



# *Article* **A Novel Multi-Objective Optimal Design Method for Dry Iron Core Reactor by Incorporating NSGA-II, TOPSIS and Entropy Weight Method**

**Yan Li <sup>1</sup> [,](https://orcid.org/0000-0002-4616-5172) Yifan Liu <sup>1</sup> , Shasha Li <sup>2</sup> , Leijie Qi <sup>1</sup> , Jun Xie 1,[\\*](https://orcid.org/0000-0001-6657-0880) and Qing Xie <sup>1</sup>**

- <sup>1</sup> School of Electrical and Electronic Engineering, North China Electric Power University, Baoding 071003, China<br><sup>2</sup> State Crid Habei Baodina Electric Power Campany Limited, Baoding 071051, China
- <sup>2</sup> State Grid Hebei Baoding Electric Power Company Limited, Baoding 071051, China
- **\*** Correspondence: junxie@ncepu.edu.cn

**Abstract:** Dry iron core reactors are widely used in various power quality applications. Manufacturers want to optimize the cost and loss simultaneously, which is normally achieved by the designers' experience. This approach is highly subjective and can lead to a non-ideal product. Thus, an objective dry iron core reactor design approach to balance the cost and loss with a scientific basis is desired. In this paper, a multi-objective optimal design method is proposed to optimize both the cost and loss of the reactor, which provides an automatic and scientific design method. Specifically, a three-dimensional finite element model of dry iron core reactor is established, based on which the dependency of cost and loss upon the wire size of the reactor's winding is studied by using joint Matlab-finite element method (FEM) simulation. The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is used to search for the Pareto optimal solution set, out of which the optimal wire size of the reactor is determined by using the fusion of the technique for order preference by similarity to ideal solution (TOPSIS) method and the entropy weight method. TOPSIS helps the designer to balance the concern between cost and loss, while the entropy weight method can determine the weight information through the dispersion degree of cost and loss. This methodology can avoid personal random subjective opinion when selecting the design solution out of the Pareto set. The calculation shows that the cost and loss can be reduced by up to 17.85% and 19.45%, respectively, with the proposed method. Furthermore, the obtained optimal design is approved by experimental tests.

**Keywords:** dry iron core reactor; multi-objective optimization; NSGA-II; matlab-finite element; TOPSIS; entropy weight method

#### **1. Introduction**

Electrical reactors are essential equipment in power systems used for reactive power compensation, current limiting, over-voltage suppression, etc. [\[1\]](#page-13-0). There are two types of reactors, namely dry-type and oil-immersed. Compared with oil-immersed reactors, the oil-free dry-type reactors eliminate the risk of oil leakage and fire hazard. Thus, dry-type reactors have gradually overtaken oil-immersed reactors in the 35 kV and below market, especially in places with a high safety awareness such as airports, railway stations, etc. Two kinds of dry-type reactors exist, namely air core and iron core reactors. Dry iron core reactors, which are the focus of this paper, are favored for applications with installation space limitations due to their small size. The great demand has urged manufacturers to pay attention to the design optimization [\[2–](#page-13-1)[4\]](#page-13-2). The current design of dry iron core reactors follows a similar process as a transformer, which relies on electromagnetic field theory to design the core and coil, with loss and temperature rise as the validation checks [\[5\]](#page-13-3). Thus, loss is a key optimization objective for reactor design and has been studied by researchers. For example, in [\[6\]](#page-13-4), the effect of air gaps on the core losses of shunt iron core reactors was analyzed to select the optimal air gap. Fating Yuan et al. optimized the rain cover structure based on loss evaluation [\[7\]](#page-13-5). Takaaki Ibuchi et al. minimized the copper loss



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for dry-type reactors in the high frequency range [\[8\]](#page-13-6). Additionally, manufactures want to reduce their costs, which makes cost also an optimized parameter. Yuan et al. proposed to design dry air core reactors with the minimum amount of metal conductors, which referred to the minimum cost [\[9\]](#page-13-7). In [\[10\]](#page-13-8), genetic algorithms were introduced to reduce the cost of single-phase dry air core reactors.

As discussed, the studies above take only the loss or the cost of reactor as the one optimized parameter. However, it is of more significant engineering importance to optimize the loss and cost simultaneously for a reactor.

