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Control of Three-Phase Two-Level Inverters: A Stochastic LPV Model Approach

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Abstract: This paper proposes a stochastic linear parameter-varying (LPV) model approach to design a state feedback controller for three-phase, two-level inverters. To deal with the parameter changes, stochastic noise, and delays faced by the inverter, it is modeled as a stochastic LPV system with time delay. Stability analysis and control synthesis are conducted for the LPV system. With parameter-dependent Lyapunov functionals, a condition of sufficient stability for asymptotical mean-square stability is obtained. In addition, the slack matrix technique is employed to improve the feasibility and reduce the conservatism of the conditions. The obtained theoretical results are applied to the three-phase, two-level inverter, whose currents are treated as state variables and are controlled to reach the equilibrium point. The simulation results validate the effectiveness of the proposed theories and demonstrate the advantages of using the slack matrix.

Keywords: three-phase two-level inverter; linear parameter-varying (LPV) system; stochastic system; time delay; mean square stability

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1. Introduction

Three-phase, two-level inverters are the most widely used power converters in power electronics and motor drives. The main components of the inverters are controllable power switches such as IGBTs, MOSFETs, GTOs, etc., [1]. By implementing appropriate control strategies, they can output active and reactive power as required while also operating with desirable performance in terms of good current quality, high power efficiency, and strong robustness against disturbances. In real applications, the inverter faces uncertain conditions including time-varying grid voltage, filter capacitance and inductance changes, measurement delays, and environmental noise. To effectively control the inverter, all of these uncertainties should be taken into account [2,3].

Linear parameter-varying (LPV) systems are used to describe systems with time-varying parameters. Since the seminal work of Shamma in 1991 [4], the LPV control and filtering approach has attracted a lot of attention from the control field. This is because (1) it is effective at coping with the nonlinearities and time-varying dynamics of the system, and (2) the maturely developed LTI control methodologies, such as sensitivity, shaping, and modeling tools, can be extended to LPV systems. Over the past few decades, several advances have been made in terms of the theoretical and practical aspects. Among other things, theoretical works are related to analysis and synthesis issues [5,6], robust filtering problems [7,8], fault detection and isolation [9], etc. Practical engineering works are also popular. For instance, an LPV static output feedback control method has been proposed to improve the lateral stability and driving comfort of narrow tilting vehicles [10], an LPV modeling method has been adopted to ensure large signal stability for multi-mode buck-boost converters [11], and an uncertain robot system has been controlled by an

LPV controller with sliding-mode optimization [12].Time delays exist in many practical systems, such as network-controlled systems, electronic networks, hydraulic systems, and chemical processes, which are mostly nonlinear systems. Time delays may cause instability and oscillation in these systems and severely degrade performance [13,14]. To target this problem, research on time-delayed LPV systems has come into focus in recent years; see [15,16].

Stochastic systems are used to model systems with stochastic perturbations. This approach is used in several fields and engineering applications, such as mechanical systems, economic systems, etc. Over the past few decades, the study of stabilization and filtering problems in stochastic systems has received increased attention, and many valuable results have been obtained. As for stabilization problems, the asymptotic stability of semi-switched stochastic systems has been investigated in [17]. The problem of input-to-state stability for nonlinear systems with stochastic impulses was systematically studied in [18]. As to filtering problems, the recursive filtering problem in a class of uncertain stochastic systems with amplification and forward relays was investigated in [19], and the problem of distributed robust filtering for switched stochastic time-delay systems with fading measurements over sensor networks was addressed in [20].

This paper aims to propose a stochastic LPV model approach to deal with the uncertain conditions and time delay faced by a three-phase, two-level inverter, where the grid voltage, filter capacitance, and inductance are treated as time-varying parameters and the environment noises as stochastic disturbances. Sufficient conditions for stability analysis and controller synthesis are obtained for the stochastic LPV system. The theoretical results obtained are applied to the inverter, whose currents are treated as state variables and are controlled to reach the equilibrium point. The contributions of this paper are three fold: (1) the LPV approach is used to model the three-phase, two-level inverter, which is a new practical method; (2) compared to previous theoretical works, this paper further considers the stochastic perturbation and parameter-varying time delay, based on which the stochastic LPV model is established, and the stability conditions are obtained; (3) the slack matrix technique is adopted to improve the feasibility of the stability conditions, which facilitates the controller design process. The remainder of this work includes system description and inverter modeling in Section 2. The main theoretical analysis is presented in Section 3. Simulations to validate the proposal are provided in Section 4. Finally, conclusions are addressed in Section 5.

2. System Description and Inverter Modeling

Figure 1 is a two-level, three-phase inverter which is normally used as grid-connecting power converter (the case in this paper) or motor drive. In an ideal case, the inverter can be modeled in a synchronous reference frame as follows [21]:

$$L\frac{di_d}{dt} = u_d V_{in} - ri_d - \omega L i_q - v_d, \tag{1a}$$

$$L\frac{di_q}{dt} = u_q V_{in} - ri_q + \omega L i_d - v_q, \tag{1b}$$

where V_{in} is the input DC power source, ω is the frequency of grid voltage, L is the filtering inductor and r is its equivalent series resistance, and the dq variables are transformed from the three-phase abc variables, i.e., v_d, v_q are grid voltages transformed from v_a, v_b and v_c; i_d, i_q are grid currents transformed from i_a, i_b, and i_c; and u_d, u_q are switching functions generated by the controller, which is to be used to generate the switching signals S_a, S_b, and S_c through the PWM modulator. To provide the required power to the grid, the dq currents must be controlled to the desired value. Taking into account the inverter parameter variations, stochastic grid perturbations, and control/communication delays, the inverter model can be formulated as a general time-delayed stochastically perturbed LPV system:

$$\begin{aligned} dx(t) =& [A(\rho(t))x(t) + A_h(\rho(t))x(t - h(\rho(t)))]dt \\ &+ B(\rho(t))u(t)dt + B_v(\rho(t))v(t)dt \\ &+ B_\omega(\rho(t))x(t)d\omega(t), \quad (2a) \\ y(t) =& C(\rho(t))x(t) + C_h(\rho(t))x(t - h(\rho(t))) \\ &+ D(\rho(t))v(t), \quad (2b) \\ x(\theta) =& \phi(\theta),\ \theta \in [-h(\rho(0)), 0], \quad (2c) \end{aligned}$$

where $x(t) = [i_d; i_q]$ is the state vector, $A(\rho(t))$ is the system matrix related to L, r, ω; $u(t) = [u_d V_{in} - v_d; u_q V_{in} - v_q]$ is the control input; $B(\rho(t))$ is the control matrix related to L; $v(t) \in \mathcal{R}^q$, belonging to $\mathcal{L}_2[0, \infty)$, is either a disturbance input or a reference signal; $\omega(t)$ is a one-dimensional (1-D) Brownian motion satisfying $E\{d\omega(t)\} = 0$ and $E\{d\omega^2(t)\} = dt$; $\rho(t) \in \mathcal{R}^s$ is vector-valued parameter evolving continuously over time and its range is limited to a compact subset; and $h(\rho(t))$ is the parameter-varying delay and satisfies:

$$0 \le h(\rho(t)) \le H < \infty, \quad |\dot{h}(\rho(t))| \le \sigma < \infty,$$

where H and σ are constant scalars, and $x(\theta)$ is the initial data function given in the time interval $[-H, 0]$. The value of $\rho(t)$ is unknown but can be measured in real time.

