellipse is at the origin,  $O$ , and the x-axis is the major axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (1)$$

With reference to Figure A1b, the values of  $p$  and  $b$  are given in Equation S2:

$$p = \frac{b^2}{a} \quad \text{and} \quad b^2 = a^2 - c^2 \quad (S2)$$

The shape parameters  $a$  and  $b$  are the semi-major axis and semi-minor axis, respectively. The proposed SSS is composed of an array of unit-cells wherein each unit cell has the shape of two overlapping half-ellipses that share the same focal point.

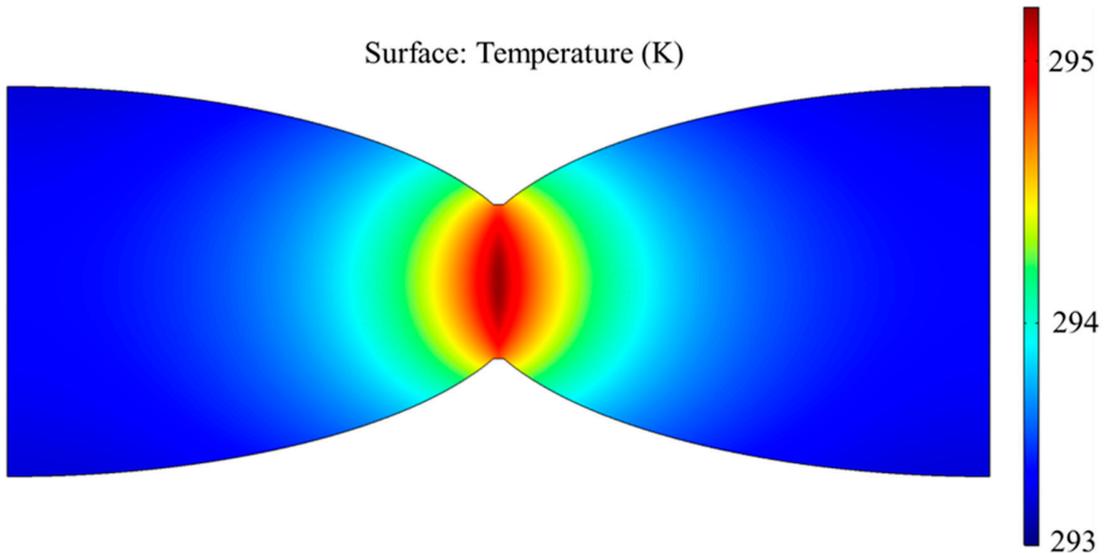
## B. Thermal analysis of the EASSS

The ray tracing and heat transfer modules in COMSOL Multiphysics software are coupled to investigate the temperature distribution in a unit cell of the EASSS structure during steady state. The optical and mechanical properties are shown and explained in Table S1. The incoming solar irradiance is  $1000 \text{ W/m}^2$ .

**Table S1:** Mechanical and optical properties of the EASSS for thermal analysis.

Property	variable	value	unit
Heat capacity at constant pressure	Cp	1420	J/(kg*K)
Density	$\rho$	1190	kg/m <sup>3</sup>
Thermal conductivity	k	0.19	W/(m*K)
Heat transfer coefficient	h	10	W/(m <sup>2</sup> *K)
Ambient temperature	T	293.15	K
Refractive index, real part	n	1.492	1
Refractive index, imaginary part	k	$3 \times 10^{-8}$	1
Thermal variation of the real part of the refractive index	dn/dT	$100 \times 10^{-6}$	1/K

Figure S1 shows the results of the simulations for the temperature distribution in an EASSS unit cell with  $h = 2 \text{ cm}$  and  $c = 5 \text{ cm}$ .



**Figure S2.** Temperature distribution within an EASSS unit cell for an incoming solar irradiance of  $1000 \text{ W/m}^2$  ( $h = 2 \text{ cm}$  and  $c = 5 \text{ cm}$ ).

Figure S2 shows the temperature profile for this case along the top surface of the EASSS as shown as the blue line in the inset (Figure S2a), and along the red horizontal line at the middle of the EASSS (Figure S2b).