The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is an effective method for solving such multi-objective optimization problems [\[11](#page-13-9)[,12\]](#page-13-10), which has been applied in different fields [\[13–](#page-13-11)[16\]](#page-13-12). It has been used to optimize designs for electrical equipment. In [\[17\]](#page-13-13), the optimal design for transformer locations, taking account of investment and operating costs, was realized by using the NSGA-II algorithm. Reference [\[18\]](#page-13-14) used the NSGA-II algorithm to optimize the transformer conductor size considering the eddy current loss and the temperature gradient of the winding. NSGA-II was also applied for the design of a dry-type air-core reactor, with the goal of minimizing production costs and operating costs [\[19](#page-13-15)[,20\]](#page-13-16). However, the NSGA-II algorithm gives a set of Pareto solutions, and the designer may arbitrarily pick one from this set for subsequent design, but not necessarily the best one. Selecting one of the Pareto solutions requires weighing the importance of different objectives, which is normally highly randomly subjective and lacks a scientific basis [\[21\]](#page-13-17). It can be imagined that different electrical reactor designers may choose different solutions from the Pareto set, which leads to products with different performance.

To this end, the research question is to find the best design solution for an electrical reactor with an objective approach, considering both the loss and cost. Thus, the designer can scientifically derive the best solution. In this paper, NSGA-II is utilized for the reactor's design with the cost and loss as two optimized objectives. To overcome the drawbacks of random subjective selection, this paper incorporates the technique for order preference by similarity to ideal solution (TOPSIS) and the entropy weight method into the NSGA-II algorithm. In this way, the designer can focus more on a certain aspect of the design process, i.e., loss or cost, eliminating the disadvantages of random selection. This proposed approach is successfully implemented in the design of a 10 kV three-phase dry iron core reactor.

The paper is organized as follows. Firstly, the three-dimensional (3D) finite element model of a dry iron core reactor is established, based on which the cost and loss of the reactor are taken as two optimized objectives. A set of Pareto optimal solutions are obtained using the joint Matlab-FEM simulation with the NSGA-II algorithm, after which the TOPSIS method and entropy weight method are implemented to determine the optimal wire size in the reactor design process. A validation experiment is performed at the end.

#### **2. Multi-Objective Optimization Modeling of Dry Iron Core Reactors**

#### *2.1. Three-Dimensional Finite Element Model*

A 3D finite element model of a 10 kV three-phase dry iron core reactor was developed using COMSOL Multiphysics software as shown in Figure [1,](#page-2-0) based on which the cost and loss were derived. The dimensions of the model geometric are shown in Appendix [A.](#page-12-0) As shown in Figure [1](#page-2-0) and Appendix [A,](#page-12-0) the structure of the reactor was constructed in detail. There are 6 air gaps in each iron core and 3 layers for each winding. The implementation of the calculation includes geometric design, material addition, electromagnetic field boundary setting and mesh dissection [\[22](#page-13-18)[,23\]](#page-13-19). The relevant material parameters are listed in Table [1.](#page-2-1)

The reactor core diameter is fixed at 160 mm to meet the rated power while the wire size is varied as a control parameter. The reactor cost includes the core cost and winding coil cost, neglecting accessories. Each result is calculated by multiplying the price of related material per unit volume and its total volume.

<span id="page-2-0"></span>

**Figure 1.** Three-dimensional finite element model of reactor.

<span id="page-2-1"></span>**Fable 1.** Relevant material parameters.



The loss of the reactor includes two parts. One is copper loss, caused by the heat generated from the current passing through the winding coil [24]. For the calculation or<br>copper losses, the following equation is derived according to Joule's law: generated from the current passing through the winding coil [\[24\]](#page-13-20). For the calculation of

$$
P_{cu} = I^2 R_1 + I^2 R_2 + I^2 R_3 \tag{1}
$$

where  $R_1 - R_3$  are the three-phase winding resistance.

