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Investigation and Analysis of Influential Parameters in Bottomhole Stick–Slip Calculation during Vertical Drilling Operations

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Abstract: The critical factors that affect bottomhole stick–slip vibrations during vertical drilling operations are thoroughly investigated and analyzed in this research. Influential factors, such as rotation speed, weight on bit (WOB), bottom hole assembly (BHA) configuration, and formation properties, were studied in order to understand their part in the stick–slip phenomena. The analysis is based on a thorough review of previous research conducted on stick–slip drilling vibrations. A mathematical model was created that not only explains axial vibrations but also includes the torsional vibrations present in stick–slip occurrences, which helps with understanding the stick–slip phenomena better. This model can be used as an analytical tool to predict and evaluate the behavior of drilling systems under various operational circumstances. Furthermore, two drilling tests using a WellScan simulator were performed to validate the research findings and assess mitigation techniques' viability. These test scenarios reflect the stick–slip vibration-producing situations, allowing us to test mitigation strategies. The finding of this study shows the effectiveness of two tactics for reducing stick–slip vibrations. First was the reduction of WOB, which successfully lowered the occurrence of stick–slip vibrations. The second was the increase in the rotation speed, which helped to control the stick–slip problem and increased the drilling speed. This study explains the complex dynamics of stick–slip vibrations during vertical drilling and offers practical, tried-and-true methods for reducing their adverse effects on drilling operations.

Keywords: stick–slip vibrations; vertical drilling; torsional vibration: influential parameters; drilling performance

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1. Introduction

Vertical drilling operations are one of the drilling practices used extensively in the oil and gas industry to extract fossil fuels from the subsurface. However, despite the apparent simplicity of vertical drilling, it is a highly complex process influenced by many factors that can significantly impact its efficiency, safety, and overall success. Bottomhole stick–slip is one of the major challenges encountered during vertical drilling operations [1]. This occurrence is mainly characterized by rapid, erratic movements of the drilling assembly, which can have costly repercussions, notably impairment of drilling equipment, decreased drilling efficiency, and elevated hazards during operation [2,3]. Bottomhole stick–slips not only increase drilling durations but also pose a risk to the equipment's integrity and the safety of the drilling crew. In vertical well drilling, stick–slip causes torsional movement of the Bottom Hole Assembly (BHA), which increases the disturbance of the drilling operations [4]. These disruptions can be from various factors including operational, geological, and mechanical [5]. For vertical drilling practices to be efficient, cost-effective, and safe, it is crucial to comprehend the complex interactions among these parameters and their combined impact on stick–slip. Figure 1 shows an illustration of a conceptual small-scale drill string used for this analysis.

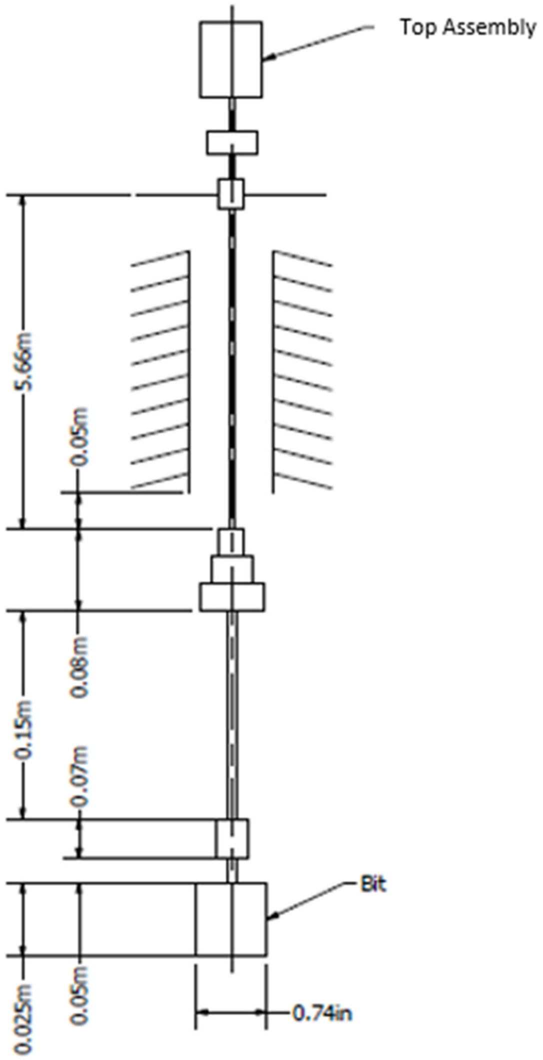

Figure 1. Setup to study torsional vibration.

Micro-drilling processes and vertical stick–slip vibrations share a critical interrelation, particularly in the context of their impact on the dynamic behavior of drilling systems. In micro-drilling, precision is paramount, and the erratic motion associated with stick–slip vibrations can introduce instability, compromising the accuracy of the drilling process. The smaller tools used in micro-drilling are more susceptible to wear, and the intermittent nature of stick–slip vibrations can exacerbate tool wear, affecting the longevity and effectiveness of these tools. Furthermore, the quality of drilled holes in micro-drilling is highly sensitive to variations in forces and torque induced by stick–slip vibrations, potentially leading to irregularities in hole geometry and surface finish. Controlling the dynamic response of the drill string becomes crucial in micro-drilling to ensure stability and prevent adverse effects on tool life and hole quality. The application of control strategies, such as feedback controllers, becomes essential to mitigate stick–slip vibrations and optimize the precision and stability required in micro-drilling processes. The challenges and solutions presented in the context of micro-drilling for high-frequency and high-speed printed circuit boards indirectly address issues that are often associated with vibrations, including tool breakage, variations in drilling forces and temperatures, and concerns about micro-hole quality [6]. The effects of cutting parameters, tool geometry, and thermal considerations may indirectly relate to the stability of the drilling process, including the potential influence of stick–slip vibrations on thrust force, tool wear, and thermal effects [7].

A thorough examination of the factors influencing bottomhole stick–slip during vertical drilling operations is necessary. Finding precise trigger mechanisms for stick–slip vibrations is an important issue. The circumstances that cause this phenomenon to occur are still under investigation despite the fact that it is frequently linked to the frictional interaction between the drill bit and geological formations. Table 1 shows a summary of previous studies on factors affecting stick–slip vibrations.

Table 1. Summary of previous studies on factors affecting stick–slip.

References	Parameters Considered	In-Depth Study	Mathematical Model	Empirical Validation	Mitigation Strategies	Practical Insight
Sadeghi et al. [8]	Rotational Speed, WOB, Damping		√	√	√	
Weiji et al. [9]	Drill Bit, BHA		√	√		√
Hongyuan et al. [10]	Drill Bit		√	√		
LIu et al. [11]	Rotary Speed, WOB	√	√	√		
Sananikone et al. [12]	Sensors		√		√	
Saldivar et al. [13]	Feedback Controllers		√	√	√	
Zakuan et al. [2]	Rotary Speed, WOB, Torque				√	

2. Studies on Stick–Slip in Drilling Operations

The existing literature has provided valuable information on the complex behavior of stick–slip vibrations, with a focus on understanding its mechanisms, effects, and mitigation strategies. Numerous studies have explored the dynamics of stick–slip vibrations in the drill string and their impact on drilling performance. These studies have often examined the mechanical and operational factors influencing stick–slip, such as weight on bit (WOB), rotary speed (RPM), drilling fluid properties, bit type, and geological formations. Researchers have developed mathematical models, conducted laboratory experiments, and analyzed field data to better understand the causes and effects of stick–slip behavior [14–16].

