

## Article

# Performance Optimisation of Public Transport Networks Using AHP-Dependent Multi-Aspiration-Level Goal Programming

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**Abstract:** This study proposes an optimisation approach to improve multiple-criteria aspiration-level public transportation performance by combining public transport criteria matrix analytic hierarchy process (PTCM-AHP) models and multi-aspiration-level goal programming. The approach uses the PTCM-AHP to calculate the system weights. Based on the weight values, the approach combines the multi-aspiration goal-level selection process in three different ways. The proposed approach was used to optimise public transportation networks in Bayswater, Cockburn, and Stonnington, Australia, to demonstrate the public transportation network performance optimisation process. By controlling the criteria goal value interval, this new approach combines decision-making plans and strategies to optimise various scenarios. The optimisation outcomes can be applied to provide guidelines for improving the performance of public transportation networks.

**Keywords:** case selection; criterion aspiration-level; optimal solution; optimisation process; public transport network



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## 1. Introduction

Governments of various countries have policies to guide the development of public transportation network performance; however, many scenarios may be far from optimal solutions. Increasing the performance of one criterion may have an impact on the performance of the others. A preference-based scheme was created to address multi-criteria public transportation network performance optimisation issues that aimed to identify the best solution by balancing multiple goals [1]. The goal programming (GP) and criteria weighting methods are two common methods for determining the optimal solution. Through the decision maker's (DM's) preference structure, these methods combine multiple criteria into a single criterion [1,2]. However, the criteria may include more than one aspiration-level. The current GP method is unable to solve the problem of multiple criteria aspiration levels. To solve this problem in practice, it is critical to use a multi-aspiration-level GP method combined with a criterion weighting method, which assigns weights to multiple criteria and combines them.

### 1.1. Literature Review

In recent years, the analytic hierarchy process (AHP) has frequently been used to calculate weights to address multiple-criteria decision-making (MCDM) problems. The ideal solution for multi-objective problems for DMs is to set the optimal objective value for each objective [3]. There are two popular methods for solving multi-objective problems: GP and fuzzy GP approaches [3]. The weights for the criteria can be assigned using the AHP, and they can then be used to optimise the scenario using the GP objective function [2]. Furthermore, the weighting approach has extensively been combined with the GP model to improve public transportation [2,4].

The criteria weighting method and GP have previously been used to solve multi-criterion problems. To solve the housing location selection problem, Ho et al. (2013) proposed AHP and multi-choice goal programming (MCGP) models [5]. Lin et al. (2014) combined AHP and MCGP to assist DMs in using the most appropriate online IT tools [6]. The AHP and GP models were utilised to solve problems regarding rail system project selection in Istanbul [4,7]. Moreover, Cyril et al. (2019) developed AHP and GP models to improve public transportation performance from the perspective of users and operators [2].

Most studies on improving public transportation performance have considered a single aspiration level for each criterion; however, in practice, these criteria frequently involve multiple goals and levels of aspiration. Chang (2007, 2008, 2011) proposed the MCGP model to assist DMs to set multiple levels of aspiration or goals for each criterion [8–10]. The model solves the problem regarding to multiple aspiration levels, but the aspiration level of the criteria may be needed to choose among different aspiration-level cases in public transportation performance optimisation.

In the recent literature, several mathematical models have been used to optimise the public transport performance. A public transportation timetable optimisation model has been developed to minimise transfer waiting time [11]. By changing the bus lines offset, this optimisation model was also applied with different criteria by many researchers [12–14]. Niu and Zhou (2013) further developed a public transport (PT) timetable optimisation model by considering the situation of crowded station passengers boarding [15]. Guihaire and Hao (2010) proposed an optimisation model to optimise the quantity and quality of passenger transfer opportunities [16]. Parbo et al. (2014) utilised a bi-level timetable optimisation model to optimise PT timetable from the user's perspectives [17]. Heyken Soares et al. (2019) applied genetic algorithms (GA) to scale down the network in order to optimise PT routes [18]. The GA model has also been used to propose a zone-based optimisation procedure for PT route optimisation [19]. Faizrahnemoon et al. (2015) described a Markov-chain-based model to optimise the efficiency of a public transportation network [20]. However, these PT optimisation models focused on a single aspect, such as timetable, PT route, or efficiency.

## 1.2. Contribution

Several aspects of improving public transportation performance have been overlooked; consequently, developing a comprehensive tool for optimising public transportation performance at multiple levels of aspiration is critical. Hence, this study aims to develop a suitable optimisation method for the performance of a variety of criteria with multiple aspiration levels for public transportation networks. The novelty of this paper is twofold. First, this paper develops an innovative AHP-dependent multi-aspiration-level GP approach, which is applied to public transportation networks. Second, this paper focuses on comprehensive public transportation performance with multiple levels of aspiration. In this study, we consider the priority structure and specified goals using public transport criteria matrix (PTCM)-AHP models, which were described by Lin et al. (2021) [21].

The remainder of this paper is organised as follows: in Section 2, the AHP method is explained, followed by the mathematical formulations for the GP and MCGP models. Section 3 proposes the multi-aspiration-level GP approach dependent on PTCM-AHP models, followed by three examples in Section 4. Section 5 discusses the results of three case study areas and the application of the model. Section 6 presents conclusions and future research directions.

## 2. Materials and Methods

### 2.1. AHP Model

AHP, as an MCDM method, enables DMs to solve complex issues involving conflicting criteria and numerous objectives [22,23]. The problem is decomposed into various levels in the AHP model. Pairwise comparisons are used to create level objectives, and weights

are assigned to each criterion [22]. The three principles of the AHP model are summarised below [24].

1. Calculate the criteria priority weight for the model structure.
2. Create the comparative judgement matrix for comparing criteria and sub-criteria.
3. Generate the weight for each objective using a pairwise comparison process.

The weighting method AHP defines the priorities of the GP's objective function criteria [2]. The AHP algorithm's main steps are as follows [21]:

(1) Identify the research problem and establish criteria. A model structure of the problem, comprising various criteria and sub-criteria, is constructed in a hierarchical fashion.

