

# *Article* **Transient Simulation of the Six-Inlet, Two-Stage Radial Turbine under Pulse-Flow Conditions**

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**Abstract:** In recent years, the automotive sector has been focused on emission reductions using hybrid and electric vehicles. This was mainly caused by political trends promoting "green energy". However, that does not mean that internal combustion engines (ICEs) should be forgotten. The ICE has still the potential of recovering energy from exhaust gases. One of the promising ways to recover energy is turbocharging. Over the years engine manufacturers have designed very efficient turbocharger systems which have greatly increased the overall engine efficiency. This led to pollutant emission reductions. This paper presents the results of the three-dimensional (3-D) numerical simulations of the two-stage, six-inlet turbocharging system under the influence of unsteady, pulsed-flow conditions. The calculations were carried out for three turbine speeds. The most interesting results of this study were the separation of exhaust gases coming from the six-exhaust pipes and the performance of both stages under pulse-flow conditions. The two-stage turbocharging system was compared against the single-stage turbocharging system and the results showed that the newly designed two-stage turbine system properly separated the exhaust gases of the adjacent exhaust pipes.

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**Keywords:** two-stage turbocharger; internal combustion engine; gas separation; computational fluid dynamic; rotor

### **1. Introduction**

The increasing trend of pollutant emissions reduction has forced engine manufacturers to focus their interest on hybrid and electric vehicles. The advantages of such vehicles are their relatively low fuel consumption and the possibility to recover energy. However, that does not mean that internal combustion engines (ICEs) should be eliminated from commercial use. One of the most important advantages of the ICE over electrically driven vehicles is the range. The limited battery charge of electric vehicles greatly constrains their maximal driving range. There are a lot of places in the ICE that can be improved to recover more energy from the exhaust gases. The hot exhaust gases contain a large portion of the energy that could be recovered with exhaust after-treatment components such as turbochargers which are driven by the hot exhaust gases from the cylinder. The first turbochargers had a single-stage rotor. However, the need for further energy recovery led to the implementation of multi-stage turbochargers. This has led to a greater energy recovery but on the other hand, also made the exhaust systems more complicated. Nowadays, two-stage turbocharging systems are used in ICEs. In such a system, the recovered energy is used to boost the air pressure at the intake of the engine. An analytical approach based on the power balance between the turbine and compressor with minimum energy consumption offers a reliable way to optimize the two-stage turbocharging system [\[1\]](#page-23-0). Key parameters such as the total expansion ratio and the total pressure ratio are often used with an analytical predesigned model [\[2,](#page-23-1)[3\]](#page-23-2). The analytical approach of a two-stage turbocharged engine with three bypass valves was also studied at different altitudes [\[4\]](#page-24-0). The results showed that the cylinder combustion pressure decreased with the increase of the altitude. The two-stage turbocharged system was also analyzed during the steady-state and transient

engine response regimes [\[5\]](#page-24-1). The results showed an improved durability of the two-stage turbocharging system with the reduction of the break-specific fuel consumption (BSFC) at low engine speeds. The analytical design of the Miller-cycle, two-stage turbocharging system leads to the reduction of the in-cylinder combustion temperature [\[6\]](#page-24-2). The 1-D simulations of series two-stage turbocharging systems allowed us to investigate the control of the pressure inside the exhaust system by the opening and closing of the variable guide vanes [\[7\]](#page-24-3). Researchers have shown that with the use of the 1-D simulations the displacement of the spark ignition V8, a two-stage turbocharged engine's pressure can be lowered by 20% without reducing the brake torque [\[8\]](#page-24-4). Such simulations used a predictive knock model. The minimum BSFC criterion was also used with the 1-D simulations of the two-stage turbocharging system [\[9\]](#page-24-5). The results were very optimistic, indicating an increase in the brake power and a reduction in fuel consumption. The 1-D simulation of the engine performance, with the two-stage turbocharging system at different altitudes, showed a greater increase in the brake power and torque when compared to the singlestage turbocharger [\[10](#page-24-6)[,11\]](#page-24-7). Such simulations neglect the pressure losses inside ducts caused by the vortices, however, the 1-D calculations are in good agreement with the experimental data and require less computational time. The more complex 3-D calculations require a lot of computational resources. To reduce computational time, the geometry of the problem can be simplified. Such calculations allow investigating a fully developed turbulent flow with the Reynolds-averaged Navier-Stokes solver [\[12\]](#page-24-8). Some computational solvers for multistage turbomachinery issues implement discrete turbulence models with coupled, mixing-plane boundary conditions [\[13\]](#page-24-9). Such an approach favors the simulations under steady-state conditions. A coupled 1-D and 3-D calculations are also performed [\[14\]](#page-24-10). The 1-D calculations are performed in order to establish inlet boundary conditions for the 3-D numerical model. Both 1-D and 3-D solvers can be run simultaneously or independently.

The turbocompounding system is a type of multi-stage turbine system and one of the approaches to recover energy from exhaust gases. In such a system, an auxiliary turbine is installed behind the turbocharger. The exhaust gases drive the turbocharger and the auxiliary turbine. The energy from the exhaust gases can be recovered mechanically or electrically [\[15,](#page-24-11)[16\]](#page-24-12). The additional electrical energy generated by the electric turbocompounder can be used to supply the vehicle's electronic units or batteries. Turbocompounding systems are mostly used in heavy-duty engines, where the reduction of fuel consumption plays an important part. Such systems are often assisted by exhaust recirculation valves which reduce the NOx concentration [\[17\]](#page-24-13). The mechanical recovery system consists of the auxiliary turbine connected with the crankshaft via gear train and fluid coupling. The electrical recovery is based on the auxiliary turbine that drives the alternating current (AC) generator. The generated electric energy can be used to supply the battery.