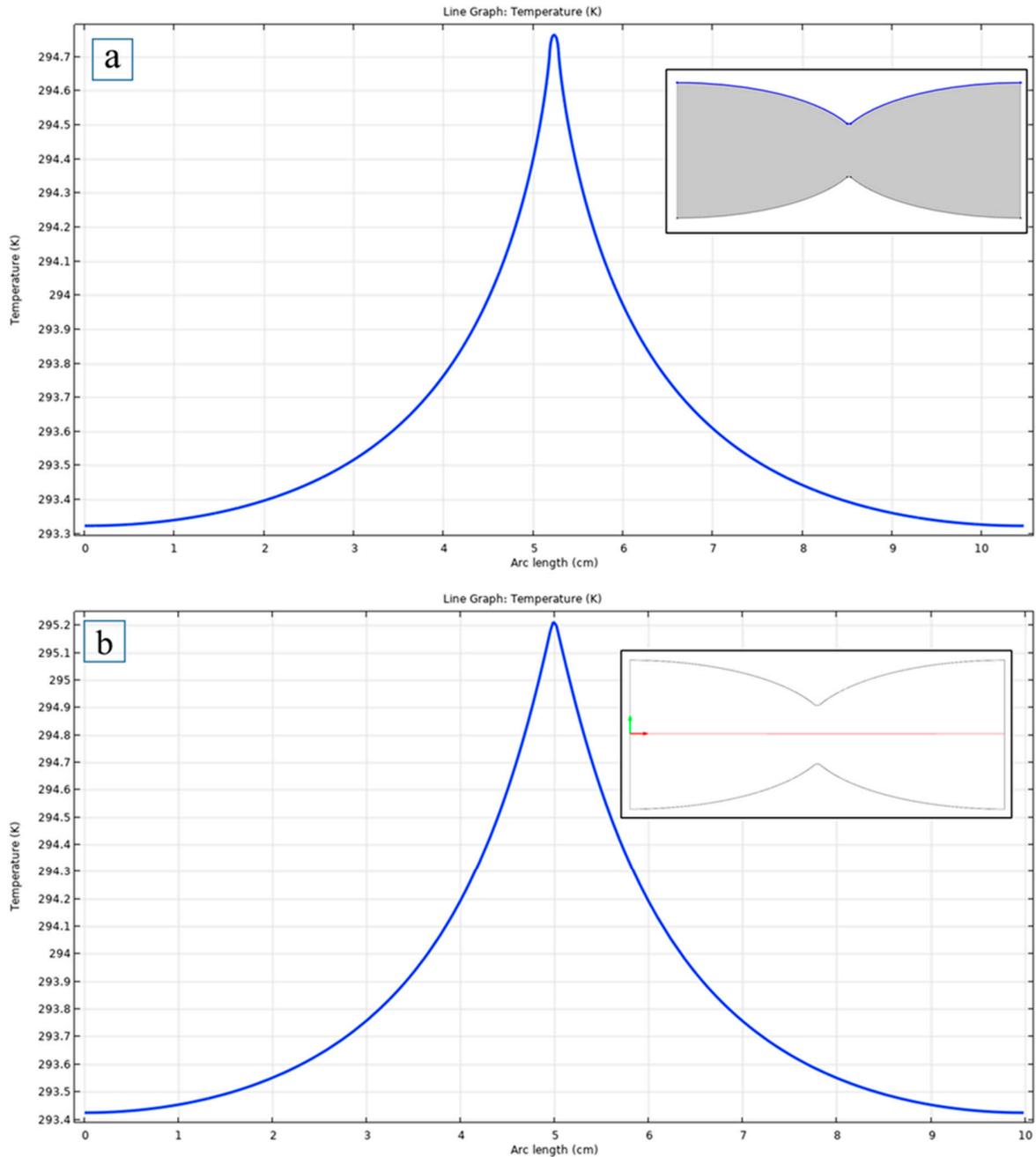


Figure S3. Temperature profile for the case study ( $h = 2$  cm and  $c = 5$  cm) of the EASSS along the a) Top surface of the EASSS shown in the inset with blue line, and b) Horizontal middle line shown in the inset with red line.

### C. Comparison of the weights of the EASSS and conventional LSC

To compare the weights of the EASSS and standard LSC we selected LSCs and EASSS with equal heights. We considered two cases: 1)  $c = 5$  cm and  $b = 1$  cm, and 2)  $c = 5$  cm and  $b = 2$  cm. Noting the

thickness of the normal LSC is  $h = 2 \cdot b$ , and taking the density of PMMA as  $\rho = 1190 \text{ Kg/m}^3$ , the results are shown below:

- 1)  $c = 5 \text{ cm}$  and  $b = 1 \text{ cm} \Rightarrow$  *weight of normal LSC (23.8 Kg) – weight of EASSS including the Petzval lens array (21Kg) = 2.8 Kg for 1m<sup>2</sup> of surface area for each structure*

This result implies that for an area of 1m<sup>2</sup> the EASSS is 2.8 Kg lighter than LSC.

- 2)  $c = 5 \text{ cm}$  and  $b = 2 \text{ cm} \Rightarrow$  *weight of normal LSC (47.6 Kg) – weight of EASSS including the Petzval lens array (41.34 Kg) = 6.26 Kg for 1m<sup>2</sup> of surface area for each structure*

This result implies that for an area of 1m<sup>2</sup> the EASSS is 6.26 Kg lighter than LSC.

It should be mentioned that the weight of the Petzval lens array can be vary between 2 Kg to 15.6 Kg based on its distance from the unit cells of the EASSS. If the lenses are close to the unit cells (~10 cm), the Petzval lenses would be thick and therefore they will be heavy (15.6 Kg for these sample cases when  $c = 5\text{cm}$ ). If the Petzval lens array is further from the EASSS (~80cm), the weight of the lens arrays would be 2 Kg for the same sample case. In the calculations above it is assumed that the weight of the Petzval lens array is 2 Kg.