Belokobylskii and Prokopov [17] conducted the initial study on the stick–slip vibration in drilling engineering. They defined this occurrence as the irregular rotation of the bit during the drilling process and theorized that this phenomenon was caused by the buildup of torsional energy during the rotation of the drill string. A solution for reducing stick–slip vibrations is described in Sampaio and Weber's [18] study of the stick–slip vibrations in slender drilling rigs. They examined torsional vibrations while considering damping effects and compared experimental testing results with those derived from the mathematical model. In order to achieve the goals of the operator, Phillips et al. [19] suggested a method to decrease stick–slip vibrations and increase penetration rate by incorporating a bit sensing vibration data tool. Chen et al. [20] presented a technique to determine the potential origins of harmful vibrations by cutting action or friction.

To explore the torsional stability of the drill string and understand the reasons for the stick–slip and possible solutions, Cunha-Lima et al. [21] created one degree of freedom model to understand the stability of the drill string. Based on this model, Tang et al. [22–24] created a mechanical model quite comparable to it to investigate the impact of drilling parameters on stick–slip vibration. Results revealed that friction coefficients, viscous damping, and rotating table velocity significantly affect the dynamics and its occurrence. A three-degree freedom stick–slip model was developed by Patil and Teodoriu [25], in which they investigated the influence of rotary table velocity and WOB on stick–slip vibration, where the nonlinear friction forces represented the bit-rock contact.

Kreuzer and Steidl [26] first presented the wave equation controlling the torsional dynamics of an actual drill string. The research uncovered two key ideas: (a) the identification of a specific range of angular velocity for drill string rotation where stick–slip motion occurs, and (b) the consistency of stick–slip vibrations and their patterns regardless of the initial conditions. The dispersed drill string model was developed by Aarsnes et al. [27] with the goal of simulating the stick–slip vibrations caused by the switch between static and dynamic frictions. To explore the occurrence and general characteristics of self-induced vibrations resulting from the regenerative influence of the bit-rock interaction, the researchers extended their study by developing a distributed axial-torsional drill string model. The investigation showed how bit rotation speed affected the activation of several axial and torsional modes in the drill string. The researcher offered a stability chart [28] detailing the frequency of stick–slip vibrations. In earlier studies, the analysis of stick–slip vibrations was mostly restricted to the torsional direction. Ritto et al. [29] disagreed with the usual idea that there is only one point where the bit touches the rock in vertical wells. The argument put forth was that axial stick–slip occurs due to frictional forces. A stochastic model was developed in response to this situation, accounting for the frictional forces between the drill string and the borehole. Integration of the Monte Carlo approach with the Finite Element method made it possible to forecast axial stick–slip behavior in the bar model. The stochastic friction forces and parameter selection are the sources of uncertainty in this bit-rock interaction model. In a recent study by Ritto [30], an investigation was made to pinpoint the elements connected to the bit-rock contact model. Table 2 highlights a summary of stick–slip drilling operations.

Table 2. A summary of studies on stick–slip in drilling operations.

Year	References	Research Findings	Testing Method	Drilling Operation Variables	Key Observation
1982	Belokobylskii and Prokopov [17]	As the dimensionless velocity increases, the rod twists more, reaching a point where the potential energy from elastic forces, despite the rising drag torque, shortens the time compared to the standard law.	Theoretical	Angular velocity	Friction is a major factor
1992	Sananikone et al. [12]	Use of feedback control system.	Field	Torque measurement	Reduces drill string vibration by 90%
2008	Raymond et al. [31]	Implementing a feedback control system enables the bit response to closely follow the predicted displacement from the drillstring model.	Laboratory/simulation	Rock bit interaction	Feedback control is crucial for achieving a quicker response time
2008	Canudas-de-wit et al. [32]	The stability of three sliding-mode controllers was investigated and demonstrated through the application of the Lyapunov direct method.	Simulation	Lumped-parameter model	The sliding-mode control technique was utilized on the drill string to mitigate stick–slip oscillations
2013	Ritto et al. [30]	Drilling dynamics are related to torsional vibrations.	Simulation	Output/input power ratio	The output/input power is bimodal
2015	Kapitaniak et al. [33]	Formulation of models for undesired vibrations.	Experiment/simulation	Rotational speed, rate of penetration, lateral force	A simple torsional pendulum model is created and adjusted using experimental measurements for the drilling assembly
2022	Liu et al. [9]	Stick–slip vibrations weaken and diminish as the distance from the drill bit increases, predominantly occurring near the BHA close to the drill bit.	Simulation	borehole, bit, rock model	High-frequency torsional impact drilling reduces bit resistance torque by 34.71% and increases drilling depth by approximately 58.28%

3. Influential Parameters

Some influential parameters that can significantly affect the occurrence of stick–slip during vertical drilling operations are presented in this section. Understanding these parameters is crucial in order to properly understand the phenomenon surrounding the occurrence and mitigation techniques. The classical drilling operation consists of adjusting WOB, RPM, and/or mud flowrate. These three parameters are considered the most important ones, while torque on bit or rate of penetration is the output of the three.

3.1. Surface Rotation Speed

Surface rotation speed is a vital drilling operation parameter significantly affecting the stick–slip behavior. When the drilling bit's rotation speed aligns with the rotation speed at the surface, the initial rotation speed tends to oscillate around the average surface rotation speed. The drill bit's rotation speed is directly influenced by surface rotation speed, which also has an impact on the dynamics of the entire drill string. The drilling bit's rotation speed might be negative, which implies it rotates in the opposite direction, or it can range from zero to twice the surface rotation speed [34]. These dynamics, in turn, have an impact on stick–slip in the following ways:

3.1.1. Resonance and Vibration

High rotation speed can cause the drill string to vibrate, which may cause the drill string's natural frequencies to reverberate with one another. The system becomes highly vulnerable to resonance-induced stick–slip when the drill string vibrates at or close to its inherent frequencies as a result of the rotation speed. Small forces can cause significant vibrations to occur at resonance. As the drill string faces alternating compressive and tensile stresses, resonance-induced vibrations might intensify stick–slip. Sticking and slipping cycles are influenced by this cyclic loading and unloading. Through the use of experimental data, Besaisow et al. [35] investigated the causes of vibrations and resonance in drilling BHA. The authors examined dynamic data from field tests and a case study. The research analyzed vibration data from a test well drilled to a depth of 244 m using three different BHA configurations. Various excitation mechanisms are identified, including those dependent on RPM and resonances like lateral, axial, and torsional modes. It proposes the presence of more fundamental excitation mechanisms not recognized before, highlighting the need to include them in models predicting drill string and BHA vibrations.

