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Hybrid Quasi-Optimal PID-SDRE Quadrotor Control

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Abstract: In the paper, a new cascade control system for an autonomous flight of an unmanned aerial vehicle (UAV) based on Proportional–Integral–Derivative (PID) and finite-time State-Dependent Riccati Equation (SDRE) control is proposed. The PID and SDRE controllers are used in a hybrid control system for precise control and stabilization, which is necessary to increase the drone's flight stability and maneuver precision. The hybrid PID-SDRE control system proposed for the quadrotor model is quasi-optimal, since the suboptimal control algorithm for the UAV stabilization is used. The combination of the advantages of PID and SDRE control gives a significant improvement in the quality of control while maintaining the simplicity of the control system. Furthermore, the use of the suboptimal control technique provides the UAV attitude tracking in finite time. These remarks are drawn from a series of simulation tests conducted for the drone model.

Keywords: state-dependent riccati equation technique; SDRE control; PID control; attitude control; UAV; quadrotor

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Citation: Giernacki, W.; Stępień, S.; Chodnicki, M.; Wróblewska, A. Hybrid Quasi-Optimal PID-SDRE Quadrotor Control. *Energies* **2022**, *15*, 4312. https://doi.org/10.3390/en15124312

Academic Editor: Adel Merabet

Received: 12 May 2022
Accepted: 10 June 2022
Published: 13 June 2022

1. Introduction

In recent years, there has been a strong trend in the development of control and estimation techniques for unmanned aerial vehicles (UAVs) [1]. This is mainly due to their wide availability, which, in combination with photo- and video-recording devices, greatly extends the scope of their applicability. To operate safely and precisely in an environment close to humans [2], drones need appropriate hardware and sensory tools as well as efficient control algorithms.

Currently, a cascade closed-loop control system is widely used [3]. The speed and precision of control is there based on the outer and inner loops for adjusting the orientation and position of the drone in 3D space. It usually uses well-known, simple, fixed-value controllers in the P, PD or PID structure. For an underactuated plant such as a drone, using four inputs expressing the expected/reference position of the drone and its orientation around the Z axis (yaw angle) in the observer (Earth) coordinate system, already roughly selected controller gains allow for a stable, controllable, autonomous flight, which in terms of image recording from a camera equipped with a stabilizer is more than enough.

The situation is quite different in the cases that require greater precision. Here, more advanced solutions are sought to ensure fast stabilization in flights with variable mass [4], mobile manipulation [5], or military missions [6]. Often in military tasks, the vector correlated with the front of the drone marks the target, and it is necessary not only to move the drone from point to point but also to orientate and stabilize it in the 3D space by tracking predefined angles that express the orientation of the drone (roll, pitch, yaw angles). This paper is devoted to this application problem, which is still recognized and classified as one of the key areas of research in the UAV community. In the following article, the proposed hybrid quasi-optimal PID-SDRE quadrotor control method will serve to achieve this goal.

The potential of the SDRE control strategy being considered and extended here has already been validated with the success by the UAV community over the last two decades. For the first time in the world scientific literature, a non-linear UAV control system based on state-dependent Riccati equations (SDRE) was proposed in [7], where its aim is to stabilize a desired velocity vector and the attitude of a multirotor UAV model. In [8], an INS/GPS sensor fusion scheme was introduced as an alternative to the extended Kalman filter (EKF). There, the state-dependent Riccati equation navigation filter was tested in the flight scenario. The aim was to minimize the influence of linearization errors on the tracking performance of the reference signals. In the paper, one may also find the stability proof of the SDRE non-linear filter and comparison with the classical EKF filter. Furthermore, in [9], through the integration process of the differential SDRE filter algorithm and the finite-horizon SDRE technique, the authors created an efficient online technique to control the missile guidance system.

The latest research trends in the use of the SDRE method in UAVs are, respectively:

- Development of a flight controller for quad tilt-wing UAV that during its transition flight (with the change of wing angle) will be able to deal with high nonlinearity in this situation and provide drone stability [10];
- Development of a suboptimal integral sliding mode trajectory tracking anti-interference controller based on the state-dependent Riccati equation [11];
- Development of non-linear controllers for cargo UAVs to obtain precise robot flight and efficient reduction of load oscillations by exploiting the natural coupling between horizontal UAV movement and payload oscillation [12].

Last but not least (to summarize the state-of-the-art of SDRE methods for UAVs) are the papers of Nekoo, Acosta and Ollero [13–15]. They are devoted to aerial–acrobatic maneuvers and collision avoidance of the SDRE controller using the artificial potential field method.

Except for the SDRE control method, state-of-the-art analysis for UAV control provides a wide spectrum of approaches, both model-free and model-based [16–18]. In this paper and research, using the advantages of both, we proposed a hybrid method, in which the model-free PID control is used to control the UAV's position, while the model-based finite-time SDRE method will increase the precision level in tracking the UAV orientation (via attitude control in inner loop).

The novelty and added value of our work is the development of an original cascade hybrid finite-time quasi-optimal PID-SDRE quadrotor control system as well as comparative simulation tests for the problem of stabilization of the set orientation of the drone in a predefined time horizon.

The new contribution of this work is described as follows:

- Optimal attitude stabilization and control with finite time;
- An increasing precise attitude control method;
- Elimination of the PID stabilizer and the tuning problem.

