Economical Optimum Gas Allocation Model Considering Different Types of Gas-Lift Performance Curves

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Abstract: The traditional optimum modes of gas-lift production are usually established by taking the injected gas rate as a decision variable and maximum oil production as the objective function. After solving the model, the injected gas rates of single wells are obtained, and then the oil productions of single wells, the total oil productions of well groups and economic profit can be obtained. However, the models do not take both different types of gas-lift performance curves (GLPCs) and the cost factors of gas-lift production technique into account. On the basis of GLPCs, this paper introduces the factors of a gas-lift production technique, which includes the water cut of crude oil, cost of gas injection and water treatment, and oil and gas prices. The concept of a gas-lift economic performance curve (GLEPC) is proposed, and an optimum gas allocation model is established, considering different types of GLPCs and taking economic benefits as the objective, and the model is solved by the method of mixed penalty function. Taking gas-lift well group JD as an example, four gas-lift gas allocation schemes are obtained, and the proposed economical optimum model is applied to optimize gas allocation and analyze profit. What is more, the oil production rate and the result of optimum gas allocation taking maximum oil production rate as the objective in the model are calculated and compared. Then the gas allocation scheme with maximum economical profit is selected, and the significance of considering different types of GLPCs and taking economic benefits as the objective to gas allocation is confirmed.

Keywords: types of GLPCs; optimum gas allocation; GLEPCs; mathematical model; mixed penalty function

1. Introduction

In the middle and later periods of oil well production, the formation energy gradually decreases, and it is usually required to artificially replenish energy to make oil flow out of the wellhead when oil wells cannot flow naturally. The gas-lift technique is usually used to take overflowing wells as one of common artificial lift technique in the oilfield. With the number of oil wells requiring gas lift increasing, the reasonable allocation of a finite gas-lift high-pressure gas source becomes the major factor that influences the economic benefits of gas lift. Because of the different wellbore configurations of oil wells and the difference of formation conditions, the response of oil production is different to the injected gas rate of gas lift, and, meanwhile, the optimum gas allocation is usually restricted by the allocable gas rate on the ground. When the gas allocation of well groups or the total oilfield are optimized, how to realize the objective of optimal economical production and maximum production of the oilfield has a significant meaning in economical and efficient oilfield development, which is based on the gas-lift characteristics of different wells and the restriction on allocable gas rate on the ground and by optimizing continuous gas lift wells and gas allocation.

Mayhill [1] analyzed the relationship between injected gas rate and oil production of continuous gas lift, and it was called gas-lift performance curve (GLPC). The optimal injected gas rate was defined as the injected gas rate when the added cost of injecting gas exactly equals to a certain proportion of the added profit. Liu [2] proposed the mathematical model of optimum gas allocation that takes maximum daily oil production and daily work cash receipt as multiple objectives. By applying linear weighing-sum method, the model was solved to obtain the optimal gas allocation scheme. However, the method is influenced by gas allocation constraint condition at the wellhead. Afterwards many scholars carried on optimum gas-lift research on single-well optimization [3], multiple-wells optimization, and whether to consider the constraint of injected gas rate and gas injection pressure, and so on [4–6].

With regard to the question of optimum gas allocation of gas lift, many scholars have studied the optimization algorithm [7–11]. At present, the optimization algorithms are classified into two types, which include numerical algorithm and heuristic algorithm [12]. The traditional numerical algorithms usually obtain the arithmetic solution on the basis of some conventional calculation or curve analysis. The solutions of the calculations are coincident at different times in a particular period. With the parameters involved in the question increasing, the complexity of the algorithms increases sharply, which includes slope method [13], gradient optimization method [14], Gauss-newton iterative method, and nonlinear programming [4]. The other type of algorithm is the heuristic algorithm, which aims at function calculation with multiple variables and selects some possible solutions in advance, then it modifies the population in the iteration till the satisfactory solution is found. The algorithm includes GA algorithm [5,15], simulated annealing algorithm [16], and TLBO algorithm [17].

With the deepening of the research on gas-lift optimization and the introduction of various modern algorithms, the optimum gas allocation of gas lift is developing from the traditional gas-lift optimization to SGLO, and among them ORAT, GUF, and NPV obtain an obvious promotion [18]. However, these types of modes just evaluate the NPV on the basis of the oil production of optimum gas allocation model, which cannot carry on the gas-lift optimization based on NPV, and the NPV result obtained from the model cannot be as the optimal NPV of gas-lift production. This paper is based on GLPC and introduces the factors that include water cut of crude oil, cost of gas injection and water treatment, and oil and gas prices. In addition, the concept of gas-lift economic performance curves is proposed, and the optimum gas allocation model is established, which takes different types of GLPCs into account and takes economic benefits as the objective. The method of mixed penalty function is used to solve the model and the Newton–Rafaison method is used to solve the system of nonlinear equations. The model modifies the maximum gas allocation rate method that estimates the initial injected gas rate, and it increases the computation speed. The application example confirmed the significance of considering different types of GLPCs and taking economic benefits as the objective to optimum gas allocation.

2. Mathematical Model of Single Well

2.1. GLPC of Single Well

The GLPC of single well refers to the relationship between injected gas flowrate q_g (10^4 m^3/d) and oil (liquid) production q_o (m^3/d) of gas lift well, and the curve shape depends on the response of injected gas rat on liquid production, and the typical GLPCs are shown in Figure 1.

Some discrete points of GLPCs can be obtained from production test on gas-lift oil wells, and these points can be fitted a curve equation to acquire the injected gas rate of single well by applying optimization algorithm. This paper fits the gas-lift performance curves to a quadratic polynomial, and the mathematical regression model is as followed.

$$q_o = Aq_g^2 + Bq_g + C \tag{1}$$

Only when the above equation must be satisfied that $A < 0$, $B > 0$, $B^2 - 4AC > 0$, the optimum gas allocation can be carried on. Because the injection capacity is finite and gas allocation often cannot reach the maximum, it just needs to fit the GLPCs before the maximum oil production to increase computation speed and accuracy.
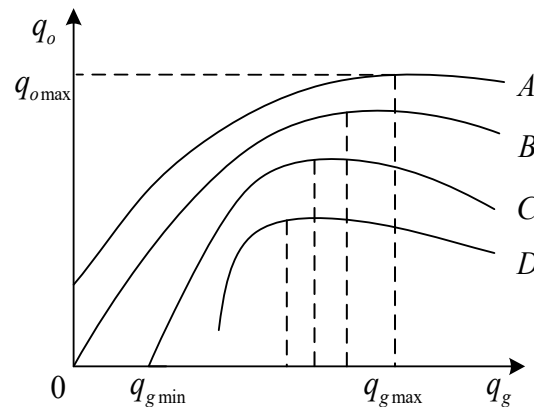

Figure 1. Types of GLPCs ($q_{g\min}$, $q_{g\max}$ are the minimum, maximum injected gas rate; ABC and D represent four different gas lift characteristic curves).