3.1.2. Hydraulic Efficiency

The term "hydraulic efficiency" describes how well the drilling mud, including cuttings removal and wellbore stability, carries out its intended tasks. Rotation speed has an impact on how well the drilling mud performs hydraulically. Higher rotation speed could affect the cleaning of wellbore cuttings by increasing the fluid flow rate in the annulus. Cuttings buildup and sticking may be the result of ineffective cuttings clearance. Drill string and wellbore dynamics may be affected by changes in hydraulic conditions, such as pressure and flow rate variations. These dynamics can alter the forces acting on the drill bit and the drill string, changing the stick–slip behavior. Several researchers have created laboratory test rigs to gain insight into drill string dynamics and the relationship between the bit and the rock. A test rig created by Raymond et al. [31] enables the recreation of BHA's dynamic features. Rock samples were extracted from the setup using a real drill bit. In an experimental setup employing genuine drill bits and actual rock samples, Kapitaniak et al. [33], along with Wiercigroch et al. [36], examined the dynamics of the drill string. The researchers assert that this experimental apparatus can replicate different phenomena such as stick–slip oscillations, whirling, drill bit bounce, and helical buckling.

3.1.3. Friction and Wear

Elevated rotation speed intensifies the frictional forces between the drill bit and the formation, resulting in heightened wear and increased heat generation due to excessive friction. This heightened friction directly contributes to sticking phenomena. As rotation speed increases, the bit undergoes accelerated wear, leading to a dulled and overheated state. The combined effects of wear and heat diminish the bit's drilling efficiency, reducing its ability to penetrate the formation effectively. Importantly, studies by Hao et al. [37] emphasize that surface deformations on the bit induced by wear play a significant role in influencing stick–slip friction. Therefore, the intricate relationship between increased rotation speed, heightened frictional forces, bit wear, and their direct impact on stick–slip underscores the importance of carefully managing rotation speeds in drilling operations.

3.1.4. Torque and Drag

More torque is needed to sustain higher rotational speed. High drag along the wellbore due to elevated torque can result in sticking, particularly in deviated or horizontal wells. Weiji et al. study [9] focused on controlling stick–slip vibration behavior in BHA. The researchers devised a finite element model to examine the characteristics of stick–slip vibrations. The research results show that the resisting torque of the drill bit primarily causes stick–slip vibration in the drilling process, while reducing the torque amplitude can lessen or eliminate this vibration.

3.2. Weight on Bit (WOB)

WOB is the force applied to the bit as it drills downward. By applying additional pressure to the formation being drilled, reducing WOB can assist in overcoming sticking. However, high WOB can also produce differential sticking when the drill string gets stuck in the wellbore wall, or it can exceed the rock's compressive strength, resulting in sticking. Stick–slip can be exacerbated by excessive WOB fluctuation. Stick–slip vibrations may result from dynamic load variations brought on by rapid changes in weight on the bit. Liping et al. [24] used a lumped drilling system torsional pendulum model to investigate the WOB on torsional stick–slip vibration. The adverse damping effect occurs during the shift from the stick phase to the slip phase during stick–slip operations. Whenever the WOB hits a certain level, the bit behavior may switch from a steady movement to a stick–slip vibration as a result of an increased WOB. The phase trajectory eventually converges to a limit cycle in the case of stick–slip vibration, which symbolizes periodic bit motion. The limit cycle grows as the WOB increases. The drill bit bounces damply and eventually converges to a uniform motion in situations with no stick–slip vibrations. A novel WOB control law was created by Canudas-de-Wit et al. [32] to guide the drill string to a globally asymptotically stable closed-loop system, which results in the oscillation killer mechanism for WOB. The WOB control law plays a significant role in the practical implications of drilling operations. The study demonstrates that by varying the WOB, a family of curves related to Hopf and grazing-sliding bifurcations can be computed. This allows for the determination of the minimum desired rotary speed for the proportional derivative controller under different WOB values. The WOB control law is crucial in influencing the controllability of the drill string system. It enables the identification of parameter windows where stick–slip vibrations and constant rotation coexist. Moreover, the study shows that the size of the parameter window, and thus the controllable range, can be controlled by adjusting the WOB. Specifically, a larger WOB widens the speed range within which the system can operate effectively. The drill bit selection can also affect stick–slip. Different bit types (roller cone bit and Polycrystalline Diamond Compact (PDC) bits), respond differently to WOB. Compared to roller cone bits, PDC bits are frequently less prone to stick–slip.

3.3. Flowrate

The hydraulic efficiency of drilling mud, influenced by rotation speed, significantly impacts torsional stick–slip in drilling vibrations. The fluid flow rate within the drill string, a critical factor in vertical drilling, establishes lubricating conditions at the drill bit. Adequate lubrication is essential to minimize friction, prevent torsional stick–slip, and ensure the drilling assembly's proper cooling and cleaning. Achieving an ideal flow rate reduces bit balling, prevents cuttings buildup, and enhances drilling efficiency. Inadequate flow rates, on the other hand, lead to insufficient lubrication, intensifying friction and torsional vibrations. This phenomenon is particularly pronounced in vertical drilling due to gravity's heightened impact on cooling and lubrication. Balancing torsional stick–slip and flow rate is delicate; excess flow can cause inefficiencies, while insufficient flow raises the risk of heat-induced damage and vibrations. Maintaining a suitable flow rate is crucial to maximize drilling efficiency, extend tool life, and minimize adverse consequences, addressing the question of how rotation speed affects hydraulic efficiency and its implications for drilling operations.

3.4. BHA Configuration

In drilling operations, BHA settings can significantly impact stick–slip vibrations. These vibrations can result in wellbore instability, equipment wear, and decreased drilling productivity. A vital part of the drill string is the BHA. It is situated close to the drill bit and performs several duties, such as steering the drill bit, stabilizing the drill string, and transferring weight and torque to the bit. Drill bits, stabilizers, drill collars, drill pipes, measurement while drilling (MWD), directional tools, and shock subs are the typical components of the BHA. The configuration and kind of BHA components can considerably influence vibrations. Stick–slip behavior can be made worse by poorly matched BHA components, such as stabilizers and collars, or by inappropriate location along the drill string. Stabilizer placement within the BHA is crucial. Stabilizers that are strategically positioned serve to center the drill string and lessen vibrations. BHAs with longer lengths and greater flexibility are more prone to torsional vibrations. The interaction between the BHA and the bit may impact vibrations. If the parts are not synchronized effectively, irregular forces may be applied to the bit, causing stick–slip. To study the stick–slip vibration behaviors of the drill bit and BHA, Weiji et al. built a finite element model (FEM) of a highly deviated well, which comprises a drill string system, a borehole, a PDC bit, and a rock model [9]. In order to control the stick–slip vibration of BHA, they proposed the torsional impact drilling technique. The study concluded that as the distance from the drill bit increases, the stick–slip vibration of BHA gradually weakens and disappears. Tianheng et al. [38] presented a FEM model to optimize the BHA design and make it vibration resistant. To assess BHA vibration, the FEM model used vibration indices such as the BHA strain energy and the stabilizer side force. Dushaishi and Stutts [39] modeled the BHA as a torsional shaft that was subjected to a point load external force from the drill bit's interaction with the formation and a localized external force from the reamer's cutting operation. Their findings demonstrated that the position of the reamer within the BHA significantly impacts the vibration response and that the symmetry exhibiting vibration modes within the reamer site has little effect on the BHA's overall torsional response.