A two-stage turbocharging system further increases the energy recovery of the exhaust gases. However, such systems are complex and require precise control over the position of the variable nozzle vanes. Another interesting fact is that the two-stage turbocharging system increases the backpressure in the exhaust duct thus leading to an increase in the pumping losses. This might affect the in-cylinder combustion process. The turbochargers of the two-stage system operate independently from each other. That is why the rotational speed of each stage is different. Also, very few works have dealt with the 3-D computational approach of such a system.

This paper presents the numerical 3-D modelling results of a six-inlet, two-stage radial inflow turbine under pulse-flow conditions. The novelty of this work is the description of a two-stage model with both rotors placed on the same shaft, thus operating with the same rotational speed. Also, the separation of the exhaust pulses was investigated by implementing a six-inlet housing. Such a two-stage system will be used as a turbocompound unit in the 2-stroke, 6-cylinder opposed piston (OP) engine which is being designed at the Warsaw University of Technology. The two-stage system will be connected mechanically to the engine crankshaft via a gear train and fluid coupling.

The paper is organized as follows. Section [2](#page-2-0) presents the theoretical approach for the estimation of the key turbine operating parameters. A detailed description of the computational geometry, mesh, and simulation setup is provided in Section [3.](#page-4-0) Section [4](#page-10-0) presents the results of the model validation against the experimental data. Section [5](#page-11-0) r<br>presents the results with an in-depth discussion. The conceptual adaptation scheme of the two-stage turbine system for the OP engine is described in Section [6.](#page-21-0) The conclusions are given in Section [7.](#page-22-0) Section 7.

at the Warsaw University of Technology. The two-stage system will be connected me-

#### <span id="page-2-0"></span>**2. Theory 2. Theory**

The typical hot section of a turbocharger consists of the volute and turbine wheel. The The typical hot section of a turbocharger consists of the volute and turbine wheel. volute might be vaneless or equipped with the nozzle guide vanes, however, most of the volutes are vaneless as they are cheaper, smaller, less complex, and easier to manufacture. However, for research purposes, the nozzle guide vanes were considered to accelerate hot exhaust gases before the rotor wheel.

Due to the rotational movement of the rotor wheel, the correlation between velocities Due to the rotational movement of the rotor wheel, the correlation between velocities in the turbine stage is represented by Equation (1): in the turbine stage is represented by Equation (1):

$$
\vec{w} = \vec{c} - u \tag{1}
$$

where *w* is the relative velocity, *c* is the absolute velocity and *u* is the linear velocity at the tip of the rotor blades. Figure [1](#page-2-1) shows the turbine configurations with the following Sections: inlet, behind nozzle vanes, behind first-stage rotor wheel, behind second-stage rotor wheel which corresponds to the second-stage rotor wheel which corresponds to the follownozzle vanes, and behind the second-stage rotor wheel which corresponds to the following indexes: 0–4. where is the relative velocity, is the absolute velocity and is the linear velocity at the  $\omega$  is the relative velocity,  $\epsilon$  is the absolute velocity and  $\mu$  is the filled velocity at

<span id="page-2-1"></span>

Figure 1. Section numbers for: (a) the exhaust pipes and the first-stage nozzle guide vanes; (b) the first-stage rotor outlet, the inter-stage pipes, the second-stage nozzle vanes and the second-stage rotor outlet.

The pressure ratio at nozzle vanes of first and second-stage is calculated using Equations (2) and (3) [\[18\]](#page-24-14):

$$
\frac{p_1}{p_0^*} = \left[1 - \frac{c_1^2}{2\varphi^2 \frac{k'}{k'-1} R' T_0^*}\right]^{\frac{k'}{k'-1}}\tag{2}
$$

$$
\frac{p_3}{p_2^*} = \left[1 - \frac{c_3^2}{2\varphi^2 \frac{k'}{k'-1} R' T_2^*}\right]^{\frac{k'}{k'-1}}\tag{3}
$$

where  $p_1$  and  $p_3$  are the pressures behind the first and second-stage nozzle vanes,  $p_0^*$  and  $p_2^*$  are the total pressures before the first and second-stage nozzle vanes,  $T_0^*$  and  $T_2^*$  are the total temperatures before the first and second-stage nozzle vanes,  $c_1$  and  $c_3$  are the absolute velocities after nozzle vanes for both stages,  $\varphi$  is the vane flow coefficient,  $k'$  is the exhaust specific heat ratio,  $R'$  is the exhaust gas constant.

The temperature behind the nozzle vane at both stages can be calculated using Equations (4) and (5):

$$
T_1 = T_0^* - \frac{c_1^2}{2\frac{k'}{k'-1}R'}
$$
\n<sup>(4)</sup>

$$
T_3 = T_2^* - \frac{c_3^2}{2\frac{k'}{k'-1}R'}
$$
\n<sup>(5)</sup>

where  $T_1$  and  $T_3$  are the temperatures behind the first and second-stage nozzle vane.

