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Archimedes Screw Design: An Analytical Model for Rapid Estimation of Archimedes Screw Geometry

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Abstract: In designing Archimedes screws, determination of the geometry is among the fundamental questions that may affect many aspects of the Archimedes screw powerplant. Most plants are run-of-river and highly depend on local flow duration curves that vary from river to river. An ability to rapidly produce realistic estimations for the initial design of a site-specific Archimedes screw plant helps to facilitate and accelerate the optimization of the powerplant design. An analytical method in the form of a single equation was developed to rapidly and easily estimate the Archimedes screw geometry for a specific site. This analytical equation was developed based on the accepted, proved or reported common designs characteristics of Archimedes screws. It was then evaluated by comparison of equation predictions to existing Archimedes screw hydropower plant installations. The evaluation results indicate a high correlation and reasonable relative difference. Use of the equation eliminates or simplifies several design steps and loops and accelerates the development of initial design estimations of Archimedes screw generators dramatically. Moreover, it helps to dramatically reduce one of the most significant burdens of small projects: the non-scalable initial investigation costs and enables rapid estimation of the feasibility of Archimedes screw powerplants at many potential sites.

Keywords: Archimedes screw design; analytical equation; quick estimation method; Archimedean screw; generator; turbine; fish safe/friendly; small/micro/pico hydropower plant; run of river powerplant; low head hydropower



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1. Introduction

The Archimedes screw (also known as an Archimedean or hydrodynamic screw) is one of the earliest hydraulic machines [1]. Using Archimedes screws as water pumps dates back many centuries. In the modern world, Archimedes screw pumps (ASP) are widely used in wastewater treatment plants and for dewatering low-lying regions. The Archimedes screw generator (ASG) is a safer hydropower technology for aquatic life, especially fish [2–9], that has been in use since the 1990s [10]. The Archimedes screw is a reversible hydraulic machine, and there are several examples of Archimedes screw installations where the screw can operate at different times as either pump or generator, depending on needs for power and watercourse flow [11].

When designing an ASG, estimation of the screw geometry is a fundamental necessity. The screw geometry affects many aspects of an Archimedes screw powerplant design. Each site has different specifications and limitations, including head, flow and available locations for the power plant installation. Most ASG installations are run-of-river, with small or no reservoirs, and plant performance is directly dependent on the local flow duration curve which varies from river to river. The lack of active water storage in run-of-river (ROR) powerplants makes the importance of the temporal distribution of volumetric flow rate an important design parameter. This means that ASG designs must be highly site-specific, requiring different designs to account for site-specific characteristics.

Recently there has been an emerging interest using computational fluid dynamics (CFD) in the modeling of Archimedes screw turbines to measure and visualize the fluid and forces within the screw. Simmons [12,13] and Shahverdi have completed CFD studies modelling several case studies to study ASG performance [14], optimize screw performance [15], or assess and predict the design parameters of the Archimedes screw generator [16]. However, these studies demonstrate that successful CFD modelling requires significant time, resources and experienced practitioners, software and hardware infrastructures. Proper CFD models must be carefully developed, validated and analyzed for each case.

Archimedes screw pumps (ASP) could be considered among the well-studied hydraulic machines, and some analytical methods are available [17], however in practice some aspects of ASP design remain based on experience. There is also a lack of analytical guidelines for ASG design. At the current time, insufficient general design guidelines could be considered as an important disadvantage for ASGs [18]. The literature shows that there are still no general analytical standards for designing ASGs [18], and the designs are still highly dependent on the designer's experience [19]. Important non-English language ASG design literature includes the works of Brada (1996) [20], Aigner (2008) [21], Schmalz (2010) [22], Lashofer et al. (2011) [23], and Nuernbergk's (2020) book [24]. The papers by Rorres [25] and Nuernbergk and Rorres [19] are among the well-known studies in English literature. However, the proposed methods in these works are not easy to understand and implement [18], particularly at the first stage of plant design.

Dragomirescu (2021) proposed a method to estimate the required screw outer diameter based on the volume of filled buckets [17]. However, there was no analytical equation to calculate this volume. To deal with this issue, Dragomirescu used regression to estimate correction factors based on a list of ASGs that were all designed by the same manufacture (Rehart Power) [25] and selected based on their high overall plant efficiencies (more than 60%) [17]. Using regression analysis for such limited case studies may affect the generality of the model and limit it to these case studies. However, in comparison to the former studies, this method resulted in a method to quickly estimate required screw size that was easier to understand and implement.

Currently, there is no generally accepted and easy to understand and implement method to rapidly determine preliminary size and operating characteristics of ASG designs. Obviously, each design requires deep studies, evaluation, modelling and optimization, which is costly and time-consuming. However, the first step of optimizing a design is to develop realistic estimates of the primary variables for the initial designs. Therefore, a model is needed for the purpose of rapidly estimating initial design parameters. This study focuses on developing an analytical method to estimate site-specific Archimedes screw geometry properties rapidly and easily.

2. Materials and Methods

2.1. Theoretical Basis

An Archimedes screw is made of a helical array of blades wrapped around a central cylinder [26] and supported within a fixed trough with small gap that allows the screw to rotate freely [18]. The most important dimensions and parameters required to define the Archimedes screws are represented in Figure 1 and described in Table 1.