The paper is organized as follows: In Section 2, the dynamical model of the quadrotor is presented. Section 3 contains a description of the control system design with the new PID-SDRE attitude controller, the P-PID attitude controller, and the finite-time SDRE stabilizer, respectively. The UAV used in simulation experiments, as well as their comprehensive report and analysis, can be found in Section 4. Finally, the conclusion is drawn in Section 5.

2. Quadrotor Model

In most mathematical models of UAVs, its dynamics is considered for the structure treated as a rigid body with the mass of the UAV placed in the center of gravity and the mass of each of four propulsion units placed symmetrically in the cross-type frame. Therefore, the rigid body equations of motion are the differential equations that describe the evolution of the basic states of the quadrotor.

Furthermore, regarding the shape of the drone (Figure 1), and its natural X-type layout configuration, the North-East-Down (NED) axes convention with regard to the observer's coordinate system (the so-called Earth frame—$\{\mathcal{EF}\}$) is used. In this convention, the x axis of the UAV's local coordinate system (body frame—$\{\mathcal{BF}\}$) follows the camera direction, the y axis is perpendicular to the right, and the z axis is looking down according to the right-hand rule, respectively.

Figure 1. AtraxASF UAV used for drone modeling and simulation experiments.

The dynamics of the quadrotor is generally defined using Newton's force and moment equations [3]. The force equation is the following

$$\boldsymbol{F} = m(\dot{\boldsymbol{v}} + \boldsymbol{\omega} \times \boldsymbol{v}), \tag{1}$$

where $\boldsymbol{v}$ is a quadrotor linear velocity, $\boldsymbol{\omega}$ is the angular velocity, m is the mass of the aircraft and $\boldsymbol{F}$ denotes the force vector. For completeness, the moment equation should also be considered. The equation describes all the moments that act on the aircraft, which are equal to the rate of change in angular momentum.

$$\boldsymbol{M} = \boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega}, \tag{2}$$

where $\boldsymbol{I}$ is an aircraft inertia matrix and $\boldsymbol{M}$ denotes the moment vector. When considering the vector $\boldsymbol{v}$ defined for all components in the direction x, y and z and $\boldsymbol{\omega}$ for roll ϕ, pitch θ and angle of yaw ψ

$$\boldsymbol{v} = \begin{bmatrix} u & v & w \end{bmatrix}^T \tag{3}$$

and

$$\boldsymbol{\omega} = \begin{bmatrix} p & q & r \end{bmatrix}^T \tag{4}$$

then equations of aircraft aerodynamics can be defined for linear and angular speeds. In addition, because of symmetry, in the inertia matrix, only the diagonal elements become nonzero

$$\boldsymbol{I} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}. \tag{5}$$

The system of non-linear equations that describes the flight dynamics of aircraft, considering gravity force g and force due to thrust F_T, is the following

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rv - qw + \frac{1}{m}F_x \\ pw - ru + \frac{1}{m}F_y \\ qu - pv + \frac{1}{m}F_z \end{bmatrix}, \tag{6}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{I_z - I_y}{I_x} rq + \frac{1}{I_x} M_x \\ \frac{I_x - I_z}{I_y} pr + \frac{1}{I_y} M_y \\ \frac{I_y - I_x}{I_z} pq + \frac{1}{I_z} M_z \end{bmatrix}, \tag{7}$$

where $F_x = k_1\dot{x}$, $F_y = k_2\dot{y}$, $F_z = k_3\dot{z}$, $M_x = k_4\dot{\varphi}^2$, $M_y = k_5\dot{\theta}^2$, $M_z = k_6\dot{\psi}^2$, and k_1, k_2, k_3 are translational air drag coefficients, while k_4, k_5, k_6 are aerodynamic friction coefficients.

Equations (6) and (7) are non-linear functions of states, and they have to be easily formed as the state-dependent coefficient (SDC) form. Therefore, the separation of (6) and (7) is not complicated because, in general, the variables in the state are in the form of products.

To describe the aircraft orientation, the kinetic equations should be considered as functions that transform its angular position from the Earth frame to the body frame

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} p + (qsin\phi + rcos\phi)tan\theta \\ qcos\phi - rsin\phi \\ (qsin\phi + rcos\phi)sec\theta \end{bmatrix}, \tag{8}$$

where $sec\theta = 1/cos\theta$.

To convert between the body frame (BF) and the Earth frame (EF), the following rotation matrix from BF to EF is used:

$$\boldsymbol{R_{BE}} = \begin{bmatrix} cos\theta cos\psi & sin\varphi\theta cos\psi - cos\varphi sin\psi & cos\psi sin\theta cos\varphi + cos\psi sin\varphi \\ cos\theta sin\psi & sin\psi sin\theta sin\varphi + cos\varphi cos\psi & sin\psi sin\theta cos\varphi - cos\psi sin\varphi \\ -sin\theta & cos\theta sin\varphi & cos\theta cos\varphi \end{bmatrix}, \tag{9}$$

where $R_X(\varphi)$, $R_Y(\theta)$, and $R_Z(\psi)$, are matrices of Euler angles: roll (φ), pitch (θ) and yaw (ψ), defined as

$$R_X(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix}, \tag{10}$$

$$R_Y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}, \tag{11}$$

$$R_Z(\psi) = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{12}$$

3. Control System Design

3.1. PID-SDRE Attitude Controller

The quadrotor is an unstable plant. Therefore, a UAV control system should contain a stabilization subsystem in design to make attitude control fast in response and free from overshoots. Then, from the point of view of practical implementation and drone usefulness, both the angular and linear speeds should stabilize. This is a reason why two blocks of controllers are proposed: one to control the orientation in space by the angular position and the other to stabilize the angular quadrotor speeds. These requirements can be achieved by using a PID attitude controller coupled to PID stabilizers. However, the use of PID-type controllers has affected efforts to tune and achieve optimal performance for the control system. Thus, a better idea is to use the PID-SDRE coupled solution or full integrated SDRE controller, which does not need to be optimized because it is optimal for control purposes.

