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Extension and Correction of Budeanu Power Theory Based on Currents' Physical Components (CPC) Theory for Single-Phase Systems

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Abstract: In 1927, the most recognized power theory in the frequency domain was proposed by Budeanu. The second power theory in the frequency domain, which is currently catching a lot of supporters, is the approach proposed by Czarnecki. Both theories have common features in the form of the description of active power and are completely different in terms of the description and interpretation of reactive power. This article presents the possibility of using mutual elements of both approaches: thus, it is possible to interpret the physical meaning of the reactive power (reactive current) proposed by Budeanu and the power before the deformation obtained from the mathematical description.

Keywords: Budeanu theory; currents' physical components (CPC) theory; equivalent parameters; nonsinusoidal systems; single-phase systems

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1. Introduction

The basic purpose of any power theory is the appropriate description of the energy properties of a load supplied, in a sense, from any voltage source. The first person who decided to describe single-phase systems with the use of power equations was professor Constantin Budeanu [1–4]. The relationships given by Budeanu are the most frequently used in electrical engineering, although a large number of people (regardless of age and skills) are not aware of the fact of using the power theory. A great advantage of the Budeanu theory is its simplicity, while its significant weakness is the misinterpretation of reactive and distortion powers [4,5]. Additionally, until the publication of this article, the power components included in the distortion power proposed by Budeanu are not known. Moreover, Budeanu was of the opinion that there should be a right triangle formed by lines whose lengths represented the magnitude of the average power, the apparent power, and the reactive power. That triangle was noted by Fleming, in the paper in which he introduced the term "power factor" in 1892 [6]. Fleming noted that the right triangle was applicable only if the signals were all sinusoidal.

In the 1980s, a publication by professor Czarnecki [7] was published, which unmistakable showed that the distortion power proposed by Budeanu has no signs of distortion and the definition of reactive power, resulting from the sum of individual harmonics, is inappropriate considering the possibility of accomplishing a zero value [2,4,8] despite the presence of a reactive current causing an increase in the RMS value of the load current. Moreover, Emanuel was responsible for including distortion power in IEEE Standard 1459. A more recent paper [9,10] shows that since the harmonic components of the current and voltage are orthogonal, the distortion power must always have zero value.

Over the years, countless scientists have attempted to describe the components of the Budeanu theory [5,11–13]. The change in the description of the Budeanu theory was and is intended to improve the value of the power factor [5,11,12,14,15]. Additionally, single-phase systems are currently used to supply wireless power transmission (WPT) devices [16–20]. The WPT system, built of appropriately selected reactance elements, is also exposed to losses resulting from the nonsinusoidal supply and nonlinearity resulting from the control system. In such systems, over time, an appropriate energetic description between voltage and current is also applicable. Additionally, such systems are also subject to passive compensation, hybrid compensation, or the use of active power filters [21–28]. So there is a strict dependence on the amount of energy transferred.

In the area of modern lighting using semiconductor LED sources, there are also problems ensuing from the incorrect consideration of these types of light points in terms of their energy characteristics. The LED power supply systems are AC/DC converters, characterized by the efficiency depending on the quality of the supply voltage, and therefore they may impact the inappropriate interpretation of the energy description [29–31]. In addition, LED sources, through their internal construction, regulate the current on the DC side, mainly due to the RMS voltage. Then most of such systems can cope and keep the value of the luminous flux at an unchanged level. The problem appears when the voltage distortion occurs, and then the current control system is no longer as effective and there is a decrease or increase in the luminous flux. The problem worsens in systems that are not compensated, as the distorted current produced by the LED sources negatively affects the supply voltage.

Another aspect to be addressed nowadays is microgrids related to renewable energy sources [32–36]. Regardless of the source of electricity obtained (e.g., sun, wind, water), the whole circuit is controlled by a frequency converter. On the power source side, such a system is a rectifier and therefore generates capacitive reactive current despite the fact that it is controlled to a power factor close to the value of 1. Microgrids of small power up to a value of less than 4 kW are single-phase systems, while the rest are configured into three-phase systems.

The purpose of this article is to present the energy properties of single-phase loads described by the developed Budeanu theory. By energy properties, the authors understand the possibility of determining equivalent parameters of the load and linking them, through the supply voltage, with the current components. Each component of the current described in the correction of Budeanu's theory has a physical interpretation. Through this description, it is possible to determine the parameters of passive compensators, which are able to compensate the currents associated with the reactive elements. For many years, the authors of the publication have been dealing with the aspects of the energetic description of three-phase systems under conditions of supply voltage distortion due to amplitude and phase asymmetry [37–39]. In addition to the description of energy properties, the authors always propose the possibility of using such a description for the construction of reactance compensators [40,41]. The development of the Budeanu theory and the interpretation of the current components is possible only due by using the currents' physical components (CPC) theory [2,4,7,8,15].

List of main names:

i—line current, i_a—active current, i_r—reactive current, i_s—scattered current,
i_{rB}—Budeanu reactive current, i_{crB}—Budeanu complemented reactive current,
G_e—equivalent conductance, G_n—harmonic conductance,
B_e—equivalent susceptance, B_n—harmonic susceptance,
$\boldsymbol{Y}_n$—load admittance for harmonics,
S—apparent power, P—active power, Q—reactive power,
D_S—scattered power, Q_B—Budeanu reactive power,
Q_{crB}—Budeanu complemented reactive power.

2. Currents' Physical Components (CPC) and Budeanu Theories

The basic assumption of the currents' physical components theory, as the name suggests, is the physical interpretation of the individual current components resulting from the equivalent parameters of the load. Only in the result (secondary value) is the transition to the power equation.

A single-phase load (Figure 1) can be supplied by the periodic voltage, whose waveform can be described as [2]:

$$u(t) = u = U_0 + \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} \quad (1)$$

where U_0—direct component of the voltage, $\boldsymbol{U}_n$—complex RMS (CRMS) value of the voltage harmonic, and n—harmonic order.

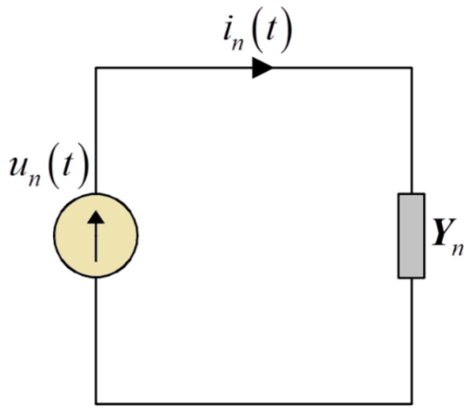

Figure 1. Scheme of a single-phase system supplied from a nonsinusoidal voltage source.

The direct component of the supply voltage can be left out, and then the relation (1) is shortened to the form [2]:

$$u(t) = u = \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} \quad (2)$$

The result of supplying voltage (2) to the terminals of the load from Figure 1 is the line current, the waveform of which is described by the equation [2]:

$$i(t) = i = \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{Y}_n \boldsymbol{U}_n e^{jn\omega_1 t} = \sqrt{2}\mathrm{Re}\sum_{n\in N} \boldsymbol{I}_n e^{jn\omega_1 t} \quad (3)$$

where $\boldsymbol{Y}_n$—load admittance for harmonics and $\boldsymbol{I}_n$—CRMS value of the current harmonic.

