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Article

Improvement of Contact Force Calculation Model Considering Influence of Yield Strength on Coefficient of Restitution

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Abstract: Aiming at the problem that the current coefficient of restitution model cannot effectively predict energy dissipation in the multi-body system collision process, a coefficient of restitution model considering the yield strength is proposed in this article. As an important parameter for energy loss and material deformation prediction during collision, the coefficient of restitution has an important influence on the accurate calculation of contact force. The current main coefficient of restitution models are compared and analyzed in this article. In view of the large difference between the results obtained by different models on the same parameter, through the use of ANSYS/LS–DYNA for dynamic simulation, the influence of different yield strengths on the coefficient of restitution is studied. Then, the article establishes a new coefficient of restitution model considering the yield strength combined with the J–G model, and verifies the effectiveness of the model in the article using experimental results. At the same time, the article compares the new coefficient of restitution model with the constant coefficient of restitution model, and further studies the effect of the coefficient of restitution on the dynamic results.

Keywords: multi-body system; coefficient of restitution model; yield strength

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1. Introduction

In a multi-body system, the hinge pair is used as a necessary connecting component to transmit the force and motion between the components, and its performance will have a direct impact on the dynamic response of the system. In actual working conditions, due to processing and installation errors, material deformation, and other issues, most hinge pairs have a certain clearance, such as the connection between the cutting head and the cutting arm of the shearer, the connection between the large and small arms of industrial robots [1], and the hinge pair of artificial joints and other parts. The existence of the clearance prevents the pin shaft and the shaft sleeve at the joint of the mechanism from moving concentrically, and the deviation of the pin shaft center makes the inner side of the shaft sleeve easily collide, which causes vibration and reduces the stability and reliability of the system. Therefore, the accuracy of the contact force model used to study contact collision is very important for the prediction of the performance of a multi-body system involving revolute joints with clearance, and it is of great significance to derive an accurate contact force model.

In recent years, researchers have conducted a lot of research on the contact force model. The researchers first took the Hertz model as the starting point of the contact force model, and calculated the contact force of the elastic body based on the theory of elastic mechanics and continuum mechanics. Due to the shortcomings of the Hertz model itself and without considering the energy loss during the collision, there are certain applications limitations. The Hunt–Crossley model [2] equivalated the revolute joint with clearance as a nonlinear spring element, and improved the energy dissipation term of the contact force model, considering that the energy in the collision process is dissipated in the form of thermal energy, without considering material deformation and kinetic energy loss. The Lankarani–Nikrvesh model [3] further analyzed the energy loss method based on the Hunt–Crossley model and established a new model. The ZhiYing–QiShao model [4] started from the definition of the coefficient of restitution, obtained the energy dissipation term caused by the collision through the law of conservation of energy, and then obtained a new damping contact force model. The Flores model [5] established the corresponding contact force model by deriving the energy consumed by the deformation of the material in the collision process, and can be effectively applied to collisions with different coefficients of restitution. In the previous calculation of the contact force model, the coefficient of restitution was considered to be a constant, but the results obtained were different from the experimental situation. Researchers have found that the coefficient of restitution, which characterizes the energy dissipation in the collision process, is also an important parameter for calculating the contact force in the collision process, which is of great significance for accurately predicting the dynamic characteristics, reliability, and friction and wear of mechanical systems [6–8]. Therefore, many researchers now focus on the study of the coefficient of restitution in the study of the contact force model [9–12].

The earliest proposed coefficient of restitution model is the Hertz model. In the Hertz model, energy loss is not considered, and the coefficient of restitution is considered to be a constant. Later, to consider the impact of material damping on energy loss, and at the same time to express the coefficient of restitution more accurately, researchers considered different factors to establish different coefficient of restitution models, among which the most commonly used are the Stronge model [13], Schafer model [14], Brilliantov model [15], Walton model [16], Thornton model [17], Johnson model [18], Wu model [19], J–G model [20], and Ma–Liu model [21]. The different coefficient of restitution models are suitable for different working conditions. Ma etc. combined the relationship between the contact force and penetration depth in the Hertz model and the Johnson model, and derived the relationship between contact force and penetration depth in the elastic–plastic collision phase by using the continuity of contact force and penetration depth and the geometric relationship in the collision process. Jackson and Green [22] used the finite element method to simulate the contact between the hemisphere and the rigid plane, obtained the empirical formula between the dimensionless contact area and the contact force, and derived the coefficient of restitution model of the collision process based on the conservation of energy. According to the principle of constant volume, Chang established a model of contact force varying with penetration depth [23]. Ling used this model to establish a new coefficient of restitution model [24]. However, the amplitudes and changing trends of the contact force calculated by different coefficient of restitution models are different, resulting in differences in the nonlinear dynamic results of the system. So, a suitable coefficient of restitution model is key to the accuracy of the dynamic solution.

This article compares and studies the existing coefficient of restitution models and finds their shortcomings so as to improve them and establish a new coefficient of restoration model. Next, the theoretical results calculated by the new coefficient of restitution model are compared with the data obtained from the experimental results to verify the reliability of the established model. The coefficient of restitution is an important parameter in the prediction of energy loss and material deformation in the collision process. Therefore, the new coefficient of restitution model established in this article is of great significance to improve the accuracy of contact force prediction and can also promote the development of the dynamics of multi-body systems involving revolute joints with clearance.

2. Comparative Study of Different Coefficient of Restitution Models

2.1. Contact Force Model

At first, Hertz calculated the contact force of the elastic body based on the theory of elasticity and continuum mechanics. During collision, it is assumed that the material only undergoes recoverable elastic deformation without energy loss, which is obviously not in line with the actual situation. Therefore, Hunt considered adding an energy dissipation term to the Hertz model, and equivalent to a non-linear spring damping element of the gap hinge joint, the model expression is:

$$F_N = K\delta^n + D\dot{\delta} \quad (1)$$

In the formula: K is the stiffness coefficient, $K = \frac{4}{3}E^*R^{*1/2}$ and E^* is the elastic modulus of the contact system, $E^* = \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)^{-1}$. E_1 and E_2 are the elastic moduli of the two contacting bodies. μ_1 and μ_2 are the Poisson's ratios of the two contacting bodies. R^* is the equivalent radius of the contact system, $R^* = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$. R_1 and R_2 are the radii of the two contacting bodies. δ is the penetration depth of the contact point and n is the power exponent, and its size depends on the type of collision material. Generally, the size of the metal material is 1.5. The unit of Equation (1) is N.