(2) Perform pairwise comparisons between criteria in a binary manner. The coefficient  $C_{ig}$  represents the importance from 1 to 9 of criteria  $i$  and  $g$  [25]. The pairwise comparison matrix  $C = (C_{ig})$  is formulated.

(3) Evaluate the weights of the criteria and sub-criteria. The pairwise comparison matrix is normalised. Matrix  $C$  is transformed into matrix  $D = (d_{ig})$ , calculated as

$$d_{ig} = \frac{c_{ig}}{\sum_{i=1}^n C_{ig}} \quad (1)$$

where  $n$  is the number of decision criteria.

The rows of the normalised matrix  $D$  will be obtained to compute the prioritisation vector  $w$ ,

$$w = \frac{\sum_{g=1}^n d_{ig}}{n} \quad (2)$$

(4) Determine the consistency index and ratio for all criteria and sub-criteria. Thus, expert evaluations must be verified. The consistency index (CI) and consistency ratio (CR) are determined for each comparison matrix  $C$  as follows:

$$CI = \frac{T_{max} - n}{n - 1} \quad (3)$$

$$CR = \frac{CI}{RI} \quad (4)$$

(5) Let  $T_{max}$  be the highest eigenvalue of the matrix.  $T_{max}$  is approximated by:

$$Cw = T_{max}w \text{ and } T_{max} \approx T = \frac{\sum_{i=1}^n T_i}{n}. \quad (5)$$

Perform the weight calculation. After the calculation of CR, the comparisons are deemed internally coherent when  $CR \leq 10\%$ . Otherwise, the comparisons are considered inconsistent [26].

## 2.2. GP Approach

GP helps DMs solve MCDM issues and find a set of satisfactory solutions [8,27]. The aim of GP is to minimise the deviation between aspiration levels and achievement of the goal [8,27]. GP can achieve many criteria goals specified by DMs and can be applied to multiple user-defined criteria priorities [1,2,4]. Furthermore, governments often set a goal or target level for criteria, rather than pursuing the optimal solution. GP enables DMs to optimise the criteria performance and aims to propose a solution that best meets their goals [1]. Furthermore, GP can provide solutions for DMs to implement. The main advantage of GP is that it offers DMs optimal processes and control over their preferences [3].

GP is an optimisation process that minimises the objective function by selecting inputs from various input values [2]. Most recent research on GP has been applied to project selection, supplier selection, and budget scenario optimisation [3]. Few studies have been conducted on the performance optimisation of PT. Cyril et al. (2019) proposed an AHP-GP model to optimise PT performance related to operation and user aspects [2].

To model PT network performance, we rate each criterion as coefficients of the objective function [2]. Details of the objective function and constraints are presented below.

We define the following notations:

$i$ : number of goals,  $i = 1, 2, \dots, n$ ,  
 $s$ : number of criteria,  $s = 1, 2, \dots, e$ ,  
 $R_i$ :  $i$ th priority,  
 $x_s$ :  $s$ th criteria,  
 $b_{is}$ : coefficient of the  $i$ th goal and  $s$ th criteria,  
 $d_i$ : goal value for goal  $i$ ,  
 $p_i$ : positive deviation,  
 $q_i$ : negative deviation.

The optimisation problem of the PT network performance can be formulated as follows:

$$\min \sum_{i=1}^n R_i(p_i + q_i) \quad (6)$$

subject to

$$\sum_{s=1}^e b_{is}x_s - p_i + q_i = d_i, \quad (7)$$

$$p_i, q_i, x_s \geq 0, \quad (8)$$

where  $R_i$  is the rating and value of the  $i$ th decision variable.

### 2.3. MCGP Approach

The MCGP approach enables DMs to specify a series of values instead of a single scalar target level or select multiple aspiration levels for each criterion [3,8,28]. This approach is more favourable than GP, because the DMs can choose multiple aspiration levels or goals for each criterion [3,28].

The basic concept of MCGP is to set multiple aspiration levels for criteria, because the criteria may have several desired target values for each goal [29]. In MCGP modelling, these aspiration levels provide multiple choices to find a satisfactory solution set during the optimisation process [29].

The GP optimisation approach relies on choosing a goal at a single aspiration level. However, a goal may involve multiple choice aspiration levels [8,28]. The original GP method cannot solve multiple choice aspiration-level issues; thus, MCGP was proposed to solve this problem. The details of the MCGP model consists of the objective function (6) and the constraints:

$$\sum_{s=1}^e b_{is}x_s - p_i + q_i = \sum_{j=1}^m g_{ij}S_{ij}(B), \quad (9)$$

$$S_{ij}(B) \in R_i(x), p_i, q_i, x_s \geq 0, \quad (10)$$

where  $g_{ij}$  is the  $j$ th aspiration level of the  $i$ th objective,  $g_{i,j-1} \leq g_{ij} \leq g_{i,j+1}$ , and  $S_{ij}(B)$  represents a binary serial function attached to multiple aspiration levels for each objective and is based on the restriction  $R_i(x)$  [3,8].  $S_{ij}(B)$  ensures that each objective selects one of the multiple goals [3,8].

### 3. AHP-Dependent Multi-Aspiration-Level GP

AHP is often combined with GP models to solve performance optimisation issues. In a real situation, the model has multiple choices for each criterion. However, the criteria aspiration level may have different aspiration-level cases. The DM requires the model to be able to select an aspiration level among different cases. The current MCGP model in PT performance optimisation lacks consideration in the selection process of different aspiration level cases. Hence, this research proposes an AHP-dependent multi-aspiration-level GP model to address this issue. The input of the model was derived from current data collected from relevant government websites. The details of the model are as follows.

