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Review

Review of Vibroacoustic Analysis Methods of Electric Vehicles Motors

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Abstract: The dynamic development of electromobility has resulted in new directions of research, one of which is the analysis of the noise of traction motors. The designs of the motors used in electric vehicles are relatively new and often modified. In addition, strong competition also forces an increase in the power generated per unit mass of the motor, often at the expense of weakening the mechanical structure. This may result in an increase in the noise level generated by the electric drive, so this issue should be analyzed at the motor design stage. Different construction and operating conditions in relation to industrial or railway traction motors make it necessary to constantly develop methods for the noise analysis of the motors for electric vehicles. The aim of this article is to review the methods used so far in an analysis of the noise generated by the motors for electric vehicles. Three main methods are used by the authors of this paper: the analytical method, the hybrid method using two-dimensional models, and the hybrid method using three-dimensional models. In addition to the review of these methods, the paper also focuses on a synthetic summary of the most important factors determining the level and nature of the noise generated by electric vehicle motors.

Keywords: noise; vibrations; electric vehicles; electric drives; NVH

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1. Introduction

The concept of noise is inherently associated with every mechanical element set in motion, while the noise level may vary depending on an element's structure. Electric drives are widely considered to be among the quietest drive systems. This might be proven by the fact that the producers of electric cars are legally forced to install special systems for generating additional sound (the so-called AVAS system) at an electric car's lower speeds [1]. This provision was introduced in the year 2021, and since then every newly manufactured electric car must be equipped with such a device [1]. Therefore, why analyze the noise generated by the electric motors used in vehicles? Since technological progress and fierce competition force motor manufacturers to increase traction parameters without increasing or even reducing the motor weight [2–7], there is often an association with a reduction in the stiffness of the structure and, thus, with an increase in the vibration level of the structure [2–5]. Vibrations are caused by deformations due to the interaction of magnetic forces [8], torque ripple [9], or cogging torque [10]. This can result in an increase in the noise level generated by the motor at a higher rotational speed [11], especially in high-power motors (over 200 kW). Such motors are used to drive trucks and buses. Particularly in buses, noise can be a nuisance for passengers. This is due not only to the level of this noise but also to its nature. It is a medium frequency signal (between 1 and 3 kHz), which is clearly audible and, additionally, unpleasant to receive [12]. It can, therefore, be concluded that the analysis of the noise level that is generated by an electric vehicle drive is an important topic, as the trend of converting combustion vehicles into electric vehicles is very dynamic [13,14]. Most manufacturers have already announced the cessation of the production of combustion vehicles in the coming years [13,14].

It is well-known that noise in electric motors has three main sources: mechanical, aerodynamic, and electromagnetic [15]. Since this review concerns the methods used in traction motors (which are usually liquid-cooled), the vast majority of publications concern methods for calculating noise from electromagnetic forces. There are also several works in which the authors analyze the issue of noise as a result of mechanical effects, for example, the eccentricity of the rotor position or the uneven distribution of masses. No papers on aerodynamic noise in traction motors were found, which is due to the liquid cooling method mentioned earlier.

We have analyzed works published in the years 1973–2022 (Figure 1). It should be noted that the authors use three main methods of analysis: analytical, hybrid using two-dimensional FEM models, and hybrid using three-dimensional FEM models. The use of individual methods in the relevant years was undoubtedly closely related to the then-current state of tools available for the computer analysis of electrical machines. The structure of this work is divided based on the type of methods used by the authors. Section 2 presents publications that used only analytical methods for vibroacoustic analysis. Section 3 summarizes works in which the authors used hybrid methods based on both analytical models and two-dimensional models for FEM analysis. Section 4 discusses and analyzes works in which the basis of the analysis was the use of hybrid two- and three-dimensional models. Section 5 discusses and summarizes the entire scope of the literature review, highlighting and listing the most important factors influencing the noise of traction electric motors. These factors are presented in a synthetic way in the form of a table.

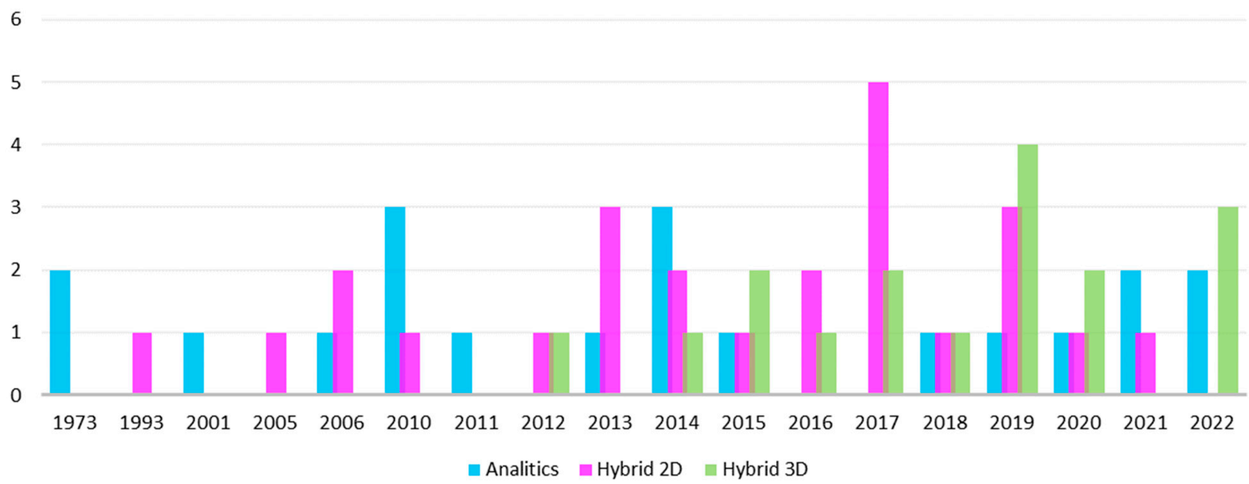
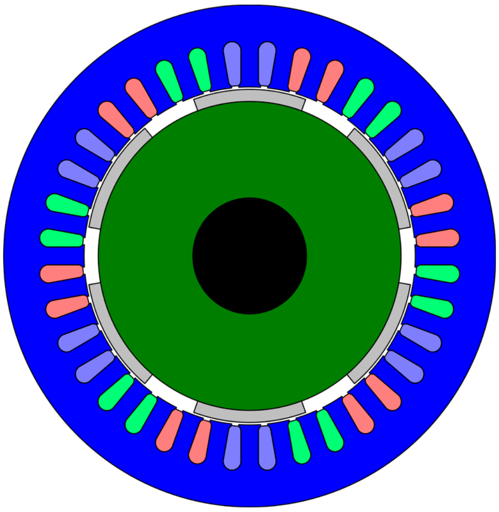


Figure 1. Number of publications on noise analysis methods of electric vehicles motors versus publication year.

Figure 1 presents which of the methods described in the review were used most often in publications from a given time period. It shows the correlation between the choice of method and the availability of IT tools (software and hardware) to facilitate engine analysis using 2D FEM and 3D FEM methods. This figure only represents the publications described in this review. Figure 2 shows the most popular types of electric motors that were the subject of the vibroacoustic analyses in the reviewed publications. These are as follows: SPM—surface permanent magnet motor, ORSPM—outer rotor surface permanent magnet motor, IPM—interior permanent magnet motor, and IM—induction motor.

The aim of this paper is to review, summarize, and evaluate the methods of the vibroacoustic analyses of traction motors used to drive vehicles and described in the literature of the subject. It should be noted that when selecting the literature, we considered only those works that concern motors with the potential for use in electric vehicles. In our review, papers dealing with noise issues in industrial motors were not included. From the conducted analysis, we selected and summarized the most important aspects and factors that, in our opinion, should be already considered at the stage of designing electromagnetic circuits. They affect both the level and nature of the noise in traction motors, which is extremely important from the point of view of the passengers of the vehicle in use. It is not always possible at the design stage to meet all the discussed guidelines to reduce noise, but, thanks to our review and summary of the key aspects, a scientist or designer could recognize which aspects to pay special attention to or what should be absolutely avoided in the context of the noise generated by traction motors for electric vehicles.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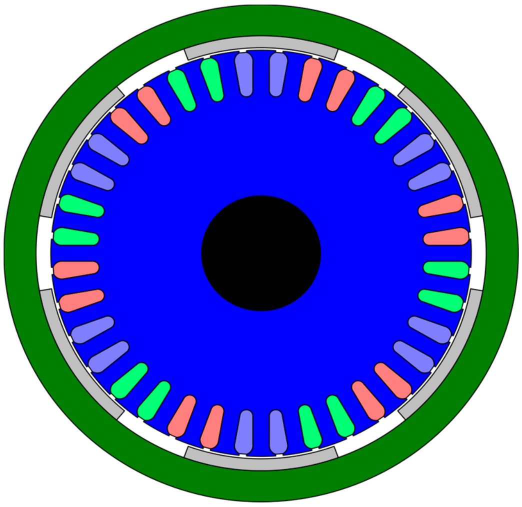
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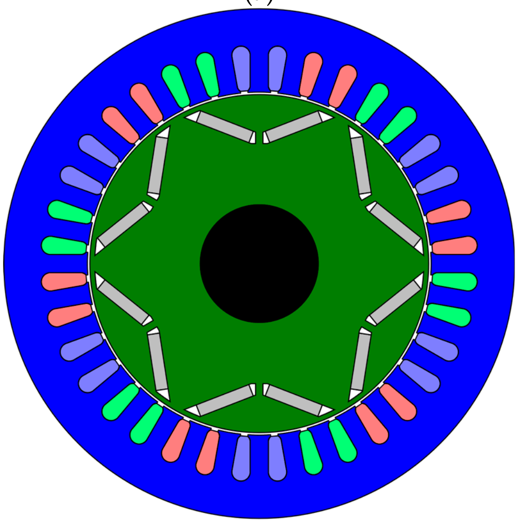
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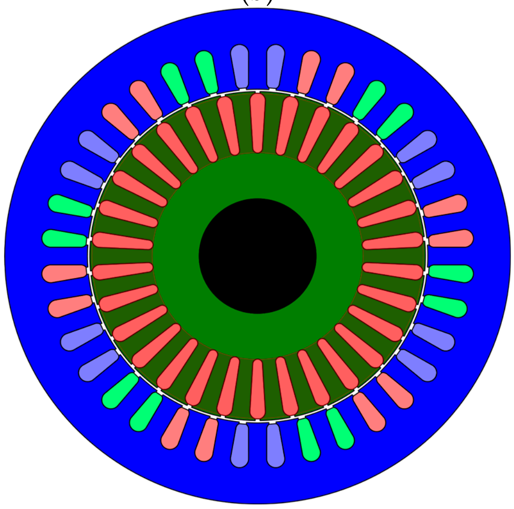
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Figure 2. Types of motors reviewed in the publications: (**a**) SPM, (**b**) ORSPM, (**c**) IPM, and (**d**) IM.

