

```

In[152]:= clear [a, b, c, A, B, C, Az, Bz, Cz,
               Rx, Az, Bz, Cz, InvAz, u1, Kcal, Qcal, Gcal, Pcal]

Out[152]= clear[a, b, c, {{0, 0}, {0, 0}}, {{6/5, 1/2}, {-7/5, -1/5}}, {{0}}, Az, Bz, Cz,
           {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}, Az, Bz, Cz, InvAz, u1, Kcal, Qcal, Gcal, Pcal]

In[153]:= Unprotect[C]
Out[153]= {}

In[154]:= A = {{-8/10, 0/10}, {-5/10, -a}}
Out[154]= {{-4/5, 0}, {-1/2, -a}}

In[155]:= B = {{-b, 0/10}, {-5/10, -6/10}}
Out[155]= {{-b, 0}, {-1/2, -3/5}}

In[156]:= C = {{-a, 0/10}, {-5/10, -7/10}}
Out[156]= {{-a, 0}, {-1/2, -7/10}}

In[157]:= Rx = {{2, 0}, {0, 3}}
Out[157]= {{2, 0}, {0, 3}}

In[158]:= I2 = {{1, 0}, {0, 1}}
Out[158]= {{1, 0}, {0, 1}}

In[159]:= Sigma = I2
Out[159]= {{1, 0}, {0, 1}}

In[160]:= Az[z_] = I2 + A * z
Out[160]= {{1 - 4z/5, 0}, {-z/2, 1 - a z}}

In[161]:= Az[z]
Out[161]= {{1 - 4z/5, 0}, {-z/2, 1 - a z}}

In[162]:= MatrixForm[Az[z]]
Out[162]//MatrixForm=

$$\begin{pmatrix} 1 - \frac{4z}{5} & 0 \\ -\frac{z}{2} & 1 - a z \end{pmatrix}$$


In[163]:= Bz[z_] := I2 + B * z

```

In[164]:= **Bz[z] // MatrixForm**

Out[164]//MatrixForm=

$$\begin{pmatrix} 1 - b z & 0 \\ -\frac{z}{2} & 1 - \frac{3 z}{5} \end{pmatrix}$$

In[165]:= **Cz[z\_] := I2 + C \* z**

In[166]:= **Cz[z] // MatrixForm**

Out[166]//MatrixForm=

$$\begin{pmatrix} 1 - a z & 0 \\ -\frac{z}{2} & 1 - \frac{7 z}{10} \end{pmatrix}$$

In[167]:= **Solve[Det[Az[z]] == 0]**

$$\text{Out[167]} = \left\{ \left\{ z \rightarrow \frac{5}{4} \right\}, \left\{ z \rightarrow \frac{1}{a} \right\} \right\}$$

In[168]:= **Solve[Det[Bz[z]] == 0]**

$$\text{Out[168]} = \left\{ \left\{ z \rightarrow \frac{5}{3} \right\}, \left\{ z \rightarrow \frac{1}{b} \right\} \right\}$$

In[169]:= **Solve[Det[Cz[z]] == 0]**

$$\text{Out[169]} = \left\{ \left\{ z \rightarrow \frac{10}{7} \right\}, \left\{ z \rightarrow \frac{1}{a} \right\} \right\}$$

In[170]:= **u1[z\_] := {{1}}**

In[171]:= **MatrixForm[u1[z]]**

Out[171]//MatrixForm=

$$\begin{pmatrix} 1 \end{pmatrix}$$

In[172]:= **InvAz[z\_] = Inverse[Az[z]]**

$$\text{Out[172]} = \left\{ \left\{ \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \left\{ \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\}$$

In[173]:= **I4 = KroneckerProduct[I2, I2]**

$$\text{Out[173]} = \{ \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\} \}$$

In[174]:= **O2 = ConstantArray[0, {2, 2}]**

$$\text{Out[174]} = \{ \{0, 0\}, \{0, 0\} \}$$

In[175]:= **O4 = I4 - I4**

$$\text{Out[175]} = \{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \}$$

In[176]:= **KroneckerProduct** [u1[z], -InvAz[z].Bz[z]]

$$\text{Out[176]} = \left\{ \left\{ -\frac{(1-az)(1-bz)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, 0 \right\}, \right. \\ \left. \left\{ \frac{\left(1-\frac{4z}{5}\right)z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{z(1-bz)}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, -\frac{\left(1-\frac{4z}{5}\right)\left(1-\frac{3z}{5}\right)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} \right\} \right\}$$

In[177]:= **KroneckerProduct** [u1[z], I2]

$$\text{Out[177]} = \{ \{1, 0\}, \{0, 1\} \}$$

In[178]:= **O2 - O2**

$$\text{Out[178]} = \{ \{0, 0\}, \{0, 0\} \}$$

In[179]:= **Gcal[z\_] = ArrayFlatten** [{**KroneckerProduct** [u1[z], -InvAz[z].Bz[z]],  
**O2**}, {**KroneckerProduct** [u1[z], I2]}]

$$\text{Out[179]} = \left\{ \left\{ -\frac{(1-az)(1-bz)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, 0 \right\}, \right. \\ \left\{ \frac{\left(1-\frac{4z}{5}\right)z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{z(1-bz)}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, -\frac{\left(1-\frac{4z}{5}\right)\left(1-\frac{3z}{5}\right)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} \right\}, \\ \{0, 0\}, \{0, 0\}, \{1, 0\}, \{0, 1\} \}$$

In[180]:= **MatrixForm** [%]

Out[180]//MatrixForm=

$$\begin{pmatrix} -\frac{(1-az)(1-bz)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} & 0 \\ \frac{\left(1-\frac{4z}{5}\right)z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{z(1-bz)}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} & -\frac{\left(1-\frac{4z}{5}\right)\left(1-\frac{3z}{5}\right)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[181]:= **Kcal[z\_] = ArrayFlatten** [{**KroneckerProduct** [u1[z], -InvAz[z].Cz[z]],  
**KroneckerProduct** [u1[z], I2]}, {**O2**}]

$$\text{Out[181]} = \left\{ \left\{ -\frac{(1-az)^2}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, 0 \right\}, \right. \\ \left\{ \frac{\left(1-\frac{4z}{5}\right)z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{z(1-az)}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, -\frac{\left(1-\frac{4z}{5}\right)\left(1-\frac{7z}{10}\right)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} \right\}, \\ \{1, 0\}, \{0, 1\}, \{0, 0\}, \{0, 0\} \}$$

In[182]:= **InvBz[z\_] = Inverse** [Bz[z]]

$$\text{Out[182]} = \left\{ \left\{ \frac{1-\frac{3z}{5}}{1-\frac{3z}{5}-bz+\frac{3bz^2}{5}}, 0 \right\}, \left\{ \frac{z}{2\left(1-\frac{3z}{5}-bz+\frac{3bz^2}{5}\right)}, \frac{1-bz}{1-\frac{3z}{5}-bz+\frac{3bz^2}{5}} \right\} \right\}$$

In[183]:= **sigma[z\_] = Together[Transpose[InvBz[z]].Inverse[Sigma].InvBz[1/z]]**

$$\text{Out[183]} = \left\{ \left\{ \frac{z(60 - 161z + 60z^2)}{4(b-z)(-5+3z)(-3+5z)(-1+bz)}, \frac{25z^2}{2(-5+3z)(-3+5z)(-1+bz)} \right\}, \right. \\ \left. \left\{ -\frac{25z}{2(-b+z)(-5+3z)(-3+5z)}, -\frac{25z}{(-5+3z)(-3+5z)} \right\} \right\}$$

In[184]:= **Gcalinv[z\_] = Gcal[1/z]**

$$\text{Out[184]} = \left\{ \left\{ -\frac{\left(1 - \frac{a}{z}\right)\left(1 - \frac{b}{z}\right)}{1 + \frac{4a}{5z^2} - \frac{4}{5z} - \frac{a}{z}}, 0 \right\}, \right. \\ \left\{ \frac{1 - \frac{4}{5z}}{2\left(1 + \frac{4a}{5z^2} - \frac{4}{5z} - \frac{a}{z}\right)z} - \frac{1 - \frac{b}{z}}{2\left(1 + \frac{4a}{5z^2} - \frac{4}{5z} - \frac{a}{z}\right)z}, -\frac{\left(1 - \frac{4}{5z}\right)\left(1 - \frac{3}{5z}\right)}{1 + \frac{4a}{5z^2} - \frac{4}{5z} - \frac{a}{z}} \right\}, \\ \{0, 0\}, \{0, 0\}, \{1, 0\}, \{0, 1\} \}$$