Disregarding the control input $u(t)$, the autonomous system of (2) is formulated as:

$$\begin{aligned} dx(t) =& [A(\rho(t))x(t) + A_h(\rho(t))x(t - h(\rho(t)))]dt \\ &+ B_v(\rho(t))v(t)dt + B_\omega(\rho(t))x(t)d\omega(t), \quad (3a) \\ y(t) =& C(\rho(t))x(t) + C_h(\rho(t))x(t - h(\rho(t))) \\ &+ D(\rho(t))v(t), \quad (3b) \\ x(\theta) =& \phi(\theta), \theta \in [-h(\rho(0)), 0]. \quad (3c) \end{aligned}$$

On the other hand, to carry out stability analysis and synthesis, the exogenous disturbance "$v(t)$" in (2) is considered to be zero, yielding following system:

$$\begin{aligned} dx(t) =& A(\rho(t))x(t) + A_h(\rho(t))x(t - h(\rho(t))) \\ &+ B(\rho(t))u(t)dt + B_\omega(\rho(t))x(t)d\omega(t), \quad (4a) \\ x(\theta) =& \phi(\theta), \theta \in [-h(\rho(0)), 0]. \quad (4b) \end{aligned}$$

Therefore, the autonomous system of (4) can be formulated as:

$$\begin{aligned} dx(t) =& [A(\rho(t))x(t) + A_h(\rho(t))x(t - h(\rho(t)))]dt \\ &+ B_\omega(\rho(t))x(t)d\omega(t), \quad (5a) \\ x(\theta) =& \phi(\theta),\ \theta \in [-h(\rho(0)), 0]. \quad (5b) \end{aligned}$$

For simplicity, ρ will be used instead of $\rho(t)$ in the rest of this paper. Before going further, some definitions and lemmas should be given, which are important for deriving main results.

Definition 1. *For any initial state $x(0) \in R^n$, the time-delayed LPV stochastic system (3) is said to be robustly stable with disturbance attenuation index γ if for all $v(t) \in \mathcal{L}_2[0, \infty)$ and all parameter trajectories, it holds that*

$$\|y(t)\|_2 < \gamma \|v(t)\|_2. \quad (6)$$

Also, the system is asymptotically mean square stable according to Definition 1 in [22].

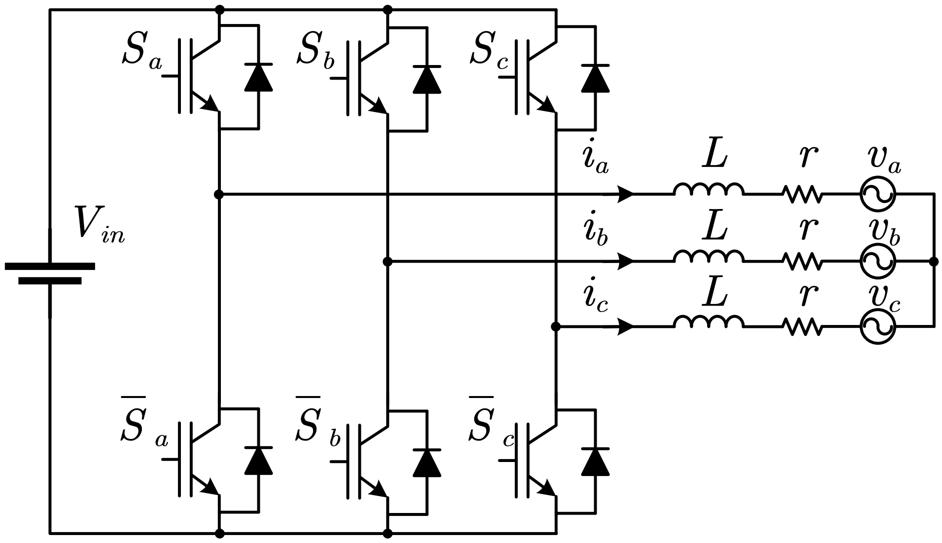

Figure 1. Two-level three-phase inverter.

3. System Analysis and Controller Design

3.1. Stability Analysis and Synthesis

Proposition 1. *If there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $P(\rho)$ and $Q(\rho)$ such that, for all the parameter trajectories, it satisfies*

$$\begin{bmatrix} \Pi_{11} & P(\rho)A_h(\rho) & B_\omega{}^T(\rho)P(\rho) \\ * & \Pi_{12} & 0 \\ * & * & -P(\rho) \end{bmatrix} < 0, \tag{7}$$

where $\Pi_{11} = \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + \mathrm{sym}\{P(\rho)A(\rho)\} + Q(\rho)$, $\Pi_{12} = -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t-h(\rho))$, *then system* (5) *is asymptotically mean square stable.*

Proof. Set the Lyapunov–Krasovskii functional as

$$V(x,\rho) = x^T(t)P(\rho)x(t) + \int_{t-h(\rho)}^{t} x^T(s)Q(s)x(s)ds. \tag{8}$$