3.5. Formation Properties

The formation parameters significantly influence stick–slip vibrations in drilling operations. The hardness, abrasiveness, consistency, and lithological composition of the subsurface rock or formation being drilled are all included in these criteria. The tendency for stick–slip vibrations is substantially affected by the interaction between the drill bit and the formation's characteristics. Sandstones and carbonates are examples of harder formations that provide more friction between the drill bit and the rock. In certain formations, the drill string may experience torsional vibrations due to the bit's intermittent gripping and releasing of the rock. Cutting tool degradation can speed up in formations containing materials like quartz or abrasive minerals. Because abrasion results in uneven bit wear, it will cause erratic cutting behavior and higher friction, amplifying stick–slip vibrations. Stick–slip vibrations can be influenced by the nature of the formation, including whether it is homogeneous or heterogeneous. Drilling in the heterogeneous formation can induce vibrations because different minerals/rocks will require varying cutting forces as they are encountered in the bit. Moreover, vibration will be worsened if the drilling fluid circulation loss occurs due to the presence of thief zones in the formation.

This paper will provide insight into the underlying factor driving stick–slip phenomena, and effective remedies will be proposed to mitigate the adverse effect of the stick–slip. This research focuses on discovering and assessing these variables (rotation speed and

WOB), understanding their influence, and detecting potential correlations and interactions. This study contributes to ongoing efforts to improve drilling technologies, reduce operational costs, and ensure the well-being of drilling personnel as the industry continues to evolve and face new challenges.

4. Mathematical Models

Stick–slip calculations are intricate and frequently use mathematical models to comprehend and anticipate this phenomenon [14]. Solving a group of partial differential equations (PDEs) that characterize the dynamic behavior of the drill string is often required to arrive at the generalized mathematical equation for models of stick–slip vibrations. These PDEs are discretized throughout the spatial and temporal domains, enabling the simulation of intricate systems such as drilling operations. The relevant equations used for the axial and torsional vibrations are given in the following sections.

4.1. Axial Vibrations

The drill string axial displacement (u) and variation in time (t) and location (x) are described by the drill string wave equation for axial vibration. Originating from Newton's second law of axial motion [40].

$$\mathrm{F = ma} \tag{1}$$

Here, "F" stands for the axial force that is exerted on a drill string component, whereas "m" denotes the component's mass and "a" denotes its axial acceleration.

Defining acceleration in terms of displacement and time:

$$a = \frac{\delta^2 u}{\delta t^2} \tag{2}$$

This equation represents acceleration, which is the second derivative of axial displacement with respect to time.

Relating force to tension,

$$\mathrm{F} = \mathrm{T}(\mathrm{x},\ \mathrm{t}) - \mathrm{F}_{\mathrm{friction}} \tag{3}$$

T(x, t) represents the tension in the drill string at position x and time t, and $\mathrm{F}_{\mathrm{friction}}$ represents the frictional force along the drill string.

Substituting Equation (1) in Equation (3)

$$\mathrm{ma} = \mathrm{T}(\mathrm{x},\ \mathrm{t}) - \mathrm{F}_{\mathrm{friction}} \tag{4}$$

Rearranging for acceleration

$$\frac{\delta^2 u}{\delta t^2} = \frac{1}{m}\mathrm{T}(\mathrm{x},\mathrm{t}) - \mathrm{F}_{\mathrm{friction}} \tag{5}$$

Introduce mass per unit length (φA) to describe the distribution of mass along one dimension

$$\varphi A \frac{\delta^2 u}{\delta t^2} = \mathrm{T}(\mathrm{x},\ \mathrm{t}) - \mathrm{F}_{\mathrm{friction}} \tag{6}$$

The wave equation for axial vibrations is expressed in this equation in its simplest form. It has to do with the drill string's axial acceleration in relation to tension and frictional forces.

4.2. Torsional Vibration

The equation for torsional vibrations describes how torsional displacement (θ) varies along the drill string with respect to both time (t) and position (x).

For Newton's second law for rotational motion [41].

$$\mathrm{M} = \mathrm{I}\alpha \tag{7}$$

In this case, M denotes the torque on a drill string component; I represent the component's moment of inertia, and α denotes the component's angular acceleration.

Using torsional displacement and time to express angular acceleration.

$$\alpha = \frac{\delta^2\theta}{\delta t^2} \tag{8}$$

This equation represents the angular acceleration, which is the second derivative of torsional displacement with respect to time.

Relating torque to applied torque and friction.

$$\mathrm{M} = \mathrm{T}(\mathrm{x},\ \mathrm{t}) - \mathrm{T}_{\mathrm{friction}} \tag{9}$$

In this case, T(x, t) represents the applied torque to the drill string at position x and time t, and $\mathrm{T}_{\mathrm{friction}}$ represents the frictional torque.

Equating Equation (7) to Equation (9)

$$\mathrm{I}\frac{\delta^2\theta}{\delta t^2} = \mathrm{T}(\mathrm{x},\ \mathrm{t}) - \mathrm{T}_{\mathrm{friction}} \tag{10}$$

Rearranging for angular acceleration

$$\frac{\delta^2\theta}{\delta t^2} = \frac{1}{\mathrm{I}}(\mathrm{T}(\mathrm{x},\mathrm{t}) - \mathrm{T}_{\mathrm{friction}}) \tag{11}$$

Introducing polar moment of inertia (J) and (G), which is the shear modulus of the drill string material:

$$\mathrm{GJ}\frac{\delta^2\theta}{\delta t^2} = \mathrm{T}\ (\mathrm{x},\ \mathrm{t}) - \mathrm{T}_{\mathrm{friction}} \tag{12}$$

These equations are the basis for modeling drill string stick–slip dynamics behavior in response to axial and torsional vibrations. Utilizing FEA, the proper boundary conditions are integrated and numerically solved.

5. Vertical Well Drilling String System's Numerical Simulation Modeling

This study utilized the WellScan simulator, a powerful tool in the realm of drilling exploration. The WellScan Software Platform (Version 3.8) served as the foundation, seamlessly integrating field-proven methodologies within various engineering modules, all interconnected through a centralized database. This sophisticated software facilitated a comprehensive analysis, leveraging its capabilities to simulate and evaluate diverse scenarios in the context of drilling operations. The finite element model employed in this study is based on modal analysis. Modal analysis is a technique specifically focused on capturing dynamic instabilities resulting from resonance phenomena. In the context of simulating real-world drilling conditions, the effectiveness of this finite element model lies in its ability to calculate the natural frequencies and mode shapes of a stabilized drill string. By utilizing modal analysis, the model provides valuable insights into the dynamic behavior of the drill string under various conditions. It allows for the decomposition of any vibration shape into a linear combination of three distinct mode types, providing a comprehensive understanding of the system's response to different forces and vibrations. Furthermore, the model's applicability to vertical wells is noteworthy. It enables the separate study of axial, lateral, and torsion modes, offering a more detailed exploration of each aspect. This capability is crucial in simulating and analyzing real-world drilling scenarios where a combination of these modes covers the entire spectrum of potential modal variations.