In[185]:= **Pcal[z\_] = Together[Gcal[z].Sigma.Transpose[Gcalinv[z]]]**

$$\text{Out[185]} = \left\{ \left\{ \frac{25(-b+z)(-1+bz)}{(-5+4z)(-4+5z)}, -\frac{5(-4+5b)(-1+bz)}{2(-a+z)(-5+4z)(-4+5z)}, 0, 0, -\frac{5(-1+bz)}{-5+4z}, 0 \right\}, \right. \\ \left\{ -\frac{5(-b+z)(-4z^2+5bz^2)}{2(-5+4z)(-4+5z)(-1+az)}, \right. \\ \left. \frac{1200 - 5180z + 8376z^2 - 1000bz^2 + 625b^2z^2 - 5180z^3 + 1200z^4}{100(-a+z)(-5+4z)(-4+5z)(-1+az)}, 0, 0, \right. \\ \left. \frac{-4z^2+5bz^2}{2(-5+4z)(-1+az)}, \frac{5-3z}{5(-1+az)} \right\}, \{0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0\}, \\ \left\{ -\frac{5(-b+z)}{-4+5z}, \frac{-4+5b}{2(-a+z)(-4+5z)}, 0, 0, 1, 0 \right\}, \left\{ 0, \frac{3-5z}{5(-a+z)}, 0, 0, 0, 1 \right\} \}$$

In[186]:= **Qcal[z\_] = Together[Kcal[z].Rx.Transpose[Kcal[1/z]]]**

$$\text{Out[186]} = \left\{ \left\{ -\frac{50(a-z)(-1+az)}{(-5+4z)(-4+5z)}, -\frac{5(-4+5a)(-1+az)}{(-a+z)(-5+4z)(-4+5z)}, -\frac{10(-1+az)}{-5+4z}, 0, 0, 0 \right\}, \right. \\ \left\{ \frac{5(a-z)(-4z^2+5az^2)}{(-5+4z)(-4+5z)(-1+az)}, \right. \\ \left. \frac{4200 - 17550z + 27527z^2 - 2000az^2 + 1250a^2z^2 - 17550z^3 + 4200z^4}{100(-a+z)(-5+4z)(-4+5z)(-1+az)}, \right. \\ \left. \frac{-4z^2+5az^2}{(-5+4z)(-1+az)}, -\frac{3(-10+7z)}{10(-1+az)}, 0, 0 \right\}, \\ \left\{ \frac{10(a-z)}{-4+5z}, \frac{-4+5a}{(-a+z)(-4+5z)}, 2, 0, 0, 0 \right\}, \left\{ 0, -\frac{3(-7+10z)}{10(-a+z)}, 0, 3, 0, 0 \right\}, \\ \{0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0\} \}$$

In[187]:= **intergrand1[z\_] = Simplify[Together[KroneckerProduct[Pcal[z], sigma[z]]]]**

$$\text{Out[187]} = \left\{ \left\{ -\frac{25z(60 - 161z + 60z^2)}{4(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, \right. \right. \\ \left. \frac{625z^2(-b+z)}{2(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, \right. \\ \left. \left. \right\} \right\}$$

$$\begin{aligned}
& - \frac{5(-4+5b)z(60-161z+60z^2)}{8(b-z)(-a+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, \\
& \frac{125(-4+5b)z^2}{4(a-z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, 0, 0, 0, 0, \\
& - \frac{5z(60-161z+60z^2)}{4(b-z)(-5+3z)(-5+4z)(-3+5z)}, \frac{125z^2}{150-460z+422z^2-120z^3}, 0, 0\}, \\
& \left\{ - \frac{625z(-1+bz)}{2(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, \right. \\
& \quad \left. - \frac{625z(-b+ z)(-1+bz)}{(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, \right. \\
& \quad \left. - \frac{125(-4+5b)z(-1+bz)}{4(b-z)(-a+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, \right. \\
& \quad \left. \frac{125(-4+5b)z(-1+bz)}{2(-a+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)}, 0, 0, 0, 0, \right. \\
& \quad \left. \frac{125z(-1+bz)}{2(-b+z)(-5+3z)(-5+4z)(-3+5z)}, \frac{125z(-1+bz)}{(-5+3z)(-5+4z)(-3+5z)}, 0, 0\}, \\
& \left\{ \frac{5(-4+5b)z^3(60-161z+60z^2)}{8(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)(-1+bz)}, \right. \\
& \quad \frac{125(-4+5b)(b-z)z^4}{4(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)(-1+bz)}, \\
& \quad \frac{z(60-161z+60z^2)(1200-5180z+(8376-1000b+625b^2)z^2-5180z^3+1200z^4)}{400(b-z)(-a+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)(-1+bz)}, \\
& \quad \frac{z^2(1200-5180z+(8376-1000b+625b^2)z^2-5180z^3+1200z^4)}{8(a-z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)(-1+bz)}, \\
& \quad 0, 0, 0, 0, \frac{(-4+5b)z^3(60-161z+60z^2)}{8(b-z)(-5+3z)(-5+4z)(-3+5z)(-1+az)(-1+bz)}, \\
& \quad \frac{25(-4+5b)z^4}{4(-5+3z)(-5+4z)(-3+5z)(-1+az)(-1+bz)}, \\
& \quad \frac{z(60-161z+60z^2)}{20(-b+z)(-3+5z)(-1+az)(-1+bz)}, - \frac{5z^2}{2(-3+5z)(-1+az)(-1+bz)} \}, \\
& \left\{ \frac{125(-4+5b)z^3}{4(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \right. \\
& \quad \frac{125(-4+5b)z^3(-b+z)}{2(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \\
& \quad \frac{z(1200-5180z+(8376-1000b+625b^2)z^2-5180z^3+1200z^4)}{8(a-z)(-b+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \\
& \quad \frac{z(1200-5180z+(8376-1000b+625b^2)z^2-5180z^3+1200z^4)}{4(a-z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \\
& \quad 0, 0, 0, 0, \frac{25(-4+5b)z^3}{4(b-z)(-5+3z)(-5+4z)(-3+5z)(-1+az)}, \\
& \quad - \frac{25(-4+5b)z^3}{2(-5+3z)(-5+4z)(-3+5z)(-1+az)}, \\
& \quad \left. \frac{5z}{2(-b+z)(-3+5z)(-1+az)}, \frac{5z}{(-3+5z)(-1+az)} \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},
\end{aligned}$$

$$\begin{aligned}
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{5 z (60 - 161 z + 60 z^2)}{4 (-5 + 3 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \right. \\
& - \frac{125 z^2 (-b + z)}{2 (-5 + 3 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
& - \frac{(-4 + 5 b) z (60 - 161 z + 60 z^2)}{8 (b - z) (-a + z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
& \frac{25 (-4 + 5 b) z^2}{4 (-a + z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, 0, 0, 0, 0, \\
& \frac{z (60 - 161 z + 60 z^2)}{4 (b - z) (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, \frac{25 z^2}{2 (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, 0, 0 \}, \\
& \left\{ \frac{125 z}{2 (-60 + 211 z - 230 z^2 + 75 z^3)}, \frac{125 z (-b + z)}{(-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}, \right. \\
& \frac{25 (-4 + 5 b) z}{4 (b - z) (-a + z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}, \\
& \frac{25 (-4 + 5 b) z}{25 (-4 + 5 b) z}, 0, 0, 0, 0, \\
& \frac{2 (a - z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}{25 z}, \frac{25 z}{2 (b - z) (-5 + 3 z) (-3 + 5 z)}, - \frac{25 z}{15 - 34 z + 15 z^2}, 0, 0 \}, \{0, 0, \\
& \frac{z (60 - 161 z + 60 z^2)}{20 (-a + z) (-b + z) (-5 + 3 z) (-1 + b z)}, \frac{5 z^2}{2 (a - z) (-5 + 3 z) (-1 + b z)}, 0, 0, 0, 0, \\
& 0, 0, \frac{25 z^2}{4 (b - z) (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, \frac{25 z^2}{2 (-5 + 3 z) (-3 + 5 z) (-1 + b z)} \}, \\
& \{0, 0, \frac{5 z}{2 (-a + z) (-b + z) (-5 + 3 z)}, \frac{5 z}{(-a + z) (-5 + 3 z)}, 0, 0, 0, \\
& 0, 0, 0, \frac{25 z}{2 (b - z) (-5 + 3 z) (-3 + 5 z)}, - \frac{25 z}{15 - 34 z + 15 z^2} \} \}
\end{aligned}$$