The other part is the core loss, including hysteresis loss, eddy current loss and additional loss [\[25\]](#page-13-21). Hysteresis loss is generated by the memory of a magnetic material. More force is necessary to demagnetize magnetic material than it takes to magnetize it, and<br>the magnetic demains in the material resist realizement. Eddy current lesses are from circulating currents in the core material, which is induced by an alternating magnetic hysteresis phenomena. Based on the magnetic field distribution in the core, the core loss<br>can be calculated by the following formula [25,27]: The other part is the core loss, including hysteresis loss, eddy current loss and addithe magnetic domains in the material resist realignment. Eddy current losses are from field [\[26\]](#page-13-22). Additional loss is the loss that cannot be directly attributed to eddy current or can be calculated by the following formula [\[25,](#page-13-21)[27\]](#page-13-23):

$$
P_{ir} = P_h + P_{ec} + P_e = afB_m^x + bf^2B_m^2 + ef^{1.5}B_m^{1.5}
$$
 (2)

additional loss; f is the frequency; and  $B_m$  is the flux density amplitude. Values of a, b, x, e where  $P_{ir}$  is the core loss;  $P_h$  is the hysteresis loss;  $P_{ec}$  is the eddy current loss;  $P_e$  is the are dependent upon the selected ferromagnetic material.

are dependent upon the selected ferromagnetic material.<br>The distribution of the final flux density is obtained using the finite element study as shown in Figure 2.

shown in Figure 2.<br>As seen in Figure [2,](#page-3-0) the flux density is larger at the corners of the core, and the magnitude and distribution of the flux density are consistent with the design.

<span id="page-3-0"></span>

**Figure 2.** Magnetic flux density distribution chart.

Figure 2. Magnetic flux density distribution chart.

# 2.2. Multi-Objective Optimization Mathematical Model

The optimal design of reactors is essential to solve a nonlinear problem with constraints. The input variables and objective function are the key to solving such optimization issues.<br>In the design process of the three-phase dry iron core reactor, the sizes of the wire were to form a multi-objective optimization function as shown in Equation (3): The input variables and objective function are the key to solving such optimization issues. In the design process of the three-phase dry iron core reactor, the sizes of the wire were<br>used as input variables. The cost and loss of the reactor were used as the optimized indexes<br>to form a multi-objective optimization solving such optimiza<br>actor, the sizes of the

$$
\begin{cases}\n\min f_1(X) = f(x_1, x_2) \\
\min f_2(X) = f(x_1, x_2)\n\end{cases}
$$
\n(3)

**3. Optimization of Reactor Incorporating NSGA-II and TOPSIS and Entropy Weight** 

where  $x_1$  is the wire length and  $x_2$  is the wire width, as shown in Figure 3.  $f_1$  is the total  $f_1$  and  $f_2$  are taken as the objectives for design optimization. cost while  $f_2$  denotes the loss, which is the sum of Equations (1) and (2), i.e.,  $P_{ir} + P_{cu}$ . The

<span id="page-3-1"></span>

**Figure 3.** Illustration of conductor wire.

#### **Figure 3.** Illustration of conductor wire. **3. Optimization of Reactor Incorporating NSGA-II and TOPSIS and Entropy Weight Method**

optimal solution set. In this paper, the optimal wire size was selected by combining the *XICCA II x*2 Th[e](#page-4-0) NAGA-II algorithm is the optimized design of the dry iron core reactor is shown in Figure 4. Firstly, the 2D model of the reactor was huilt begond on which the reactor less and got were obtained with different wires via FEM. Then, the NSGA-II algorithm was used for iterative The NAGA-II algorithm is widely used in the design of multi-objective optimization;<br>the algorithm performs a non-dominated sorting of all design solutions to find the Pareto encement to both the optimized sets of wires differentiations. This process combining with the entropy weight method was applied to derive an optimal wire size. the algorithm performs a non-dominated sorting of all design solutions to find the Pareto TOPSIS method and entropy weight method on the basis of the NSGA-II algorithm. The the 3D model of the reactor was built, based on which the reactor loss and cost were calculation to obtain the optimized sets of wires' dimensions. Finally, TOPSIS combined

<span id="page-4-0"></span>

**Figure 4.** Dry iron core reactor optimization design process. **Figure 4.** Dry iron core reactor optimization design process.

*3.1. Implementation of NSGA-II Algorithm*

The NSAG-II algorithm was applied to the 3D finite element model of a three-phase dry core reactor with the following procedure.

