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Arrhenius Equation for Calculating Viscosity in Assessing the Dilution Level of Lubricating Oil with Diesel Oil—A Case Study of SAE 30 and SAE 40 Grade Marine Lubricating Oils

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Abstract: This article proposes using the Arrhenius model for estimating the viscosity of a mixture of two liquids in the quantitative assessment of the physicochemical properties of lubricating oils in the context of assessing the level of dilution of lubricating oil with diesel oil. Dynamic are made of mixtures of lubricating oil and diesel oil with known concentrations of 0, 1, 2, 5, 10, 20, 50, and 100% m/m of the diesel oil content in the mixture. Mixtures of the most popular oils (viscosity classes SAE 30 and SAE 40) with diesel oil that meet the requirements of the DMX of the marine distillate fuels category are prepared and tested. Viscosity measurements are performed at 40, 50, 60, 70, 80, 90, and 100 °C temperatures. The Arrhenius model is used to estimate the viscosity of the mixtures with an assumed diesel oil content and to estimate the diesel oil concentration in the mixtures with a known measured viscosity value. In both cases, the absolute estimation error is determined, and the accuracy of the estimation is assessed against the known concentration of diesel oil in the mixture with lubricating oil and the temperature at which the viscosity is measured. The estimated concentrations of diesel oil in mixtures with lubricating oil are useful to assess the condition of the lubricating oil (for concentrations of diesel oil lower than 5% m/m). The method is proposed to be used in practice to support standard laboratory oil analysis.

Keywords: fuel dilution; lubricating oil; diesel oil; dynamic viscosity; mixture viscosity; marine diesel engine

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1. Introduction

One of the threats that may occur during the operation of marine piston combustion engines is the deterioration of lubrication conditions, which is influenced by the design [1–3] and technical condition of the engine [4–6], and type of oil [7–9] and fuel [10–12] with which the engine is powered. There are many research results devoted to assessing the efficiency and reliability of internal combustion engines with contaminated circulating lubricating oil (LO) [13–15], including fuel-diluted lube oil (both distillation and residual) [16–18]. Operating experience and test results clearly show that diluting LO with fuel is unfavorable for the engine, deteriorating the engine's operating efficiency, shortening the life of the oil, and reducing the reliability and safety of the engine [19,20]. Over time, the excessive and progressive dilution of oil with fuel can lead to significant wear [7,21] and serious engine failure [22], resulting in explosions in the crankcase, among other issues [13,23]. Castrol indicates in a publication that diluting LO with fuel would result in a crankcase explosion [23].

In operational practice, both macro- and micro-analytical methods are used [24–31], and some of these methods are proprietary [16,28,29,32,33]. A list of the most important methods with an indication of their application variants is presented in Figure 1. In operational practice, the most employed methods for assessing the contamination of LOs with fuels are routinely used, including oil viscosity analysis and measurement of the ignition temperature of oil periodically to collect tests from the engine lubrication system [34–36]. These macro-analytical methods are relatively cheap and generally available in most industrial laboratories.

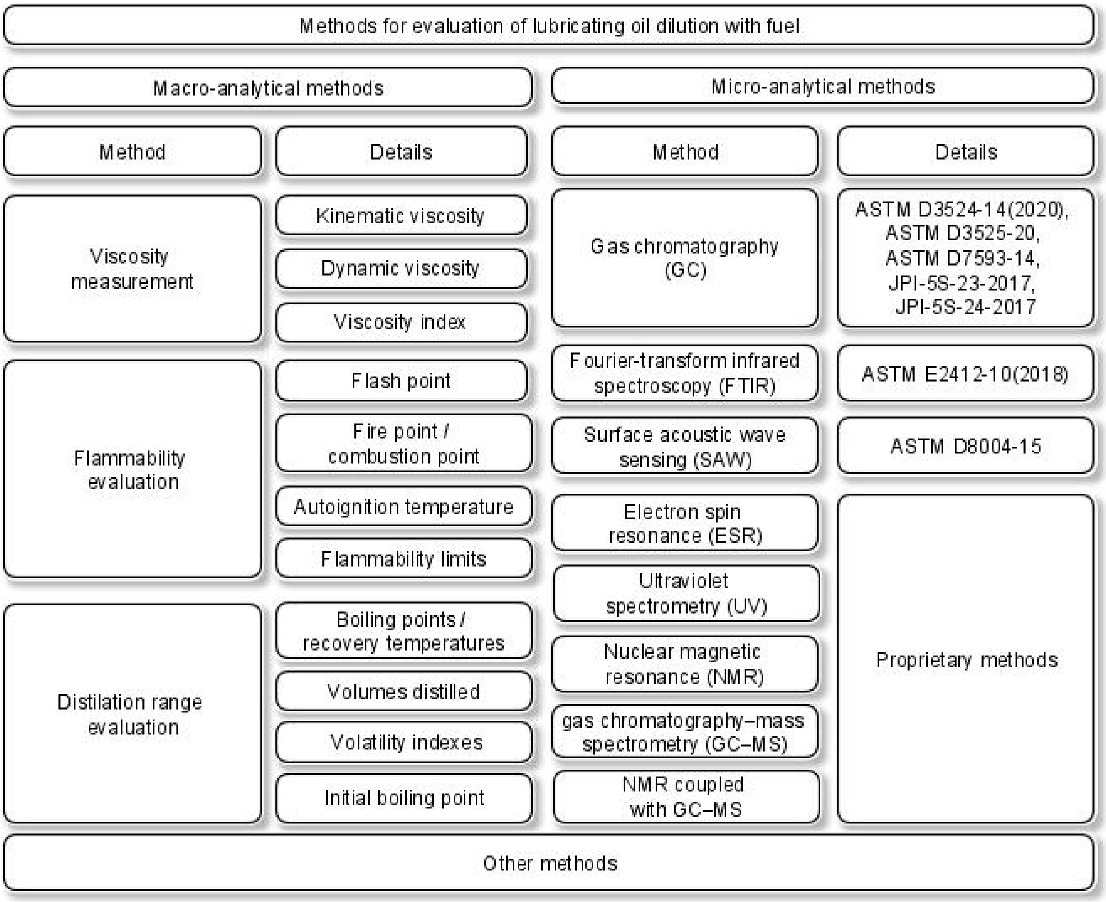

Figure 1. List of methods for detecting and assessing the degree of dilution of engine LOs with fuels (all mentioned standards are listed in the references [25–31]).

Oil viscosity changes due to contamination of the LO with fuel and may increase when the LO is diluted with distillation fuel (light or medium diesel oil (DO), gasoline, etc.), or increase due to dilution of the LO with residual fuel (i.e., heavy fuel oil). To draw conclusions regarding the dilution of oil with fuel, it is necessary to know the type of fuel used by a given engine from which the assessed LO was taken.

The advantages of using viscosity as a measure of the level of dilution of LO with DO include the general availability of both portable and laboratory equipment for viscosity measurements [17,37,38]. For this reason, this method is used routinely to periodically check LOs. Moreover, the technique makes it possible to detect ethanol and biofuels such as biodiesel.

When using this method, the age and operating conditions of the oil in a given engine must be considered because a change in oil viscosity value may relate to the following factors [13,39]:

- Accumulation of pollutants (wear products of interacting machine elements, products of the combustion process, dust, and dust present in the air);
- The oil being diluted with other oils, grease, or coolants;
- Oil aging due to oxidation and thermal decomposition of components;

- Depletion of additives that modify the properties of the oil (chemical reactions);
- Infection of the oil with protozoa, bacteria, fungi, and viruses.

Considering the influence of the above-mentioned factors is possible using additional indicators such as base number, the content of sediments, and the share of metallic components, as well as the water content in the tested oil, which is periodically determined for oil samples from circulating lubrication installations of marine and industrial engines in connection with routine tests.

With the above in mind, the use of oil viscosity as a measure of the degree of dilution of LO with fuel should be used as a complementary indicator determined together with the oil ignition temperature, or possibly with the simultaneous use of other methods.

Determining the viscosity of a mixture of two or more liquids with different viscosities is a complex issue because different liquids mixed in varying proportions can have very different viscosity characteristics, which, depending on the volume fraction of the ingredients, may show high nonlinearity. So far, no universal model has been developed to describe the viscosity of a mixture of liquids with known viscosities when the share of the liquids in the mixture is known.

