

## Article

# Compensation of Data Loss Using ARMAX Model in State Estimation for Control and Communication Systems Applications

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**Abstract:** Compensation of data loss in the state estimation plays an indispensable role in efficient and stable control and communication systems. However, accurate compensation of data loss in the state estimation is extremely challenging issue. To cater this challenging issue, two techniques such as the open-loop Kalman filter and the compensating closed-loop Kalman filter have emerged. The closed-loop technique compensates for the missing data using the autoregressive model. However, the autoregressive model used only past measurements for data loss compensation. Considering only one parameter, i.e., the past measurements, is insufficient and leads to inaccurate state estimation. Thus, in this work, autoregressive moving average with exogenous inputs model considers three parameters, i.e., the past measurements, the input signal, and the sensor noise, simultaneously to compensate data loss in state estimation. To endorse the effectiveness and applicability of the proposed model, a standard mass-spring-damper is employed in the case study. Simulation results show that the proposed model outperforms the existing autoregressive models in terms of performance parameters.

**Keywords:** autoregressive moving average with exogenous input model; Kalman filter; linear prediction theory; loss of observation; open-loop estimation; closed-loop estimation

## 1. Introduction

State estimation has been an interesting and active research area over the past decades due to its important role in systems where direct access to the measured state of system is either impossible or very difficult [1]. There are particularly high demands on the design of robust algorithms that provide bounded estimation errors when a data loss occurs at the system output, such as when the measurements channels fail (sensors) [2]. In both the control and communication literatures, data loss is an important case study. Temporary defective sensors, limited bandwidth of communication lines, limited memory sizes, and mismatching of measurement devices, to name a few, are all possible causes of such loss. In order, to avoid the adverse effects arising due to loss of data in control and communication systems has been one of the most challenging research problems for the last few decades [3].

Different estimation techniques are used to extract the optimal information signal from a noise contaminated signal when both the information and noise signal and have overlapping spectrum. In [4], machine learning techniques are employed for prediction

purpose. In literature the Kalman filter is one of the best estimation tools among them [5]. Kalman filter is a least square error method for the estimation of a signal, or a parameter vector distorted in transmission through a channel and observed in noise. Kalman filter can be used with time-varying as well as time-invariant processes [6]. Kalman filter is a two-step recursive filter, i.e., a priori step and a posteriori step. In a priori step, it predicts the state and covariance of the system, and then updates it with the help of measurements in the a posteriori step. A priori step is a time-dependent step, whereas the a posteriori step heavily depends on noisy measurement.

The performance of state estimation is affected when subjected to data loss. This data loss may be due to limited memory space, faulty sensors, or insufficient bandwidth of the channel [7]. To address the problem, open-loop estimation (OLE) and closed-loop estimation (CLE) have been proposed. In OLE, only a priori step is carried out and a posteriori step is avoided due to the unavailability of measurement. However, OLE cannot produce bounded estimation error if the data loss occurs for a longer period [8–10]. Some estimation techniques are required to produce acceptable state estimation with bounded estimation error in case of loss of observation. Some researchers have replaced OLE technique with other methods, i.e., zero-order hold (ZOH) method or first-order hold technique [11–13]. In CLE, the most prevailed used in data loss is the autoregressive (AR) model [14,15]. The above discussed techniques are valuable assets of the literature, and all are capable of performing data loss compensation in the state estimation. However, the AR model compensates for the missing data using a minimum amount of information, i.e., only past measurement. Considering past measurements and control input parameters is insufficient and leads to inaccurate state estimation. With this motivation, in this paper, the autoregressive–moving-average with exogenous inputs (ARMAX) model is developed to compensate data loss in state estimation for control and communication systems applications. The novelty and main contribution of the paper are highlighted as follows.

The ARMAX model compensates for the lost signal and missing data by using an auxiliary vector employing the input to the system, the measurement values, and the sensor noise. The proposed ARMAX model for the compensation of data loss uses the system input and sensor noise in addition to the measurement used by the existing AR model. Linear prediction theory has been used in integration with Kalman filtering, to deal with the problem of data loss in the state estimation. The intent is to improve the compensation of data loss by replacing the AR model with the ARMAX model with the reason of using more information associated with the missing signal. A detailed description of Kalman filtering and linear prediction theory has been provided in the upcoming sections.

A detailed explanation of Kalman filtering is presented in Section 2. The theory of linear prediction has been also covered in Section 2. Section 3 narrates the existing remedies employed to reconstruct the lost missing signal and the mathematical description and calculation of Linear Prediction Coefficients (LPCs) of the proposed model. Section 4 contains the simulation results and comparison of existing and proposed schemes. The conclusive remarks of this paper and future work are given in Section 5.

## 2. Filtering with Data Loss

The two normally used customs are filtering and linear prediction theory in the event of unavailability of output data or inaccurate measurement data [16]. The well-known estimation for linear time-invariant (LTI) system is the Kalman filter, with some generic assumptions. A general review of Kalman filter and linear prediction theory is discussed in this section, suitable for an article level. Each iteration of the Kalman filter consists of two cycles without loss of generality: prediction or time update, and filtration or measurement update [6,17].

### 2.1. Basic Structure of Kalman Filter

A discrete linear time-invariant system has the following basic structure:

$$\mathbf{X}_{m+1} = \mathbf{A}\mathbf{X}_m + \mathbf{B}\mathbf{u}_m + \mathbf{d}_m\mathbf{V}_m = \mathbf{C}\mathbf{X}_m + \mathbf{W}_m$$

where  $\mathbf{A}$ :  $\mathbf{O}^k \mathbf{O}^k \times k$ ; being the transition matrix,  $\mathbf{B}$ :  $\mathbf{O}^k \mathbf{O}^k \times 1$ ; represent the input matrix,  $\mathbf{C}$ :  $\mathbf{O}^k \mathbf{O}^m \times k$ ; is the output matrix,  $\mathbf{x}$  is the state vector,  $\mathbf{u}$  is input to the system,  $\mathbf{d}$  is the system disturbance,  $\mathbf{w}$  is the sensor noise, and  $\mathbf{v}$  is the system output.

$$\mathbf{d}_m = (0, Q_m)\mathbf{W}_m = (0, R_m)$$

where  $Q_m$  and  $R_m$  the system and measurement noise covariance, respectively. If the Gaussian noise does not allow accurate measurement of the state variables, the estimation process must be performed, for example, by Kalman filter. The detailed process of Kalman filtering is given as follows [18].

### 2.1.1. A Priori Step

This stage is dependent on the dynamics of the system from which the estimated state and associated error covariance are determined as

$$\mathbf{X}_{m+1|m} = \mathbf{A}\mathbf{X}_{m|m} + \mathbf{B}\mathbf{u}_m \quad (1)$$

$$\mathbf{P}_{m+1|m} = \mathbf{A}\mathbf{P}_{m|m}\mathbf{A}^T + \mathbf{Q}_m \quad (2)$$

where

$$\mathbf{P}_{m+1|m} = \text{Exp}(\mathbf{e}_{m+1|m} \mathbf{e}_{m+1|m}^T) \mathbf{e}_{m+1|m} = \mathbf{X}_{m+1} - \mathbf{X}_{m+1|m}$$

These two variables must be corrected utilizing Kalman filter gain computed through the measurement vector.

### 2.1.2. Kalman Filter Gain

To calculate the optimal value of Kalman filter gain the following procedure has been carried out

$$\mathbf{K}_m = \mathbf{P}_{m+1|m} \mathbf{C}^T [\mathbf{C}\mathbf{P}_{m+1|m} \mathbf{C}^T + \mathbf{R}_m]^{-1}$$

The state prediction and covariance prediction are given by Equations (1) and (2) are corrected based on Kalman filter gain as given in the next subsection.