- (1) Population initialization. Since this paper aims to analyze the effect of different wire size on the optimized indexes, i.e., cost and loss, the common standard wire size was  $\frac{1}{2}$ selected as the minimized population, as shown in facte 2. Twenty-eight combinations of different wire lengths and wire widths were formed. These 28 individuals were entered into the COMSOL model as the initial population to calculate the cost and loss under various wire sizes. selected as the initialized population, as shown in Table [2.](#page-5-0) Twenty-eight combinations
- (2) Fast non-domination sorting and crowding distance calculation of the initial population. Dominance and non-dominance are the focus of the process, as shown in Figure [5.](#page-5-1) If both the cost and loss for individual *i* are smaller than individual *j*, then the cost and loss of individual *i* are less than individual *j*, indicating that *i* dominates *j*. However, though the loss of individual *i* is smaller than that of  $\overrightarrow{i}$  + 1, the cost of individual *i* is higher than that of  $i + 1$ . Thus,  $i$  and  $i + 1$  does not constitute a dominant relationship. This is the same for individuals *i* and  $i - 1$ . All the solutions in the population that are not dominated by other solutions constitute the non-dominated population that are not dominated *frontier, i.e., the Pareto optimal solution set. The rank of all the individuals in the range dominated frontiers and the 1. A flow that all the individuals with some dominated from the solution set. The range of the stat* removed and the above operation was repeated for the rest of the population until the non-dominated sorting of all the individuals was completed. The crowding distance of individuals under the same dominance rank was sorted and calculated according to the magnitude of the optimized indexes, where the crowding distance of two boundary solutions is infinity and the crowding distance of the *i*th individual can be defined as the sum of its distances to individual  $i - 1$  and individual  $i + 1$  on each original as the sum of the sum of the sum of the sum of individual  $\overline{\phantom{a}}$ axis component, which is half of the perimeter of the rectangle in Figure [5.](#page-5-1) *i* dominates *j*. Otherwise it is a non-dominated relationship. As shown in Figure [5,](#page-5-1) non-dominated frontier was set to 1. After that, all the individuals with rank 1 were
- (3) Selection, crossover and mutation. The parent that is suitable for reproduction is selected from the original population to form the elite population. Each selection is a consected from the original population to voint the one population. Each observer is a competition between two randomly selected individuals from the original population to compare their non-dominance rank, and the individual with smaller rank will be selected for the elite population. If the non-dominance rank of two individuals is the same, the individual with the larger crowding is selected to enter the elite population to ensure the diversity of the population. Since there were 28 individuals population to obtain the final of spring population. in the initial population, an elite population containing 14 individuals was obtained after the selection process. After that, a suitable cross-variance operator was selected after the selection process. After that, a suitable cross-variance operator was selected to cross-variance this population to obtain the offspring population, and the final offspring were 14-28 because the probability of cross-variance was different. to compare their non-dominance rank, and the individual with smaller rank will be selection, crossover and mutation. The parent that is suitable for reproduction is
	- (4) Merging parent and child generations to generate a new population. The parentchild population was merged into a new population, and the elite retention strategy was used to calculate the new population's non-dominated sorting and crowding  $\frac{1}{2}$ distance. Afterwards, a new population containing 28 individuals was selected to matrice: The rwards, a new population containing 20 marviolation was selected to participate in the subsequent evolution based on the sorting and crowding distance. The subsequent step was to repeat the process of  $(3)$ – $(4)$  until the number of iterations was satisfied to obtain the Pareto optimal solution set. The above process of NSGA-II can be represented as Figure [6,](#page-6-0) where Gen indicates the number of generations of genetic evolution.

<span id="page-5-0"></span>**Table 2.** Selected line length and line width. **Table 2.** Selected line length and line width.



<span id="page-5-1"></span>

Figure 5. Schematic diagram of dominance relationships and crowding distance. (There is no dominance relationship between the blue dots. The black dot is dominated by the blue dot. The perimeter of the rectangle represents 2 times crowding distance of the *i*th individual).

