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Supplement A

Detailed methodology for deriving a marginal Beta distribution from expert opinion of risk impact on cost and schedule.

Step 1: Define the root causes of a risk factor (Ni) and the possible scenarios for each root cause.

Step 2: For each root cause or scenario (Mi), subjectively evaluate the frequency of occurrence and adverse consequence using the corresponding fuzzy membership function.

Step 3: Using Equation 3, form a fuzzy relationship between the adverse consequence and frequency of occurrence for each root cause/scenario.

$$\boldsymbol{\mu_R(x_i, y_i) = min[\mu_F(x_i), \mu_C(y_i)]} \tag{3}$$

where x_i = an element of universe X
y_i= an element of universe
Y; $\mu_R(x_i, y_i)$ = the membership value of element (x_i, y_i) in the fuzzy relation R
min = the minimum values of both elements x_i and y_i
$\mu_F(x_i)$ = the membership value of element x_i in fuzzy set F
$\mu_C(y_i)$ = the membership value of element y_i in fuzzy set C.

Step 4: Using Equations 4 and 5, develop a fuzzy union matrix between all relationships.

$$\boldsymbol{\mu_{S \cup Z}(x_i, y_i) = max[\mu_s(x_i, y_i), \mu_z(x_i, y_i)]} \tag{4}$$

where *max* = the maximum value of both relations s and z.

Union **U**, between the fuzzy relation matrices R (F, C), is then computed as:

$$\boldsymbol{U = max\ [(F_1 \times\ C_1)\ \cup\ (F_2 \times\ C_2)\ \cup\ (F_3 \times\ C_3)\ \ldots\ldots\ldots\ldots..\cup\ (F_k \times\ C_k)]} \tag{5}$$

where *max* = the maximum value of the two relations.

Step 5: Define the minimum and maximum (A, B) values for the risk impact.

Step 6: Divide the impact range (between minimum and maximum) into equal subsets.

Step 7: Evaluate the mapping value of each range (example provided as follows):

Impact range	Small impact					Medium impact					Large impact				
Impact value	10	12.5	15.0	17.5	20.0	20.0	22.5	25.0	27.5	30.0	30.0	32.5	35.0	37.5	40
Mapping degree	1.0	0.9	0.8	0.7	0.6	0.7	0.85	1.0	0.85	0.7	0.5	0.7	0.8	0.9	1.0

Step 8: Using Equation 3, form a fuzzy relationship between the adverse consequence of a root cause/scenario and the range of a risk impact.

$$\boldsymbol{\mu_R(x_i, y_i) = min[\mu_F(x_i), \mu_C(y_i)]} \quad \textbf{(3)}$$

where x_i = an element of universe X
y_i= an element of universe
Y; $\mu_R(x_i, y_i)$ = the membership value of element (x_i, y_i) in the fuzzy relation R
min = the minimum values of both elements x_i and y_i
$\mu_F(x_i)$ = the membership value of element x_i in fuzzy set F
$\mu_C(y_i)$ = the membership value of element y_i in fuzzy set C.

Step 9: Using Equations 4 and 5, develop a fuzzy union matrix between all relationships in Step 8.

$$\boldsymbol{\mu_{S\cup Z}(x_i, y_i) = max[\mu_s(x_i, y_i), \mu_z(x_i, y_i)]} \quad \textbf{(4)}$$

where *max* = the maximum value of both relations s and z.

Union **U**, between the fuzzy relation matrices R (F, C), is then computed as:

$$\boldsymbol{U = max\ [(F_1 \times\ C_1)\ \cup\ (F_2 \times\ C_2)\ \cup\ (F_3 \times\ C_3)\ \ldots\ldots\ldots\ldots..\cup\ (F_k \times C_k)]} \quad \textbf{(5)}$$

where *max* = the maximum value of the two relations.

Step 10: Using Equation 6, develop the fuzzy composition matrix between the fuzzy union matrices developed in Step 4 and Step 9.

$$\boldsymbol{U \circ V(x_i, z_k) = \max_{y_j}\{min[\mu_U(x_i, y_i), \mu_V(y_j, z_k)]\}} \quad \textbf{(6)}$$

where $U \circ V(x_i, z_k)$ = membership value of element (x_i, z_k) in composition matrix between U and V
$\mu_U(x_i, y_i)$ = membership value of element (x_i, y_i) in union matrix U
$\mu_V(y_j, z_k)$= membership value of element (y_j, z_k) in union matrix V.