In[188]:= `intergrand2[z_] = Simplify[Together[KroneckerProduct[Qcal[z], sigma[z]]]]`

$$\begin{aligned}
\text{Out[188]} = & \left\{ \left\{ \frac{25 (a - z) z (-1 + a z) (60 - 161 z + 60 z^2)}{2 (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \right. \right. \\
& - \frac{625 (a - z) z^2 (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
& - \frac{5 (-4 + 5 a) z (-1 + a z) (60 - 161 z + 60 z^2)}{4 (a - z) (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
& - \frac{125 (-4 + 5 a) z^2 (-1 + a z)}{2 (a - z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
& - \frac{5 z (-1 + a z) (60 - 161 z + 60 z^2)}{2 (b - z) (-5 + 3 z) (-5 + 4 z) (-3 + 5 z) (-1 + b z)}, \\
& - \frac{125 z^2 (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-3 + 5 z) (-1 + b z)}, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ \frac{625 (a - z) z (-1 + a z)}{(-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \right. \\
& \frac{1250 (a - z) z (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)},
\end{aligned}$$

$$\begin{aligned}
& - \frac{125 (-4 + 5a) z (-1 + az)}{2 (a - z) (-b + z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z)}, \\
& - \frac{125 (-4 + 5a) z (-1 + az)}{(a - z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z)}, \\
& \frac{125 z (-1 + az)}{(-b + z) (-5 + 3z) (-5 + 4z) (-3 + 5z)}, \\
& \frac{250 z (-1 + az)}{(-5 + 3z) (-5 + 4z) (-3 + 5z)}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{5 (-4 + 5a) (a - z) z^3 (60 - 161z + 60z^2)}{4 (b - z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az) (-1 + bz)}, \right. \\
& \frac{125 (-4 + 5a) (a - z) z^4}{2 (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az) (-1 + bz)}, \\
& \left. (z (60 - 161z + 60z^2) \right. \\
& \left. (4200 - 17550z + (27527 - 2000a + 1250a^2) z^2 - 17550z^3 + 4200z^4) \right) / \\
& (400 (b - z) (-a + z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az) (-1 + bz)), \\
& \frac{z^2 (4200 - 17550z + (27527 - 2000a + 1250a^2) z^2 - 17550z^3 + 4200z^4)}{8 (a - z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az) (-1 + bz)}, \\
& \frac{(-4 + 5a) z^3 (60 - 161z + 60z^2)}{4 (b - z) (-5 + 3z) (-5 + 4z) (-3 + 5z) (-1 + az) (-1 + bz)}, \\
& \frac{25 (-4 + 5a) z^4}{2 (-5 + 3z) (-5 + 4z) (-3 + 5z) (-1 + az) (-1 + bz)}, \\
& \frac{3 z (-10 + 7z) (60 - 161z + 60z^2)}{40 (b - z) (-5 + 3z) (-3 + 5z) (-1 + az) (-1 + bz)}, \\
& \frac{15 z^2 (-10 + 7z)}{4 (-5 + 3z) (-3 + 5z) (-1 + az) (-1 + bz)}, 0, 0, 0, 0\}, \\
& \left\{ \frac{125 (-4 + 5a) (a - z) z^3}{2 (b - z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \right. \\
& \frac{125 (-4 + 5a) (a - z) z^3}{(-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \\
& \frac{z (4200 - 17550z + (27527 - 2000a + 1250a^2) z^2 - 17550z^3 + 4200z^4)}{8 (a - z) (-b + z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \\
& \frac{z (4200 - 17550z + (27527 - 2000a + 1250a^2) z^2 - 17550z^3 + 4200z^4)}{4 (a - z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \\
& \frac{25 (-4 + 5a) z^3}{2 (b - z) (-5 + 3z) (-5 + 4z) (-3 + 5z) (-1 + az)}, \\
& \frac{25 (-4 + 5a) z^3}{(-5 + 3z) (-5 + 4z) (-3 + 5z) (-1 + az)}, \\
& \frac{15 z (-10 + 7z)}{15 z (-10 + 7z)}, \frac{15 z (-10 + 7z)}{2 (-5 + 3z) (-3 + 5z) (-1 + az)}, \\
& 0, 0, 0, 0\}, \left\{ \frac{5 (a - z) z (60 - 161z + 60z^2)}{2 (b - z) (-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)}, \right. \\
& \frac{125 (a - z) z^2}{(-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)}, \\
& \frac{(-4 + 5a) z (60 - 161z + 60z^2)}{4 (a - z) (-b + z) (-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)},
\end{aligned}$$

$$\begin{aligned}
& - \frac{25 (-4 + 5 a) z^2}{2 (a - z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
& \frac{z (60 - 161 z + 60 z^2)}{2 (b - z) (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, \\
& \frac{25 z^2}{(-5 + 3 z) (-3 + 5 z) (-1 + b z)}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{125 (a - z) z}{(b - z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}, - \frac{250 (a - z) z}{(-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}, \right. \\
& \frac{2 (a - z) (-b + z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}{25 (-4 + 5 a) z}, \\
& \frac{(a - z) (-5 + 3 z) (-4 + 5 z) (-3 + 5 z)}{25 z}, \\
& \left. \frac{(b - z) (-5 + 3 z) (-3 + 5 z)}{15 - 34 z + 15 z^2}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{ 0, 0, - \frac{3 z (-7 + 10 z) (60 - 161 z + 60 z^2)}{40 (b - z) (-a + z) (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, \right. \\
& \frac{15 z^2 (-7 + 10 z)}{4 (a - z) (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, 0, \\
& 0, \frac{3 z (60 - 161 z + 60 z^2)}{4 (b - z) (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, \\
& \frac{75 z^2}{2 (-5 + 3 z) (-3 + 5 z) (-1 + b z)}, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, \frac{15 z (-7 + 10 z)}{4 (-a + z) (-b + z) (-5 + 3 z) (-3 + 5 z)}, \frac{15 z (-7 + 10 z)}{2 (-a + z) (-5 + 3 z) (-3 + 5 z)}, \right. \\
& 0, 0, \frac{75 z}{2 (b - z) (-5 + 3 z) (-3 + 5 z)}, - \frac{75 z}{15 - 34 z + 15 z^2}, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}
\end{aligned}$$

In[189]:= intergrand1expim[t\_] = intergrand1[Exp[I t]]

$$\begin{aligned}
\text{Out[189]} = & \left\{ \left\{ - \frac{25 e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{4 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \right. \right. \\
& \frac{625 e^{2 i t} (-b + e^{i t})}{2 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& - \frac{5 (-4 + 5 b) e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8 (b - e^{i t}) (-a + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& \frac{125 (-4 + 5 b) e^{2 i t}}{4 (a - e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& 0, 0, 0, 0, - \frac{5 e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{4 (b - e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t})}, \\
& \left. \frac{125 e^{2 i t}}{150 - 460 e^{i t} + 422 e^{2 i t} - 120 e^{3 i t}}, 0, 0\right\}, \\
& \left\{ - \frac{625 e^{i t} (-1 + b e^{i t})}{2 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \right.
\end{aligned}$$



$$\begin{aligned}
& - \frac{625 e^{i t} (-b + e^{i t}) (-1 + b e^{i t})}{(-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& - \frac{125 (-4 + 5 b) e^{i t} (-1 + b e^{i t})}{4 (b - e^{i t}) (-a + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& \frac{125 (-4 + 5 b) e^{i t} (-1 + b e^{i t})}{2 (-a + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& 0, 0, 0, 0, \frac{125 e^{i t} (-1 + b e^{i t})}{2 (-b + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t})}, \\
& \frac{125 e^{i t} (-1 + b e^{i t})}{(-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t})}, 0, 0 \}, \\
& \left\{ \frac{5 (-4 + 5 b) e^{3 i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t})}, \right. \\
& \frac{125 (-4 + 5 b) e^{4 i t} (b - e^{i t})}{4 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t})}, \\
& \frac{e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{(1200 - 5180 e^{i t} + (8376 - 1000 b + 625 b^2) e^{2 i t} - 5180 e^{3 i t} + 1200 e^{4 i t})} \Big/ \\
& \frac{(400 (b - e^{i t}) (-a + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t})}{(-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t}))}, \\
& - \left( \frac{e^{2 i t} (1200 - 5180 e^{i t} + (8376 - 1000 b + 625 b^2) e^{2 i t} - 5180 e^{3 i t} + 1200 e^{4 i t})}{(8 (a - e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t})}{(-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t}))} \right) \Big/ \\
& \frac{(-4 + 5 b) e^{3 i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8 (b - e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t})}, \\
& \frac{25 (-4 + 5 b) e^{4 i t}}{4 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t})}, \\
& \frac{e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{20 (-b + e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t})}, \\
& \frac{5 e^{2 i t}}{2 (-3 + 5 e^{i t}) (-1 + a e^{i t}) (-1 + b e^{i t})} \Big\}, \\
& \left\{ \frac{125 (-4 + 5 b) e^{3 i t}}{4 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \right. \\
& \frac{125 (-4 + 5 b) e^{3 i t} (-b + e^{i t})}{2 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \\
& \frac{e^{i t} (1200 - 5180 e^{i t} + (8376 - 1000 b + 625 b^2) e^{2 i t} - 5180 e^{3 i t} + 1200 e^{4 i t})}{8 (a - e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \\
& \frac{e^{i t} (1200 - 5180 e^{i t} + (8376 - 1000 b + 625 b^2) e^{2 i t} - 5180 e^{3 i t} + 1200 e^{4 i t})}{4 (a - e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \\
& \frac{25 (-4 + 5 b) e^{3 i t}}{0, 0, 0, 0, \frac{25 (-4 + 5 b) e^{3 i t}}{4 (b - e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \\
& \frac{25 (-4 + 5 b) e^{3 i t}}{2 (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \\
& \frac{5 e^{i t}}{2 (-b + e^{i t}) (-3 + 5 e^{i t}) (-1 + a e^{i t})}, \frac{5 e^{i t}}{(-3 + 5 e^{i t}) (-1 + a e^{i t})} \Big\},
\end{aligned}$$