Turbine isentropic enthalpy drop at the first and second-stage can be calculated using the following equation:

$$
\Delta h_{Tiz}^{1,2} = \frac{c_{iz1,2}^2}{2} \tag{6}
$$

where  $c_{iz1,2}$  is the theoretical velocity during isentropic expansion from pressure  $p_0^*$  to pressure  $p_2$  for the first-stage turbine and from the  $p_2^*$  to  $p_4$  for the second-stage turbine. The reaction ratio *ρ* for both stages indicates rotor enthalpy drop compared to total enthalpy drop and it is calculated as:

$$
\rho_{1,2} = \frac{\Delta h_{Wiz1,2}}{\Delta h_{Tiz} 1,2} \tag{7}
$$

where ∆*hWiz*1,2 is the rotor enthalpy drop. Indexes 1 and 2 represent the first and secondstage. The expansion work in rotor blades, as well as expansion efficiency for both stages, can be calculated with the equations:

$$
l_{uT1} = u_1 c_{1u} \mp u_2 c_{2u}
$$
 (8)

$$
l_{uT2} = u_3 c_{3u} \mp u_4 c_{4u} \tag{9}
$$

$$
\eta_{uT1,2} = \frac{l_{uT1,2}}{\Delta h_{Tiz1,2}} \tag{10}
$$

Also, the total–static efficiency of both stages can be obtained using equations:

$$
\eta_{T1} = \frac{h_0^* - h_2^*}{h_0^* - h_{2iz}}\tag{11}
$$

$$
\eta_{T2} = \frac{h_2^* - h_4^*}{h_2^* - h_{4iz}}\tag{12}
$$

where  $h_0^*$  and  $h_2^*$  are total enthalpy at the inlet to the first and second-stage,  $h_2^*, h_{2iz}, h_4^*$  and *h*4*iz* are total and isentropic enthalpies at the outlet of the first and second-stage.

The blade tip speed of both stages can be calculated using the formula: e up speed of both stages can be calculated using the formula.

$$
u_{1,2} = \frac{D_{1,2}\pi n}{60} \tag{13}
$$

where  $D_{1,2}$  outer diameter of the turbine wheel. Index 1 stands for the first-stage rotor and 2 stands for the second-stage rotor. and 2 stands for the second-stage rotor.

#### <span id="page-4-0"></span>**3. Computational Approach 3. Computational Approach**

Figure [2a](#page-4-1) presents the B&W K44 turbine wheel which was used to create the 3-D Figure 2a presents the B&W K44 turbine wheel which was used to create the 3-D model for the computational domain. The presented turbine wheel was scanned using model for the computational domain. The presented turbine wheel was scanned using the the AKON smartSCAN 3D HE scanner and turntable. A total number of 60 scans per AKON smartSCAN 3D HE scanner and turntable. A total number of 60 scans per 360 360 degrees was done to scan as many surfaces as possible. However, the inter-blade zones degrees was done to scan as many surfaces as possible. However, the inter-blade zones were difficult to process, especially close to the hub. In those areas, computer-aided design were difficult to process, especially close to the hub. In those areas, computer-aided design software was used to create accurate surf[ace](#page-5-0)s. Table  $1$  shows the general parameters of the turbine wheel.

<span id="page-4-1"></span>







Figure 2. (a) The turbine rotor that was used for computational domain, (b) scanning process of the turbine wheel (c) the scanned geometry, (d) the generated 3-D model of the turbine wheel.

> Figure 3a–c shows the set-up of the computational domain which consisted of nine Figure [3a](#page-5-1)–c shows the set-up of the computational domain which consisted of nine domains: the exhaust pipes, the first-stage nozzle vanes, the first-stage turbine rotor, the tip clearance gap, the diffuser, the inter-stage pipes, the second-stage nozzle vanes (variable), the second-stage turbine rotor and the outlet. The same rotor geometry was used for the<br>Geotecal second-stage. The sature was used by an existely dealer the fourth contained first and second-stage. The rotors were installed on a single shaft, thus the rotational<br>relative of the raters were the some velocity of the rotors was the same.

<span id="page-5-1"></span>

<span id="page-5-0"></span>**Table 1.** The parameters of B&W K44 turbine wheel. **Table 1.** The parameters of B&W K44 turbine wheel.

**Figure 3.** *Cont*.

(**b**)



Figure 3. Computational domain set-up: (a) isometric view with 6 exhaust pipes; (b) cross-Section of the first and secondstage; (**c**) close-up at the clearance gap domain. stage; (**c**) close-up at the clearance gap domain.

As can be seen from Figure 3a the six exhaust pipes are equally placed on the circum-As can be seen from Figure [3a](#page-5-1) the six exhaust pipes are equally placed on the circumference of the first-stage rotor. The number of exhaust pipes corresponded to the number ference of the first-stage rotor. The number of exhaust pipes corresponded to the number of engine cylinders. Each pipe transported the exhaust gases from the cylinder to the rotor. The hydraulic diameter of each exhaust pipe was 80 mm and the length was 1500 mm. The hydraulic diameter of each exhaust pipe was 80 mm and the length was 1500 mm. The nozzle guide vanes were used to accelerate the exhaust gases before entering the first-stage rotor. The trailing edge (TE) angle of the nozzle vane was set to 30°. To investigate the latter the trailing edge (TE) angle of the nozzle vane was set to 30°. To investigate the r investigate the the leakage between the exhaust pipes during the pulse-flow, a small gap between the leakage between the exhaust pipes during the pulse-flow, a small gap between the rotor rotor blade tip and shroud was modeled and shown in Figure 3c. The height of the gap set to 3 mm. To reduce the pressure losses between the two stages, the length and cross-Sectional area of the diffuser were iteratively adjusted. The inter-stage pipes consisted of the upstream straight segment, the elbow, and the downstream straight segment. The  $\frac{1}{\sqrt{10}}$  since upstream straight segment, the upstream straight segment, the downstream straight segment. The second-stage nozzle vanes consisted of 19 vanes and were fixed at the angle of 31° outlet domain was extended up to seven diameters of the second-stage rotor outlet. second-stage nozzle vanes consisted of 19 vanes and were fixed at the angle of 31°. The second-stage notation was extended up to seven diameters of the second stage rotar outlet. blade tip and shroud was modeled and shown in Figure [3c](#page-5-1). The height of the gap was

A structured hexahedral mesh was generated on every domain except for the turbine A structured hexahedral mesh was generated on every domain except for the turbine rotor of the first and second stages. The ICEM CFD software was used for mesh generation. The rotor domains contained tetrahedral mesh which was later transformed into the polyhedral mesh in FLUENT. The mesh independence study of the rotor domains was performed before the beginning of the calculations. Figure [4](#page-7-0) shows the plot of the volumeaveraged total pressure values during the single revolution of the crankshaft for the different number of grid densities.