### 2.1.3. A Posteriori Step

The measurement update step is described as

$$\mathbf{X}_{m+1|m+1} = \mathbf{X}_{m+1|m} + \mathbf{K}_m [\mathbf{V}_m - \mathbf{C}\mathbf{X}_{m+1|m}] \mathbf{P}_{m+1|m+1} = (\mathbf{I} - \mathbf{K}_m)\mathbf{P}_{m+1|m}\mathbf{C}$$

The description of this traditional Kalman filter can be found in different recommended graduate books, e.g., [6].

## 2.2. Linear Prediction Theory

The basic idea in linear prediction is that a signal is modeled as a linear combination of the present and past samples of the signal. A major portion of work has been carried out on system modeling in the field of control systems under subjects of system identification and estimation. The weights used for computation of the linear combination are calculated by reducing the mean-square prediction error [7,19]. The two major classes of linear prediction are external and internal linear prediction. A detailed explanation of internal and external linear prediction is given in the following sub-sections.

### 2.2.1. Internal Linear Prediction

In this type of prediction, prediction coefficients for a certain data frame are computed from the data inside that frame. The LPCs in internal prediction capture the data frame statistics accurately. The data frame may be static or dynamic. The advantage associated with longer frame size is its low computational complexity because LPCs are calculated and hence transmitted less often. However, the coding delay for longer frame sizes may

grow larger as the system needs to wait for a longer period of time to collect too many samples [20,21]. Additionally, LPCs of a long frame may fail to achieve good prediction gain in case of the changing nature of non-stationary systems. Alternatively, a shorter data frame requires more frequent LPC updates, which results in a more accurate portrayal of the input signal statistics compared with the longer data frame. Mostly, the internal prediction techniques rely on non-recursive autocorrelation methods for estimation, which uses a window of a finite length for obtaining the signal samples. The internal linear prediction does not predict the signal; rather, it just computes the coefficients of the input signal to be transmitted. The transmission of LPCs of a signal requires less bandwidth and storage compared with the original signal, thus saving the useful bandwidth and memory space [4].

### 2.2.2. External Linear Prediction

The LPCs derived in external prediction are used in the future data frame, i.e., the coefficients related to a frame are not computed from data samples located inside that frame; instead, LPCs are derived from the past samples of the signal [8]. External prediction is more useful where the statistical properties of the signal change slowly with time. The frame size needs to be long enough such that in case of data loss it can recover the signal. In this paper, the external prediction has been used. In external prediction, the calculation of LPCs is based on the following two suppositions:

- There must exist a correlation between the data samples [5].
- The statistical characteristics of the signal slowly change over time [5].

If the above two suppositions are met, the future values of the data can be computed. We will also establish and apply the external prediction method in the event of a loss of observation that could match many engineering applications to our best knowledge, including our case study discussed in the following section. The proposed model for our work has the following representation:

$$\bar{\mathbf{v}} = \sum_{i=1}^o \mathbf{v}_i \mathbf{v}_{m-i} + \sum_{j=1}^p \boldsymbol{\phi}_j \mathbf{u}_{m-j} + \sum_{k=1}^q \boldsymbol{\varphi}_k \mathbf{w}_{m-k} \quad (3)$$

where  $\bar{\mathbf{v}}$  is the required data vector to be compensated,  $\mathbf{v}_i$ ,  $\boldsymbol{\phi}_j$ , and  $\boldsymbol{\varphi}_k$  are the required LPCs to be computed optimally.  $\mathbf{v}_m$ ,  $\mathbf{u}_m$ , and  $\mathbf{w}_m$  are the system output data, input signal, and sensor noise, respectively. The variables  $o$ ,  $p$ , and  $q$  in Equation (3) show the order of the past samples and are to be decided optimally [3].

## 3. Existing Solutions Employed for Data Loss

Open-loop estimation and closed-loop estimation are employed to reconstruct the lost measurement in the estimation process. A detailed explanation including its pros and cons has been presented in the next subsections.

### 3.1. Open-Loop Kalman Filtering

In the literature, the most prevailed method used in the case of output data loss is open-loop Kalman filtering. Here, only a priori step is performed, i.e., a posteriori step is ignored due to the unavailability of measurement. The structure of open-loop filtering, which is derived from the conventional Kalman filter, is presented by the following equations.

- a. A priori Step

$$\mathbf{X}_{m+1|m} = \mathbf{A}\mathbf{X}_{m|m} + \mathbf{B}\mathbf{u}_m \quad \text{state prediction} \quad \mathbf{P}_{m+1|m} = \mathbf{A}\mathbf{P}_{m|m}\mathbf{A}^T + \mathbf{Q}_m \quad \text{covariance prediction}$$

- b. Gain Calculation

If  $\mathbf{v}$  is not available,  $\mathbf{K}_m = 0$

c. A posteriori Step

$$\mathbf{X}_{m+1|m} = \mathbf{X}_{m+1|m} \text{ state update } \mathbf{P}_{m+1|m} = \mathbf{P}_{m+1|m} \text{ covariance update}$$

As OLE only performs the prediction step, it results in unbounded estimation errors if the observation data is missing for a long period of time. Some estimation techniques are required to produce satisfactory state estimation with restricted estimation errors in case of a loss of observation. Some researchers have replaced the OLE technique with other methods i.e., closed-loop Kalman filtering using the AR model. A detailed description of the closed-loop method has been provided in the next subsection.

### 3.2. Improved Compensation Using the AR Model (Normal Equations)

In a system identification application, the FIR filter output shows an estimate of the present output sample from an unknown system as a sum of linearly weighted past and/or future samples from the input to an unknown system [22,23]. The normal equation derivation is based on the minimization of the mean square error. Conventionally, several methods have been used to solve normal equations; most common are the covariance method which is used for non-stationary processes, and the autocorrelation method is appropriate for a stationary process. In the normal equation method, the predicted signal is represented by an equation given below [9]:

$$\bar{\mathbf{v}} = \sum_{i=1}^P \mathbf{v}_i \mathbf{v}_{m-i} \quad (4)$$

In the above equation,  $\mathbf{v}_i$  and  $\mathbf{v}_{m-i}$  are the measurement vector and linear prediction coefficients, respectively.  $P$  shows the order of the filter and has to be chosen optimally. The results achieved using the model described by Equation (4) and the associated topics can be read in [3,14]. The main problem associated with this model that it contains a limited amount of information for the compensation of loss measurement. By taking more information, it is believed that improved results can be achieved. Therefore, the model of ARMAX has been proposed in this paper which contains sufficient information for compensation of missing measurement. The necessary explanation, derivation, and terms based on the ARMAX model have been presented in the next section.

## 4. Improved Compensation Using Proposed ARMAX Model

The proposed scheme for the compensation of lost observations is based on the ARMAX model employing three different information regarding the missing signal, i.e., input, measurement, and system disturbance. By using more information for compensation, more efficient results are expected. Mathematical representation of the model is

$$\bar{\mathbf{v}} = \sum_{i=1}^o \mathbf{v}_i \mathbf{v}_{m-i} + \sum_{j=1}^p \boldsymbol{\phi}_j \mathbf{u}_{m-j} + \sum_{k=1}^q \boldsymbol{\varphi}_k \mathbf{w}_{m-k}$$

The same order is taken here for simplicity purpose.

$$\bar{\mathbf{v}} = \sum_{j=1}^o \mathbf{v}_j \mathbf{v}_{m-j} + \sum_{j=1}^o \boldsymbol{\phi}_j \mathbf{u}_{m-j} + \sum_{j=1}^o \boldsymbol{\varphi}_j \mathbf{w}_{m-j}$$

Compensation error is calculated as follows.

$$\mathbf{e}_m = \mathbf{v}_m - \bar{\mathbf{v}}_m$$

where  $\bar{v}_m$  and  $\mathbf{v}_m$  are the compensated and actual vector, respectively. The optimal values of LPCs are achieved using cost function consisting of mean square prediction as presented below

$$\mathbf{J} = \mathbb{E}[\mathbf{e}^2] \mathbf{J} = \mathbb{E}[\mathbf{v}_m - \bar{v}_m] \quad (5)$$