Figure 2 illustrates the plotted spatial trajectory of the drill string. The parameters for the BHA used in this analysis are as follows:

- 0.74 inch × 0.03 m in PDC
- 0.25 inch × 0.07 m in Near Bit Stabilizer (NBS)

- 0.25 inch × 0.05 m in Short Drill Collar (SDC)
- 0.25 inch × 0.15 m in Drill Collar (DC)
- 0.25 inch × 0.08 m in MWD
- 0.12 inch × 5.65 m in Drill pipe (DP). The drill pipe is made of nylon string.

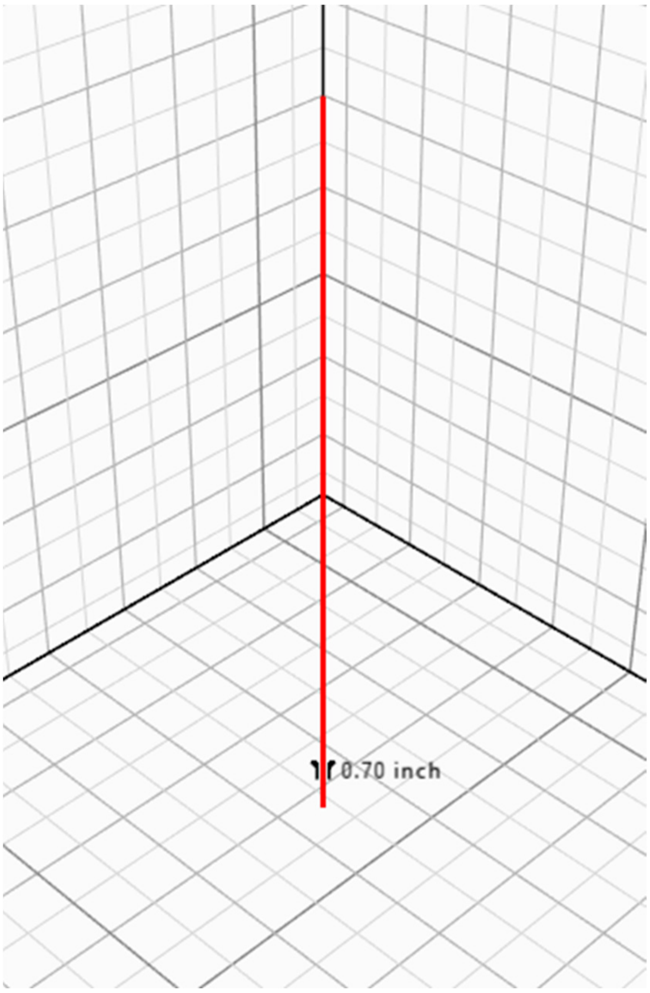

Figure 2. Well Trajectory.

Table 3 shows a comprehensive detail of the string, and Figure 3 shows an image of the string.

Table 3. String details.

Type	Length	OD	ID	Guage	Total Length	Contact	Mass	Total Mass	Linear Mass	OD Tool Joint
	(m)	(inch)	(inch)	(inch)	(m)	(m)	(kg)	(kg)	(kg/m)	(inch)
PDC	0.03	0.74	-	0.74	0.03	-	0	0	0.09	-
NBS	0.07	0.25	0.2	0.74	0.1	0.1	0.01	0.01	0.09	-
SDC	0.05	0.25	0.2	-	0.15	-	0.01	0.01	0.09	-
DC	0.15	0.25	0.2	-	0.3	-	0.03	0.03	0.09	-
MWD	0.08	0.25	0.2	-	0.38	-	0.03	0.03	0.09	-
DP	5.65	0.12	0.1	-	6.03	-	0.13	0.13	0.02	0.125

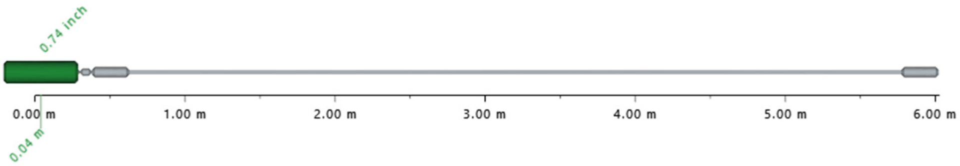

Figure 3. Modeled String.

The arrangement involved creating a model in which essential factors were methodically modified to trigger stick–slip behavior in the drill string. Several parameters, such as WOB applied to the drill bit, rotation speed, and well trajectory at various measured depths (MD) along the wellbore, were altered and compared.

In the simulation, a series of two runs were conducted to analyze and calculate the stick–slip behavior of the system. The primary objective was to understand how varying factors influenced the stick–slip phenomenon. The simulator was configured to maintain a constant rotation speed in the two test runs. This approach helps to understand the stick–slip behavior when the RPM is kept constant. The WOB was carefully managed and observed, providing insights into the impact of varying loads on stick–slip tendencies. Table 4 shows the parameters used for Tests 1 and 2.

Table 4. Considered parameters.

Parameters				
Test	**WOB**	**RPM**	**TOB**	**TOB/WOB**
1	0.01	50	0.01	1
2	0.005	30	0.01	2

The specific gravity of the drilling fluid was set at 0.01, indicating an extremely low density since the string is in air. Additionally, the viscosity of the fluid, which refers to its resistance to flow, was specified as 0.01 cp, indicating that it had a very low resistance to flow, making it highly fluid in nature. The reason for choosing these values for specific gravity and viscosity was to make them negligible for the simulation, ensuring that they had minimal impact on the overall results. The casing material selected for this setup was Polyvinyl Chloride (PVC), a widely used thermoplastic known for its durability and corrosion resistance. This choice of casing material offers several advantages, including its cost-effectiveness and compatibility with various wellbore conditions. The casing has an outer diameter (OD) measurement of 0.73 inches, representing the external size of the PVC casing (Figure 4). Table 5 shows the mechanical properties of the strings. This dimension of the strings was carefully selected to ensure compatibility with the wellbore and other downhole components. The casing serves a vital role in maintaining well integrity by providing essential structural support and facilitating controlled fluid containment, as required for the setup.

Table 5. Mechanical properties of the strings.