The subsequent contact force models are mostly based on the calculation formula of the Hunt model, and the calculation formula of the damping coefficient has been continuously improved. For example, Flores established the corresponding contact force model by deriving the energy consumed due to material deformation during the collision. Therefore, the Flores model is more accurate. Its damping coefficient expression is:

$$D = \frac{8K(1 - C_r)\delta^n}{5C_r\dot{\delta}^-} \quad (2)$$

In the formula, C_r is the coefficient of restitution. $\dot{\delta}^-$ is the initial penetration velocity. The unit of Equation (2) is N/(m/s). It can be seen that the coefficient of restitution affects the contact force model by affecting the damping coefficient. Therefore, the correctness of the coefficient of restitution model is very important to the dynamics of the multi-body system.

2.2. Summary of Main Coefficient of Restitution Models

The contact collision of a multi-body system is a complicated process. The coefficient of restitution is not only affected by the material of the collision body, but also by its initial velocity, surface morphology, and connection mode. The coefficient of restitution has a great influence on the result of the collision process. Therefore, a suitable coefficient of restitution model is of great significance to the prediction of system dynamics results. It can be seen from the definition of the coefficient of restitution that the separation kinetic energy of the collision body is caused by the release of the elastic strain energy stored in the material during the impact loading process. Based on this idea, researchers have established some coefficient of restitution models; the units of the following contact force equations are all N, and the units of the coefficient of restitution equations are all dimensionless.

2.2.1. Hertz Coefficient of Restitution Model

The Hertz model is the first proposed contact force calculation model. According to the research theory of elasticity and continuum mechanics, Hertz obtained the contact force calculation model of the elastic body in the collision process.

In the formula: E^* is the equivalent elastic modulus of the contact system and R^* is the equivalent radius of the contact system.

$$F = \frac{4}{3}E^*R^{*1/2}\delta^{3/2} \quad (3)$$

In the derivation process, he assumed that the collision body material only deforms elastically during the collision and there is no energy loss. Therefore, the coefficient of restitution in the Hertz model is 1, that is:

$$C_r = 1 \tag{4}$$

2.2.2. Johnson Coefficient of Restitution Model

Johnson assumed that the deformation of the matrix during the loading stage is completely plastic; the relationship between contact force and penetration depth is as follows:

$$F = \begin{cases} 5.5F_y\left(\delta/\delta_y\right) \\ 0.38F_y\left(\delta/\delta_y\right)^2 \end{cases} \tag{5}$$

In the formula: F_y and δ_y are the critical contact force and critical depth at initial yielding.

During the collision process, the material of the collision body has no influence on the coefficient of restitution of the elastic and mixed elastic–plastic phases. Thus, the new coefficient of restitution model is derived as follows:

$$C_r = 1.72\left(\frac{\sigma_y^5}{E^{*4}\rho}\right)^{1/8} {V_1}^{-1/4} \tag{6}$$

In the formula: σ_y is the yield strength, ρ is the material density, and V_1 is the initial contact velocity.

2.2.3. Thornton Coefficient of Restitution Model

Thornton believed that elastic deformation occurs first, and then elastic–plastic deformation occurs during collision. The Hertz contact force model was used in the elastic loading stage and the rebound stage, and the assumed contact force model was used in the mixed elastic–plastic stage. The expression of the relationship between the contact force and the penetration depth during collision is as follows:

$$F = \begin{cases} \frac{4}{3}E^*R^{*1/2}\delta^{3/2} \\ F_y + \pi\sigma_y R^*\left(\delta - \delta_y\right) \\ \frac{4}{3}E^*{R^*_{res}}^{1/2}\left(\delta - \delta_{res}\right)^{3/2} \end{cases} \tag{7}$$

In the formula: R^*_{res} is the residual deformation radius, $R^*_{res} = \frac{4E^*R^{*1/2}\delta_{max}^{3/2}R^*}{3F_{max}}$, and δ_{res} is the residual deformation depth.

The new coefficient of restitution model is derived as follows:

$$C_r = \left(\frac{6\sqrt{3}}{5}\right)^{1/2}\left[1 - \frac{1}{6}\left(\frac{V_y}{V_1}\right)^2\right]^{1/2} \times \left[\frac{\left(\frac{V_y}{V_1}\right)}{\frac{V_y}{V_1} + 2\sqrt{\frac{6}{5} - \frac{1}{5}\left(\frac{V_y}{V_1}\right)^2}}\right]^{1/4} \tag{8}$$

In the formula: V_y is the critical velocity at initial yielding, $V_y = 3.194\left(\frac{\left(1.61\sigma_y\right)^5 R^{*3}}{E^{*4}m^*}\right)^{1/2}$. m^* is the equivalent mass, $m^* = \frac{m_1 m_2}{m_1 + m_2}$.