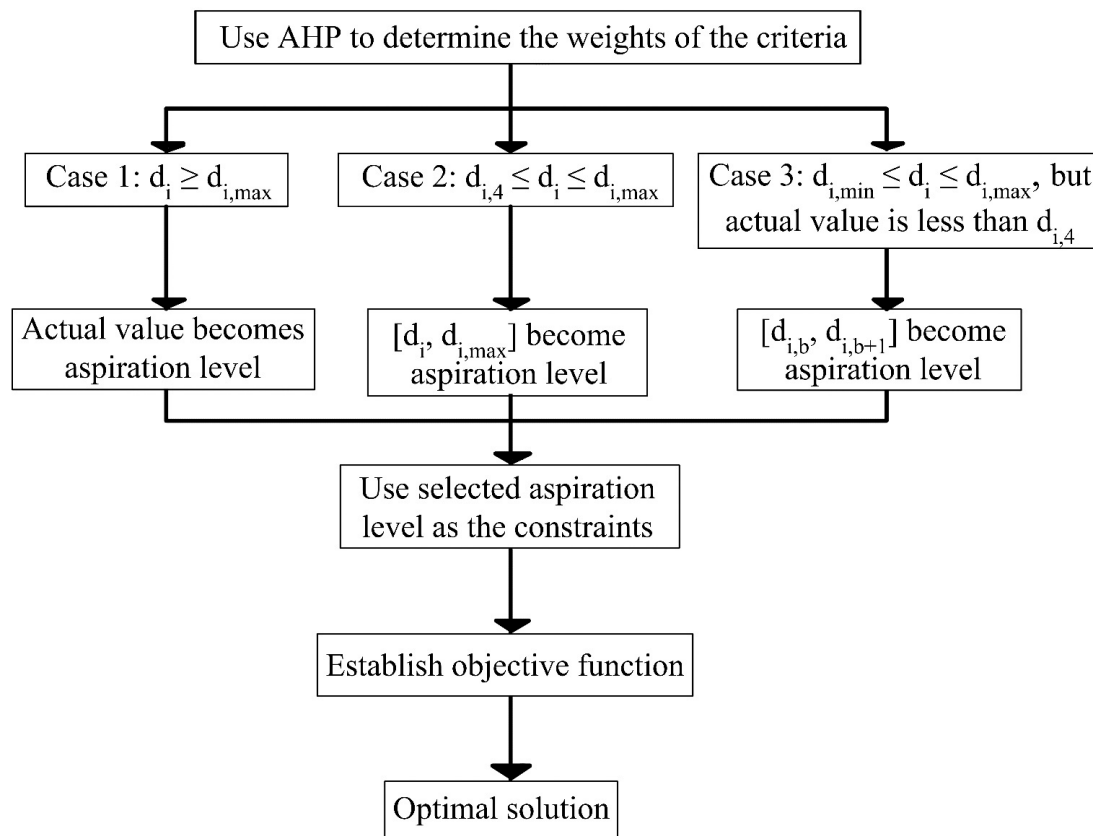
The proposed model consists of three steps:

(1) As mentioned in Section 2.1, the weights for the model are obtained from the PTCM-AHP model [21].

(2) The formulated constraints consider the upper and lower bounds of the criteria by assigning positive and negative deviations in the form of inequalities. The model considers three cases for the aspiration level selection criterion.

(3) The model uses the selected aspiration levels as constraints to establish the objective function and calculate the optimal solution.

The approach minimises the sum of the deviations in which the optimal values are close to the goal value. A flowchart of the model is shown in Figure 1.



**Figure 1.** Flowchart of AHP-dependent multi-aspiration-level GP model.

### 3.1. AHP

The proposed model applies the PTCM-AHP model to determine the weights of the criteria. The model identifies the weights by studying the local council policies of the case study areas [21]. Based on the AHP, the PTCM-AHP model considers the basic PT infrastructure, PT services, economic benefits, and sustainable development levels. These criteria are further divided into 15 factors. Details of the 15 sub-criteria can be found in Lin et al. (2021) [21]. The 15 decision variables are the PT network ratio ( $X_1$ ), PT coverage ratio ( $X_2$ ), green PT vehicle rate ( $X_3$ ), PT energy intensity ( $X_4$ ), PT priority lane setting ratio ( $X_5$ ), PT land area per capita ( $X_6$ ), PT on-time rate ( $X_7$ ), passenger freight rate ( $X_8$ ), coverage rate ( $X_9$ ), peak hours intersection blocking rate ( $X_{10}$ ), harbour-type bus stop setting ratio ( $X_{11}$ ), bus ownership rate ( $X_{12}$ ), PT utilisation rate ( $X_{13}$ ), PT driving accident rate ( $X_{14}$ ), and intact car rate ( $X_{15}$ ). Based on the established AHP model, the weights for the criteria are used in the multi-aspiration-level GP objective function. The weights for each sub-criterion are listed in Table A1.

### 3.2. Criteria Aspiration-level Case Selection

Criterion case selection was based on the criterion of aspiration levels. Table A2 presents the level grades for all the sub-criteria according to Lin et al. (2021) [20]. This

study utilises Levels 1–5 to represent Levels E–A. The aspiration level selection for the cases is listed in Figure 2.

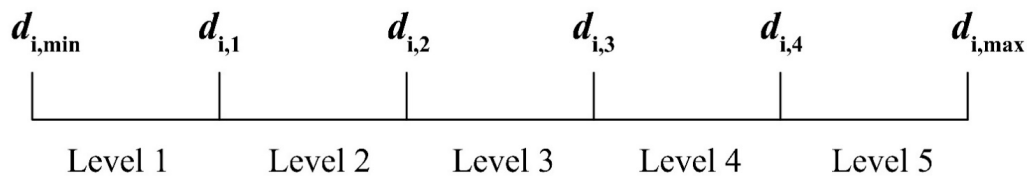


Figure 2. Sub-criteria grade level.

Case 1:

If the actual value of the  $i$ th criterion is higher than  $d_{i,max}$ , then the actual value becomes the  $i$ th criterion aspiration goal value.

Case 2:

If the  $i$ th criterion's actual value is higher than  $d_{i,4}$  but less than  $d_{i,max}$ , the  $i$ th criterion's aspiration goal value should be higher than the actual value but less than  $d_{i,max}$ .

Case 3:

If the  $i$ th criterion's actual value belongs to levels 1, 2, 3, or 4, the aspiration level of the  $i$ th criterion becomes the  $(i + 1)$ th goal level. After the case selection process, the formulas for the three cases are as given in Section 3.3.

### 3.3. Establish Multi-Aspiration-Level GP

This model focuses on the criteria index value interval selection that enables the government to control the optimisation process.

Let  $d_i$  be the  $i$ th criterion grade level,  $i = 1, 2, \dots, 5$ . The new multi-aspiration-level GP is described below.