The inlet depth of the Archimedes screw can be represented in a dimensionless form as:

$$\Xi = h_u (D_O \cos \beta)^{-1} \quad (1)$$

The available head (H) and volumetric flow rate (Q) and are two important parameters in hydropower plants. In Archimedes screws, the flow always has a free surface (exposed to atmospheric pressure). In addition, the cross-sectional areas at the inlet and outlet of a screw are equal. Applying continuity and the Bernoulli equation, it can be shown that ideally, the available head at an ASG is the difference of free surface elevations at the

upstream (Z_U) and downstream (Z_L) of the AST, where Z_U and Z_L are both measured from the same datum:

$$H = Z_U - Z_L \quad (2)$$

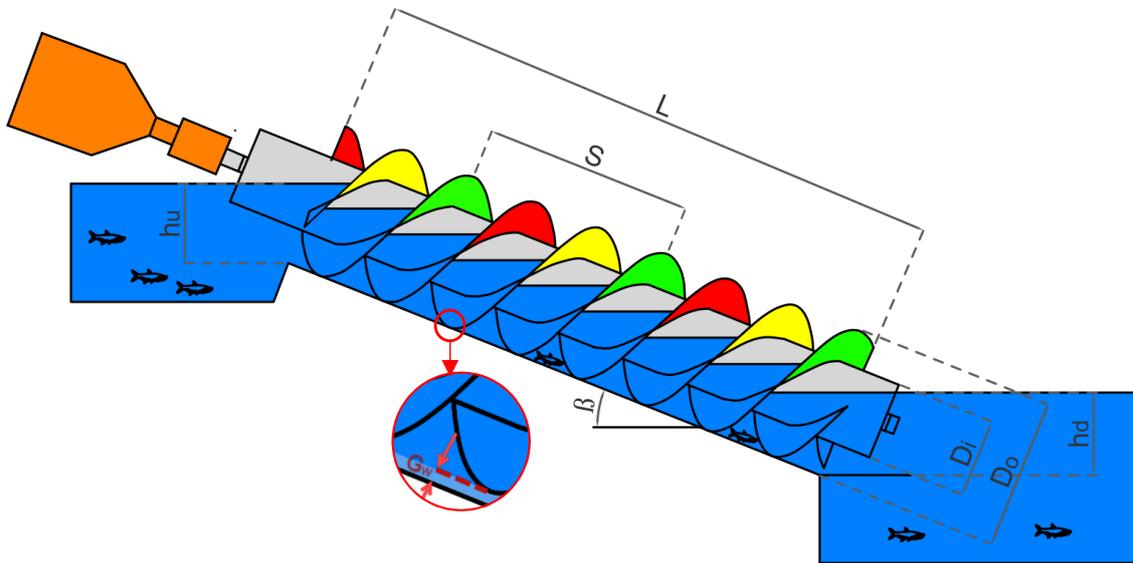


Figure 1. Required parameters to define the geometry of Archimedes screws [18,28].

Table 1. Required parameters to define Archimedes screws' geometry and operating variables.

Parameter	Description	Variable	Description
L (m)	Total length of the screw	ω (rad/s)	Rotation speed of screw *
D_o (m)	Outer diameter	h_u (m)	Upper (inlet) water level
D_i (m)	Inner diameter	h_L (m)	Lower (outlet) water level
S (m)	Screw's pitch or period [27] (The distance along the screw axis for one complete helical plane turn)	Q (m ³ /s)	Volumetric flow rate passing through the screw
N (1)	Number of helical planed surfaces (also called blades, flights or starts [27])		
β (rad)	Inclination Angle of the Screw		
G_w (m)	The gap between the trough and screw.		

* Note: In the fixed speed Archimedes screws rotation speed is a constant.

The inclination angle of the Archimedes screw (β) is sometimes restricted based on slope or geometry. Considering Figure 1, for a known head the screw length (L) is:

$$L = H / \sin \beta \quad (3)$$

For development of the current predictive model, application of the continuity equation suggests that the flow rate passing through the screw (Q) is dependent on the flow depth at the entrance (h_u) overall (outer) diameter (D_o) and the rotation speed (ω) of the screw. This assumption is applied and evaluated formerly in studies such as Nuernbergk and Rorres [19] and YoosefDoost and Lubitz [28].

In Archimedes screws, a water bucket is a volume of entrapped water between two adjacent helical plane surfaces. For an ideal screw operating under steady-state conditions (steady flow, constant rotational speed), all buckets will have the same shape

and volumetric size [29]. Moreover, it could be assumed that the flow has a speed equal to the screw axial translation speed (V_T) which is equal to:

$$V_T = \frac{S\omega}{2\pi} \quad (4)$$

In 1932 Muysken proposed the required equations and design parameters for Archimedes screws used as pumps [30]. Muysken proposed a maximum recommended rotation speed (ω_M) for Archimedes screws [30], and Lashofer et al. [10] confirmed that many current industrial ASGs are designed with this rotation speed which is close to:

$$\omega_M = \frac{5\pi}{3D_o^{2/3}} \quad (5)$$

YoosefDoost and Lubitz proposed a nondimensional equation to estimate the total flow rate passing through an Archimedes screw for rotation speeds equal or different than Muysken's maximum rotation speed, different inlet water levels and screw sizes as [28]:

$$\frac{Q}{Q_{Max}} = a \left(\frac{A_E}{A_{Max}} \right)^b \left(\frac{\omega}{\omega_M} \right)^c \quad (6)$$

where $A_{Max} = \pi(D_o - D_i)^2/4$, $Q_{Max} = A_{Max}\omega_M$ and a , b and c are constants related to the screw's properties. The effective area (A_E) is calculated using the following equations, with the definitions of additional variables defined in Figure 2.