If the linear load from Figure 1 is built on resistance and reactance, then its admittance, for harmonic frequencies, is [2]:

$$\boldsymbol{Y}_n = Y_n e^{-j\varphi_n} = G_n + jB_n \quad (4)$$

where G_n and B_n mean conductance and susceptance for harmonics.

Based on the dependence (2) and (4), it is possible to determine the waveform of the active current i_a, for which the load is equivalent to the original load due to the active power P resulting from the sum of the active powers of individual harmonics, namely [2]:

$$i_a = \sqrt{2}\mathrm{Re}\sum_{n\in N} G_e \boldsymbol{U}_n e^{jn\omega_1 t} \quad (5)$$

where the equivalent conductance G_e of the whole system is described by the formula [2]:

$$G_e = \frac{\sum\limits_{n\in N} P_n}{\|u\|^2} = \frac{P}{\|u\|^2} \quad (6)$$

The waveform of the reactive current i_r, for which the susceptances of individual harmonics are responsible, takes the form [2]:

$$i_r = \sqrt{2}\text{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n e^{jn\omega_1 t} \tag{7}$$

The RMS value of the active current $\|i_a\|$ and the reactive current $\|i_r\|$ are [2]:

$$\|i_a\| = G_e\|u\| = \frac{P}{\|u\|} \tag{8}$$

$$\|i_r\| = \sqrt{\sum_{n\in N} (B_n U_n)^2} \tag{9}$$

The active current i_a is proportional to the supply voltage, while the reactive current i_r is shifted by a quarter of a period in regard to this voltage. It does not mean, however, that the system consists only of two components of the current. As a result of changing the conductance value for individual harmonics in the system, the scattered current i_s may also be present, which waveform is defined as follows [2]:

$$i_s = \sqrt{2}\text{Re}\sum_{n\in N} (G_n - G_e)\boldsymbol{U}_n e^{jn\omega_1 t} \tag{10}$$

The RMS value of the scattered current $\|i_s\|$ is [2]:

$$\|i_s\| = \sqrt{\sum_{n\in N} [(G_n - G_e)\boldsymbol{U}_n]^2} \tag{11}$$

Based on the dependencies (5), (7), and (10), the current of a single-phase load has three components [2]:

$$i = i_a + i_r + i_s \tag{12}$$

and due to the orthogonality of components [2,7]:

$$\|i\|^2 = \|i_a\|^2 + \|i_r\|^2 + \|i_s\|^2 \tag{13}$$

This means that the load current is the sum of active current, reactive current, and scattered current. Any of the components are related to a different physical phenomenon. The active current i_a is related to the consistent flow of energy from the source to the load. The reactive current i_r is related to the displacement of the load current in relation to the supply voltage. The scattered current i_s is the result of the change in the conductance value with the order of harmonics. The three abovementioned currents can be accurately related to distinctive physical phenomena in the system. They can, therefore, be treated as the currents' physical components of the current of a load supplied with a single-phase nonsinusoidal voltage.

The theory that is the most widespread and most frequently used in practice and in classes, with students, is the Budeanu theory. According to Budeanu's theory, the active power PB of a single-phase load supplied from a nonsinusoidal voltage source is [2,7]:

$$P_B = \sum_{n=0}^{\infty} \boldsymbol{U}_n I_n \cos\varphi_n \tag{14}$$

By analogy with the definition of the active power of P_B, the professor from Bucharest proposed a description of the reactive power Q_B, namely [2,7]:

$$Q_B = \sum_{n=1}^{\infty} U_n I_n \sin\varphi_n \tag{15}$$

Based on the dependence (14) and (15), the apparent power S of the load is or should be [2,7]:

$$S^2 \geq P_B^2 + Q_B^2 \tag{16}$$

To explain why the apparent power S is bigger than the root of the sum of the squares of the active power P_B and the reactive power Q_B have been added (on the basis of mathematical notation only) in the distortion power [2,7]:

$$D_B = \sqrt{S^2 - \left(P_B^2 + Q_B^2\right)} \tag{17}$$

In the publication from 1987 [7], it was presented why the distortion power D_B has no signs of distortion, and in the works [2,4,7] the inaccuracy of the Formula (15) of the reactive power of the considered system supplied from a single-phase nonsinusoidal voltage source has been described.

From time to time there are works [5,11,12] trying to develop the Budeanu theory in a way more or less understandable by the average reader. At this point, the authors of the publication raise questions, the answers to which are presented in the following chapters of the publication:

(a) Question 1—Is it possible to determine the equivalent parameters of a single-phase load that allows the physical interpretation of the current components?
(b) Question 2—Does the formula of the reactive power (15) proposed by Budeanu have the limitations presented by Czarnecki [7] or maybe more limitations? If so, why and what are these limitations?
(c) Question 3—Why, using the Budeanu theory in the description of compensators' parameters, is it not possible to achieve the power factor equal to unity? What is the reason for this situation, and how can it be changed?

3. Currents' Components of a Single-Phase Load Based on Extension and Correction of Budeanu's Theory

As has been already mentioned, the authors of many publications over the years have tried, and are probably still trying, to develop the Budeanu theory into a description that allows it to be effectively used to describe the energy properties of single-phase loads and to use it to improve the value of the power factor. Czarnecki's approach was used to demonstrate the well-defined elements in Budeanu theory.

The first step in correcting the Budeanu theory is to present what is actually right in the theory. The definition of the active power P is certainly well-defined. Both in the Budeanu and Czarnecki approaches, the active power, and thus also the active current, are equal:

$$P_{CPC} = P_B = P \tag{18}$$

and

$$i_{aCPC} = i_{aB} = i_a \tag{19}$$

In view of the equivalence of active powers in both approaches, it will be marked as P. This will be the same with the use of active current i_a.

The interpretation and the values of reactive powers are completely different, because:

$$Q_{CPC} \neq Q_B \tag{20}$$

and

$$i_{rCPC} \neq i_{rB} \tag{21}$$

It is possible to describe the dependencies (20) and (21) more precisely:

$$Q_{CPC} > Q_B \tag{22}$$

and

$$i_{rCPC} > i_{rB} \tag{23}$$

Therefore, reactive powers and reactive currents will be marked with appropriate subscripts.