Taking into account the above, this paper deals with the hybrid PID-SDRE controller dedicated to attitude control and finite-time stabilization. The control system schema is presented in Figure 2.

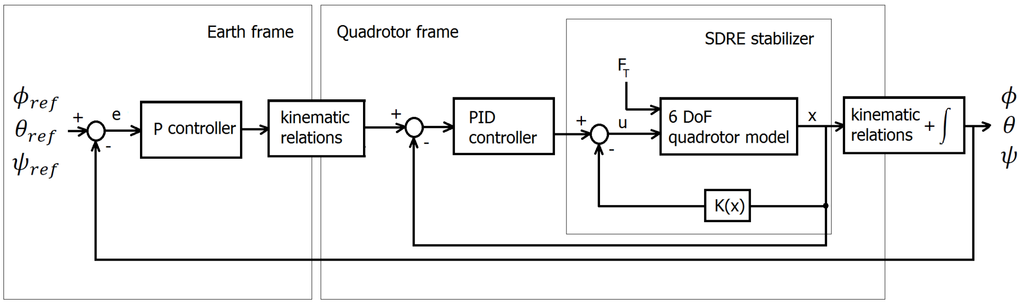

Figure 2. PID-SDRE control schema of the 6 DoF quadcopter model.

As shown, the controller consists of three control units. The attitude control system is implemented in outer closed-loop systems using the P controller, but the speed stabilization problem is performed by the inner closed-loop subunit with the PID controller and the feedback compensator employing the finite-time SDRE control technique. The stabilization problem can also be realized by the following:

- PID controller without SDRE stabilizer;
- SDRE feedback compensator neglecting PID stabilizer.

This means that the PID speed controller or SDRE speed compensator is redundant and the system can work as a two-unit and two-closed-loop control system. In this case, a thrust force F_T is set as constant and allows one to obtain the desired altitude. The other variables contained in Figure 2 denote: $\boldsymbol{x} = \begin{bmatrix} v & \omega \end{bmatrix}^T = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T$—state vector of the 6 DoF model, $\boldsymbol{u} = \begin{bmatrix} M_x & M_y & M_z \end{bmatrix}^T$—attitude control vector and error vector of the attitude angles $\boldsymbol{e} = \begin{bmatrix} \phi_{ref} - \phi & \theta_{ref} - \theta & \psi_{ref} - \psi \end{bmatrix}^T$.

3.2. P-PID Attitude Controller

The control system presented in Figure 2 includes two PID-based controllers: situated in the main loop P controller for attitude control and located in the inner loop PID controller for angular speed control (stabilization). The main P controller operates in the Earth frame and performs the UAV space orientation task, controlling the attitude angles: roll ϕ, pitch θ, and yaw ψ to the reference values. The inner-loop PID controller is used to stabilize the attitude speeds to zero. The PID-based control system works when fine and optimal tuning of P and PID controllers is achieved; however, sometimes it is problematic and not easy.

Considering the kinematic relations from Earth to the quadrotor frame, the control law for the main P controller is as follows

$$\boldsymbol{u_p} = \begin{bmatrix} p_p \\ q_p \\ r_p \end{bmatrix} = \begin{bmatrix} (e_\phi - e_\psi sin\theta)k_{p\phi} \\ (e_\theta cos\phi - e_\psi sin\phi cos\theta)k_{p\theta} \\ (e_\psi cos\phi cos\theta - e_\theta sin\phi)k_{p\psi} \end{bmatrix}, \tag{13}$$

where

$$\boldsymbol{e} = \begin{bmatrix} e_\phi \\ e_\theta \\ e_\psi \end{bmatrix} = \begin{bmatrix} \phi_{ref} - \phi \\ \theta_{theta} - \phi \\ \psi_{ref} - \psi \end{bmatrix} \tag{14}$$

is the error signal $\boldsymbol{e}$, which is a vector of three elements fed to the P controller. The PID controller used to stabilize the quadrotor space consists of three independent controllers for the rolling speed p, the pitching speed q, and the yawing speed r. The output of a PID controller $\boldsymbol{u_{PID}} = \begin{bmatrix} M_{xPID} & M_{yPID} & M_{zPID} \end{bmatrix}^T$ is calculated in the time domain from the feedback speed error as follows:

$$\boldsymbol{u_{PID}} = \boldsymbol{k_P}\omega + \boldsymbol{k_I}\int \omega dt + \boldsymbol{k_D}\frac{d\omega}{dt}. \tag{15}$$