In order to inspect the mesh independence study for the transient issue, the total pressure values were investigated during the single revolution of the crankshaft. It was found that for the exhaust pipes the volume-average pressure plot remained almost unchanged for the number of the elements of 7200. For the first and second-stage rotor, the number of 1.34 million cells provided suitable pressure changes. The pressure changes inside the second-stage nozzle vanes remained unchanged with the number of 711,000 cells. Table [2](#page-6-0) shows the number of cells for the rest of the domains.

<span id="page-6-0"></span>**Table 2.** The number of cells for each domain.



As a result, the complete computational domain contained 2.1 million cells. Figure [5](#page-8-0) shows the polyhedral mesh generated on the rotor domains.

<span id="page-7-0"></span>

Figure 4. The total pressure plot against different number of elements during the single revolution of the crankshaft for the: (a) exhaust pipe, (b) first-stage rotor, (c) second-stage nozzle vanes, (d) second-stage rotor.

<span id="page-8-0"></span>

**Figure 5.** The polyhedral mesh generated on the rotor domain. **Figure 5.** The polyhedral mesh generated on the rotor domain. **Figure 5.** The polyhedral mesh generated on the rotor domain.

The ANSYS FLUENT software was used for transient simulations. For this reason, the sliding mesh method was used to capture the instantaneous parameters during the single time step. For the inlet of each exhaust pipe, the mass flow inlet boundary condition was set. The mass flow rate profile at the inlet to each exhaust pipe is shown in F[ig](#page-8-1)ure 6.

As a result, the computational domain computational domain contained  $2.1$  million cells. Figure 5.1 million cells. F

 $\overline{\phantom{a}}$  result, the computational domain computational domain contained 2.1 million cells. Figure 5.1 million cel

<span id="page-8-1"></span>

**Figure 6.** The inlet mass flow rate profile. **Figure 6.** The inlet mass flow rate profile. **Figure 6.** The inlet mass flow rate profile.

The mass flow rate profile was obtained from the 1-D, transient simulations of 6cylinder, 1500 kW, 2-stroke engine. The rapid increase of the mass flow rate from 0 to 1 kg/s represented the beginning of the pulse-flow during the opening of the cylinder exhaust valve. The interval between the exhaust pulses was  $60^{\circ}$  which was adequate for the firing interval of the 6-cylinder, 2-stroke engine which parameters can be seen in Ta[ble](#page-9-0) 3. 3. Table 4 shows the parameters of the boundary conditions. 3. Ta[ble](#page-9-1) 4 shows the parameters of the boundary conditions. Table 4 shows the parameters of the boundary conditions.

step of  $1.1 \times 10^{-5}$  s was chosen. Such a time step was chosen due to the strongly warring **Engine Parameters** cross-section and high velocity of the exhaust gases. Based on the previous simulations, e step leads to stable calculations with the pressure-v nculated using the single-equational Spalart-Allman The SA furbulence model is based on the transport equation for a modified turbulent kinematic viscosity  $\vartheta$ . The modified turbulent viscosity is assumed **Engine Parameters** such a small time step leads to stable calculations with the pressure-velocity coupling scheme. Each case was calculated using the single-equational Spalart-Allmaras (SA) turbu- $T_{\rm{2}}$   $T_{\rm{2}}$  lence model [\[19\]](#page-24-15). The SA turbulence model is based on the transport equation that is solved To accurately capture the transient changes inside the whole exhaust system, a time to be linear in near-wall region which greatly reduces computational time. Most of the turbomachinery simulations are done with k-ω shear stress transport or k-ε turbulence models. However, the SA model is also capable of capturing near-wall shockwave [\[20](#page-24-16)[–22\]](#page-24-17). The SA model can also be extended for subsonic and transonic flows which also makes it suitable for capturing wake inside the stator domain [\[23\]](#page-24-18). Due to the linearization of the turbulent kinematic viscosity, the computational time is reduced [\[24\]](#page-24-19). The SA model tends to be more stable during calculations with the unstructured grid. With such a small time step size, the SA turbulence model greatly reduces the computational time.



<span id="page-9-0"></span>**Table 3.** The operating parameters of the 6-cylinder, 2-stroke engine.

<span id="page-9-1"></span>**Table 4.** The parameters of the boundary conditions.



The calculations were carried out at the three different turbines rotational speed. Table [5](#page-9-2) shows the defined operating points of both rotors and the blade tip speed, which was calculated with Equation (13).

<span id="page-9-2"></span>**Table 5.** The turbine operating points with calculated blade tip speed.



The calculations were carried for three revolutions of the engine crankshaft. The reason behind this was to achieve the same pressure variations inside each exhaust pipe during the single revolution of the crankshaft. Table [6](#page-9-3) shows the parameters of the node device that was used for computing purposes.

<span id="page-9-3"></span>**Table 6.** The parameters of the node device.



With such computational restraints and such a low time step, the time needed for the calculation of the single revolution of the crankshaft was about 1.5 weeks.

Figure [7](#page-10-1) shows the mass flow average pressure variations at the first-stage turbine inlet during the 3 revolutions of the crankshaft.