Case 1: When the goal value is greater than  $d_{i,max}$ ,

$$d_i \geq d_{i,max}, \quad (11)$$

where the aspiration level of  $d_i$  is the actual value of the criterion.

Case 2: When the goal value is less than  $d_{i,max}$  but higher than  $d_{i,4}$ ,

$$d_{i,4} \leq d_i \leq d_{i,max}, \quad (12)$$

where the constraints of  $d_i$  are selected between the actual value of the criterion and  $d_{i,max}$ .

Case 3: When the goal value is less than  $d_{i,max}$  but the actual value is less than  $d_{i,4}$ ,

$$d_{i,min} \leq d_i \leq d_{i,max}, \quad (13)$$

where the constraints of  $d_i$  are selected from the next level of the criterion goal value. For example, if the actual value of  $d_i$  achieves goal 1, then goal 2 should be the aspiration level for  $d_i$ .

Further aspiration levels can be added by DMs to define the relationships between each goal for multiple criteria performance optimisation problems.

## 4. Illustrative Examples

To explain the process and outcome of the proposed model, this study used the PTCM-AHP model-based multi-aspiration-level GP model on three case studies. The case studies were used to explain how the multi-aspiration-level GP model is able to optimise PT network performance in three cities in Australia, considering basic PT infrastructure, PT services, economic benefits, and sustainable development levels. The goal value of the case study areas is to choose the selection process of the aspiration level for optimisation based on the actual value.

The formulated constraints were different for each of the three case study areas. The constraints of the objective function were based on the criteria-level grade selection (for

details, see Table A2). Hence, this study assumed the conditions for three case studies in which the DMs optimise the performance based on the criteria aspiration level. The details of the actual values and goals are listed in Tables 1–3. The formulations are as follows:

**Table 1.** Bayswater’s actual and goal values for decision variables.

Variable	Criteria	Actual Value	Goal Value
X <sub>1</sub>	PT network ratio	17.64	50–55
X <sub>2</sub>	PT coverage ratio	46.82	50–55
X <sub>3</sub>	Green PT vehicle rate	100	100
X <sub>4</sub>	PT energy intensity	25.45	0–25.45
X <sub>5</sub>	PT priority lane setting ratio	0	10–15
X <sub>6</sub>	PT land area per capita	20.47	20.47
X <sub>7</sub>	PT on-time rate	91.03	95–100
X <sub>8</sub>	Passenger freight rate	1.75	1.75
X <sub>9</sub>	Coverage rate	98.8	100
X <sub>10</sub>	Peak hours intersection blocking rate	21	8–11
X <sub>11</sub>	Harbour-type bus stop setting ratio	19.04	25–35
X <sub>12</sub>	Bus ownership rate	7	18–19
X <sub>13</sub>	PT utilisation rate	0.8	0.8–2
X <sub>14</sub>	PT driving accident rate	2.38	1.5–2
X <sub>15</sub>	Intact car rate	100	100

**Table 2.** Cockburn’s actual and goal values for decision variables.

Variable	Criteria	Actual Value	Goal Value
X <sub>1</sub>	PT network ratio	19.21	50–55
X <sub>2</sub>	PT coverage ratio	50.42	55–100
X <sub>3</sub>	Green PT vehicle rate	100	100
X <sub>4</sub>	PT energy intensity	25.45	0–25.45
X <sub>5</sub>	PT priority lane setting ratio	0.31	10–15
X <sub>6</sub>	PT land area per capita	26.23	26.23
X <sub>7</sub>	PT on-time rate	91.03	95–100
X <sub>8</sub>	Passenger freight rate	1.75	1.75
X <sub>9</sub>	Coverage rate	98.8	100
X <sub>10</sub>	Peak hours intersection blocking rate	8.1	5–8
X <sub>11</sub>	Harbour-type bus stop setting ratio	9.2	15–25
X <sub>12</sub>	Bus ownership rate	7	18–19
X <sub>13</sub>	PT utilisation rate	0.8	0.8–2
X <sub>14</sub>	PT driving accident rate	2.38	1.5–2
X <sub>15</sub>	Intact car rate	100	100



**Table 3.** Stonnington’s actual and goal values for decision variables.

Variable	Criteria	Actual Value	Goal Value
$X_1$	PT network ratio	60.78	60.78–70
$X_2$	PT coverage ratio	83.72	83.72
$X_3$	Green PT vehicle rate	100	100
$X_4$	PT energy intensity	83.59	30–80
$X_5$	PT priority lane setting ratio	25.38	25.38–30
$X_6$	PT land area per capita	9.28	11–14
$X_7$	PT on-time rate	84.68	85–95
$X_8$	Passenger freight rate	2.33	2.33
$X_9$	Coverage rate	101.5	150–200
$X_{10}$	Peak hours intersection blocking rate	1.5	0–1.5
$X_{11}$	Harbour-type bus stop setting ratio	26.71	35–100
$X_{12}$	Bus ownership rate	7.36	18–19
$X_{13}$	PT utilisation rate	0.78	0.78–2
$X_{14}$	PT driving accident rate	4.54	2.5–3
$X_{15}$	Intact car rate	100	100

Objective function for Bayswater:

$$\min 14.3p_1 + 14.3q_1 + 14.3p_2 + 14.3q_2 + 9p_3 + 9q_3 + 7.9p_4 + 7.9q_4 + 6.5p_5 + 6.5q_5 + 5.5p_6 + 5.5q_6 + 5.05p_7 + 4.6p_8 + 4.6q_8 + 4.5p_9 + 4.5q_9 + 4.3p_{10} + 4.3q_{10} + 2.25p_{11} + 2.25q_{11}$$

Constraints for Bayswater:

Constraint 1: Improve PT network ratio

$$X_1 + p_1 = 55$$

$$X_1 - q_1 = 50$$

Constraint 2: Increase PT coverage ratio

$$X_2 + p_2 = 55$$

$$X_2 - q_2 = 50$$

Constraint 3: Minimise PT energy intensity and increase green PT vehicle rate

$$X_3 + X_4 + p_3 = 125.45$$

$$X_3 + X_4 - q_3 = 100$$

Constraint 4: Maximise PT priority lane setting ratio

$$X_5 + p_5 = 15$$

$$X_5 - q_5 = 10$$

Constraint 5: Improve PT on-time rate

$$X_7 + p_5 = 100$$

$$X_7 - q_5 = 95$$

Constraint 6: Improve PT utilisation rate and increase PT land area per capita

$$X_6 + X_{13} + p_6 = 22.47$$

$$X_6 + X_{13} - q_6 = 21.27$$



Constraint 7: Optimise financial resources by decreasing passenger freight rate and increasing coverage rate

$$X_8 + X_9 + p_7 = 101.75$$

Constraint 8: Reduce peak hours intersection blocking rate

$$X_{10} + p_8 = 11$$

$$X_{10} - q_8 = 8$$

Constraint 9: Increase harbour-type bus stop setting ratio

$$X_{11} + p_9 = 35$$

$$X_{11} - q_9 = 25$$

Constraint 10: Maximise bus ownership rate

$$X_{12} + p_{10} = 19$$

$$X_{12} - q_{10} = 18$$

Constraint 11: Maximise intact car rate and reducing PT driving accident rate

$$X_{14} + X_{15} + p_{11} = 102$$

$$X_{14} + X_{15} - q_{11} = 101.5$$

The objective function for Cockburn is the same as that of Bayswater.

Constraints for Cockburn:

Constraints 1, 3, 4, 5, 7, 10, and 11 are the same as those for Bayswater.

Constraint 2: Increase PT coverage ratio

$$X_2 + p_2 = 100$$

$$X_2 - q_2 = 55$$

Constraint 6: Improve PT utilisation rate and increase PT land area per capita

$$X_6 + X_{13} + p_6 = 28.23$$

$$X_6 + X_{13} - q_6 = 27.03$$

Constraint 8: Reduce peak hours intersection blocking rate

$$X_{10} + p_8 = 8$$

$$X_{10} - q_8 = 5$$

Constraint 9: Increase harbour-type bus stop setting ratio

$$X_{11} + p_9 = 25$$

$$X_{11} - q_9 = 15$$

Objective function for Stonnington:

$$\min 14.3p_1 + 14.3q_1 + 9p_2 + 9q_2 + 7.9p_3 + 7.9q_3 + 7.8p_4 + 7.8q_4 + 6.5p_5 + 6.5q_5 + 5.05p_6 + 5.05q_6 + 4.6p_7 + 4.6q_7 + 4.5p_8 + 4.5q_8 + 4.3p_9 + 4.3q_9 + 3.2p_{10} + 3.2q_{10} + 2.25p_{11} + 2.25q_{11}$$

Constraints for Stonnington:

Constraint 1: Maximise accessibility by improving PT network and coverage ratios

$$X_1 + X_2 + p_1 = 153.72$$

$$X_1 + X_2 - q_1 = 144.5$$

Constraint 2: Minimise PT energy intensity and increase green PT vehicle rate

$$X_3 + X_4 + p_2 = 180$$

$$X_3 + X_4 - q_2 = 130$$

Constraint 3: Maximise PT priority lane setting ratio

$$X_5 + p_3 = 30$$

$$X_5 - q_3 = 25.38$$

Constraint 4: Increasing PT land area per capita

$$X_6 + p_4 = 14$$

$$X_6 - q_4 = 11$$

Constraint 5: Improve PT on-time rate

$$X_7 + p_5 = 95$$

$$X_7 - q_5 = 85$$

Constraint 6: Optimise financial resources by decreasing passenger freight rate and increasing coverage rate

$$X_8 + X_9 + p_6 = 202.33$$

$$X_8 + X_9 - q_6 = 152.33$$

Constraint 7: Reduce peak hours intersection blocking rate

$$X_{10} + p_7 = 1.5$$

$$X_{10} - q_7 = 0$$

Constraint 8: Increase harbour-type bus stop setting ratio

$$X_{11} + p_8 = 100$$

$$X_{11} - q_8 = 35$$

Constraint 9: Maximise bus ownership rate

$$X_{12} + p_9 = 19$$

$$X_{12} - q_9 = 18$$

Constraint 10: Improve PT utilisation rate

$$X_{13} + p_{10} = 2$$

$$X_{13} - q_{10} = 0.78$$

Constraint 11: Maximise intact car rate and reducing PT driving accident rate

$$X_{14} + X_{15} + p_{11} = 103$$

$$X_{14} + X_{15} - q_{11} = 102.5$$

The optimisation results were obtained using MATLAB to obtain the optimal solution for the case study areas which are shown in Tables 4–6.

**Table 4.** Optimal solution for Bayswater.

Decision Variable	Criteria	Actual Value	Optimal Value	Increase/Decrease Percentage
$X_1$	PT network ratio	17.64	50	183.34
$X_2$	PT coverage ratio	46.82	50	6.79
$X_3$	Green PT vehicle rate	100	100	0
$X_4$	PT energy intensity	25.45	0	-
$X_5$	PT priority lane setting ratio	0	10	-
$X_6$	PT land area per capita	20.47	20.47	0
$X_7$	PT on-time rate	91.03	95	4.36
$X_8$	Passenger freight rate	1.75	1.75	0

**Table 4.** *Cont.*

Decision Variable	Criteria	Actual Value	Optimal Value	Increase/Decrease Percentage
X <sub>9</sub>	Coverage rate	98.8	100	1.21
X <sub>10</sub>	Peak hours intersection blocking rate	21	8	−61.9
X <sub>11</sub>	Harbour-type bus stop setting ratio	19.04	25	31.3
X <sub>12</sub>	Bus ownership rate	7	18	157.14
X <sub>13</sub>	PT utilisation rate	0.8	0.8	0
X <sub>14</sub>	PT driving accident rate	2.38	1.5	−36.97
X <sub>15</sub>	Intact car rate	100	100	0