One will obtain the following equation by incorporating the value-compensated vector in Equation (5).

$$\mathbf{J} = \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} + \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} + \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \quad (6)$$

Take the derivative of Equation (6) with respect to  $\mathbf{v}_i$  as

$$\begin{aligned} \frac{\partial \mathbf{J}}{\partial \mathbf{v}_i} &= \frac{\partial}{\partial \mathbf{v}_i} \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] = 0 \\ 2 \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \mathbf{v}_{m-i} &= 0 \\ \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \mathbf{v}_{m-i} &= 0 \\ \mathbb{E}[\mathbf{v}_m \mathbf{v}_{m-i}] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} \mathbf{v}_{m-i} \right] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} \mathbf{v}_{m-i} \right] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \mathbf{v}_{m-i} \right] &= 0 \end{aligned} \quad (7)$$

Let

$$\mathbb{E}[\mathbf{v}_m \mathbf{v}_{m-i}] = \mathbf{m}_1 \quad (8)$$

$$\mathbb{E}[\mathbf{v}_{m-j} \mathbf{v}_{m-i}] = \mathbf{M}_1 \quad (9)$$

$$\mathbb{E}[\mathbf{u}_{m-j} \mathbf{v}_{m-i}] = \mathbf{M}_2 \quad (10)$$

$$\mathbb{E}[\mathbf{w}_{m-j} \mathbf{v}_{m-i}] = \mathbf{M}_3 \quad (11)$$

Substituting Equations (8)–(11) in Equation (7) will reduce to the following form:

$$\mathbf{m}_1 - \mathbf{v}_j \mathbf{M}_1 - \boldsymbol{\phi}_j \mathbf{M}_2 - \boldsymbol{\varphi}_j \mathbf{M}_3 = 0 \mathbf{v}_j \mathbf{M}_1 + \boldsymbol{\phi}_j \mathbf{M}_2 + \boldsymbol{\varphi}_j \mathbf{M}_3 = \mathbf{m}_1 \quad (12)$$

Differentiating Equation (6) with respect to  $\boldsymbol{\phi}_i$  will result in the following equation:

$$\begin{aligned} \frac{\partial \mathbf{J}}{\partial \boldsymbol{\phi}_i} &= \frac{\partial}{\partial \boldsymbol{\phi}_i} \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] = 0 \\ 2 \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \mathbf{u}_{m-i} &= 0 \\ \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \mathbf{u}_{m-i} &= 0 \\ \mathbb{E}[\mathbf{v}_m \mathbf{u}_{m-i}] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} \mathbf{u}_{m-i} \right] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} \mathbf{u}_{m-i} \right] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \mathbf{u}_{m-i} \right] &= 0 \end{aligned} \quad (13)$$

Let

$$\mathbb{E}[\mathbf{v}_m \mathbf{u}_{m-i}] = \mathbf{m}_2 \quad (14)$$

$$\mathbb{E} \left[ \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} \mathbf{u}_{m-i} \right] = \mathbf{M}_4 \quad (15)$$

$$\mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} \mathbf{u}_{m-i} \right] = \mathbf{M}_5 \quad (16)$$

$$\mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \mathbf{u}_{m-i} \right] = \mathbf{M}_6 \quad (17)$$

Substitution of Equations (14)–(17) in Equation (13) will take the following form:

$$\mathbf{m}_2 - \mathbf{v}_j \mathbf{M}_4 - \boldsymbol{\phi}_j \mathbf{M}_5 - \boldsymbol{\varphi}_j \mathbf{M}_6 = 0 \mathbf{v}_j \mathbf{M}_4 + \boldsymbol{\phi}_j \mathbf{M}_5 + \boldsymbol{\varphi}_j \mathbf{M}_6 = \mathbf{m}_2 \quad (18)$$

Next, we differentiate Equation (6) with respect to  $\varphi_i$ .

$$\begin{aligned} \frac{\partial J}{\partial \varphi_i} &= \frac{\partial}{\partial \varphi_i} \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] = 0 \\ 2 \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \mathbf{w}_{m-i} &= 0 \\ \mathbb{E} \left[ \mathbf{v}_m - \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} - \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \right] \mathbf{w}_{m-i} &= 0 \\ \mathbb{E}[\mathbf{v}_m \mathbf{d}_{m-i}] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} \mathbf{d}_{m-i} \right] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} \mathbf{w}_{m-i} \right] - \mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \mathbf{w}_{m-i} \right] &= 0 \end{aligned} \quad (19)$$

Let

$$\mathbb{E}[\mathbf{v}_m \mathbf{d}_{m-i}] = \mathbf{m}_3 \quad (20)$$

$$\mathbb{E} \left[ \sum_{j=1}^{\circ} \mathbf{v}_j \mathbf{v}_{m-j} \mathbf{w}_{m-i} \right] = \mathbf{M}_7 \quad (21)$$

$$\mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\phi}_j \mathbf{u}_{m-j} \mathbf{w}_{m-i} \right] = \mathbf{M}_8 \quad (22)$$

$$\mathbb{E} \left[ \sum_{j=1}^{\circ} \boldsymbol{\varphi}_j \mathbf{w}_{m-j} \mathbf{w}_{m-i} \right] = \mathbf{M}_9 \quad (23)$$

Incorporating Equations (20)–(23) into Equation (19) will give the following equation

$$\mathbf{m}_3 - \mathbf{v}_j \mathbf{M}_7 - \boldsymbol{\phi}_j \mathbf{M}_8 - \boldsymbol{\varphi}_j \mathbf{M}_9 \mathbf{v}_j \mathbf{M}_7 + \boldsymbol{\phi}_j \mathbf{M}_8 + \boldsymbol{\varphi}_j \mathbf{M}_9 = \mathbf{m}_3 \quad (24)$$

The values of LPCs can be computed easily by solving Equations (12), (18) and (24) simultaneously. There are two different methods for solving simultaneous equations, i.e., the substitution method and the elimination method. The elimination method is used here for this purpose.

$$\mathbf{v}_j \mathbf{M}_4 \mathbf{M}_1 + \boldsymbol{\phi}_j \mathbf{M}_4 \mathbf{M}_2 + \boldsymbol{\varphi}_j \mathbf{M}_4 \mathbf{M}_3 = \mathbf{M}_4 \mathbf{m}_1$$

$$\mathbf{v}_j \mathbf{M}_4 \mathbf{M}_1 + \boldsymbol{\phi}_j \mathbf{M}_5 \mathbf{M}_1 + \boldsymbol{\varphi}_j \mathbf{M}_6 \mathbf{M}_1 = \mathbf{M}_1 \mathbf{m}_2 \quad (25)$$

$$\boldsymbol{\phi}_j [\mathbf{M}_4 \mathbf{M}_2 - \mathbf{M}_5 \mathbf{M}_1] + \boldsymbol{\varphi}_j [\mathbf{M}_4 \mathbf{M}_3 - \mathbf{M}_6 \mathbf{M}_1] = \mathbf{M}_4 \mathbf{m}_1 - \mathbf{M}_1 \mathbf{m}_2$$

Let

$$[\mathbf{M}_4 \mathbf{M}_2 - \mathbf{M}_5 \mathbf{M}_1] = \mathbf{N}_1 \quad (26)$$

$$[\mathbf{M}_4 \mathbf{M}_3 - \mathbf{M}_6 \mathbf{M}_1] = \mathbf{N}_2 \quad (27)$$

$$[\mathbf{M}_4 \mathbf{m}_1 - \mathbf{M}_1 \mathbf{m}_2] = \mathbf{n}_1 \quad (28)$$

After incorporating these values in Equation (25), we will obtain the following equation.

$$\boldsymbol{\phi}_j \mathbf{N}_1 + \boldsymbol{\varphi}_j \mathbf{N}_2 = \mathbf{n}_1 \quad (29)$$