It has been known, from Equation (17), that there is some power D_B in the system, which is caused by some undefined current components. This is where Czarnecki's approach and the scattered current he defined (10) come with help. Writing at this point the equation of the current of a single-phase load defined in the developed Budeanu theory received:

$$i = i_a + i_{rB} + i_s + i_X \tag{24}$$

the waveform of the Budeanu's reactive current i_{rB} is obtained as follows:

$$i_{rB} = \sqrt{2}\mathrm{Re}\sum_{n\in N} jB_e \boldsymbol{U}_n e^{jn\omega_1 t} \tag{25}$$

In Formula (25), a new quantity B_e appears, which is the equivalent susceptance of the entire system. It is achieved in the same way as the equivalent conductance G_e, that is:

$$B_e = \frac{-\sum\limits_{n\in N} Q_n}{\|u\|^2} = \frac{-Q_B}{\|u\|^2} \tag{26}$$

The RMS value of the Budeanu's reactive current $\|i_{rB}\|$ is defined as follows:

$$\|i_{rB}\| = |B_e|\|u\| = \frac{|Q_B|}{\|u\|} \tag{27}$$

In Formula (24), one more current is present, marked as i_X. It can be easily obtained by:

$$i_X = i - (i_a + i_{rB} + i_s) \tag{28}$$

However, in this way, it is still not known what equivalent parameter of a single-phase load is responsible for the generation of this current, and whether it is realizable to influence this parameter—whether it is a purely susceptive element, purely conductive, or whether it has both of these quantities.

In order to be able to obtain the equivalent parameter of the load, which would be responsible for the generation of the current described as i_X, it is necessary to look at the generation of the scattered current defined by Czarnecki. As it has already been written, it obtains as a result of the differences between the equivalent conductance G_e and the conductances for individual harmonics G_n. Since the Budeanu's reactive current i_{rB} is formed from the sum of the reactive powers of the individual harmonics Q_{Bn}, then in the search for the equivalent parameter of the load responsible for the generation of the current i_X, the procedure should be the same:

$$i_X = \sqrt{2}\mathrm{Re}\sum_{n\in N} j(B_n - B_e)\boldsymbol{U}_n e^{jn\omega_1 t} = i_{crB} \tag{29}$$

As can be noticed from the dependence (29), the current denoted as i_X is associated only with susceptance elements, so in the following, it will be determined as i_{crB} and is called the Budeanu complemented reactive current. The RMS value of this current is:

$$\|i_{crB}\| = \sqrt{\sum_{n\in N}[(B_n - B_e)\boldsymbol{U}_n]^2} \tag{30}$$

Equation (24) can be presented as follows:

$$i = i_a + i_{rB} + i_s + i_{crB} \tag{31}$$

According to the fact that the components of the Formula (31) are mutually orthogonal (Appendix A), the following is correct:

$$\|i\|^2 = \|i_a\|^2 + \|i_{rB}\|^2 + \|i_s\|^2 + \|i_{crB}\|^2 \tag{32}$$

Multiplying the current components by the square of the RMS value of the nonsinusoidal voltage $\|u\|$ obtains the power equation describing a single-phase load:

$$S^2 = P^2 + {Q_B}^2 + {D_S}^2 + {Q_{crB}}^2 \tag{33}$$

It can be noticed that the deformation power D_B, defined in relation (17), consists of two powers:

$${D_B}^2 = {D_S}^2 + {Q_{crB}}^2 \tag{34}$$

As can be noticed from the Formula (34), the distortion power defined by Budeanu consists of the power related to the scattered current, i.e., it is related to the change of conductance along with the order of harmonics, and the complemented reactive current of the Budeanu related to the reactive elements of the load, i.e., the change of susceptance along with the order of the harmonic.

Additionally, as it turns out, reactive power, defined by Czarnecki, also has two power components:

$${Q_{CPC}}^2 = {Q_B}^2 + {Q_{crB}}^2 \tag{35}$$

Perhaps not everything is easily identifiable on the basis of the formulas themselves, and therefore, in Sections 4–6 of this article, several theoretical illustrations which clearly explain the issues of the physical interpretation of current components in the developed Budeanu theory are presented.

4. Theoretical Illustration 1

For the theoretical illustration, a single-phase system supplied with a nonsinusoidal voltage, from Figure 2, has been chosen. All calculations were accomplished assuming the linearity of the load and the invariability of its parameters in time.

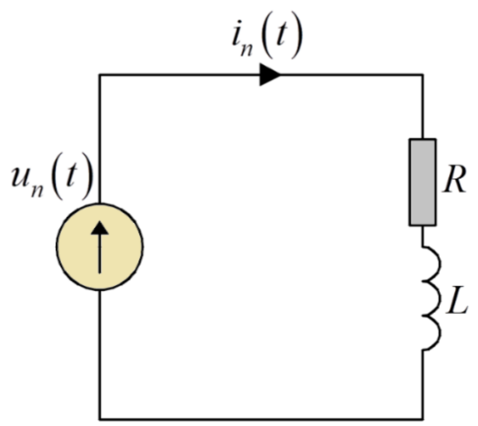

Figure 2. Scheme of the single-phase system used for theoretical calculations.

The values of resistances and inductances, shown in Figure 2, are listed in Table 1.

Table 1. List of values of resistance and inductance.

Harmonic Order	R [Ω]	L [mH]	C [mF]
$n = 1$	2	5	-

The nonsinusoidal voltage supplying the load, shown in Figure 2, has the following waveform:

$$u = \sqrt{2}\mathrm{Re}\left(230e^{j\omega_1 t} + 15e^{j3\omega_1 t} + 25e^{j5\omega_1 t} + 10e^{j7\omega_1 t}\right)V$$

Based on the described values of the supply voltage and the load parameters, Table 2 lists the equivalent values of the load parameters, described by the dependencies (4), (6), and (26).

Table 2. List of equivalent values of the load's parameters.

Admittance [S]	Harmonic Order			
	1	**3**	**5**	**7**
$\boldsymbol{Y}_n$	0.309–0.243i	0.076–0.18i	0.03–0.12i	0.016–0.088i
G_e	0.304			
G_n	0.309	0.076	0.03	0.016
B_e	−0.241			
B_n	−0.243	−0.18	−0.12	−0.088

As a result of applying a voltage to the load terminals, the currents of individual harmonics and powers of the active (14) and reactive (15) for harmonics have been obtained. All the obtained results, increased by the RMS values or the sum, are listed in Table 3.

Table 3. List of values of currents and active and reactive powers.

Harmonic Order	Voltage [V]	Current [A]	Active Power P [W]	Reactive Power Q [var]
$n = 1$	$230e^{j0^\circ}$	$90.44e^{-j38.2^\circ}$	16,359	12,848.3
$n = 3$	$15e^{j0^\circ}$	$2.93e^{-j67^\circ}$	17.2	40.5
$n = 5$	$25e^{j0^\circ}$	$3.09e^{-j75.7^\circ}$	19	74.7
$n = 7$	$10e^{j0^\circ}$	$0.9e^{-j79.7^\circ}$	1.6	8.8
Sum/RMS	232.056	90.545	16,396.8	12,972.3

The waveform of the phase voltage is shown in Figure 3, while Figure 4 presents the waveform of a single-phase current of a load.

Based on the dependencies (8), (9), (11), and (13), the RMS values of individual component currents, defined in the CPC theory, are presented in Table 4. Table 5 summarizes the RMS values of the component currents, described by the Formulas (8), (11), (27), (30), and (32), defined in the developed Budeanu theory.

Table 4. List of RMS values of currents in CPC theory.