Figure [7](#page-10-1) shows that for every turbine speed the pressure variations for each exhaust pipe were almost similar during the 3rd revolution of the crankshaft except for the 40,000 rpm case. The shapes of the pressure plot for each exhaust pipe were different in this case. For the 40,000 rpm case, the maximal pressure values during the 3rd revolution of the crankshaft changed from 485,000 Pa for the exhaust pipe 2 to 502,500 Pa for the exhaust pipe 5. However, such a difference was within the acceptable range. The minor changes can be observed between the 3rd and 2nd revolution of the crankshaft. However, based on the author's previous experiences, the results for the 4th revolution of the crankshaft

<span id="page-10-1"></span>

were not presented as further changes did not exceed 1%. That is why the results were presented during the 3rd revolution of the crankshaft.

Figure 7. The mass flow average pressure variations at the turbine inlet at: (a)  $n = 40,000$  rpm; (b)  $n = 50,000$  rpm; (**c**) n = 60,000 rpm. (**c**) n = 60,000 rpm.

#### <span id="page-10-0"></span>Figure 7 shows that for every turbine speed the pressure variations for each exhaust **4. Model Validation**

The numerical model was validated against the experimental data. The parameter used for the comparison between the real data and the predicted values is the root mean square error (RMSE). The RMSE gives the difference between the actual and the predicted values. The RMSE is calculated using the formula below:

$$
\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (X_{obs,i} - X_{model,i})^2}{n}}
$$
(14)

where, the  $X_{obs,i}$  is the measured value,  $X_{model,i}$  is the predicted value and *n* is the number at the outlet from the second-stage rotor was used. The validation results are shown in Figure 8 for t[he](#page-11-1) 60,000 rpm case during the three revolutions of the crankshaft. of data points. For the measured and predicted values, the area-averaged total pressure

<span id="page-11-1"></span>

**Figure 8.** Numerical model validation against the experimental results using the RMSE parameter. **Figure 8.** Numerical model validation against the experimental results using the RMSE parameter.

The RMSE of the total pressure for the numerical simulation fall within 2% for the The RMSE of the total pressure for the numerical simulation fall within 2% for the 3rd revolution of the crankshaft. Such results are acceptable for validation purposes.

## <span id="page-11-0"></span>**5. Results and Discussion 5. Results and Discussion**

Figure [9](#page-12-0) shows the plots of the area-averaged mass flow rate that was monitored in Figure 9 shows the plots of the area-averaged mass flow rate that was monitored in the Section 1-1 for three different turbine speeds. The plots were captured during the 3rd the Section 1-1 for three different turbine speeds. The plots were captured during the 3rd revolution of the crankshaft. For every turbine speed, the shapes of the pressure plots were almost similar and shifted by 60° crank angle degree (CAD). From Figure [8](#page-11-1) one can see that the shape of the pressure plot of the exhaust pipe 6 during the exhaust phase differs from the rest of the pressure plots. This pressure plot comes from the 2nd revolution of the crankshaft because of the 60° CAD shift between the exhaust pulses. The shapes of the pressure plots of 40,000 rpm case were different when compared to the pressure plots of  $50,000$ of 50,000 rpm and 60,000 rpm cases. The dashed rectangle shows the deteriorated mass flow rate at the end of the pulse for every exhaust pipe. Such a phenomenon is shown in<br>Figures of the pulse in the pulse of the pulse for every exhaust pipe. Such a phenomenon is shown in Figure [9](#page-12-0) in much greater detail.

Figure 9 in much greater detail.<br>Figure [10](#page-12-1) shows the mass flow rate and the total pressure of exhaust pipes 1 and 2 during the 3rd revolution of the crankshaft at three different turbine speeds.

(**a**) of the exhaust pulse, while the pressure inside the exhaust pipe 1 was at its peak values From Figure [10](#page-12-1) it can be seen that for 50,000 rpm and 60,000 rpm case the mass flow rate of the exhaust pipe 2 decreased between the 30◦ and 88◦ CAD and for the 40,000 rpm case, between the 37◦ and 96◦ CAD. This was associated with the increase of the mass flow rate inside the exhaust pipe 1 due to the opening of the exhaust valve. The lowest value of the pipe 2 mass flow rate was  $0.084 \text{ kg/s}$  at  $41°$  CAD for  $40,000$  rpm,  $0.092 \text{ kg/s}$ at 31.6◦ CAD for 50,000 rpm and 0.094 kg/s at 32.1◦ CAD for 60,000 rpm. The pipe 2 exhaust valve opened at the 83<sup>°</sup> CAD for every case, this increased the mass flow rate. The interaction between the pipe 1 and pipe 2 mass flow rate after the opening of the pipe 1 exhaust valve was caused by the leakage between those pipes. As it can be seen from Figure [10a](#page-12-1)–c, between the opening of the pipe 1 exhaust valve and opening of the pipe 2 exhaust valve, first the exhaust pipe 2 operated in lower pressure values due to the end due to the opening of the exhaust valve. Although, the height of the tip clearance gap was only 3 mm, part of the exhaust gases passed from exhaust pipe 1 to exhaust pipe 2. Also Figure [10d](#page-12-1)–f show the velocity vectors at the state represented by the points A–C on the mass flow rate plots. The figures clearly show the leakage between the adjacent exhaust pipes with velocity vectors entering exhaust pipe 2 in the upstream direction. The lowest leakage was observed at point C for 60,000 rpm while the highest leakage was observed at point A for 40,000 rpm. At the 40,000 rpm rotor caused the lowest acceleration of the exhaust gases. That is why a larger portion of the exhaust gases passed through the tip clearance gap. However, no negative values of the pipe 2 mass flow rate were observed. This means that no backflow of the exhaust gases from pipe 1 to pipe 2 occurred.

<span id="page-12-0"></span>

is shown in Figure 9 in much greater detail.