$$\begin{aligned}
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{5 e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{4 (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \right. \\
& \quad - \frac{125 e^{2 i t} (-b + e^{i t})}{2 (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \\
& \quad \frac{(-4 + 5 b) e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8 (b - e^{i t}) (-a + e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \\
& \quad \frac{25 (-4 + 5 b) e^{2 i t}}{4 (-a + e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \\
& \quad \frac{e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{0, 0, 0, 0, \frac{4 (b - e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}{25 e^{2 i t}}}, \\
& \quad \left. \frac{25 e^{2 i t}}{2 (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, 0, 0\right\}, \\
& \left\{ \frac{125 e^{i t}}{2 (-60 + 211 e^{i t} - 230 e^{2 i t} + 75 e^{3 i t})}, \frac{125 e^{i t} (-b + e^{i t})}{(-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \right. \\
& \quad \frac{25 (-4 + 5 b) e^{i t}}{4 (b - e^{i t}) (-a + e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& \quad \frac{25 (-4 + 5 b) e^{i t}}{2 (a - e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, 0, 0, 0, 0, \\
& \quad \frac{25 e^{i t}}{2 (b - e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, - \frac{25 e^{i t}}{15 - 34 e^{i t} + 15 e^{2 i t}}, 0, 0\right\}, \\
& \left\{ 0, 0, \frac{e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{20 (-a + e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t}) (-1 + b e^{i t})}, \right. \\
& \quad \frac{5 e^{2 i t}}{2 (a - e^{i t}) (-5 + 3 e^{i t}) (-1 + b e^{i t})}, 0, 0, 0, 0, 0, \\
& \quad \frac{e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{0, \frac{4 (b - e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}{25 e^{2 i t}}}, \\
& \quad \left. \frac{25 e^{2 i t}}{2 (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})} \right\}, \\
& \left\{ 0, 0, \frac{5 e^{i t}}{2 (-a + e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t})}, \frac{5 e^{i t}}{(-a + e^{i t}) (-5 + 3 e^{i t})}, 0, 0, \right. \\
& \quad \left. 0, 0, 0, 0, \frac{25 e^{i t}}{2 (b - e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, - \frac{25 e^{i t}}{15 - 34 e^{i t} + 15 e^{2 i t}} \right\}
\end{aligned}$$

In[190]:= intergrand2expim[t\_] = intergrand2[Exp[I t]]

$$\begin{aligned}
\text{Out[190]} = & \left\{ \frac{25 e^{i t} (a - e^{i t}) (-1 + a e^{i t}) (60 - 161 e^{i t} + 60 e^{2 i t})}{2 (-b + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \right. \\
& - \frac{625 e^{2 i t} (a - e^{i t}) (-1 + a e^{i t})}{(-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \\
& - \frac{5 (-4 + 5 a) e^{i t} (-1 + a e^{i t}) (60 - 161 e^{i t} + 60 e^{2 i t})}{4 (a - e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t}) (-5 + 4 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}
\end{aligned}$$

$$\begin{aligned}
& \frac{125 (-4+5a) e^{2it} (-1+a e^{it})}{2 (a-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+b e^{it})}, \\
& - \frac{5 e^{it} (-1+a e^{it}) (60-161 e^{it}+60 e^{2it})}{2 (b-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-3+5 e^{it}) (-1+b e^{it})}, \\
& - \frac{125 e^{2it} (-1+a e^{it})}{(-5+3 e^{it}) (-5+4 e^{it}) (-3+5 e^{it}) (-1+b e^{it})}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{625 e^{it} (a-e^{it}) (-1+a e^{it})}{(-b+e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it})}, \right. \\
& \frac{1250 e^{it} (a-e^{it}) (-1+a e^{it})}{(-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it})}, \\
& - \frac{125 (-4+5a) e^{it} (-1+a e^{it})}{2 (a-e^{it}) (-b+e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it})}, \\
& - \frac{125 (-4+5a) e^{it} (-1+a e^{it})}{(a-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it})}, \\
& \frac{125 e^{it} (-1+a e^{it})}{(-b+e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-3+5 e^{it})}, \\
& \left. \frac{250 e^{it} (-1+a e^{it})}{(-5+3 e^{it}) (-5+4 e^{it}) (-3+5 e^{it})}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{ \frac{5 (-4+5a) e^{3it} (a-e^{it}) (60-161 e^{it}+60 e^{2it})}{4 (b-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}, \right. \\
& \frac{125 (-4+5a) e^{4it} (a-e^{it})}{2 (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}, \\
& \left. \frac{e^{it} (60-161 e^{it}+60 e^{2it})}{(4200-17550 e^{it}+(27527-2000a+1250a^2) e^{2it}-17550 e^{3it}+4200 e^{4it})} \right) / \\
& \frac{(400 (b-e^{it}) (-a+e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it})}{(-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}), \\
& - \left( \frac{e^{2it} (4200-17550 e^{it}+(27527-2000a+1250a^2) e^{2it}-17550 e^{3it}+4200 e^{4it})}{(8 (a-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it}))}, \right. \\
& \left. \frac{(-4+5a) e^{3it} (60-161 e^{it}+60 e^{2it})}{4 (b-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}, \right. \\
& \frac{25 (-4+5a) e^{4it}}{2 (-5+3 e^{it}) (-5+4 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}, \\
& \frac{3 e^{it} (-10+7 e^{it}) (60-161 e^{it}+60 e^{2it})}{40 (b-e^{it}) (-5+3 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}, \\
& - \frac{15 e^{2it} (-10+7 e^{it})}{4 (-5+3 e^{it}) (-3+5 e^{it}) (-1+a e^{it}) (-1+b e^{it})}, \\
& 0, 0, 0, 0\}, \\
& \left\{ \frac{125 (-4+5a) e^{3it} (a-e^{it})}{2 (b-e^{it}) (-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+a e^{it})}, \right. \\
& \frac{125 (-4+5a) e^{3it} (a-e^{it})}{(-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+a e^{it})}, \\
& \left. - \frac{125 (-4+5a) e^{3it} (a-e^{it})}{(-5+3 e^{it}) (-5+4 e^{it}) (-4+5 e^{it}) (-3+5 e^{it}) (-1+a e^{it})}, \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{e^{it} (4200 - 17550 e^{it} + (27527 - 2000a + 1250a^2) e^{2it} - 17550 e^{3it} + 4200 e^{4it})}{8 (a - e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \\
& \frac{e^{it} (4200 - 17550 e^{it} + (27527 - 2000a + 1250a^2) e^{2it} - 17550 e^{3it} + 4200 e^{4it})}{4 (a - e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \\
& \frac{25 (-4 + 5a) e^{3it}}{2 (b - e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \\
& - \frac{25 (-4 + 5a) e^{3it}}{25 (-4 + 5a) e^{3it}}, \\
& - \frac{(-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}{15 e^{it} (-10 + 7 e^{it})}, \\
& \frac{4 (-b + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}{15 e^{it} (-10 + 7 e^{it})}, \\
& \frac{2 (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}{5 e^{it} (a - e^{it}) (60 - 161 e^{it} + 60 e^{2it})}, 0, 0, 0, 0\}, \\
& \left\{ \frac{2 (b - e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{125 e^{2it} (a - e^{it})}, \right. \\
& \frac{(-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{(-4 + 5a) e^{it} (60 - 161 e^{it} + 60 e^{2it})}, \\
& \frac{4 (a - e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{25 (-4 + 5a) e^{2it}}, \\
& - \frac{2 (a - e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{e^{it} (60 - 161 e^{it} + 60 e^{2it})}, \\
& \frac{2 (b - e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{25 e^{2it}}, \\
& \frac{(-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{125 e^{it} (a - e^{it})}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{(b - e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}{250 e^{it} (a - e^{it})}, \right. \\
& - \frac{(-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}{25 (-4 + 5a) e^{it}}, \\
& \frac{2 (a - e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}{25 (-4 + 5a) e^{it}}, \\
& \frac{(a - e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}{25 e^{it}}, \\
& \frac{(b - e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it})}{15 - 34 e^{it} + 15 e^{2it}}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, - \frac{3 e^{it} (-7 + 10 e^{it}) (60 - 161 e^{it} + 60 e^{2it})}{40 (b - e^{it}) (-a + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \right. \\
& \frac{15 e^{2it} (-7 + 10 e^{it})}{4 (a - e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, 0, \\
& 0, \frac{3 e^{it} (60 - 161 e^{it} + 60 e^{2it})}{4 (b - e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \\
& \frac{75 e^{2it}}{2 (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, \frac{15 e^{it} (-7 + 10 e^{it})}{4 (-a + e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it})}, \right.
\end{aligned}$$

$$\frac{15 e^{i t} (-7 + 10 e^{i t})}{2 (-a + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, 0, 0,$$

$$\frac{75 e^{i t}}{2 (b - e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, -\frac{75 e^{i t}}{15 - 34 e^{i t} + 15 e^{2 i t}}, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}\}$$