CPC Component	$\|\boldsymbol{i}_a\|$	$\|\boldsymbol{i}_r\|$	$\|\boldsymbol{i}_s\|$	$\|\boldsymbol{i}\|$
Value [A]	70.659	56.014	8.256	90.545

Table 5. List of RMS values of currents in developed Budeanu theory.

Budeanu Component	$\|i_a\|$	$\|i_{rB}\|$	$\|i_s\|$	$\|i_{crB}\|$	$\|i\|$
Value [A]	70.659	55.902	8.256	3.547	90.545
Value of distortion current [A]			$\|i_d\| = 8.986$		

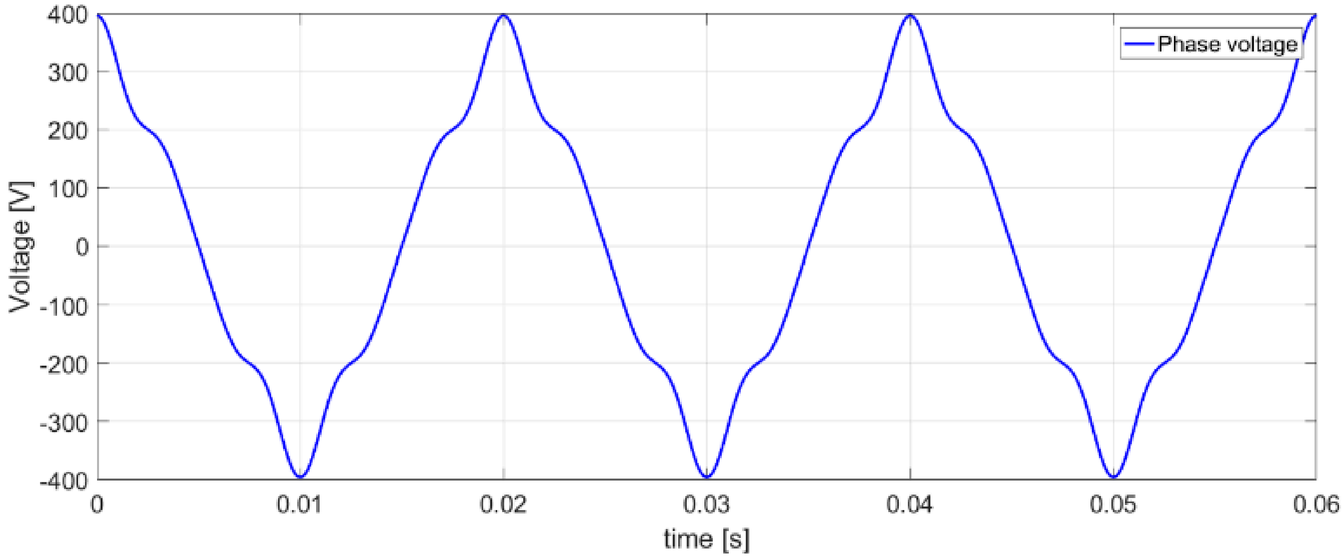

Figure 3. The waveform of phase voltage value at the terminals of load.

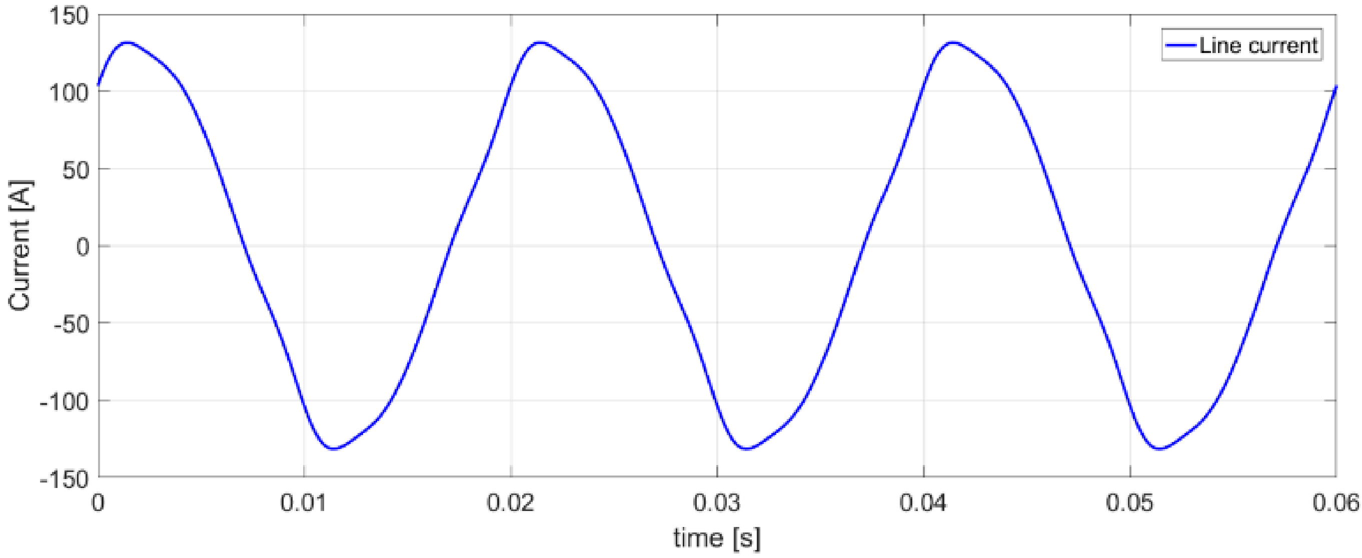

Figure 4. The waveform of the line current of the load.

As can be noticed from Tables 4 and 5, the RMS values of the active and scattered currents are identical, as this is the assumption of both approaches. The RMS value of the load current $\|i\|$ is also the same. The difference in the RMS values of the component currents appear between the reactive current defined by Czarnecki and the reactive current defined by Budeanu. Additionally, in the development of the Budeanu approach, an additional current appears associated with the susceptance change around the equivalent susceptance of the load. It should also be noted that the root of the sum of the squares of the RMS values of the Budeanu reactive current and the complemented reactive current of Budeanu are the same as the RMS value of the reactive current defined by Czarnecki:

$$\sqrt{\|i_{rB}\|^2 + \|i_{crB}\|^2} = \|i_r\| = 56.014A$$

The RMS value of the distortion current $\|i_d\|$ resulting from the root of the sum of the squares of the RMS values of the scattered current and the complemented reactive current is 7.729 A and is completely responsible for the value of the distortion power D_B.

Based on the relationships (5), (7), (10), and (13), Table 6 summarizes the CRMS values of the component currents defined by Czarnecki. In Table 7, on the basis of the relationships (5), (10), (25), (29), and (31), the CRMS values defined according to the developed Budeanu theory have been listed.

Table 6. List of complex values of currents in CPC theory.

Harmonic Order	CPC Current [A]			
	I_a	I_r	I_s	I
1	$70.03e^{j0°}$	$55.86e^{-j90°}$	$1.09e^{j0°}$	$90.44e^{-j38.2°}$
3	$4.57e^{j0°}$	$2.70e^{-j90°}$	$3.42e^{j180°}$	$2.93e^{-j67°}$
5	$7.61e^{j0°}$	$2.99e^{-j90°}$	$6.85e^{j180°}$	$3.09e^{-j75.7°}$
7	$3.06e^{j0°}$	$0.88e^{-j90°}$	$2.89e^{j180°}$	$0.9e^{-j79.7°}$

Table 7. List of complex values of currents in developed Budeanu's theory.