$$A_E = \frac{1}{8} \left[D_o^2(2\theta_o - \sin 2\theta_o) - D_i^2(2\theta_i - \sin 2\theta_i) \right] \quad (7)$$

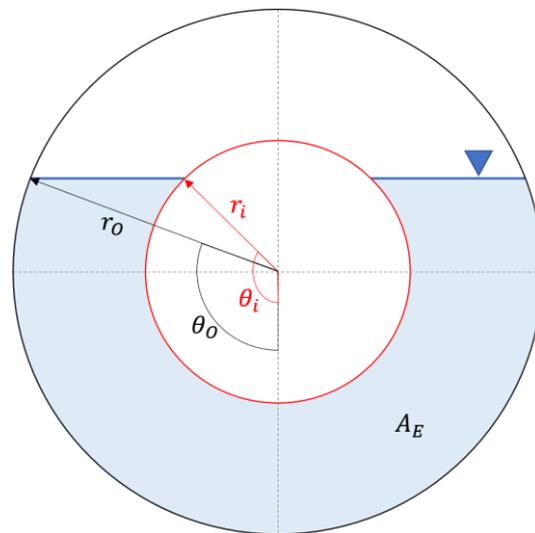


Figure 2. Required parameters to define the effective area.

Although the optimum value may be different for each screw, studies on a range of lab-scale (small) and full-scale Archimedes screw sizes showed that values of $a = 1.242$, $b = 1.311$, and $c = 0.822$ resulted in reasonable predicted flow rates [28].

For optimal design of industrial full-scale Archimedes screws running at a fixed speed near Muysken's maximum recommended rotation speed (ω_M) [30], Nuernbergk and Rorres proposed an analytical method based on the screw's water-inflow conditions [19]. This method can be simplified to an equation which is a function of the inlet depth, rotation speed and geometry of the screw using the method proposed by YoosefDoost and Lubitz

(2021) based on the concept of the effective cross-sectional area within the screw (A_E), and using the axial transport velocity of Archimedes screws (V_T) [28]:

$$Q = A_E V_T \quad (8)$$

$$\frac{Q}{Q_O} = \frac{A_E}{A_O} \quad (9)$$

where $A_O = \pi D_O^2/4$ and $Q_O = A_O V_T$. Using the concept of effective area (A_E) with Equations (4) and (8) and defining $\delta = D_i/D_O$ and $\sigma = S/D_O$ results in the following analytical equation for the volume of flow passing through the Archimedes screw:

$$Q = D_O^3 \frac{\sigma \omega}{16\pi} \left(2\theta_O - \sin 2\theta_O - \delta^2 (2\theta_i - \sin 2\theta_i) \right) \quad (10)$$

The overall (outer) diameter of the screw (D_O) can then be determined based on the flow rate passing through the screw. Solving for the outer diameters results in the following equation that enables estimation of the required Archimedes screw outer diameter to accommodate the specified flow rate passing through the screw:

$$D_O = \sqrt[3]{\frac{16\pi}{\sigma \omega \left(2\theta_O - \sin 2\theta_O - \delta^2 (2\theta_i - \sin 2\theta_i) \right)} Q} \quad (11)$$

These equations could be simplified for industrial full-scale Archimedes screws running at a fixed speed near to the Muysken's maximum recommended rotation speed (ω_M) as:

$$Q = \frac{5}{48} \sigma D_O^{7/3} \left(2\theta_O - \sin 2\theta_O - \delta^2 (2\theta_i - \sin 2\theta_i) \right) \quad (12)$$

$$D_O = \left(\frac{48}{5\sigma \left(2\theta_O - \sin 2\theta_O - \delta^2 (2\theta_i - \sin 2\theta_i) \right)} Q \right)^{3/7} \quad (13)$$

It is notable that for a specific screw at a specific fill level, the variables other than Q are constant, and the form of Equation (13) could be represented as a power function with two constants of η and ψ :

$$D_O = \eta Q^\psi \quad (14)$$

The inner diameter of the screw (D_i) has an important effect in A_E and the flow rate passing through the screw. In smaller screws, it is possible to deal with technical constraints such as permissible deflection by increasing the thickness of the shaft tube wall. However, to maximize the shaft length, it may be necessary to increase the outer diameter. Nagel in 1968 indicated that reasonable filling of ASPs would be achieved for $\delta = D_O/D_i$ between 0.4 and 0.6 [31]. Theoretical studies and experimental investigations on models and full-scale ASPs indicate that maximizing the water volume in the screw occurs with δ between 0.45 and 0.55. This ratio is reported as the economically optimum ratio as well, due to optimum the usage of material [31]. Lashofer et al. [10] confirmed that for most ASG powerplants, δ is usually very close to 0.5.