The speed error signal is equal to ω, because the reference angular speed is equal to zero. Then, a three-element vector fed to the PID controller is computed that performs the proportional, derivative, and integral functions of this signal with respect to time. $\boldsymbol{k_P}$, $\boldsymbol{k_I}$, and $\boldsymbol{k_D}$ are proportional, integral, and derivative gain matrices:

$$\begin{aligned} \boldsymbol{k_P} &= diag\left(k_{P_p}, k_{P_q}, k_{P_r}\right), \\ \boldsymbol{k_I} &= diag\left(k_{I_p}, k_{I_q}, k_{I_r}\right), \\ \boldsymbol{k_D} &= diag\left(k_{D_p}, k_{D_q}, k_{D_r}\right). \end{aligned} \tag{16}$$

The integral matrix gain $\boldsymbol{k_I}$ times the integral of the error vector plus the derivative matrix gain $\boldsymbol{k_D}$ times the derivative of the error vector are calculated using its approximation and creating the digital form of the PID. This is a standard formulation of digital PID that uses the bilinear transformation of the continuous integral and derivative action [1].

3.3. Finite-Time SDRE Stabilizer

The state-dependent Riccati equation (SDRE) optimal control method is a promising and rapidly emerging tool for the control of non-linear systems. The technique with further improvement and a modified approach is widely described in recent literature [19–23]. Scientists can follow the state-dependent Riccati equation (SDRE) approach in the context of the non-linear control problem with a quadratic objective function [24–27]. The formulation based on a quadratic objective function is commonly used in practical solutions because the objective function defines energy, i.e., energy lost and delivered to the system, which is compatible with practical applications.

The finite-time control problem consists of finding an optimal control law that minimizes the following objective function defined for control time t_f [28]

$$J(\boldsymbol{u}) = 1/2\boldsymbol{x}^{\boldsymbol{T}}(t_f)\boldsymbol{S}(\boldsymbol{x}(\boldsymbol{t_f}))\boldsymbol{x}(t_f) + 1/2\int_0^{t_f}\left(\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{Q}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{u}^{\boldsymbol{T}}\boldsymbol{R}(\boldsymbol{x})\boldsymbol{u}\right)dt \tag{17}$$

subject to non-linear dynamics for affine systems

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}. \tag{18}$$

Non-linear dynamics (18) can be written using the state-dependent coefficient (SDC) form [29]

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u}, \tag{19}$$

where $\boldsymbol{S}(\boldsymbol{x})$ and $\boldsymbol{Q}(\boldsymbol{x})$ are symmetric, positive semi-definite weighting matrices for states, and $\boldsymbol{R}(\boldsymbol{x})$ is the symmetric, positive definite weighting matrix for control inputs. Equation (18) includes the vector $\boldsymbol{F}(\boldsymbol{x})$, which is piecewise continuous in time and smooth with respect to its arguments, and that satisfies the Lipschitz condition.

Taking into account the SDC approximation (19), if the pair $\boldsymbol{A}(\boldsymbol{x}), \boldsymbol{B}(\boldsymbol{x})$ is a stabilizable parameterization of the system, then to check the controllability of the affine system, this pair in the linear sense should be controllable. On the other hand, checking the controllability of that pair does not require state or control input information [19,21,27]. It can be simply checked by the matrix

$$\boldsymbol{M}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{B}(\boldsymbol{x}) & \boldsymbol{A}(\boldsymbol{x})\boldsymbol{B}(\boldsymbol{x}) & \ldots & \boldsymbol{A}^{\boldsymbol{n-1}}\boldsymbol{B}(\boldsymbol{x}) \end{bmatrix} \tag{20}$$

often called the controllability matrix. Then, the system (18) or (19) is controllable if the controllability matrix (20) has full rank.

Employing Hamiltonian theory, the optimal control law is as follows

$$\boldsymbol{u} = -\boldsymbol{R}(x)^{-1}\boldsymbol{B}(x)^{T}\boldsymbol{K}(x)x, \tag{21}$$

where $\boldsymbol{K}(x)$ is a state-dependent feedback compensator that can be obtained from the solution of a state-dependent differential Riccati equation (SDDRE)

$$\dot{\boldsymbol{K}}(x) + \boldsymbol{K}(x)\boldsymbol{A}(x) + \boldsymbol{A}(x)^{T}\boldsymbol{K}(x) - \boldsymbol{K}(x)\boldsymbol{B}(x)\boldsymbol{R}(x)^{-1}\boldsymbol{B}(x)^{T}\boldsymbol{K}(x) + \boldsymbol{Q}(x) = \boldsymbol{0}. \tag{22}$$

Equation (22) is in the form of a differential SDRE for affine systems and must be solved many times for each $\boldsymbol{x}$ throughout the control process with the final condition $\boldsymbol{K}(\boldsymbol{x}(t_f)) = \boldsymbol{S}(\boldsymbol{x}(t_f))$. The solution of the equation exactly results in suboptimal control because it neglects the so-called 'SDRE necessary condition for optimality', which tends to zero [19,23,27]. Equation (22) known as differential SDRE or shortly SDDRE (State-Dependent Differential Riccati Equation); it can be solved numerically employing different algorithms. In the literature, there are many efficient algorithms dedicated to finding a solution of the SDDRE. The most popular are: backward iteration, state transition matrix approach, Lyapunov-based method, Riccati root method, etc. [30].