 $r_{\rm{10}}$  rate at the pulse for every exhaust pipe. Such a phenomenon pipe. Such a phenomenon

(**c**)

Figure 9. The area-average mass flow rate monitored at the Section 1-1 for: (a)  $n = 40,000$  rpm; (b)  $n = 50,000$  rpm; (**c**) n = 60,000 rpm. (**c**) n = 60,000 rpm.

<span id="page-12-1"></span>

**Figure 10.** *Cont*.



**Figure 10.** *Cont*.



Figure 10. The plots of mass flow rate and total pressure monitored at the Section 1-1 of the exhaust pipes 1 and 2 for: (a)  $n = 40,000$  rpm; (b)  $n = 50,000$  rpm; (c)  $n = 60,000$  rpm; and the corresponding velocity vectors at: (d) point A; (e) point B; (**f**) point C. (**f**) point C.

Figure [11](#page-15-0) shows the pressure variations in the Section 1-1 during the 3rd revolution of the crankshaft for three different turbine speeds. For each turbine speed, the shapes of the pressure plots of each exhaust pipe were almost similar apart from exhaust pipe 6 which came from the 2nd revolution of the crankshaft. The pressure plots of each exhaust pipe were shifted by 60° CAD which is associated with the inlet boundary conditions shown in Figure [6.](#page-8-1) Figure [11d](#page-15-0)–f show the pressure contours and represent the turbine state at lines A–C respectively. By looking at the pressure contours, it can be seen that the highest pressure occurred inside the exhaust pipe 5.

Unlike the ordinary scroll turbine, the rotor was not supplied on the whole circumference during a single pulse. The rotor was supplied with exhaust gases only on the  $1/6$  of its circumference. However, due to the short intervals between pulses, it can be seen that the rotor was constantly supplied by the exhaust gases. The intersection of the pressure plots at the Section 1-1 during the 3rd revolution of the crankshaft for three turbine speeds is shown in Figure 12.

From Figure 12 it can be seen that during the 3rd revolution of the crankshaft the pressure was changing in a quasi-constant manner for every turbine speed. The average pressure values were 422,000 Pa, 460,000 Pa, and 476,000 Pa for 40,000 rpm, 50,000 rpm and 60,000 rpm case, respectively. Such differences between the pressure values for 40,000 rpm, 50,000 rpm and 60,000 rpm were caused by the increasing pressure ratio.

The pressure variations behind the first and second-stage rotor are shown in Figure [13a](#page-17-0),b. One can see that the pressure at the outlet of the first-stage rotor was quasi-constant with minor fluctuations during the 3rd revolution of the crankshaft. The pressure at the Section 2-2 increased with the increase of the turbine rotational speed which was caused by the increase of the inlet pressure as shown in Figure [12.](#page-16-0) The pressure monitored at the Section 4-4 reached lower values with the increase of the turbine rotational speed. However, comparing the pressure plots from Figure [13a](#page-17-0),b, one can assume that the pressure ratio of the second-stage rotor should further increase with the increase of the turbine rotational speed. The inlet boundary conditions of the in

<span id="page-15-0"></span>

highest pressure occurred inside the exhaust pipe 5.







(**c**)







Figure 11. The plots of the total pressure monitored at the Section 1-1 during the 3rd revolution of the crankshaft for: (a)  $n = 40,000$  rpm; (b)  $n = 50,000$  rpm; (c)  $n = 60,000$  rpm; and contours of the total pressure at the first-stage rotor during: (**d**) state A; (**e**) state B; (**f**) state C. (**d**) state A; (**e**) state B; (**f**) state C. that the rotation was constant the  $\alpha$  intersection  $\mathbf{r}$  and  $\mathbf{r}$  are section of the changes for the present of the present  $\mathbf{r}$ see  $\mathcal{C}$ .

<span id="page-16-0"></span>

Figure 12. The inter Section of pressure plots monitored at the Section 1-1 for three different turbine speeds during the 3rd revolution of the crankshaft. revolution of the crankshaft.

the Section 2.1-2.1. The highest variations of the pres[sure](#page-18-0) losses can be seen in Figure 14a revolution of the crankshaft. caused by the lower acceleration of the exhaust gasses behind the first-stage rotor and vortices occurring inside the inter-stage pipes. For the turbine speed of 50,000 rpm and 60,000 rpm case, the pressure losses were the lowest. In those cases, the highest value of To investigate the pressure losses caused by the inter-stage pipes, the pressure values were monitored at the Sections 2.1-2.1 and 2.2-2.2. The results are shown in Figure 14 during the 3rd revolution of the crankshaft for three different turbine speeds. The instantaneous pressure losses were marked as percentage loss of the pressure monitored at for 40,000 rpm case. The pressure losses, in this case, reached up to 12%. This might be the instantaneous pressure losses reached up to 6%.

Figure 15 presents the plots of the pressure ratio for the first and second-stage rotor at three different turbine speeds during the 3rd revolution of the crankshaft. As it was at three different targing speeds daring the state revolution of the entireshand. The transformationed previously, the pressure ratio of the first-stage rotor increased with the increase of the turbine speed. The same phenomenon was observed for the pressure ratio of the ext the can see that the same pressure at the second-stage rotor. Higher changes in the pressure ratio values were observed at the first-stage rotor which was caused by the rapid pressure changes at the rotor inlet. On the first-stage rotor which was caused by the rapid pressure changes at the rotor inlet. On the other hand, changes in the pressure ratio for the second-stage rotor were lower. This was caused which was caused which was caused which was caused which was a caused which was a caused with a variance with  $\frac{1}{2}$  and mainly a consequence of the quasi-constant character of the pressure behind the first and second-stage rotor. For the second-stage rotor, the pressure ratio fluctuations tend to be lower with the increase of the turbine speed. Such a phenomenon allows the second-stage rotor to operate in almost steady-stage conditions. rotational speed.