Nagel in 1968 indicated the ratio of $\sigma = S/D_O$ is related directly to the number of blades (N) and reversely to the inclination angle (β) of the ASPs (higher β or lower N results in lower δ , and vice versa). From the hydraulic point of view, Nagel recommends [10]:

$$\sigma = \begin{cases} 1.2, & \beta < 30^\circ \\ 1, & \beta = 30^\circ \\ 0.8, & \beta > 30^\circ \end{cases} \quad (15)$$

Figure 3 compares the results of Equation (14) for the proposed σ values across the full range of dimensionless fill heights (Ξ) of screws with $\delta = 0.5$. An analysis of this

result indicates that $\eta_{\sigma=0.8}/\eta_{\sigma=1} \approx 1.1$ and $\eta_{\sigma=1.2}/\eta_{\sigma=1} \approx 0.925$. Due to manufacturing considerations, Nagel proposed to consider $\sigma = 1$ as a fixed ratio (constant) and the inclination angle as a parameter to optimize [10]. The $\sigma = 1$ is proven as a correct ratio for ASPs with three blades and inclination angles up to 35° [31]. Lashofer et al. confirmed that two-thirds of AST installations follow this ratio and the rest utilized larger variations, most likely as a result of the installation conditions [10].

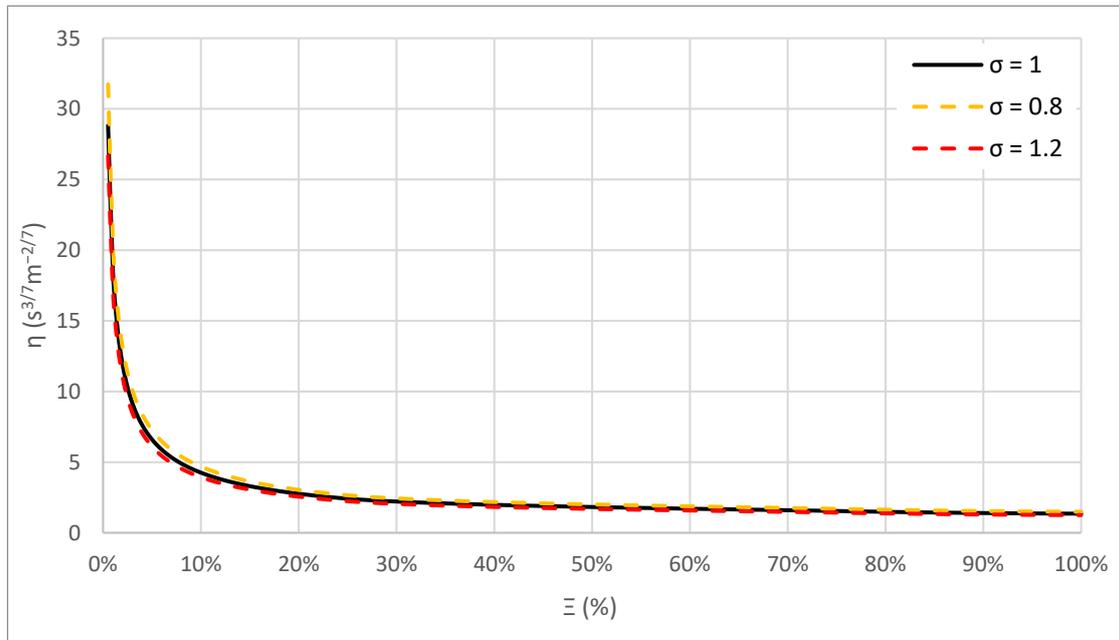


Figure 3. Comparison of Equation (14) results for $\delta = 0.5$ and different σ values.

As a general analytical method to estimate the Archimedes screw outer diameter based on the volumetric flow rate for all AST inlet depths, Equation (14) could be applied for $\delta = 0.5$, $\sigma = 1$, $\theta = 3/7$ and the corresponding η value of each dimensionless inlet depth.

The observations above can be used to determine an overall relationship between volume flow rate and outer diameter for a screw. The general form of Equation (15) is:

$$D_O = \eta Q^{3/7} \quad (16)$$

where η is a constant accounting for screw geometry and fill level. Assuming the commonly used values of $\delta = 0.5$ and $\sigma = 1$, a value of η can be determined for each dimensionless inlet depth, which is turn is related to volume flow rate.

This resulting analytical equation eliminates the requirement of many design steps and loops and simplifies and accelerates the design of Archimedes screw dramatically. For example, for the known flow rate of Q , Equation (16) could be used for any Archimedes screw with $\delta = 0.5$, and Figure 3 could be used to determine the corresponding η value of each dimensionless inlet depths. It is noteworthy that for $\delta = 0.5$ and $\sigma = 1$, Equations (12) and (13) could be simplified as:

$$Q = \frac{5}{192} (8\theta_O - 4 \sin 2\theta_O - 2\theta_i + \sin 2\theta_i) D_O^{7/3} \quad (17)$$

$$D_O = \left(\frac{192}{5 (8\theta_O - 4 \sin 2\theta_O - 2\theta_i + \sin 2\theta_i)} \right)^{3/7} Q^{3/7} \quad (18)$$

Moreover, it is worth mentioning that as a check, solving the same problem for different ASG inlet fill heights leads to results that are in complete agreement with Equation (16).