Harmonic Order	Budeanu Current [A]				
	I_a	I_{rB}	I_s	I_{crB}	I
1	$70.03e^{j0°}$	$55.41e^{-j90°}$	$1.09e^{j0°}$	$0.46e^{-j90°}$	$90.44e^{-j38.2°}$
3	$4.57e^{j0°}$	$3.61e^{-j90°}$	$3.42e^{j180°}$	$0.92e^{j90°}$	$2.93e^{-j67°}$
5	$7.61e^{j0°}$	$6.02e^{-j90°}$	$6.85e^{j180°}$	$3.03e^{j90°}$	$3.09e^{-j75.7°}$
7	$3.06e^{j0°}$	$2.41e^{-j90°}$	$2.89e^{j180°}$	$1.53e^{j90°}$	$0.9e^{-j79.7°}$

Based on the values in Table 6, Figure 5 presents the waveforms of the component currents and the waveform of the load current, defined by the CPC theory, from Figure 2.

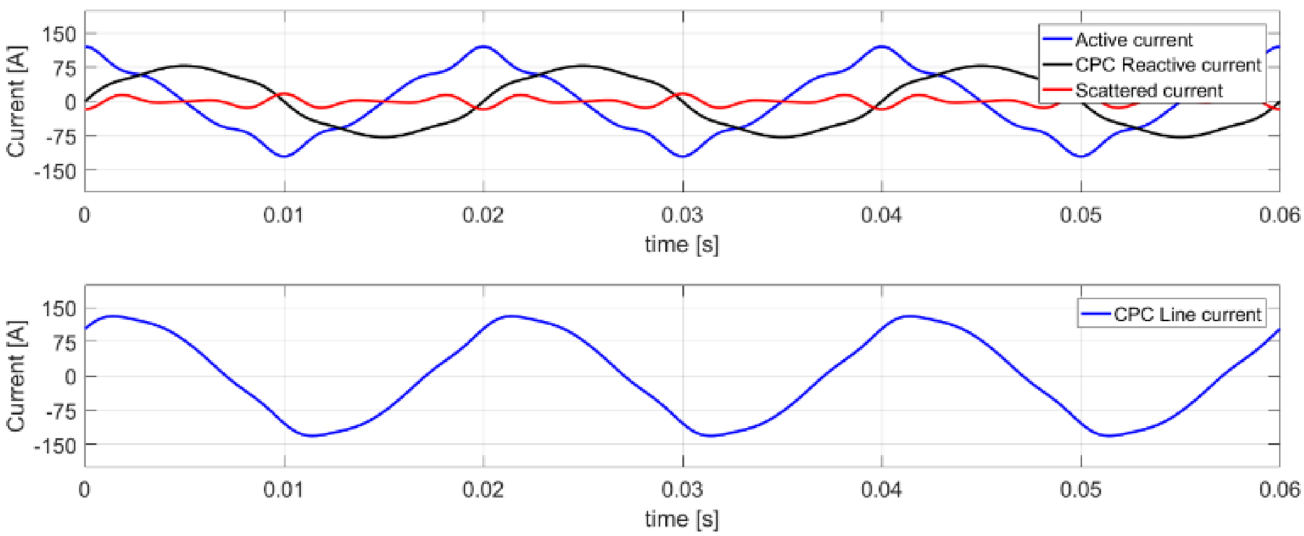

Figure 5. The waveform of the currents' components and current of the load in CPC theory.

Figure 6 shows the waveforms of the component currents and the load current defined by the developed Budeanu theory and presented in Table 7.

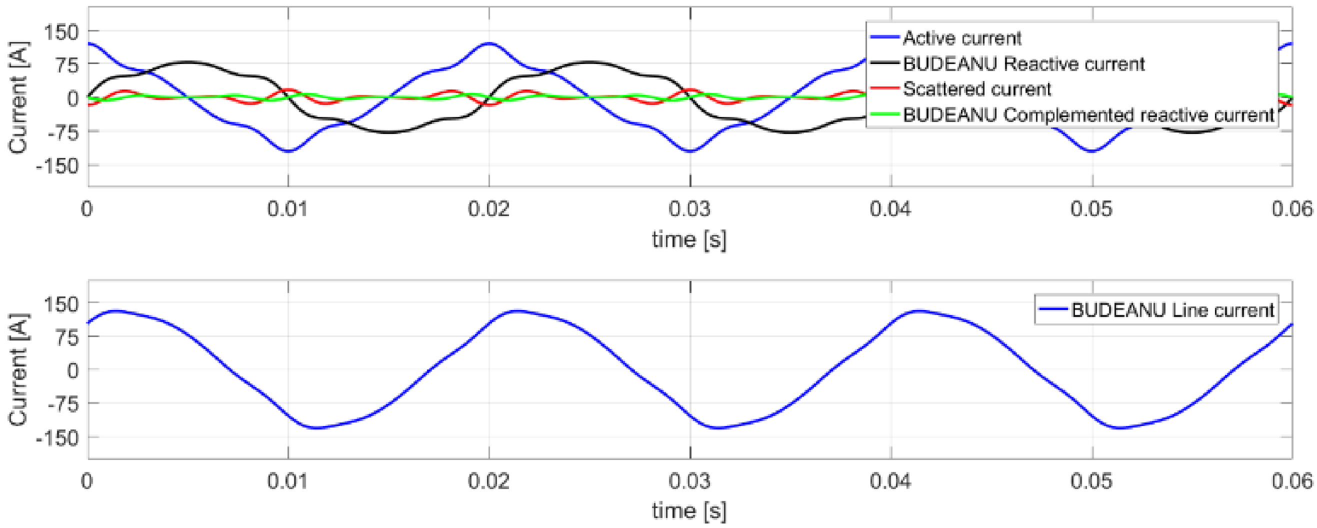

Figure 6. The waveform of the currents' components and current of the load in developed Budeanu's theory.

From the presented calculations from the Theoretical Illustration no. 1, it can be noticed that the waveforms of the load current, presented in Figures 4–6, are identical. It is also worth paying attention to the character of reactive power presented in Table 3. The reactive power for each harmonic is inductive, so it cannot be mitigated by the value of the reactive power of harmonics of different orders. Despite this, Czarnecki's reactive current has a different RMS value and a different CRMS value than the Budeanu reactive current.

At this point, the following question can be asked:

(d) Question 4—If the load is purely capacitive or purely inductive, without a resistive element, will the reactive powers and the reactive currents be equal in both approaches?

5. Theoretical Illustration 2

The analyzed system in Theoretical Illustration no. 2 is shown in Figure 7, the load parameter in the form of a capacitor with a value of 2 mF, CRMS values of currents for individual harmonics (Tables 8 and 9), and waveforms of the component currents (Figures 8 and 9). The supply voltage is identical to the system presented in Figure 2.