2. Analytical Methods

The oldest approach to noise analysis was the analytical method, which was also the most difficult from the point of view of complexity. The development of an analytical model of a specific motor for vibroacoustic calculations is complicated and requires very good knowledge of mechanics as well as issues related to the magnetic field and the action of electrodynamic forces. The analytical model is intended to create a certain abstraction that allows for carrying out analyses, not simulations or even to run the tested mechanical structure. Any form of analysis involves certain simplifications, the aim of which is to remove unnecessary details. It is important that they are irrelevant from the point of view of the analyzed issues because the model itself is a kind of simplification of reality. Of course, at the same time, it should faithfully reflect a given phenomenon with the necessary accuracy while maintaining the greatest possible simplicity. The most commonly used analytical models are mathematical models. They describe a given system using variables whose values may belong to different sets (real, integer, and complex). These variables represent certain features of the system, and a properly developed model links them with a group of functions, which in this case represents the relationships between these quantities. In the case of an electric motor, the simplifications [8,16] assume that its structure can be represented as a cylinder body with fixed ends. The shapes and sizes of the slots and the windings are usually omitted, replacing them with rings of equivalent mass. In this case, each of the rings is stressed in the same way and a mathematical model can be developed using a cylindrical coordinates system.

In order to estimate the vibration and noise levels, it is necessary to develop equations to determine the radial displacement caused by the stress source. This displacement is transferred to the motor housing, which, thus, becomes the cause of the sound, and, when its level exceeds a certain value defined by the standards, it is considered as noise. In the case of an electric motor, torque pulsations, cogging torque, and electromagnetic forces are considered as the main sources of stress. The authors' experience shows that the latter are most often the cause of noise generation (in the absence of mechanical damage). The normal or radial component of the electromagnetic force between the rotor magnets and the stator teeth causes radial vibrations of the stator structure. The equivalent magnetizing current (EMC) method [17,18] or Maxwell's stress tensor method [19] can be used to determine the value of electromagnetic forces. The EMC method uses a magnetizing current to directly calculate the electromagnetic force that affects the surface of the structure. Maxwell's stress tensor method, on the other hand, uses a second-order tensor to represent the interaction between electromagnetic forces and mechanical torque using the Lorentz force law [20]. Finally, the radial force acting on an element of unit thickness is calculated as the product of the stress and the cross-sectional area of the ring representing the motor structure. This, in turn, allows the development of a differential equation describing the axial displacement of the stator as a result of the stresses caused by electromagnetic forces.

In the next step, it is necessary to determine the equations to calculate the natural frequency of the stator. This is one of the most important issues in the area of vibration and noise analysis. These frequencies, as a result of the resonance phenomenon, can significantly amplify the vibrations caused by electromagnetic forces. This is particularly important in motors operating with variable rotational speed, which is the case with motors used in electric vehicles. A change in the rotational speed is associated with a change in the frequency of vibrations and the resonance phenomena that can occur at various points of the motor's operation. The basic method of determining these frequencies assumes that the stator is a freely vibrating ring, which also considers the mass of the teeth and windings. Finally, the formulas for estimating natural frequencies are based on the effects of shear and rotary inertia. Unfortunately, the use of simplifications can lead to significant errors, which is one of the biggest disadvantages of the analytical method. The last task of the model is to determine the level of the acoustic power generated for the points with the highest radial displacement determined by the previously developed equations. This power is easily converted to the noise level, which is defined in decibels.

Of course, the above approach can be modified, because the goal is not always to determine the maximum noise level. This approach is also used to analyze the impact of design factors or control methods on the level of generated noise. Therefore, in the further part of this section, short summaries of articles are described, in which the authors used not only the analytical model for the issues of analysis but also construction optimization, in order to minimize the level of generated noise.

The first publications on the magnetic noise analysis of electric motors were published in the 1950s. In 1973, a two-part publication [16,21] was published on methods for the analytical determination of the resonant frequencies of the stators of medium- and low-power electric machines, in which the stator is placed in a thin-walled cylindrical frame. This used the issues of three-dimensional elastic theory and Flugge's theory of thin shells [22] for the frame. The frequency equations derived in the first part [16] made it possible to evaluate not only radial but also torsional and axial vibrations. The equations developed in the paper required a large computational effort. Therefore, simplified equations were also proposed in the paper; these could allow for reducing the calculation time and effort required to solve them. Due to the simplifications adopted, they could be applied to electrical machines in which the ratio of the thickness of the frame to its average radius is no greater than 1/12. Part two [21] contained an experimental verification of the equations derived in part one [16], including the simplified version. The authors confirmed the sufficient accuracy of the results of the developed equations for the purpose of determining the resonant frequencies of the stators of electric machines of various powers.

Ref. [23] presented an analysis of the noise and vibration of surface permanent magnet (SPM) motors. Motors with both radial-flux and axial-flux constructions were analyzed. To achieve the goal, the authors proposed a linear model of an exciting force wave. This was intended to provide the possibility of optimizing the stator winding design, rotor shape, and rotor skew to minimize the sound power level. The paper showed that the rotor skew has a significant effect on minimizing the sound level generated by a motor. On the other hand, the stator yoke height is crucial for reducing the vibration. Further conclusions from the analyses indicated that in order to estimate the spectrum and dominant components of noise and vibrations, both the mode number and frequency of the exciting force wave as well as the natural frequency resulting from the geometry of the machine and the elastic modulus of its components should be considered.

Ref. [24] dealt with methods of minimizing the noise of permanent magnet synchronous motors (PMSM) with magnets already placed inside the rotor (IPM) at the design stage. As a solution to the problem, the authors proposed methods to optimize the mechanical design and electromagnetic circuit. The modification of the design was aimed at eliminating the sources of noise by strengthening the stiffness and, thus, transferring the resonant frequencies above the frequency of the forces acting in the electromagnetic circuit. On the other hand, in the process of optimizing the electromagnetic circuit, two objective functions were proposed. The first was related to the harmonics of the magnetic forces and the second to the torque ripple. A model was developed for each function, which was then subjected to simulation studies, which were compared with experimental studies on a prototype. As a result of the analyses, it was shown that the best results in noise reduction can be obtained by optimizing the harmonics of magnetic forces in combination with increasing the stiffness of the mechanical structure. These conclusions, although valid, are difficult to use in modern traction motor designs in which the aim is to minimize the weight of the drive. This is because it results in a natural reduction in the stiffness of the overall structure.

The results of the analyses presented in [25] showed that the main causes of mechanical vibration and acoustic noise are electromagnetic sources of vibration, such as cogging torque and radial force change, which are generated by harmonics according to the non-ideal shape of the radial-flux density distribution in the air gap. In this publication, the fractional slot concentrated winding (FSCW) motor was analyzed. As a proposal to eliminate mechanical vibration and acoustic noise, by changing the shape of the rotor of the BLDC IPM motor, it was proposed to use notches in the rotor in the d axis; these are made to obtain spatial alteration of the radial-flux density for reductions in the cogging torque and radial force. The shape and size of a notch should be selected for the specific case as a result of the analysis carried out. As a result of the rotor modification proposed in [25], a reduction in the negative third-order harmonics in the radial-flux density in the d axis was achieved. Experimental tests showed that this method is useful for reducing the vibrations of this type of motor.

In contrast, electromagnetic forces as the main sources of noise in a PMSM motor, instead of torque ripple or cogging torque, were considered in [26]. The paper showed that they cause radial displacements of the stator along the teeth. An analytical model was developed to estimate the levels of these displacements. The radial pressure [8,27,28] was used as input data. The determination of displacement levels can then be used to determine the levels of the sound power generated by the machine. FEM analysis and accelerometric tests predicting radial displacements were used for verification purposes. Four different PMSM topologies were used in the conducted analyses. The tests confirmed that radial forces, rather than torque ripples, are the main cause of noise and vibration in PMSM motors. We can also find similar conclusions in the summary of another publication [29]. Its authors used the Maxwell stress tensor method [19] for calculating this radial component of electromagnetic force. The results of this method were then used to develop an analytical model for the radial displacements prediction; this model assumed that the stator could be considered as a ring.

An analysis of radial forces occurring in PMSM SPM (Figure 2a) motors is also available in another paper [30]. For this purpose, the modal superposition method and Love's thin shell theory were used. The aim was to develop a simple method to minimize the vibration level already at the motor design stage. The developed model treated the motor as a continuous structure and indicated the relationship between radial force, natural frequency, modal shape, and vibration.

Another group of traction motors, induction motors, were the object of research in [31]. This publication presented a full vibroacoustic model for the analysis of an induction motor. The proposed model was integrated with a simulation model that can be used to design quiet motors of this type over a wide range of speeds. The presented solutions were compared with both experimental measurements of vibrations and the level of noise generated. The authors confirmed that the proposed model correctly determines the predicted main excitation forces, motor natural frequencies, and resonance phenomena during variable speed operation. Furthermore, using this model, a 350 kW commercial motor was designed, showing a significant reduction in noise compared to previous designs. A valuable element from the analysis of this work is the demonstration that the use of an analytical model significantly reduces the calculation time, which may allow the use of additional techniques such as optimization with different objective functions or artificial intelligence methods.

Another article [32] on the analysis of sources of excessive noise levels in PMSM motors used the so-called black box method. The method was divided into three steps. In the first, a theoretical model was proposed, with rotational speed as the input signal and noise frequency as the output signal. The theoretical model considered the various noise sources that can occur in PMSM motors, i.e., the higher harmonics of the supply current, switching frequency of the inverter supplying the motor, eccentricity, bearings, and resonance phenomenon. In the second step of the black box algorithm, the noise was tested under acceleration conditions. Acoustic and psychoacoustic indicators are used for this purpose. In the third step, noise sources were identified using the black box testing theory [33] and the black box testing experiment. The approach proposed in [32] is interesting because neither the parameters of the mechanical structure nor the parameters of the electromagnetic circuit are necessary for the analysis. However, the correctness of the analysis results should be verified by a series of experiments including laboratory tests.

In [34], the authors performed an analysis of the noise levels emitted by four sample electric commercial vehicles. This showed that the main source of noise from motors with many pole pairs is vibration caused by the breathing component (mode 0 of the radial forces when atoms are vibrating along the bonds, 'stretching' them). This confirms the results of the analyses from the previously reviewed publications [29,31].

The author of [35] is the CEO of EOMYS, and this company provides a comprehensive tool for the vibroacoustic and electromagnetic analysis of electric motors. The tool is called MANATEE, which was one of the first available on the market and is still being developed. The article deals with the Maxwell tensor analysis [19] of the tangential and radial magnetic forces in PMSM motors operating without any load. The analysis results can be a valuable source of information for the designers of such motors. It shows that both the contribution of cogging torque and zero-order radial forces are related to the smallest common multiple of the number of stator slots and the number of rotor poles. The analyses also show that the smallest non-zero spatial order of tangential and radial force harmonics are given by the greatest common divider of the number of slots and the number of poles. Motor designers can, therefore, already use the results of these analyses at the design stage by properly selecting the pole and slot numbers' combination.

Modern electric car designs mainly use a center drive, such as IC engine vehicles. However, research work is underway, and vehicle designs with motors located in the wheels are being developed. The analysis of the vibration and noise levels of such motors was described in [36]. For this purpose, the authors developed an analytical model of an axial-flux motor with an external rotor. Then, they evaluated the forces acting on the surfaces of the magnets. As a result of the analysis, it was found that electromagnetic vibration and noise are induced only when the spatial order of the axial electromagnetic force coincides with the circumferential order of the modal shapes. The proposed model allowed efficient evaluation of the vibration and noise for a large range of speeds; therefore, it can be useful for the designers of axial-flux motors.