In[191]:= Assuming[a > -1 && a < 1 && b > -1 && b < 1,

Gmatrix1 = Integrate[(1/(2 Pi)) intergrand1expim[t], {t, 0, 2 Pi}]]

$$\text{Out[191]} = \left\{ \left\{ \frac{43925}{7488}, -\frac{625 (-35 + 37 b)}{3744}, \right. \right.$$

$$\frac{5 (-4 + 5 b) (-820925 + a (462180 - 253008 b) + 462180 b)}{14976 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)},$$

$$\frac{125 (-875 + 444 a) (-4 + 5 b)}{7488 (-5 + 3 a) (-5 + 4 a)}, 0, 0, 0, 0, \frac{5 (-7285 + 3396 b)}{832 (-5 + 3 b) (-5 + 4 b)}, -\frac{375}{416}, 0, 0\},$$

$$\left\{ -\frac{625 (-35 + 37 b)}{3744}, \frac{625 (37 + b (-70 + 37 b))}{1872}, \right.$$

$$\frac{125 (-4 + 5 b) (-16835 + (28897 - 10500 b) b + 12 a (781 + 37 b (-35 + 12 b)))}{7488 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)},$$

$$\frac{125 (-4 + 5 b) (-781 + a (420 - 444 b) + 875 b)}{3744 (-5 + 3 a) (-5 + 4 a)}, 0, 0, 0,$$

$$0, \frac{25}{416} \left( -15 + \frac{144}{5 - 4 b} + \frac{208}{-5 + 3 b} \right), \frac{125}{208} (-5 + 3 b), 0, 0\},$$

$$\left\{ \frac{5 (-4 + 5 b) (-820925 + a (462180 - 253008 b) + 462180 b)}{14976 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)}, \right.$$

$$\frac{125 (-4 + 5 b) (-16835 + (28897 - 10500 b) b + 12 a (781 + 37 b (-35 + 12 b)))}{7488 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)},$$

$$(25 (-47709200 + 102369880 b - 49400941 b^2 - 20010655 b^3 + 13177500 b^4) +$$

$$a^2 (-937070400 + 2724188560 b - 2978197292 b^2 + 1318277075 b^3 + 25503125 b^4 -$$

$$116512500 b^5) + 12 a^3 (22464000 - 79158800 b + 90688600 b^2 - 17951341 b^3 -$$

$$30793375 b^4 + 13177500 b^5) - 5 a (-292224400 + 642897160 b -$$

$$501750837 b^2 + 287393120 b^3 - 205349375 b^4 + 65887500 b^5) \Big) /$$

$$(748800 (-5 + 3 a) (-5 + 4 a) (-1 + a^2) (-5 + 3 b) (-5 + 4 b) (-1 + a b) (-1 + b^2)),$$

$$(-625 (4 - 5 b)^2 (-875 + 444 b) +$$

$$25 a (379600 + b (-1180440 + b (1661481 + 125 b (-9019 + 2220 b)))) -$$

$$12 a^3 (-187200 + b (-87920 + b (962744 + 125 b (-7927 + 2220 b)))) +$$

$$5 a^2 (-2150400 + b (3548560 + b (-539692 + 125 b (-13561 + 5460 b)))) \Big) /$$

$$(14976 (-5 + 3 a) (-5 + 4 a) (-1 + a^2) (-5 + 3 b) (-5 + 4 b) (-1 + a b)), 0, 0, 0, 0,$$

$$(-4 + 5 b) (-5625 + b (-4875 + 5625 a + 3 a (5785 - 3396 b) b + 20 b (-1165 + 624 b))),$$

$$\frac{1664 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b) (-1 + b^2)}{675 (-4 + 5 b)},$$

$$\frac{832 (-5 + 3 a) (-5 + 3 b)}{-625 + 60 (10 - 3 b) b + 3 a (100 + b (-120 + 61 b))},$$

$$\frac{20 (-5 + 3 a) (-5 + 3 b) (-1 + a b) (-1 + b^2)}{20 (-5 + 3 a) (-5 + 3 b) (-1 + a b) (-1 + b^2)},$$

$$\begin{aligned}
& - \frac{15}{2(-5+3a)(-5+3b)} \Bigg\}, \\
& \left\{ \frac{125(-875+444a)(-4+5b)}{7488(-5+3a)(-5+4a)}, \frac{125(-4+5b)(-781+a(420-444b)+875b)}{3744(-5+3a)(-5+4a)}, \right. \\
& (-625(4-5b)^2(-875+444b) + \\
& 25a(379600+b(-1180440+b(1661481+125b(-9019+2220b)))) - \\
& 12a^3(-187200+b(-87920+b(962744+125b(-7927+2220b)))) + \\
& 5a^2(-2150400+b(3548560+b(-539692+125b(-13561+5460b)))) \Bigg) / \\
& (14976(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)), \\
& (-455a(-44+25b)(4+25b)+12a^2(7312+4625b(-8+5b)) - \\
& 25(22288+4625b(-8+5b))) / (7488(-5+3a)(-5+4a)(-1+a^2)), \\
& 0, 0, 0, 0, \frac{25(-4+5b)(375+b(100+3a(-125+36b)))}{832(-5+3a)(-5+3b)(-5+4b)(-1+ab)}, \\
& - \frac{225(-4+5b)}{416(-5+3a)}, - \frac{5a}{2(-5+3a)(-1+ab)}, \frac{5}{-5+3a} \Bigg\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{5(-7285+3396b)}{832(-5+3b)(-5+4b)}, \frac{25}{416} \left( -15 + \frac{144}{5-4b} + \frac{208}{-5+3b} \right), \right. \\
& (-4+5b)(-5625+b(-4875+5625a+3a(5785-3396b)b+20b(-1165+624b))) \\
& \frac{1664(-5+3a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)}{25(-4+5b)(375+b(100+3a(-125+36b)))}, \\
& \frac{832(-5+3a)(-5+3b)(-5+4b)(-1+ab)}{445-117b}, 0, \\
& 0, 0, 0, \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b}, 0, 0 \Bigg\}, \\
& \left\{ -\frac{375}{416}, \frac{125}{208}(-5+3b), \frac{675(-4+5b)}{832(-5+3a)(-5+3b)}, \right. \\
& -\frac{225(-4+5b)}{416(-5+3a)}, 0, 0, 0, 0, \frac{75}{160-96b}, \frac{25}{16}, 0, 0 \Bigg\}, \\
& \left\{ 0, 0, \frac{-625+60(10-3b)b+3a(100+b(-120+61b))}{20(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}, \right. \\
& -\frac{5a}{2(-5+3a)(-1+ab)}, 0, 0, 0, 0, 0, 0, \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b} \Bigg\}, \\
& \left\{ 0, 0, -\frac{15}{2(-5+3a)(-5+3b)}, \frac{5}{-5+3a}, 0, 0, 0, 0, 0, 0, \frac{75}{160-96b}, \frac{25}{16} \right\} \Bigg\}
\end{aligned}$$

In[192]:= Assuming[a > -1 && a < 1 && b > -1 && b < 1,

Gmatrix2 = Integrate[(1/(2 Pi)) intergrand2expim[t], {t, 0, 2 Pi}]]

Out[192]= 
$$\left\{ \left\{ \frac{(25(-5(8785+a(-15406+8785a))-13(1195+a(-3514+1195a))b+12(1757+a(-3830+1757a))b^2)}{(3744(-5+3b)(-5+4b)(-1+b^2))} \right\}, \right.$$

$$\frac{625(875-444b+a(-1706+a(875-444b)+840b))}{1872(-5+3b)(-5+4b)},$$

$$\left. \left( 5(-4+5a)(-820925+12a^3b(43925+(15535-21084b)b)+5b(-100139+ \right. \right.$$