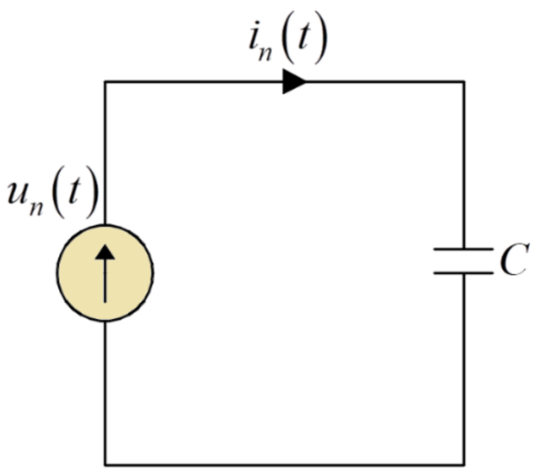

Figure 7. Scheme of the single-phase system used for theoretical calculations.

Table 8. List of complex values of currents in CPC theory.

Harmonic Order	CPC Current [A]			
	I_a	I_r	I_s	I
1	-	$144.51e^{j90°}$	-	$144.51e^{j90°}$
3	-	$28.27e^{j90°}$	-	$28.27e^{j90°}$
5	-	$78.54e^{j90°}$	-	$78.54e^{j90°}$
7	-	$43.98e^{j90°}$	-	$43.98e^{j90°}$

Table 9. List of complex values of currents in developed Budeanu's theory.

Harmonic Order	Budeanu Current [A]				
	I_a	I_{rB}	I_s	I_{crB}	I
1	-	$154.04e^{j90°}$	-	$9.53e^{-j90°}$	$144.51e^{j90°}$
3	-	$10.05e^{j90°}$	-	$18.23e^{j90°}$	$28.27e^{j90°}$
5	-	$16.74e^{j90°}$	-	$61.8e^{j90°}$	$78.54e^{j90°}$
7	-	$6.7e^{j90°}$	-	$37.29e^{j90°}$	$43.98e^{j90°}$

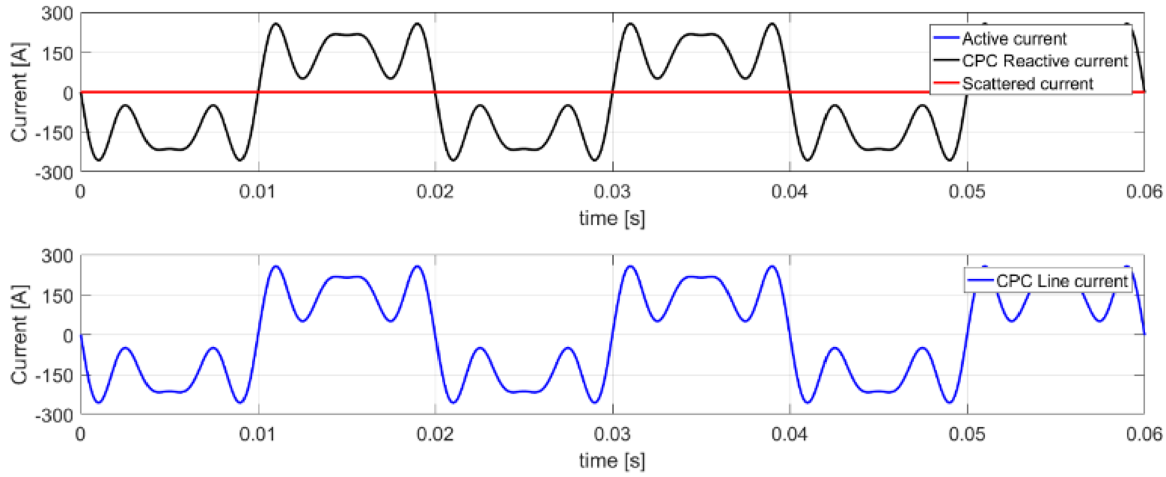

Figure 8. The waveform of the currents' components and current of the load in CPC theory.

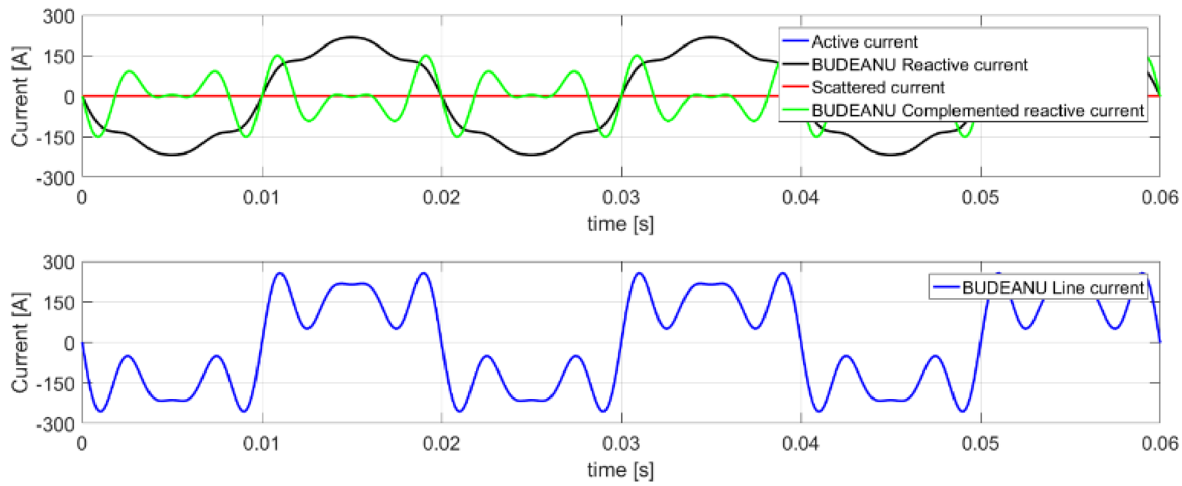

Figure 9. The waveform of the currents' components and current of the load in development Budeanu's theory.