4. Experimental Results

4.1. UAV Used in Simulation Experiments

In the conducted experiments with the use of MATLAB/Simulink environment, a dynamical model of a military AtraxASF drone (shown in Figure 1) was used. AtraxASF is a quadrotor specially designed to perform precise test flights to inspect wild animals, especially in terms of detecting wild boars suffering from ASF (African swine fever). It was built as part of the research and development project financed by the National Center for Research and Development (Poland) and constructed by the Air Force Institute of Technology (ITWL, Warsaw, Poland). The UAV is equipped with a high-resolution thermal imaging sensor and has the following parameters:

- Take-off mass: 13 kg,
- Max. flight time: 40 min,
- Flight range: 4.5 kg,
- Optimal flight speed: 30 km/h,
- Max. flight speed: 60 km/h.

This military UAV was chosen to be modeled, as the authors of this article have all the UAV data (some can be provided on request) and its hardware and software characteristics gathered and verified during laboratory, test stand, and flight tests with AtraxASF.

4.2. Simulation Experiments

The non-linear UAV model is applied to check the PID-SDRE control for attitude and stabilization when it tries to find the desired angular position during flight or take-off. Using the governing equations that describe the UAV aerodynamics in SDC form (19), the control problem consists of finding the ϕ, θ, and ψ moments with trust generated by UAV rotors. As defined in (6) and shown in Figure 2, the thrust acts positively along the positive body axis z. To perform the attitude control, to adjust its ψ angle, or to make it turn left or right, the vehicle applies more thrust to one set of motors generating ψ moment. ϕ and θ are adjusted by applying more thrust to one rotor and less to the other opposing rotor, generating ϕ and θ moments. In this simple way, rolling, pitching, or yawing moments are generated. According to the control schema proposed in Figure 2, the control applied to the UAV is a sum of the PID control and the SDRE stabilizator control, where the controller outputs are ϕ, θ, and ψ moments. Z-axis force related to altitude is assumed to be constant, and the forces on the x and y axes generated by the controller are neglected. Therefore, the output of an SDRE subcontroller $\boldsymbol{u_{SDRE}} = \begin{bmatrix} M_{xSDRE} & M_{ySDRE} & M_{zSDRE} \end{bmatrix}^T$ is calculated from (21). Using the described UAV model, the PID-SDRE control technique is applied to control the UAV attitude considering finite-time horizon SDRE feedback compensation for stabilization. To be exact, as shown in Figure 2, the attitude is controlled by the P controller (13), but the PID stabilization works, zeroing angular speeds (15). An additional SDRE feedback compensator additionally stabilizes the UAV angular position and makes it possible not only in finite time but also for rapid attitude changes. Briefly, the PID-SDRE method makes possible rapid response for user commands and moreover enables rapid stabilization of the path of flight when unexpected external forces try to change its position and orientation during flying action. Taking into account the above, the control problem consists of finding the state dynamics of the UAV and the PID-SDRE controls for the prescribed attitude for $\phi_{ref} = 30$ deg, $\theta_{ref} = 45$ deg, $\psi_{ref} = 15$ deg with and reference angular speed $p_{ref} = 0$ deg/s, $q_{ref} = 0$ deg/s, $r_{ref} = 0$ deg/s.

In association with (13), the gains of the P attitude controller are: $k_{P\phi} = 10$, $k_{P\theta} = 20$, $k_{P\psi} = 100$. PID stabilizer gains (15) are chosen as: $\boldsymbol{k_P} = 0,3\boldsymbol{I_{3\times3}}$, $\boldsymbol{k_I} = 0,1\boldsymbol{I_{3\times3}}$, $\boldsymbol{k_D} = diag(0.01, 0.01, 0.0)$ and finally, the quadratic functional cost weighting matrices defined in (22) are as follows: $\boldsymbol{S} = 2\boldsymbol{I_{6\times6}}$, $\boldsymbol{Q} = 0,5\boldsymbol{I_{6\times6}}$ and $\boldsymbol{R} = 0,1\boldsymbol{I_{6\times6}}$.

The dynamics of the state of the UAV, in other words, the speed response, including its orientation to the desired angle position, is shown in Figures 3 and 4. The UAV attitude control has been activated at time $t = 1$ s; then, the UAV angulary has been moved from the "zero" attitude to the reference angular position. First, simulations are performed for the UAV controlled by the P and PID stabilizer only, neglecting the SDRE stabilizer.

Looking at the above figures (Figures 3 and 4), the angular position and speed responses are quick due to the large gains in the P-controller. However, the presented P-PID technique controls the attitude with overshoots and oscillations. Generally, the control works and is easy to implement; however, the system fails in precision operation in airspace. In this type of control, stabilization and improvement of accuracy seems to be necessary.

Next, simulations are performed for the complete PID-SDRE controller to show how the UAV can stabilize in a finite time t_f in the context of angular speeds. To verify precision and rapidity and to compare the proposed technique considering the SDRE-based method with the commonly used PID technique, a numerical experiment is performed three final times: $t_f = 4$ s, $t_f = 2$ s, and $t_f = 1$ s with the same reference attitude. The simulation results are presented in Figures 5–10 with the impact of the successively reduced control time t_f from 4 to 1 s.