2.2. Gas-Lift Economic Performance Curve (GLEPC) of Single Well

From GLPCs it is not difficult to find that with the injected gas rate increasing, the increasing trend of oil production slows down and the benefits of unit-injected gas rate gradually decreases. To more intuitively analyze the influence rule of gas-lift cost factors on the economic benefits of oil wells, the economic objective function of single-well daily work cash income (NPV, $/d) is established. According to the actual situation of production wells, the function considers oil price, nature gas cost and compression cost, and water treatment cost (considering the influence of the water cut of crude oil), which is as follows:

$$y = p_1 q_o + p_2 q_{pg} - p_3 q_w - p_4 q_g - p_5 q_o \tag{2}$$

where p_1, p_3, and p_5 are, respectively, oil price per unit, produced water treatment cost and oil degassing cost, $/m^3$; p_2, p_4 are, respectively, nature gas price and injected gas pressurization cost, $/10^4$ m^3; q_o, q_w are, respectively, oil production rate, and water production rate, m^3/d; q_{pg}, q_g are, respectively, gas production rate and injected gas rate, 10^4 m^3/d. Among them, q_{pg} and q_g, q_o, and q_w, respectively, satisfy Equations (3) and (4):

$$q_{pg} = q_o GOR_f \tag{3}$$

$$q_w = q_o \times \frac{F_w}{1 - F_w} \tag{4}$$

where GOR_f are formation gas–oil ratio, 10^4 m^3/m^3; F_w are water cut, %.

Equations (1), (3) and (4) are substituted into Equation (2), then it can be obtained that

$$y = \left(A q_g^2 + B q_g + C \right) \left(p_1 + p_2 GOR_f - p_3 \frac{F_w}{1 - F_w} - p_5 \right) - p_4 q_g \tag{5}$$

Suppose that

$$\alpha = \left(p_1 + p_2 GOR_f - p_3 \frac{F_w}{1 - F_w} - p_5 \right) A \tag{6}$$

$$\beta = \left(p_1 + p_2 GOR_f - p_3 \frac{F_w}{1 - F_w} - p_5 \right) B - p_4 \tag{7}$$

$$\gamma = (p_1 + p_2 GOR_f - p_3 \frac{F_w}{1 - F_w} - p_5) C \tag{8}$$

The economic objective function can be simplified as

$$y = \alpha q_g^2 + \beta q_g + \gamma \tag{9}$$

According to the characteristics of Equation (9) and combining the GLPC of single well, the economical performance curve of single well can be obtained, which is shown in Figure 2.
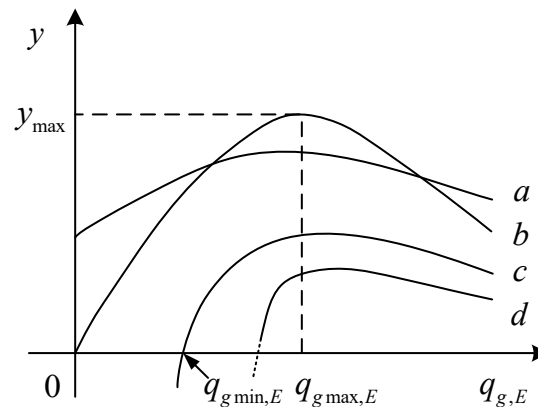

Figure 2. Types of economical performance curves of gas lift ($q_{g\min,E}$, $q_{g\max,E}$ are the minimum, maximum injected gas rate; *abc* and *d* represent four different gas lift economical performance curves).

The wells of Type *a* refer to the flowing wells or the oil wells with strong production capacity, they can make a profit in the condition of no gas allocation or as soon as gas injection. The wells of Type *b* refer to the oil wells that have obvious response to gas injection, which have high efficiency of gas lift and fast-growing profit. The wells of Type *c* refer to the oil wells whose gas lift cost is larger than income and production is in the red in the early stage, which begin to make profit with the gas injection increasing. The wells of Type *d* refer to the oil wells have no obvious response to gas injection, which need a certain gas injection to make profit.

For the wells of Type *a* and *b*, the injected gas rate corresponding to the maximum economic benefits of single wells is

$$q_{g\max,E} = -\frac{\beta}{2\alpha} \tag{10}$$

The corresponding maximum economic benefit is

$$y_{\max} = \frac{4\alpha\gamma - \beta^2}{4\alpha} \tag{11}$$

For the wells of Type *c*, when $y_{\max} < 0$, the wells have no economic benefits of gas lift and do not carry on gas-lift gas allocation; when $y_{\max} > 0$, the maximum injected gas rate and the optimal economic benefits can be calculated by the same method as the wells of Type *a* and *b*, and meanwhile it is required to calculate the injected gas rate $q_{g\min,E}$ when the revenue equals to 0. The economical gas allocation of the types of wells require that the injected gas rate must satisfy $q_{g,E} > q_{g\min,E}$.

$$q_{g\min,E} = \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \tag{12}$$

For the wells of Type *d*, it is required to set the minimum injected gas rate. If the allocated gas rate is smaller than the minimum injected gas rate, the well does not carry on gas allocation.

3. Block Optimum Gas Allocation Model

Suppose that a certain block have n wells that constitute the set N, and the injected gas rate is $q_{gi,E}$, the oil production rate of single well is q_{oi}, the total oil production rate of the block is Q_t, the single well revenue is y_i, the total block revenue is Y, so the daily work cash income of the block is

$$Y = \sum_{i=1}^{n} y_i = f\left(q_{g1,E}, q_{g2,E}, \cdots, q_{gn,E}\right) \tag{13}$$

The maximum NPV of the block is expressed as

$$\max Y = \max f\left(q_{g1,E}, q_{g2,E}, \cdots, q_{gn,E}\right) \tag{14}$$

Substituting Equation (9) into Equation (14) can obtain that

$$\max Y = \max \sum_{i=1}^{n} \left(\alpha_i q_{gi,E}^2 + \beta_i q_{gi,E} + \gamma_i\right) \tag{15}$$

According to Equation (9), the injected gas rate corresponding to the maximum NPV of single well is

$$q_{gi\max,E} = -\frac{\beta_i}{2\alpha_i} \tag{16}$$

Therefore, the total injected gas rate corresponding to the maximum NPV of the block is

$$Q_{g\max,E} = \sum_{i=1}^{n} \left(-\frac{\beta_i}{2\alpha_i}\right) \tag{17}$$

Form Equation (17), it can be concluded that if the attainable maximum injected gas rate $Q_{\max}$ is larger than $Q_{g\max,E}$, the total injected gas rate will not be the constraint to the system and it will be on the contrary. Hence the constraint condition of the total injected gas rate is

$$\sum_{i=1}^{n} q_{gi,E} = Q_{\max} \quad Q_{\max} \leq Q_{g\max,E} \tag{18}$$

The wells of Type a and b performances curve in the Figure 2 constitute the set I, and the constraint condition of single well is

$$0 \leq q_{gi,E} \leq q_{gi\max,E} \tag{19}$$

The wells of Type c and d performances curve constitute the set J, and the constraint condition is

$$0 \leq q_{gj\min,E} \leq q_{gj,E} \leq q_{gi\max,E} \ or \ q_{gj,E} = 0 \tag{20}$$

For the wells of Type c performance curve, $q_{gj\min,E}$ can be calculated by the following equation

$$q_{gj\min,E} = \frac{-B_j - \sqrt{B_j^2 - 4A_j C_j}}{2A_j} \tag{21}$$

For the wells of Type d performance curve, $q_{gj\min,E}$ need to be given, or the wells will not be allocated gas.