As can be noticed from the analyzed illustration no. 2 and answering the question (d) in linear systems, it is not possible to obtain identical reactive powers and identical reactive currents in the Czarnecki and Budeanu approaches despite the fact that the load is purely inductive (or capacitive). In order to obtain identical reactive powers and reactive currents in both approaches, one of the basic principles of electrical engineering should be broken, namely, the inductive reactance should not be increased or the capacitive reactance should not be decreased along with the harmonic order. Only then would it be possible to obtain identical values of reactive power and reactive currents in both approaches. The reason for this, in addition to answering question (b), is a more significant restriction of the formula of reactive power (14) than what Czarnecki showed in [7]. Apart from the possibility of subtracting the reactive power of individual harmonics [2,7] (capacitive or inductive character), the formula for the reactive power of harmonics is not an arithmetic sum, but a weighted sum, which makes it possible to use the relationship (14) only in the area of a given harmonic and on the basis of the defined parameter; for the equivalent load, calculate the value of the reactive current as Czarnecki does in his approach. Otherwise, the apparent power resulting from the product of voltage and current of the load is always higher than the sum of reactive powers:

$$(S = \|u\|\|i\|) > \left(Q_B = \sum_{n \in N} Q_n\right) \tag{36}$$

and the RMS value of the load current consists of the Budeanu's complemented reactive current and the Budeanu's reactive current:

$$\|i\|^2 = \|i_{rB}\|^2 + \|i_{rcB}\|^2 \tag{37}$$

At this point, it is also worth noting the assumption made by Czarnecki in 1987 [3]. This assumption concerns the existence of the distortion power D_B. The lack of presence is conditioned by [2,7]:

$$\boldsymbol{Y}_n = \boldsymbol{Y}_h \tag{38}$$

where n and h denote the orders of harmonics in the set $N = \{1, 2, 3 \ldots\}$. The assumption (38) can be presented in the form [2,7]:

$$\boldsymbol{Y}_1 = \boldsymbol{Y}_{h \in N \setminus \{1\}} \tag{39}$$

As can be observed from the assumption that the distortion power D_B (17) is not present in the system, there can be no admittance change along with the harmonic order. From illustration no. 1 it can be noticed that the distortion current (distortion power) is composed of the scattered current (10) and the complement reactive current of the Budeanu (29). From illustration no. 2 it can be noticed that the distortion current is composed solely of the Budeanu's complemented reactive current. If we analyzed a resistive system, in which the conductance would change with the order of the harmonic, it would turn out that the distortion current consists only of the scattered current.

6. Theoretical Illustration 3

The analyzed system in Theoretical Illustration no. 3 is shown in Figure 10, the load parameter in the form of a inductor with a value of 5 mH, CRMS values of currents for individual harmonics (Tables 10 and 11), and waveforms of the component currents (Figures 11 and 12). The supply voltage is identical to the system presented in Figure 2.

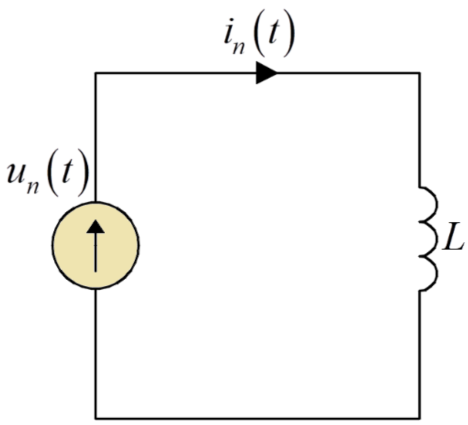

Figure 10. Scheme of the single-phase system used for theoretical calculations.

Table 10. List of complex values of currents in CPC theory.

Harmonic Order	CPC Current [A]			
	I_a	I_r	I_s	I
1	-	$146.42e^{-j90^\circ}$	-	$146.42e^{-j90^\circ}$
3	-	$9.55e^{-j90^\circ}$	-	$9.55e^{-j90^\circ}$
5	-	$15.92e^{-j90^\circ}$	-	$15.92e^{-j90^\circ}$
7	-	$6.37e^{-j90^\circ}$	-	$6.37e^{-j90^\circ}$

Table 11. List of complex values of currents in developed Budeanu's theory.

Harmonic Order	Budeanu Current [A]				
	I_a	I_{rB}	I_s	I_{crB}	I
1	-	$146.42e^{-j90^\circ}$	-	-	$146.42e^{-j90^\circ}$
3	-	$9.55e^{-j90^\circ}$	-	-	$9.55e^{-j90^\circ}$
5	-	$15.92e^{-j90^\circ}$	-	-	$15.92e^{-j90^\circ}$
7	-	$6.37e^{-j90^\circ}$	-	-	$6.37e^{-j90^\circ}$

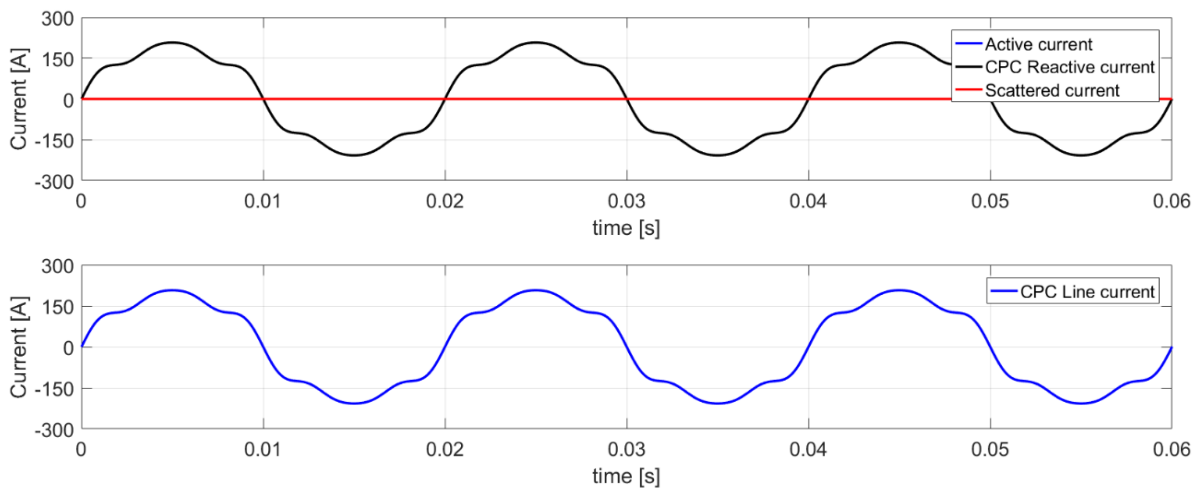

Figure 11. The waveform of the currents' components and current of the load in CPC theory.

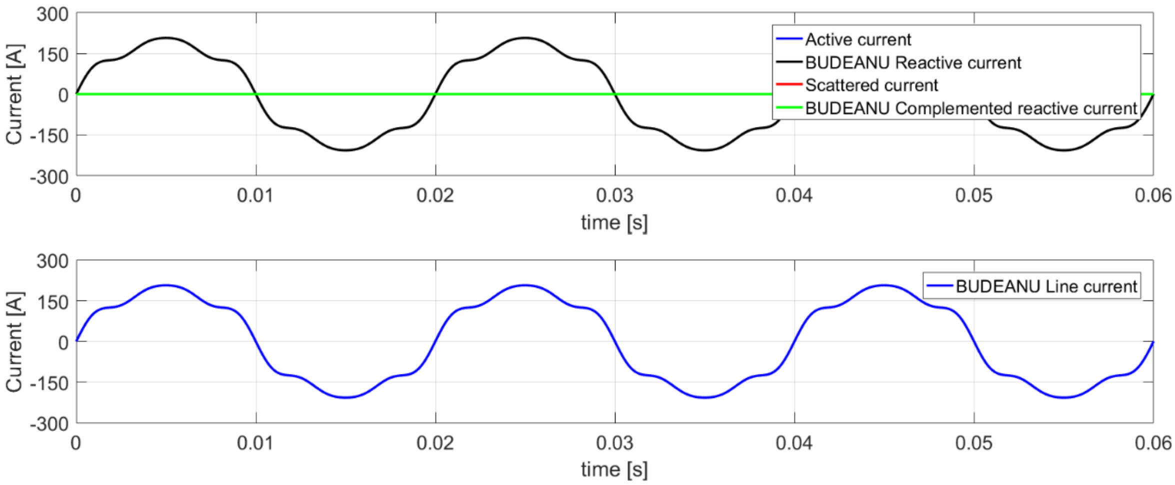

Figure 12. The waveform of the currents' components and current of the load in development Budeanu's theory.