**Table 5.** Optimal solution for Cockburn.

Decision Variable	Criteria	Actual Value	Optimal Value	Increase/Decrease Percentage
X <sub>1</sub>	PT network ratio	19.21	50	160.28
X <sub>2</sub>	PT coverage ratio	50.42	55	9.08
X <sub>3</sub>	Green PT vehicle rate	100	100	0
X <sub>4</sub>	PT energy intensity	25.45	0	-
X <sub>5</sub>	PT priority lane setting ratio	0.31	10	3125.8
X <sub>6</sub>	PT land area per capita	26.23	26.23	0
X <sub>7</sub>	PT on-time rate	91.03	95	4.36
X <sub>8</sub>	Passenger freight rate	1.75	1.75	0
X <sub>9</sub>	Coverage rate	98.8	100	1.21
X <sub>10</sub>	Peak hours intersection blocking rate	8.1	8	−1.23
X <sub>11</sub>	Harbour-type bus stop setting ratio	9.2	15	63.04
X <sub>12</sub>	Bus ownership rate	7	18	157.14
X <sub>13</sub>	PT utilisation rate	0.8	0.8	0
X <sub>14</sub>	PT driving accident rate	2.38	1.5	−36.97
X <sub>15</sub>	Intact car rate	100	100	0

**Table 6.** Optimal solution for Stonnington.

Decision Variable	Criteria	Actual Value	Optimal Value	Increase/Decrease Percentage
X <sub>1</sub>	PT network ratio	60.78	60.78	0
X <sub>2</sub>	PT coverage ratio	83.72	83.72	0
X <sub>3</sub>	Green PT vehicle rate	100	100	0
X <sub>4</sub>	PT energy intensity	83.59	30	−64.11
X <sub>5</sub>	PT priority lane setting ratio	25.38	25.38	0
X <sub>6</sub>	PT land area per capita	9.28	11	18.53
X <sub>7</sub>	PT on-time rate	84.68	85	0.37
X <sub>8</sub>	Passenger freight rate	2.33	2.33	0
X <sub>9</sub>	Coverage rate	101.5	150	47.78
X <sub>10</sub>	Peak hours intersection blocking rate	1.5	0	-

Table 6. Cont.

Decision Variable	Criteria	Actual Value	Optimal Value	Increase/Decrease Percentage
X <sub>11</sub>	Harbour-type bus stop setting ratio	26.71	35	31.03
X <sub>12</sub>	Bus ownership rate	7.36	18	144.56
X <sub>13</sub>	PT utilisation rate	0.78	0.78	0
X <sub>14</sub>	PT driving accident rate	4.54	2.5	−44.93
X <sub>15</sub>	Intact car rate	100	100	0

## 5. Discussion

The optimal solutions for the three cities are presented in Tables 4–6. These scenarios indicate that the criteria performances significantly improved, such as the PT network ratio, PT coverage ratio, PT energy intensity, PT priority lane setting ratio, PT on-time rate having a higher priority than coverage rate, peak hours intersection blocking rate, harbour type bus stop setting ratio, bus ownership rate, and PT driving accident rate.

The optimal solutions for Bayswater are listed in Table 4. At the basic PT infrastructure level, an increase of 183.34, 6.79, and 31.3% in the PT network, PT coverage, and harbour-type bus stop setting ratios, respectively, would improve the PT network performance for Bayswater. Reducing the peak hours intersection blocking rate by 61.9%, decreasing the PT driving accident rate by 36.97%, and improving the PT on-time rate by 4.36% would improve the PT service level in Bayswater. Improving the coverage rate by 1.21% and bus ownership rate by 157.14% would optimise Bayswater's economic benefit level.

Table 5 shows that increasing the PT network, PT coverage, and harbour-type bus stop setting ratios by 160.28, 9.08, and 63.04%, respectively, would improve Cockburn's basic PT infrastructure level. In terms of Cockburn's PT service level, increasing the PT on-time rate to 95%, decreasing the peak hours intersection blocking rate by 1.23%, and reducing the PT driving accident rate to 1.5 times per million kilometres would help to achieve the optimal PT service level scenario. Increasing the coverage rate to 100% and bus ownership rate to eighteen cars per ten thousand people would improve Cockburn's economic benefit level. Both Bayswater and Cockburn's optimal solution suggests decreasing the PT energy intensity to 0 g standard coal per person-kilometre and improving the PT priority lane setting ratio to 10%.

The optimal solutions for Stonnington are listed in Table 6. In terms of the PT infrastructure level, increasing the harbour-type bus stop setting ratio by 31.03% would improve PT network performance. An increase in the PT on-time rate of 0.37% and a reduction of 44.93% in the PT driving accident rate would improve Stonnington's PT service level. The optimal solution was achieved with an intersection blocking rate of 0% during peak hours. Increasing the coverage rate by 47.78% and bus ownership rate by 144.56% would improve the economic benefit level. A reduction of 64.11% in PT energy intensity and an increase of 18.53% in PT land area per capita would improve the optimal value for the sustainable development level.

PT performance optimisation can offer an optimal solution for the government to implement. The optimal model shows that the PTCM-AHP model-based multi-aspiration-level GP approach enables DMs to propose an optimal solution for PT network performance incorporating the criteria of basic PT infrastructure, PT service, economic benefit, and sustainable development levels. DMs can consider multi-aspiration levels or interval goals while considering relative importance criteria. Furthermore, the governments may propose the new policy and strategy. DMs can adjust and change the criteria importance and the selection process of the aspiration-level to optimise PT network performance. In addition, the model can also add more constraints for the optimisation process which are based on DMs' requirements.

## 6. Conclusions

The proposed model was formulated as a multi-aspiration-level GP model for PT network performance optimisation. The proposed model is a further development of the GP and MCGP models. The criteria for optimising a PT network's performance often contains multiple aspiration levels. Hence, this study considered optimising the PT network performance with criteria with multiple aspiration levels. The PTCM-AHP model-based multi-aspiration-level GP approach involves three steps. First, the DM's criteria preferences are implemented to express each criterion weight. Subsequently, the DM grades the criteria performance based on the level grade for all sub-criteria and finds each criterion aspiration level for performance optimisation. Finally, the multi-aspiration-level GP method is used to optimise the city's PT network performance and provide an optimal solution.

Compared to the GP and MCGP approaches, this study combined the multi-aspiration goal-level selection process in three different situations to create a PTCM-AHP model-based multi-aspiration-level GP approach. The three examples illustrated the PT network performance optimisation process. This model combines the DM's plans and strategies for optimising the scenario by controlling the criteria goal value interval. The proposed model can be used to provide guidelines for optimising PT network performance scenarios. GP model can also consider and add new requirements and constraints to control the PT network performance optimisation.