Suppose that there are m wells of Type d and some wells among them is considered to allocate gas (the wells join the allocation category, and the allocated gas rate must be not less than the corresponding lower limit $q_{Li,E}$). Therefore, the gas allocation scheme has $C_m^0 + C_m^1 + \ldots \ldots + C_m^m$ possibilities, and it is required to compare the maximum NPV of each allocation scheme to select the gas allocation scheme corresponding to the optimal NPV.

In conclusion, the block optimum gas allocation model taking the maximum NPV as the objective can be obtained as followed

$$
\begin{cases}
\max Y = \max \sum\limits_{i=1}^{n} \left(\alpha_i q_{gi,E}^2 + \beta_i q_{gi,E} + \gamma_i \right) \\
\sum\limits_{i=1}^{n} q_{gi,E} = Q_{\max}, Q_{\max} \leq Q_{g\max,E} \\
0 \leq q_{gk,E} \leq q_{gk\max,E} \\
0 < q_{gj\min,E} \leq q_{gj,E} \leq q_{gj\max,E} \, or \, q_{gj,E} = 0
\end{cases}
\tag{22}
$$

where $k \in I, j \in J$, $J \cup I = N$, $J \cap I = \Phi$, Φ is null set.

4. Model Solution

After establishing the mathematical model of optimum gas allocation, the applicability or the choice of the model solution method becomes the biggest difficulty or the most important influence factor. Penalty function method is the common method to process constraint conditions, and the basic idea is to construct a new function by using objective function and constraint conditions, so the original optimization problem will be converted to the unconstrained optimization problem of the new function. Penalty function method is applied to solve Equation (22).

The form of Penalty function is

$$
p\left(q_{g,E}, r\right) = Y\left(q_{g,E}\right) + rN\left(q_{g,E}\right) + \frac{1}{r}\left[E\left(q_{g,E}\right) + L\left(q_{g,E}\right)\right]
\tag{23}
$$

where $N\left(q_{g,E}\right)$, $E\left(q_{g,E}\right)$, $L\left(q_{g,E}\right)$ are, respectively, the quadratic loss items of logarithmic barrier term, equality penalty term, and inequality penalty term, which are, respectively, expressed as

$$
\begin{aligned}
N\left(q_{g,E}\right) &= \sum_{i \in I_1} \ln \frac{1}{g_i\left(q_{g,E}\right)} \\
E\left(q_{g,E}\right) &= \sum_{j=1}^{m} h_j^2\left(q_{g,E}\right) \\
L\left(q_{g,E}\right) &= \sum_{i \in I_2} \left\{ \min\left[0, g_i\left(q_{g,E}\right)\right] \right\}^2
\end{aligned}
\tag{24}
$$

where r is penalty factor that is a series of definite positive values, and r constitutes the sequence $\{r_k\}$ that is a monotone decreasing infinitesimal sequence When $k \to \infty$, and the subscript set I_1 and I_2 are defined as

$$
I_1 = \left\{ i \big| g_i\left(q_{g,E}\right) > 0, 1 \leq i \leq p \right\}, \quad I_2 = \left\{ i \big| g_i\left(q_{g,E}\right) \leq 0, 1 \leq i \leq p \right\}
\tag{25}
$$

Substituting the block gas allocation model into Equation (27) can obtain the penalty function.

When $q_{gi,E} - q_{Li,E} > 0$,

$$
p\left(q_{g,E}, r\right) = \sum_{i=1}^{n} \left(\alpha_i q_{gi,E}^2 + \beta_i q_{gi,E} + \gamma_i \right) + r\sum_{i \in I_1} \ln \frac{1}{q_{gi,E} - q_{Li,E}} + \frac{1}{r}\left(\sum_{i=1}^{n} q_{gi,E} - Q_{\max} \right)^2
\tag{26}
$$

When $q_{gi,E} - q_{Li,E} \leq 0$,

$$
p\left(q_{g,E}, r\right) = \sum_{i=1}^{n} \left(\alpha_i q_{gi,E}^2 + \beta_i q_{gi,E} + \gamma_i \right) + \frac{1}{r}\left(\sum_{i=1}^{n} q_{gi,E} - Q_{\max} \right)^2 + \frac{1}{r}\sum_{i \in I_2} \left[\min\left(0, q_{gi,E} - q_{Li,E}\right) \right]^2
\tag{27}
$$

Equations (26) and (27) are, respectively, taken their partial derivatives of $q_{gi,E}$, it can be obtained as followed

$$\frac{\partial p}{\partial q_{gi,E}} = 2\alpha_i q_{gi,E} + \beta_i - \frac{r}{q_{gi,E} - q_{Li,E}} + \frac{2}{r}\left(\sum_{i=1}^{n} q_{gi,E} - Q_{\max}\right)$$

$$\frac{\partial p}{\partial q_{gi,E}} = 2\alpha_i q_{gi,E} + \beta_i + \frac{2}{r}\left(\sum_{i=1}^{n} q_{gi,E} - Y\right) + \frac{2}{r}\left(q_{gi,E} - q_{Li,E}\right)$$

From $\frac{\partial p}{\partial q_{gi,E}} = f\left(q_{gi,E}, r\right) = 0$, the Newton–Rafaison method [19,20] can be used to solve the equation set. In addition, using LU decomposition method, the algebraic equation set can be expressed as

$$\begin{pmatrix} y\left(q_{g1,E}\right) & 2/r & \cdots & 2/r & 2/r \\ 2/r & y\left(q_{g2,E}\right) & \cdots & 2/r & 2/r \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 2/r & 2/r & \cdots & y\left(q_{gi,E}\right) & 2/r \\ 2/r & 2/r & \cdots & 2/r & y\left(q_{gn,E}\right) \end{pmatrix}_{n \times n}^{(k)} \bullet \begin{pmatrix} \delta q_{g1,E} \\ \delta q_{g2,E} \\ \vdots \\ \delta q_{gi,E} \\ \delta q_{gn,E} \end{pmatrix}^{(k)} = \begin{pmatrix} -f\left(q_{g1,E}, r\right) \\ -f\left(q_{g2,E}, r\right) \\ \vdots \\ -f\left(q_{gi,E}, r\right) \\ -f\left(q_{gn,E}, r\right) \end{pmatrix}^{(k)} \quad (28)$$

$$q_{g,E}^{(k+1)} = q_{g,E}^{(k)} + \delta q_{g,E}^{(k)}$$

The approximation solution of the equation set can be obtained by using loop iteration. When r tends to small enough, the approximation optimal solution of the problem can be obtained.

When $q_{gi,E} - q_{Li,E} > 0$,

$$y\left(q_{gi,E}\right) = 2\alpha_i + \frac{r}{\left(q_{gi,E} - q_{Li,E}\right)^2} + \frac{2}{r} \quad (29)$$

When $q_{gi,E} - q_{Li,E} \leq 0$,

$$y\left(q_{gi,E}\right) = 2\alpha_i + \frac{4}{r} \quad (30)$$

where $q_{g,E} = \left(q_{g1,E}, q_{g2,E}, \ldots, q_{gn,E}\right)^T$.