<span id="page-6-0"></span>

**Figure 6.** NSGA-II algorithm flowchart. **Figure 6.** NSGA-II algorithm flowchart.

## *3.2. Implementation of TOPSIS Method 3.2. Implementation of TOPSIS Method*

ITHE NSGA-II algorithm produces an optimal solution set. In order to obtain the optimal solution from it, the TOPSIS method [\[28\]](#page-14-0) was used. The basic principle of TOPSIS timal solution from it, the TOPSIS method [28] was used. The basic principle of TOPSIS is is to rank the evaluation objects by detecting the distance between the ideal solution and to rank the evaluation objects by detecting the distance between the ideal solution and the the negative ideal solution. If the evaluation object is closest to the ideal solution and far negative ideal solution. If the evaluation object is closest to the ideal solution and far from from the negative ideal solution, the object is the best design solution, and vice versa. The NSGA-II algorithm produces an optimal solution set. In order to obtain the

the negative ideal solution, the object is the best design solution, and vice versa. In order to obtain the optimal individual in the optimal solution set obtained by the In order to obtain the optimal individual in the optimal solution set obtained by the NSGA-II algorithm, the samples to be evaluated and their evaluation indexes are collected  $\frac{1}{2}$  algorithm, the samples to be evaluated and the samples to be evaluated and the evaluation indexes are collected and the evaluation indexes are collected and the evaluation indexes are collected and the evaluatio at first. If there are *m* evaluation samples and *n* evaluation indexes, the original matrix as in (4) can be formed in (4) can be formed. in (4) can be formed.

$$
X = \begin{pmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{pmatrix}
$$
 (4)

The collected data are normalized for the indicators. Specifically for this paper, the  $\sigma$  are normalized for the indicators. Specifically for the indicators. Specifically for this paper, the indicators. Specifically for the indicators. Specifically for the indicators. Specifically for the indicators. Spec two evaluation indicators of cost and loss are normalized according to Equation (5):

$$
x_{ij} = \max_{1 \le i \le m} \{x_{ij}\} - x_{ij}
$$
\n<sup>(5)</sup>

*i* the *i*th evaluation object. where *xij* is the *j*th evaluation index of the *i*th evaluation object.

In order to eliminate the influence of different indicators, we need to use the method of vector normalization to obtain the normative decision matrix, as shown in Equation (6), where *zij* is the *j*th evaluation index of the *i*th evaluation object after normalization, and *Z* is the normative decision matrix.

$$
z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}
$$
 (6)

$$
Z = \begin{pmatrix} z_{11} & \dots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{m1} & \dots & z_{mn} \end{pmatrix}
$$
 (7)

Based on Equations (8) and (9), the ideal solution  $Z^+$  and the negative ideal solution *Z* <sup>−</sup> of each index can be found.

$$
Z^{+} = (max\{z_{11}, \cdots, z_{m1}\}, \cdots, max\{z_{1n}, \cdots, z_{mn}\})
$$
(8)

$$
Z^{-} = (\min\{z_{11}, \cdots, z_{m1}\}, \cdots, \min\{z_{1n}, \cdots, z_{mn}\})
$$
\n(9)

The distance between the *i*th evaluation object and the ideal/negative ideal solution, are defined as follows, respectively:

$$
D_i^+ = \sqrt{\sum_{j=1}^n (Z_j^+ - z_{ij})^2}
$$
 (10)

$$
D_i^- = \sqrt{\sum_{j=1}^n (Z_j^- - z_{ij})^2}
$$
 (11)

where  $D_i^+$  is the distance between the *i*th evaluation object and the ideal solution,  $D_i^ \int_{i}^{-}$  is the distance between the *i*th evaluation object and the negative ideal solution,  $Z_j^+$  is the ideal solution of the *j*th evaluation index, and  $Z_i^{\text{-}}$ *j* is the negative ideal solution of the *j*th evaluation index.