The future research work is planned as follows: (1) We will consider more suitable criteria and sub-criteria for performance optimisation for the real requirements. (2) During the performance optimisation process, there is uncertainty regarding the performance optimisation in a real situation. This uncertainty will be considered for the optimisation problem. The uncertainty management model can combine with the current model which mitigate the influence of uncertainty.

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## Appendix A

**Table A1.** The distribution matrix of city score [21].

Criteria	Local Weight (%)	Global Weight (%)
Basic PT infrastructure level: 41%		
PT network ratio	35	14.3
PT coverage ratio	35	14.3
Harbour-type bus stop setting ratio	11	4.5
PT priority lane setting ratio	19	7.9
PT service level: 19%		
PT on-time rate	34	6.5
Peak hours intersection blocking rate	24	4.6

**Table A1.** *Cont.*

Criteria	Local Weight (%)	Global Weight (%)
Passenger freight rate	28	5.3
PT driving accident rate	14	2.6
Economic benefit level: 11%		
Coverage rate	44	4.8
Bus ownership rate	39	4.3
Intact car rate	17	1.9
Sustainable development level: 29%		
PT land area per capita	27	7.8
PT utilisation rate	11	3.2
Green PT vehicle rate	31	9
PT energy intensity	31	9

## Appendix B

**Table A2.** Level grade for all sub-criteria [30–34].

	Level Grade	Level A	Level B	Level C	Level D	Level E
PT network ratio (unit: %)	Index value interval	[60, 70]	[55, 60]	[50, 55]	[0, 50]	—
	Score interval	[90, 100]	[75, 90]	[60, 75]	[0, 60]	—
PT coverage ratio (unit: %)	Index value interval	≥55	[50, 55]	[45, 50]	[35, 45]	<35
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
Harbour-type bus stop setting ratio (unit: %)	Index value interval	[35, 100]	[25, 35]	[15, 25]	[0, 15]	—
	Score interval	[90, 100]	[75, 90]	[60, 75]	[0, 60]	—
PT priority lane setting ratio (unit: %)	Index value interval	≥ 25	[20, 25]	[15, 20]	[10, 15]	[0, 10]
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
PT on-time rate (unit: %)	Index value interval	[95, 100]	[85, 95]	[70, 85]	[0, 70]	—
	Score interval	[90, 100]	[75, 90]	[60, 75]	[0, 60]	—
Peak hours intersection blocking rate (unit: %)	Index value interval	[0, 2]	(2, 5]	(5, 8]	(8, 11]	>11
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
Passenger freight rate (unit: %)	Index value interval	<3.5	[3.5, 4.5]	[4.5, 5.5]	[5.5, 6.5]	≥6.5
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
PT driving accident rate (unit: times /million kilometres)	Index value interval	[0, 1.5]	[1.5, 2)	[2, 2.5)	[2.5, 3)	>3
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
Coverage rate (unit: %)	Index value interval	>150	(100, 150]	= 100	[50, 100)	<50
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
Bus ownership rate (unit: car/10,000)	Index value interval	[20, 25]	[19, 20)	[18, 19)	[0, 18)	—
	Score interval	[90, 100]	[75, 90]	[60, 75]	[0, 60]	—
Intact car rate (unit: %)	Index value interval	≥ 92	[88, 92)	[84, 88)	[80, 84)	<80
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
PT land area per capita (unit: m <sup>2</sup> /person)	Index value interval	≥11	[8, 11)	[6, 8)	[4, 6)	<4
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]

Table A2. Cont.

	Level Grade	Level A	Level B	Level C	Level D	Level E
PT utilisation rate (unit: %)	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
	Score interval	[90, 100]	[75, 90]	[60, 75]	[0, 60]	—
	Index value interval	[0.17, 2)	[0.14, 0.17)	[0.11, 0.14)	[0.08, 0.11)	<0.08
Green PT vehicle rate (unit: %)	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
	Index value interval	$\geq 95$	[95, 92)	[88, 92)	[85, 88)	<85
PT energy intensity (unit: g standard coal/person-km)	Score interval	[90, 100]	[80, 90]	[70, 80]	[60, 70]	[0, 60]
	Index value interval	[0, 30)	[30, 80)	[80, 130)	[130, 200)	—
	Score interval	[90, 100]	[75, 90)	[60, 75)	[0, 60)	—