According to Itô's formula [23],

$$dV(x,\rho) = \mathcal{L}V(x,\rho)dt + 2x^T(t)P(\rho)B_\omega(\rho)x(t)d\omega(t), \tag{9}$$

where

$$\begin{aligned}
&\mathcal{L}V(x,\rho) = \\
&x^T(t)\dot{P}(\rho)x(t) + 2x^T(t)P(\rho)A(\rho)x(t) \\
&+ 2x^T(t)P(\rho)A_h(\rho)x(t-h(\rho)) \\
&+ x^T(t)B_\omega{}^T(\rho)P(\rho)B_\omega(\rho)x(t) + x^T(t)Q(\rho)x(t) \\
&- \left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) \cdot x^T(t-h(\rho))Q(t-h(\rho))x(t-h(\rho)) \\
&= \begin{bmatrix} x(t) \\ x(t-h(\rho)) \end{bmatrix}^T \Pi \begin{bmatrix} x(t) \\ x(t-h(\rho)) \end{bmatrix},
\end{aligned}$$

with $\Pi = \begin{bmatrix} \Pi_{11} & P(\rho)A_h(\rho) \\ * & \left(1-\sum\limits_{i=1}^{s}\tau_i\frac{\partial h(\rho)}{\rho_i}\right)Q(t-h(\rho)) \end{bmatrix}$, and $\Pi_{11} = \sum\limits_{i=1}^{s}\tau_i\frac{\partial P(\rho)}{\rho_i} + \mathrm{sym}\{P(\rho)A(\rho)\} + B_\omega{}^T(\rho)P(\rho)B_\omega(\rho) + Q(\rho)$. Thus, according to [24], system (5) is asymptotically mean square stable if $\Pi < 0$, which is equal to the inequality (7) with Schur complement transformation. Proof is completed. □

Proposition 2. *If there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $R(\rho)$, $\bar{Q}(\rho)$, and matrices $F(\rho)$ such that, for all the parameter trajectories, it holds that*

$$\begin{bmatrix} \Pi_{11} & A_h(\rho)R(t-h(\rho)) & R(\rho)B_\omega{}^T(\rho) \\ * & \Pi_{22} & 0 \\ * & * & -R(\rho) \end{bmatrix} < 0, \tag{10}$$

where

$$\begin{aligned} \Pi_{11} =& \mathrm{sym}\{A(\rho)R(\rho)+B(\rho)F(\rho)\} - \sum_{i=1}^{s}\tau_i\frac{\partial R(\rho)}{\rho_i} + \bar{Q}(\rho), \\ \Pi_{22} =& -\left(1-\sum_{i=1}^{s}\tau_i\frac{\partial h(\rho)}{\rho_i}\right)\bar{Q}(t-h(\rho)), \end{aligned}$$

then system (4) is mean square stable under $u(t) = K(\rho)x(t)$ with $K(\rho) = F(\rho)R^{-1}(\rho)$.

Proof. Consider system (4) with $u(t) = K(\rho)x(t)$; $K(\rho)$ is a parameter-dependent state feedback controller.

Substitute $A(\rho)$ in (7) with $A(\rho)+B(\rho)K(\rho)$, and perform a congruence transformation to it with $\mathrm{diag}\{P^{-1}(\rho), P^{-1}(t-h(\rho)), P^{-1}(\rho)\}$. It is obtained that

$$\begin{bmatrix} \bar{\Pi}_{11} & A_h(\rho)P^{-1}(t-h(\rho)) & P^{-1}(\rho)B_\omega{}^T(\rho) \\ * & \bar{\Pi}_{12} & 0 \\ * & * & -P^{-1}(\rho) \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned} \bar{\Pi}_{11} =& P^{-1}(\rho)\sum_{i=1}^{s}\tau_i\frac{\partial P(\rho)}{\rho_i}P^{-1}(\rho) + P^{-1}(\rho)Q(\rho)P^{-1}(\rho) \\ &+ \mathrm{sym}\{(A(\rho)+B(\rho)K(\rho))P^{-1}(\rho)\}, \\ \bar{\Pi}_{12} =& -\left(1-\sum_{i=1}^{s}\tau_i\frac{\partial h(\rho)}{\rho_i}\right) \\ &\cdot P^{-1}(t-h(\rho))Q(t-h(\rho))P^{-1}(t-h(\rho)). \end{aligned}$$

Let matrix $R(\rho) = P^{-1}(\rho)$, $\bar{Q}(\rho) = P^{-1}(\rho)Q(\rho)P^{-1}(\rho)$, $F(\rho) = K(\rho)P^{-1}(\rho)$. Then, it is obtained that (10), and $K(\rho) = F(\rho)R^{-1}(\rho)$. Proof is completed. □

Remark 1. *τ_i denotes the varying rate of parameter i, which is assumed to be measurable in real time. If τ_i is not measurable, but its bound is known a priori, i.e., $|\tau_i| \le v_i$, then $\sum_{i=1}^{s}\tau_i\frac{\partial R(\rho)}{\partial\rho_i}$ can be approximated by $\sum_{i=1}^{s}\pm v_i\frac{\partial R(\rho)}{\partial\rho_i}$ to obtain a new stability condition. $\sum_{i=1}^{s}\pm(\cdot)$ represents the sum of every combination of $+(\cdot)$ and $-(\cdot)$, which contains a total of 2^s combinations.*

Proposition 3. *If there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $P(\rho)$, $Q(\rho)$ and matrix W such that, for all the parameter trajectories, it satisfies the condition that*

$$\begin{bmatrix} -(W+W^T) & P(\rho)+W^T A(\rho) & W^T A_h(\rho) & 0 & W^T \\ * & \Pi_{22} & 0 & {B_\omega}^T(\rho) & 0 \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & -P(\rho) \end{bmatrix} < 0, \tag{12}$$

where $\Pi_{22} = -P(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + Q(\rho)$, $\Pi_{33} = -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t-h(\rho))$, *then system* (5) *is asymptotically mean square stable.*

Proof. The inequality (12) can be written as

$$\begin{bmatrix} 0 & P(\rho) & 0 & 0 & 0 \\ * & \Pi_{22} & 0 & {B_\omega}^T(\rho) & 0 \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & -P(\rho) \end{bmatrix} + \begin{bmatrix} -I \\ A^T(\rho) \\ A_h^T(\rho) \\ 0 \\ I \end{bmatrix} W \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + (*) < 0. \tag{13}$$

The null spaces of $\begin{bmatrix} -I & A^T(\rho) & A_h^T(\rho) & 0 & I \end{bmatrix}$ and $\begin{bmatrix} I & 0 & 0 & 0 & 0 \end{bmatrix}$ are

$$\begin{bmatrix} A(\rho) & A_h(\rho) & 0 & I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \text{and} \begin{bmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