The score of the evaluation object is finally determined with Equation (12). Obviously,  $0 \le C^i \le 1.$  The default  $n$  evaluation indicators of the formula have the same weight, but different designers may place different weight on loss or cost. One can use an analytic hierarchy process to give weights to these *n* evaluation indicators according to Equation (13). The analytic hierarchy process needs weight information as input, which is normally carried out by subjective assignment and can be biased. The entropy weight method instead can be used to obtain objective values through the dispersion degree of loss and cost.

$$
C^i = \frac{D_i^-}{D_i^+ + D_i^-}
$$
 (12)

$$
\sum_{j=1}^{n} w_j = 1
$$
\n(13)

#### *3.3. Implementation of the Entropy Method*

Entropy is a measure of the degree of disorder of the system [\[29\]](#page-14-1). Thus, for a certain index (such as cost and loss in this paper), the entropy value can be used to determine its dispersion degree. The smaller its entropy value, the greater the dispersion degree of the index and the greater the influence of the index on the design evaluation (i.e., the weight). The entropy method effectively avoids the interference of subjective factors and makes the weight determination of evaluation indicators more objective [\[30](#page-14-2)[,31\]](#page-14-3). In this paper, TOPSIS combined with the entropy weighting method was used to obtain the weights.

(1) First, the weight of the *j*th index value of the *i*th sample was calculated and the weight matrix *P* was built.

$$
p_{ij} = \frac{x_{ij}}{\sum\limits_{i=1}^{m} x_{ij}} \tag{14}
$$

$$
P = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mn} \end{pmatrix}
$$
 (15)

(2) Calculate the entropy  $e_i$  of the *j*th index according to Equation (16). A larger value means less variation. The redundancy value *d<sup>j</sup>* of the *j*th index is obtained referring to Equation (17), and finally the entropy weight  $w_j$  of each index is obtained by normalizing it based on Equation (18). Obviously, the entropy weight sum of all indicators is 1.

$$
e_j = -\frac{1}{\ln m} \sum_{i=1}^{m} p_{ij} \ln p_{ij} \, \mathbf{e}_j \in [0, 1] \tag{16}
$$

$$
d_j = 1 - e_j \tag{17}
$$

$$
w_j = \frac{d_j}{\sum\limits_{j=1}^n d_j} \tag{18}
$$

Based on the entropy weighting  $w_j$ , the weighting matrix  $z_{ij}$ \* and the weighting matrix are constructed as shown in Equations (19) and (20).

$$
z_{ij}^* = z_{ij} \cdot w_j \tag{19}
$$

$$
Z^* = \begin{pmatrix} z_{11} \cdot w_1 & \dots & z_{1n} \cdot w_n \\ \vdots & \ddots & \vdots \\ z_{m1} \cdot w_1 & \dots & z_{mn} \cdot w_n \end{pmatrix}
$$
 (20)

#### **4. Algorithm Implementation**

In this paper, the finite element model of a three-phase iron core reactor was established by Comsol software(Manufacturer: COMSOL, city: Stockholm, country: Sweden). The NSGA-II algorithm, TOPSIS method and entropy weight method were implemented in Matlab(Manufacturer: MathWorks, city: Massachusetts, country: USA). The joint simulation derives the cost and loss of the reactor with different wire sizes from the finite element model. The optimized wire size was selected by integrating NSGA-II, TOPSIS and the entropy weight method, which can significantly save human operation time and avoid subjective design preference.

In this example, 28 individuals were selected, and the convergence of NSGA-II algorithm was achieved by iteration. Figure [7](#page-9-0) shows the loss and cost distribution of the initial population. The 28 initial individuals were dispersed with multiple domination relationships. Figure [8](#page-9-1) shows the same result for the fifth generation. It can be seen that after the algorithm iteration is completed, there is no domination relationship between all individuals, which indicates the dominated individuals are removed and the remaining ones constitute the Pareto optimal solution set. When the number of evolutionary generations exceeds 5, the offspring have all been located in the Pareto dominance frontier. Therefore, the Pareto frontier curve at the 5th generation was selected as the optimal solution set in this study [\[32\]](#page-14-4).

<span id="page-9-0"></span>

this study  $\frac{1}{2}$ .

Figure 7. Distribution of the initial population in space. (An asterisk corresponds to an individual's cost and loss).  $\frac{1}{1}$ contra  $\frac{1}{1}$ 

<span id="page-9-1"></span>

Figure 8. Distribution of populations in space after 5 generations of genetic evolution. (An asterisk corresponds to an individual's cost and loss.) corresponds to an individual's cost and loss.) corresponds to an individual's cost and loss).