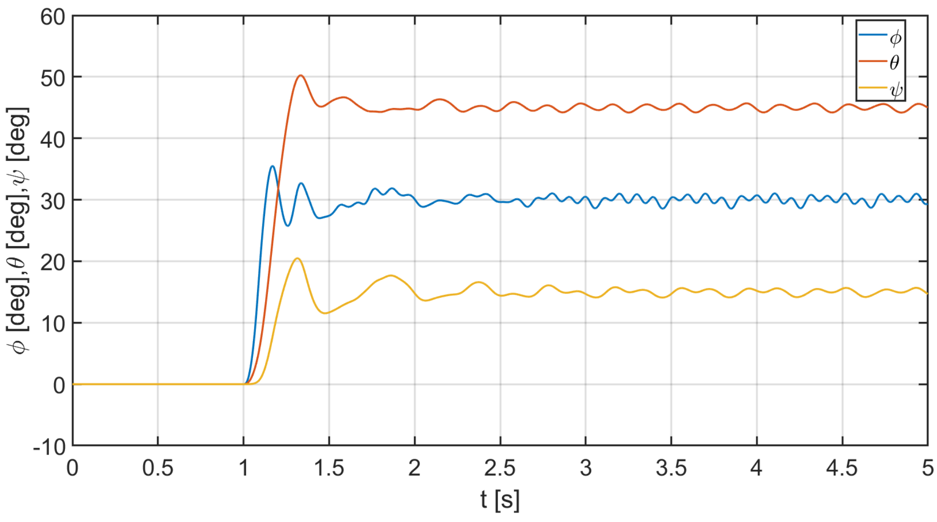

Figure 3. UAV angular response—PID control mode.

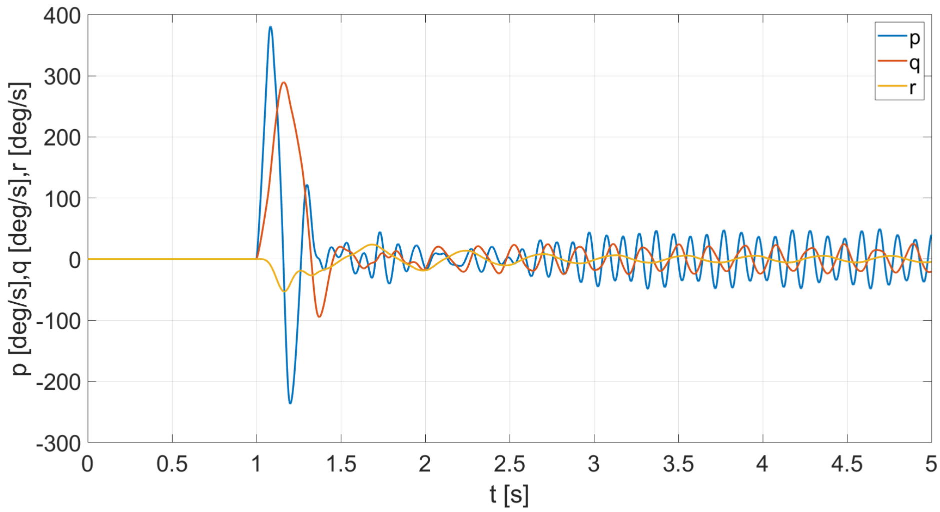

Figure 4. UAV angular speed response—PID control mode.

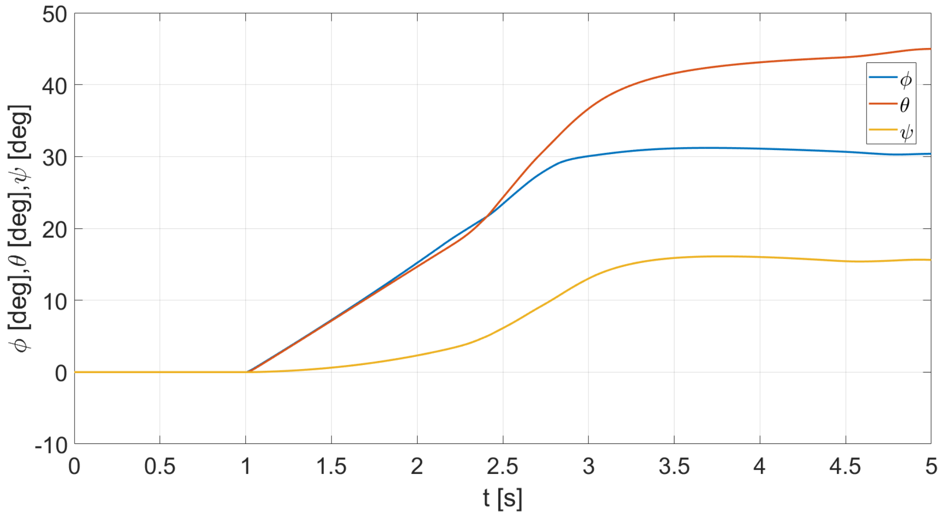

Figure 5. Angular response of UAV—PID-SDRE control mode, $t_f = 4$ s.

