

Supplementary Materials for

Research on the Formation Characteristics of Fog and Frost on the Optical Windows of Unsealed Equipment Compartments in Aircraft

Section S1. Fogging model

S1.1. Liquid film conservation governing equations

The liquid film continuity equation is given as follows:

$$\frac{\partial}{\partial t} \int_V \rho_{lf} dV + \int_A \rho_{lf} v_{lf} \cdot da = \int_V \frac{S_m}{h_{lf}} dV, \quad (S1)$$

where ρ_{lf} is the density of the liquid film, v_{lf} is the velocity of the liquid film, and h_{lf} is the thickness of the liquid film. The subscript lf stands for the liquid film. S_m is the mass source per unit area, which can result from mass transfer processes such as droplet impingement and liquid film stripping.

The liquid film momentum conservation equation is given as follows:

$$\frac{\partial}{\partial t} \int_V \rho_{lf} v_{lf} dV + \int_A \rho_{lf} v_{lf} \otimes v_{lf} \cdot da = \int_A T_{lf} \cdot da - \int_A p_{lf} \cdot da + \int_V \left(f_v + \frac{S_p}{h_{lf}} \right) dV, \quad (S2)$$

where T_{lf} represents the viscous stress tensor within the liquid film, p_{lf} represents the pressure, f_v represents the body force (such as gravity, drag force, centrifugal force, etc.), and S_p represents the momentum source corresponding to the mass source.

The liquid film energy conservation equation is given as follows:

$$\frac{\partial}{\partial t} \int_V \rho_{lf} E_{lf} dV + \int_A \rho_{lf} H_{lf} v_{lf} \cdot da = \int_A q_{lf}^* \cdot da + \int_A T_{lf} \cdot v_{lf} da - \int_V f_v \cdot v_{lf} dV + \int_V \frac{S_E}{h_{lf}} dV, \quad (S3)$$

where E_{lf} represents the total energy of the liquid film, H_{lf} represents the total enthalpy of the liquid film, q_{lf}^* represents the heat flux of the liquid film, T_{lf} represents the viscous stress tensor of the liquid film. S_E represents the energy source term per unit area of the liquid film. The energy source term comes from droplet impingement, liquid film stripping, evaporation and condensation.

The liquid film is located in the fluid region, and the volume occupied by the liquid film will be subtracted from the gas volume in the grid cells adjacent to the liquid film. The volume fraction of the liquid film in the adjacent gas grid cells is calculated as follows:

$$\alpha_w = \min\left(\frac{V_w}{V}, \alpha_{w,\max}\right), \quad (S4)$$

where V_w represents the volume of the liquid film, V represents the volume of the adjacent grid cell, and $\alpha_{w,\max}$ represents the maximum volume fraction of the liquid film.

S1.2. Evaporation mass conservation governing equations at the liquid film interface

The mass flux of single-component water vapor is conserved at the gas-liquid interface. Therefore:

$$\rho Y_v (v - \dot{h}) - \rho D_v \frac{dY_v}{dy} = \rho_w Y_w (v_w - \dot{h}) - \rho_w D_w \frac{dY_v}{dy} \Big|_w, \quad (S5)$$

where ρ and ρ_w represent the densities of water vapor and the liquid film (liquid water), respectively. Y_v and Y_w represent the mass fractions of water vapor and the liquid film, respectively. v and v_w represent the normal velocity components of water vapor and the liquid film, respectively. D_v and D_w represent the molecular diffusion coefficients of water vapor and the liquid film, respectively. \dot{h} represents the rate of change of the thickness of the liquid film.

The evaporation rate of the liquid film (liquid water) is defined as:

$$\dot{m}_v = -\rho_w \dot{h}, \quad (S6)$$

regarding the normal velocity component of the liquid film as zero, combining Equations (S5) and (S6), we can obtain:

$$-\left[1 - Y_{v,j}\right] \dot{m}_v = \rho D_v \frac{dY_v}{dy}, \quad (S7)$$

The total evaporation rate of the liquid film (liquid water) can be expressed as:

$$\dot{m}_v = -\frac{\rho D_v \frac{dY_v}{dy}}{1 - Y_{v,j}}, \quad (S8)$$

However, Equation (S8) is only applicable when the water vapor at the interface is below the saturation condition, that is, $Y_{v,j} < 1$

The vapor pressure of water vapor p_v is expressed as:

$$p_v = a p_v^*, \quad (S9)$$

where p_v^* represents the vapor pressure of pure component water vapor, and a represents the activity, which depends on the liquid mixture.