Step 11: Using Equations 7 through 9, select one row from the fuzzy composition matrix to calculate the mean and variance of the distribution.

$$\boldsymbol{P(R_I = z_k) = \frac{\mu_o(z_k)}{\sum_1^m \mu_o(z_k)}} \quad \textbf{(7)}$$

$$\boldsymbol{\mu_I = \sum_{k=1}^{m} (z_k) *\ \ P(R_I = z_k)} \quad \textbf{(8)}$$

$$\boldsymbol{\sigma^2{}_I = [\textstyle\sum_{k=1}^{m} (z_k)^2 *\ \ P(R_I = z_k)]\text{-}\mu_I{}^2} \quad \textbf{(9)}$$

where R_I = risk impact
z_k = element of the risk impact
$P(R_I = z_k)$ = probability of occurrence of the risk impact to be element z_k

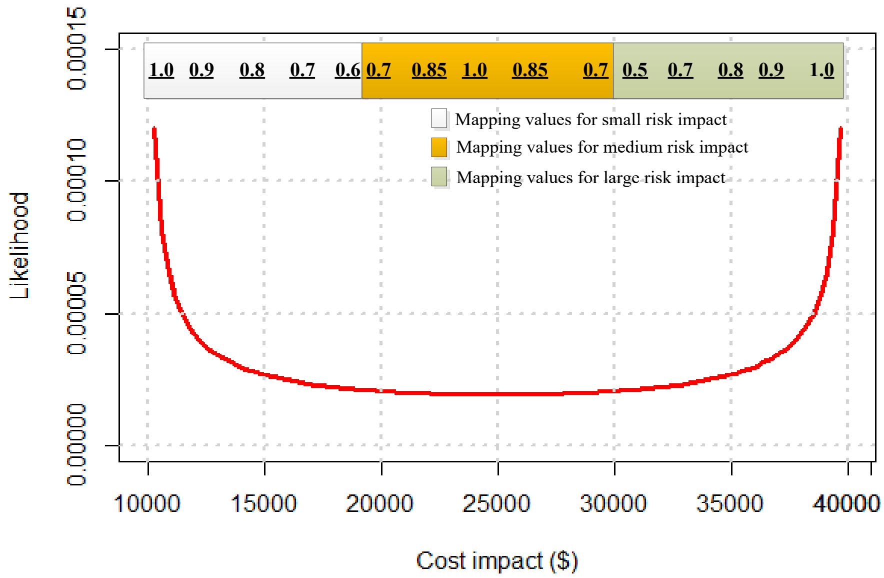
$\mu_o(z_k)$ = membership value of element z_k in subset O
m = number of risk impact elements in subset O

Step 12: Using the mean and variance, fit the results to a Beta distribution, using Equations 10 and 11, to calculate the shape and scale parameters.

$$\alpha = \frac{\mu_I - A}{B - A}\left[\frac{(\mu_I - A)(B - \mu_I)}{\sigma^2{}_I} - 1\right] \quad \textbf{(10)}$$

$$ß = \alpha \left[\frac{B - \mu_I}{\mu_I - A}\right] \quad \textbf{(11)}$$

Step 13: Visualize the marginal Beta distribution of cost or schedule impact using the minimum and maximum derived in Step 5 and the scale and shape parameters derived in Step 12 (example of a resulting marginal Beta distribution is provided as follows):



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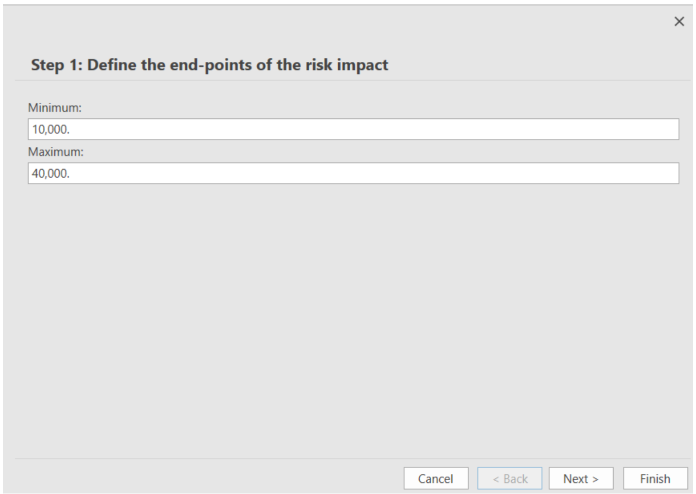
Supplement B