Many semi-empirical mathematical models have been created to describe this issue with improved approximation, including the Kendall–Monroe cubic-root equation as the more important one [40], the Refutas equation [41] used with the viscosity blending index (described in ASTM D7152-11 (2016) [42]), the Arrhenius equation [43], the Grundberg–Nissan equation [44], the Oswal–Desai equation [45], the McAllister equation [46], the Lederer–Roegiers equation, and so on [41].

One of the oldest methods was developed in the second half of the 19th century. This model, developed by Svante Arrhenius, is relatively simple and is often used to roughly calculate the viscosity of a mixture of liquids with known viscosities [43]. The Arrhenius model is described by the following equation:

$$\ln\eta_{mix} = x_A \ln\eta_A + x_B \ln\eta_B, \tag{1}$$

where η_{mix} denotes the dynamic viscosity of the liquid mixture, η_A signifies the dynamic viscosity of liquid A, η_B is the dynamic viscosity of liquid B, x_A is the mole fraction of liquid A in the mixture, x_B is the mole fraction of liquid B in the mixture, and $x_A + x_B = 1$.

However, the Arrhenius equation shows various inaccuracies for mixed substances with very different chemical compositions [47]. This resulted in the development of several models derived from the Arrhenius equation with a larger number of parameters, including corrections relating to the properties of individual component substances. However, the disadvantages of these models include the significant complexity of the relationships and the need to know the material constants that depend on the composition of the substances used to prepare the mixture [41].

This practice has shown that for substances with a similar composition, e.g., mineral oils, the equation can be used for various purposes, and the mole shares of the mixture components can be replaced in the equation by mass fractions. Moreover, in approximate calculations for mixtures of liquids with similar densities (such as LOs of various types), it is possible to replace dynamic viscosity with kinematic viscosity in Equation (1). This fact was pointed out in the publication of ref. [41]. Also, in ref. [48], the above-mentioned formula was simplified by changing mole fractions to mass fractions.

The above observations motivated the author to build a mathematical model based on the Arrhenius equation describing the viscosity of a mixture of lubricating and DOs that can be used in operational practice to assess the occurrence and quantitative assessment of the dilution of lubricating oils with DOs. To build a detailed model, the author used dynamic viscosity as the input and output quantity. Due to the difficulty in precisely determining the average molar mass of hydrocarbon mixtures, often with significantly different chemical compositions, a simplification was adopted involving the use of the actual mass percentage of DO in mixture C (% m/m):

$$\eta_{ULO} \approx \exp\left[\left(\frac{C}{100}\right)\ln\eta_{DO} + \left(\frac{100-C}{100}\right)\ln\eta_{FLO}\right], \tag{2}$$

where η_{ULO} is the calculated dynamic viscosity of the mixture of lubricating and DOs (in practice, it may be, e.g., in-service or used oil), η_{DO} is the dynamic viscosity of the DO that powers the engine, and η_{FLO} is the dynamic viscosity of fresh oil lubricant.

Transformations make it possible to derive a formula from Equation (2) for determining the estimated percentage of the mass share C_{est} of DO in the mixture with LO, which for the adopted assumption is:

$$C_{est} \approx \frac{\ln\left(\frac{\eta_{ULO}}{\eta_{FLO}}\right)}{\ln\left(\frac{\eta_{DO}}{\eta_{FLO}}\right)} \cdot 100\%, \tag{3}$$

The quantitative results of the application of Formulas (2) and (3) are presented in Appendix A, while a detailed description of how to derive these models is included in Appendix B.

To assess the suitability of Equations (2) and (3) for a quick, approximate assessment of the level of dilution of circulating LO with DO, an experiment was carried out in which the measured viscosity value for the known concentration of DO in a mixture with LO was compared with the value obtained using Formula (2), and the known DO content was compared with the calculated concentration value (Equation (3)). The level of reproduction of reality by the analyzed computational models was also assessed.

2. Materials and Methods

The research methodology of the experiment is presented in Figure 2. The exact dynamic viscosity value was measured for the prepared mixtures of lubricating and DOs with a known mass fraction of DO in the mixtures. Using the models described in Equations (2) and (3), the estimated values of dynamic viscosity and the concentrations of DO in the mixtures were determined. The estimated values were compared with the exact values, and the value of the absolute estimation error was determined. Based on the results obtained, conclusions were drawn regarding the usefulness of the Arrhenius model in operational practice as a tool for assessing the degree of dilution of LO with DO.

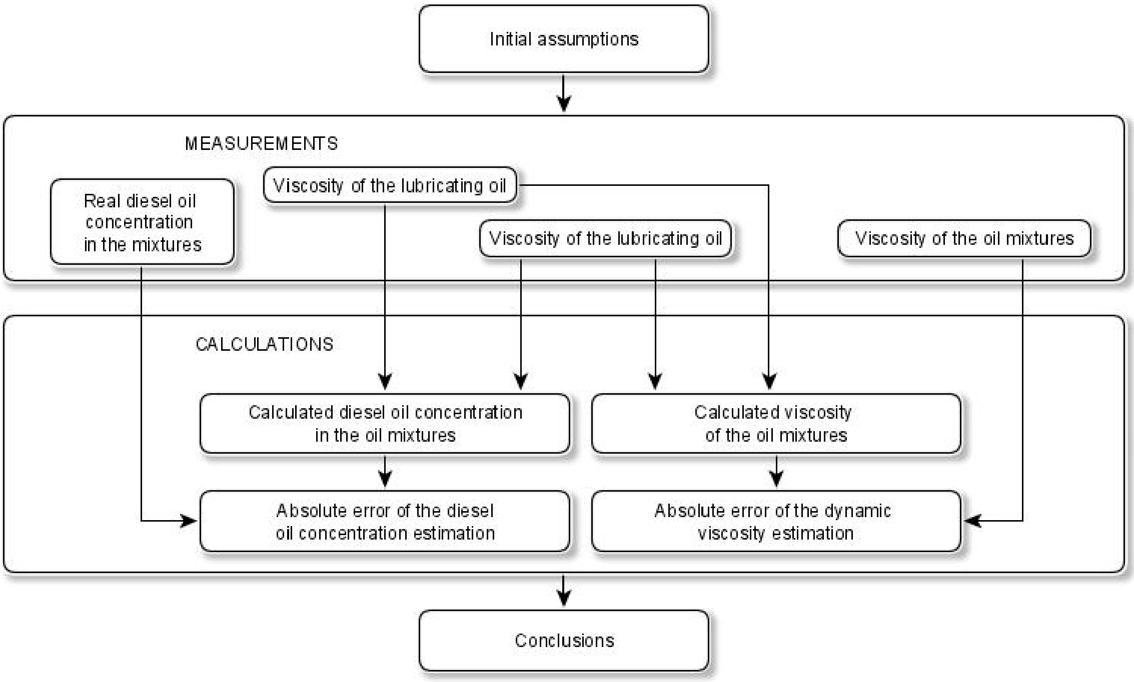

Figure 2. Research methodology adopted in the experiment.

The experiment used samples of clean LO contaminated with DO at given, known concentrations. To carry out the experiment, mixtures of single-grade oil belonging to viscosity classes SAE 30 and SAE 40 were made [49], with the DO meeting the requirements of the DMX of the marine distillate fuels category [50]. Agip/Eni Cladium SAE 30 CD/CF LOs [51], Agip/Eni Cladium SAE 40 CD/CF [52], and Orlen Efecta Diesel Biodiesel (designation CN27102011D) [53] were used. Both LOs met the requirements of the API CD/CF quality class (Series III) [54] and specifications of marine engine manufacturers. The physicochemical properties of the lubricating and DOs used to prepare the mixtures tested in this experiment, declared by the manufacturers, are presented in Tables 1 and 2.

Table 1. Physicochemical properties of the Agip/Eni Cladium 120 CD/CF LOs used in the tests (declared by the manufacturer) [51,52,55].