Applying the method of mixed penalty function needs to set the initial injected gas rate $q_{g,E}^0$ that must satisfy all the constraint conditions. Good or bad initial value will directly influence the iterations, even whether the iteration converges or not, and convergence rate. The author applies the maximum gas allocation rate method to modify it, which can be expressed as

$$q_{gi,E}^0 = \left(Q_{g\max,E} - \sum_{i=1}^{n} q_{Li,E}\right) \frac{q_{gi\max,E} - q_{Li,E}}{\sum\limits_{i=1}^{n} q_{gi\max,E} - \sum\limits_{i=1}^{n} q_{Li,E}} + q_{Li,E} \quad (31)$$

5. Example Calculation Results and Discussions

Production test was carried on 8 gas lift wells in JD Oilfield and their GLPCs were fitted, and the various coefficients of the curves, water cut, and formation gas–oil ratio are shown in Table 1.

Table 1. Coefficients of GLPCs, water cut and formation gas–oil ratio of various wells.

Well NO.	Coefficient A	Coefficient B	Coefficient C	F_W, Fraction	GOR_f (10^4 m^3/m^3)
W1	−0.58	3.86	1.082	0.52	0.016
W2	−0.92	4.24	3.53	0.45	0.01
W3	−1.42	6.53	2.091	0.48	0.0125
W4	−1.35	4.68	6.889	0.39	0.015
W5	−0.287	2.36	2.893	0.58	0.018
W6	−1.106	5.68	0.788	0.5	0.016
W7	−1.62	7.36	−1.945	0.443	0.02
W8	−2.048	9.74	−3.022	0.55	0.0175

Take market oil price, gas price, water treatment cost, injected gas pressurization cost, and oil degassing cost into account, which are shown in Table 2.

Table 2. Gas-lift economical parameters.

Oil Price p_1 $/m^3	Gas Price p_2 $/10^4$ m^3	Water Treatment Cost p_3 $/m^3	Injected Gas Pressurization Cost p_4 $/10^4$ m^3	Oil Degassing Cost p_5 $/m^3
301.89	2012.38	0.077	309.60	0.0077

From Tables 1 and 2, the GLEPC of each well can be obtained by calculation according to Equation (9), and the various coefficients are sown as Table 3. Form Table 3, it can be seen that the wells of W1 to W6 belong to GLEPCs of Type *a* and *b*, and the wells of W7 and W8, respectively, belong to GLEPCs of Type *c* and *d*.

Table 3. Coefficients of GLEPCs of various wells.

Well NO.	Coefficient a	Coefficient β	Coefficient γ
W1	−193.81	980.26	361.56
W2	−296.31	1055.96	1136.89
W3	−464.50	1826.40	683.98
W4	−448.36	1244.70	2287.94
W5	−96.49	488.58	978.44
W6	−362.41	1588.40	263.31
W7	−540.86	2208.20	−665.55
W8	−671.19	2975.99	−1018.99

(1) Gas allocation taking the maximum oil production rate as the objective

According to the coefficients of GLPCs in Table 1, the wells of W7 and W8, respectively, belong to Type *c* and *d*, and there are the following 4 gas allocation schemes. In Scheme 1, all wells will be allocated gas; in Scheme 2, W8 of Type *d* will not be allocated gas; in Scheme 3, both the wells of W7 and W8 of Type *c* and *d* will not be allocated gas; in Scheme 4, W7 of Type *c* will not be allocated gas.

Allocating gas, respectively, according to Schemes 1–4 and calculating the corresponding daily cash income, the results are, respectively, listed in Tables 4–7. The corresponding block GLPCs and GLEPCs are shown in Figures 3 and 4.

Table 4. Gas allocation result of various total injected gas rate of Scheme 1.

Scheme 1	Well NO.	Gas allocation Rate, 10^4 m^3/d; Oil Production Rate, m^3/d; Daily Cash Income, $/d																	
		2.00			4.00			8.00			12.00			16.00			Unconstrained		
		qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi
	W1	0.00	1.08	361.4	0.00	1.08	361.4	0.88	4.04	1075.3	1.74	6.04	1478.5	2.45	7.05	1598.5	3.33	7.50	1476.0
	W2	0.00	3.53	1136.5	0.11	3.99	1250.9	0.76	6.23	1769.3	1.30	7.49	2008.5	1.75	8.13	2076.3	2.30	8.42	1995.9
	W3	0.35	4.20	1263.9	0.88	6.73	1929.5	1.30	8.18	2272.8	1.65	9.00	2431.7	1.94	9.41	2477.8	2.30	9.60	2426.5
	W4	0.00	6.89	2287.3	0.24	7.93	2559.0	0.68	9.46	2928.0	1.05	10.31	3099.5	1.35	10.75	3150.2	1.73	10.94	3097.4
	W5	0.00	2.89	977.8	0.00	2.89	977.8	0.00	2.89	977.8	0.90	4.78	1337.6	2.33	6.83	1588.3	4.11	7.74	1344.9
	W6	0.06	1.14	362.6	0.74	4.40	1239.8	1.29	6.26	1693.8	1.73	7.31	1905.3	2.11	7.84	1968.1	2.57	8.08	1903.9
	W7	0.56	1.67	398.9	1.03	3.90	1016.1	1.40	5.17	1336.2	1.70	5.89	1486.3	1.96	6.25	1532.1	2.27	6.41	1489.6
	W8	1.03	4.82	1307.4	1.00	4.67	1265.6	1.69	7.59	2034.7	1.93	8.15	2150.5	2.13	8.44	2185.2	2.38	8.57	2150.9
	Total	2.00	26.23	8095.7	4.00	35.61	10,600.1	8.00	49.82	14,088.0	12.00	58.97	15,897.9	16.00	64.71	16,576.4	20.99	67.27	15,885.0

Table 5. Gas allocation result of various total injected gas rate of Scheme 2.

Scheme 2	Well NO.	Gas Allocation Rate, 10^4 m^3/d; Oil Production Rate, m^3/d; Daily Cash Income, $/d																	
		2.00			4.00			8.00			12.00			16.00			Unconstrained		
		qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi
	W1	0.00	1.08	361.4	0.19	1.81	543.2	1.35	5.24	1331.7	2.10	6.62	1563.6	2.84	7.37	1580.9	3.33	7.50	1476.0
	W2	0.00	3.53	1136.4	0.33	4.82	1450.8	1.06	6.99	1922.0	1.53	7.86	2057.6	2.00	8.33	2062.9	2.30	8.42	1995.9
	W3	0.67	5.82	1695.8	1.02	7.27	2061.7	1.49	8.67	2373.7	1.80	9.24	2464.6	2.10	9.54	2469.7	2.30	9.60	2426.5
	W4	0.02	6.97	2308.9	0.39	8.50	2701.6	0.88	9.97	3036.9	1.20	10.57	3135.6	1.52	10.89	3142.4	1.73	10.94	3097.4
	W5	0.00	2.89	977.8	0.00	2.89	977.8	0.12	3.17	1034.1	1.62	5.97	1514.4	3.13	7.47	1555.4	4.11	7.74	1344.9
	W6	0.47	3.23	932.0	0.92	5.09	1414.7	1.53	6.89	1828.3	1.92	7.62	1949.8	2.31	8.01	1958.8	2.57	8.08	1903.9
	W7	0.84	3.10	799.4	1.15	4.37	1139.3	1.56	5.60	1431.5	1.83	6.09	1518.3	2.10	6.36	1526.6	2.27	6.41	1489.5
	Total	2.00	26.62	8211.7	4.00	34.75	10,289.2	8.00	46.53	12,958.1	12.00	53.97	14,204.0	16.00	57.96	14,296.8	18.61	58.70	13,734.1

Table 6. Gas allocation result of various total injected gas rate of Scheme 3.