According to the projection lemma [25], the following inequalities are obtained:

$$\begin{bmatrix} \bar{\Pi}_{11} & P(\rho)A_h(\rho) & {B_\omega}^T(\rho) & P(\rho) \\ * & \bar{\Pi}_{12} & 0 & 0 \\ * & * & P^{-1}(\rho) & 0 \\ * & * & * & -P(\rho) \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + Q(\rho) - P(\rho) & 0 & {B_\omega}^T(\rho) & 0 \\ * & -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t-h(\rho)) & 0 & 0 \\ * & * & P^{-1}(\rho) & 0 \\ * & * & * & -P(\rho) \end{bmatrix} < 0, \tag{15}$$

where

$$\bar{\Pi}_{11} = \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + \text{sym}\{P(\rho)A(\rho)\} + Q(\rho) - P(\rho),$$
$$\bar{\Pi}_{12} = -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t-h(\rho)).$$

The inequality (14) is equal to the inequality (7) with Schur complement transformation, which indicates that (12) can ensure the system is asymptotically mean square stable. Proof is completed. □

Remark 2. *With the introduction of a new additional matrix W, the matrix $P(\rho)$ in the Lyapunov function is separated from the system matrix. This extra degree of freedom reduces the usually strong interrelations between plant data and Lyapunov variables, thereby improving the solution feasibility and reducing the conservatism.*

Theorem 1. *If there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $X(\rho)$, $Y(\rho)$, $\bar{P}(\rho)$ and matrices $R(\rho)$, V such that, for all the parameter trajectories, it holds that*

$$\begin{bmatrix} -\left(V+V^T\right) & \Pi_{12} & A_h(\rho)V & 0 & V \\ * & \Pi_{22} & 0 & V^T {B_\omega}^T(\rho) & 0 \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & -\bar{P}(\rho) & 0 \\ * & * & * & * & -X(\rho) \end{bmatrix} < 0, \tag{16}$$

where

$$\begin{aligned} \Pi_{12} =& X(\rho) + A(\rho)V + B(\rho)R(\rho), \\ \Pi_{22} =& - X(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial X(\rho)}{\rho_i} + Y(\rho), \\ \Pi_{33} =& - \left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Y(t-h(\rho)), \end{aligned}$$

then system (4) is mean square stable under $u = K(\rho)x(t)$ with $K(\rho) = R(\rho)V^{-1}(\rho)$.

Proof. Consider system (4) with $u(t) = K(\rho)x(t)$, substituting $A(\rho)$ in (12) with $A(\rho) + B(\rho)K(\rho)$, and perform a congruence transformation with $\mathrm{diag}\{W^{-1}, W^{-1}, W^{-1}, I, W^{-1}\}$. It is obtained that

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & A_h(\rho)W^{-1} & 0 & W^{-1} \\ * & \bar{\Pi}_{22} & 0 & W^{-T} {B_\omega}^T(\rho) & 0 \\ * & * & \bar{\Pi}_{33} & 0 & 0 \\ * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & -W^{-T}P(\rho)W^{-1} \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{aligned} \bar{\Pi}_{11} =& - (W^{-1} + W^{-T}), \\ \bar{\Pi}_{12} =& W^{-T}P(\rho)W^T + (A(\rho) + B(\rho)K(\rho))W^{-1}, \\ \bar{\Pi}_{22} =& W^{-T}\left(-P(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + Q(\rho)\right)W^{-1}, \\ \bar{\Pi}_{33} =& - \left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) W^{-T}Q(t-h(\rho))W^{-1}. \end{aligned}$$

Let matrix $X(\rho) = W^{-T}P(\rho)W^{-1}$, $Y(\rho) = W^{-T}Q(\rho)W^{-1}$, $V = W^{-1}$, $\bar{P}(\rho) = P^{-1}(\rho)$. (16) is obtained, and $K(\rho) = R(\rho)V^{-1}$. Proof is completed. □

3.2. Performance Analysis and Synthesis

Proposition 4. *The time-delayed LPV stochastic system (3) is said to be robustly stable with disturbance attenuation γ if there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $R(\rho)$ and $Q(\rho)$ such that, for all the parameter trajectories, it holds that*

$$\begin{bmatrix} \Pi_{11} & P(\rho)A_h(\rho) & P(\rho)B_v(\rho) & C^T(\rho) & {B_\omega}^T(\rho)P(\rho) \\ * & \Pi_{22} & 0 & C_h^T(\rho) & 0 \\ * & * & -\gamma^2 I & D^T(\rho) & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P(\rho) \end{bmatrix} < 0. \tag{18}$$

where

$$\begin{aligned} \Pi_{11} =& P(\rho)A(\rho) + A^T(\rho)P(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + Q(\rho), \\ \Pi_{22} =& -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t - h(\rho)). \end{aligned}$$

Proof. Assume that the system has a zero initial state, i.e., $x(t) = 0$ when $t \in [-h(\rho), 0]$, then, according to Itô's formula, it can be obtained that

$$E\{V(x(t), t)\} = E\left\{\int_0^t \mathcal{L}V(x(s), s)\right\} ds. \tag{19}$$

Let

$$\begin{aligned} J(t) =& E\left\{\int_0^t \left[y^T(s)y(s) - \gamma^2 v^T(s)v(s)\right] ds\right\} \\ =& E\left\{\int_0^t \begin{bmatrix} x(s) \\ x(s - h(\rho)) \\ v(s) \end{bmatrix}^T \bar{\Pi} \begin{bmatrix} x(s) \\ x(s - h(\rho)) \\ v(s) \end{bmatrix} ds\right\}, \end{aligned} \tag{20}$$

where

$$\bar{\Pi} = \begin{bmatrix} \bar{\Pi}_{11} & P(\rho)A_h(\rho) & P(\rho)B_v(\rho) \\ * & \bar{\Pi}_{22} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} C^T(\rho) \\ C_h^T(\rho) \\ D^T(\rho) \end{bmatrix} \begin{bmatrix} C^T(\rho) \\ C_h^T(\rho) \\ D^T(\rho) \end{bmatrix}^T,$$

with

$$\begin{aligned} \bar{\Pi}_{11} =& P(\rho)A(\rho) + A^T(\rho)P(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} \\ & + Q(\rho) + {B_\omega}^T(\rho)P(\rho)B_\omega(\rho), \\ \bar{\Pi}_{22} =& -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t - h(\rho)). \end{aligned}$$

With the Schur complement, condition (18) ensures $\bar{\Pi} < 0$; thus, $J(t) < 0$, meaning $y^T(t)y(t) < \gamma^2 v^T(t)v(t)$, and therefore, system (3) is robustly stable in the sense of Definition 2. Proof is completed. □