Specification	Parameter	
Oil	SAE 30 CD/CF	SAE 40 CD/CF
Kinematic viscosity (according to PN-EN ISO 3104) [56]	108 mm^2/s at 40 °C 12.0 mm^2/s at 100 °C	160 mm^2/s at 40 °C 15.7 mm^2/s at 100 °C
Viscosity index [57]	100	100
Total base number	12 mg KOH/g	12 mg KOH/g
Flash point (marked in a closed cup) [58]	225 °C	235 °C
Pour point	−18 °C	−15 °C
Density	895 kg/m^3 at 15 °C	900 kg/m^3 at 15 °C

Table 2. Physicochemical properties of Orlen Efecta Diesel Biodiesel oil declared by the manufacturer [59].

Specification	Parameter
Cetane number	≥51
Initial boiling point	75–180 °C
Boiling point range	95% vol. distills to 360 °C
Flash point (determined in a closed cup)	>56 °C
Auto-ignition temperature (according to DIN 51794:2003-05 [60])	Approx. 240 °C
Kinematic viscosity (according to PN-EN ISO 3104 [56])	1.5–4.5 mm^2/s (2.549 mm^2/s) at 40 °C Approx. 2.151 mm^2/s at 50 °C
Density	820–845 kg/m^3 at 15 °C
Relative vapor density	Approx. 6 (air = 1)
Cloud point	−7 °C
Cold filter lock temperature	−28 °C

For mixtures of each of the tested LOs, the dynamic viscosity of mixtures with a known mass fraction of DO in the LO equal to 0, 1, 2, 5, 10, 20, 50, and 100% m/m of DO was determined. The measurements were performed at temperatures of 40, 50, 60, 70, 80, 90, and 100 °C (temperatures of 40, 50, and 100 °C were used as reference temperatures for determining viscosity and derived values in accordance with the requirements of various standards). The dynamic viscosity of the tested mixtures η_{ULO} (mPa·s) at a given measurement temperature t was determined according to the following relationship:

$$\eta_{ULO} = \frac{\nu_{ULO} \cdot \rho_{ULO}}{1000}, \tag{4}$$

where ν_{ULO} is the kinematic viscosity of the mixture (mm^2/s) determined at temperature t and ρ_{ULO} is the density of the tested mixture (kg/m^3) at temperature t.

The density of individual samples ρ_{ULO} was measured using a DMA 4500 apparatus (Anton Paar GmbH, Graz, Austria) with an oscillating U-tube according to the ISO 12185:2002 standard [61]. The accuracy of the camera's setting was 0.02 °C. However, the density measurement accuracy was 5×10^{-5} g/cm^3. The kinematic viscosity was measured using a Cannon-Fenske Opaque glass capillary apparatus (Paradise Scientific Company Ltd., Dhaka, Bangladesh) and a TV 2000 viscometer bath (Labovisco bv, Zoetermeer, the Netherlands) in accordance with the ISO 3104:2004 standard [56]. The accuracy of the measurement temperature setting was 0.01 °C. The accuracy of the device was 0.1 mm^2/s. Appendix A, including Tables A1 and A2, presents the dynamic viscosity value of the tested mixtures determined as a result of the measurement.

For the known concentrations of DO in mixture C (% m/m), the known value of the dynamic viscosity of pure LO, η_{FLO} (mPa·s), the dynamic viscosity of pure DO, η_{DO} (mPa·s), and the estimated value of the dynamic viscosity of each of the tested substances, η_{ULO} (mPa·s), were calculated according to the proposed model described with Formula (2). The obtained results are presented in Appendix A in Tables A3 and A4. The actual value of the dynamic viscosity was compared with the calculated (estimated) value, and the absolute estimation error δ_η (mPa·s) of dynamic viscosity was determined, the value of which is listed in Appendix A in Tables A5 and A6. The error δ_η was calculated according to the following relationship:

$$\delta_\eta = |\eta_{real} - \eta_{ULO}|, \tag{5}$$

Similarly, using relationship (3), the estimated value of the mass share of DO C_{est} (% m/m) in the mixture was determined, calculations of which were based on the measured value of the dynamic viscosity of the tested mixture, η_{ULO} (mPa·s), the known dynamic viscosity of the LO, η_{FLO} (mPa·s), and the dynamic viscosity of the DO, η_{DO} (mPa·s). The obtained results are presented in Appendix A in Tables A7 and A8. The actual value of the dynamic viscosity was compared with the calculated value, and the absolute estimation error δ_C (% m/m) of diesel oil concentration was determined, the value of which is listed in Appendix A in Tables A9 and A10. The error δ_C was calculated according to the following relationship:

$$\delta_C = |C - C_{est}|, \tag{6}$$

The obtained results were used to draw final conclusions and assess the usefulness of the Arrhenius model in assessing the properties of mixtures of the analyzed type.

3. Results and Discussion

Figures 3 and 4 compare the measured value of dynamic viscosity with the value calculated using the previously presented model based on the Arrhenius equation for mixtures of SAE 30 and SAE 40 LO with DO, respectively, depending on the measurement temperature. Apart from the DO concentration in the mixtures of 0% and 100%, in all cases, an overestimation of the calculated values compared to the actual values is observed. The exact representation of the values for extreme concentrations of 0% and 100% of DO in a mixture with LO results from the specificity of the calculation model, in which the exact value of the dynamic viscosity of lubricating and DOs must be entered among the input values.

At the same time, the difference between the calculated and measured values decreases as the share of one of the components in the mixture increases or decreases relative to the value of 50% m/m. The overestimation of the calculated value relative to the actual value decreases as the measurement temperature increases. This nature of the changes applies to both the SAE 30 and SAE 40 oil mixtures.

Figures 5 and 6 present the values of absolute error in the estimation of dynamic viscosity for the tested mixtures of lubricating and DOs, respectively, as a function of the mass fraction of the mixture components and as a function of the measurement temperature.

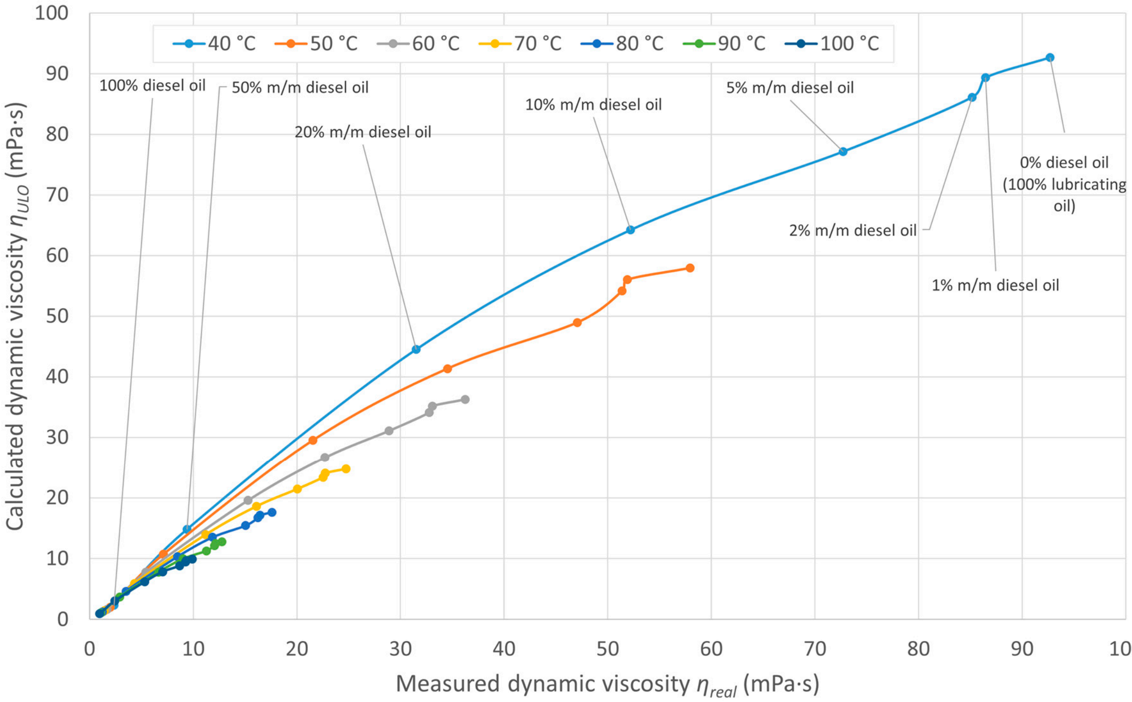

Figure 3. Relationship between the calculated and measured values of dynamic viscosity of mixtures of SAE 30 LO with DO at different measurement temperatures (DO concentrations for temperatures higher than 40 °C are the same as marked for the measurement temperature of 40 °C; the order of values is the same).