Table 3 [sh](#page-10-0)ows the final iterative results. The loss was given by copper loss and iron loss, respectively. The sum of both is the total loss. The price of iron core material was taken as CNY 16/kg, and the price of copper conductor material was taken as CNY 55/kg. The maximum loss was 1.5954 kW ( $P_{max}$ ) and the minimum value was 1.0786 kW ( $P_{min}$ ). Based on Equation (21), the difference of loss  $\Delta P$  was calculated and the result was 32.39%. Similarly, the variation of cost can be calculated to be 29.62%. Similarly, the variation of cost can be calculated to be 29.62%. Similarly, the variation of cost can be calculated to be 29.62%.

$$
\Delta P = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}}} \tag{21}
$$

Figur[e](#page-10-1) 9 were obtained. It can be seen that with the increase of cost weight, cost decreases in Figure 9 were obtained. It can be seen that with the increase of cost weight, cost while loss increases. The trend validates the TOPSIS method applied to the wire selection. Using the TOPSIS method to assign different weights to the two indexes, the results Using the TOPSIS method to assign different weights to the two indexes, the results in The TOPSIS method integrated with the entropy weight method provides a scientific way to obtain the weight of the indicators, avoiding the subjectivity of random selection. Based on Equations (16)–(18), the cost weight of 0.5051 and loss weight of 0.4949 were obtained, which provides an objective basis for the selection of wire size. The corresponding result is also shown in Figure [9,](#page-10-1) with the detailed values in Appendix [B.](#page-12-1) The loss and cost results

obtained by the TOPSIS–entropy weight method are shown in the 7th row of Appendix [B.](#page-12-1) Compared with the other design results, this solution balances the cost and loss. It is located near the crossing point of the loss and cost curve in Figure [9,](#page-10-1) which represents the optimal design solution. Following the calculation in Equation (21), the final design reduced loss up to 19.45% and cost up to 17.85%, respectively.

**Wire Length (mm) Wire Width (mm) Copper Loss (kW) Iron Core Loss (kW) Total Loss (kW) Core Quality (kg) Copper Quality (kg) Cost (CNY 10,000) Overcrowding Distance** 3.8070 1.9994 0.8030 0.2756 1.0786 716.97 348.20 3.0622 Inf 3.1459 1.3873 1.3586 0.2368 1.5954 673.86 195.81 2.1551 Inf 3.1503 1.4846 1.2739 0.2369 1.5108 674.23 210.05 2.2340 0.2298 3.4231 2.0134 0.8867 0.2530 1.1397 691.95 315.48 2.8422 0.1878 3.3500 2.0000 0.9114 0.2487 1.1601 687.23 306.40 2.7848 0.1721 3.1599 1.4258 1.3186 0.2375 1.5561 674.71 202.17 2.1915 0.1690 3.1377 1.6171 1.1816 0.2365 1.4181 673.44 228.94 2.3367 0.1674 3.6784 1.9838 0.8368 0.2678 1.1045 708.45 333.57 2.9681 0.1575 3.7500 2.0000 0.8149 0.2720 1.0869 713.13 343.08 3.0279 0.1539 3.1632 1.9212 1.0008 0.2388 1.2396 675.07 277.14 2.6044 0.1521 3.5645 2.0024 0.8560 0.2614 1.1175 701.13 326.46 2.9173 0.1493 3.1500 2.0000 0.9689 0.2375 1.2064 674.15 288.21 2.6638 0.1479 3.0589 1.7147 1.1482 0.2321 1.3803 668.22 237.37 2.3747 0.1444 3.1713 2.0382 0.9461 0.2390 1.1851 675.51 296.13 2.7095 0.1412 3.1351 1.8900 1.0250 0.2366 1.2616 673.18 269.75 2.5607 0.1311 3.1500 1.7944 1.0697 0.2374 1.3071 674.17 256.42 2.4890 0.1309 3.1447 1.5271 1.2431 0.2366 1.4796 673.80 215.95 2.2658 0.1100 3.1449 1.5331 1.2385 0.2366 1.4751 673.75 216.92 2.2711 0.1054 3.1492 1.7106 1.1179 0.2370 1.3549 674.11 243.76 2.4193 0.1034 3.3877 2.0000 0.9013 0.2509 1.1523 689.60 309.94 2.8080 0.1029 3.3308 1.4819 1.2073 0.2466 1.4539 685.97 221.57 2.3162 0.0995 3.1412 1.7553 1.0945 0.2372 1.3317 673.57 249.87 2.4520 0.0947 3.1487 1.9188 1.0066 0.2375 1.2441 673.97 275.19 2.5919 0.0908 3.1347 1.8575 1.0414 0.2368 1.2783 673.11 264.74 2.5330 0.0901 3.1506 1.4171 1.3301 0.2369 1.5670 674.14 200.14 2.1794 0.0833 3.5416 2.0276 0.8518 0.2597 1.1115 699.65 328.85 2.9281 0.0811 3.0693 1.8805 1.0516 0.2335 1.2851 668.95 262.75 2.5155 0.0769 3.1494 1.6784 1.1376 0.2374 1.3750 674.14 238.81 2.3921 0.0733 2 0693 **18805 10516 0.2335 1.2851 668 95 262 75 2.5155 0.0769** 