		Gas Allocation Rate, 10^4 m³/d; Oil Production Rate, m³/d; Daily Cash Income, \$/d
\multicolumn{2}{c}{Scheme 3}	\multicolumn{3}{c}{2.00}	\multicolumn{3}{c}{4.00}	\multicolumn{3}{c}{8.00}	\multicolumn{3}{c}{12.00}	\multicolumn{3}{c}{16.00}	\multicolumn{3}{c}{Unconstrained}
		qgi	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*
Well NO.	W1	0.00	1.08	361.4	0.58	3.12	862.1	1.66	5.90	1454.7	2.46	7.07	1599.0	3.26	7.50	1496.5	3.33	7.50	1476.0
	W2	0.12	4.03	1259.0	0.57	5.65	1641.8	1.26	7.40	1994.5	1.76	8.14	2076.5	2.26	8.41	2008.6	2.30	8.42	1995.9
	W3	0.88	6.75	1934.6	1.18	7.81	2188.3	1.62	8.94	2422.5	1.95	9.42	2477.8	2.27	9.60	2434.8	2.30	9.60	2426.5
	W4	0.25	7.95	2565.0	0.55	9.06	2837.5	1.02	10.25	3089.4	1.36	10.76	3150.4	1.70	10.94	3106.0	1.73	10.94	3097.4
	W5	0.00	2.89	977.8	0.00	2.89	977.8	0.75	4.50	1288.6	2.36	6.87	1589.4	3.97	7.74	1385.7	4.11	7.74	1344.9
	W6	0.75	4.43	1247.1	1.13	5.78	1581.6	1.70	7.24	1892.7	2.11	7.85	1968.3	2.53	8.08	1914.5	2.57	8.08	1903.9
	Total	2.00	27.14	8344.9	4.00	34.30	10,089.2	8.00	44.23	12,142.5	12.00	50.10	12,861.4	16.00	52.27	12,346.1	16.34	52.29	12,244.5

Table 7. Gas allocation result of various total injected gas rate of Scheme 4.

| | | Gas Allocation Rate, 10^4 m³/d; Oil Production Rate, m³/d; Daily Cash Income, \$/d |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \multicolumn{2}{c}{Scheme 4} | \multicolumn{3}{c}{2.00} | \multicolumn{3}{c}{4.00} | \multicolumn{3}{c}{8.00} | \multicolumn{3}{c}{12.00} | \multicolumn{3}{c}{16.00} | \multicolumn{3}{c}{Unconstrained} |
| | | *qgi* | *qo* | *yi* | *qgi* | *qo* | *yi* | *qgi* | *qo* | *yi* | *qgi* | *qo* | *yi* | *qgi* | *qo* | *yi* | *qgi* | *qo* | *yi* |
| Well NO. | W1 | 0.00 | 1.08 | 361.4 | 0.09 | 1.42 | 447.0 | 1.30 | 5.13 | 1308.9 | 2.06 | 6.57 | 1557.0 | 2.81 | 7.35 | 1584.1 | 3.33 | 7.50 | 1476.0 |
| | W2 | 0.00 | 3.53 | 1136.4 | 0.26 | 4.58 | 1392.6 | 1.03 | 6.92 | 1908.4 | 1.50 | 7.83 | 2053.8 | 1.98 | 8.32 | 2065.0 | 2.30 | 8.42 | 1995.9 |
| | W3 | 0.54 | 5.19 | 1530.4 | 0.98 | 7.11 | 2023.1 | 1.47 | 8.63 | 2364.8 | 1.78 | 9.22 | 2462.1 | 2.09 | 9.54 | 2471.0 | 2.30 | 9.60 | 2426.5 |
| | W4 | 0.00 | 6.89 | 2287.3 | 0.34 | 8.33 | 2660.4 | 0.86 | 9.92 | 3027.7 | 1.19 | 10.54 | 3132.8 | 1.51 | 10.88 | 3143.8 | 1.73 | 10.94 | 3097.4 |
| | W5 | 0.00 | 2.89 | 977.8 | 0.00 | 2.89 | 977.8 | 0.02 | 2.94 | 988.0 | 1.55 | 5.86 | 1500.7 | 3.07 | 7.44 | 1561.6 | 4.11 | 7.74 | 1344.9 |
| | W6 | 0.31 | 2.42 | 713.5 | 0.87 | 4.89 | 1363.8 | 1.51 | 6.83 | 1816.2 | 1.90 | 7.59 | 1946.3 | 2.30 | 8.00 | 1960.6 | 2.57 | 8.08 | 1903.9 |
| | W8 | 1.16 | 5.51 | 1526.9 | 1.46 | 6.85 | 1889.3 | 1.81 | 7.90 | 2141.8 | 2.02 | 8.30 | 2215.0 | 2.23 | 8.52 | 2224.6 | 2.38 | 8.57 | 2194.3 |
| | Total | 2.00 | 27.51 | 8533.7 | 4.00 | 36.07 | 10,754.0 | 8.00 | 48.27 | 13,555.8 | 12.00 | 55.91 | 14,867.8 | 16.00 | 60.04 | 15,010.7 | 18.72 | 60.86 | 14,438.8 |
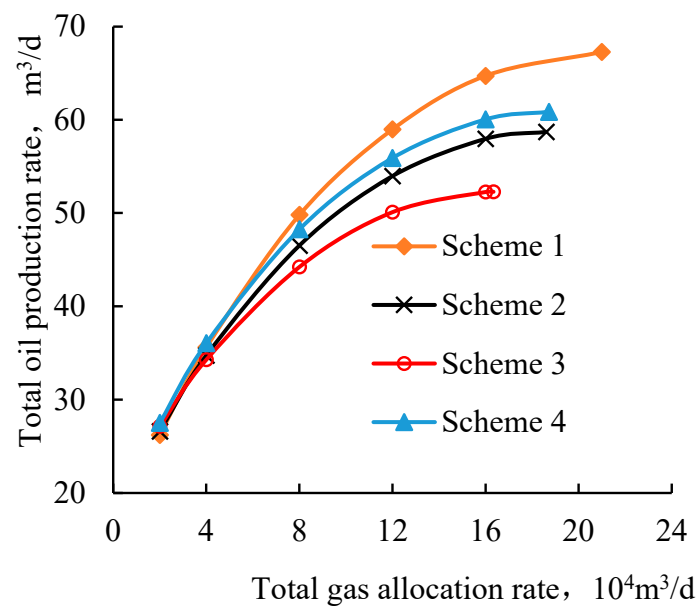
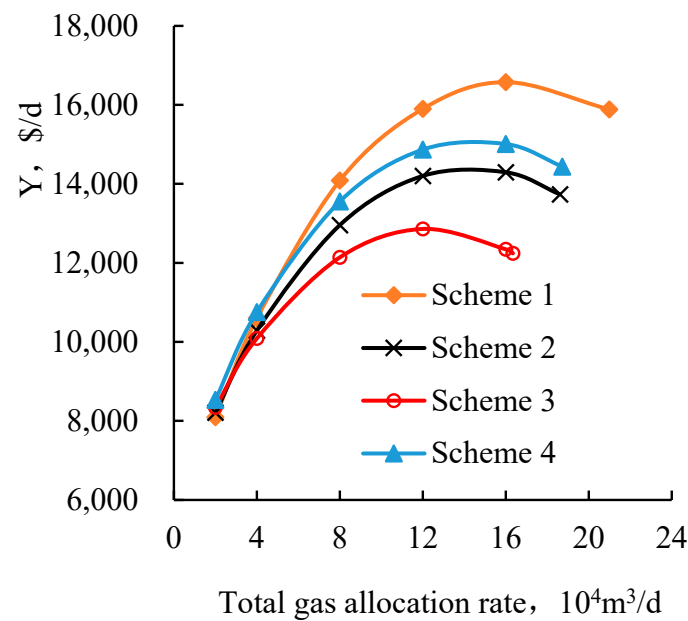**Figure 3.** Block GLPCs of various schemes with the maximum oil production rate.