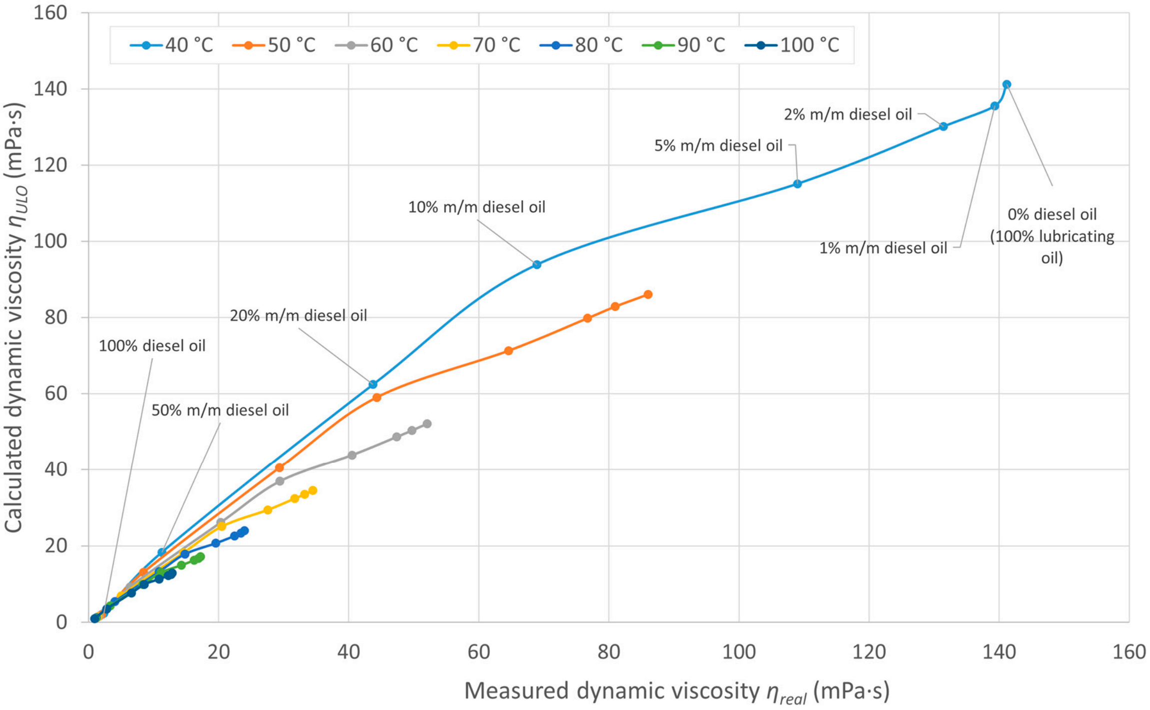

Figure 4. Relationship between the calculated and measured values of dynamic viscosity of mixtures of SAE 40 LO with DO at different measurement temperatures (DO concentrations for temperatures higher than 40 °C are the same as marked for the measurement temperature of 40 °C; the order of values is the same).

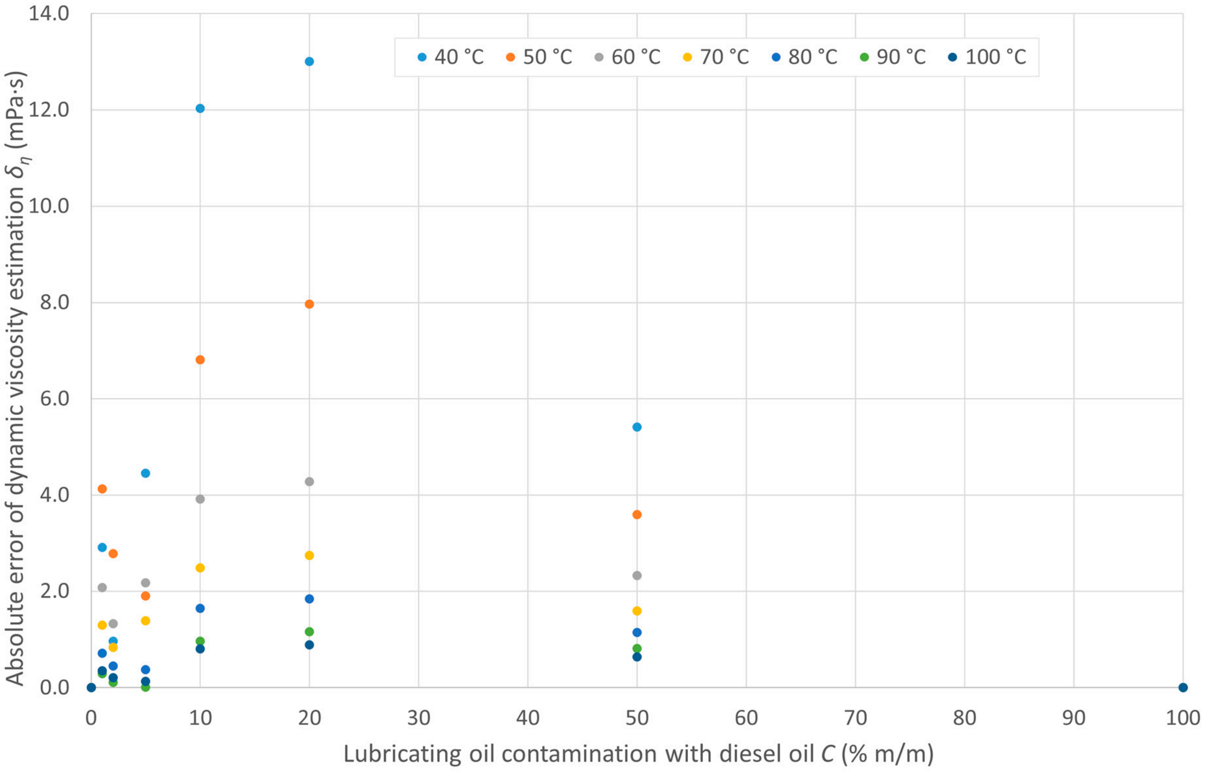

Figure 5. Absolute error in estimating the dynamic viscosity of SAE 30 LO mixtures with different proportions of DO.