One of the most important issues for analyzing the noise level of motors is the process of determining the natural frequencies. They have a significant impact when the phenomenon of the resonance of the electromagnetic forces with elements of the mechanical structure occurs. Three methods of determination are possible:

- an experimental method using a hammer with which the motor housing is struck, and exemplary results of such an experiment are shown in Figure 3 (spectral analysis of the sound generated by the housing allows to identify its natural frequencies);
- a method using FEM analysis;
- an analytical method.

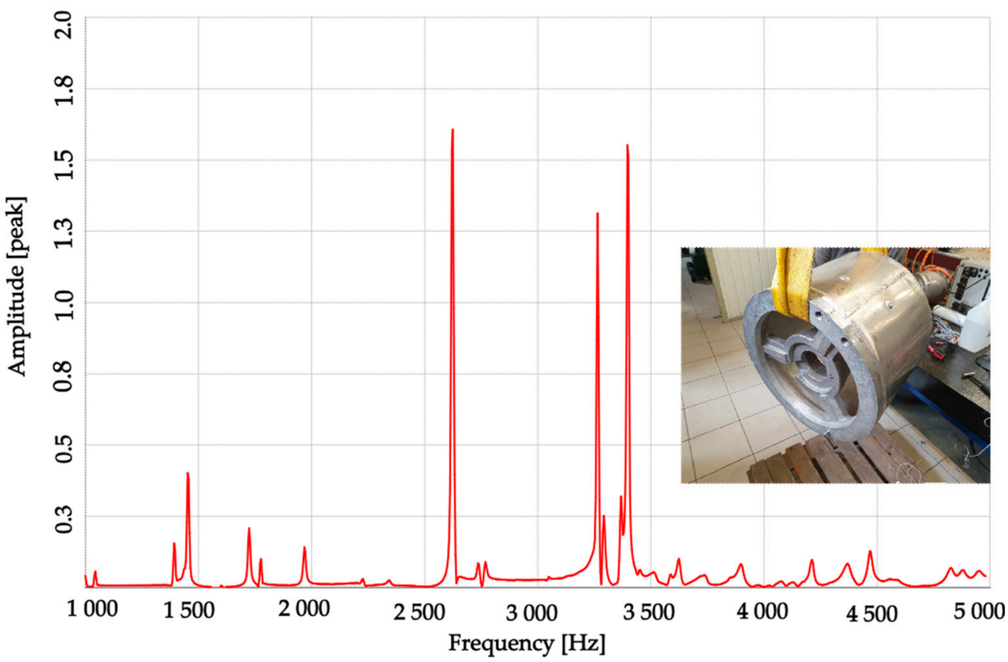


Figure 3. Modal analysis of the motor housing, x axis: frequency and y axis amplitude of sound response of the housing.

The prior method was described in [37]. To improve the efficiency of the calculations, the usually ignored influence of the mass and stiffness of the stator teeth were included in the developed model. This was implemented by the design of a double ring: one represents the stator, and the other represents the stator teeth with a given mass and stiffness. The results show a greater accuracy than previously used methods. An analysis of similar issues can also be found in [38], where it was proposed to increase the precision of determining the natural frequencies of the stator by representing the stator as a ribbed cylindrical housing. The center surface displacement functions in the cylindrical housing are dispersed as a superposition of orthogonal functions using the Gram–Schmidt orthogonalization method [39]. On the other hand, the characteristic equations of the stator's natural frequency under different boundary conditions are derived based on the energy method. As a result, natural frequency calculations involving axial and circumferential modal shapes are possible.

Ref. [40] is not a typical article on noise source analysis. It presented a specific approach to modify the noise level generated by an interior permanent magnet synchronous motor (PMSM IPM). In this publication, the authors proposed injecting an additional current component into the motor to model the sixth radial force because sixth radial force spatially excites the zeroth annular mode. In this paper, the sixth component of the radial force on the stator teeth is expressed mathematically based on the flux linkage. This expression explains the correlation between the harmonic current in the *dq* axes and the sixth radial force. First, assumptions are made to model the sixth radial force, and then the model necessary to determine the additional current component is developed and injected into the model in the *dq* coordinates. The developed model includes complete mathematical equations defining the sixth component of the radial force acting on the stator tooth. This approach makes it possible to fully control the component of the current to be injected into the motor. According to the authors, the use of such a method makes it possible to dispense with the use of a skew in motor design. Making a motor without a skew is much less time consuming; this, in turn, entails a reduction in production costs. The analyses presented in the article were verified using both simulation and laboratory experiments.

The majority of the present-day inverters used in PMSM motor drive systems use a fixed PWM (pulse-width modulation) switching frequency. If this frequency is relatively high, e.g., 10–12 kHz, it does not significantly affect the noise level generated by the electric drive system. However, in the high-power systems usually used in heavy-duty vehicles, the switching frequency may need to be reduced to as low as a few kHz (due to the level of inverter losses). This can result in additional forces acting on the motor structure in the audible frequency range. In such a case, it would be better to use a variable switching frequency. An analysis of such a solution was presented in [41]. It presented the possibility of using the pseudo-random triangular pulse width modulation (PTPWM) technique for this purpose, in a converter feeding a PMSM motor. This method can significantly minimize torque ripple and, thus, motor vibration and noise, as the publication demonstrated. Furthermore, it analyzed the relationship between the stator current harmonics and non-sinusoidal flux distribution with torque ripple. The proposed solutions were verified not only by simulation studies but also by laboratory experiments.

Minimizing the impact of PWM modulation can also be achieved by modifying the design of the motor and inverter. This was shown in [42], in which a two-level inverter and a two-winding motor were proposed. The publication verified the possibility of reducing the noise level by introducing a phase shift between the carrier waves of each level of the inverter. According to the results of the analyses, this approach significantly reduces noise. The use of a multiphase (more than three) motor involves many advantages. It is possible to significantly increase the maximum torque of the motor, which is usually limited by the parameters of the power electronic components used in the inverter design. Multiphase motors can also allow operation, of course with reduced parameters, in the situation of the partial failure of the motor windings or one of the inverter levels. However, this requires a much more complex control algorithm. On the other hand, the need to use a special design for the inverter and motor significantly reduces the practical use of this method. There are single commercial offerings of multiphase inverters on the market. However, for drive manufacturers with both the ability to freely shape the motor windings and their own inverter circuit designs, this approach could be interesting from the point of view of competition in the electric drive market.

The proper determination of the rotor position of a PMSM motor has a very large impact on its traction performance. This is particularly important in the multi-pole motors used in electric vehicle drives. In [43], the effect of the error in the determination of the rotor position, based on the resolver signal, on the electromagnetic noise generated in the motor was evaluated. To this end, a theoretical analysis of the effect of the rotor position error on the harmonics of the supply current was first carried out. Then, the spatial distributions and temporal characteristics of the electromagnetic force were analyzed. The results showed that the rotor position error introduces additional sideband harmonics to both the current and electromagnetic force. The simulations and tests performed confirmed the results obtained during the process of the theoretical analysis. The results presented in the article can be used at the stage of the laboratory testing of a PMSM motor to assess the correctness of the rotor position determination in a drive system using the field-oriented control (FOC) algorithm (Figure 4) [44].

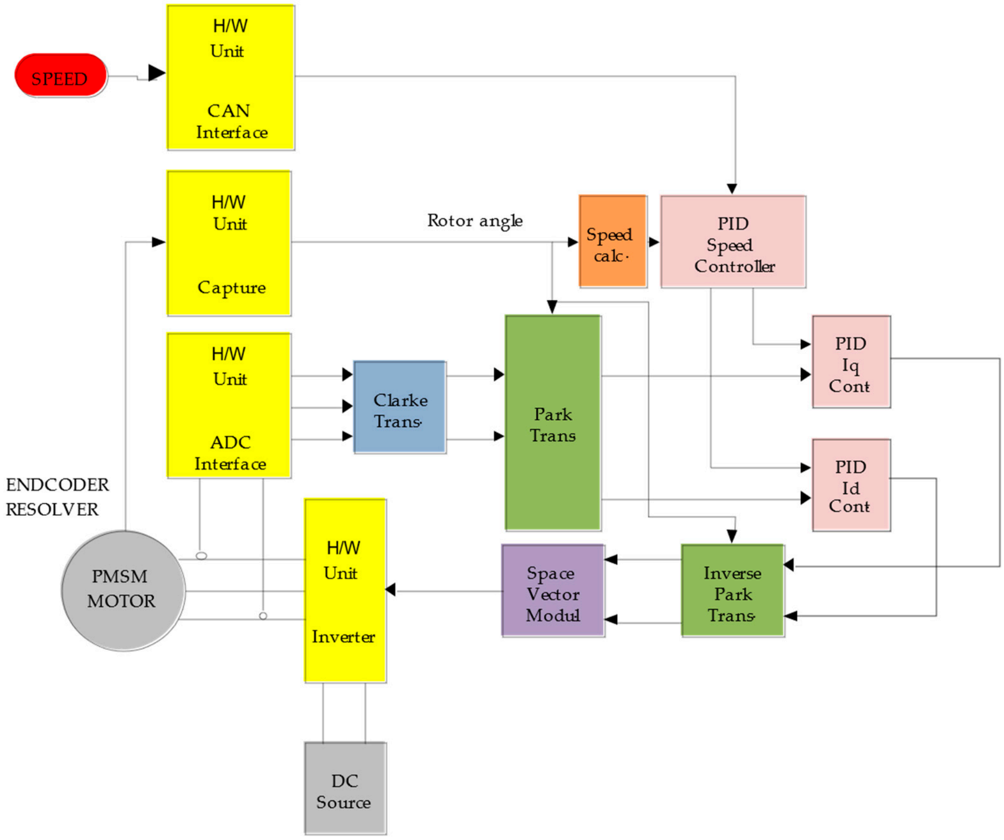


Figure 4. Block diagram of FOC algorithm.

The last article in this section [45] contained a very interesting approach to this problem. It dealt with the surface permanent magnet synchronous motor (PMSM SPM), but the approach could be applied to any type of machine by only changing the analysis parameters. First, a sensitivity analysis was performed to determine the effect of the individual structural and electromagnetic parameters on the torque ripple, dominant spatial distribution force in the air gap, dominant mode frequency, friction torque, and structural unit response, which can indirectly affect the noise level generated in the motor. The analysis conducted showed a large influence of the airgap force component. Further work was aimed at minimizing this influence using optimization methods. Simulation analyses as well as laboratory tests confirmed the theoretical analyses.