Proposition 5. *The time-delayed LPV stochastic system (2) is said to be robustly stabilized by state feedback controller $K(\rho)$ with disturbance attenuation γ if there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $R(\rho)$ and $\bar{Q}(\rho)$ and matrices $F(\rho)$ such that, for all of the parameter trajectories, it holds that*

$$\begin{bmatrix} \Pi_{11} & A_h R(t-h(\rho)) & B_v(\rho) & R(\rho)C^T & R(\rho){B_\omega}^T(\rho) \\ * & \Pi_{22} & 0 & R(t-h(\rho))C_h^T & 0 \\ * & * & -\gamma^2 I & D^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -R(\rho) \end{bmatrix} < 0, \tag{21}$$

where

$$\begin{aligned} \Pi_{11} =& \mathrm{sym}\{A(\rho)R(\rho) + B(\rho)F(\rho)\} - \sum_{i=1}^{s} \tau_i \frac{\partial R(\rho)}{\rho_i} + \bar{Q}(\rho), \\ \Pi_{22} =& -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) \bar{Q}(t-h(\rho)). \end{aligned}$$

and $K(\rho) = F(\rho)R^{-1}(\rho)$.

Proof. Consider system (2) with $u(t) = K(\rho)x(t)$, substituting $A(\rho)$ in (18) with $A(\rho) + B(\rho)K(\rho)$, and perform a congruence transformation with $\mathrm{diag}\{P^{-1}(\rho), P^{-1}(t-h(\rho)), I, I, P^{-1}(\rho)\}$. It is obtained that

$$\begin{bmatrix} \bar{\Pi}_{11} & A_h P^{-1}(t-h(\rho)) & B_v(\rho) & P^{-1}(\rho)C^T & P^{-1}(\rho){B_\omega}^T(\rho) \\ * & \bar{\Pi}_{22} & 0 & P^{-1}(t-h(\rho))C_h^T & 0 \\ * & * & -\gamma^2 I & D^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P^{-1}(\rho) \end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned} \bar{\Pi}_{11} =& \mathrm{sym}\Big\{(A(\rho) + B(\rho)K(\rho))P^{-1}(\rho)\Big\} \\ &+ P^{-1}(\rho)\left(Q(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i}\right) P^{-1}(\rho), \\ \bar{\Pi}_{22} =& -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) \\ &\cdot P^{-1}(t-h(\rho))Q(t-h(\rho))P^{-1}(t-h(\rho)). \end{aligned}$$

Let matrix $R(\rho) = P^{-1}(\rho)$, $\bar{Q}(\rho) = P^{-1}(\rho)Q(\rho)P^{-1}(\rho)$, $F(\rho) = K(\rho)P^{-1}(\rho)$. Then, (21) is obtained, and $K(\rho) = F(\rho)R^{-1}(\rho)$. Proof is completed. □

Proposition 6. *The time-delayed LPV stochastic system (3) is said to be robustly stable with disturbance attenuation* γ *if there exists a family of parameter-dependent continuous differentiable symmetric positive matrices* $P(\rho)$ *and* $Q(\rho)$ *and matrix* W *such that, for all the parameter trajectories, it holds that*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & W^T A_h(\rho) & W^T B_v(\rho) & 0 & 0 & W^T \\ * & \Pi_{22} & 0 & 0 & C^T(\rho) & {B_\omega}^T(\rho) & 0 \\ * & * & \Pi_{33} & 0 & C_h^T(\rho) & 0 & 0 \\ * & * & * & -\gamma^2 I & D^T(\rho) & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & * & * & -P(\rho) \end{bmatrix} < 0, \tag{23}$$

where $\Pi_{11} = -(W + W^T)$, $\Pi_{12} = P(\rho) + W^T A(\rho)$, $\Pi_{22} = -P(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + Q(\rho)$, $\Pi_{33} = -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t - h(\rho))$.

Proof. The inequality (23) can be written as

$$\begin{bmatrix} 0 & P(\rho) & 0 & 0 & 0 & 0 & 0 \\ * & \Pi_{22} & 0 & 0 & C^T(\rho) & {B_\omega}^T(\rho) & 0 \\ * & * & \Pi_{33} & 0 & C_h^T(\rho) & 0 & 0 \\ * & * & * & -\gamma^2 I & D^T(\rho) & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & * & * & -P(\rho) \end{bmatrix} +$$

$$\begin{bmatrix} -I \\ A^T(\rho) \\ A_h^T(\rho) \\ B_v^T(\rho) \\ 0 \\ 0 \\ I \end{bmatrix} W(\rho) \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} -I \\ A^T(\rho) \\ A_h^T(\rho) \\ B_v^T(\rho) \\ 0 \\ 0 \\ I \end{bmatrix}^T W^T(\rho) \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} < 0.$$

The null spaces of $\begin{bmatrix} -I & A(\rho) & A_h(\rho) & B_v(\rho) & 0 & 0 & I \end{bmatrix}$ and $\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ are $\begin{bmatrix} A(\rho) & A_h(\rho) & B_v(\rho) & 0 & 0 & I \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}$ respectively. According to the projection lemma, (24) is obtained:

$$\begin{bmatrix} \bar{\Pi}_{11} & P(\rho)A_h(\rho) & P(\rho)B_v(\rho) & C^T(\rho) & {B_\omega}^T(\rho) & P(\rho) \\ * & \bar{\Pi}_{12} & 0 & C_h^T(\rho) & 0 & 0 \\ * & * & -\gamma^2 I & D^T(\rho) & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & * & -P(\rho) \end{bmatrix} < 0, \tag{24}$$

with

$$\bar{\Pi}_{11} = \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + P(\rho)A(\rho) + A^T(\rho)P(\rho) + Q(\rho) - P(\rho),$$
$$\bar{\Pi}_{12} = -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t - h(\rho)),$$

and

$$\begin{bmatrix} \tilde{\Pi}_{11} & 0 & 0 & C^T(\rho) & {B_\omega}^T(\rho) & 0 \\ * & \tilde{\Pi}_{22} & 0 & C_h^T(\rho) & 0 & 0 \\ * & * & -\gamma^2 I & D^T(\rho) & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & * & -P(\rho) \end{bmatrix} < 0, \tag{25}$$

with $\tilde{\Pi}_{11} = \sum_{i=1}^{s} \tau_i \frac{\partial P(\rho)}{\rho_i} + Q(\rho) - P(\rho)$ and $\tilde{\Pi}_{22} = -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Q(t - h(\rho))$.