For example, analytical solving of the problem or using Equation (18) for $\Xi = 50\%$ and $\Xi = 75\%$ gives the following values, respectively:

$$D_{O_{50\%}} = \left(\frac{64}{5\pi}Q\right)^{3/7} \approx 1.8258 Q^{3/7} \quad (19)$$

$$D_{O_{75\%}} = \left(\frac{288}{5(3\sqrt{3} + 5\pi)}Q\right)^{3/7} \approx 1.5440 Q^{3/7} \quad (20)$$

2.2. Evaluation Criteria

In this study the estimations (E_i) are compared with the industrial ASGs represented in Table 2 as observations (O_i), by visualizations and statistical tests. Correlation is compared by Pearson correlation (R), and the relative difference is compared by the percentage error (PE) and mean absolute percentage error (MAPE). In the following equations, n is the number of data points and \bar{E} and \bar{O} are the average of the estimations and observations respectively [28].

In statistics, correlation refers to any statistically significant relationship between two variables. The Pearson correlation coefficient measures the linear relationship between two random variables [32] and describes it in a range between -1 to $+1$. Values close to $+1$ indicate a good and direct correlation, while values closer to -1 refer to a good but inverse relation between datasets. Values near zero indicate a lack of correlation [33]. These ranges could be represented in percent by multiplying the Pearson correlation by 100. The Pearson correlation is defined as [34]:

$$R = \frac{\sum_{i=1}^n (E_i - \bar{E})(O_i - \bar{O})}{\sqrt{\sum_{i=1}^n (E_i - \bar{E})^2} \sqrt{\sum_{i=1}^n (O_i - \bar{O})^2}} \quad (21)$$

The percentage (percent) error (PE) is a dimensionless error measure defined as the difference between the model estimations and the experimentally measured value:

$$PE = \frac{E_i - O_i}{O_i} \times 100 \quad (22)$$

The mean absolute percentage error (MAPE) is the average of absolute percentage errors and one of the most common accuracy measures [35] that is recommended in many textbooks (e.g. [36,37]). MAPE considers errors regardless of their sign, so positive and negative errors cannot cancel each other. MAPE is calculated as:

$$MAPE = \frac{100}{n} \sum_{i=1}^n \left| \frac{E_i - O_i}{O_i} \right| \quad (23)$$

Table 2 provides a detailed list of the Archimedes screw hydro powerplant installations that are used for evaluation purposes in this paper. Table 2 is a compilation of published data from multiple sources. References for each plant are listed in the table.

Table 2. Details of Archimedes screw hydropower plants used for model evaluation.

ID	Name	D _O (m)	H (m)	Q (m ³ /s)	P (kW)	Note	Ref.
1	Haddo	1.4	5	0.5	15.9	*	[17,25]
2	Indore	1.4	5.3	0.6	19	*	[17,25]
3	Mühlen	1.5	3	1	21	*	[17,25]
4	Bischofsmais	1.6	3.16	1	21	*	[17,25]
5	Gennkikungou	1.6	1.05	0.99	7.3	*	[17,25]
6	Herrenhof	1.6	2.1	0.9	13.9	*	[17,25]
7	Schnaittach	1.6	1.35	0.8	7.5	*	[17,25]
8	Vierhöfen	1.6	1	1.2	8	*	[17,25]
9	St. Michael	1.7	3.2	1.2	26.92	*	[17,25]
10	Vadodara	1.7	5	1	33	*	[17,25]
11	Eitting	1.8	3.57	1.2	29	*	[17,25]
12	Erding	1.8	1.75	1.2	13.9	*	[17,25]
13	Flatford Mill	1.9	1.1	1.6	12.6	*	[17,25]
14	Niedermühle	1.9	3.17	1.5	33	*	[17,25]
15	Gescher	2	3.45	1.8	46	*	[17,25]
16	Yvoir	2.1	1.8	2	26	*	[17,25]
17	Colditz	2.2	3	1.5	33	*	[17,25]
18	Ahornweg	2.3	1.45	2	21		[38]
19	Solvay	2.3	2	2.5	35		[39]
20	Stimpfach	2.3	2.55	2.3	44	*	[17,25]
21	Linton Falls	2.4	2.7	2.6	50		[40]
22	Untermünkheim	2.4	1.8	2.5	31	*	[17,25]
23	Turbury Mill	2.5	2.1	2.8	43	*	[17,25]
24	Dautphetal	2.6	2.55	2.5	45.8	*	[17,25]
25	Hannoversch-Münden	2.8	2.6	2	35.455		[41]
26	Wiener Neustadt	2.8	4.05	3.5	98	*	[17,25]
27	Pilsing	2.9	3.6	3.2	8	*	[17,25]
28	Linton Plant	3	3.2	4.5	110	☆	[18,42]
29	Low Wood	3	7.2	4	200		[40]
30	Marengo	3	1.6	3.7	51		[43,44]
31	Baiersdorff	3.2	1.5	4.5	48.1	*	[17,25]
32	Crescenzago	3.2	2.1	5	75		[38]
33	Hausen	3.4	5.8	6	250	*	[17,25]
34	Hausen III Neumatt	3.4	5.8	5.5	235		[38]
35	Kirchberg	3.4	2.97	6	130	*	[17,25]
36	Shanes Castle	3.4	5	5.5	192	*	[17,25]
37	Radyr	3.5	3.5	11	200		[45]
38	Maple Durham	3.6	1.73	8	99	*	[17,25]
39	Wien	3.6	1.7	7.1	84	*	[17,25]
40	Totnes	3.7	3.45	6.5	160		[46]
41	Künzelsau	4.1	1.72	8.95	132		[38]
42	Plana	4.1	3.5	8.73	220		[38]
43	Gunthorpe Weir	4.3	2.03	14.15	165		[47]
44	Ham	4.3	10	5	360		[38,48]
45	Höllthal	4.3	2.22	10.5	220		[38,49]
46	Olen	4.3	10	5	360		[38,48]
47	Hasselt	5	10	5	400		[11,50]
48	Widdington Plant	5	3	14.5	335	☆	[18,42]

Notes: * Used in Dragomirescu [17]; ☆. Two different Archimedes screws installed in Linton Lock hydropower plant.