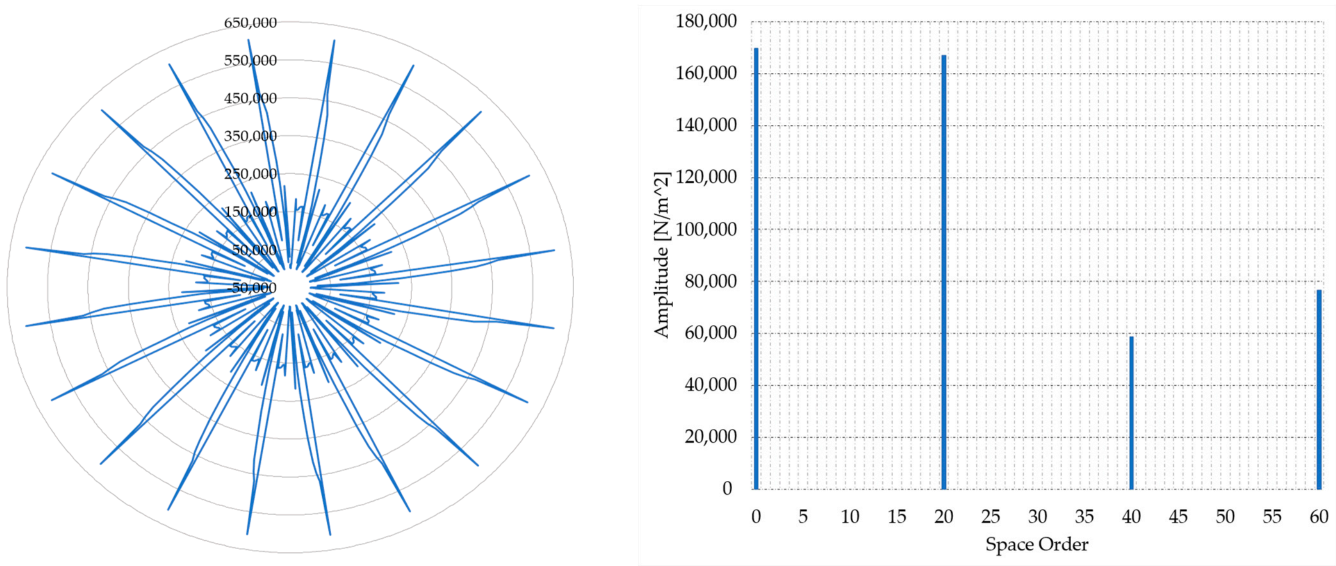
3. Hybrid Methods Using 2D Models

The hybrid method using 2D finite element method (FEM) analysis is a combination of the analytical method and FEM calculations using two-dimensional (2D) models. The analytical method performs calculations of the possible space modes of forces and their frequencies for a motor. The analytical calculation can significantly shorten the next step of the FEM calculation by predetermining the rotational speed and load ranges of the motor for which increased noise is possible. In the first step of the FEM calculations, the forces occurring in the air gap of the motor are calculated for the entire rotational speed range in which the motor operates and for the entire load range. These forces are calculated using the Maxwell stress tensor [19] for the 2D model, which considers only radial forces and ignores axial forces, which are only considered in three-dimensional models. The next step calculates the harmonics of the forces occurring in the air gap as well as the structural response of the individual components that occur in the motor. In hybrid method using 2D models, the structural responses of the winding stator, rotor, and motor housing are usually calculated, and the structural responses of the bearing plate and the motor suspension system are ignored. In this method, the structural response to vibration (harmonic air-gap forces) is based on the linearity of the machine in terms of its vibration characteristics. The next step is the process of synthesizing the structural vibrations, from which only those space modes that exist for the motor are selected. Space modes with $r = 0$ are so-called breathing mode (pulsating) waves, which are mainly directed to the stretching of the stator core; space modes with $r \neq 0$ are rotating waves, and $\pm r$ determines the direction of the rotation of the stress waves. The frequency of a breathing mode is proportional to the smallest common multiple of the number of slots and the number of motor poles. The frequency of the successive modes present in the motor is proportional to the greatest common divisor of the quotient of the number of slots, the number of motor phases, and the number of pole pairs. The existing space modes are represented on the so-called Campbell diagram, and these calculations are performed using the analytical method. Based on the synthesis of structural vibrations calculated using FEM 2D calculations, the deformations of the stator's outer surface are determined, which are transferred to the motor housing. Deformations of the outer surface of the motor housing are classified as vibrations or motor noise depending on the frequency.

The hybrid 2D FEM method combines the speed of calculation from the analytical method and the ease of model creation with a relatively short calculation time. The use of the finite element method for electromagnetic calculations and for the calculations of radial forces acting in the motor gap requires the preparation of an appropriate 2D model. Such a model is prepared by the calculation program based on the geometric dimensions and other structural data of the motor (type, location of magnets, dimensions, and number of slots). As a result, the preparation of a simple model is quick and trouble-free. The use of the 2D finite element method forces some simplifications of the model. However, by using these small simplifications, which are related to the conversion of the radial forces of the motor occurring with a model with a skewed stator or rotor, we can obtain solutions with a high accuracy and an acceptable calculation time. The following section describes brief summaries of the articles in which the authors used not only the hybrid methods using 2D models for issues of analysis but also design optimization to minimize the level of noise generated by traction motors.

In [46], on the hybrid methods for analyzing noise sources in synchronous motors, the authors analyzed the impact of model simplifications on the accuracy of the calculation results. In the first part of the paper, they analyzed the effect of modeling windings and stator teeth on stator modal frequencies. They calculated the natural frequencies of the stator using the 2D finite element method. The calculations were performed using MOSAIC software. It was shown that the model simplifications used in the analytical calculations as well as in the finite element calculations have a significant impact on the final simulation result. It was also shown that the inclusion of teeth and windings reduces the natural frequencies of stator vibrations. This is especially important for higher modes of stator deformation. For modes above the eighth, the differences in calculations for the simplified model and those including the teeth and windings can reach up to 60%. In the second part of the article, the electromagnetic forces working on the stator teeth in electric machines were analyzed and determined. The effect of the number of teeth on the modulation of the electromagnetic force and, thus, the noise of the machine were initially analyzed. The 2D finite element method was used to calculate the electromagnetic forces. The calculations were performed using the FLUXMECA program.

In [47], the authors analyzed the effect of stator current in the *d* and *q* axes on the radial and tangential components of the flux density in the motor's air gap. Using the Maxwell stress tensor [19], the radial and tangential components of the flux density in the motor's air gap were calculated. The calculations were performed using the Maxwell package for an SPM motor (Figure 2a), for which the current in the *d* axis does not affect the generated electromagnetic torque. The publication showed that the *d* axis current component has a significant effect on the average radial force component (Figure 5), with the radial force value being a quadratic function of the *d* axis current component. It was also shown that the component of current in the *q* axis affects both the radial forces and tangential forces (torque). In this case, the value of the radial force is also a quadratic function of the *q* axis current, and the value of the tangential component depends linearly on the *d* axis current. It was further shown that under standard motor operating conditions, the radial force significantly exceeds the tangential component of the force. It was investigated, with the help of analytical methods, how motor excitation affects the flux density to produce torque and radial force. The authors of the publication concluded that only a small percentage of the motor's air-gap space actively participates in the torque-generation process.



(**a**) (**b**)

Figure 5. (**a**) Force space distribution and (**b**) force harmonic analysis of the motor.

In [48], the authors presented the design and optimization process of a high-power and high-torque-traction motor designed for direct wheel drive. A motor with two rotors was used; in each rotor, magnets were placed inside, and both rotors shared the electromagnetic part of the stator. Due to the high use of the magnetic circuit, the authors of the publication found a problem with the noise of the motor. The paper also presented a process of reducing motor noise by minimizing the cogging torque. Finite element calculations were used to verify the assumptions made for calculating traction motors. A model motor was made that had 50% more power density, and the noise was reduced by 5 dB, which also confirmed the calculated electromagnetic and vibro-acoustic parameters.

The influence of electromagnetic and mechanical factors on the noise and vibration of permanent magnet synchronous motors was discussed in [49], which was published in 2006. The mechanical factors classified there were rotor unbalance, the misalignment of bearings, the thermal deformation of the rotor, and rotor eccentricity. The investigated electromagnetic factors were the harmonics of the supply current, the effect of stator use, the resonance of the magnetic forces from permanent magnets, and the supply current with the stator's natural frequency and vibration. The authors analyzed the influence of the above factors on the noise and vibration of a motor with 11 kW of a power and $2p = 4$ magnetic poles. The main conclusion drawn by the authors was that the motor noise and vibration are strongest when the stator's natural frequencies coincide with the frequencies of the forces and their harmonics generated by the current supplying the stator slotting. Therefore, the overlap of force vibration frequencies, with a source that is the current in the motor windings, permanent magnets, air gap, or other mechanical factors, should be avoided.

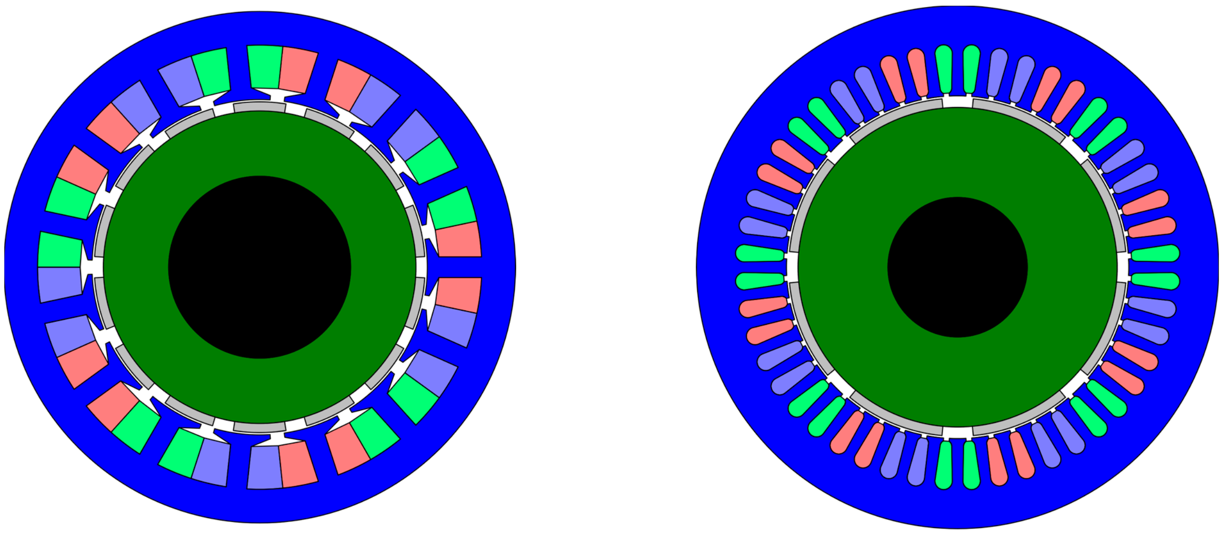
In [50], the authors applied a hybrid method of noise calculation to a permanent magnet synchronous motor. They analyzed a concentrated winding permanent magnet synchronous motor (CWPMSM). In this motor, the modification of the number of stator slots, Q_s, as well as the number of motor poles must be closely matched, and usually a change in Q_s necessitates changing other motor parameters as well. To reduce the noise of the motor, the authors modified the thickness of the stator yoke, the width of the tooth, and the opening of the stator slots. Based on their calculations, they proposed two new stator cores, keeping the number of motor poles as $2p$ and the number of stator slots as Q_s. The proposed new shape of the stator package (changed the dimensions of the stator slots and stator yoke) was used to make new models of the motors and then compare the noise produced by each of them. The modifications made it possible to reduce the noise of the motor by about 5 dB without degrading the electromagnetic parameters of the motor.