Again, using the method of elimination for simultaneous Equations (12) and (24), we are able to obtain an equation in two variables.

$$\boldsymbol{\phi}_j [\mathbf{M}_7 \mathbf{M}_2 - \mathbf{M}_8 \mathbf{M}_1] + \boldsymbol{\varphi}_j [\mathbf{M}_7 \mathbf{M}_3 - \mathbf{M}_9 \mathbf{M}_1] = [\mathbf{M}_7 \mathbf{m}_1 - \mathbf{M}_1 \mathbf{m}_3] \quad (30)$$

Let

$$[\mathbf{M}_7 \mathbf{M}_2 - \mathbf{M}_8 \mathbf{M}_1] = \mathbf{N}_3 \quad (31)$$

$$[\mathbf{M}_7 \mathbf{M}_3 - \mathbf{M}_9 \mathbf{M}_1] = \mathbf{N}_4 \quad (32)$$

$$[\mathbf{M}_7 \mathbf{m}_1 - \mathbf{M}_1 \mathbf{m}_3] = \mathbf{n}_2 \quad (33)$$

Equation (30) reduces to the following simple form after incorporating these values.

$$\phi_j \mathbf{N}_3 + \varphi_j \mathbf{N}_4 = \mathbf{n}_2 \quad (34)$$

Multiply Equation (29) by  $\mathbf{N}_3$  and Equation (34) by  $\mathbf{N}_1$ , and subtraction will give the following equation:

$$\begin{aligned} \phi_j \mathbf{N}_3 \mathbf{N}_1 + \varphi_j \mathbf{N}_3 \mathbf{N}_2 &= \mathbf{N}_3 \mathbf{n}_1 \\ \phi_j \mathbf{N}_3 \mathbf{N}_1 + \varphi_j \mathbf{N}_4 \mathbf{N}_1 &= \mathbf{N}_1 \mathbf{n}_2 \\ \phi_j [\mathbf{N}_3 \mathbf{N}_2 - \mathbf{N}_4 \mathbf{N}_1] &= [\mathbf{N}_3 \mathbf{n}_1 - \mathbf{N}_1 \mathbf{n}_2] \\ \phi_j &= [\mathbf{N}_3 \mathbf{N}_2 - \mathbf{N}_4 \mathbf{N}_1]^{-1} [\mathbf{N}_3 \mathbf{n}_1 - \mathbf{N}_1 \mathbf{n}_2] \end{aligned} \quad (35)$$

Multiply Equation (29) by  $\mathbf{N}_4$  and Equation (34) by  $\mathbf{N}_2$ , and subtraction will give the following equation:

$$\begin{aligned} \phi_j \mathbf{N}_4 \mathbf{N}_1 + \varphi_j \mathbf{N}_4 \mathbf{N}_2 &= \mathbf{N}_4 \mathbf{n}_1 \\ \phi_j \mathbf{N}_3 \mathbf{N}_2 + \varphi_j \mathbf{N}_4 \mathbf{N}_2 &= \mathbf{N}_2 \mathbf{n}_2 \\ \phi_j [\mathbf{N}_4 \mathbf{N}_1 - \mathbf{N}_3 \mathbf{N}_2] &= [\mathbf{N}_4 \mathbf{n}_1 - \mathbf{N}_2 \mathbf{n}_2] \\ \phi_j &= [\mathbf{N}_4 \mathbf{N}_1 - \mathbf{N}_3 \mathbf{N}_2]^{-1} [\mathbf{N}_4 \mathbf{n}_1 - \mathbf{N}_2 \mathbf{n}_2] \end{aligned} \quad (36)$$

To compute the value of  $\mathbf{v}_j$  now, the following procedure is to be carried out. Equation (12)  $\times$   $\mathbf{M}_5$ —Equation (24)  $\times$   $\mathbf{M}_2$

$$\mathbf{v}_j [\mathbf{M}_5 \mathbf{M}_1 - \mathbf{M}_4 \mathbf{M}_2] + \varphi_j [\mathbf{M}_5 \mathbf{M}_3 - \mathbf{M}_6 \mathbf{M}_2] = [\mathbf{M}_5 \mathbf{m}_1 - \mathbf{M}_2 \mathbf{m}_2] \quad (37)$$

Let

$$[\mathbf{M}_5 \mathbf{M}_1 - \mathbf{M}_4 \mathbf{M}_2] = \mathbf{N}_5 \quad (38)$$

$$[\mathbf{M}_5 \mathbf{M}_3 - \mathbf{M}_6 \mathbf{M}_2] = \mathbf{N}_6 \quad (39)$$

$$[\mathbf{M}_5 \mathbf{m}_1 - \mathbf{M}_2 \mathbf{m}_2] = \mathbf{n}_3 \quad (40)$$

After putting these values Equation (37) becomes

$$\mathbf{v}_j \mathbf{N}_5 + \varphi_j \mathbf{N}_6 = \mathbf{n}_3 \quad (41)$$

and Equation (12)  $\times$   $\mathbf{M}_8$ —Equation (24)  $\times$   $\mathbf{M}_2$

$$\mathbf{v}_j [\mathbf{M}_8 \mathbf{M}_1 - \mathbf{M}_7 \mathbf{M}_2] + \varphi_j [\mathbf{M}_8 \mathbf{M}_3 - \mathbf{M}_9 \mathbf{M}_2] = [\mathbf{M}_8 \mathbf{m}_1 - \mathbf{M}_2 \mathbf{m}_3] \quad (42)$$

Let

$$[\mathbf{M}_8 \mathbf{M}_1 - \mathbf{M}_7 \mathbf{M}_2] = \mathbf{N}_7 \quad (43)$$

$$[\mathbf{M}_8 \mathbf{M}_3 - \mathbf{M}_9 \mathbf{M}_2] = \mathbf{N}_8 \quad (44)$$

$$[\mathbf{M}_8 \mathbf{m}_1 - \mathbf{M}_2 \mathbf{m}_3] = \mathbf{n}_4 \quad (45)$$

The above values are shown in Equation (42) as

$$\mathbf{v}_j \mathbf{N}_7 + \varphi_j \mathbf{N}_8 = \mathbf{n}_4 \quad (46)$$

Elimination of  $\varphi_j$  from Equations (41) and (46) will give the value of  $\mathbf{v}_j$  as

$$\mathbf{v}_j [\mathbf{N}_8 \mathbf{N}_5 - \mathbf{N}_7 \mathbf{N}_6] = [\mathbf{N}_8 \mathbf{n}_3 - \mathbf{N}_6 \mathbf{n}_4] \mathbf{v}_j = [\mathbf{N}_8 \mathbf{N}_5 - \mathbf{N}_7 \mathbf{N}_6]^{-1} [\mathbf{N}_8 \mathbf{n}_3 - \mathbf{N}_6 \mathbf{n}_4] \quad (47)$$



All the equations used are summarized in Algorithm 1.

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**Algorithm 1** Summary of Equations
 

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1. **Initialize variables:**  $x = 0, p = 0, \text{Loss period} = 2235 - 2650 \text{ ms}$
  2. **Repeat:** For  $m = 1, 2, 3, \dots, p$
  3. **Prediction step**
    - 3.1 State prediction using Equation (1)
    - 3.2 Covariance Prediction using Equation (2)
  4. **v available?**
    - 4.1 Yes: Jump to step 5
    - 4.2 No:
      - (a) Compute:  $\mathbf{v}, \Phi, \varphi$  using Equations (5)–(47)
      - (b) Compensate  $\bar{v}$  using ARMAX model (Equation (3))
  5. **Update step:** Using the update set of Equations
  6. **Return:** To step 2
- 

Equations (35), (36) and (47) provide the desired linear prediction coefficients. Equation (3) is used to compensate for the missing data with the help of using these LPCs. A comparison of existing and proposed techniques is made in the next section.