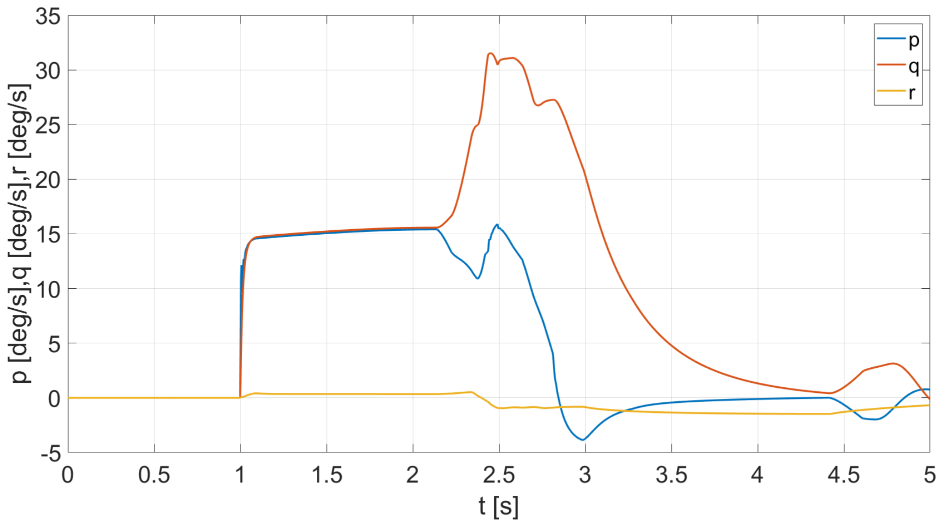

Figure 6. UAV angular speed response—PID-SDRE control mode, $t_f = 4$ s.

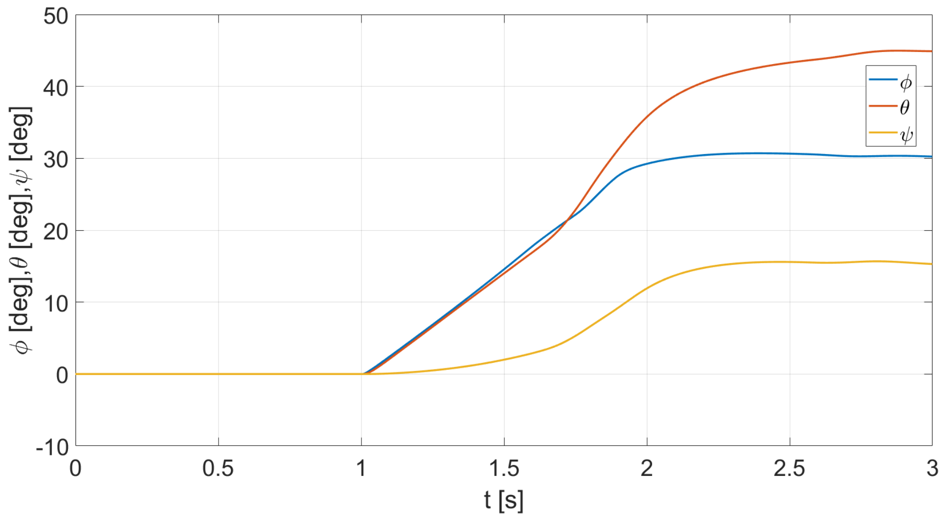

Figure 7. Angular response of UAV—PID-SDRE control mode, $t_f = 2$ s.

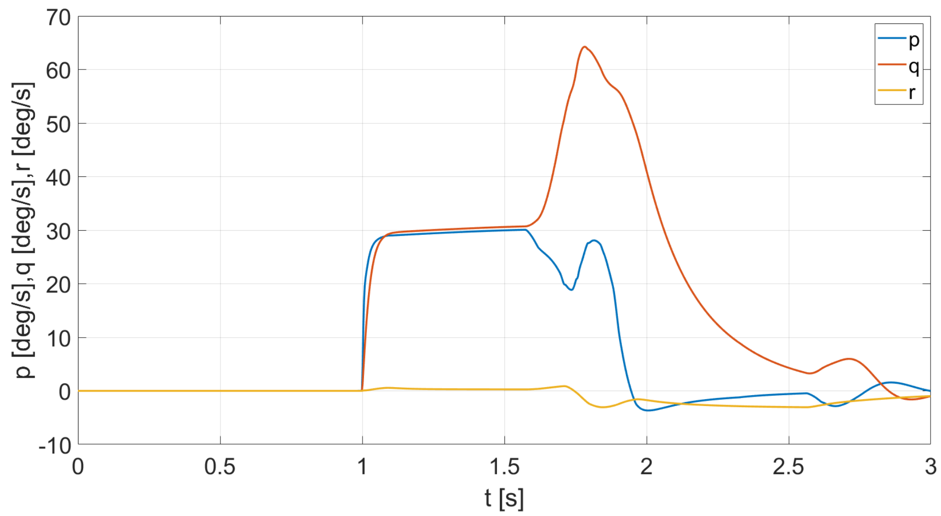

Figure 8. UAV angular speed response—PID-SDRE control mode, $t_f = 2$ s.