3. Results and Analysis

To find the most representative dimensionless fill height for the current Archimedes screw installations (Table 2), Equation (16) was used to compute all η values for the full range of dimensionless fill heights (Ξ). Then a numerical experiment was performed to estimate the overall diameter of the Archimedes screw hydro powerplant installations that are represented in Table 2 by using Equation (16). Technically, the highest agreement of the studied cases with the equation results where there is the minimum difference between the estimations and the actual observations. As shown in Figure 4, the minimum relative difference of MAPE = 2.31% occurs when $\Xi = 69\%$, with a resulting value of η of 1.61. Therefore, this η value was used for the rest of the investigation.

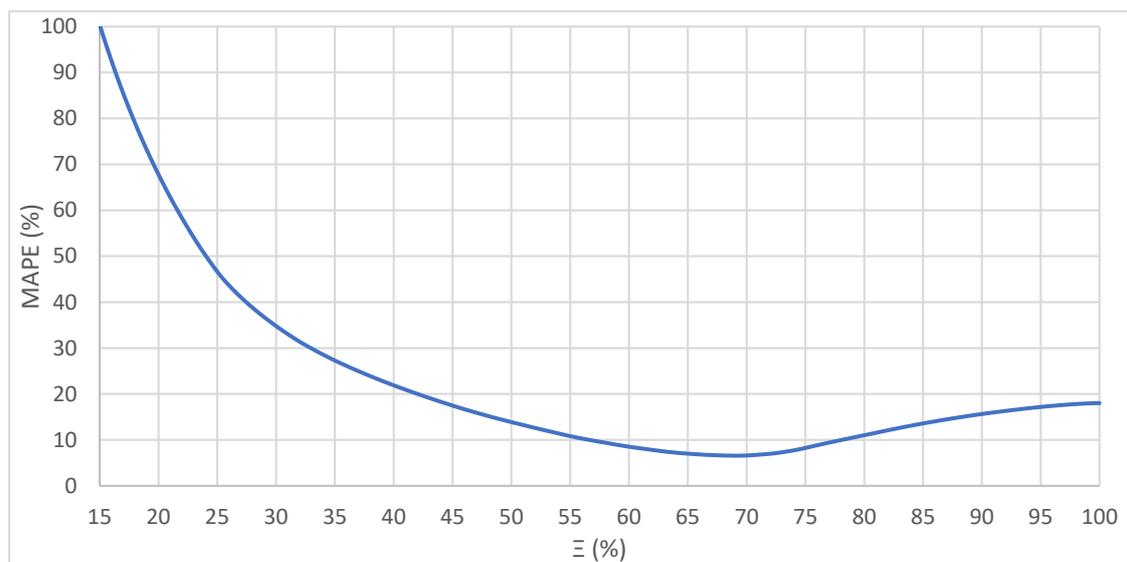


Figure 4. Equation (16) results for the whole range of the dimensionless inlet depth of the screw (Ξ) values in comparison with the currently installed AST designs of Table 2.

Figure 5 shows the outer diameter for each plant in Table 2 compared to the corresponding outer diameter predicted using Equation (16) with $\eta = 1.61$. The evaluation of the proposed analytical equation indicates a reasonable accuracy for the developed equation by a correlation high as high as $R = 91.80\%$ and a relative difference as low as $MAPE = 6.58\%$ on average. In these results, the point with highest relative difference is the Hannoversch Münden multi-ASG hydropower plant which has a unique design allowing 0° to 28° adjustable inclination angle for operation with variable tailwater levels: it is possible that the data published for this plant is not as representative of operation in conventional conditions.

Figure 5 also compares the accuracy of predictions using the Dragomirescu method [17] with Equation (16) results. According to this figure, both methods indicate a good agreement for these Archimedes screw designs. For Dragomirescu's specific cases and method, analysis the evaluation criteria indicate a high correlation ($R = 96.96\%$) and low relative difference ($MAPE = 5.72\%$) for this method. For the same cases, Equation (16) produced a higher correlation ($R = 98.63\%$) and lower relative difference ($MAPE = 4.54\%$) than the Dragomirescu method results.

In summary, the results of both methods are arguably good given the assumptions and small number of variables used in both methods. However, considering the analytical nature of Equation (16), the slightly better performance in term of some evaluation criteria, as well as its significant ease of use may make Equation (16) more practical. Equation (16) is developed based on a strong theoretical basis that makes it a general equation. Moreover, evaluation of Equation (16) with a wide range of different cases indicates a reasonable accuracy. Finally, using the Dragomirescu method requires predefining the number of

blades (N) and length (L) or the inclination angle (β) of screw even for initial estimations while Equation (16) may be used without requiring these variables as outlined below.