<span id="page-10-0"></span>**Table 3.** NSGA-II algorithm combined with finite element model simulation results.

<span id="page-10-1"></span>

Figure 9. The cost and loss with/without integrating entropy weight method. Figure 9. The cost and loss with/without integrating entropy weight method.<br> **Figure 9.** The cost and loss with/without integrating entropy weight method.

#### **5. Experiment Validation 5. Experiment Validation**

The optimization result gave the desired wire dimension as  $3.07$  mm  $\times$  1.88 mm, which is not a standard wire size. Thus, the closest size of 3.15 mm × 2 mm was selected. The trial reactor was made with this wire, and the loss test was conducted. The principle The trial reactor was made with this wire, and the loss test was conducted. The principle is shown in Figure [10a](#page-11-0) and the measurement is shown in Figure 10b. The test result was is shown in Figure 10a and the measurement is shown in Figure 10b. The tes[t re](#page-11-0)sult was 1.25 kW, with an error of less than 5% compared to the calculated value, which verified 1.25 kW, with an error of less than 5% compared to the calculated value, which verified the accuracy of the simulation calculation. The cost was also calculated, showing that the accuracy of the simulation calculation. The cost was also calculated, showing that the the maximum cost saving of this design solution was about CNY 4000 compared to other maximum cost saving of this design solution was about CNY 4000 compared to other wire wire options.

<span id="page-11-0"></span>

**Figure 10.** (**a**) The principle of loss test. (**b**) Loss measurement for the trial reactor. **Figure 10.** (**a**) The principle of loss test. (**b**) Loss measurement for the trial reactor.

#### **6. Conclusions**

In this paper, an optimal design approach for a dry iron-core reactor was proposed<br>
In this paper, an optimal design approach for a dry iron-core reactor was proposed to balance its loss and cost. To implement the method, a three-dimensional FEM was constructed with the detailed geometry of the reactor. Joint Matlab-FEM calculation was utilized to obtain the magnetic field distribution and the reactor loss with arbitrary wire dimensions. The calculated result matches the tested value with less than 5% error. Additionally, the cost can be easily derived with the materials' information. to balance its loss and cost. To implement the method, a three-dimensional FEM was

Based on the three-dimensional model, the influence of wire size on a three-phase dry iron core reactor's cost and loss was analyzed. To achieve an optimal design, cost and loss were used as the objective functions to perform the multi-objective optimization. The approach of integrating NSGA-II, TOPSIS and the entropy weight method is proposed for the design of the reactor. This novel approach can not only optimize the two parameters simultaneously, but also avoid the randomness of personal choice, which offers better quality guarantee for manufacture. The method was validated via a 10 kV reactor design. The trial product validates the method by comparing the loss and cost. The results show that the cost and loss of the reactor can be reduced by 17.85% and 19.45%, respectively.

> This method can also be used for the design of other types of reactors and other electrical equipment, such as transformers. More constraints or objectives can be integrated into the algorithm for future research work, for example the size/weight of the product and the percentage of environmentally friendly materials that are used.

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## <span id="page-12-0"></span>**Appendix A**



**Figure A1.** The dimensions of the model geometric. **Figure A1.** The dimensions of the model geometric.

## <span id="page-12-1"></span>**Appendix B**

**Table A1.** The detailed values in Figure [9.](#page-10-1)



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