$$\begin{aligned}
& \frac{12b(4165 + 1872b) + a^2(12 + 35b)(-43925 + b(-15535 + 21084b)) + a(1425055 + b(1666042 - b(237245 + 551292b)))}{(7488(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)(-1 + ab)(-1 + b^2))}, \\
& \frac{125 \left( -\frac{135(-5+3a)(-4+5a)}{(-3+5a)(-5+3b)} + \frac{320(-5+4a)}{-5+4b} - \frac{1872a(-1+a^2)}{(-5+3a)(-5+4a)(-3+5a)(-1+ab)} \right)}{3744}, \\
& \frac{5(7285 - 3b(507 + 500b) + a(-1875 + b(-5785 + 3396b)))}{416(-5 + 3b)(-5 + 4b)(-1 + b^2)}, \\
& -\frac{375(-5 + 3a)}{208(-5 + 3b)}, \\
& \{0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{625(875 - 444b + a(-1706 + a(875 - 444b) + 840b))}{1872(-5 + 3b)(-5 + 4b)}, \right. \\
& \frac{625}{936}(37 + a(-70 + 37a)), \\
& \left. \frac{25(-4 + 5a) \left( \frac{243(3-5a)}{(-5+3a)(-5+3b)} + \frac{1024(-4+5a)}{(-5+4a)(-5+4b)} \right)}{3744}, \right. \\
& \frac{125(-4 + 5a)(-781 + 37(35 - 12a)a)}{1872(-5 + 3a)(-5 + 4a)}, \\
& \left. -\frac{125(139 - 60b + a(-125 + 36b))}{208(-5 + 3b)(-5 + 4b)}, \right. \\
& \left. \frac{125}{104}(-5 + 3a), 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ (5(-4 + 5a)(-820925 + 12a^3b(43925 + (15535 - 21084b)b) + 5b(-100139 + \right. \\
& 12b(4165 + 1872b)) + a^2(12 + 35b)(-43925 + b(-15535 + 21084b)) + a(1425055 + b(1666042 - b(237245 + 551292b)))) / \\
& (7488(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)(-1 + ab)(-1 + b^2))), \\
& \frac{25(-4 + 5a) \left( \frac{243(3-5a)}{(-5+3a)(-5+3b)} + \frac{1024(-4+5a)}{(-5+4a)(-5+4b)} \right)}{3744}, \\
& (15000a^5b(-43925 - 15535b + 21084b^2) - \\
& 250a^4(-2635500 - 8213975b - 204025b^2 + 2956164b^3) + \\
& 25(-133077775 + 205853635b - 142252332b^2 + 37739520b^3) - \\
& 2a^3(298090375 + 1945968250b - 1746928805b^2 + 336089892b^3) - \\
& 5a(-1468518175 + 2342889320b - 1905168349b^2 + 583563300b^3) + \\
& a^2(-4152214550 + 9094972995b - 9213408409b^2 + 3033829860b^3)) / \\
& (748800(-5 + 3a)(-5 + 4a)(-1 + a^2)(-5 + 3b)(-5 + 4b)(-1 + ab)(-1 + b^2))), \\
& -((25(-649105 + a(754109 + 2a(526835 + a(-877873 + 262500a)))) + 5(1572420 + \\
& a(4089089 + 5a(-2697125 + 2a(554737 - 3125a(-173 + 84a))))))b + \\
& 12a(-1310375 + a(2386315 + 2a(-123131 + 125a(-5827 + 2220a))))b^2) / \\
& (14976(-5 + 3a)(-5 + 4a)(-1 + a^2)(-5 + 3b)(-5 + 4b)(-1 + ab))), \\
& (-4 + 5a)(-5625 + b(-4875 + 5625a + 3a(5785 - 3396b)b + 20b(-1165 + 624b))) \\
& \frac{832(-5 + 3a)(-5 + 3b)(-5 + 4b)(-1 + ab)(-1 + b^2)}{675(-4 + 5a)}, \\
& \frac{416(-5 + 3a)(-5 + 3b)},
\end{aligned}$$

$$\begin{aligned}
& \frac{3 (25 (-785 + 384 a) - 4285 (-5 + 3 a) b + 3 (-2240 + 2069 a) b^2)}{640 (-5 + 3 a) (-5 + 3 b) (-1 + a b) (-1 + b^2)}, \\
& - \frac{1305}{64 (-5 + 3 a) (-5 + 3 b)}, 0, 0, 0, 0\}, \\
& \left\{ \frac{125 \left( -\frac{135 (-5+3 a) (-4+5 a)}{(-3+5 a) (-5+3 b)} + \frac{320 (-5+4 a)}{-5+4 b} - \frac{1872 a (-1+a^2)}{(-5+3 a) (-5+4 a) (-3+5 a) (-1+a b)} \right)}{3744}, \right. \\
& \frac{125 (-4 + 5 a) (-781 + 37 (35 - 12 a) a)}{1872 (-5 + 3 a) (-5 + 4 a)}, \\
& - \left( (25 (-649105 + a (754109 + 2 a (526835 + a (-877873 + 262500 a)))) + 5 (1572420 + \right. \\
& \quad a (4089089 + 5 a (-2697125 + 2 a (554737 - 3125 a (-173 + 84 a)))) b + \\
& \quad 12 a (-1310375 + a (2386315 + 2 a (-123131 + 125 a (-5827 + 2220 a)))) b^2 \Big) / \\
& \quad \left. (14976 (-5 + 3 a) (-5 + 4 a) (-1 + a^2) (-5 + 3 b) (-5 + 4 b) (-1 + a b) \right), \\
& \frac{-1310375 + a (2386315 + 2 a (-123131 + 125 a (-5827 + 2220 a)))}{7488 (-5 + 3 a) (-5 + 4 a) (-1 + a^2)}, \\
& \frac{25 (-4 + 5 a) (375 + b (100 + 3 a (-125 + 36 b)))}{416 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b)}, \\
& \frac{225 (-4 + 5 a)}{208 (-5 + 3 a)}, \\
& - \frac{15 (25 + a (-160 + 87 b))}{64 (-5 + 3 a) (-5 + 3 b) (-1 + a b)}, \\
& \frac{435}{32 (-5 + 3 a)}, 0, 0, 0, 0\}, \\
& \left\{ \frac{5 (7285 - 3 b (507 + 500 b) + a (-1875 + b (-5785 + 3396 b)))}{416 (-5 + 3 b) (-5 + 4 b) (-1 + b^2)}, \right. \\
& \frac{125 (139 - 60 b + a (-125 + 36 b))}{208 (-5 + 3 b) (-5 + 4 b)}, \\
& \frac{(-4 + 5 a) (-5625 + b (-4875 + 5625 a + 3 a (5785 - 3396 b) b + 20 b (-1165 + 624 b)))}{832 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b) (-1 + b^2)}, \\
& \frac{25 (-4 + 5 a) (375 + b (100 + 3 a (-125 + 36 b)))}{416 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b)}, \\
& \frac{445 - 117 b}{75}, \frac{75}{80 - 48 b}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ -\frac{375 (-5 + 3 a)}{208 (-5 + 3 b)}, \frac{125}{104} (-5 + 3 a), \frac{675 (-4 + 5 a)}{416 (-5 + 3 a) (-5 + 3 b)}, \right. \\
& \frac{225 (-4 + 5 a)}{208 (-5 + 3 a)}, \frac{75}{80 - 48 b}, \frac{25}{8}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, \frac{3 (25 (-785 + 384 a) - 4285 (-5 + 3 a) b + 3 (-2240 + 2069 a) b^2)}{640 (-5 + 3 a) (-5 + 3 b) (-1 + a b) (-1 + b^2)}, \right. \\
& \frac{15 (25 + a (-160 + 87 b))}{64 (-5 + 3 a) (-5 + 3 b) (-1 + a b)}, 0, 0, \\
& \frac{3 (445 - 117 b)}{64 (-5 + 3 b) (-1 + b^2)}, \frac{225}{160 - 96 b}, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, -\frac{1305}{64 (-5 + 3 a) (-5 + 3 b)}, \frac{435}{32 (-5 + 3 a)}, \right.
\end{aligned}$$



$$\begin{aligned} &0, 0, \frac{225}{160 - 96b}, \frac{75}{16}, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \end{aligned}$$

In[193]:= Gmatrix12 = Gmatrix1 + Gmatrix2

$$\begin{aligned} \text{Out[193]} = & \left\{ \left\{ \frac{43925}{7488} + \left( 25 \left( -5 \left( 8785 + a \left( -15406 + 8785a \right) \right) - 13 \left( 1195 + a \left( -3514 + 1195a \right) \right) b + \right. \right. \right. \\ & \left. \left. \left. 12 \left( 1757 + a \left( -3830 + 1757a \right) \right) b^2 \right) \right) / \left( 3744 \left( -5 + 3b \right) \left( -5 + 4b \right) \left( -1 + b^2 \right) \right), \right. \\ & \left. - \frac{625 \left( -35 + 37b \right)}{3744} + \frac{625 \left( 875 - 444b + a \left( -1706 + a \left( 875 - 444b \right) + 840b \right) \right)}{1872 \left( -5 + 3b \right) \left( -5 + 4b \right)}, \right. \\ & \left. \frac{5 \left( -4 + 5b \right) \left( -820925 + a \left( 462180 - 253008b \right) + 462180b \right)}{14976 \left( -5 + 3a \right) \left( -5 + 4a \right) \left( -5 + 3b \right) \left( -5 + 4b \right)} + \right. \\ & \left. \left( 5 \left( -4 + 5a \right) \left( -820925 + 12a^3b \left( 43925 + \left( 15535 - 21084b \right) b \right) + 5b \left( -100139 + \right. \right. \right. \right. \\ & \left. \left. \left. 12b \left( 4165 + 1872b \right) \right) + a^2 \left( 12 + 35b \right) \left( -43925 + b \left( -15535 + 21084b \right) \right) + \right. \right. \\ & \left. \left. \left. a \left( 1425055 + b \left( 1666042 - b \left( 237245 + 551292b \right) \right) \right) \right) \right) / \right. \\ & \left. \left( 7488 \left( -5 + 3a \right) \left( -5 + 4a \right) \left( -5 + 3b \right) \left( -5 + 4b \right) \left( -1 + ab \right) \left( -1 + b^2 \right) \right), \right. \\ & \left. \frac{125 \left( -875 + 444a \right) \left( -4 + 5b \right)}{7488 \left( -5 + 3a \right) \left( -5 + 4a \right)} + \right. \\ & \left. \frac{125 \left( -\frac{135 \left( -5 + 3a \right) \left( -4 + 5a \right)}{\left( -3 + 5a \right) \left( -5 + 3b \right)} + \frac{320 \left( -5 + 4a \right)}{-5 + 4b} - \frac{1872a \left( -1 + a^2 \right)}{\left( -5 + 3a \right) \left( -5 + 4a \right) \left( -3 + 5a \right) \left( -1 + ab \right)} \right)}{3744}, \right. \\ & \left. \frac{5 \left( 7285 - 3b \left( 507 + 500b \right) + a \left( -1875 + b \left( -5785 + 3396b \right) \right) \right)}{416 \left( -5 + 3b \right) \left( -5 + 4b \right) \left( -1 + b^2 \right)}, \right. \\ & \left. - \frac{375 \left( -5 + 3a \right)}{208 \left( -5 + 3b \right)}, \right. \\ & \left. 0, \right. \\ & \left. 0, \frac{5 \left( -7285 + 3396b \right)}{832 \left( -5 + 3b \right) \left( -5 + 4b \right)}, \right. \\ & \left. - \frac{375}{416}, 0, 0 \right\}, \\ & \left\{ - \frac{625 \left( -35 + 37b \right)}{3744} + \frac{625 \left( 875 - 444b + a \left( -1706 + a \left( 875 - 444b \right) + 840b \right) \right)}{1872 \left( -5 + 3b \right) \left( -5 + 4b \right)}, \right. \\ & \frac{625}{936} \left( 37 + a \left( -70 + 37a \right) \right) + \frac{625 \left( 37 + b \left( -70 + 37b \right) \right)}{1872}, \\ & \frac{25 \left( -4 + 5a \right) \left( \frac{243 \left( 3 - 5a \right)}{\left( -5 + 3a \right) \left( -5 + 3b \right)} + \frac{1024 \left( -4 + 5a \right)}{\left( -5 + 4a \right) \left( -5 + 4b \right)} \right)}{3744} + \\ & \frac{125 \left( -4 + 5b \right) \left( -16835 + \left( 28897 - 10500b \right) b + 12a \left( 781 + 37b \left( -35 + 12b \right) \right) \right)}{7488 \left( -5 + 3a \right) \left( -5 + 4a \right) \left( -5 + 3b \right) \left( -5 + 4b \right)}, \\ & \frac{125 \left( -4 + 5a \right) \left( -781 + 37 \left( 35 - 12a \right) a \right)}{1872 \left( -5 + 3a \right) \left( -5 + 4a \right)} + \\ & \frac{125 \left( -4 + 5b \right) \left( -781 + a \left( 420 - 444b \right) + 875b \right)}{3744 \left( -5 + 3a \right) \left( -5 + 4a \right)}, \end{aligned}$$