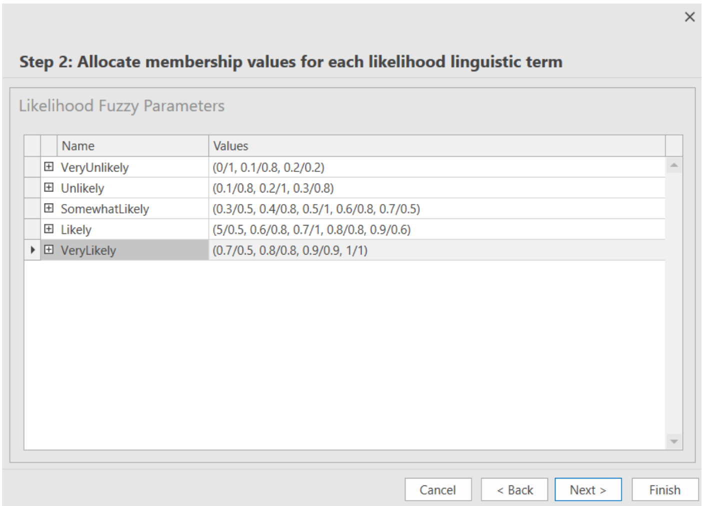
Derivation of the marginal distribution using *SimphonyProject.NET*

The steps for deriving the marginal distribution in *SimphonyProject.NET*, and a graphical user interface created to make the method easy for analyst lacking expertise in fuzzy logic, are presented as follows:

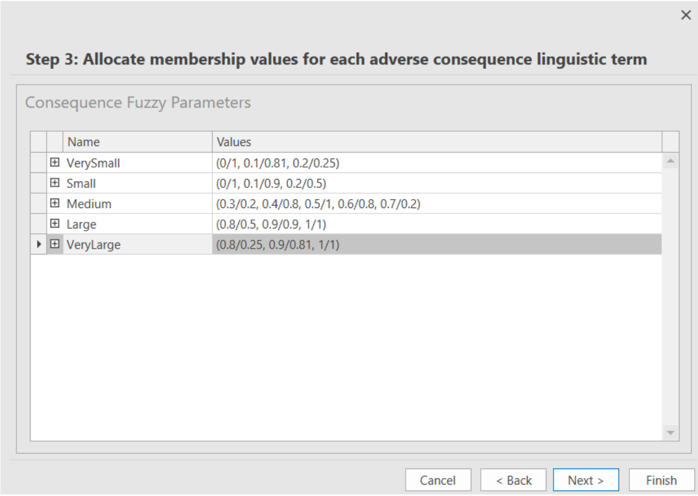
Step 1:



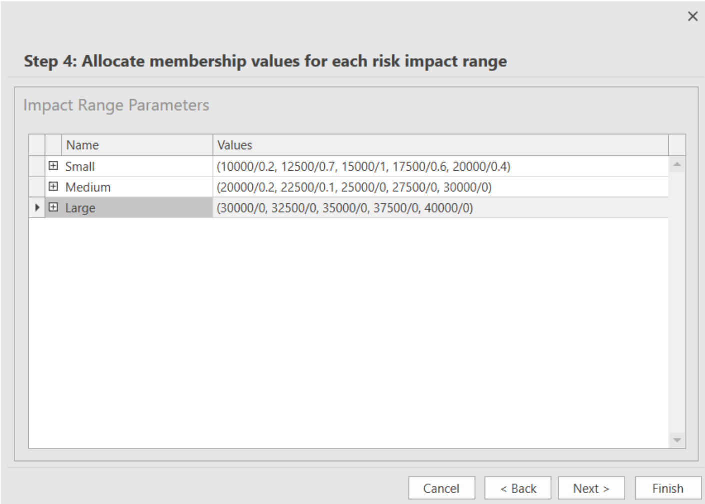
Step 2:



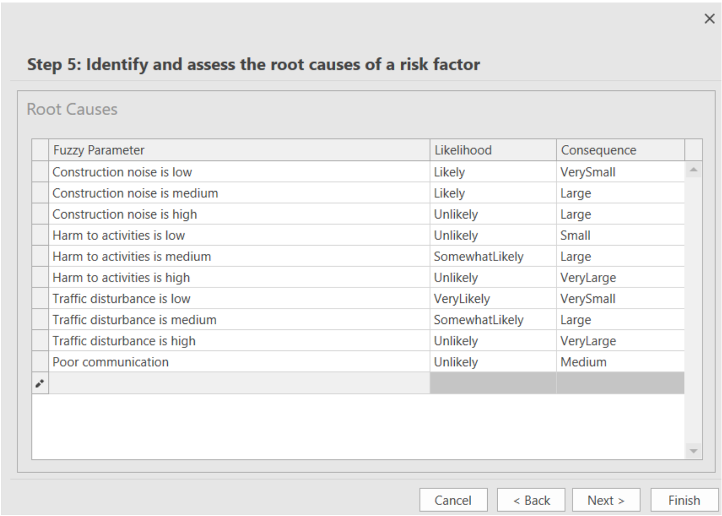
Step 3:



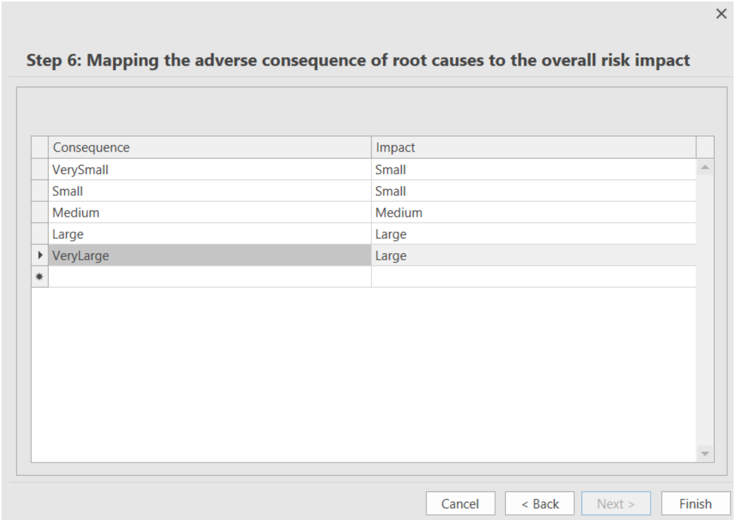
Step 4:



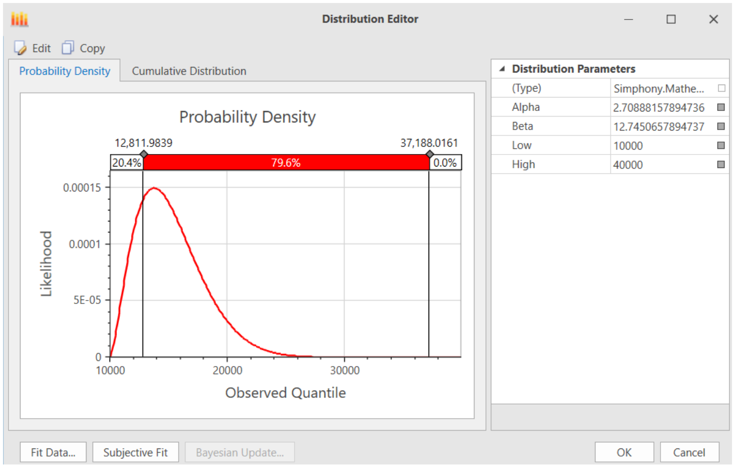
Step 5:



Step 6:


Step 6: Mapping the adverse consequence of root causes to the overall risk impact
Consequence
Impact
VerySmall
Small
Small
Small
Medium
Medium
Large
Large
VeryLarge
Large
Cancel
< Back
Next >
Finish

Step 7:


Distribution Editor
Edit
Copy
Probability Density
Cumulative Distribution
Probability Density
12,811.9839
37,188.0161
20.4%
79.6%
0.0%
Likelihood
Observed Quantile
Distribution Parameters
(Type)
Simphony.Mathe...
Alpha
2.70888157894736
Beta
12.7450657894737
Low
10000
High
40000
Fit Data...
Subjective Fit
Bayesian Update...
OK
Cancel