2.2.4. J–G Coefficient of Restitution Model

The difference between the J–G model and the Thornton model is the relationship between the contact force and the penetration depth in the mixed elastic–plastic stage. Jackson and Green used finite element software to simulate the contact process between the elastic–plastic ball and the rigid plate, and obtained the expression of the contact force changing with the penetration depth:

$$F = \begin{cases} \frac{4}{3}E^* R^{*1/2} \delta^{3/2} \\ F_y \left\{ \left[\exp\left(-\frac{1}{4} \left(\frac{\delta}{\delta_y} \right)^{\frac{5}{12}} \right) \right] \left(\frac{\delta}{\delta_y} \right)^{\frac{3}{2}} + \frac{4H}{C\sigma_y} \left[1 - \exp\left(-\frac{1}{25} \left(\frac{\delta}{\delta_y} \right)^{\frac{5}{9}} \right) \right] \frac{\delta}{\delta_y} \right\} \\ \frac{4}{3}E^* {R^*_{res}}^{1/2} (\delta - \delta_{res})^{3/2} \end{cases} \quad (9)$$

In the formula: C is the critical yield stress coefficient, $C = 1.295e^{0.736\mu}$, and μ is the Poisson's ratio. H is the material hardness.

According to the obtained in-depth relationship between the contact force and penetration, and based on the law of the conservation of energy, the coefficient of restitution model of the collision process was deduced as follows:

$$C_r = 1 - 0.1 \ln\left(\frac{V_1}{V_y} \right) \left(\frac{V_1/V_y - 1}{59} \right)^{0.156} \quad (10)$$

2.2.5. Wu Coefficient of Restitution Model

Wu used finite element software to study the energy loss of an elastic ball and elastic–plastic matrix during normal collision. He believed that, when a mixed elastic–plastic collision occurs between colliding bodies, the energy dissipation of the material is mainly caused by limited plastic deformation, and through dynamic simulation, the coefficient of restitution of the colliding body in the mixed elastic–plastic collision was obtained. Its model is as follows:

$$C_r = 0.62 \left(\left[\frac{V_1}{V_y} \right] \left[\frac{\sigma_y}{E^*} \right] \right)^{-1/2} \quad (11)$$

In the formula: V_1 is the initial contact velocity and V_y is the yield velocity.

2.2.6. Ma–Liu Coefficient of Restitution Model

Ma–Liu believed that the collision process is divided into four stages: the elastic stage, mixed elastic–plastic stage and fully plastic stage, and elastic rebound stage. Using the continuity of the contact force and the penetration depth, and the geometric relationship during the collision process, the relationship between the two was deduced as follows:

$$F = \begin{cases} \frac{4}{3}E^* R^{*1/2} \delta^{3/2} \\ \delta(c_1 + c_2 \ln(\delta/\delta_y)) + c_3 \\ F_p + k_1(\delta - \delta_p) \\ \frac{4}{3}E^* R^{*1/2}_{res} (\delta - \delta_{res})^{3/2} \end{cases} \quad (12)$$

In the formula: F_p and δ_p respectively are the load and normal deformation at the onset of a fully plastic stage. $c_1 = \frac{p_y(1+\ln(\tau^2/2)) - 2\psi\sigma_y}{\ln(\tau^2/2)} \pi R^*$, $c_2 = \frac{(2\psi\sigma_y - p_y)}{\ln(\tau^2/2)} \pi R^*$, $c_3 = F_y - c_1\delta_y$, $F_y = 1.61\sigma_y$, ψ, and τ are given dimensionless values.

He combined the relationship between contact force and penetration depth in the elastic rebound phase to solve the coefficient of restitution model in the collision process, and the model is shown in Formula (13):

$$C_r = 0.81 E^{*-\frac{1}{3}} (R^*_{res})^{-\frac{1}{6}} k_1^{\frac{5}{12}} (m^*)^{-\frac{1}{12}} V_1^{-\frac{1}{6}} \quad (13)$$

In the formula: $k_1 = 2\pi R^* \psi \sigma_y$, ψ is a given dimensionless value.

2.3. Comparison of Main Coefficient of Restitution Models

As shown in Figure 1, the classic ball–plate collision model was taken as the research object and a small ball with a radius of 0.03 m was dropped from a certain height. The material elastic modulus of the ball and the plate was 206 Gpa, and the Poisson's ratio is 0.3.

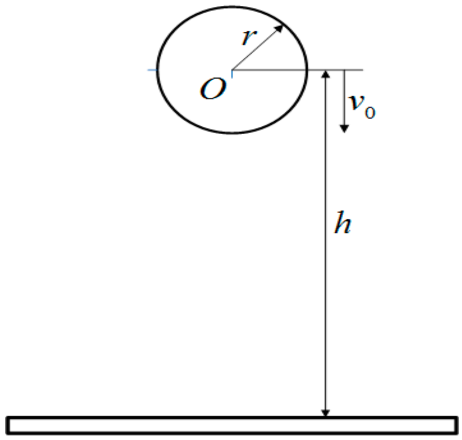


Figure 1. Classic ball–plate collision model.

Based on the above summary of the coefficient of restitution models, it can be seen that the establishment of the coefficient of restitution model is based on the law of the conservation of energy. By establishing the relationship between different contact forces and penetration depths during the different phases of contact, different coefficient of restitution models during the collision process can be derived. These coefficient of restitution models are an important parameter for calculating the collision process dynamics. It is particularly important to choose a coefficient of restitution model suitable for a multi-body system involving revolute joints with clearance. Therefore, the different coefficient of restitution models under the same parameter were further compared; the material parameters are shown in Table 1, and the result is shown in Figure 2.

Table 1. Calculation parameters.

Parameter	m/kg	$E_{stell\ ball}$/GPa	$E_{stell\ plate}$/Gpa	$\mu_{stell\ ball}$	$\mu_{stell\ plate}$	R/m	σ_y/Mpa
value	0.01	210	210	0.3	0.3	0.00675	400

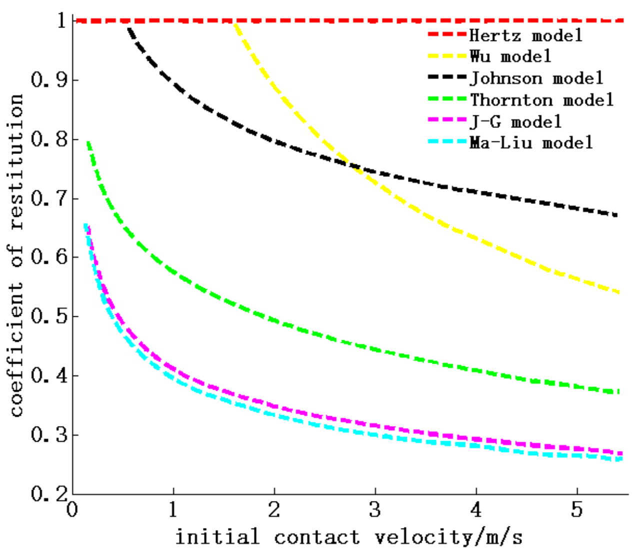


Figure 2. Comparison of the coefficient of restitution models.