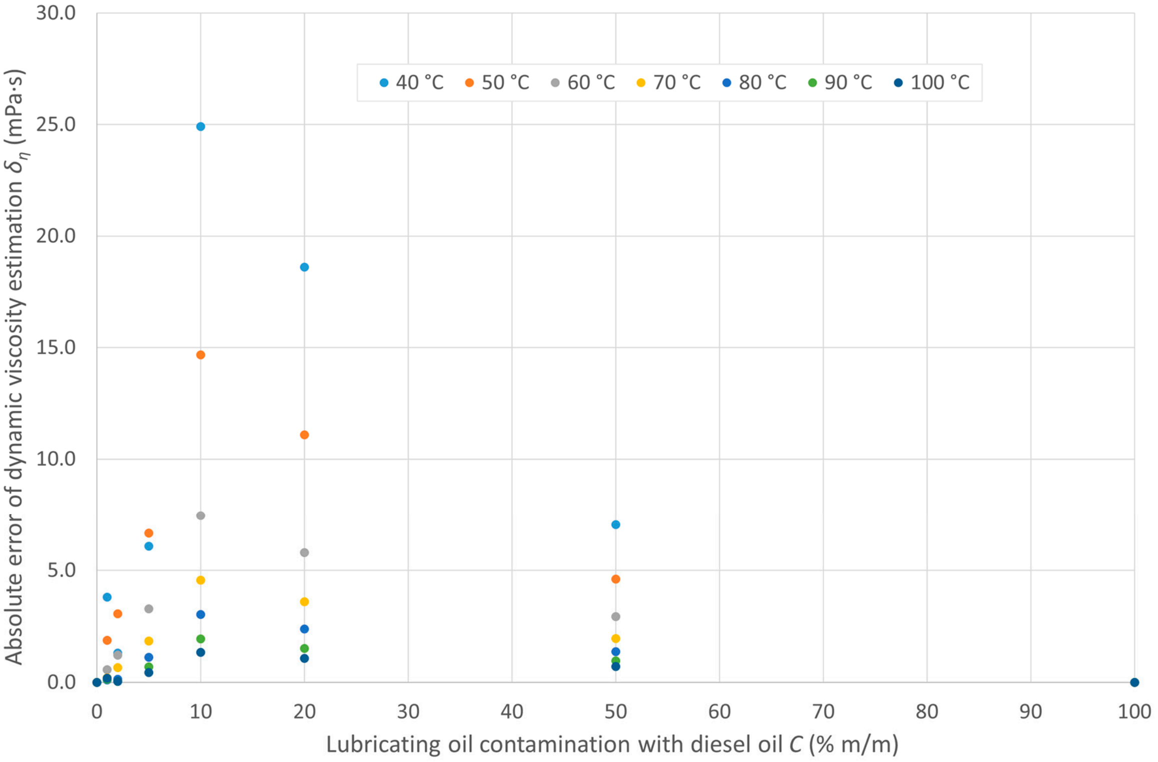

Figure 6. Absolute error in estimating the dynamic viscosity of SAE 40 LO mixtures with different proportions of DO.

The largest measurement error is observed for 10–20% m/m of DO concentration in the tested mixtures with LO, for both the SAE 30 and SAE 40 viscosity classes (Figures 5 and 6). The estimation error reaches the maximum increase as the measurement temperature decreases, which proves the sensitivity of the mathematical model to large differences in the viscosity values of mixture components. As the measurement temperature increases, the difference in the viscosity of the components decreases and, thus, the value of the estimation error decreases.

The maximum recorded values of absolute estimation error in the case of the tested mixtures of LO with DO at a measurement temperature of 40 °C do not exceed the value of 13.1 mPa·s in the case of LO mixtures of the SAE 30 class, and the value of 25 mPa·s in the case of mixtures of LO class SAE 40. For the range of DO in LO concentrations lower than 10% m/m, which are encountered under operating conditions, the maximum absolute estimation error does not exceed a value of 4.5 mPa·s for the SAE 30 oil mixtures and 6.7 mPa·s for the SAE 40 oil mixtures at the measured temperatures.

Figures 7 and 8 compare the actual and calculated values of the mass share of DO in mixtures with SAE 30- and SAE 40-class LO, respectively, depending on the measurement temperature. In all the analyzed cases, regardless of the measurement temperature and LO type, a very good fit of the computational model to the actual value of the mass concentration of DO in the mixtures with LO is observed.

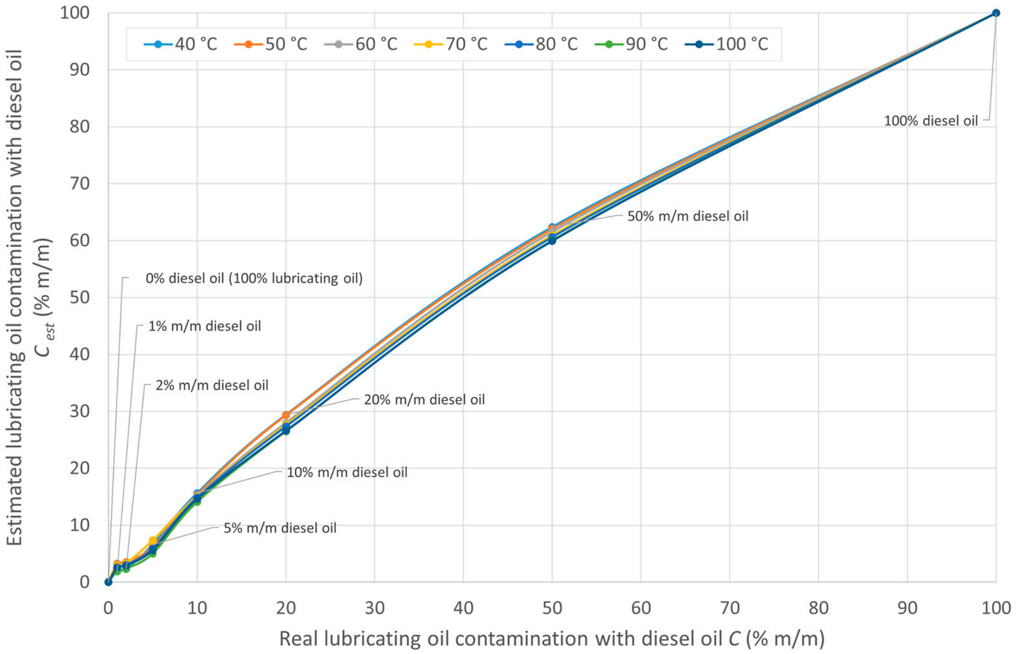

Figure 7. Relationship between the calculated and actual values of DO concentrations in mixtures with SAE 30 LO at different measurement temperatures (DO concentrations for temperatures higher than 40 °C are the same as marked for the measurement temperature of 40 °C; the order of values is the same).

Figures 9 and 10 show the value of absolute error in estimating the DO concentration in the tested mixtures as a function of the actual concentration of DO in the mixtures and as a function of the measurement temperature. The results show fluctuations in the absolute error value in the concentration range of 0–10% m/m DO in mixtures with LO at different measurement temperatures.

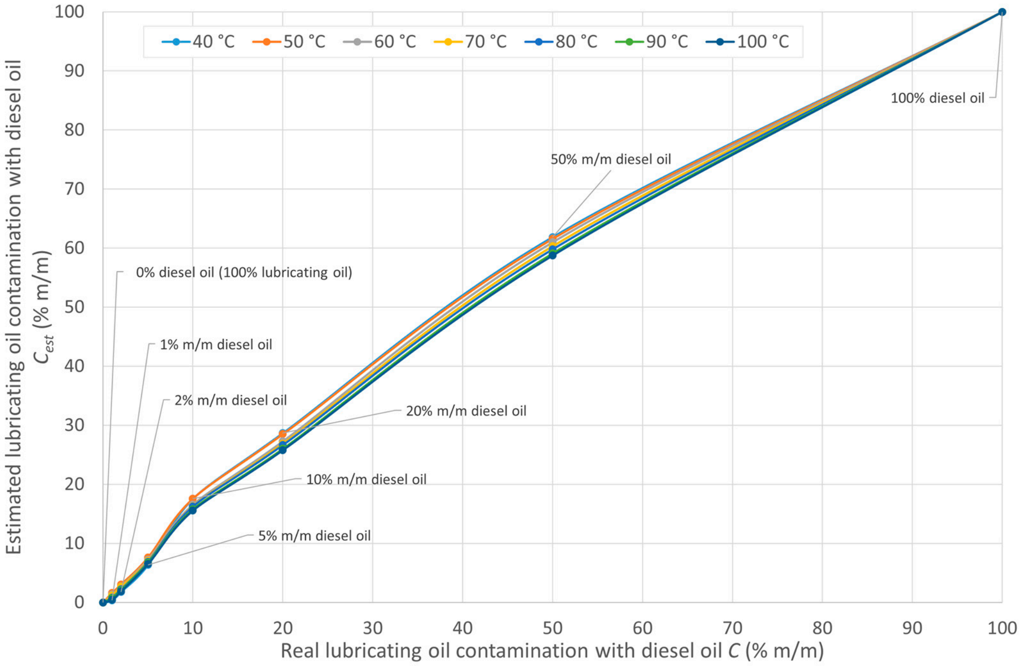

Figure 8. Relationship between the calculated and actual values of DO concentrations in mixtures with SAE 40 LO at different measurement temperatures (DO concentrations for temperatures higher than 40 °C are the same as marked for the measurement temperature of 40 °C; the order of values is the same).