Figure 4. Block GLEPCs of various schemes with the maximum oil production rate.

From Tables 4–7 and Figures 3 and 4, it can be concluded that when the gas-lift allocable gas rate is unconstrained, the various gas-lift single wells can be allocated gas according to the optimal injected gas rate of single well (which can be obtained from single well GLPC), and the block optimal gas-lift gas allocation rate is the summation of the optimal injected gas rate of single wells 20.993×10^4 m^3/d. Under the circumstance of considering gas allocation rate as restricted, from the block GLPCs of various schemes it is known that the maximum oil production rate of Scheme 1 is optimal when the allocable gas rate is larger than 5×10^4 m^3/d, and the maximum oil production rate of Scheme 4 is superior to Scheme 1 when the allocable gas rate is smaller than 5×10^4 m^3/d. Therefore, it is necessary to optimize gas lift wells to improve gas-lift production efficiency.

The block GLEPC of various schemes indicates that with the total gas allocation rate increasing, the block economic benefits shows the trend of increasing firstly and then decreasing and it start to descend at the time when the gas allocation rate increases to a

certain value. It is mainly because of the nonlinear change in gas lift oil production rate and injected gas rate. With the injected gas rate increasing, the injection cost increases and it results in the descending trend of economic benefits. Therefore, it is necessary to apply the NPV optimum gas allocation model to optimize the gas allocation of various schemes, so then to seek the optimal injected gas rate of the block and single wells, which can improve the economic benefits of gas lift production and acquire the gas allocation scheme with optimal economic benefits.

(2)   Gas allocation taking the maximum daily cash income NPV as the objective

   According to the coefficients of GLEPCs in Table 3, the above four gas allocation schemes are applied for comparison, and the gas allocation is carried on, respectively, according to Schemes 1–4. The results are, respectively, listed in Tables 8–11, and the block GLPCs and the GLEPCs corresponding to the schemes are shown in Figures 5 and 6.
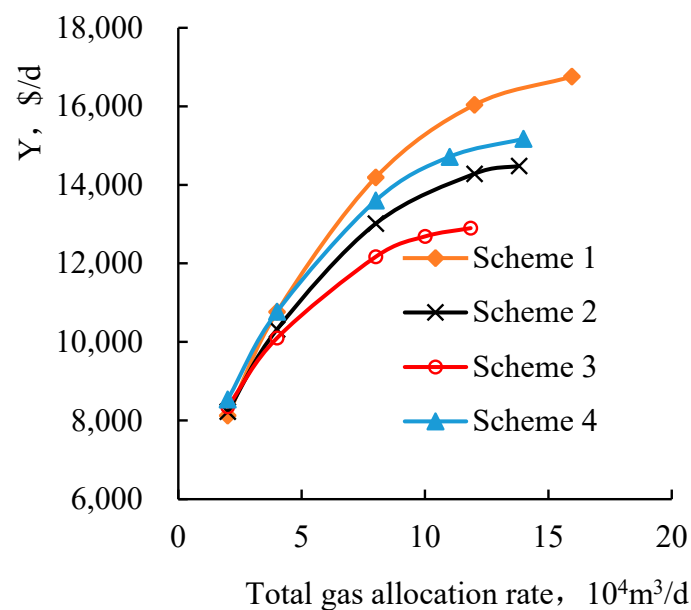

Figure 5. Block GLEPCs of various schemes with the maximum daily cash income NPV.
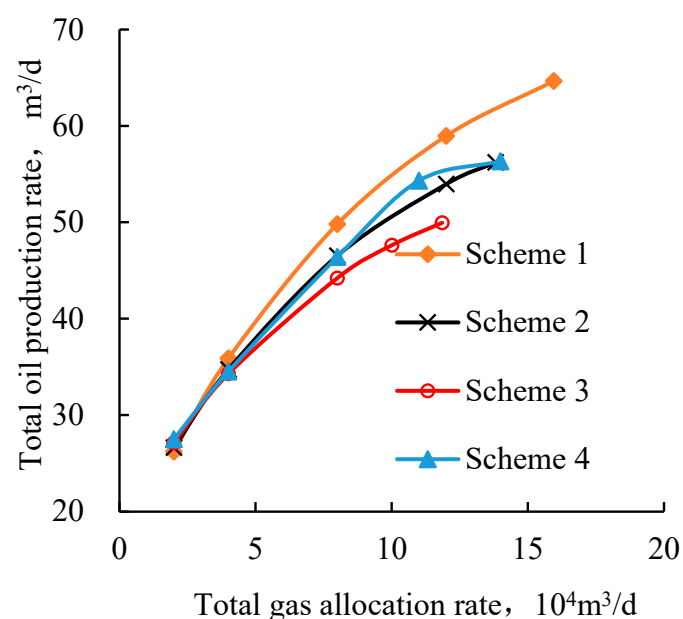

Figure 6. Block GLPCs of various schemes with the maximum daily cash income NPV.

Table 8. Gas allocation results of various injected gas rate of Scheme 1.