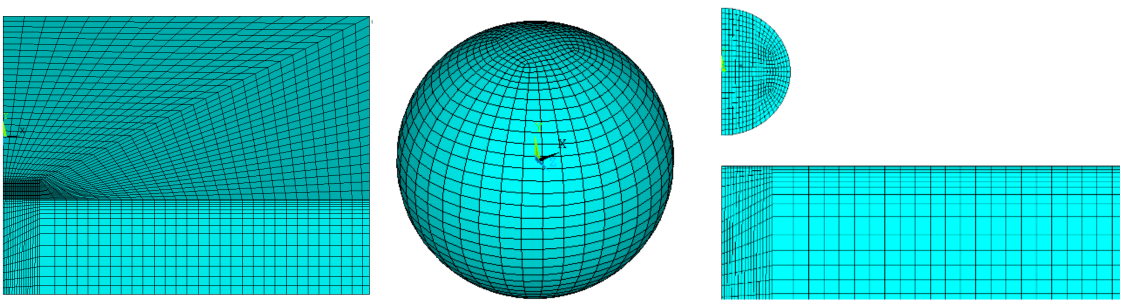
Through the above comparison, it can be seen that there is a large difference between the results of different coefficient of restitution models. Researchers have analyzed a large amount of experimental data and found that the current establishment of the coefficient of restitution model believes that, in the collision process, only the material with the lower yield strength of the two collision bodies deforms when a collision occurs. Therefore, in the derivation, the collision process is simplified to the collision between the elastic–plastic ball and the rigid plate, and the yield strength value adopts the smaller of the two collision bodies. Through numerous experimental data analyses, it is believed that during a collision, the coefficient of restitution is related to the yield strength of the two collision bodies. Especially in the collision process, when the radius of curvature of the two colliding bodies differs greatly, the yield strength of the two materials cannot be ignored. Therefore, it is necessary to further research coefficient of restitution models related to the yield strength.

3. Research on the Model of Coefficient of Restitution Considering Yield Strength

The density functional theory is a method to study the electronic structure of multi-electron systems, which has a wide range of applications in the fields of physics and chemistry. It is a method to reveal the properties of materials from the microscopic level. In recent years, many literatures have improved the density functional method in order to better characterize the mechanical properties of materials [25,26]. From a microscopic point of view, a piece of material is composed of a large number of nuclei and electrons that are free between the nuclei. Thus, the properties of the material, such as its hardness, yield strength, and the physical and chemical processes that take place within a solid, are determined by the behavior of the nuclei and their electrons contained in the material. As the yield strength characterizes the ability of a material to resist plastic deformation, the intrinsic factors affecting it at the atomic level are: bond, organization, structure, and atomic nature. In this article, the influence of yield strength on the restitution coefficient is studied from a macroscopic perspective. Therefore, the article uses dynamic simulation software to simulate the collision process, select an appropriate coefficient of restitution model to improve, and provide a new coefficient of restitution model related to yield strength at the same time.

3.1. Dynamic Simulation Model

In order to verify the accuracy of the above-mentioned coefficient of restitution models, the classic ball–plate collision model was taken as a research object, and finite element software was used to simulate and analyze the collision process. In the case of the initial contact speed, the separation speed after the collision was calculated and the simulation result of the coefficient of restitution was calculated according to the definition of the coefficient of restitution. The calculation results of the existing coefficient of restitution model were compared, and an appropriate coefficient of restitution model was selected. The simulation model is shown in Figure 3.

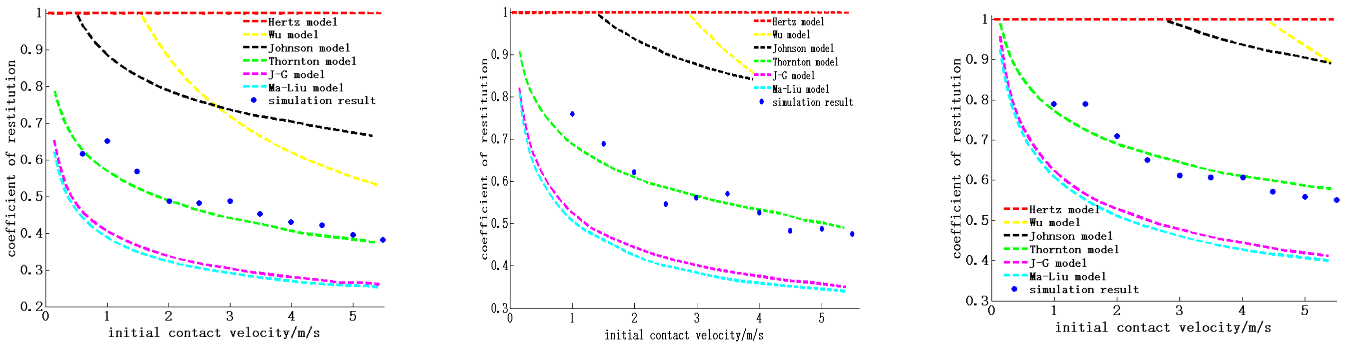


(**a**) (**b**) (**c**)

Figure 3. Mesh division during ball–plate collision: (**a**) matrix meshing, (**b**) ball meshing, (**c**) three-dimensional model of ball–plate collision.