The vapor pressure of water vapor can also be expressed as:

$$p_v = \gamma_v X_w p_v^*(T_s), \quad (S10)$$

Raoult's law stipulates that the mole fraction of water vapor at the interface is the ratio of the vapor pressure of water vapor to the ambient pressure:

$$X_v = \frac{p_v}{p}, \quad (S11)$$

The mass fraction of water vapor at the interface is given as follows:

$$Y_v = \frac{X_v W_v}{X_{v,j} W_v + \sum_j^{N_{P,G}} X_j W_j}, \quad (S12)$$

where X_v and W_j are the mole fraction and molar mass of water vapor, respectively. $X_{v,j}$ is the mole fraction of water vapor at the interface. X_j and W_j are the mole fraction and molar mass of the inert component at the interface, respectively.

Usually, since the mole fraction of the inert component is difficult to calculate, the background molar quantity is introduced here and approximated at a position a short distance away from the interface, the background molar quantity is given by:

$$W_{bg} = \frac{\sum_j^{N_{P,G}} X_j W_j}{\sum_j^{N_{P,G}} X_j}, \quad (S13)$$

where $N_{P,G}$ is the inert component gas. In this model, the inert component gas is air.

The background mole fraction at the interface position is given by:

$$X_{bg} = 1 - X_{v,j}, \quad (S14)$$

Therefore, the vapor mass fraction of water vapor at the interface position can be approximated as follows:

$$Y_v \approx \frac{X_v W_v}{X_{v,j} W_v X_{bg} W_{bg}}, \quad (S15)$$

It is assumed that the mole fraction $X_{lf,i}$ of component i in the liquid film is known, and the normal component distribution in the liquid film is approximately constant. Therefore, the mole fraction at the interface position is equal to the value at the center of the nearest grid cell as follows:

$$X_{lf,i} = X_{lf_c,i}, \quad (S16)$$

where $X_{lf_c,i}$ is the value at the center of the grid cell for the mole fraction at the boundary of the liquid film.

The evaporation rate of the liquid film (liquid water) can also be expressed as follows:

$$\dot{m}_v = Y_v \dot{m}_v - \rho D_v \frac{dY_v}{dy}, \quad (S17)$$

Notably, S16 is valid under any conditions.

S1.3. Energy conservation in the evaporation at the liquid film interface

The heat flux conservation equation at the interface position is given as follows:

$$k_v \frac{dT}{dy} - k_w \frac{dT}{dy} \Big|_{lf} - \dot{Q} = 0, \quad (S18)$$

where k_v and k_w are the thermal conductivities of water vapor and liquid water, respectively. Moreover, the total heat can also be expressed as follows:

$$\dot{Q} = \Delta H_v \dot{m}_v, \quad (S19)$$

where ΔH_v is the latent heat of vaporization of liquid water.

The total evaporation rate of the liquid film (liquid water) is defined as follows:

$$\dot{m}_v = \frac{\dot{Q} + \Delta H_v \rho D_v \frac{dY_i}{dy}}{\Delta H_v Y_v}, \quad (S20)$$

S1.4. phase change mass transfer coefficient

The molecular diffusivity has a great influence on the results of the numerical simulation of the condensation process. The Schmidt number is used in the calculation to specify the molecular diffusivity in the components, which is defined as follows:

$$Sc = \frac{\nu}{D}, \quad (S21)$$

where ν is the kinematic viscosity and D is the mass diffusivity which can be calculated by:

$$D = 2.302 \frac{p_0}{p} \left(\frac{T}{T_0} \right)^{1.81} \times 10^{-5} \text{ m}^2/\text{s}, \quad (S22)$$

Where p_0 is 0.98 bar, T_0 is 256 K, and p and T are the local pressure and the ambient temperature respectively.

S1.5. The governing equation for the condensation of water vapor at the interface

Firstly, check whether it is saturated through S23 which is given as follows:

$$Y_{v,j} = 1, \quad (S23)$$

where $Y_{v,j}$ is the mass fraction of water vapor at the interface. Only when the temperature of the water vapor reaches the saturation temperature can the water vapor reach a quasi-steady-state equilibrium at the interface. If the saturation condition is detected, the water vapor will

stop transferring towards or away from the cold wall surface, and the interface temperature at this time will be set as T_{sat} .

The heat transfer coefficient in the liquid film is defined as the ratio of the thermal conductivity of the liquid film to half of the thickness of the liquid film, the equations are given as follows:

$$h_{\text{lf}} = \frac{2k_{\text{lf}}}{h}, \quad (\text{S24})$$

$$k_l \left. \frac{dT}{dy} \right|_{l_s} \approx \frac{2k_{\text{lf}}}{h} (T_c - T_s), \quad (\text{S25})$$

where T_c is the temperature of the liquid film at the center of the grid cell, and T_s is the temperature of the cold wall surface. To ensure that the condensation rate will not be infinitely large, the minimum thickness of the liquid film is set to 10^{-8} m.