Part	Material	Density (kg/m^3)	Young Modulus of Elasticity (GPa)	Poisson's Coefficient	Minimum Yield Strength (MPa)	Minimum Tensile Strength (MPa)
Drillstring	Nylon	1140	4.2	0.3	45	31
BHA	Aluminum	2700	3.7	0.33	2400	2400
Casing	PVC	1330	4.83	0.38	57.4	60

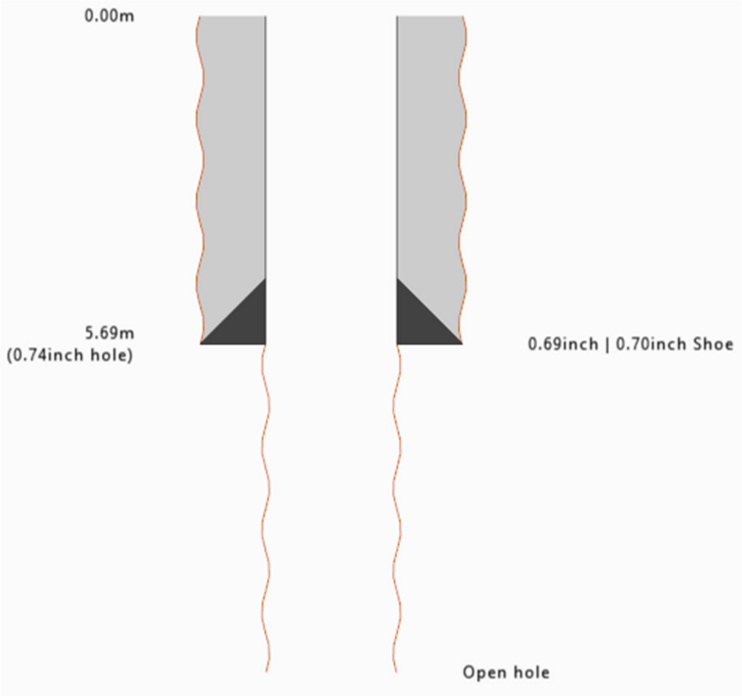

Figure 4. Open-hole casing considered for the simulation.

These simulations were conducted over a time span of 200 s, encompassing a total of 20 drilling cycles or periods, allowing for total exploration of the system's response to different operational conditions in a detailed and extensive manner. Utilizing the dynamics module of the WellScan simulator, the study implemented a comprehensive model of the drilling configuration [42–44].

6. Results and Discussions

A dynamic numerical simulation model has been effectively constructed in the section above to represent the drill string system in a vertical well. A thorough examination of the stick–slip vibration phenomena demonstrated by the drill bit, along with the mitigation strategies, will be presented in the following section.

6.1. Drill Bit Vibration Reactions to Stick–Slip

Figure 5 shows how the drill bit's rotation speed changes during the simulation in Test 1. The drill bit frequently switches between sticking and slipping states while drilling, which is a very noticeable manifestation of the stick–slip vibration phenomena. Drill bit sticking is indicated when the rotation speed of the drill bit falls to zero. The drill bit's RPM then increases, as seen in Figure 6, where the maximum recorded rotation speed is 105 RPM, and the rotary table runs at just 50 RPM. The drill bit is slipping, as this observation demonstrates. Due to the drill bit's fast rotational speed, the bit cutter encounters significant impact stresses during the slipping phase, which could lead to a premature cutter failure.

The simulation was carried out under controlled circumstances in test 2 as in test 1, with a constant rotation speed of 30 RPM and a WOB of 0.005 tf. A clear pattern can be observed in Figure 6, in which, during the first 30 s of the test, the drill bit maintains a constant, stable rotation speed before beginning to deviate. Notably, the drill bit exhibits stick–slip behavior, or sporadic sticking and slipping motions, at the 120-s mark and continues till 200 s. The drill bit's performance during this period is characterized by cyclic stick–slip behavior, which may have an impact on the effectiveness and longevity of the drilling process. The drill string builds up energy while torque is still transferred from the rotary table to the drill bit until the next slipping phase happens.

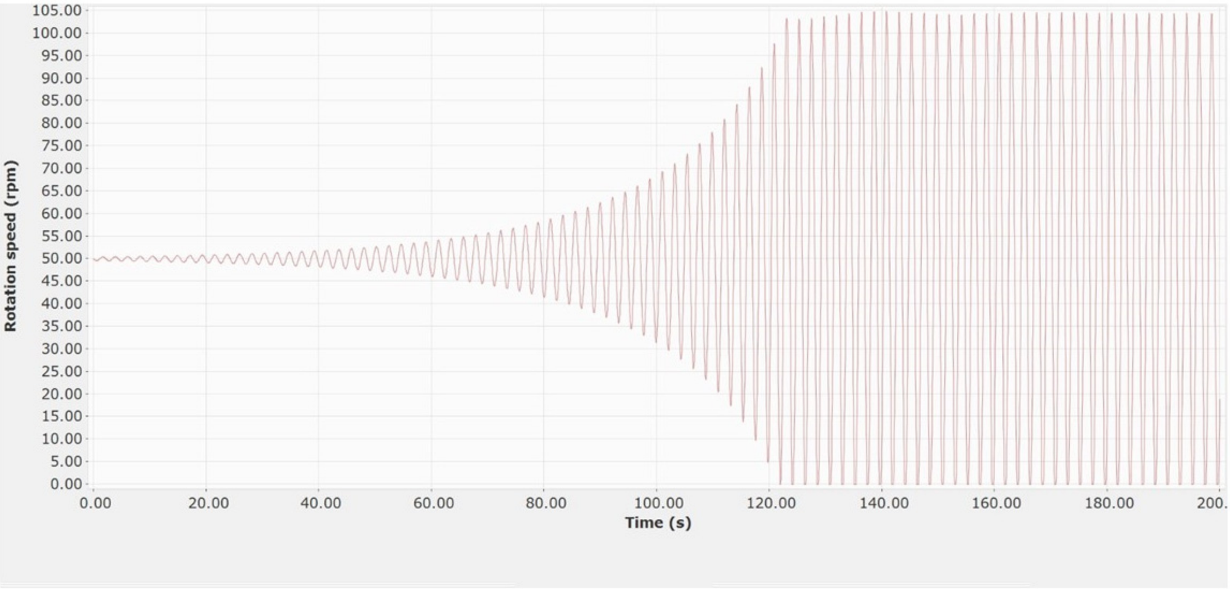

Figure 5. Rotation speed against time for test 1.