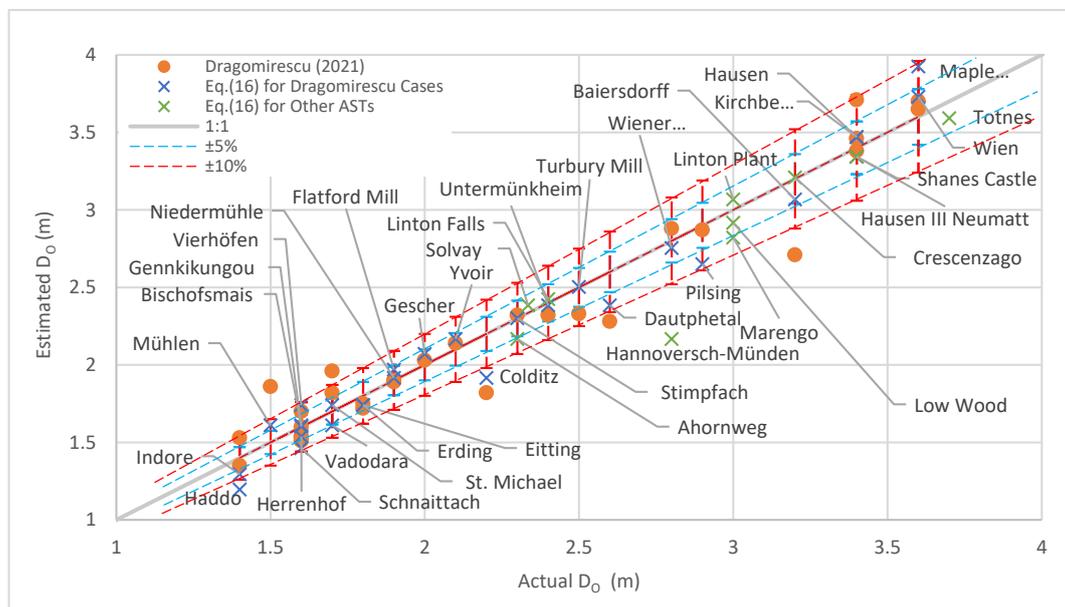


Figure 5. Comparison of Equation (16) results with Dragomirescu [17] and the other Archimedes screw installations (Table 2).

4. Analytical Method for Designing Archimedes Screws

This section proposes a rapid and simple method to estimate the design properties of Archimedes screw generators based on the analytical equations that are proposed in this study. The step-by-step design process of the Archimedes screw is:

- (1) Determine the site properties: available volumetric flow rate (Q), head (H) and the inclination angle of the Archimedes screw (β). Lashofer et al. [10] confirmed that many current industrial ASTs are installed at $\beta = 22^\circ$ [10].
- (2) Use Equation (3) to determine the Archimedes screw's length.
- (3) Use Equation (11) to determinate the overall (outer) diameter D_O of the Archimedes screw based on the desired ω , δ , σ and Ξ values. Or, Use Equation (16) to design Archimedes screw similar to the current installed ASGs in hydropower plants ($\delta = 0.5$, $\sigma = 1$, $\Xi = 69\%$, $\eta \approx 1.61$ and $\psi = 3/7$). For example, for $Q = 9 \text{ m}^3/\text{s}$ using Equation (16) results $D_O = 1.61 \times 9^{3/7} \approx 4.128 \text{ m}$. Comparison of the calculated D_O with Table 2. indicates that this is a very close outer diameter to the Künzelsau hydropower plant Archimedes screw ($D_O = 4.1 \text{ m}$) with is designed for almost the same flow rate. Or, for $Q = 1 \text{ m}^3/\text{s}$, Equation (18) gives $D_O = 1.61 \text{ m}$ which is almost the same as the average of Bischofsmais, Mühlen and Vadodara ASTs' outer diameters (1.6 m, 1.5 m and 1.7 m respectively).
- (4) Determinate the inner diameter (D_i) and screw pitch (S) based on the estimated (D_O using the following equations:

$$D_i = \delta D_O \quad (24)$$

$$S = \sigma D_O \quad (25)$$

This method is summarized in the represented flow chart in Figure 6.