3.2. Selection of Coefficient of Restitution Model When Elastic–Plastic Ball Collides with Rigid Plate

First, in the simulation process, it was assumed that the plate used in the collision was a rigid plate, which was used for the selection of an appropriate coefficient of restitution model to improve. Since materials with the same elastic modulus and Poisson's ratio may also have different yield strengths, taking steel as an example, materials with the same elastic modulus may have different yield strengths. Therefore, the coefficient of restitution models needed to effectively predict the coefficients of restitution under the same elastic modulus and different yield strengths. ANSYS/LS–DYNA simulation software was used to calculate the coefficients of restitution under different yield strengths and compare them with the calculation results of existing coefficient of restitution models. The yield strengths of the elastic–plastic balls were 400 MPa, 600 MPa, and 800 MPa. The comparison results are shown in Figure 4.



(**a**) (**b**) (**c**)

Figure 4. Influence of the yield strength of the ball on the coefficient of restitution: (**a**) yield strength of 400 MPa, (**b**) yield strength of 600 MPa, (**c**) yield strength of 800 MPa.

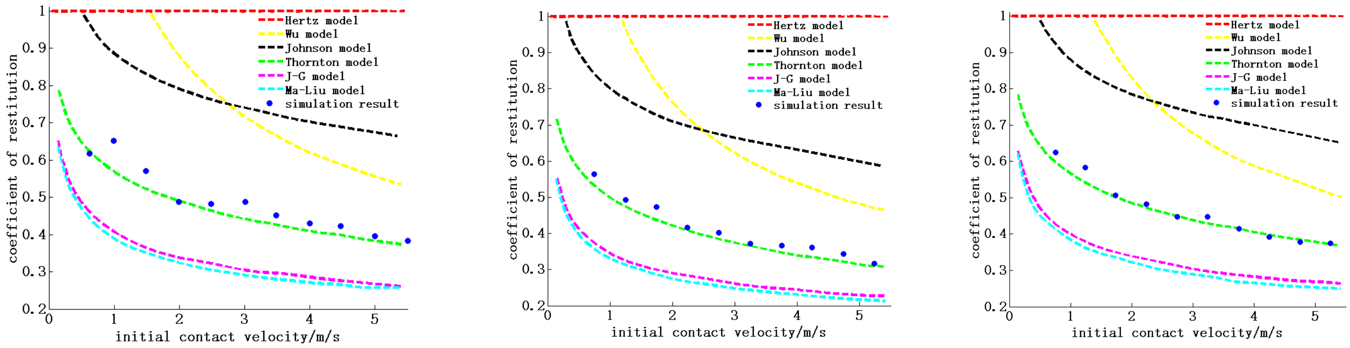
It can be found from Figure 4 that the J–G model could effectively predict the collision coefficient of restitution, and the calculation results were in good agreement with the simulation results. The Ma–Liu model, Johnson model, and Thornton model had similar trends to the simulation results, but the coefficient of restitution predicted by the Ma–Liu model was too high, and the coefficients of restitution predicted by the Johnson model and Thornton model were too low. Therefore, the J–G model had better predictability compared to the prediction results of the coefficient of restitution of the other models under different yield strengths.

The theoretical calculation results and simulation results of the coefficient of restitution were compared considering the different yield strengths of the same material. In practice, it is more important to predict the coefficient of restitution of different materials. Therefore, the next step is to study the comparison between the theoretical calculation results of the coefficient of restitution and the ANSYS simulation results of different materials. The properties of different materials are shown in Table 2 (R = 0.00675 m):

Table 2. Material parameters.

Material	E/GPa	Poisson's Ratio	σ_y/MPa	Density/kg/m^3	Tangent Modulus
Steel ball	210	0.3	400	7850	6100
Aluminum ball	76	0.34	145	2720	25
Copper ball	110	0.32	300	8600	400

Figure 5 shows the coefficient of restitution obtained when steel balls, aluminum balls, and copper balls collide with rigid plates at different initial contact velocities. In the collision of different materials, the calculation results of the coefficient of restitution predicted by the Wu model and Ma–Liu model were higher than the simulation value, while the calculation results of the coefficient of restitution predicted by the Johnson model and Thornton model were lower than the simulated results. Only the J–G model's predictions of the coefficient of restitution were close to the simulation results.



(**a**) (**b**) (**c**)

Figure 5. Influence of different materials on the coefficient of restitution: (**a**) collision of steel balls and rigid plates, (**b**) collision of aluminum balls and rigid plates, (**c**) collision of copper balls and rigid plates.

From the comparison between the above simulation results and theoretical calculation results, it can be found that, compared with other coefficient of restitution models, the theoretical calculation results of the J–G model were more consistent with the simulation results under different yield strengths of the same material and different materials, which also shows that the J–G model may be more accurate when an elastic–plastic ball collides with a rigid plate. However, in the actual collision process, the material of the collision plate cannot be a rigid plate, and neither the J–G model nor the simulation process believed that the material with higher yield strength will not undergo elastic–plastic or plastic deformation during the collision process. However, the coefficient of restitution is related to the materials of the two collision bodies, so further research is needed.

3.3. Establishment of a Model Considering the Yield Strength Coefficient of Restitution

In order to consider the influence of the yield strength of the material on the coefficient of restitution, this article studies the results of the coefficient of restitution when the yield strength of the ball is constant and the yield strength of the plate is greater than the yield strength of the ball, and when the yield strength of the plate is constant and the yield strength of the ball is greater than the yield strength of the plate. The collision between a steel ball and steel plate is taken as a research object, and the collision parameters are the initial velocity of 5 m/s, the material density of 7850 kg/m^3, the Poisson's ratio of 0.3, the elastic modulus of 210 GPa, and the tangent modulus of 6100 MPa.