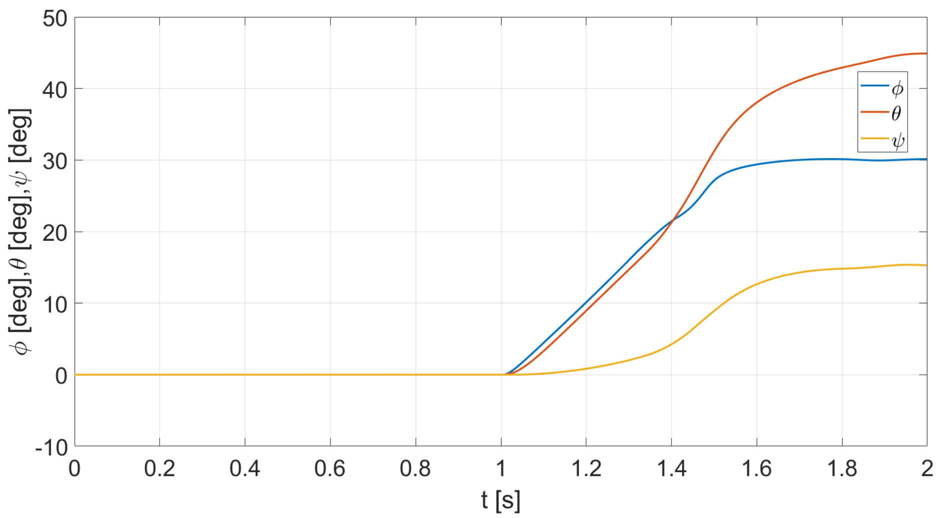

Figure 9. Angular response of the UAV—PID-SDRE control mode, t_f = 1 s.

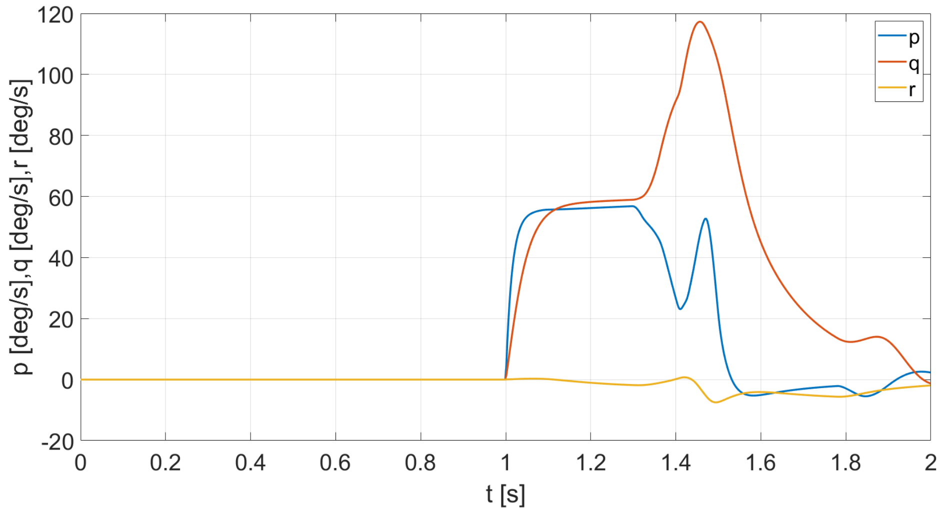

Figure 10. UAV angular speed response—PID-SDRE control mode, $t_f = 1$ s.

When looking at and analyzing Figures 5 and 6, the proposed PID-SDRE control shows that the quadrotor can be successfully controlled to referenced angles, zeroing angular speed, and reducing or eliminating overshoots. As expected, the referenced attitude is reached at the control time $t_f = 4$ s. When considering the following Figures 7–10, the same results are obtained for different control times $t_f = 2$ s and $t_f = 1$ s. Therefore, the insertion and use of the SDRE optimal stabilizer in the standard PID control system increases the complexity of the controller, making a hybrid PID-SDRE controller appropriate, because it allows for avoiding oscillations and allows the possibility of operating in airspace with high precision and adjustable control time t_f. The results presented as an effect of performed numerical experiments prove the usefulness and correctness of the proposed technique; moreover, they allow us to verify its behavior.

5. Conclusions

The hybrid PID-SDRE finite-time control technique is formulated and solved for the UAV-quadrotor attitude control problem. The UAV non-linear 6 DoF state-dependent parametrized model is proposed. The P-PID fine-tuned control methodology with an optimal non-linear SDRE feedback speed stabilizer, performing attitude control and stabilization task, is analyzed. The effectiveness of the presented technique is demonstrated in a numerical example in which a UAV response is found using a finite-time SDRE-based technique. The presented results show that the proposed technique can be successively applied to UAV flight control systems when it must operate precisely in airspace. The next step of the analysis and research performed is preparation for application in a real UAV control system.

Author Contributions: Methodology, W.G. and S.S.; software, M.C.; validation, W.G. and S.S.; writing—original draft preparation, W.G. and M.C.; writing—review and editing, W.G., M.C. and S.S.; supervision, A.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was financially supported as a statutory work of the Poznan University of Technology (No. 0214/SBAD/0237).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

BF	Body Frame
EF	Earth Frame
EKF	Extended Kalman Filter
GPS	Global Positioning System
INS	Inertial Navigation System
NED	North-East-Down
PID	Proportional–Integral–Derivative Controller
QTW UAV	Quad Tilt-Wing Unmanned Aerial Vehicle
SDC	State-Dependent Coefficient
SDRE	State-Dependent Riccati Equation
UAV	Unmanned Aerial Vehicle

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