Ref. [51] as well as [47] focused on the effect of radial force versus the noise and vibration of traction motors, recognizing radial force as the main cause of noise. They focused on trying to reduce the force harmonic content and amplitude of the radial forces and estimating the natural frequency and frequency of the radial forces, so they are far from each other without causing resonances. Ansoft Maxwell software was used for the 2D electromagnetic calculations and forces acting in the motor's air gap. The calculation of the motor housing's natural frequencies was accomplished in 3D using Hyper Mesh software. The authors then compared the natural frequencies of the housing, stator, and radial force frequencies and their modes. From the comparison, the speeds and frequencies for which likely resonances and likely increases in traction motor noise can occur were selected. In the analysis, the authors of the publication did not consider tangential forces, axial forces, or axial forces that can also have a significant effect on resonance frequencies as well as the noise of motors, especially those operating in a wide speed range.

In [52], the authors compare motors with permanent magnets placed inside the rotor (IPM) and different types of stator windings. They use distributed windings with different ratios of the number of stator slots to the number of motor poles. The authors stated that with an appropriate ratio of the number of stator slots to the number of poles, motors with concentrated windings can be successfully used in traction drives. Since the electromagnetic parameters and noise of motors do not increase, the use of concentrated windings in the motor allows for making much shorter motors with less weight and a comparable electromechanical performance to motors with distributed windings.

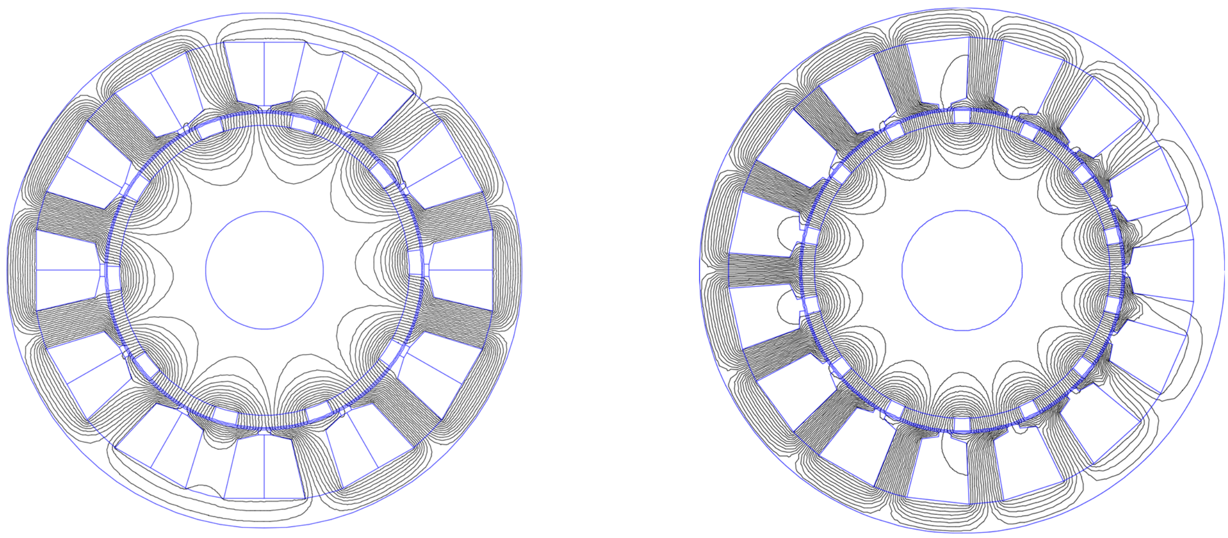
In [53], permanent magnet synchronous motors with concentrated windings (Figure 6a) and surface permanent magnets (SPM) were analyzed. The analysis was focused on the ratio of the number of stator slots to the number of poles, while also not omitting the so-called asymmetric motors (e.g., those with 14 poles and 15 stator slots) (Figure 6a).

The selection of the number of stator slots and the number of poles, especially in motors with concentrated windings (Figure 7), is very important because of the uneven magnetic tension (the force pulling the rotor to a particular part of the stator, which is caused by the imbalance of forces) [54,55], which, if the number of stator slots and the number of rotor poles are improperly selected, can cause noise as well as machine vibration. The authors of [53] performed calculations of the motor using Flux 2D software to further compare the results of finite element calculations with their own software using only analytical methods. The authors, knowing the peculiarities of noise generation in electric motors, especially those with permanent magnets, decided to compare the calculation methods for machines operating under load. The algorithm they developed allowed them to identify the interaction of the rotor with permanent magnets and the space orders of the forces in the air gap. The paper did not present any information on the errors in the results of calculations using the analytical method and the finite element method. A very similar methodology of operation was presented in [8], but in this case the results of the tests of the motor were presented.



(**a**) (**b**)

Figure 6. SPM motor with (**a**) concentrated windings and (**b**) distributed windings.



(**a**) (**b**)

Figure 7. Drive motor with concentrated winding: (**a**) $2p$ = 10, and Q_s = 12 and (**b**) $2p$ = 14, and Q_s = 15—unbalanced.

The authors of [56] used a classical hybrid method to analyze the noise of a permanent magnet synchronous motor mounted on the rotor surface (Figure 7b). The motor they analyzed was a 2p = 44 poles machine, so the thickness of the stator yoke was relatively small in relation to the diameter of the stator. Multipole motors do not require a thick stator yoke for magnetic reasons, but a thicker stator yoke is often required for mechanical reasons and to reduce motor noise. First, calculations of the main harmonics of the magnetic field and the forces acting in the motor's air gap were carried out using an analytical method.

Based on the above calculations, it was found that the deformation of the stator core, which is directly responsible for the noise generated by the motor, is inversely proportional to four times the square of the order of the force. For this reason, those electromagnetic forces that had a spatial order greater than 4 were omitted from the analysis. Further calculations to investigate the sources of motor noise were performed in the Ansys environment, extending the analytical calculations and investigating the effect of electromagnetic forces up to the fourth order. The final conclusions of the study, for the analyzed motor, are that two of the main sources of noise are the zero and fourth space order electromagnetic forces.

The noise of electric motors, especially those used for traction drives in electric vehicles, affects both driving comfort and the safety of bystanders. In [57], the authors presented a method for evaluating the electromagnetic noise in permanent magnet traction motors. In the method used, the structural vibration response was based on the linearity of the machine in terms of its vibration characteristics. The electromagnetic force actions and then the structural response of the motor components were calculated. The calculations were performed using the finite element method.

The resultant noise was calculated by superimposing the structural response on the successive harmonics of the forces from electromagnetic calculations. As in [56], the authors chose to omit higher structural space orders that are larger than the number of motor poles. The publication showed that for the type of motor analyzed, the zero spatial order forces, the so-called breathing forces, have the greatest impact on noise and are its main source.

The authors of [58] also considered a permanent magnet motor, in this case a motor with an external rotor (Figure 2b) and a stator with concentrated coils (Figure 6a). The use of concentrated coils makes it possible to significantly reduce the length of the motor's end windings and minimize the axial dimensions, which is especially important for traction motors that are located directly in the wheels of the driven vehicle. The authors realized that in this type of machine there are a lot of harmonic electromagnetic forces, which cause noise and vibration. In order to minimize noise (to comply with noise requirements), they used two different methods. The first was to use special software in the inverter, which minimizes the pulsation of the motor torque and radial force on the external rotor. The second method was the design limitation of radial forces in the motor by the proper selection of the ratio of the number of poles to the number of stator slots. Calculations were performed using the finite element method and the Maxwell stress tensor [19]. The authors demonstrated, only computationally, that through an appropriate control strategy, torque pulsations can be significantly reduced, and radial force can be partially reduced. This leads us to believe that the proposed noise reduction method can also be applied to other types of permanent magnet traction motors.

In [59,60], the authors attempted to minimize the noise of a IPM motor (Figure 2c) through mechanical modifications and changes in the shape of the rotor surface. For the existing motor, the authors designed a modified symmetrical rotor. The modification was aimed at introducing symmetrical irregularities in the rotor surface, which has the effect of introducing the damping of selected harmonics of the magnetic flux and, at the same time, the radial forces with which the rotor interacts with the stator. The introduced modifications to the rotor resulted in an increase in other harmonics in the air gap, so [59] also examined the effect of the modifications on the torque of the motor as well as the effect on increasing the iron losses of the motor's magnetic circuit. Iterative calculations on various rotor surface modifications were performed using the finite element method and the Maxwell stress tensor [19] (Figure 5). The authors of [59] managed to reduce the sound pressure level [61] of the motor by more than 6dB, which is a very good result. The authors of [60] did not state by how much they managed to reduce the noise of the motor, only by how much they reduced the value of the radial forces. However, the noise reduction methods used require lengthy calculations, so the modifications introduced in existing machines should not lead to unexpected resonances with, for example, the gearbox or the motor frame.

In [11], the author concentrated on a method for the rapid prediction of the noise of motors operating at variable speed. The author focused on describing the method based on the influence of Maxwell's forces (Figure 5) on the noise of SPM motors and induction motors (Figure 2a,d). Based on their calculations and tests, the author concluded that there is no need to analyze the effect of magnetostrictive forces, since they have a negligible effect on the noise generated by motors. For vibroacoustic calculations, the author used MANATEE software, extensively describing the program's capabilities and its advantages; among the most important advantages, the author counted the speed of operation and the high compatibility of the calculations with the test results. They performed a validation of the calculation results for three different types of motors through experimental tests, where they obtained the height compatibility of the calculations with the tests. However, the author did not mention that they are an employee of the company that develops and sells MANATEE software. The same software was used in another publication [62], by employees of EOMYS, which owns the MANATEE software. Again, the advantages of the program in the rapid vibroacoustic analysis of permanent magnet traction motors were presented. The vibroacoustic calculations of a 2004 Toyota Prius traction motor are presented as an example of the analysis.

The authors of [63] analyzed the effect of air-gap deformation on the vibration and noise of permanent magnet synchronous motors. The authors considered the deformation of the stator and rotor due to radial forces in the motor's air gap as the primary cause of the gap irregularity. The analyzed motor has a stator with concentrated coils and a rotor with magnets inside (IPM). Using the analytical and finite element method and testing of the analyzed motor, a mathematical relationship between the air-gap deformations and electromagnetic vibrations was obtained. A detailed analysis was carried out for both the motor running at idle and the motor under load. It is noteworthy that the authors were the only ones to consider the effect of temperature on geometric dimensions during motor operation in their calculations. In their calculations, the authors did not consider the case in which the deformation of the gap could result from inaccurate motor manufacturing or other mechanical causes.

In [64], the authors analyzed the effect of the load angle in a permanent magnet synchronous motor with magnets mounted on the rotor surface (Figure 2a) on the radial and tangential forces. These forces are mainly responsible for motor noise and vibration. For the motor analyzed, the optimum load angle is 90°, and a change in this angle results in the motor not providing optimum performance at the operating point. In the first control zone, this can be due to an error in the configuration of the inverter or damage to the speed sensor; in the second control zone, the value of the angle changes depending on the required speed and, therefore, the value of the current in the d axis does too. The authors analyzed the noise of the motor by analytical and finite element methods using MANATEE software. Based on the analysis, the authors concluded that the load angle neither minimizes nor maximizes the harmonic magnetic forces responsible for the noise.