The inequality (24) is equal to the inequality (18) with Schur complement transformation, which indicates that (23) can ensure that the system is robustly stable. Proof is completed. □

Theorem 2. *The time-delayed LPV stochastic system (2) is said to be robustly stabilized by state feedback controller $K(\rho)$ with disturbance attenuation γ if there exists a family of parameter-dependent continuous differentiable symmetric positive matrices $X(\rho)$, $Y(\rho)$, and $\bar{P}(\rho)$ and matrices $R(\rho)$ and V, such that for all the parameter trajectories, it holds that*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & B_v(\rho) & 0 & 0 & V \\ * & \Pi_{22} & 0 & 0 & V^T C^T(\rho) & V^T {B_\omega}^T(\rho) & 0 \\ * & * & \Pi_{33} & 0 & V^T C_h^T(\rho) & 0 & 0 \\ * & * & * & -\gamma^2 I & V^T D^T(\rho) & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\bar{P}(\rho) & 0 \\ * & * & * & * & * & * & -X(\rho) \end{bmatrix} < 0 \tag{26}$$

where

$$\begin{aligned} \Pi_{11} &= -\left(V + V^T\right),\ \Pi_{12} = X(\rho) + A(\rho)V + B(\rho)R(\rho), \\ \Pi_{13} &= A_h(\rho)V,\ \Pi_{22} = -X(\rho) + \sum_{i=1}^{s} \tau_i \frac{\partial X(\rho)}{\rho_i} + Y(\rho), \\ \Pi_{33} &= -\left(1 - \sum_{i=1}^{s} \tau_i \frac{\partial h(\rho)}{\rho_i}\right) Y(t - h(\rho)). \end{aligned}$$

and $K(\rho) = R(\rho)V^{-1}(\rho)$.

Proof. Consider system (2) with $u(t) = K(\rho)x(t)$, substituting $A(\rho)$ in (23) with $A(\rho) + B(\rho)K(\rho)$, and perform a congruence transformation with $\mathrm{diag}\{W^{-1}, W^{-1}, W^{-1}, I, I, I, W^{-1}\}$. It is obtained that

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & B_v(\rho)W^{-1} & 0 & 0 & W^{-1} \\ * & \bar{\Pi}_{22} & 0 & 0 & W^{-T} C^T(\rho) & 0 & 0 \\ * & * & \bar{\Pi}_{33} & 0 & W^{-T} {C_h}^T(\rho) & 0 & 0 \\ * & * & * & -\gamma^2 I & W^{-T} D^T(\rho) & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -P^{-1}(\rho) & 0 \\ * & * & * & * & * & * & \bar{\Pi}_{77} \end{bmatrix} < 0, \tag{27}$$

where

$$\begin{aligned}
\bar{\Pi}_{11} =& -(W^{-1} + W^{-T}),\\
\bar{\Pi}_{12} =& W^{-T}P(\rho)W^{T} + (A(\rho) + B(\rho)K(\rho))W^{-1},\\
\bar{\Pi}_{13} =& A_h(\rho)W^{-1},\\
\bar{\Pi}_{22} =& W^{-T}\left(-P(\rho) + \sum_{i=1}^{s}\tau_i\frac{\partial P(\rho)}{\rho_i} + Q(\rho)\right)W^{-1},\\
\bar{\Pi}_{33} =& -\left(1 - \sum_{i=1}^{s}\tau_i\frac{\partial h(\rho)}{\rho_i}\right)W^{-T}Q(t-h(\rho))W^{-1},\\
\bar{\Pi}_{77} =& -W^{-T}P(\rho)W^{-1}.
\end{aligned}$$

Let matrix $X(\rho) = W^{-T}P(\rho)W^{-1}$, $Y(\rho) = W^{-T}Q(\rho)W^{-1}$, $V = W^{-1}$, $\bar{P}(\rho) = P^{-1}(\rho)$; (26) is obtained as well as $K(\rho) = R(\rho)V^{-1}$. Proof is completed. □

4. Simulation Results

For the following simulation, the grid and inverter parameters were chosen as follows: $v_d = 400$ V, $v_d = 0$ V, $V_{in} = 750$ V, $L = 0.002$ H, $r = 0.004\ \Omega$, and $\omega = 314$ rad/s. These values were chosen to comply with the European low-voltage grid, which has a voltage of 400 V, a frequency of 50 Hz, and small line resistance and inductance. The matrices of the LPV inverter model (2) are as follows:

$$\begin{aligned}
A(\rho) =& \begin{bmatrix} -\frac{r}{L} + 0.1\rho_1(t) & -\omega \\ \omega & -\frac{r}{L} + 0.1\rho_1(t) \end{bmatrix},\\
A_h(\rho) =& \begin{bmatrix} -\frac{r}{L} + 0.1\rho_2(t) & -\omega \\ \omega & -\frac{r}{L} + 0.1\rho_2(t) \end{bmatrix},\\
B(\rho) =& \begin{bmatrix} \frac{1}{L} + 0.1\rho_1(t) \\ \frac{1}{L} + 0.1\rho_1(t) \end{bmatrix}, B_v(\rho) = \begin{bmatrix} 0.2 + 0.1\rho_1(t) \\ 0.1 + 0.1\rho_2(t) \end{bmatrix},\\
B_\omega(\rho) =& \begin{bmatrix} 1.4 + 0.1\rho_1(t) & 1.0 + 0.2\rho_1(t) \\ 0.7 & 0.9 \end{bmatrix},\\
C(\rho) =& \begin{bmatrix} 1.2 + 0.2\rho_1(t) + 0.1\rho_2(t) \\ 0.8 \end{bmatrix}^{T},\\
C_h(\rho) =& \begin{bmatrix} 0.6 + 0.1\rho_1(t) + 0.1\rho_2(t) & 0.4 \end{bmatrix},\\
D(\rho) =& 1 + 0.1\rho_1(t) + 0.1\rho_2(t),\ h(\rho) = 0.01\rho_2(t),
\end{aligned}$$

where $\rho_1(t) = \sin(t)$ and $\rho_2(t) = |\cos 5t|$ are time-varying parameters satisfying $\rho_1(t) \in [-1, 1]$, $\rho_2(t) \in [0, 1]$, $\tau_1 \in [-1, 1]$, and $\tau_2 \in [-5, 5]$; $h(\rho)$ is the parameter varying delay with a bounded varying rate. The initial states are set as $x(t) = [1.0, -1.0]^T$, where $t \in [-1, 0]$.