## References

- Chen, A.; Xu, X. Goal programming approach to solving network design problem with multiple objectives and demand uncertainty. *Expert. Syst. Appl.* **2012**, *39*, 4160–4170. [\[CrossRef\]](#)
- Cyril, A.; Mulangi, R.H.; George, V. Performance optimization of public transport using integrated AHP–GP methodology. *Urban Rail Transit.* **2019**, *5*, 133–144. [\[CrossRef\]](#)
- Jadidi, O.; Cavalieri, S.; Zolfaghari, S. An improved multi-choice goal programming approach for supplier selection problems. *Appl. Math. Model.* **2015**, *39*, 4213–4222. [\[CrossRef\]](#)
- Hamurcu, M.; Eren, T. Transportation planning with analytic hierarchy process and goal programming. *Int. Adv. Res. Eng. J.* **2018**, *2*, 92–97. Available online: <https://dergipark.org.tr/en/download/article-file/522658> (accessed on 1 May 2021).
- Ho, H.P.; Chang, C.T.; Ku, C.Y. On the location selection problem using analytic hierarchy process and multi-choice goal programming. *Int. J. Syst. Sci.* **2013**, *44*, 94–108. [\[CrossRef\]](#)
- Lin, T.C.; Ho, H.P.; Chang, C.T. Evaluation model for applying an e-learning system in a course: An analytic hierarchy process—Multi-choice goal programming approach. *J. Educ. Comput. Res.* **2014**, *50*, 135–157. [\[CrossRef\]](#)
- Hamurcu, M.; Alağaç, H.M.; Eren, T. Selection of rail system projects with analytic hierarchy process and goal programming. *Sigma J. Eng. Nat. Sci.* **2017**, *8*, 291–302. Available online: <https://eds.yildiz.edu.tr/ArticleContent/Journal/sigma/Volumes/2017/Issues/6/YTUJENS-2017-8-4.3350.pdf> (accessed on 1 May 2021).
- Chang, C.T. Multi-choice goal programming. *Omega* **2007**, *35*, 389–396. [\[CrossRef\]](#)
- Chang, C.T. Revised multi-choice goal programming. *Appl. Math. Model.* **2008**, *32*, 2587–2595. [\[CrossRef\]](#)
- Chang, C.T. Multi-choice goal programming with utility functions. *Eur. J. Oper. Res.* **2011**, *215*, 439–445. [\[CrossRef\]](#)
- Wong, R.C.; Yuen, T.W.; Fung, K.W.; Leung, J.M. Optimizing timetable synchronization for rail mass transit. *Transport. Sci.* **2008**, *42*, 57–69. [\[CrossRef\]](#)
- Cevallos, F.; Zhao, F. Minimizing transfer times in public transit network with genetic algorithm. *Transp. Res. Record.* **2006**, *1971*, 74–79. [\[CrossRef\]](#)
- Hadas, Y.; Ceder, A.A. Optimal coordination of public-transit vehicles using operational tactics examined by simulation. *Transport. Res. C-Emer.* **2010**, *18*, 879–895. [\[CrossRef\]](#)
- Petersen, H.L.; Larsen, A.; Madsen, O.B.; Petersen, B.; Ropke, S. The simultaneous vehicle scheduling and passenger service problem. *Transport. Sci.* **2012**, *47*, 603–616. [\[CrossRef\]](#)
- Niu, H.; Zhou, X. Optimizing urban rail timetable under time-dependent demand and oversaturated conditions. *Transport. Sci.* **2013**, *36*, 212–230. [\[CrossRef\]](#)
- Guihaire, V.; Hao, J.K. Improving timetable quality in scheduled transit networks. In Proceedings of the International Conference on Industrial, Engineering and Other Applications of Applied Intelligent Systems 2010, Cordoba, Spain, 1–4 June 2010.
- Parbo, J.; Nielsen, O.A.; Prato, C.G. User perspectives in public transport timetable optimisation. *Transport. Res. C-Emer.* **2014**, *48*, 269–284. [\[CrossRef\]](#)
- Heyken Soares, P.; Mumford, C.L.; Amponsah, K.; Mao, Y. An adaptive scaled network for public transport route optimisation. *Public Transport.* **2019**, *11*, 379–412. [\[CrossRef\]](#)
- Heyken Soares, P. Zone-based public transport route optimisation in an urban network. *Public Transport.* **2021**, *13*, 197–231. [\[CrossRef\]](#)
- Faizrahneem, M.; Schlote, A.; Maggi, L.; Crisostomi, E.; Shorten, R. A big-data model for multi-modal public transportation with application to macroscopic control and optimisation. *Int. J. Control* **2015**, *88*, 2354–2368. [\[CrossRef\]](#)



21. Lin, G.; Wang, S.; Lin, C.; Bu, L.; Xu, H. Evaluating performance of public transport networks by using public transport criteria matrix analytic hierarchy process models—Case study of Stonnington, Bayswater, and Cockburn public transport network. *Sustainability* **2021**, *13*, 6949. [[CrossRef](#)]
22. Jain, S.; Aggarwal, P.; Kumar, P.; Singhal, S.; Sharma, P. Identifying public preferences using multi-criteria decision making for assessing the shift of urban commuters from private to public transport: A case study of Delhi. *Transp. Res. F* **2014**, *24*, 60–70. [[CrossRef](#)]
23. Boujelbene, Y.; Derbel, A. The performance analysis of public transport operators in Tunisia using AHP method. *Procedia Comput. Sci.* **2015**, *73*, 498–508. [[CrossRef](#)]
24. Nassereddine, M.; Eskandari, H. An integrated MCDM approach to evaluate public transportation systems in Tehran. *Transp. Res. A* **2017**, *106*, 427–439. [[CrossRef](#)]
25. Saaty, T.L. A scaling method for priorities in hierarchical structures. *J. Math. Psychol.* **1977**, *15*, 234–281. [[CrossRef](#)]
26. Sadeghi, M.; Ameli, A. An AHP decision making model for optimal allocation of energy subsidy among socio-economic subsectors in Iran. *Energy Policy* **2012**, *45*, 24–32. [[CrossRef](#)]
27. Larbani, M.; Aouni, B. A new approach for generating efficient solutions within the goal programming model. *J. Oper. Res. Soc.* **2011**, *62*, 175–182. [[CrossRef](#)]
28. Ho, H.P. The supplier selection problem of a manufacturing company using the weighted multi-choice goal programming and MINMAX multi-choice goal programming. *Appl. Math. Model.* **2019**, *75*, 819–836. [[CrossRef](#)]
29. Hocine, A.; Zhuang, Z.Y.; Kouaissah, N.; Li, D.C. Weighted-additive fuzzy multi-choice goal programming (WA-FMCGP) for supporting renewable energy site selection decisions. *Eur. J. Oper. Res.* **2020**, *285*, 642–654. [[CrossRef](#)]
30. GB50220-1995; Code for Transport Planning on Urban Road. Available online: <https://wenku.baidu.com/view/fa103f6b0b4c2e3f57276369.html> (accessed on 1 May 2021).
31. Evaluation Index System of Public Transportation City Assessment. Available online: <https://wenku.baidu.com/view/1015f28a360cba1aa811dac1.html> (accessed on 1 May 2021).
32. GBT 22484-2016; Passenger Transport Services Specifications for Urban Bus/Trolleybus. Available online: <https://pan.baidu.com/s/1eSeT2N4> (accessed on 1 May 2021).
33. GB/T22484-2008; Passenger Transport Services for Bus/Trolleybus. Available online: <https://wenku.baidu.com/view/9c319528e2bd960590c677e6.html> (accessed on 1 May 2021).
34. Urban Road Traffic Management Evaluation Index System 2012 Edition. Available online: <https://wenku.baidu.com/view/20e4368f84868762caaed5a1.html> (accessed on 1 May 2021).