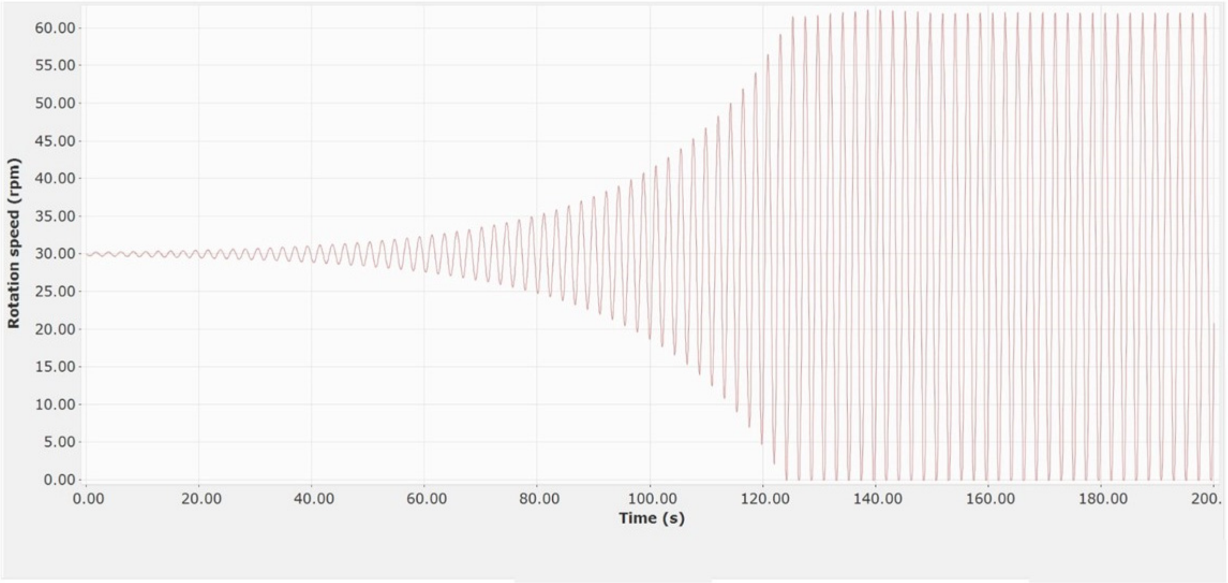

Figure 6. Rotation speed against time for test 2.

6.2. Strategy to Regulate Stick–Slip Vibrations

It is clear from the prior discussion that stick–slip vibrations can negatively impact drilling operations, including a decrease in the effectiveness of rock breaking, deformations in the drilling string system, and increases the chance of BHA fracture. Therefore, addressing and reducing stick–slip vibrations in the BHA is essential. In order to achieve this, the numerical simulation model of the vertical well that was described in the previous section is used to explore the tactics that can help to reduce stick–slip vibrations.

The strategy employed to mitigate stick–slip in test 1 encompassed a dual-pronged approach. Initially, the WOB was reduced by 50%. This modification led to the stick–slip mitigation, as shown in Figure 7. Additionally, the rotation speed was elevated to 100 RPM, as illustrated in Figure 8, and notably, no stick–slip occurred during the entire test duration.

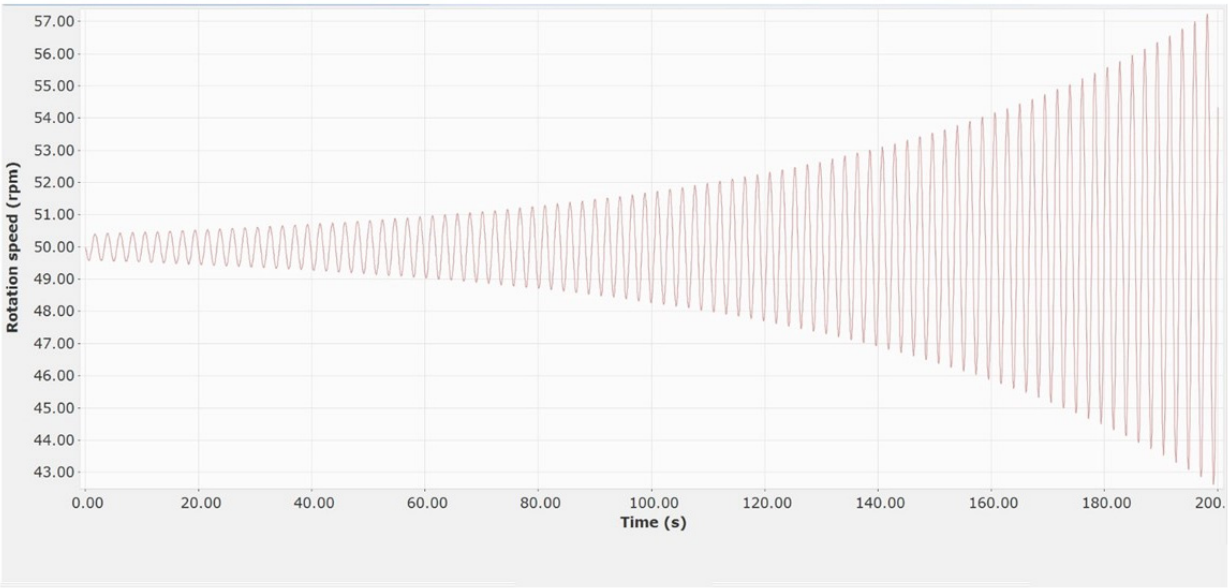

Figure 7. Rotation speed against time with halved WOB for test 1.

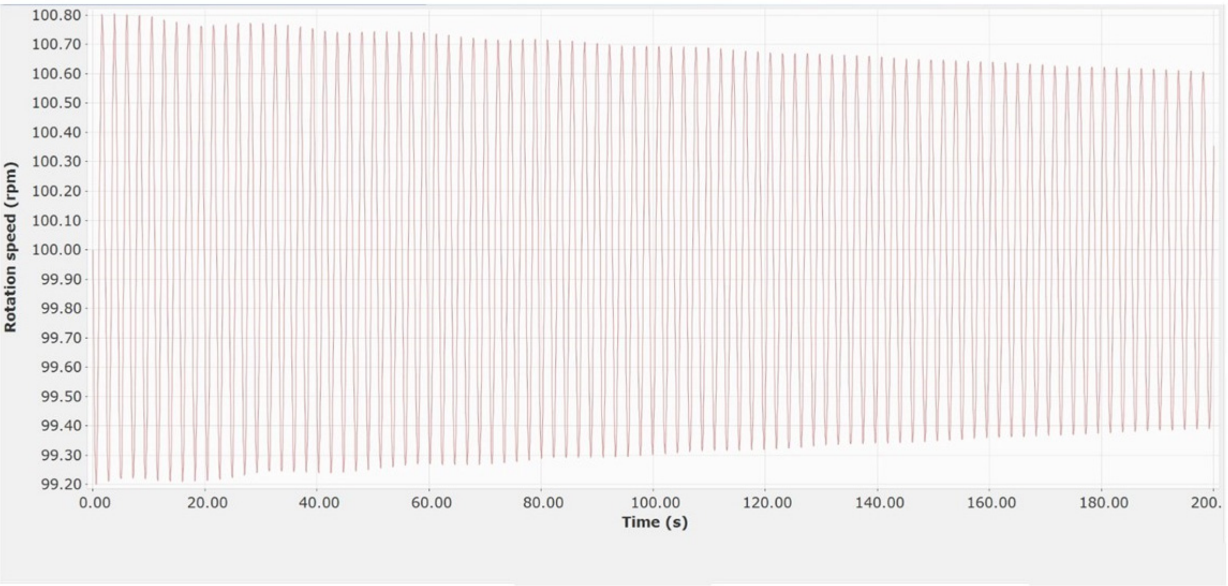

Figure 8. Rotation speed against time with an increased rotation speed of 100 RPM for test 1.