To deal with the problem of infinite LMIs to be solved during controller synthesis, gridding technique and basis functions should be adopted. The basis functions are chosen as $f_1(\rho) = 1, f_2(\rho) = \rho_1(t), f_3(\rho) = \rho_2(t)$, leading to $R(\rho) = \sum_{j=1}^{3} f_j(\rho)R_j$, $F(\rho) = \sum_{j=1}^{3} f_j(\rho)F_j$, therefore the controller synthesis problem becomes finding matrices $R_j(j = 1, 2, 3)$ and $F_j(j = 1, 2, 3)$ to satisfy (10), (16), (21), and (26).

4.1. Stabilization

The state feedback control $K(\rho)$ is designed to ensure that system (4) is asymptotically mean square stable. By solving the conditions in (10) in Proposition 2, it is obtained that

$$
\begin{aligned}
R_1 &= 10^{-26}\begin{bmatrix} 0.1402 & 0.0557 \\ 0.0557 & 0.1030 \end{bmatrix}, R_2 = 10^{-28}\begin{bmatrix} 0.3142 & 0.2191 \\ 0.2191 & 0.1663 \end{bmatrix}, \\
R_3 &= 10^{-28}\begin{bmatrix} 0.5181 & -0.5006 \\ -0.5006 & 0.5802 \end{bmatrix}, \\
F_1 &= 10^{-26}\begin{bmatrix} -0.1251 & -0.1250 \end{bmatrix}, \\
F_2 &= 10^{-28}\begin{bmatrix} -0.1031 & -0.0825 \end{bmatrix}, \\
F_3 &= 10^{-28}\begin{bmatrix} -0.4335 & -0.4369 \end{bmatrix}. \\
K_1 &= \begin{bmatrix} -0.5222 & -0.9310 \end{bmatrix}, K_2 = \begin{bmatrix} 0.2204 & -0.7866 \end{bmatrix}, \\
K_3 &= \begin{bmatrix} -9.3997 & -8.8628 \end{bmatrix}.
\end{aligned}
$$

Figure 2a shows the state response of one arbitrarily chosen set, which means a random set of currents can be controlled at a certain reference. Figure 2b shows the state response of ten arbitrarily chosen sets, which shows that the states of all the sets converge to zero. As can be seen, the stochastic phenomenon can influence the trajectories of currents but does not affect the stability. Therefore, the time-delayed LPV inverter system is asymptotically stabilized under the stochastic perturbations.

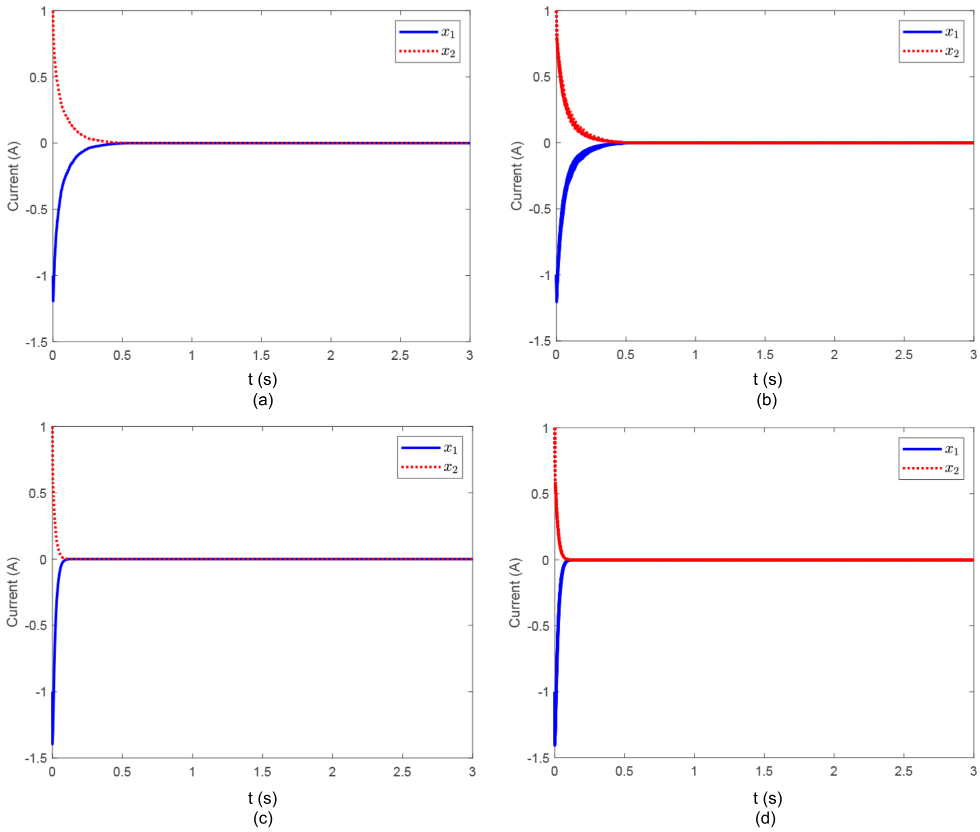

Figure 2. Stabilization: (**a**) one set of state responses without slack matrix; (**b**) ten sets of state responses without slack matrix; (**c**) one set of state responses with slack matrix; (**d**) ten sets of state responses with slack matrix.

4.2. Stabilization with Slack Matrix

Using a slack matrix technique and solving the conditions in (16) in Theorem 1, it is obtained that

$$
\begin{aligned}
V =& 10^{-28}\begin{bmatrix}0.2004 & 0.1983; 0.2192 & 0.2194\end{bmatrix},\\
R_1 =& 10^{-26}\begin{bmatrix}-0.2288 & -0.2284\end{bmatrix},\\
R_2 =& 10^{-28}\begin{bmatrix}-0.1545 & -0.1342\end{bmatrix},\\
R_3 =& 10^{-30}\begin{bmatrix}0.2733 & 0.2741\end{bmatrix}.\\
K_1 =& \begin{bmatrix}-28.4143 & -78.3968\end{bmatrix}, K_2 = \begin{bmatrix}-8.8494 & 7.3862\end{bmatrix},\\
K_3 =& \begin{bmatrix}-0.0018 & 0.0141\end{bmatrix}.
\end{aligned}
$$

Similarly to the previous subsection, Figure 2c shows the state response of one arbitrarily chosen set, and Figure 2d shows the state response of ten arbitrarily chosen sets. As can be seen, with the controller obtained by adopting a slack matrix, the currents converge to the equilibrium faster as compared to the controller obtained in the previous subsection, which demonstrates the advantage of the slack matrix technique.