The authors of [65] chose a completely different method of eliminating the noise of motors with concentrated windings. The method consisted of modifying the outer surface of the motor stator, so it contacts the housing as little as possible; therefore, any deformation of the stator is not transferred to the housing and does not cause excessive motor noise. The authors analyzed a motor with an outer diameter of 85 mm, 12 stator slots, and 10 poles (Figure 7a). In their analysis, the authors did not take into account that isolating the stator from the housing would worsen the heat transfer from the motor and, at the same time, increase its operating temperature.

In [66], the authors analyzed a 70 kW motor that was designed as a drive for an electric vehicle. They proved that the ratio of the number of stator slots to the number of poles in the motor has a significant effect on traction torque, torque ripple, and motor noise. The motor analyzed had 42 stator slots and 8 rotor poles. Three additional versions of the motor with stator slots numbers of 48, 54, and 60 were analyzed. The basic conclusion the authors made was that when analyzing several optimization conditions, there is not a universally perfect number of stator slots given the number of poles. Each version has its advantages and disadvantages, so the final version of the motor must be a compromise between motor parameters and vibroacoustic parameters.

In [67,68], the authors analyzed various simulation models of permanent magnet synchronous motors, aiming to develop a model that would give comparable results with the finite element method but that would not be as computationally expensive. Electromagnetic torque and radial forces, which are the main sources of noise and vibration in electric drives, were chosen as comparative parameters. Two types of electromagnetic models with state variables of current (CSVMs) and state variables of magnetic flux (FSVMs) were analyzed [67]. In the next step, different methods were analyzed for calculating the forces operating in motors; these data were further used for vibroacoustic calculations. In [68], a reduced model was used that utilizes the basic characteristics of the motor. The obtained calculation results were compared with the FEM calculations. From the results presented, the proposed models give similar results to the finite element method.

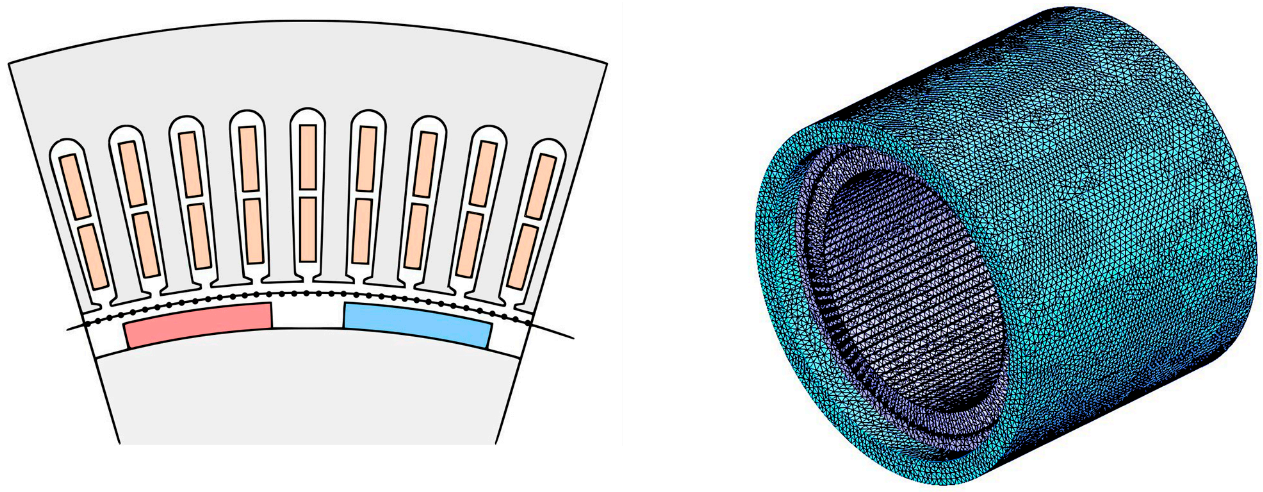
In [69], the authors analyzed a synchronous motor with salient poles in the rotor with windings. This type of motor is increasingly common in traction drives because it does not use permanent magnets. In this article, a method for calculating the deformation of the outer surface of the stator for further vibroacoustic analysis of the motor was proposed. Forces from the analytical calculations and simplified numerical calculations were used to calculate the deformation of the outer surface of the stator and to calculate the stator's natural frequency. The developed model does not take into consideration the housing or bearing plate of the motor. The calculations were verified by the finite element method.

In [70], the authors analyzed the effect of the uneven magnetization of the permanent magnets installed in a traction synchronous motor on motor noise and vibration. The authors analyzed a Toyota Prius motor with 48 stator slots and eight poles and used analytical equations as well as the MANATEE program for the analysis. The analyzed motor is a motor with magnets located inside the rotor (IPM, Figure 2c), so even the absence of one magnet or its inadequate magnetization is very difficult to notice during quality control. The authors noted that the analyzed motor, even for the equal magnetization of magnets, has a noise problem from mode 0 (breathing mode), so a further introduction of uneven magnet magnetization significantly affects the noise level of the motor over the entire speed range. Uneven magnetization of the rotor magnets causes additional structural resonances that cannot be predicted by the analytical method, so the analytical analysis of a motor with unevenly magnetized magnets has a very large error.

Ref. [71] analyzed different types of electric motors in terms of application in traction drives as well as in terms of noise and vibration. Permanent magnet synchronous motors were rated among the best for traction applications in terms of noise, while switched reluctance motors (SRM) were rated as the worst, which are very rarely used as traction drives in electric vehicles due to their parameters. Next, the authors focused on optimizing the shape of the additional slots in the rotor of a permanent magnet synchronous motor (PMSM IPM). The optimization aimed to reduce the noise of the motor for mode 24 and mode 48, but the authors completely ignored the noise of mode 0, which, according to [70], is the primary and dominant mode that generates noise in motors that have eight poles and 48 stator slots, which is exactly the type of traction motor that the authors of [71] optimized.

4. Hybrid Methods Using 3D Models

Apart from the hybrid method using the analytical method and 2D FEM models, some authors use hybrid methods using two-dimensional (2D) and three-dimensional (3D) FEMM models for the noise calculations of motors for electric vehicles. In the case of this method, the authors usually follow a general procedure. The electromagnetic forces, taken as the source and main cause of electromagnetic noise, are calculated using two-dimensional models. Often, the wave of the electromagnetic force is mapped circumferentially from the center of the air gap (Figure 8a). Then, the obtained results are applied to the structure mesh of the three-dimensional model, for which the target vibroacoustic calculations are performed (Figure 8b).



(**a**) (**b**)

Figure 8. (**a**) Two-dimensional model for calculations of the electromagnetic force and (**b**) three-dimensional model for structural calculations.

The use of the three-dimensional finite element method for vibroacoustic calculations gives the possibility to analyze complex motor designs without building a prototype. However, this method is not suitable for the preliminary vibroacoustic analysis of a motor. This is because, in a preliminary analysis, we do not yet know the design of the housing as well as the mounting of the motor. In this case, building a complex 3D FEM model and the lengthy calculations of the motor produces an effect comparable to the 2D hybrid FEM methods, with a simultaneous and significant increase in the calculation time. The accuracy of the calculation results significantly depends on the level of mapping of the model and the definition of the correct mechanical parameters of the motor components. Mapping the mechanical parameters, especially of the motor housing, which is cast as a single piece, is very difficult, and these parameters can vary depending on the company casting the housing and motor shields as well as depending on the composition of the alloy or its heat treatment. Under such conditions, the best method for analyzing the vibroacoustic parameters of motors may be to make a prototype and then test it.

The first recognized publication that used a hybrid approach to the calculation of vibroacoustic motors with the use of two and three-dimensional models is [72]. In this work, the analysis was carried out for a spoke-type, fractional-slot IPM motor with ferrite magnets. The motor has 10 magnetic poles, 12 stator slots, and maximum power for a rotational speed of n = 2500 rpm. The authors pointed out that the methods of acoustic calculations using only 2D models do not fully reflect all the phenomena taking place. The study considered the combined effect on noise of the fundamental harmonics and stator teeth harmonics of higher orders. The following calculation methodology was adopted. First, the radial magnetic force acting on the stator teeth was calculated, and the modal analysis was performed. The frequency of the force was found to be twice as high as the fundamental frequency of the motor. Then, an acoustic noise prediction model was developed by including the solution for solving the intermediate boundary elements. It was assumed that the static force has a slight influence on the vibrations, because this force does not change with time. In the obtained results, a difference in the amplitude of force between even and uneven slots was observed. The results of the calculations were verified with the results of the experimental tests. Good convergence of the results was obtained.

Ref. [73] presented the simulation methodology considering the multidisciplinary approach to determining dynamic forces and noise. The subject of the analysis is a four-pole inset type permanent magnet motor rated at 50 kW, and the number of stator slots is Q_s = 48 with distributed winding. The calculation procedure was similar to that presented in the previous publication. First, an electromagnetic calculation was performed on a

2D model in order to designate Maxwell's air-gap pressure [8,27,28]. These results were projected onto the structural mesh of the 3D model in order to calculate the stator vibration velocity, which is taken as a boundary condition in further calculations. The final step as to calculate the radiation of the vibrating structure, where the output is the acoustic power emitted by the machine. A similar methodology was used in [74], although the case was analyzed for a classical salient-pole synchronous motor. This work also provided a general review of the methods to optimize noise levels in electric motors.

Ref. [73] also focused on the possibility of including motor defects such as rotor eccentricity in simulations and the assessment of their impact on noise. The work was limited only to simulation studies, and the results were not verified by the laboratory model. In the summary of the work, the authors stated that static eccentricity causes the modulation of the excitation with respect to the ideal model by ± 1, only for the spatial harmonics, while dynamic eccentricity causes the modulation of the excitation by ± 1 for both the spatial order and harmonic order of the motor.

The influence of rotor eccentricity was also analyzed in [75]. In contrast to [73], in this paper the simulation results on a specially prepared test stand were validated. The authors pointed out that the eccentricity, unbalance, or misalignment of the shaft, to a certain extent, occurs in each motor and is a natural consequence of the inaccuracies in the performance of the motor, which are directly related to mechanical issues. The analysis was carried out for an eight-pole fractional slot concentrated winding (FSCW) motor with two slot combinations, 9 slots and 12 slots. The summary stated that the nine-slot motor has a higher level of vibration. The rotor eccentricity tests were carried out in a wide range of 0–1.5 mm. When analyzing the results of the simulation tests and measurements, it is not entirely possible to agree with the authors' claim that the results were in good agreement. Figures 14 and 15 in paper [75], which present the radiated power level sub-summary, show that for some harmonics the discrepancy between the calculations and the test results is quite significant. Moreover, a certain confusion is also caused by the fact that the presented results do not show a clear increase in the noise level due to the introduced rotor eccentricity, especially for 12-slot model. The summary also lacks a measurable assessment of the impact of rotor eccentricity on noise.