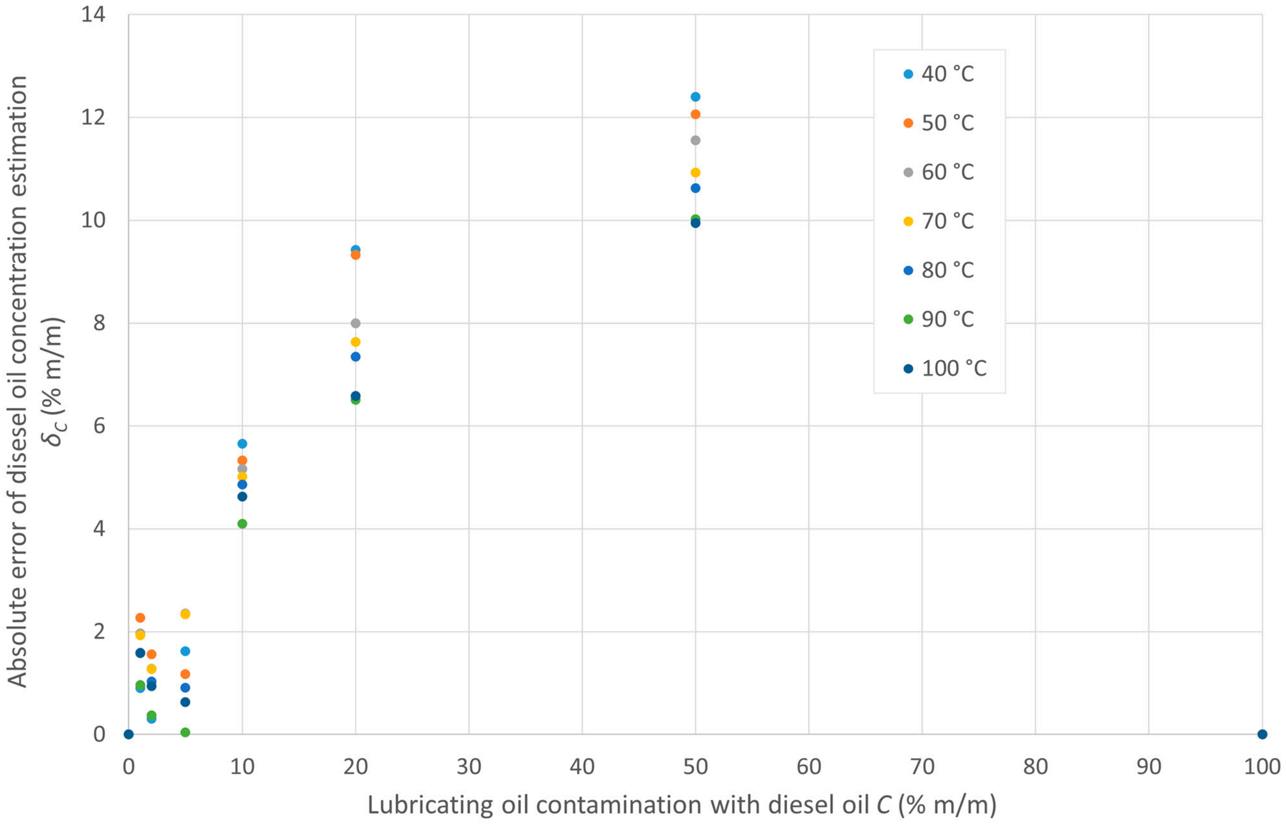

Figure 9. Absolute error in estimating the concentration of DO in mixtures with SAE 30 LO as a function of the actual concentration of DO in the mixture for different mixture viscosity measurement temperatures.

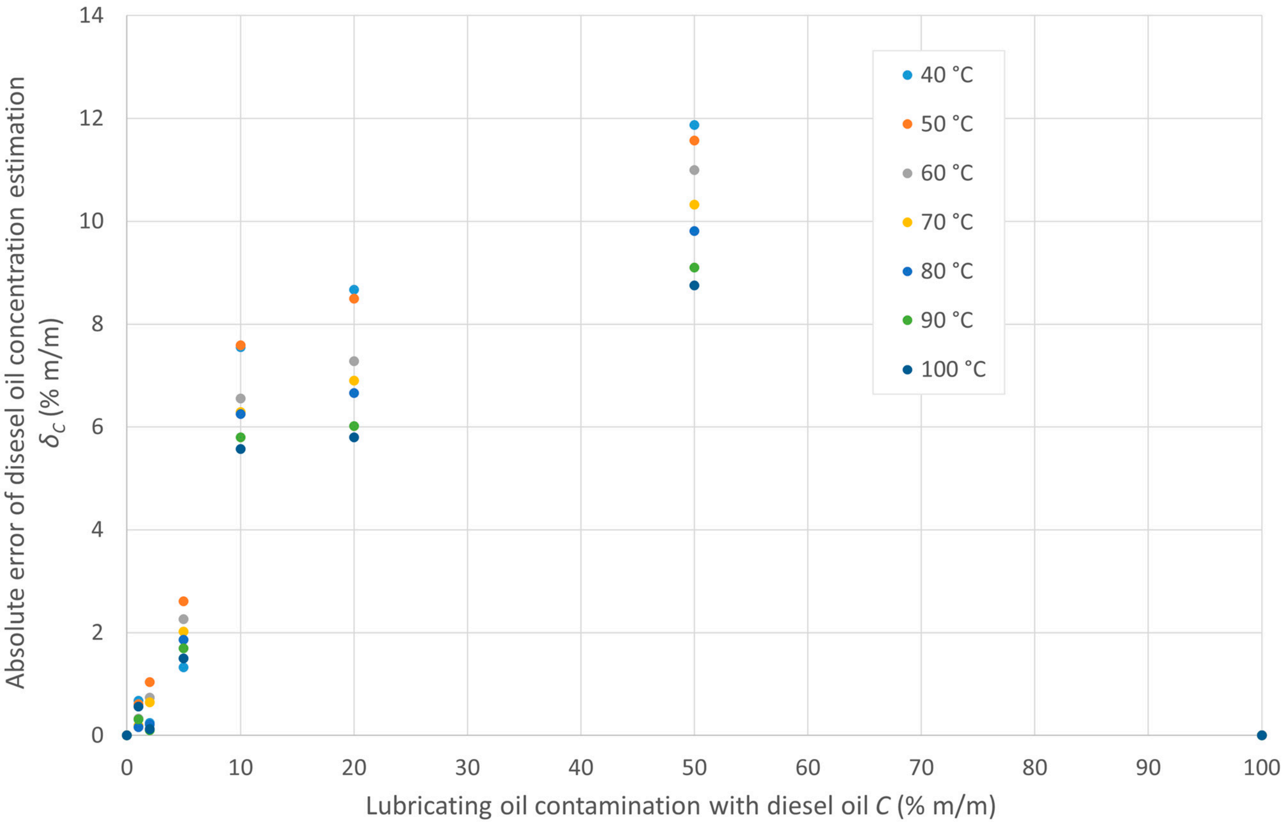

Figure 10. Absolute error in estimating the concentration of DO in mixtures with SAE 40 LO as a function of the actual concentration of DO in the mixture for different mixture viscosity measurement temperatures.

In the case of the SAE 30 oil mixtures, the largest absolute estimation error was 12.4% m/m and concerned the measurement of the mixture viscosity at a temperature of 40 °C for the mixture containing 50% m/m DO.

In the case of the SAE 40 oil mixtures, the maximum recorded absolute estimation error in the entire analyzed concentration range was 11.87% m/m, and also concerned the mixture with 50% m/m DO with the mixture viscosity measurement taken at a temperature of 40 °C. In normal conditions, for DO concentrations between 0 and 5% m/m, the absolute estimation error did not exceed 3% m/m.

4. Conclusions

An estimation of dynamic viscosity using the Arrhenius model can be used in the quantitative assessment of LO dilution with DO for single-grade LOs of SAE 30 and SAE 40 classes as a complementary element of analysis next to the measurement of the flash point. The obtained analysis results showed a good representation of this model. The accuracy of the presented method increases as the measurement temperature increases.

The use of the Arrhenius equation to estimate the mass concentration of DO in a mixture with LO shows a practically acceptable reflection of reality. Moreover, the obtained value of the absolute estimation error supports the validity of using the presented model in operational practice in the general assessment of the physicochemical properties of engine oils subjected to periodic, routine control.