$$\begin{aligned}
& - \frac{125 (139 - 60b + a (-125 + 36b))}{208 (-5 + 3b) (-5 + 4b)}, \frac{125}{104} (-5 + 3a), 0, 0, \\
& \frac{25}{416} \left( -15 + \frac{144}{5 - 4b} + \frac{208}{-5 + 3b} \right), \frac{125}{208} (-5 + 3b), 0, 0 \}, \\
& \left\{ \frac{5 (-4 + 5b) (-820925 + a (462180 - 253008b) + 462180b)}{14976 (-5 + 3a) (-5 + 4a) (-5 + 3b) (-5 + 4b)} + \right. \\
& \quad (5 (-4 + 5a) (-820925 + 12a^3b (43925 + (15535 - 21084b)b) + 5b (-100139 + \\
& \quad \quad 12b (4165 + 1872b)) + a^2 (12 + 35b) (-43925 + b (-15535 + 21084b)) + \\
& \quad \quad a (1425055 + b (1666042 - b (237245 + 551292b)))) / \\
& \quad (7488 (-5 + 3a) (-5 + 4a) (-5 + 3b) (-5 + 4b) (-1 + ab) (-1 + b^2)), \\
& \quad \frac{25 (-4 + 5a) \left( \frac{243 (3 - 5a)}{(-5 + 3a) (-5 + 3b)} + \frac{1024 (-4 + 5a)}{(-5 + 4a) (-5 + 4b)} \right)}{3744} + \\
& \quad \frac{125 (-4 + 5b) (-16835 + (28897 - 10500b)b + 12a (781 + 37b (-35 + 12b)))}{7488 (-5 + 3a) (-5 + 4a) (-5 + 3b) (-5 + 4b)}, \\
& (15000a^5b (-43925 - 15535b + 21084b^2) - \\
& \quad 250a^4 (-2635500 - 8213975b - 204025b^2 + 2956164b^3) + \\
& \quad 25 (-133077775 + 205853635b - 142252332b^2 + 37739520b^3) - \\
& \quad 2a^3 (298090375 + 1945968250b - 1746928805b^2 + 336089892b^3) - \\
& \quad 5a (-1468518175 + 2342889320b - 1905168349b^2 + 583563300b^3) + \\
& \quad a^2 (-4152214550 + 9094972995b - 9213408409b^2 + 3033829860b^3)) / \\
& \quad (748800 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab) (-1 + b^2)) + \\
& \quad (25 (-47709200 + 102369880b - 49400941b^2 - 20010655b^3 + 13177500b^4) + \\
& \quad a^2 (-937070400 + 2724188560b - 2978197292b^2 + 1318277075b^3 + 25503125b^4 - \\
& \quad \quad 116512500b^5) + 12a^3 (22464000 - 79158800b + 90688600b^2 - 17951341b^3 - \\
& \quad \quad 30793375b^4 + 13177500b^5) - 5a (-292224400 + 642897160b - \\
& \quad \quad 501750837b^2 + 287393120b^3 - 205349375b^4 + 65887500b^5)) / \\
& \quad (748800 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab) (-1 + b^2)), \\
& - ((25 (-649105 + a (754109 + 2a (526835 + a (-877873 + 262500a)))) + 5 (1572420 + \\
& \quad a (4089089 + 5a (-2697125 + 2a (554737 - 3125a (-173 + 84a))))))b + \\
& \quad 12a (-1310375 + a (2386315 + 2a (-123131 + 125a (-5827 + 2220a))))b^2) / \\
& \quad (14976 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab)) + \\
& \quad (-625 (4 - 5b)^2 (-875 + 444b) + \\
& \quad 25a (379600 + b (-1180440 + b (1661481 + 125b (-9019 + 2220b)))) - \\
& \quad 12a^3 (-187200 + b (-87920 + b (962744 + 125b (-7927 + 2220b)))) + \\
& \quad 5a^2 (-2150400 + b (3548560 + b (-539692 + 125b (-13561 + 5460b)))))) / \\
& \quad (14976 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab)), \\
& \quad (-4 + 5a) (-5625 + b (-4875 + 5625a + 3a (5785 - 3396b)b + 20b (-1165 + 624b))) \\
& \quad \frac{832 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab) (-1 + b^2)}{675 (-4 + 5a)}, \\
& \frac{416 (-5 + 3a) (-5 + 3b)}{3 (25 (-785 + 384a) - 4285 (-5 + 3a)b + 3 (-2240 + 2069a)b^2)}, \\
& \frac{640 (-5 + 3a) (-5 + 3b) (-1 + ab) (-1 + b^2)}{1305}, \\
& - \frac{64 (-5 + 3a) (-5 + 3b)}{1305},
\end{aligned}$$