In [76], an analysis of the impact of slot–pole combination on the noise of the PMSM motor with surface permanent magnets (SPM) was presented. The analyzed motor has a stator outer diameter of 150 mm and an axial length of the core of 75 mm. The authors provided some general relationships that minimize noise by the correct selection of the number of slots with the number of poles, although it should also be noted that their analysis was limited to only four cases: $2p = 8$, and $Q_s = 48$; $2p = 8$, and $Q_s = 72$; $2p = 46$, and $Q_s = 48$; $2p = 50$, and $Q_s = 48$. The methodology of the calculations was very similar to the one presented in [72,73], but the authors ignored the modeling of the rotor because they stated that it only affects noise at low frequencies. It should be noted that the slot–pole combinations affect several motor parameters, not only noise. Therefore, the selection of the appropriate combination must always be a compromise taking into account the impact of numerous important factors.

In [77], the problem of electromagnetic noise and the possibility of its limitation in a permanent magnet motor with an external rotor were analyzed. The great advantage of the presented simulation results is their verification on a laboratory model. The object of the analysis is the fractional slot concentrated winding (FSCW) motor, which, due to several advantages, is often used in traction drives. One of the disadvantages of FSCW, however, is the increased noise level in relation to motors with distributed winding. The analyzed model is characterized by a slot–pole combination of $2p = 24$, and $Q_s = 27$. The authors pointed out that, in contrast to classic designs with an external stator, noise in a motor with an external rotor comes mainly from the rotor and is the effect of the forces acting on the surface of the permanent magnets. In their analysis, the authors also noted that the electromagnetic force acting between the stator and rotor surfaces is a spatial, rotating wave of variable amplitude; therefore, simplifying it to a point-concentrated force on the stator surface may introduce a large calculation error. In the publication, particular attention was paid to the dependence of the influence of the opening width of the slot on the noise of electromagnetic origin. As a result of the optimization and appropriate selection of the slot opening width, the noise level was reduced by 6 dB.

In [78], the problem of noise was analyzed for a 2.1 kW permanent magnet motor with a stator core that is made of amorphous material. At the same time, a comparison of the results with a motor with a stator core made of silicon electrical sheet was presented. In this work, the phenomenon of magnetostriction was considered. The characteristics of magnetostriction were determined experimentally for the classic and amorphous cores. The authors analyzed the noise issue for the entire motor structure (including the housing). In the summary results, which were verified with real laboratory models, the authors stated that the noise of the motor with an amorphous core is about 25% higher compared to the motor with silicon sheets.

In [79], the authors, based on multi-physical models combining the 2D model for electromagnetic calculations and the 3D model for structural and acoustic calculations, predicted the vibration and noise for the fractional slot concentrated winding motor with the number of poles as $2p = 6$ and the number of stator slots as $Q_s = 9$. The authors carried out the analysis for a wide range of rotational speeds of the motor and, characteristically, they assess the quality of the generated sound with four psychoacoustic indicators for various shapes of the supply current. In summary, the authors stated that the current harmonics significantly deteriorate the sound quality; therefore, they should be fully considered in the calculation models, especially when the appropriate sound quality is expected.

In [80], with the use of three-dimensional models, the impact of structural changes in the stator and the housing on changes in the natural frequency was analyzed. The authors showed that the natural frequencies of the simplified housing differ significantly from the housing with cooling ribs, feet, and even a junction box. In view of the above, the authors proposed a vibroacoustic analysis on a simplified model of the housing but with corrected dimensions. However, the authors of the work did not provide a method for correcting the dimensions, which would lead to the creation of a simplified model considering the target components of the stator housing. Furthermore, the authors of the study further analyzed the influence of the slot opening, the width and height of the tooth, and the thickness of the stator yoke on the natural frequency. In the last part of the publication, the correct modeling of the stator coils was considered. In conclusion, it was found that the correct method is to model the active parts of the winding, considering the mass of the end connections.

Ref. [81] is an interesting work, which presented numerical models and a methodology for predicting the noise inside the car caused by the electromagnetic forces of the PMSM motor. The initially adopted methodology of the authors is like the one presented in [72,73,76–80]. Using the FEM electromagnetic solver, the input in the form of electromagnetic forces was calculated and then projected onto the 3D mesh structure of the motor to determine the dynamic responses. On the other hand, an innovative approach was presented in the final stage of work, where, by combining transfer path analysis (TPA) techniques [82], the electromagnetic noise inside the electric vehicle was accurately determined. The authors concluded that the electromagnetic noise emitted by the PMSM motor has a significant impact on the acoustics of the vehicle interior, particularly high-frequency noise. The authors also concluded that the top of the motor and the junction box were the main motor components contributing to the noise inside the vehicle.

Ref. [83] presented a comprehensive noise analysis of an IPM motor with permanent magnets and distributed winding. The maximum power of the motor is 100 kW, the maximum speed is 12,000 rpm, the number of magnetic poles is $2p = 8$, and the number of stator slots is $Q_s = 48$. In this paper, note that nodal electromagnetic forces were needed as input for the vibroacoustic simulations, 3D mesh structure of the motor, and modal properties: the natural frequency, vibration waveform, and modal damping factors. The mass and stiffness of the motor structure are of key importance for the accuracy of the simulations. The housing and endcap were somewhat simplified for the purposes of simulation, but their main dimensions were retained, so that the weight does not differ from the real motor. Particular attention was paid to the analysis of the influence of the two-segment skew in the rotor on the amplitudes of the dominant harmonics of radial force densities, and attempts were made to reduce noise by the appropriate selection of the skew angle. The final results showed that the two-segment skew of the magnets helps to reduce the magnitude of the dominant harmonics of the radial force densities of the stator, especially between 3000 and 6000 rpm. It was simulated that the highest SPL (sound pressure level) [61] occurs in the same plane in which radial vibrations and acoustic pressure arise, i.e., in the transverse plane of the motor (Arc.1)—virtual microphones 1, 6, 8, 11, 16, and 18 in Figure 9.

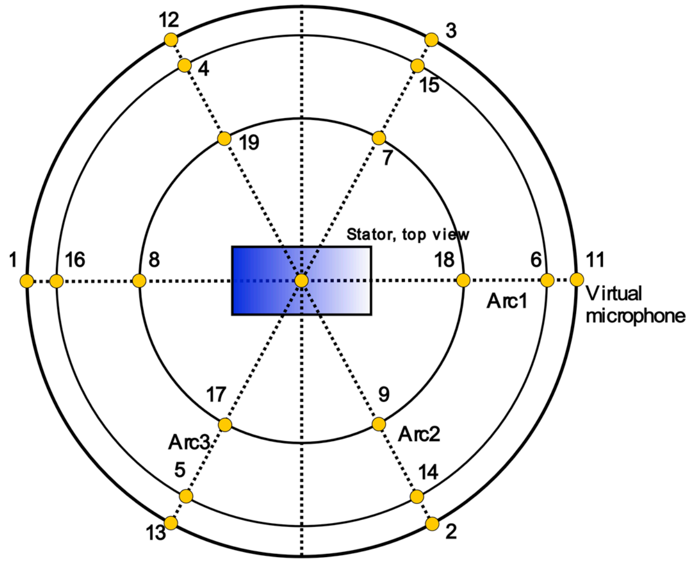


Figure 9. Arrangement of virtual microphones in the space of the stator core [83].

The analysis of the impact of the step skewing in the rotor on the noise level of the IPM motor was also analyzed in [84]. The authors emphasized that the use of skew on permanent magnets changes the distribution of electromagnetic force in the stator, which causes changes in the noise level of the motor. The approach and methodology used by the authors do not differ from previous works. Two types of skews were analyzed: diagonal skew and V-shaped skew. As in [83], the authors showed that for a certain range of rotational speeds, the use of a skew allows to reduce the noise level of the motor.

In [85], the authors used a methodology of vibroacoustic calculations similar to that in the previously discussed papers. The subject of the analysis is a permanent magnet motor with $2p = 8$ and $Q_s = 48$. As the main factor of noise reduction, the authors assumed an improvement in the stator structure stiffness, which at the same time reduces the radial electromagnetic force.

The authors of [86,87] analyzed the influence of the zero-sequence component on the level of electromagnetic noise. This component increases when the motor winding is delta-connected. The authors showed that the zero-sequence component changes the amplitudes of high-frequency vibration. Therefore, it should be considered when analyzing the operation of the delta-connected motor.

Ref. [88] focused on improving the shape of the electromagnetic circuit to improve NVH (noise, vibration, and harshness). The main objective of the work is to reduce the noise emissions during the creep driving of the LF Sonata HEV. According to the authors, at low rotational speeds, the cogging torque and torque pulsations due to the applied interior permanent magnet concentrated winding (IPMCW) motor were the main cause of noise in this driving mode. The subject of the analysis is a motor with the number of poles as $2p = 16$ and the number of slots as $Q_s = 24$, with a maximum power of 38 kW and a maximum rotational speed of 6000 rpm. The main scope of optimization is the slotted part of the stator and rotor cores using appropriate cutouts in the cores. Following the simulations, an optimized motor model was made, and laboratory measurements were conducted. For the optimized motor model, a significant reduction in the noise level in the creep driving mode and, at the same time, a slight improvement in motor efficiency were obtained.

A similar issue was analyzed in [89], in which the authors, using 3D structural dynamics analysis utilizing the vibration synthesis method, focused on optimizing the shape of the stator teeth in order to reduce NVH emissions. The forces in the air gap, the shape of the uniform force, and the final vibration responses for the three stator structures were analyzed.

The authors showed that the main factor that causes the vibration response are the electromagnetic forces of the air gap.

5. Discussion

The dynamic development of electromobility and, thus, the competitiveness of electric vehicle models is leading to the need to optimize the design of electric motors. Currently, improvements in electromechanical performance are often combined with a reduction in the weight of motors, which leads to a reduction in construction rigidity. This can produce a deterioration in the acoustic performance of electric motors. The presented literature review indicates not only that this topic has been addressed for a relatively long time but also that it is often overlooked at the design stage of new structures. In part, this may be due to the lack of tools for vibroacoustic analysis; additionally, in industrial drives, with the designs that dominated during this period, it may be less relevant.

From the review of the methods, it is also observed that some authors, while analyzing the issue of noise and vibration, and, at the same time, giving options for their mitigation, do not consider or even mention the consequences of the proposed changes; for example, the electromagnetic parameters of the motor or losses in permanent magnets can be significantly affected by the proposed noise mitigation treatments. This includes, for example, the proper matching of the number of stator slots to the number of magnetic poles of the motor. It should be concluded that the noise reduction methods proposed in this way are not inappropriate.