<span id="page-17-0"></span>

Figure 13. The pressure plots monitored for three different turbine speeds for the 3 revolutions of the crankshaft at: (a) the Section 2-2; (**b**) the Section 4-4. Section 2-2; (**b**) the Section 4-4.

The temperatures at the outlet from both rotors are shown in Figure [16.](#page-19-0) From Figure [16](#page-19-0) one can see that the temperature behind the first and second-stage rotor became quasiconstant during the 3rd revolution of the crankshaft. It was found that, with the increase of the turbine speed, the temperature at the outlet increased. The reason for such a phenomenon is not known. A great difference between the temperatures at the Sections 2-2 and 4-4 can be observed which resulted in the recovery of the energy from the exhaust gases on the f[irst](#page-19-0) and second-stage rotor. Figure 16c-e show the contours of the total temperature behind the first-stage rotor at state A for the  $40,000$  rpm,  $50,000$  rpm and  $60,000$  rpm case respectively. On the other hand, Figure 16f-h show the temperature contours behind the second-stage rotor at state B. It can be seen that the temperature drops behind the second-stage rotor. However, minor temperature changes can be seen behind<br>the finite the first stage rotor.



**Figure 14.** *Cont*.

<span id="page-18-0"></span>

Figure 14. The pressure plots monitored at the Sections 2.1-2.1 and 2.2-2.2 with the percentage pressure losses between both Sections for: (a)  $n = 40,000$  rpm; (b)  $n = 50,000$  rpm; (c)  $n = 60,000$  rpm.

<span id="page-18-1"></span>

Figure 15. The pressure ratio at three different turbine speeds during the 3rd revolution of the crankshaft: (a) first-stage rotor; (**b**) second-stage rotor. rotor; (**b**) second-stage rotor.

In Figure [17](#page-21-1) one can see the variation of the total-static efficiency of the first and secondstage rotor during the 3rd revolution of the crankshaft for the three different turbine speeds. The efficiency during the 3rd revolution of the crankshaft was oscillating in a narrow range for every turbine speed. With the increase of the turbine speed, the efficiency of the first-stage rotor decreased. It was mainly caused by the presence of the second-stage rotor which increases the pressure behind the first-stage rotor and thus, lowers its pressure ratio. For the first-stage rotor, the highest efficiency was 0.89 for 40,000 rpm case. A completely different situation was observed for the efficiency values of the second-stage rotor. The efficiency values increased with the increase of the turbine speed. The highest efficiency value was 0.62 for the 60,000 rpm case. The high-efficiency values of the first-stage rotor were caused by the considerably higher inlet pressure and higher enthalpy drop. The efficiency of the complete two-stage system can be calculated using the formula below:

$$
\eta_{t-s}^{T1-2} = \eta_{T1} * \eta_{T2} \tag{15}
$$

Figure [18](#page-21-2) shows the total-static efficiency of the complete two-stage system com-1. Figure 16 shows the total static emerging of the complete two stage system compared to the efficiency of the single-stage system with the same turbine model which was previously presented by the authors in [\[25\]](#page-24-20).  $\epsilon$  revolution of the 3rd revolution of the 3rd revolution of the  $\epsilon$  revolution that, with the increase  $\epsilon$  revolution that, with the surface in  $[25]$ 

The efficiency values of the two-stage system were higher than those in the single-<br>The efficiency values of the two-stage system were higher than those in the single-stage system [\[25\]](#page-24-20). The average efficiency values of the two-stage system were 0.491 for the  $40,000$  rpm,  $0.496$  for the 50,000 rpm and 0.504 for the 60,000 rpm case. For the single-stage system, the efficiency values were  $0.442$ ,  $0.475$  and  $0.484$  for the  $40,000$  rpm,  $50,000$  rpm and  $60,000$  rpm case, respectively. Such differences lead to the conclusion that the two-stage system is more efficient than the single-stage system at the low turbine speed, thus at the low-load operating points of the engine however, the overall efficiency of the two-stage system is relatively low when compared to the modern scroll turbochargers.

<span id="page-19-0"></span>

**Figure 16.** *Cont*.



Figure 16. The temperature plots for three different turbine speeds monitored at: (a) the Section 2-2; (b) the Section 4-4; and the contours of the total temperature behind the:  $(c_{-}e)$  first-stage rotor at the state  $A \cdot (f_{-}h)$  second-stage rotor at the state R state B. the contours of the total temperature behind the: (**c**–**e**) first-stage rotor at the state A; (**f**–**h**) second-stage rotor at the state B.

<span id="page-21-1"></span>

<span id="page-21-2"></span>Figure 17. The total-static efficiency for three different turbine speeds, during the 3rd revolution of the crankshaft of the: (a) first-stage rotor; (b) second-stage rotor. pared to the equilibrary to the single-stage system with the single-stage system with the same ture.



Figure 18. The total-static efficiency of the two-stage exhaust system and single-stage exhaust system for three different turbine speeds during the 3rd revolution of the crankshaft. turbine speeds during the 3rd revolution of the crankshaft.

## 6. Adaptation of Two-Stage System for OP Engine

 $\frac{1}{2}$  The recovered energy from the hot exhaust gasses will be transmitted back to the engine through turbocompounding. Such a conceptual scheme is shown in Figure [19.](#page-22-1)

<span id="page-21-0"></span>Most of the turbocompound systems consist of the turbocharger turbine and free power turbine [\[26\]](#page-24-21). In this case, the end of the single-shaft, two-stage turbine system would be connected with the engine crankshaft with a gear train and fluid coupling. Such a concept has its drawback as the mechanical losses of the gear train and the fluid coupling are included [\[27–](#page-24-22)[30\]](#page-24-23). Nowadays, most turbocompound solutions are focused on the generation of electrical power. The electric turbocompounding does not require a gear train or fluid coupling, and the generated energy could be used to supply other devices [\[31](#page-25-0)[–34\]](#page-25-1). It was found that the electric turbocompounding is more efficient than the mechanical turbocompound especially for heavy-duty engines if the turbines are connected in parallel [\[35\]](#page-25-2). However, for this project, the series mechanical turbocompound system will be used. The type of gear train and fluid coupling is still a concern for future research.