A similar strategy was implemented in test 2, where a stable rotation speed of 30 RPM was maintained. Figure 9 shows that reducing the WOB to 0.001 tf effectively prevented stick–slip occurrences. While increasing the rotation speed to 100 RPM and maintaining the initial WOB also resulted in the absence of stick–slip during the entire cycle, as presented in Figure 10. These findings show the effectiveness of the mitigation strategy in eliminating stick–slip vibrations and improving drilling performance.

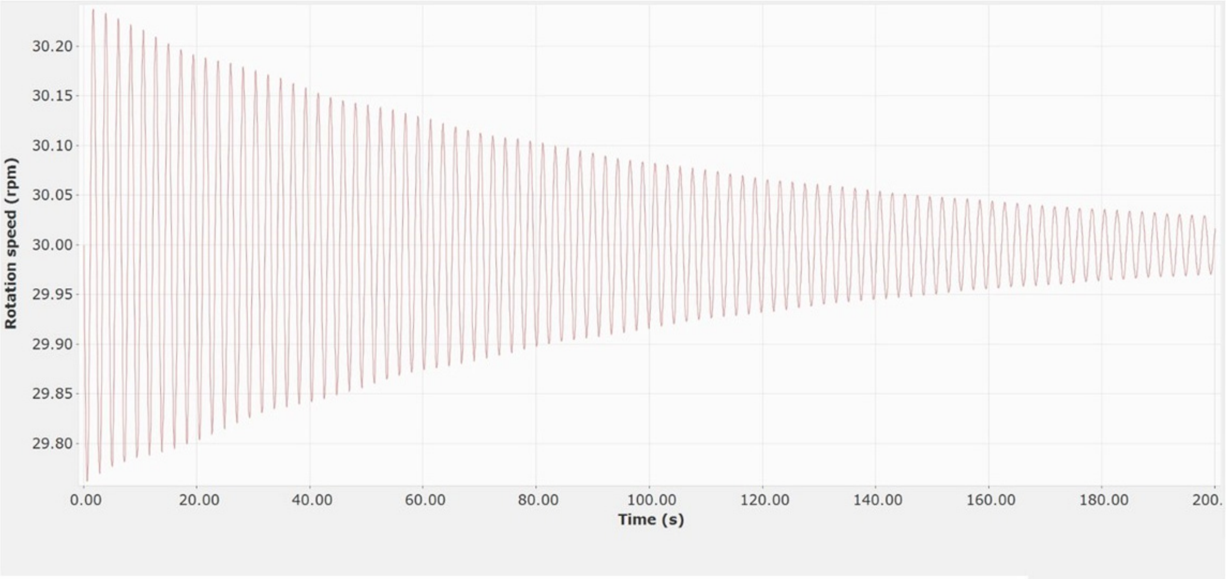

Figure 9. Rotation speed against time with halved WOB for test 2.

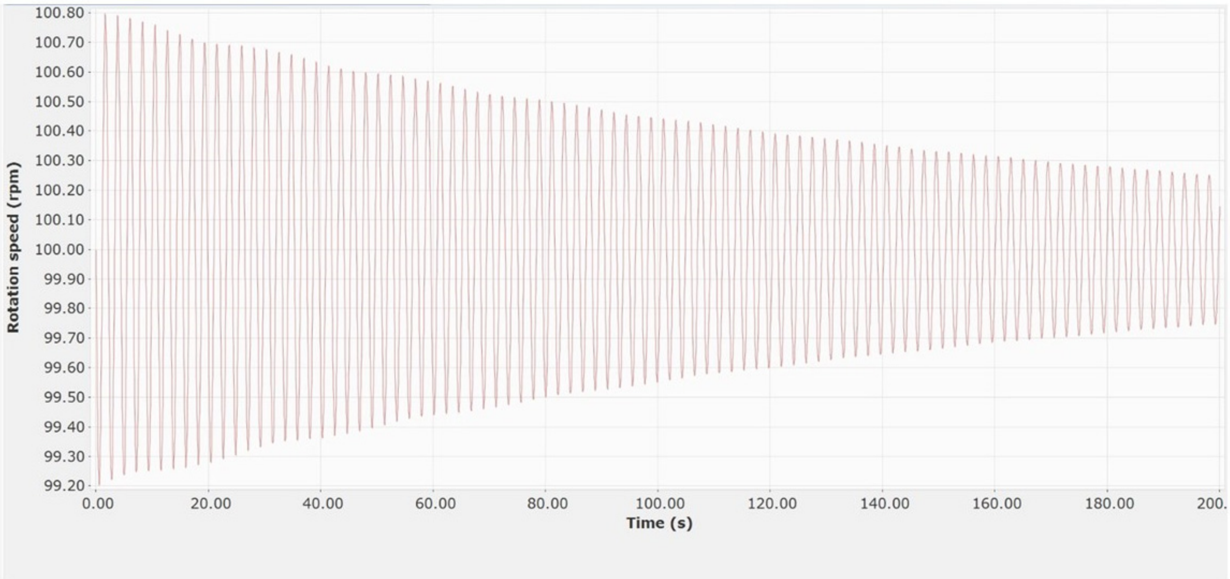

Figure 10. Rotation speed against time with an increased rotation speed of 100 RPM for test 2.

7. Conclusions

The conclusions made from the study are as follows

- Rotation speed, WOB, BHA assembly arrangement, and formation qualities are the critical factors affecting stick–slip vibrations during vertical drilling operations.
- The mathematical model from this study can help to understand and predict the axial and torsional vibrations associated with stick–slip.
- Wellscan software package was used for two drilling tests that produced empirical proof of stick–slip vibrations and gave mitigation measures.
- Reducing the WOB can improve drilling system stability by reducing vibrations caused by stick–slip.
- Another effective strategy for preventing stick–slip and eventually improving drilling performance is increasing the rotation speed of the drilling process.
- This study provides useful suggestions for reducing stick–slip vibrations' negative impacts on drilling operations and gives insights into the intricacies of stick–slip vibrations.

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Abbreviations

BHA	Bottom Hole Assembly
RPM	Rotation Per minutes
WOB	Weight on Bit
PDC	Polycrystalline Diamond Compact
MWD	Measurement While Drilling
FEM	Finite Element Model
PDEs	Partial Differential Equations
u	Displacement
t	Time
x	Position
F	Axial Force
M	Component Mass
a	Axial Acceleration
T	Tension
Θ	Torsional Displacement
M	Torque
I	Inertia
α	Angular Acceleration
T (x, t)	Applied Torque in x and t direction
$T_{friction}$	Frictional Torque
J	Polar Moment of Inertia
G	Shear Modulus
NBS	Near Bit Stabilizer
SDC	Short Drill Collar
DC	Drill Collar
DP	Drill Pipe
m	Meters
inch	Inches
kg	Kilogram
kg/m	Kilogram Per Meter
tf	Tonne Force
kg/m^3	Kilogram per Meter Cube
GPa	Gigapascal
MPa	Megapascal
deg	Degree
SG	Specific Gravity
cp	Centipoise
DoFs	Degree of Freedom
PVC	Polyvinyl Chloride
OD	Outer Diameter
MD	Measured Depth

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