## 5. Numerical Simulation Results and Analysis

The example employed for the evaluation of the above analysis is a mass-spring-damper (MSD) system. The dynamics of the MSD system are described by the following mathematical equations:

$$m_j \ddot{\mathbf{S}}_i + d_j \dot{\mathbf{S}}_i + k_j \mathbf{S}_i = u_i$$

In the above equation, the terms are

$m_1$  and  $m_2$  are being the masses,

$\mathbf{S}_1$  and  $\mathbf{S}_2$  represent the displacement of masses  $m_1$  and  $m_2$ , respectively,

$d_1$  and  $d_2$  are the damper constants,

$k_1$  and  $k_2$  are the spring constants and

$u_1$  and  $u_2$  are the forces on  $m_1$  and  $m_2$ , respectively.

$$\ddot{\mathbf{S}}_1 = -\frac{k_1}{m_1} d_1 + \frac{k_1}{m_1} d_1 - \frac{d_1}{m_1} \dot{\mathbf{S}}_1 + \frac{d_1}{m_1} \dot{\mathbf{S}}_2 + \frac{u}{m_1}$$

$$\ddot{\mathbf{S}}_2 = \frac{k_1}{m_2} d_2 - \frac{(k_1+k_2)}{m_2} d_2 + \frac{d_1}{m_2} \dot{\mathbf{S}}_1 - \frac{(d_1+d_2)}{m_2} \dot{\mathbf{S}}_2$$

$$\mathbf{S}^T = \left[ \mathbf{S}_1 \quad \mathbf{S}_2 \quad \dot{\mathbf{S}}_1 \quad \dot{\mathbf{S}}_2 \right] \mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{d_1}{m_1} & \frac{d_1}{m_1} \\ \frac{k_1}{m_2} & -\frac{(k_1+k_2)}{m_2} & \frac{d_1}{m_2} & -\frac{(d_1+d_2)}{m_2} \end{bmatrix}$$

Additionally,

$$\mathbf{I}^T = \left[ 0 \quad 0 \quad 0 \quad \frac{1}{m_2} \right]$$

With output equation:

$$\mathbf{v}_m = \mathbf{C}_m \mathbf{d}_m + \vartheta$$

The second state (i.e., the position of second mass  $m_2$ ) is taken as output in this work:

$$\mathbf{C} = \left[ 0 \quad 1 \quad 0 \quad 0 \right]$$

The mass-spring-damper system of order two is illustrated in Figure 1.

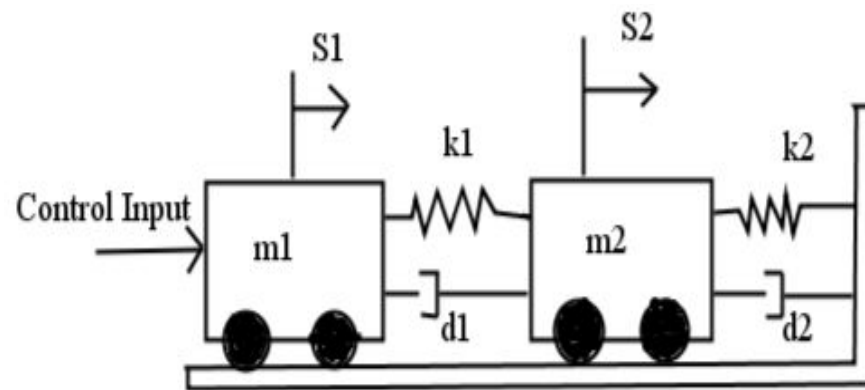


Figure 1. Case Study (MSD).

In the next subsection, the proposed compensated closed-loop KF (CCLKF) using ARMAX, and existing models, are implemented in the MSD system, and the results are presented. In closed-loop KF, the lost observation samples are predicted by using linear prediction schemes; hence, the estimation error is reduced as compared with the existing open-loop method.

### 5.1. Simulation Results

To perform the simulation, the values of different parameters are taken as follows from [10].

$$m_1 = m_2 = 1, k_1 = k_2 = 0.15, d_1 = d_2 = 0.1, T_s = 1 \text{ ms.}$$

By incorporating these known parameters, the following matrices are obtained:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & -0.1 & 0.1 \\ 0.1 & -1.15 & 0.1 & -0.2 \end{bmatrix} \quad \mathbf{B}^T = [0 \quad 0.1 \quad 0] \quad \mathbf{I}^T = [0 \quad 0.1 \quad 1]$$

The above case study is simulated on MATLAB 2018b CORE i5, 4.GB RAM, and 64-bit operating system.

### 5.2. Simulation Results without Data Loss

Before introducing the data loss in the estimation process, we present the performance of conventional Kalman filtering. These results have been shown in Figures 1–3, demonstrating three different results for the position of mass  $m_2$  of MSD, i.e., measured state shown by the solid line; the actual state represented by the dashed line, and estimated state by convention Kalman filtering given by the dash-dotted line in Figure 3 consists of three subplots, i.e., first state, third state, and fourth state, each with the actual state shown by the dotted line and the estimated state by convention Kalman filtering represented by the dashed line.

In case of no data loss, the convention Kalman filtering offers satisfactory results obvious from the above figures. The results for some existing remedies and a proposed one and their comparison is made in the next subsection.

### 5.3. Simulation Results with Data Loss Using Open-Loop Estimation, AR Model, and Proposed ARMAX Model

In this section, data loss has been introduced for a period of 415 samples and is inserted from 2235 ms to 2650 ms. The period of data loss can be introduced in any place during the estimation process. Results for existing techniques (OLE, and CLE using AR Model) in comparison with the ARMAX model are presented in the following subsection.

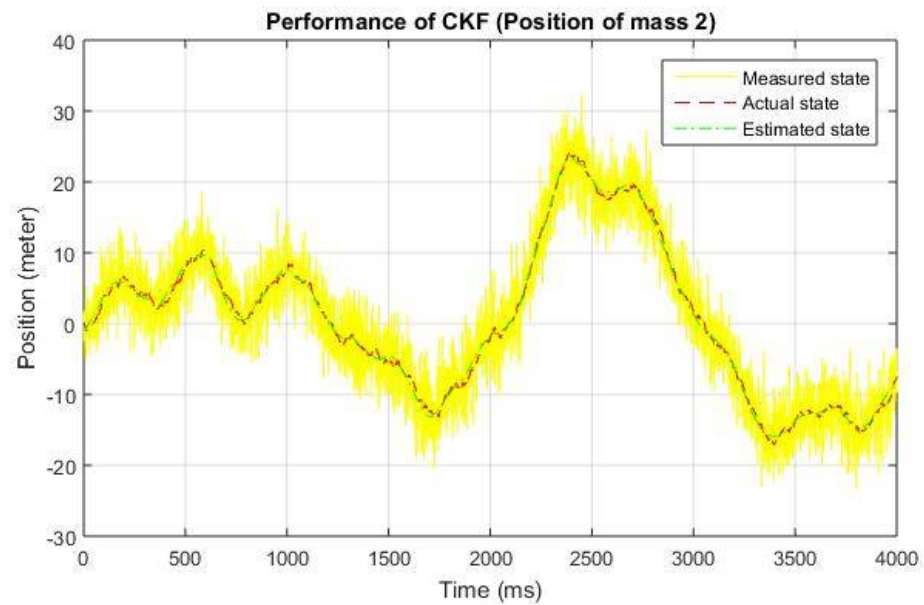


Figure 2. Simulation results with no data loss (measured state).