<span id="page-22-1"></span>

**Figure 19.** A schematic adaptation of two-stage turbine system for OP engine with turbocompounding unit. **Figure 19.** A schematic adaptation of two-stage turbine system for OP engine with turbocompounding unit.

#### <span id="page-22-0"></span>**7. Conclusions**

The paper deals with the unsteady simulation of the two-stage, radial turbine under pulse-flow conditions. The novelty of this research was the development of a transient, 3-D simulation of the two-stage turbine with two rotors placed on a single shaft. The be simulation of the two suge tablies with two follows placed on a slight shaft. The separation of the exhaust gases at the inlet of the first-stage rotor was also investigated. The simulations were carried out for the pulse-flow conditions at the inlet. The turbine wheel was scanned with the use of the 3-D scanner to create a virtual model. The same wheel was scanned with the use of the 3-D scanner to create a virtual model. The same turbine model was used for the first and second-stage. The computational domain was generated using 3-D CAD software. The simulations were carried out using Ansys Fluent software with a small time step. The simulations lasted until the 3rd revolution of the crankshaft. Three different turbine speeds were investigated. The two-stage system was capable of recovering energy from the exhaust gasses. The temperature between the inlet the total pressure values changed from about 450,000 Pa at the inlet of the first-stage rotor to 120,000 Pa behind the second-stage rotor. It was also found that despite the pulsatile operation of the two-stage system, the first-stage rotor was constantly supplied with exhaust gases. The pressure at the inlet to the first-stage rotor and behind the second-stage rotor was changing in a quasi-constant manner. The pressure changes decreased with the increase of the turbine speed. It was also found that the presence of the second-stage rotor limited the pressure behind the first-stage rotor and thus, limited its pressure ratio. That is why the total-static efficiency of the first-stage rotor decreased with the increase of the turbine speed. A great efficiency difference between the two-stage system and single-stage system at the low turbine speed was observed. This showed that such a system might be efficient at low-loads of the ICE. However, at the higher turbine speed, the efficiency difference between the two-stage system and single-stage system was lower and reached up to 2%. On the other hand, the two-stage system with newly designed housing provided to the first-stage rotor and behind the second-stage rotor dropped by about 100 K. Also, limited the pressure behind the first-stage rotor and thus, limited its pressure ratio. That<br>is why the total-static efficiency of the first-stage rotor decreased with the increase of the<br>turbine speed. A great efficiency proper separation between exhaust pulses. A relatively low leakage between the adjacent exhaust pipes was observed during the simulation. Also, no negative values of the mass flow rate were observed which meant that no backflow occurred during the simulation. The low total-static efficiency values of the two-stage system give room for improvement. However, for multicylinder engines, more than one scroll turbocharger needs to be used. Because of that, the exhaust system is more complex. On the other hand, with such a two-stage turbocharging system, the exhaust system would be easier to design. Such a system satisfies the demands of both the variable turbine geometry turbocharger and scroll turbocharger. The two-stage turbocharging system is a very promising device with its reduced leakage between the exhaust pipes and variable turbine geometry features. That is why further research will focus on the influence of the different positions of the

second-stage nozzle vanes to control the backpressure in the exhaust system. The final second-stage nozzle vanes to control the backpressure in the exhaust system. The final milestone is the manufacturing of the prototype device with a two-stage expansion system milestone is the manufacturing of the prototype device with a two-stage expansion system and adjustable nozzle vanes at the second-stage. Such a system will be used in a 2-stroke, and adjustable nozzle vanes at the second-stage. Such a system will be used in a 2-stroke, 6-cylinder OP engine which is being developed at the Warsaw University of Technology. 6-cylinder OP engine which is being developed at the Warsaw University of Technology.

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#### **Nomenclature Nomenclature**

- 
- 
- 
- 
- *p*  $\dot{T}^*$
- 
- *k*
- 
- $R'$ <br> $\Delta h_{Tiz}$
- ∆*hTiz* Turbine isentropic enthalpy drop Theoretical velocity during the
- *ciz* isentropic expansion
- 
- *ρ* The turbine reaction ratio  $Δh<sub>Wiz</sub>$  The rotor isentropic entha
- The rotor isentropic enthalpy drop
- *IuT* The expansion work of the rotor blades
- $\eta_{uT}$  The expansion efficiency of the rotor blades<br> $h^*$  Total enthalpy
- *h*\* Total enthalpy<br>*D* Diameter of th Diameter of the turbine
- *n* The rotational speed of the turbine

#### **Subscripts**

- 0 Conditions before first-stage nozzle vanes
- 1 Conditions after first-stage nozzle vanes
- 2 Conditions after first-stage rotor
- 3 Conditions before second-stage rotor
- 4 Conditions after second-stage rotor
- 5 Conditions at the outlet

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#### **Notations Abbreviations**

- *w* Relative velocity **ICE** Internal combustion engine
- *c* Absolute velocity 3-D Three-dimensional
- *u* Linear velocity **COV** CRC Organic Rankine cycle
- *p* Static pressure CFD Computational fluid dynamics
	- Total pressure AC Alternating current
	- Total temperature OP Opposed piston (engine)
- *φ* Vane flow coefficient TE Trailing edge
	- Exhaust specific heat ratio SA Spalart-Allmaras turbulence model
	- Exhaust gas constant CAD Crankshaft angle degree
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