The condensation model only adopts the simplified dropwise condensation method. The contact angle is assumed to be 90° . And all droplet radii are exactly the same and can be calculated by:

$$R = \left(\frac{3h}{2\pi N} \right)^{\frac{1}{3}}, \quad (\text{S26})$$

where h is the liquid film thickness, and N is the droplet nucleation density.

S1.6. The calculation formula for saturated pressure

Buck formula which is applicable in the temperature range from -50°C to 10°C is utilized as follows:

When $T < 0^\circ\text{C}$, saturated pressure is calculated as follows:

$$P_{\text{sat}} = 6.1115 \times e^{\frac{(23.036 - \frac{T}{333.7}) \times T}{279.82 + T}}, \quad (\text{S27})$$

When $T > 0^\circ\text{C}$, saturated pressure is calculated as follows:

$$P_{\text{sat}} = 6.1121 \times e^{\frac{(18.678 - \frac{T}{234.5}) \times T}{257.14 + T}}, \quad (\text{S28})$$

It is consistent with formulas (1) and (2) in the main manuscript.

Section S2. Frosting model.

S2.1. solidification and melting

The solidification and melting model adopts the enthalpy formula to determine the distribution of the liquid film (liquid water) - solid film (frost). The total enthalpy consists of

two parts: sensible enthalpy and the latent heat of solidification/melting which can be represent as follows:

$$h_{ls}^* = h_{ls} + (1 - \alpha_s^*) h_{\text{latent}}, \quad (\text{S29})$$

where h_{ls} is the sensible enthalpy and h_{latent} is the latent enthalpy. α_s^* is the relative solid phase volume fraction, which represents the volume fraction of the frost layer in the part of the liquid film - solid film. In the enthalpy model, the relative solid phase volume fraction α_s^* is expressed as a temperature-dependent function, its value in different ranges is given as follows:

$$\alpha_s^* = \begin{cases} 1, & \text{if } T^* < 0 \\ f(T^*), & \text{if } 0 < T^* < 1 \\ 0, & \text{if } T^* > 1 \end{cases} \quad (\text{S30})$$

with:

$$T^* = \frac{T - T_s}{T_l - T_s}, \quad (\text{S31})$$

and the solidification function is defined as follows:

$$f(T^*) = 1 - T^*, \quad (\text{S32})$$

where T_s and T_l refer to the temperatures of the cold wall surface and the liquid part respectively.

S2.2. The governing equation for the solidification source term

During each time step, the increment of the frost layer thickness Δh_s will be calculated and updated at each internal iteration. Δh_s can be calculated by:

$$\Delta h_s' = \begin{cases} -\frac{\Delta H(t) - \Delta H_{\text{liq}}}{\Delta H_{\text{latent}}} (h + \Delta h_s), & \text{if } T \geq T_{\text{liq}}, \\ h \cdot \alpha_s, & \text{if } T < T_{\text{liq}} \end{cases} \quad (\text{S33})$$

where, $\Delta h_s'$ is the correction of Δh_s . During each time step, when the temperature of the frost layer near the wall is lower than the liquidus temperature of water, the mass of the frost layer will increase. However, if the temperature of the frost layer near the wall is higher than the liquidus temperature of water, the mass of the frost layer will decrease. Nevertheless, there is a lower limit $\Delta h_s \geq 0$ for the solidification model. Therefore, the solidification process only allows solidification to occur and does not allow melting.

The increment of the frost layer thickness can be used to calculate the mass flux in the mass transfer process of solidification and melting as follows:

$$\dot{m} = \frac{\rho_{\text{liq}} \Delta h_s}{\Delta t}, \quad (\text{S34})$$

The mass flux can be calculated for the liquid film, and the mass of liquid water that solidifies into the mass of the frost layer can be obtained. This mass flux can also be used to calculate the moving speed of the interface between the frost layer and liquid water by taking into account the density of the solid mass.

The moving speed of the interface between the frost layer and liquid water can be used to calculate the increment of the solid thickness in this time step, that is:

$$\Delta h_s = V \cdot dt, \quad (\text{S35})$$

Under normal circumstances, the solidified frost layer will gradually accumulate on the liquid film wall and become fixed at a certain position without flowing anymore, thus causing obvious deformations on the surfaces of the gas film and the liquid film. This effect can be modeled by applying Δh_s in the deformation.