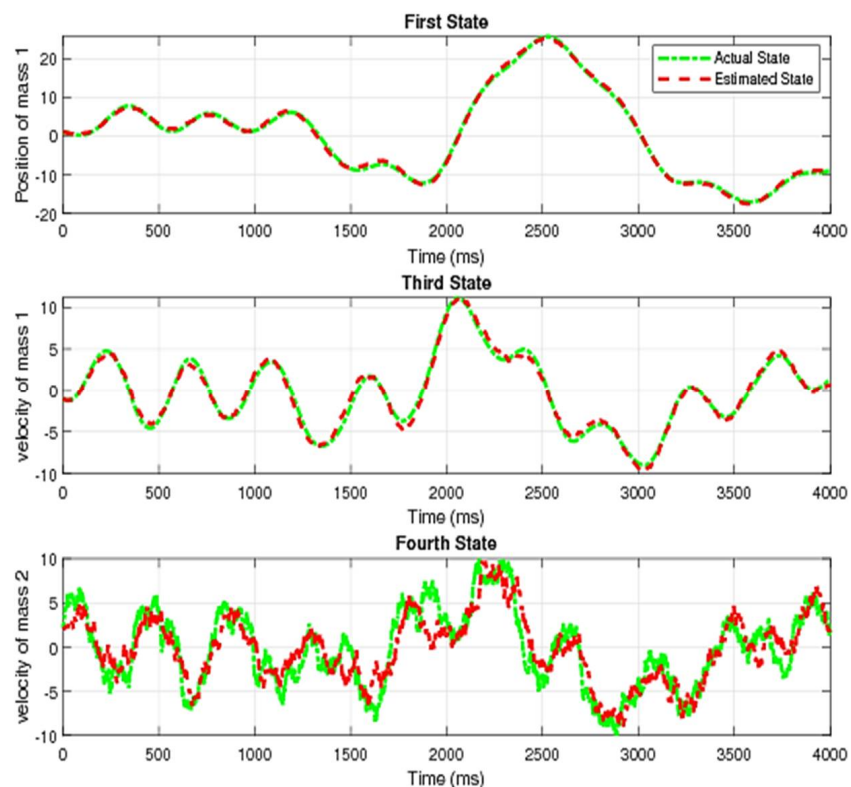
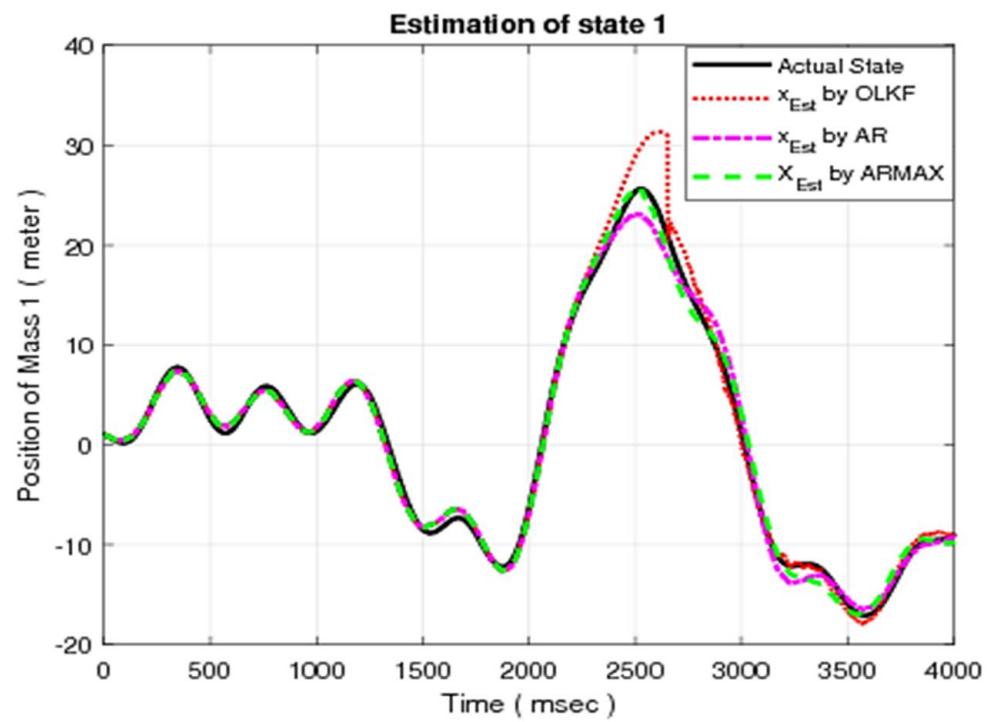
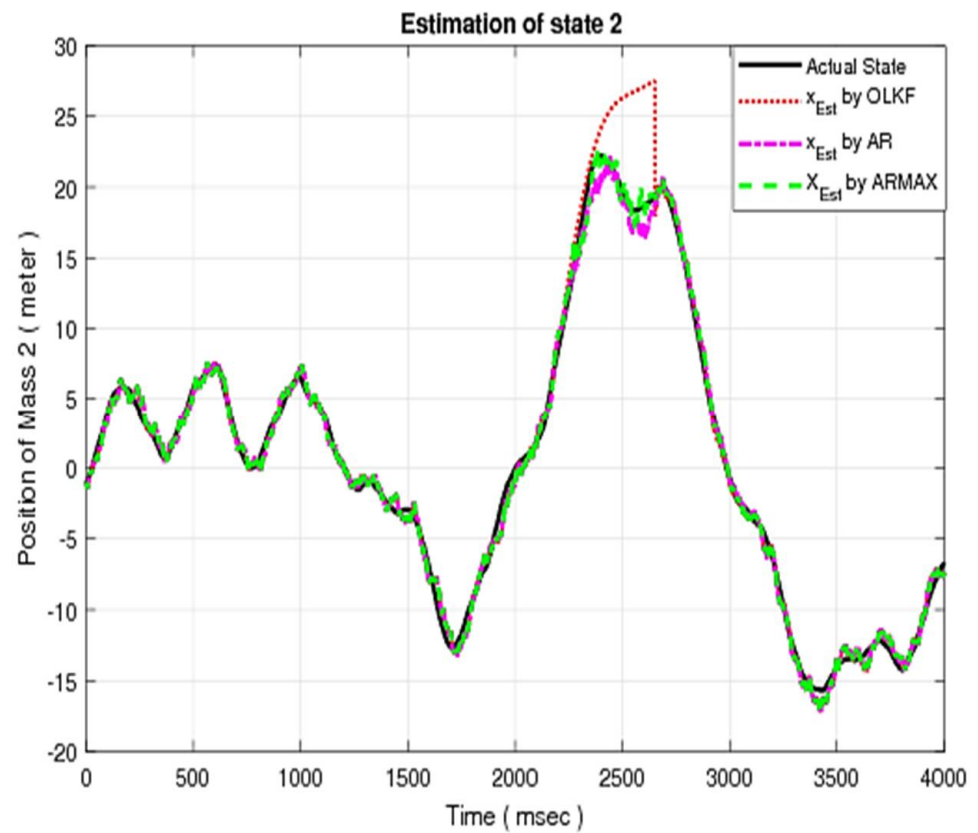


Figure 3. Simulation results with no data loss (other three states).

### 5.3.1. State Estimation

The simulation results for the first state of MSD using the OLE, AR model, and ARMAX model are given in Figure 4. Similarly, the simulation results for other states using OLE, compensated closed-loop (AR model and ARMAX) models are depicted in Figures 5–7, respectively.

Figure 4. Position of mass  $m_1$ .Figure 5. Position of mass  $m_2$ .