Figure 6 is a graph of the influence when the material has different yield strengths on the coefficient of restitution calculated by LS–DYNA simulation software. It can be found from the figure that, when the yield strength of the steel ball was constant at 400 MPa, the increase in yield strength of the plate had almost no effect on the coefficient of restitution; that is, when the yield strength of the plate was greater than that of the ball, the result of the coefficient of restitution was the same as that of most current references. The calculation of the coefficient of restitution was based on the minimum yield strength of the collision body. When the yield strength of the steel plate was constant at 400 MPa and the yield strength of the ball is greater than the yield strength of the plate, as the yield strength of the steel ball increased, the coefficient of restitution gradually increased. When the yield strength of the steel ball reached three times the yield strength of the steel plate, the coefficient of restitution was no longer affected by the change in the yield strength of the steel ball. The above calculation results can be fitted to obtain the collision curves at different yield strengths during the collision between the steel ball and the steel plate. According to the fitted curve, a coefficient formula related to the yield strength, which was applied to the coefficient of restitution model, could be obtained as follows.

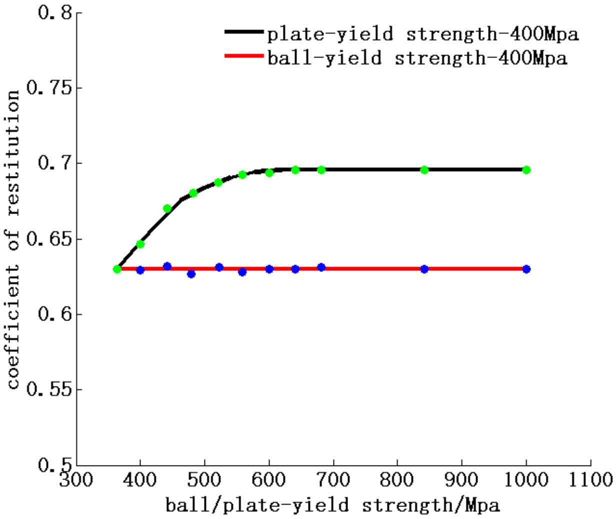
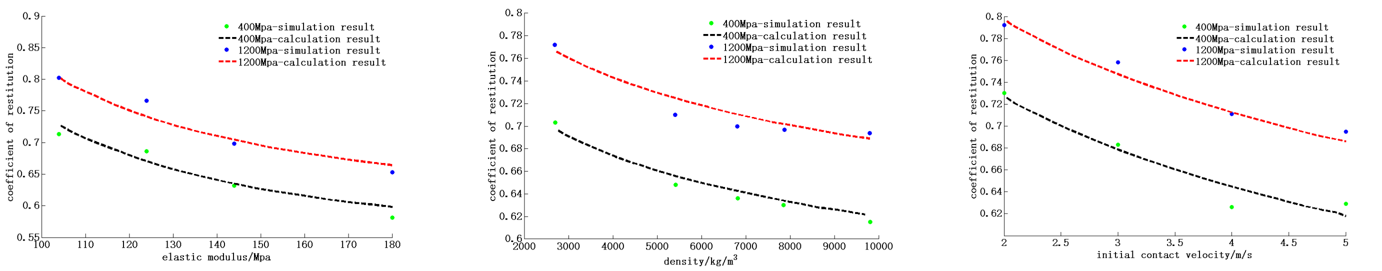


Figure 6. Influence of different yield strength ratios on the coefficient of restitution.

$$K = \begin{cases} \frac{k-1}{2+k^{1.65}} + 1 & \sigma_{plate} < \sigma_{ball} < 3\sigma_{plate} \\ 1.246 & 3\sigma_{plate} < \sigma_{ball} \\ 1 & \sigma_{ball} < \sigma_{plate} \end{cases} \quad (14)$$

In the formula: $k = \sigma_{ball}/\sigma_{plate}$.

Since the coefficient formula related to the yield strength was established according to the collision between the steel ball and the steel plate, it did not consider whether the model conformed to the collision between other materials and the collision at different initial contact velocities. The material properties of the collision body are related to parameters such as the elastic modulus and density. Therefore, this article establishes a new coefficient of restitution model through the combination of the coefficient formula related to the yield strength and the J–G model, and uses the controlled variable method to compare the calculation results of the new model with the simulation results under different elastic moduli, densities, and initial contact velocities to verify the effectiveness of the new model. The results are shown in Figure 7. Among them, the yield strength of the steel plate was 400 Mpa, and the yield strengths of the steel ball were 400 Mpa and 1200 Mpa.



(**a**) (**b**) (**c**)

Figure 7. Influence of the different parameters on the coefficient of restitution: (**a**) influence of the elastic modulus, (**b**) influence of the density, (**c**) influence of the velocity.

Through the above research, it can be found that, under different materials and different initial contact velocities, the new coefficient of restitution model has a good predictability for collision bodies with different yield strength ratios. Therefore, the newly established coefficient of restitution model is as follows:

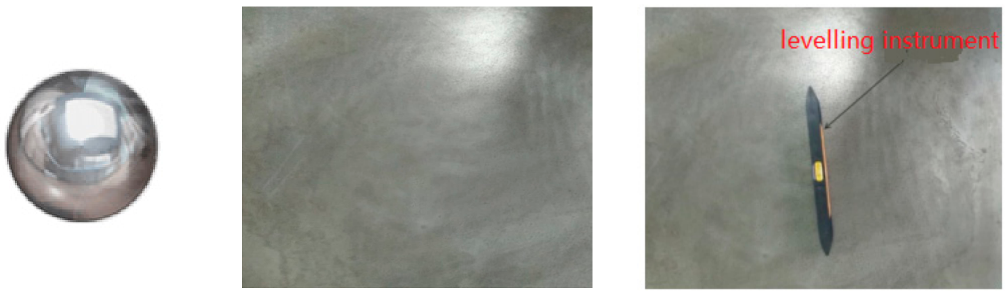
$$C_r = 1 - 0.1\ln\left(\frac{V_1}{V_y}\right)\left(\frac{V_1/V_y - 1}{59}\right)^{0.156} \tag{15}$$

In the formula: $V_y = 3.194\left(\frac{(1.61K\sigma_y)^5 R^{*3}}{E^{*4} m^*}\right)^{1/2}$ and $K = \begin{cases} \frac{k-1}{2+k^{1.65}} + 1 & \sigma_{plate} < \sigma_{ball} < 3\sigma_{plate} \\ 1.246 & 3\sigma_{plate} < \sigma_{ball} \\ 1 & \sigma_{ball} < \sigma_{plate} \end{cases}$.