Scheme 1		Gas Allocation Rate,10^4 m^3/d; Oil Production Rate, m^3/d; Daily Cash Income, \$/d													
		2.00			4.00			8.00			12.00			Unconstrained		
		qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi
Well NO.	W1	0.00	1.08	361.6	0.00	1.08	361.6	0.90	4.08	1085.5	1.74	6.04	1479.5	2.44	7.04	1599.4
	W2	0.00	3.53	1136.9	0.00	3.53	1136.9	0.71	6.07	1736.0	1.26	7.41	1996.7	1.72	8.10	2076.5
	W3	0.30	3.90	1183.9	0.78	6.33	1828.2	1.28	8.13	2261.4	1.63	8.96	2427.2	1.92	9.40	2478.5
	W4	0.00	6.89	2287.9	0.17	7.63	2482.2	0.68	9.45	2927.6	1.04	10.30	3098.7	1.35	10.74	3151.0
	W5	0.00	2.89	978.4	0.00	2.89	978.4	0.00	2.89	978.4	0.93	4.83	1347.8	2.31	6.82	1592.3
	W6	0.06	1.11	354.2	0.67	4.10	1166.0	1.30	6.30	1715.1	1.74	7.32	1930.2	2.11	7.85	2001.3
	W7	0.60	1.91	470.9	1.01	3.84	1014.1	1.43	5.26	1384.9	1.72	5.92	1532.8	1.97	6.26	1585.2
	W8	1.04	4.91	1353.2	1.37	6.48	1798.4	1.71	7.64	2103.9	1.94	8.18	2228.7	2.14	8.45	2275.7
	Total	2.00	26.22	8127.1	4.00	35.89	10,765.7	8.00	49.82	14,192.9	12.00	58.96	16,041.7	15.95	64.66	16,759.8

Table 9. Gas allocation result of various injected gas rate of Scheme 2.

Scheme 2		Gas allocation Rate,10^4 m^3/d; Oil Production Rate, m^3/d; Daily Cash Income, \$/d													
		2.00			4.00			8.00			12.00			Unconstrained		
		qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi	qgi	qo	yi
Well NO.	W1	0.00	1.08	361.6	0.22	1.90	566.9	1.35	5.24	1333.1	2.10	6.63	1565.2	2.44	7.04	1599.4
	W2	0.00	3.53	1136.9	0.26	4.58	1393.2	1.01	6.87	1899.8	1.50	7.82	2053.6	1.72	8.10	2076.5
	W3	0.63	5.64	1650.2	1.00	7.19	2043.2	1.47	8.62	2365.5	1.78	9.22	2463.6	1.92	9.40	2478.5
	W4	0.01	6.93	2297.9	0.39	8.50	2703.4	0.88	9.96	3035.1	1.20	10.56	3135.9	1.35	10.74	3151.0
	W5	0.00	2.89	978.4	0.00	2.89	978.4	0.17	3.27	1056.4	1.64	6.00	1520.7	2.31	6.82	1592.3
	W6	0.48	3.25	941.0	0.94	5.16	1438.0	1.54	6.91	1849.5	1.93	7.63	1979.2	2.11	7.85	2001.3
	W7	0.88	3.29	863.8	1.19	4.52	1197.2	1.59	5.65	1476.7	1.85	6.12	1567.9	1.97	6.26	1585.2
	Total	2.00	26.62	8229.9	4.00	34.74	10,320.4	8.00	46.53	13,016.1	12.00	53.97	14,286.1	13.81	56.21	14,484.1

Table 10. Gas allocation result of various injected gas rate of Scheme 3.

| Scheme 3 | | Gas allocation Rate,10^4 m^3/d; Oil Production Rate, m^3/d; Daily Cash Income, \$/d | | | | | | | | | | | | | |
| | | 2.00 | | | 4.00 | | | 8.00 | | | 10.00 | | | Unconstrained | | |
		qgi	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*
Well NO.	W1	0.00	1.08	361.6	0.61	3.23	888.9	1.67	5.91	1458.1	2.07	6.59	1560.1	2.44	7.04	1599.4
	W2	0.07	3.83	1210.4	0.52	5.49	1605.9	1.22	7.32	1982.5	1.48	7.79	2050.1	1.72	8.10	2076.5
	W3	0.88	6.72	1927.5	1.16	7.76	2178.3	1.60	8.91	2418.1	1.77	9.20	2461.3	1.92	9.40	2478.5
	W4	0.26	8.03	2584.3	0.56	9.08	2842.9	1.02	10.25	3089.4	1.19	10.54	3133.7	1.35	10.74	3151.0
	W5	0.00	2.89	978.4	0.00	2.89	978.4	0.79	4.58	1304.9	1.58	5.91	1510.0	2.31	6.82	1592.3
	W6	0.79	4.58	1291.0	1.15	5.85	1609.9	1.71	7.26	1918.0	1.92	7.61	1976.0	2.11	7.85	2001.3
	Total	2.00	27.13	8353.1	4.00	34.30	10,104.3	8.00	44.23	12,170.9	10.00	47.63	12,691.2	11.85	49.95	12,898.9

Table 11. Gas allocation result of various injected gas rate of Scheme 4.

| Scheme 4 | | Gas Allocation Rate,10^4 m^3/d; Oil Production Rate, m^3/d; Daily Cash Income, \$/d | | | | | | | | | | | | | |
| | | 2.00 | | | 4.00 | | | 8.00 | | | 11.00 | | | Unconstrained | | |
		qgi	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*	*qgi*	*qo*	*yi*
Well NO.	W1	0.00	1.08	361.6	0.12	1.54	477.3	1.31	5.14	1311.7	1.87	6.28	1517.7	2.44	7.04	1599.4
	W2	0.00	3.53	1136.9	0.20	4.33	1334.4	0.98	6.79	1885.7	1.35	7.58	2022.2	1.72	8.10	2076.5
	W3	0.50	5.01	1483.8	0.96	7.04	2006.4	1.45	8.58	2356.1	1.69	9.07	2443.4	1.92	9.40	2478.5
	W4	0.00	6.89	2287.9	0.35	8.35	2664.9	0.86	9.91	3025.8	1.10	10.41	3115.4	1.35	10.74	3151.0
	W5	0.00	2.89	978.4	0.00	2.89	978.4	0.07	3.06	1013.1	1.20	5.30	1424.5	2.31	6.82	1592.3
	W6	0.32	2.48	730.4	0.89	4.97	1390.9	1.51	6.85	1837.4	1.81	7.45	1951.6	2.11	7.85	2001.3
	W8	1.18	5.63	1559.5	1.49	5.41	1922.2	1.82	6.08	2174.1	1.98	8.24	2242.1	2.14	6.38	2275.7
	Total	2.00	27.51	8538.5	4.00	34.54	10,774.5	8.00	46.41	13,604.0	11.00	54.32	14,716.8	13.99	56.33	15,174.6

From Tables 8–11 and Figures 5 and 6 it can be concluded that when the allocable gas rate is larger than 4×10^4 m^3/d, the block economic benefits obtained from Scheme 1 are superior to the other three schemes all along, and when the allocable gas rate is smaller than 4×10^4 m^3/d, the block economic benefits obtained from Scheme 4 is superior to Scheme 1, which is the optimal gas allocation scheme. Therefore, it is required to optimize scheme firstly before blocking economical optimum gas allocation.

Taking Scheme 1 as an example, analysis and comparison are conducted on ORAT and NPV optimum model. The block maximum total injected gas rate required by NPV optimum model is 5.04×10^4 m^3/d less than ORAT optimum model. Though the ultima oil production rate of NPV optimum model is 2.61 m^3/d less than the ORAT optimum model, the economic benefit is 874.8 \$/d more than the latter. Simultaneously, the total injected gas rate required by block NPV optimum model is less than the ORAT optimum model, hence the remaining high-pressure gas can be allocated to the other wells that require gas lift, so the oil production and revenue will increase.