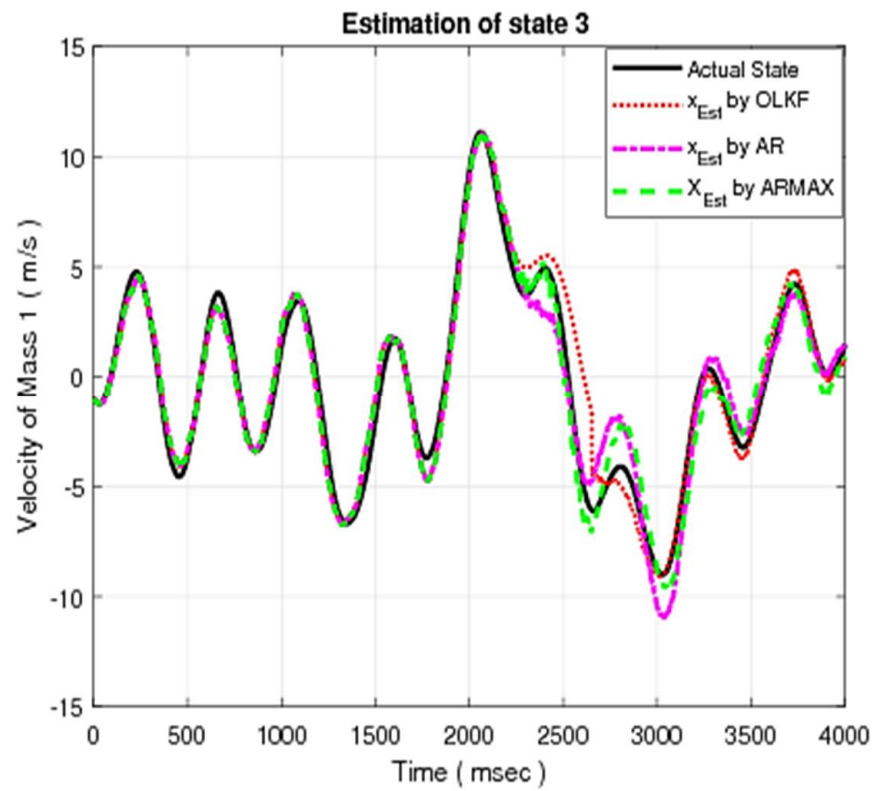


Figure 6. Velocity of mass  $m_1$ .

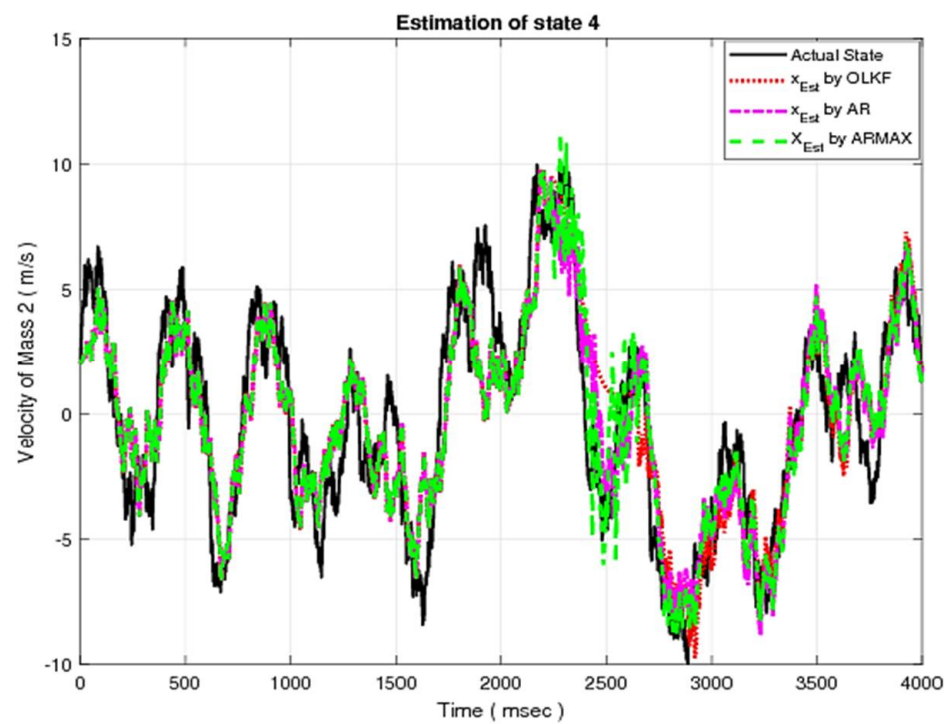


Figure 7. Velocity of mass  $m_2$ .

In each figure, the actual state is shown by the solid line. On the other hand, the estimated states using the OLE, AR model, and ARMAX model are represented by dotted, dashed-dotted, and dashed lines, respectively. It is obvious from the above results that the

ARMAX model offers improved compensation as compared with OLE and CLE using the AR model.

### 5.3.2. Error Analysis

The error resulting due to compensation for the position of mass  $m_1$  of MSD is depicted in Figure 8. Similarly, the resulting errors for the other three states are given in Figures 9–11, respectively.

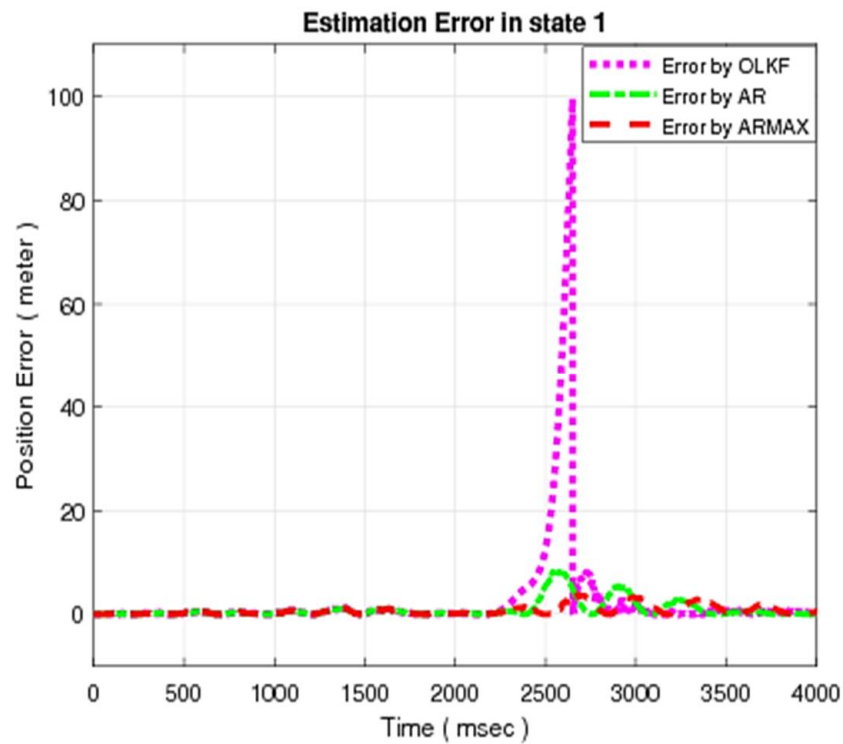


Figure 8. Position error,  $S_1$  (meter).

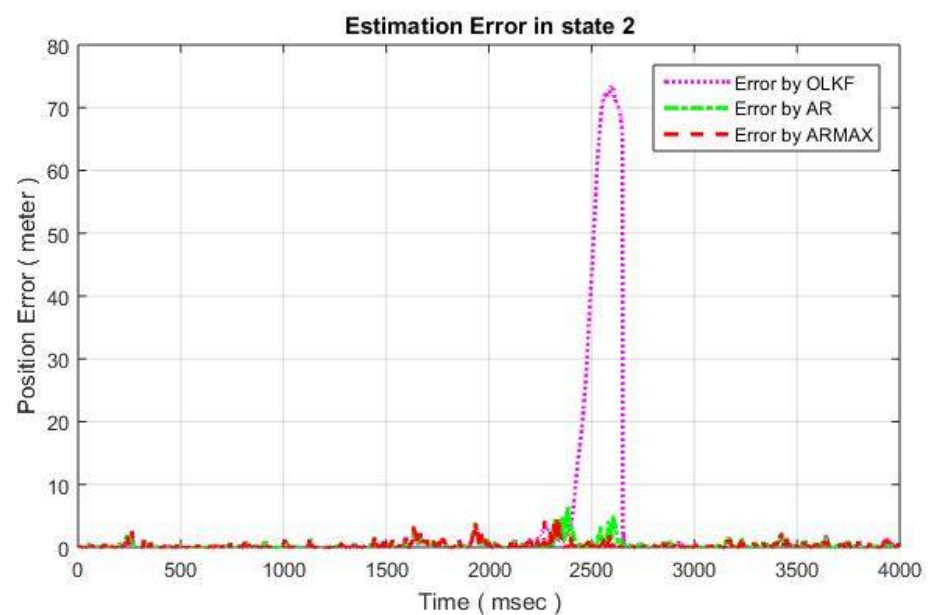


Figure 9. Position error,  $S_2$  (meter).