4. Experimental Verification of Coefficient of Restitution and Analysis of Kinetic Results

4.1. Experimental Design

There are many collisions between steel bodies and steel bodies in a revolute joint with clearance. So, in the experiment, a steel ball was used as the collision ball, and a steel plate with a size of 0.1 m × 0.1 m × 0.01 m was used as a plane to study the coefficient of restitution. The experiment facilities is shown in Figure 8. The material parameters of the steel balls and steel plates are shown in Table 3. The experiment used a steel ball falling from a specified height and measured the bounce height after the steel ball collided with the surface of the steel plate. According to the conservation of energy, the separation velocity after the collision was derived, and the ratio of the velocities after the collision and before the collision was selected as the coefficient of restitution to obtain the experimental results.



(**a**) (**b**) (**c**)

Figure 8. Experiment facilities: (**a**) experimental steel ball, (**b**) experimental steel plate, (**c**) leveling instrument.

Table 3. Material parameters of the ball plate.

Material	Radius/m	E/GPa	Poisson's Ratio	σ_y/MPa	Density/kg/m^3	Mass/kg
Steel ball	0.00675	210	0.3	540	7800	0.01
Steel plate		210	0.3	1300	7800	0.78

4.2. Statistics of Experimental Results

During the ball–to–plate collision experiment, a high-precision camera was used to measure the bounce height of the ball. At the same time, in order to improve the accuracy of the experiment, multiple collisions were performed at the same initial height to obtain the data results. Figure 9 shows the measured value of the bounce height after multiple ball–plate collisions when the initial height was 0.5 m. The experimental data in the figure are relatively concentrated, and the range is mainly 0.16–0.17 m.

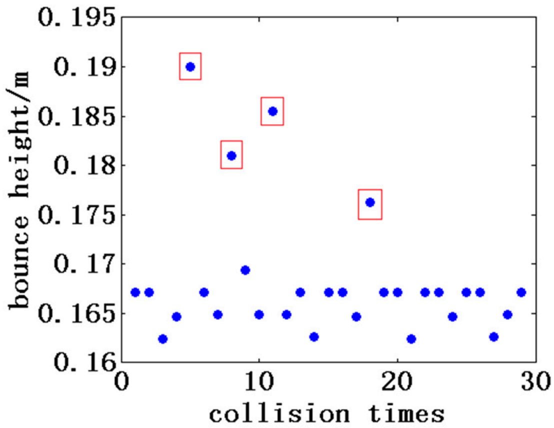


Figure 9. Test data of the steel plate collision.

For the individual mutation results in the square frame, it was believed that they were mainly due to the residual stress on the surface of the steel body after the collision combined with the literature [27]. When the collision occurs again at the same position, the residual stress causes the bounce height value to increase. In order to verify the correctness of this theory, this article selected four different positions and repeated two ball–plate collision experiments at each same position. The experimental results are shown in Figure 10. It was found that the bounce height of the second ball was significantly higher than that of the first ball, thus proving that residual stress can cause an increase in the ball bounce height value. This article did not consider the influence of residual stress, so these data were ignored.

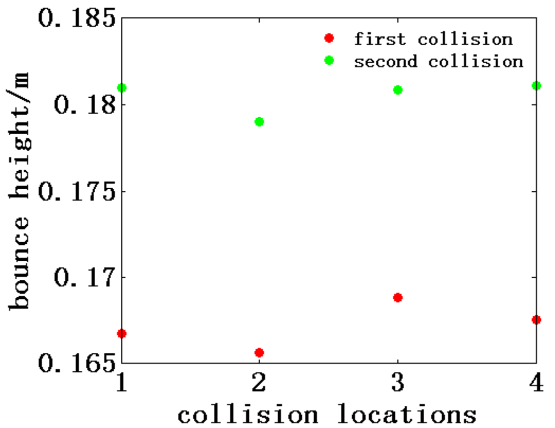


Figure 10. Test data of the steel plate collision repeated experiments.

4.3. Comparison of Theoretical and Experimental Results

In order to verify the accuracy and validity of the new coefficient of restitution model established in this article, single-ball collision was used as the research object and the material parameters shown in Table 3 were used to calculate the coefficient of restitution. The results are shown in Figure 11.

Figure 11 is a comparison diagram between the calculation results of the new coefficient of restitution model and the experimental results during the collision process. It can be seen from the figure that the experimental results were consistent with the calculation results, and both decreased with the increase in the initial height, and the maximum error was 5%. This error had little effect on the result of the coefficient of restitution. According to the current research, it is believed that there are many reasons for the error, such as: the strain hardening of the material, propagation of elastic waves, surface roughness, and rotation of the ball during collision. These factors may lead to inconsistencies between theoretical model calculations and experimental data. Therefore, it is acceptable to control errors between calculation results and experimental results within 5%.

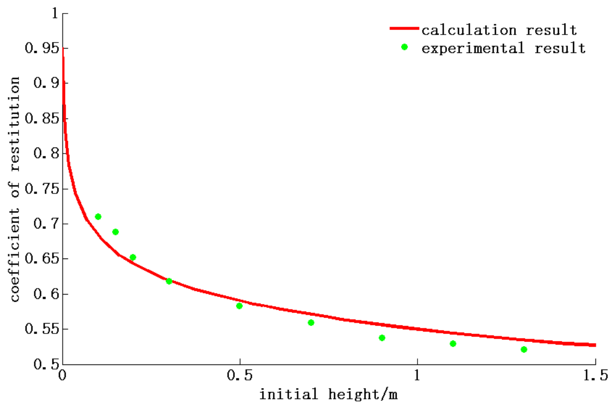


Figure 11. Comparison of the calculation results and experimental results.