When concluding on the impact of the potential dilution of LO with DO on changes in viscosity, one must consider several factors, apart from the dilution of LO with fuel, that affect viscosity in real conditions. However, if we are certain that the LO is diluted with fuel, the presented methodology is a convenient and easy-to-use tool for the quantitative assessment of the level of dilution of LO with DO. Nevertheless, due to the large relative error, the use of the presented model for the accurate quantitative assessment of oil dilution level is not recommended.

Further research may be related to the testing of other types of liquids (including impure crankcase oils, biodiesels, and recycled fuels). Moreover, we advise researchers to analyze other models of physical characteristics of oil mixtures in terms of their applicability to estimate lubricating oil dilution with fuels. It is also worthwhile to develop methods connected to the proposed approach, together with standardized analytical tests, in order to obtain comprehensive data on lubricating oils.

Author Contributions: Conceptualization, L.C.; methodology, L.C.; software, L.C., M.S. and K.G.; validation, L.C., M.S. and K.G.; formal analysis, L.C., M.S. and K.G.; investigation, L.C., M.S. and K.G.; resources, L.C., M.S. and K.G.; data curation, L.C.; writing—original draft preparation, L.C., M.S. and K.G.; writing—review and editing, L.C., M.S. and K.G.; visualization, L.C., M.S. and K.G.; supervision, L.C. and K.G.; project administration, L.C.; funding acquisition, L.C. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

A	designation of the liquid that is a component of the mixture
API	American Petroleum Institute
ASTM	American Society for Testing and Materials
B	designation of the liquid that is a component of the mixture
C	actual mass concentration of diesel oil in the mixture with lubricating oil
CD	API quality class of lubricating oils for self-ignition engines
C_{est}	estimated mass concentration of diesel oil in the mixture with lubricating oil
CF	API quality class of lubricating oils for self-ignition engines
DO	diesel oil
DMX	marine distillate fuels category according to the ISO 8217:2017 standard
ESR	electron spin resonance
GC	gas chromatography
GC-MS	gas chromatography–mass spectrometry
FTIR	Fourier-transform infrared spectroscopy
ISO	International Organization for Standardization
LO	lubricating oil
JPI	Japan Petroleum Institute
NMR	nuclear magnetic resonance
SAE	Society of Automotive Engineers
SAE 30, SAE 40	viscosity grades of lubricating oils according to the SAE J300-2021 standard
SAW	surface acoustic wave
t	measurement temperature
UV	ultraviolet
x_A	mole fraction of liquid A
x_B	mole fraction of liquid B
x_{DO}	mole fraction of diesel oil
x_{FLO}	mole fraction of lubricating oil

x_{mDO}	mass fraction of diesel oil
x_{mFLO}	mass fraction of lubricating oil
δ_C	absolute error in estimating the mass concentration of diesel oil in the mixture with lubricating oil
δ_η	absolute error in estimating the dynamic viscosity of the mixture
η_A, η_B	dynamic viscosity of liquids *A*, *B*
η_{DO}	dynamic viscosity of diesel oil
η_{FLO}	dynamic viscosity of fresh lubricating oil
η_{mix}	dynamic viscosity of the mixture of liquids *A* and *B*
η_{ULO}	calculated dynamic viscosity of lubricating oil (mixture of lubricating and diesel oils)

Appendix A. Data Tables

Table A1. Measured values of the dynamic viscosity ηreal of the tested SAE 30 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Measured dynamic viscosity of the mixture η_{real} (mPa s)							
Measurement temperature *t* (°C)	40	92.69	86.45	85.17	72.71	52.21	31.52	9.41	2.37
	50	57.95	51.89	51.39	47.05	34.54	21.54	7.13	1.98
	60	36.26	33.09	32.77	28.90	22.72	15.29	5.43	1.66
	70	24.73	22.74	22.52	20.04	16.09	11.21	4.32	1.41
	80	17.61	16.44	16.25	15.05	11.85	8.50	3.50	1.23
	90	12.77	12.16	12.04	11.27	9.01	6.63	2.89	1.08
	100	9.91	9.33	9.25	8.69	7.04	5.32	2.44	0.96

Table A2. Measured values of the dynamic viscosity η_{real} of the tested SAE 40 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Measured dynamic viscosity of the mixture η_{real} (mPa s)							
Measurement temperature *t* (°C)	40	141.19	139.35	131.42	109.02	68.92	43.75	11.26	2.37
	50	86.00	80.95	76.69	64.56	44.32	29.37	8.44	1.98
	60	52.04	49.72	47.37	40.52	29.42	20.33	6.36	1.66
	70	34.50	33.21	31.70	27.56	20.49	14.60	5.01	1.41
	80	23.99	23.40	22.46	19.56	14.80	10.85	4.05	1.23
	90	17.20	16.88	16.23	14.29	11.11	8.37	3.35	1.08
	100	12.87	12.72	12.25	10.87	8.58	6.58	2.79	0.96

Table A3. Estimated values of the dynamic viscosity η_{ULO} of the tested SAE 30 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Estimated dynamic viscosity of the mixture η_{ULO} (mPa s)							
Measurement temperature *t* (°C)	40	92.69	89.35	86.13	77.16	64.24	44.52	14.82	2.37
	50	57.95	56.02	54.16	48.95	41.35	29.50	10.72	1.98
	60	36.26	35.16	34.09	31.08	26.64	19.57	7.76	1.66
	70	24.73	24.03	23.35	21.43	18.57	13.95	5.91	1.41
	80	17.61	17.15	16.70	15.42	13.49	10.34	4.65	1.23
	90	12.77	12.45	12.15	11.28	9.97	7.79	3.71	1.08
	100	9.91	9.68	9.46	8.81	7.84	6.21	3.08	0.96

Table A4. Estimated values of the dynamic viscosity η_{ULO} of the tested SAE 40 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Estimated dynamic viscosity of the mixture η_{ULO} (mPa s)							
Measurement temperature *t* (°C)	40	141.19	135.54	130.11	115.10	93.83	62.35	18.30	2.37
	50	86.00	82.82	79.76	71.23	58.99	40.46	13.06	1.98
	60	52.04	50.28	48.58	43.81	36.88	26.13	9.30	1.66
	70	34.50	33.41	32.36	29.40	25.06	18.20	6.97	1.41
	80	23.99	23.29	22.61	20.68	17.82	13.23	5.42	1.23
	90	17.20	16.73	16.28	14.98	13.04	9.88	4.31	1.08
	100	12.87	12.54	12.22	11.30	9.92	7.65	3.51	0.96

Table A5. Absolute error in estimating the dynamic viscosity δ_η (mPa s) of the SAE 30 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Absolute error in estimating dynamic viscosity of the mixture δ_η (mPa s)							
Measurement temperature *t* (°C)	40	0.00	2.90	0.96	4.45	12.03	13.01	5.42	0.00
	50	0.00	4.13	2.77	1.90	6.81	7.97	3.58	0.00
	60	0.00	2.07	1.32	2.17	3.92	4.28	2.33	0.00
	70	0.00	1.29	0.84	1.39	2.48	2.74	1.59	0.00
	80	0.00	0.71	0.45	0.37	1.64	1.84	1.15	0.00
	90	0.00	0.29	0.11	0.01	0.96	1.16	0.81	0.00
	100	0.00	0.35	0.21	0.13	0.80	0.89	0.64	0.00

Table A6. Absolute error in estimating the dynamic viscosity δ_η (%) of the SAE 40 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Absolute error in estimating dynamic viscosity of the mixture δ_η (mPa s)							
Measurement temperature *t* (°C)	40	0.00	3.81	1.31	6.08	24.91	18.60	7.03	0.00
	50	0.00	1.87	3.06	6.67	14.67	11.09	4.62	0.00
	60	0.00	0.56	1.21	3.29	7.46	5.80	2.93	0.00
	70	0.00	0.20	0.66	1.84	4.57	3.61	1.96	0.00
	80	0.00	0.11	0.15	1.11	3.02	2.38	1.37	0.00
	90	0.00	0.14	0.04	0.69	1.93	1.52	0.96	0.00
	100	0.00	0.19	0.04	0.43	1.34	1.07	0.71	0.00

Table A7. Estimated values of diesel oil concentration C_{est} (% m/m) in the tested SAE 30 oil mixtures.