Each Figure consists of a dotted line that give the estimation error of the open-loop technique, dashed-dotted which depicts the estimation error resulting due to AR model

compensation, and dashed line which presents the error in case of proposed ARMAX model compensation.

### 5.3.3. Analysis of Kalman Filter Gain

The variations of Kalman filter gain in case of data loss for the existing techniques (OLE and AR model) and proposed ARMAX are shown in Figure 12.

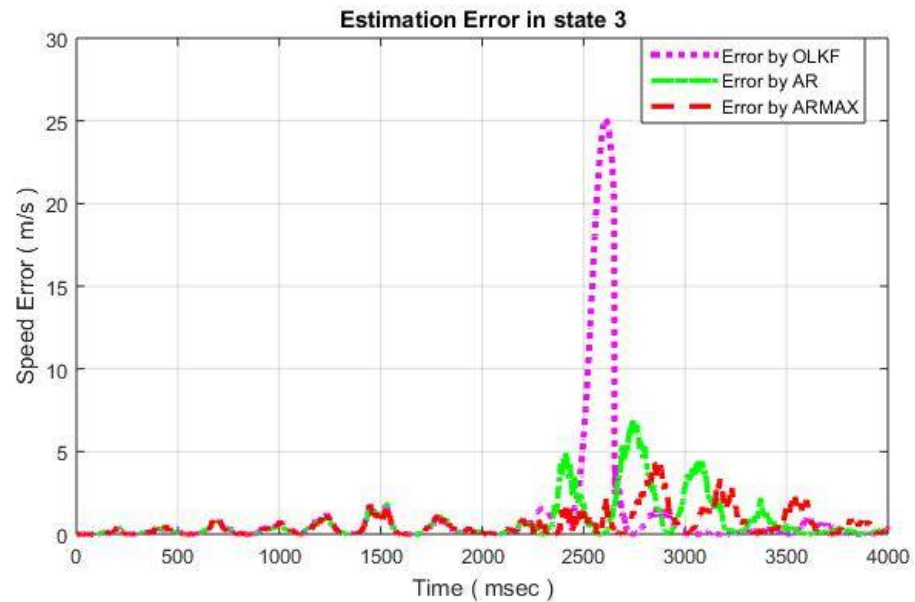


Figure 10. Velocity error,  $S_3$  (meter).

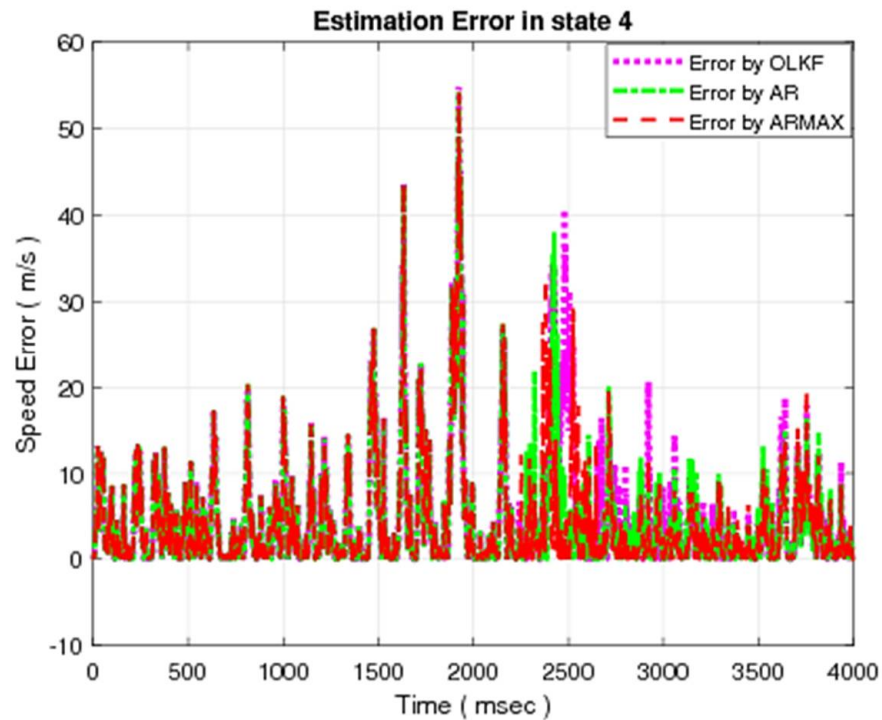


Figure 11. Velocity error,  $S_4$  (meter).

During the period of lossy compression, the value of Kalman filter gain is zero for open-loop estimation until the data loss is recovered. In the case of compensated closed-loop techniques using the Kalman filter gain, this is calculated with help of a compensated measurement vector using AR or ARMAX.

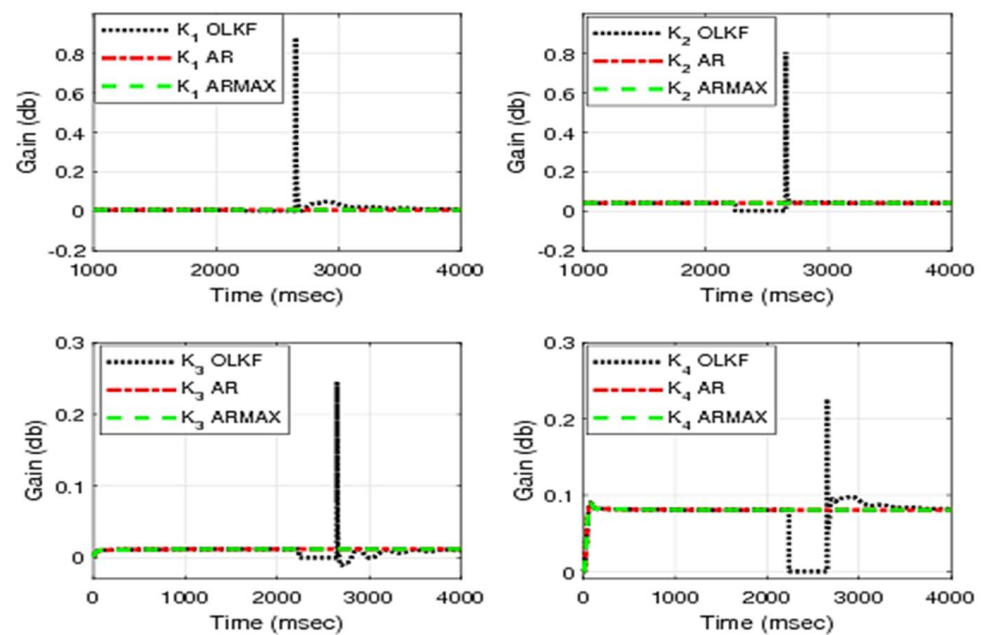


Figure 12. Kalman filter gain for all four states.

## 6. Conclusions and Future Work

This article presents the estimation technique and proposes an improved compensation when the estimation process is subjected to data loss. In the beginning, the problem of measurement loss is addressed using open-loop estimation with different shortcomings. These disadvantages are overcome by proposing different closed-loop techniques. The earlier closed-loop techniques were based on the autoregressive model with a minimal number of pieces of information. In the proposed model, the amount of information has been increased by taking the measurement, input, and system disturbance for the compensation of missing data samples.

In this work, existing and proposed schemes are based on a linear system. However, most of the time the system under consideration is nonlinear. Therefore, the problem of data loss for a non-linear system is the possible extension of this work.

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