4.4. Consider the Dynamic Response of the Coefficient of Restitution Model

This section compares the dynamic results of the revolute joint with clearance between the constant coefficient of restitution model and the new coefficient of restitution model in the article, as shown in Figure 12.

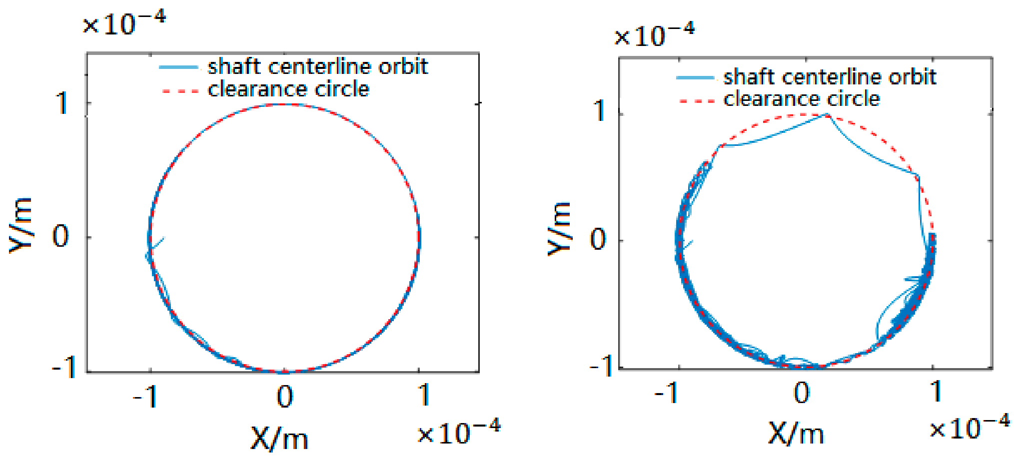


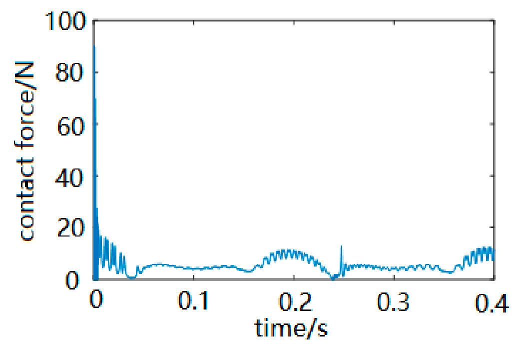
Figure 12. Shaft centerline orbit: (**a**) the constant coefficient of restitution model, (**b**) the new coefficient of restitution model.

When the rotation velocity of the revolute joint with clearance was 300 r/min, the clearance value was 0.1 mm, the rotation period was two circles, and the revolute joint with clearance was made of steel with a yield strength of 350 MPa; its shaft centerline orbit is shown in Figure 12. The shaft centerline orbit in Figure 12 is the actual shaft centerline orbit during the simulation process, and the clearance circle is the theoretical shaft centerline orbit.

Figure 12a is the simulation result of the constant coefficient of restitution model with a constant of 0.9. The shaft centerline orbit was relatively smooth and stable as a whole, except for the small vibration at the initial moment.

Figure 12b is the simulation result of the new coefficient of restitution model. The shaft centerline orbit was rather chaotic, and did not reach a stable state after two circles.

Figure 13 is the contact force diagram corresponding to the shaft centerline orbit diagram. According to the relationship of the contact force with time in Figure 13a, it can be found that the contact force was in the oscillating state at the initial moment and then entered a stable state, which corresponded to the shaft centerline orbit diagram. From Figure 13b, it can be found that the contact force was in the oscillating state with time and could not be stabilized, which led to confusion in its shaft centerline orbit diagram. The main reason for this phenomenon is that, when the coefficient of restitution is a constant value of 0.9, it is believed that plastic deformation occurs during the collision process, resulting in energy dissipation; that is, the collision process tends to become stable. When the coefficient of restitution of the new coefficient of restitution model changed with velocity, the initial contact velocity was relatively small due to the small clearance in the multi-body system involving revolute joint with clearance, so the coefficient of restitution value tended to 1; that is, no energy dissipation occurred during the collision. As a result, the vibration was large during rotation process, and it was difficult to stabilize, which is in line with the actual situation.



(**a**) (**b**)

Figure 13. Change in contact force with time: (**a**) constant coefficient of restitution model, (**b**) new coefficient of restitution model.

According to the above analysis, it can be seen that the state of collision under a certain working condition can be determined according to the shaft centerline orbit diagram and contact force diagram. When the contact force diagram was in an oscillating state, that is to say, there was no energy consumption during the collision process, the collision process of the collision body was an elastic collision. If the collision process entered a stable state, it was an elastic–plastic collision and the materials of the collision bodies underwent plastic deformation. At this time, materials with higher strength should be used to ensure that no deformation occurs in collision.

5. Conclusions

This article studies the coefficient of restitution in the calculation process of the contact force of the multi-body system involving a revolute joint with clearance, and compares and summarizes the coefficient of restitution models in the current mechanical dynamics calculation process. In view of the large difference between the results obtained by different coefficient of restitution models under the same parameters, this article used LS–DYNA software to simulate the influence of different material yield strength ratios on the coefficient of restitution, and established a new coefficient of restitution model combined with the J–G model. Compared with the experimental results, the maximum error was 5%, which proved the validity of the new coefficient of restitution model. At the same time, the dynamic results of the constant coefficient of restitution model and the new coefficient of restitution model were compared. Through comparison, it was found that the new coefficient of restitution model considering the material yield strength ratio was more in line with the actual situation.

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