Diesel Oil Concentration in the Mixture *C* (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Estimated diesel oil concentration in the mixture C_{est} (% m/m)							
Measurement temperature *t* (°C)	40	0.00	1.90	2.31	6.62	15.66	29.42	62.40	100.00
	50	0.00	3.27	3.56	6.17	15.33	29.32	62.06	100.00
	60	0.00	2.97	3.28	7.35	15.16	28.00	61.55	100.00
	70	0.00	2.93	3.27	7.33	15.01	27.63	60.93	100.00
	80	0.00	2.59	3.03	5.91	14.86	27.35	60.63	100.00
	90	0.00	1.97	2.37	5.04	14.10	26.51	60.02	100.00
	100	0.00	2.58	2.94	5.63	14.63	26.58	59.94	100.00

Table A8. Estimated values of diesel oil concentration C_{est} (% m/m) in the tested SAE 40 oil mixtures.

Diesel Oil Concentration in the Mixture C (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Estimated diesel oil concentration in the mixture C_{est} (% m/m)							
Measurement temperature t (°C)	40	0.00	0.32	1.75	6.33	17.55	28.67	61.87	100.00
	50	0.00	1.61	3.04	7.61	17.59	28.50	61.57	100.00
	60	0.00	1.33	2.73	7.26	16.56	27.28	61.00	100.00
	70	0.00	1.19	2.65	7.02	16.29	26.90	60.32	100.00
	80	0.00	0.84	2.22	6.86	16.25	26.66	59.81	100.00
	90	0.00	0.69	2.10	6.69	15.79	26.02	59.10	100.00
	100	0.00	0.43	1.88	6.50	15.57	25.79	58.75	100.00

Table A9. Absolute error in estimating the value of the diesel oil concentration δ_C (%) in the tested SAE 30 oil mixtures.

Diesel Oil Concentration in the Mixture C (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Absolute error in estimating dynamic viscosity of the mixture δ_C (%)							
Measurement temperature t (°C)	40	0.00	0.90	0.31	1.62	5.66	9.42	12.40	0.00
	50	0.00	2.27	1.56	1.17	5.33	9.32	12.06	0.00
	60	0.00	1.97	1.28	2.35	5.16	8.00	11.55	0.00
	70	0.00	1.93	1.27	2.33	5.01	7.63	10.93	0.00
	80	0.00	1.59	1.03	0.91	4.86	7.35	10.63	0.00
	90	0.00	0.97	0.37	0.04	4.10	6.51	10.02	0.00
	100	0.00	1.58	0.94	0.63	4.63	6.58	9.94	0.00

Table A10. Absolute error in estimating the value of the diesel oil concentration δ_C (%) in the tested SAE 40 oil mixtures.

Diesel Oil Concentration in the Mixture C (% m/m)		**0**	**1**	**2**	**5**	**10**	**20**	**50**	**100**
		Absolute error in estimating dynamic viscosity of the mixture δ_C (%)							
Measurement temperature t (°C)	40	0.00	0.68	0.25	1.33	7.55	8.67	11.87	0.00
	50	0.00	0.61	1.04	2.61	7.59	8.50	11.57	0.00
	60	0.00	0.33	0.73	2.26	6.56	7.28	11.00	0.00
	70	0.00	0.19	0.65	2.02	6.29	6.90	10.32	0.00
	80	0.00	0.16	0.22	1.86	6.25	6.66	9.81	0.00
	90	0.00	0.31	0.10	1.69	5.79	6.02	9.10	0.00
	100	0.00	0.57	0.12	1.50	5.57	5.79	8.75	0.00

Appendix B. Derivation of Mathematical Models

Appendix B.1. Viscosity of the Lubricating Oil and Diesel Oil Mixture

The initial relationship proposed by Arrhenius [43] can be written in the following form:

$$\ln\eta_{mix} = x_A \ln\eta_A + x_B \ln\eta_B, \tag{A1}$$

where η_{mix} is the dynamic viscosity of the liquid mixture, η_A is the dynamic viscosity of liquid A, η_B is the dynamic viscosity of liquid B, x_A is the mole fraction of liquid A in the mixture, x_B is the mole fraction of liquid B in the mixture, and $x_A + x_B = 1$.

For a mixture of lubricating and diesel oils, we can assume that η_{mix} is the viscosity of oil contaminated with distillation fuel η_{ULO}, η_A is the viscosity of pure diesel oil η_{DO}, η_B is the dynamic viscosity of pure lubricating oil η_{FLO}, x_{DO} is the mole fraction of diesel oil in the mixture, x_{FLO} is the mole fraction of lubricating oil, and x_{DO} + x_{FLO} = 1 in the mixture. Then, the relationship takes the following form:

$$\ln\eta_{ULO} = x_{DO}\ln\eta_{DO} + x_{FLO}\ln\eta_{FLO}. \tag{A2}$$

Assuming the simplification [47] that the molar values of the components are close to the mass fractions of the components x_{mDO} and x_{mFLO}, the relationship (A2) takes the following form:

$$\ln\eta_{ULO} \approx x_{mDO}\ln\eta_{DO} + x_{mFLO}\ln\eta_{FLO}, \tag{A3}$$

where x_{mDO} + x_{mFLO} = 1.

If C(% m/m) is the mass percentage of diesel oil in the mixture with lubricating oil, the following substitution can be made:

$$x_{mDO} = \frac{C}{100\%}, \tag{A4}$$

$$x_{mFLO} = 1 - x_{mDO} = \frac{100\%}{100\%} - \frac{C}{100\%} = \frac{100\% - C}{100\%}. \tag{A5}$$

Substituting the dependencies (A4) and (A5) into the dependency (A3), we obtain:

$$\ln\eta_{ULO} \approx \left(\frac{C}{100}\right)\ln\eta_{DO} + \left(\frac{100 - C}{100}\right)\ln\eta_{FLO}. \tag{A6}$$

After determining the inverse function of the logarithm, we determine a formula of the following form:

$$\eta_{ULO} \approx \exp\left[\left(\frac{C}{100}\right)ln\eta_{DO} + \left(\frac{100 - C}{100}\right)ln\eta_{FLO}\right]. \tag{A7}$$

The dependency (A7) is the model described in the article via Equation (2).

Appendix B.2. Mass Concentration of Diesel Oil in a Mixture with Lubricating Oil

Since x_{mDO} + x_{mFLO} = 1, Equation (A3) can be expressed as:

$$\ln\eta_{ULO} \approx x_{mDO}\ln\eta_{DO} + (1 - x_{mDO})\ln\eta_{FLO}. \tag{A8}$$

This can be transformed into:

$$\ln\eta_{ULO} \approx x_{mDO}\ln\eta_{DO} + \ln\eta_{FLO} - x_{mDO}\ln\eta_{FLO}. \tag{A9}$$

After leaving the products on one side and taking the x_{mDO} value in front of the brackets, we obtain the following:

$$x_{mDO}(\ln\eta_{DO} - x_{mDO}\ln\eta_{FLO}) \approx \ln\eta_{ULO} - \ln\eta_{FLO}. \tag{A10}$$

From this, we can derive the value of x_{mDO}, which is:

$$x_{mDO} \approx \frac{\ln\eta_{ULO} - \ln\eta_{FLO}}{\ln\eta_{DO} - x_{mDO}\ln\eta_{FLO}}. \tag{A11}$$

After substituting the relationship (A4) into Equation (A11) and adopting the notation for the estimated mass concentration of diesel oil in the mixture with lubricating oil C_{est} = C, Formula (A11) then takes the following form:

$$C_{est} = \frac{ln\eta_{ULO} - ln\eta_{FLO}}{ln\eta_{DO} - ln\eta_{FLO}} \cdot 100\% \tag{A12}$$

After applying the relationships and simplifying the difference of logarithms, we obtain dependency (A13), which is the model described in this article via Equation (3):

$$C_{est} = \frac{ln\left(\frac{\eta_{ULO}}{\eta_{FLO}}\right)}{ln\left(\frac{\eta_{DO}}{\eta_{FLO}}\right)} \cdot 100\% \tag{A13}$$

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