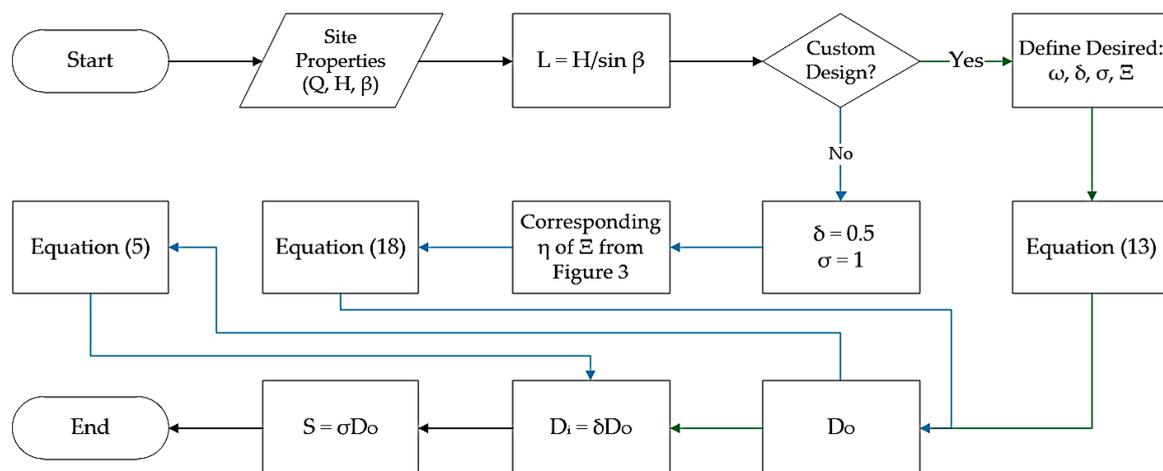


Figure 6. A fast and easy analytical method for designing Archimedes screws.

5. Conclusions

This study developed an analytical method to produce a rapid initial estimate of the geometry of an Archimedes screw power plant. Analytical equations were developed based on mathematical, physical and hydraulic facts from the literature. Finally, a general analytical equation is proposed to estimate the overall diameter of the ASG, and a method is proposed to estimate the other design properties of the geometry of the Archimedes screws.

The developed analytical equation has been evaluated by 48 industrial Archimedes screws designed by different manufacturers and currently installed and operating in hydropower plants. For this process, the values of parameters were chosen based on the accepted, proved, reported or observed common aspects of Archimedes screws ($\delta = 0.5$, $\sigma = 1$, $\Xi = 69\%$ and $\psi = 3/7$). This process also led to simplifying the general analytical equation (Equation (12)) into a much simpler form, resulting in a considerably easier and faster to use equation (Equation (16)) that could be used to design Archimedes screw turbines in a similar basis of current Archimedes screw installations in hydro powerplants.

The evaluation of the proposed analytical equation indicates a reasonable accuracy for the developed equation with a correlation as high as 91.80% and a MAPE as low as 6.61%.

Moreover, results using this equation were compared with Dragomirescu's method results. Both methods are arguably good given the assumptions and small number of variables used in both methods. However, considering the slightly better performance in terms of the evaluation criteria besides the single-equation nature of Equation (16) as well as its significant ease of use, it seems that Equation (16) is more practical. The strong theoretical basis of Equation (16) results in a general and potentially more reliable method. Moreover, evaluation of Equation (16) with a wide range of different cases indicates a reasonable accuracy. Equation (16) does not require to predefining of the number of blades (N) and length (L) or the inclination angle (β) of screw for initial estimations, which makes it easier to use for initial estimates.

The proposed analytical equation not only showed a reasonable accuracy based on the evaluations but also simplifies and could eliminate several design steps and loops and accelerate the design of Archimedes screws.

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Nomenclature

The following symbols are used in this paper:

A_E	Effective cross-sectional area at the screw's inlet	(m ²)
A_{Max}	Maximum cross-sectional water area at the screw's inlet	(m ²)
A_O	The outer diameter's cross-sectional area	(m ²)
a	Coefficient of dimensionless flow rate	(-)
b	Coefficient of dimensionless area constant	(-)
c	Coefficient of dimensionless rotation speed constant	(-)
D_i	The inner diameter of the Archimedes screw	(m)
D_O	The outer diameter of the Archimedes screw	(m)
E_i	The estimated value	
\bar{E}	The average of the estimations	
f	Fill height of water in a bucket of screw	(-)
h_u	Upper (inlet) water level of the screw	(m)
h_L	Lower (outlet) water level of the screw	(m)
H	The available head	(m)
G_w	Gap width (The gap between the trough and screw)	(m)
L	T total length of the screw	(m)
MAPE	The mean absolute percentage error	(%)
n	The number of data points in the dataset	
N	Number of helical planed surfaces	(-)
O_i	The observed value	
\bar{O}	The average of the observed data	
PE	The percentage (percent) error	(%)
Q	Total flow rate passing through the screw	(m ³ /s)
Q_{Max}	The maximum flow rate that could pass through a screw when $\omega = \omega_M$ and $A_E = A_{Max}$	(m ³ /s)
Q_O	The volumetric flow rate that passes through the cross-sectional area of A_O at the speed of V_T .	(m ³ /s)
r	Radios	(m)
R	Pearson correlation	(%)
S	Pitch of the screw (Distance along the screw axis for one complete helical plane turn)	(m)
V_T	Axial transport velocity	(m/s)
y	The cross section fill height	(rad)
Z_U	The free surface elevations at the upstream	(m)
Z_L	The free surface elevations at the downstream	(m)
β	Thinclination angle of the screw	(rad)
δ	The screw's pitch to outer diameter ratio (S/D_O)	(-)
η	The constant accounting for screw geometry, rotation speed and fill level in the power function form of the diameter equation	(s ^{3/7} m ^{-2/7})
θ	Angle of sector	(rad)
σ	The screw's inner to outer diameter ratio (D_i/D_O)	(-)
Ξ	The dimensionless inlet depth of the screw	(-)
ω	The rotation speed of the screw	(rad/s)
ω_M	The maximum rotation speed of the screw (Muysken limit)	(rad/s)
Subscripts		
i	inner	
min	minimum	
Max	Maximum	
O	Outer	

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