The results of the research described earlier show that:

- Theoretical analysis requires the development of a complex mathematical model of the machine, so simplifications of the design are often used at this stage. It is important that they do not contribute to a large calculation error. On the other hand, the advantage of this approach is the very short calculation time (once the model has been developed). Another advantage of mathematical description is the possibility of sensitivity analysis, which can allow the evaluation of the main sources of noise in individual structural solutions. It is also possible to use a few optimization approaches that can allow the motor design to be optimized to minimize the generated noise in a relatively short time. Such approaches can be identified as a direction for future research that could lead to the development of new design methods. Another interesting direction for future research seems to be the use of artificial intelligence methods for the design process of quiet and efficient electric motors.
- Vibroacoustic analysis carried out by hybrid methods allows calculations to be carried out with more complex models, but this is at the expense of calculation time. In this method, the aim is that the quantities that can be calculated by analytical methods with little error and without developing a complex analytical model are the input for the finite element calculations. However, to already correctly represent the motor design at the design stage, we need to have tools that allow rapid vibroacoustic calculations for any version of the design, and the two-dimensional finite element method helps with this. By using it, we can significantly shorten the calculation time compared to 3D calculations, while maintaining a sufficient level of correspondence between the calculations and the actual motor. The hybrid method can never be as accurate

as the method using 3D FEA, nor can it be as fast as analytical calculations, but, because it uses the best of both methods, it allows vibration-acoustic calculations to be carried out in a relatively short time with acceptable error. This method is especially recommended for design calculations of new types of motors, where we must perform multiple calculations to look for the optimal motor version.

- Vibroacoustic analysis using a hybrid FEM method based on two- and three-dimensional models allows mapping and analysis of the target and often complex structural solutions, though it should be borne in mind that these calculations are very time-consuming and require a lot of computing power. Even though the method itself allows calculations on faithfully reproduced models, in most works the authors use some simplification of the models, due to the target calculation time and, simultaneously, their low impact on the final calculation error. The methodology of vibroacoustic calculations in most works follows a similar pattern. Two-dimensional models are used for electromagnetic calculations to determine the Maxwell pressure [8,27,28] in the air gap. In subsequent steps, these results are projected onto the 3D model's structural grid to calculate the stator vibration velocity and, ultimately, the acoustic power emitted by the machine. It seems that the use of a hybrid method using 3D models for vibroacoustic analysis is expedient for motors in which the length of the stator iron package is larger than its diameter. In addition, this method can be useful for unusual motor solutions, where analysis with analytical models or only two-dimensional models would be impossible or subject to high computational error. From the literature review, it is noted that the main benefit of using 3D models is not so much the improvement in computational accuracy over other methods, but the extension of functionality and the possibility of analyzing more complex systems.

Table 1 compares the most important features of the different noise analysis methods in terms of their practical application. This can be useful for engineers developing new designs of electric motors. Of course, the choice of method can be determined by other factors, such as the availability of analytical tools (software).

Table 1. Summary of properties of methods used to analyze noise levels in electric motors.

Method	Model Development Time	Calculation Error	Calculation Time	Sensitivity Analysis	Optimization Approach	Best Usage
Analytical	Long	Large	Short	Easy	Easy	Series of motors
Hybrid with 2D MES	Medium	Medium	Medium	Difficult	Difficult	New typical motors
Hybrid with 3D MES	Long	Low	Long	Difficult	Very difficult	New unusual motors

The methods of analyzing noise sources in the traction motors used in electric vehicles require examining the influence of the many factors affecting the level of noise generated by the motor. Some of the most important factors affecting the noise and vibration of the motor are the frequency resonance of the magnetic forces generated by the permanent magnets, the natural frequencies of the stator and the housing, and the harmonics originating from the supply current. The resonance of these frequencies leads to excessive deformation of the stator, which is transferred to the housing and radiates in the form of acoustic noise. Therefore, the material of the motor housing should be properly selected, because it, as part of the motor, can either amplify or dampen the noise coming from the stator.

A very important issue affecting the level of the noise generated in traction motors is the appropriate selection of the number of slots and poles. The ratio of the number of slots to the number of poles is particularly important in motors with concentrated windings. In these motors, the source of noise may be uneven magnetic tension, which occurs with an inappropriate ratio between the number of stator teeth and the number of rotor poles.

Other important factors that should be considered when designing a motor is the impact of the thickness of the stator yoke, the shapes of the stator teeth, and the mass of the winding on the natural frequencies of the stator. Noise in traction motors can also be caused by the interaction of the permanent magnets located in the rotor or on its surface with the teeth of the stator (cogging torque). In many motors, especially traction motors with a power of up to 100 kW, an important element of noise reduction is the use of the skew of the stator slots or rotor magnets. However, for high-torque motors with a higher power operating in the speed range from 0 to 3000 rpm, the influence of the skew on the noise is negligible, and the main source of noise is mode 0 (breathing) forces, which for this size of machine have frequencies such as the natural vibration of the housing as well as the motor stator. In order to reduce noise in this type of traction drive, the ranges of the rotational speeds at which the motor is to operate are limited.

Another noise reduction method used, in addition to the above-described ones, is changing the shape of the rotor surface. The modification is aimed at introducing symmetrical irregularities (notches) on the surface of the rotor, which results in the introduction of the damping of selected harmonics of the magnetic flux and, at the same time, of the radial forces with which the rotor interacts with the stator.

Human error, which may already occur during the production of the motor, can also contribute to the increase in the noise level. One such error is making an off-center rotor or using unequally magnetized magnets. Many traction motors, even for the even magnetization of magnets, have a problem with noise from the 0 mode (breathing), since the introduction of additional non-uniform magnetization of the magnets significantly affects the noise level of the motor in the entire range of rotational speeds. Furthermore, such unevenness or eccentricity may cause additional structural resonances, the effects of which are difficult to predict without detailed laboratory tests of the motor.

In addition to design solutions, in the analysis of noise sources, the influence of the control system should also be considered. In this case, an important source of information seems to be the assessment of the impact of the stator current in the d and q axes on the radial and tangential components of the force in the motor air gap. The current component in the q axis affects both radial forces and tangential forces. The use of an appropriate control model that minimizes the pulsation of the motor torque and radial force can reduce motor noise without changing the motor design. Using the modified control system, it is possible to actively influence the level of generated noise by also injecting additional current components, with the task of shaping the radial forces. In such a case, however, it is necessary to develop a circuit model with the task of determining the reference current components. From the point of view of the power supply, the PWM modulation frequency of the inverter should also be properly selected, since choosing too low a frequency may result in the appearance of additional noise sources, such as, e.g., stator sheets or rotor packages. If the design of the inverter prevents the use of a higher PWM frequency, then one of the solutions may be using pseudo-random modulation.

In the methods of the noise analysis of traction motors, simplifications are also important, regardless of the analysis method used. In many publications, the authors proved that one of the most important simplifications in motor noise analysis is the omission of electromagnetic forces with a spatial order higher than four. It was proven that the deformation of the stator core, which is directly responsible for the noise generated by the motor, is inversely proportional to the square of the order of force. Another simplification is the omission of the influence of magnetostrictive forces, which, due to their frequencies, have a negligible effect on the noise generated by the motor.

Additional sources of noise and vibration in traction motors are mechanical and aerodynamic sources. Mechanical noise and vibration can come from bearings or gears, but their frequencies are usually lower than those of electromagnetically induced noise, and their levels are not high. Methods of eliminating mechanical noise usually involve replacing damaged bearings or optimizing the structure (e.g., reinforcement).

Aerodynamic noise in electric motors is caused by periodic changes in the air pressure from mounted fans or air turbulence in the air gap caused by stator slots. However, electric traction drives are usually liquid-cooled, or ventilators are not used at all. Usually, the traction motor housing is sealed. It can, therefore, be concluded that, for this type of motor, the ventilation and aerodynamic noise are negligible.

Table 2 contains a synthetic summary of the most important factors influencing the phenomenon of noise and vibration in motors for electric vehicles. For each of them, the type of noise origin and a reference to the literature in which the authors analyzed the phenomenon are also provided. The last column of Table 2 lists the types of motors covered by the analysis.

Table 2. The influence of various factors on NVH of motors for electric vehicles.

Factor Type	Noise Origin	Analysis Methods	References	Type of Analyzed Motors
Optimization of the stator winding and the elements of the electromagnetic circuit	Magnetic	Analytical methods Hybrid 2D Hybrid 3D	[23,46,50,52,56,65,80,89]	Radial-flux SPM motor [23] Axial-flux SPM motor [23] FSCW motor [50,65] IPM motor [52,56]
Slot–pole combinations	Magnetic, design faults	Analytical methods Hybrid 2D Hybrid 3D	[8,35,50,52,53,58,66,76]	PMSM motor [35], FSCW motor [8,50,52,53,58,66,76]
Slot—opening width	Magnetic	Hybrid 2D Hybrid 3D	[50,77,80]	FSCW motor [50] PMSM with outer rotor [77]
Notches in the stator or rotor magnetic sheets	Magnetic	Analytical methods Hybrid 2D Hybrid 3D	[25,59,60,71,88]	BLDC motor [25], IPM motor [59,60,71] FSCW motor [88]
Skew in the stator or rotor	Magnetic, design faults	Analytical methods Hybrid 3D	[23,83,84]	Radial-flux SPM motor [23] Axial-flux SPM motor [23] FSCW IPM motor [83,84]
Type of rotor core material—amorphous core	Magnetic	Hybrid 3D	[78]	PMSM motor [78]
Cogging torque	Magnetic	Analytical methods Hybrid 2D Hybrid 3D	[25,48,58,62,66,88]	BLDC motor [25] IPM outer rotor [48] SPM outer rotor [58] IPM motor [62] FSCW motor [66,88]
Torque ripple	Magnetic	Analytical methods Hybrid 2D Hybrid 3D	[24,56,58,63,88]	IPM motor [24,56,63] SPM outer rotor [58], FSCW motor [88]
Rigidity of the mechanical structure	Mechanical	Analytical methods Hybrid 3D	[24,80,85]	IPM motor [24] PMSM motor [85]
Rotor eccentricity or unbalance	Mechanical, manufacturing mistakes	Hybrid 2D Hybrid 3D	[49,63,73,75]	IPM motor [49,63] FSCW motor [75]
Radial force change	Magnetic, mechanical	Analytical methods Hybrid 2D Hybrid 3D	[11,25,26,29,30,47,51,57–60,62,63,68,68,70,72–75,80,87,89]	BLDC motor [25] PMSM motor [11,26,29,30,47] IPM motor [11,51,57,59,60,62,63,67,68,72,73], SPM outer rotor [58] Induction motor [11] FSCW motor [75,79,87] Synchronous salient-pole machine [74]
Harmonics of magnetic forces	Magnetic, power supply	Analytical methods Hybrid 2D Hybrid 3D	[24,51,57,58,63,69,70,88]	IPM motor [24,51,57,63,70] SPM outer rotor [58] PMSM motor [69] FSCW motor [88]

Table 2. *Cont.*

Factor Type	Noise Origin	Analysis Methods	References	Type of Analyzed Motors
Structural mode 0 (breathing mode)	Magnetic, mechanical	Analytical methods Hybrid 2D Hybrid 3D	[34,62,69–71,86,87]	IPM motor [62,70,71] PMSM motor [69] FSCW motor [86,87]
Additional harmonics in current waveforms	Power supply	Analytical methods Hybrid 2D Hybrid 3D	[40,49,59,67,68]	IPM motor [40,49,67,68] FSCW motor [59,79]
Inverter switching frequency	Power supply	Analytical methods	[41]	PMSM motor [41]
Use of multilevel inverter and more than 3 phase motors	Power supply	Analytical methods	[42]	PMSM motor [42]
Incorrect determination of the rotor position	Mechanical manufacturing mistakes, power supply	Analytical methods Hybrid 2D	[43,64]	PMSM motor [43,64]

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