As can be seen, the currents in both theories are the same. This is because the equivalent admittance of the load is not variable (it behaves like pure resistance or a resistance connected in series with another reactive element). Only such theoretical consideration of the circuit guarantees equality of the currents in both approaches and the lack of Budeanu's complemented reactive current.

The purpose of the article was to answer the questions presented in the publication. The answer to question 1 about the possibility of determining the equivalent parameters of a single-phase load based on the developed Budeanu theory is in the affirmative. As can be seen from the relationships (4) and (26), the susceptance for individual harmonics and the equivalent susceptance of the entire system creates the Budeanu's reactive current and the Budeanu's developed reactive current in the appropriate configuration, and these currents are then shifted relative to the voltage by half a period. After summing up the waveforms, they are Czarnecki's reactive current, which is closely related to the physical phenomenon. The inability to obtain the power factor equal to 1 (question 3) using the Budeanu theory is not possible due to two aspects, i.e., the first aspect is the presence of a scattered current related to the conductance and therefore the indelibility of this current by inductance or capacitance. The second aspect is the presence of the developed reactive current (29), which is not taken into account when determining the parameters of the reactance compensators because the relationship (17) does not reveal its presence in the system. The answers to question 2 and question 4 are the same. As shown in illustration 2, a purely capacitive circuit (the same is true for purely inductive circuits), as expected, will not only have reactive current and reactive power, as defined by Budeanu (15). In order for the apparent power S, resulting from the RMS voltage and current values, to be equal to the Budeanu reactive power, condition (38) must be accomplished. It means that the susceptance cannot change with the order of the harmonic. Accepting the invariability of susceptance would be undermined the assumptions of theoretical electrical engineering and real measurements. The formula for the sum of reactive powers (15) should be applied only to a single harmonic in order to determine the equivalent parameter of the load. It cannot be used to get the reactive power of a single-phase system supplied by a nonsinusoidal voltage because it is impossible to achieve a correct result.

7. Conclusions

As presented in the article, it is possible to develop the Budeanu theory to a form in which it is possible to determine the components currents of a single-phase load supplied by a nonsinusoidal voltage source. Additionally, as shown in the correction of the Budeanu theory, it is possible to have a physical interpretation of each current component and a further limitation of the formula for the sum of the reactive powers of individual harmonics. The entire physical interpretation and determination of the current components in the developed Budeanu theory are possible due to the use of the CPC theory.

Due to the fact that the Budeanu theory has current components related to the equivalent elements of the load, it is possible to determine the parameters of the passive compensator. The reactance compensator can be built in such a way as to compensate the reactive current (Czarnecki's definition), the reactive current of Budeanu, or Budeanu's complemented reactive current.

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Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Vectors are mutually orthogonal when their scalar product [2]:

$$(x, y) = \frac{1}{T}\int_{0}^{T} x(t)y(t)dt \tag{A1}$$

is equal to zero and then [2]:

$$(x, y) = \text{Re}\sum_{n\in N_0} \boldsymbol{X}_n \boldsymbol{Y}_n^* \tag{A2}$$

Of course, assuming that single-phase quantities are defined in the form of CRMS values [2]:

$$x = \sqrt{2}\text{Re}\left\{ \boldsymbol{X}_n e^{jn\omega t} \right\}, \quad y = \sqrt{2}\text{Re}\left\{ \boldsymbol{Y}_n e^{jn\omega t} \right\} \tag{A3}$$

1. Orthogonality of the Budeanu's reactive current and the active current:

$$\begin{aligned}(i_{rB}, i_a) &= \text{Re}\sum_{n\in N_0} \boldsymbol{I}_{rBn}\boldsymbol{I}_{an}^* = \text{Re}\sum_{n\in N_0} (jB_e\boldsymbol{U}_n)(G_e\boldsymbol{U}_n)^* \\ &= \text{Re}\sum_{n\in N_0} jB_eG_eU_n^2 = 0\end{aligned} \tag{A4}$$

2. Orthogonality of the Budeanu's reactive current and the scattered current:

$$\begin{aligned}(i_{rB}, i_s) &= \text{Re}\sum_{n\in N_0} \boldsymbol{I}_{rBn}\boldsymbol{I}_{sn}^* = \text{Re}\sum_{n\in N_0} (jB_e\boldsymbol{U}_n)[(G_n - G_e)\boldsymbol{U}_n]^* \\ &= \text{Re}\sum_{n\in N_0} jB_e(G_n - G_e)U_n^2 = 0\end{aligned} \tag{A5}$$

3. Orthogonality of the Budeanu's complemented reactive current and the active current:

$$\begin{aligned}(i_{crB}, i_a) &= \text{Re}\sum_{n\in N_0} \boldsymbol{I}_{crBn}\boldsymbol{I}_{an}^* = \text{Re}\sum_{n\in N_0} [j(B_n - B_e)\boldsymbol{U}_n](G_e\boldsymbol{U}_n)^* \\ &= \text{Re}\sum_{n\in N_0} j(B_n - B_e)G_eU_n^2 = 0\end{aligned} \tag{A6}$$

4. Orthogonality of the Budeanu's complemented reactive current and the scattered current:

$$\begin{aligned}(i_{crB}, i_s) &= \mathrm{Re}\sum_{n\in N_0} \boldsymbol{I}_{crBn}\boldsymbol{I}^*_{sn} = \mathrm{Re}\sum_{n\in N_0}[j(B_n - B_e)\boldsymbol{U}_n][(G_n - G_e)\boldsymbol{U}_n]^* \\ &= \mathrm{Re}\sum_{n\in N_0} j(B_n - B_e)(G_n - G_e)U_n^2 = 0\end{aligned} \tag{A7}$$

5. Orthogonality of the Budeanu's complemented reactive current and the reactive current Budeanu:

$$\begin{aligned}(i_{crB}, i_{rB}) &= \mathrm{Re}\sum_{n\in N_0} \boldsymbol{I}_{crBn}\boldsymbol{I}^*_{rBn} = \mathrm{Re}\sum_{n\in N_0}[j(B_n - B_e)\boldsymbol{U}_n](jB_e\boldsymbol{U}_n)^* \\ &= \mathrm{Re}\sum_{n\in N_0} j^2 B_e^*(B_n - B_e)U_n^2 = B_e^*\left(\sum_{n\in N_0} B_n U_n^2 - B_e\sum_{n\in N} U_n^2\right) \\ &= B_e^*(Q - Q) = 0\end{aligned} \tag{A8}$$

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