This paper aims to propose the concept of gas-lift economical optimum gas allocation and establish an optimum gas allocation model on the basis of NPV, which will more directly reflect the relationship between injected gas rate and NPV of gas-lift technique in oil wells. Contrastive analyses were conducted on the case calculation results of ORAT and NPV optimum gas allocation models. Though NPV, optimum models reduced 3.88% oil production but increased 5.51% NPV and saved 24.01% block of the total injected gas rate.

6. Conclusions

(1) On the basis of GLPCs, this paper introduces the factors of a gas-lift production technique, which include gas injection cost, oil price, water cut, and water treatment cost. The concept of GLEPC and the mathematical model are proposed, and the types of GLEPCs are analyzed.

(2) Considering different types of GLPCs, this paper establishes the block optimum gas allocation model that takes economic benefits as the objective, and the model is solved by the method of mixed penalty function. In addition, the maximum gas allocation rate method that estimates the initial injected gas rate is modified.

(3) The parameters of eight wells in the JD Oilfield are applied to carry on example calculation, and the economic benefits of various schemes corresponding to two types of models are analyzed. The various gas allocation schemes are conducted to compare, and the optimal gas allocation scheme is selected. The block performance curves and economic performance curves corresponding to various gas allocation schemes are given, and the calculation result indicates that it has a significant meaning to consider different types of GLPCs and take economic benefits as the objective for gas allocation is confirmed.

(4) Since oil and gas prices are greatly influenced by market factors, it is necessary to adjust the gas allocation scheme of the gas lift wells in real time, which may affect the stability of the gas lift well.

Author Contributions: Data curation, C.Z. (Chengjie Zhao) and Z.X.; Formal analysis, C.Z. (Chengjie Zhao) and C.Z. (Chuangen Zheng); Investigation, H.Z.; Methodology, H.Z.; Project administration, Z.X.; Software, H.Z.; Writing—original draft, C.Z. (Chengjie Zhao); Writing—review & editing, H.Z. and C.Z. (Chuangen Zheng) All authors have read and agreed to the published version of the manuscript.

Data Availability Statement: The authors declare that the data of this research are available from the correspondence author on request.

References

1. Mayhill, T.D. Simplified Method for Gas Lift Well Problem Identification and Diagnosis, Paper SPE 5151. In Proceedings of the SPE 49th Annual Fall Meeting, Houston, TX, USA, 6–9 October 1974.
2. Liu, X.; Zhang, B.; Wang, J.; Liao, R.; Guan, D. A Multi-objective Optimum Approach to the Allocation of Gas Quantity for a Continuous Gas Lift Unit. *Pet. Explor. Dev.* **1995**, *22*, 59–62.
3. Fang, W.Y.; Lo, K.K. A generalized well-management Scheme for reservoir simulation. *SPE Reserv. Eng.* **1996**, *11*, 116–120. [CrossRef]
4. Alarcón, G.A.; Torres, C.F.; Gómez, L.E. Global optimization of gas allocation to a group of wells in artificial lift using nonlinear constrained programming. *J. Energy Resour. Technol.* **2002**, *124*, 262–268. [CrossRef]
5. Ray, T.; Sarker, R. Genetic algorithm for solving a gas lift optimization problem. *J. Pet. Sci. Eng.* **2007**, *59*, 84–96. [CrossRef]
6. Khamehchi, E.; Abdolhosseini, H.; Abbaspour, R. Prediction of maximum oil production by gas lift in an Iranian field using auto-designed neural network. *Int. J. Pet. Geosci. Eng.* **2014**, *2*, 138–150.
7. Mahmudi, M.; Sadeghi, M.T. The optimization of continuous gas lift process using an integrated compositional model. *J. Pet. Sci. Eng.* **2013**, *108*, 321–327. [CrossRef]
8. Al-Lawati, M. Gas-Lift Nodal Analysis Model—Economical Optimization Approach. In Proceedings of the SPE Artificial Lift Conference & Exhibition-North America, Houston, TX, USA, 6–8 October 2014.
9. Dutta-Roy, K.; Kattapuram, J. A new approach to gas-lift allocation optimization. In Proceedings of the SPE Western Regional Meeting, Long Beach, CA, USA, 25–27 June 1997.
10. Khamehchi, E.; Mahdiani, M.R. *Gas Allocation Optimization Methods in Artificial Gas Lift*; Springer: Berlin, Germany, 2017.
11. Buitrago, S.; Rodriguez, E.; Espin, D. Global optimization techniques in gas allocation for continuous flow gas lift systems. In Proceedings of the SPE Gas Technology Symposium, Calgary, AB, Canada, 28 April–1 May 1996.
12. Peixoto, A.J.; Pereira-Dias, D.; Xaud, A.; Secchi, A.R. Modelling and extremum seeking control of gas lifted oil wells. *IFAC-PapersOnLine* **2015**, *48*, 21–26. [CrossRef]
13. Kanu, E.P.; Mach, J.; Brown, K.E. Economic approach to oil production and gas allocation in continuous gas lift (includes associated papers 10,858 and 10,865). *J. Pet. Technol.* **1981**, *33*, 1887–1892. [CrossRef]
14. Fletcher, R. *Practical Methods of Optimization*, 2nd ed.; Wiley: New York, NY, USA, 2013.
15. Deng, L.; Olalotiti-Lawal, F.; Davani, E.; Castieira, D. Hypervolume-Based Multi-objective Optimization for Gas Lift Systems. In Proceedings of the SPE Oklahoma City Oil and Gas Symposium, Oklahoma City, OK, USA, 9–10 April 2019.
16. Raoufi, M.H.; Farasat, A.; Mohammadifard, M. Application of simulated annealing optimization algorithm to optimal operation of intelligent well completions in an offshore oil reservoir. *J. Pet. Explor. Prod. Technol.* **2015**, *5*, 327–338. [CrossRef]
17. Rao, R.V.; Savsani, V.J.; Vakh Aria, D.P. Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems. *Comput. Aided Des.* **2011**, *43*, 303–315. [CrossRef]
18. Ismail, A.M.M.; Alnaqbi, J.M.R.; Mustafa, H.; Su, S.; Alfonzo, A. Efficient and Flexible Reservoir Field Management Gas-Lift Optimization Approach to Unlock the Production Potential & Economic Investment of Giant Carbonate Reservoir. In Proceedings of the Abu Dhabi International Petroleum Exhibition & Conference, Abu Dhabi, United Arab Emirates, 9–12 November 2020.
19. Li, Q.; Zhong, H.; Wang, Y.; Leng, Y.; Guo, C. Integrated development optimization model and its solving method of multiple gas fields. *Pet. Explor. Dev.* **2016**, *43*, 268–274. [CrossRef]
20. Zhong, H.; Zheng, C.; Li, M.; Liu, T.; He, Y.; Li, Z. Transient Pressure and Temperature Analysis of a Deepwater Gas Well during a Blowout Test. *Processes* **2022**, *10*, 846. [CrossRef]