4.3. Disturbance Attenuation

Let us consider in the system a disturbance signal $v(t) = 1/(1 + 0.5t^2)$. Then, the state feedback control $K(\rho)$ is designed to ensure the system (2) is stabilized and achieves some disturbance attenuation index γ. By solving the conditions in (21) in Proposition 5, it is obtained that

$$
\begin{aligned}
R_1 =& \begin{bmatrix}0.0846 & 0.0846\\ 0.0846 & 0.0847\end{bmatrix}, R_2 = \begin{bmatrix}-0.0033 & -0.0035\\ -0.0035 & -0.0037\end{bmatrix},\\
R_3 =& \begin{bmatrix}-0.0111 & -0.0111\\ -0.0111 & -0.0111\end{bmatrix}, F_1 = \begin{bmatrix}-0.3018 & -0.3017\end{bmatrix},\\
F_2 =& \begin{bmatrix}-0.0023 & -0.0019\end{bmatrix}, F_3 = \begin{bmatrix}0.0031 & 0.0031\end{bmatrix}.\\
K_1 =& \begin{bmatrix}-3.8245 & 0.2580\end{bmatrix}, K_2 = \begin{bmatrix}-65.2790 & 62.5179\end{bmatrix},\\
K_3 =& \begin{bmatrix}1.3953 & -1.6769\end{bmatrix}.
\end{aligned}
$$

With this controller, the currents converge to equilibrium and the system is stabilized under stochastic perturbations, as shown in Figure 3a,b, and the achieved disturbance attenuation index is γ is 21.4113.

4.4. Disturbance Attenuation with Slack Matrix

Similar to Section 4.3, let us consider in the system a disturbance signal $v(t) = 1/(1 + 0.5t^2)$. By solving the conditions in (26) in Theorem 2, it is obtained that

$$
\begin{aligned}
V =& \begin{bmatrix}0.1136 & 0.1150; 0.1167 & 0.1185\end{bmatrix},\\
R_1 =& \begin{bmatrix}-0.1501 & -0.1500\end{bmatrix}, R_2 = \begin{bmatrix}0.0075 & 0.0076\end{bmatrix},\\
R_3 =& \begin{bmatrix}0.0012 & 0.0012\end{bmatrix},\\
K_1 =& \begin{bmatrix}-5.8506 & 4.4128\end{bmatrix}, K_2 = \begin{bmatrix}-0.0117 & 0..0758\end{bmatrix},\\
K_3 =& \begin{bmatrix}0.0369 & -0.0260\end{bmatrix}.
\end{aligned}
$$

Applying the controller to the inverter system, as shown in Figure 3c,d, the currents converge faster as compared to the controller obtained without adopting the slack matrix technique. Furthermore, the achieved disturbance attenuation index γ is 12.0578, which is smaller than the previous method. This means that the controller obtained with the slack matrix has better disturbance rejection performance than the controller obtained without the slack matrix. These results again demonstrate the advantage of the slack matrix technique.

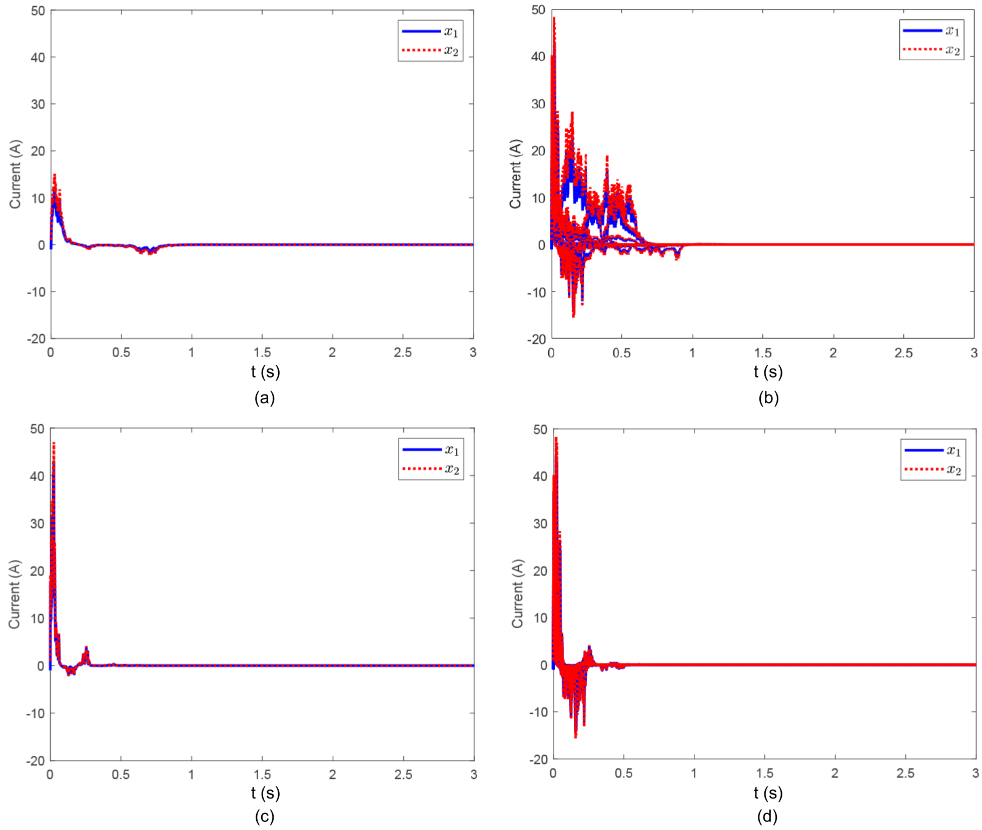

Figure 3. Disturbance attenuation: (**a**) one set of state responses without slack matrix; (**b**) ten sets of state responses without slack matrix; (**c**) one set of state responses with slack matrix; (**d**) ten sets of state responses with slack matrix.

5. Conclusions

In this paper, a stochastic LPV approach has been proposed for controlling the three-phase, two-level inverter. First, a time-delayed stochastic LPV system was established with which the analysis and synthesis of the system were carried out. With the parameter-dependent Lyapunov functionals, sufficient conditions were proposed for stability analysis and control synthesis in terms of parameter-dependent LMIs. The slack matrix approach was used to derive a new set of stability LMIs, which improved the feasibility and reduced the conservatism of the solution. The theoretical results were verified on the inverter, whose currents were controlled to reach the equilibrium point. Simulation results validated the effectiveness of the proposed theories and demonstrated the advantages of adopting a slack matrix in terms of faster state response and lower disturbance attenuation index.

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