$$\begin{aligned}
& \frac{(-4 + 5b) (-5625 + b (-4875 + 5625a + 3a (5785 - 3396b) b + 20b (-1165 + 624b)))}{1664 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab) (-1 + b^2)}, \\
& \frac{675 (-4 + 5b)}{832 (-5 + 3a) (-5 + 3b)}, \\
& \frac{-625 + 60 (10 - 3b) b + 3a (100 + b (-120 + 61b))}{20 (-5 + 3a) (-5 + 3b) (-1 + ab) (-1 + b^2)}, \\
& - \frac{15}{2 (-5 + 3a) (-5 + 3b)}, \\
& \left\{ \frac{125 (-875 + 444a) (-4 + 5b)}{7488 (-5 + 3a) (-5 + 4a)} + \right. \\
& \left. \frac{125 \left( -\frac{135 (-5+3a) (-4+5a)}{(-3+5a) (-5+3b)} + \frac{320 (-5+4a)}{-5+4b} - \frac{1872a (-1+a^2)}{(-5+3a) (-5+4a) (-3+5a) (-1+ab)} \right)}{3744}, \right. \\
& \frac{125 (-4 + 5a) (-781 + 37 (35 - 12a) a)}{1872 (-5 + 3a) (-5 + 4a)} + \\
& \frac{125 (-4 + 5b) (-781 + a (420 - 444b) + 875b)}{3744 (-5 + 3a) (-5 + 4a)}, \\
& - \left( (25 (-649105 + a (754109 + 2a (526835 + a (-877873 + 262500a)))) + 5 (1572420 + \right. \\
& \quad a (4089089 + 5a (-2697125 + 2a (554737 - 3125a (-173 + 84a)))) b + \\
& \quad 12a (-1310375 + a (2386315 + 2a (-123131 + 125a (-5827 + 2220a)))) b^2) / \\
& \quad (14976 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab)) + \\
& \quad (-625 (4 - 5b)^2 (-875 + 444b) + \\
& \quad 25a (379600 + b (-1180440 + b (1661481 + 125b (-9019 + 2220b)))) - \\
& \quad 12a^3 (-187200 + b (-87920 + b (962744 + 125b (-7927 + 2220b)))) + \\
& \quad 5a^2 (-2150400 + b (3548560 + b (-539692 + 125b (-13561 + 5460b)))) ) / \\
& \quad (14976 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab)), \\
& \left. \frac{-1310375 + a (2386315 + 2a (-123131 + 125a (-5827 + 2220a)))}{7488 (-5 + 3a) (-5 + 4a) (-1 + a^2)} + \right. \\
& \quad (-455a (-44 + 25b) (4 + 25b) + 12a^2 (7312 + 4625b (-8 + 5b)) - \\
& \quad 25 (22288 + 4625b (-8 + 5b))) / (7488 (-5 + 3a) (-5 + 4a) (-1 + a^2)), \\
& \frac{25 (-4 + 5a) (375 + b (100 + 3a (-125 + 36b)))}{416 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab)}, \\
& \frac{225 (-4 + 5a)}{208 (-5 + 3a)}, \\
& - \frac{15 (25 + a (-160 + 87b))}{64 (-5 + 3a) (-5 + 3b) (-1 + ab)}, \\
& - \frac{435}{32 (-5 + 3a)}, \\
& \frac{25 (-4 + 5b) (375 + b (100 + 3a (-125 + 36b)))}{832 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab)}, \\
& \frac{225 (-4 + 5b)}{416 (-5 + 3a)}, \\
& - \frac{5a}{2 (-5 + 3a) (-1 + ab)}, \\
& - \frac{5}{-5 + 3a} \Big\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{5 (7285 - 3 b (507 + 500 b) + a (-1875 + b (-5785 + 3396 b)))}{416 (-5 + 3 b) (-5 + 4 b) (-1 + b^2)}, \right. \\
& \quad \left. - \frac{125 (139 - 60 b + a (-125 + 36 b))}{208 (-5 + 3 b) (-5 + 4 b)}, \right. \\
& \quad \left. \frac{(-4 + 5 a) (-5625 + b (-4875 + 5625 a + 3 a (5785 - 3396 b) b + 20 b (-1165 + 624 b)))}{832 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b) (-1 + b^2)}, \right. \\
& \quad \left. \frac{25 (-4 + 5 a) (375 + b (100 + 3 a (-125 + 36 b)))}{416 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b)}, \right. \\
& \quad \left. \frac{445 - 117 b}{32 (-5 + 3 b) (-1 + b^2)}, \right. \\
& \quad \left. \frac{75}{80 - 48 b}, 0, \right. \\
& \quad \left. 0, 0, 0, 0, 0 \right\}, \\
& \left\{ -\frac{375 (-5 + 3 a)}{208 (-5 + 3 b)}, \frac{125}{104} (-5 + 3 a), \right. \\
& \quad \left. \frac{675 (-4 + 5 a)}{416 (-5 + 3 a) (-5 + 3 b)}, \right. \\
& \quad \left. -\frac{225 (-4 + 5 a)}{208 (-5 + 3 a)}, \frac{75}{80 - 48 b}, \right. \\
& \quad \left. \frac{25}{8}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, \frac{3 (25 (-785 + 384 a) - 4285 (-5 + 3 a) b + 3 (-2240 + 2069 a) b^2)}{640 (-5 + 3 a) (-5 + 3 b) (-1 + a b) (-1 + b^2)}, \right. \\
& \quad \left. -\frac{15 (25 + a (-160 + 87 b))}{64 (-5 + 3 a) (-5 + 3 b) (-1 + a b)}, \right. \\
& \quad \left. 0, 0, \frac{3 (445 - 117 b)}{64 (-5 + 3 b) (-1 + b^2)}, \right. \\
& \quad \left. \frac{225}{160 - 96 b}, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{1305}{64 (-5 + 3 a) (-5 + 3 b)}, \frac{435}{32 (-5 + 3 a)}, \right. \\
& \quad \left. 0, 0, \frac{225}{160 - 96 b}, \frac{75}{16}, 0, 0, 0, 0 \right\}, \\
& \left\{ \frac{5 (-7285 + 3396 b)}{832 (-5 + 3 b) (-5 + 4 b)}, \frac{25}{416} \left( -15 + \frac{144}{5 - 4 b} + \frac{208}{-5 + 3 b} \right), \right. \\
& \quad \left. \frac{(-4 + 5 b) (-5625 + b (-4875 + 5625 a + 3 a (5785 - 3396 b) b + 20 b (-1165 + 624 b)))}{1664 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b) (-1 + b^2)}, \right. \\
& \quad \left. \frac{25 (-4 + 5 b) (375 + b (100 + 3 a (-125 + 36 b)))}{832 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b)}, \right. \\
& \quad \left. \frac{445 - 117 b}{64 (-5 + 3 b) (-1 + b^2)}, \right. \\
& \quad \left. 0, 0, 0, 0, \frac{75}{160 - 96 b}, 0, 0 \right\}, \\
& \left\{ -\frac{375}{416}, \frac{125}{208} (-5 + 3 b), \frac{675 (-4 + 5 b)}{832 (-5 + 3 a) (-5 + 3 b)}, \right.
\end{aligned}$$

$$\begin{aligned}
& -\frac{225(-4+5b)}{416(-5+3a)}, 0, 0, 0, \\
& 0, \frac{75}{160-96b}, \frac{25}{16}, 0, 0\}, \\
& \left\{0, 0, \frac{-625+60(10-3b)b+3a(100+b(-120+61b))}{20(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}, \right. \\
& \quad \left. -\frac{5a}{2(-5+3a)(-1+ab)}, 0, 0, 0, 0, 0, \right. \\
& \quad \left. 0, \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b}\right\}, \\
& \left\{0, 0, -\frac{15}{2(-5+3a)(-5+3b)}, \frac{5}{-5+3a}, 0, 0, \right. \\
& \quad \left. 0, 0, 0, 0, \frac{75}{160-96b}, \frac{25}{16}\right\}
\end{aligned}$$

In[194]:= Gmatrix1288 = Gmatrix12 /. {a → 8 / 10, b → 8 / 10}

$$\begin{aligned}
\text{Out[194]} = & \left\{ \left\{ \frac{43925}{2496}, \frac{1125}{416}, 0, 0, -\frac{43925}{3744}, -\frac{375}{208}, 0, 0, -\frac{43925}{7488}, -\frac{375}{416}, 0, 0 \right\}, \right. \\
& \left\{ \frac{1125}{416}, \frac{75}{16}, 0, 0, -\frac{375}{208}, -\frac{25}{8}, 0, 0, -\frac{375}{416}, -\frac{25}{16}, 0, 0 \right\}, \\
& \left\{ 0, 0, \frac{1414633075}{31539456}, \frac{14914625}{1752192}, 0, 0, -\frac{12984425}{584064}, -\frac{32625}{10816}, 0, 0, -\frac{490325}{54756}, -\frac{375}{338} \right\}, \\
& \left\{ 0, 0, \frac{14914625}{1752192}, \frac{66775}{7488}, 0, 0, -\frac{875}{192}, -\frac{2175}{416}, 0, 0, -\frac{250}{117}, -\frac{25}{13} \right\}, \\
& \left\{ -\frac{43925}{3744}, -\frac{375}{208}, 0, 0, \frac{43925}{3744}, \frac{375}{208}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ -\frac{375}{208}, -\frac{25}{8}, 0, 0, \frac{375}{208}, \frac{25}{8}, 0, 0, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{12984425}{584064}, -\frac{875}{192}, 0, 0, \frac{43925}{2496}, \frac{1125}{416}, 0, 0, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{32625}{10816}, -\frac{2175}{416}, 0, 0, \frac{1125}{416}, \frac{75}{16}, 0, 0, 0, 0 \right\}, \\
& \left\{ -\frac{43925}{7488}, -\frac{375}{416}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416}, 0, 0 \right\}, \\
& \left\{ -\frac{375}{416}, -\frac{25}{16}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16}, 0, 0 \right\}, \\
& \left\{ 0, 0, -\frac{490325}{54756}, -\frac{250}{117}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416} \right\}, \\
& \left. \left\{ 0, 0, -\frac{375}{338}, -\frac{25}{13}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16} \right\} \right\}
\end{aligned}$$

In[195]:= Det[Gmatrix1288]

Out[195]= 0

In[196]:= Simplify[%]

Out[196]= 0

In[197]:= Gmatrix12a8 = Gmatrix12 /. {b → 8 / 10}

$$\begin{aligned}
\text{Out}[197] = & \left\{ \left\{ \frac{43\,925}{7488} - \frac{1}{3\,942\,432} \times \right. \right. \\
& 15\,625 \left( -\frac{52}{5} (1195 + a (-3514 + 1195 a)) + \frac{192}{25} (1757 + a (-3830 + 1757 a)) - \right. \\
& \left. \left. 5 (8785 + a (-15\,406 + 8785 a)) \right), \frac{375}{416} + \frac{15\,625 \left( \frac{2599}{5} + a \left( -1034 + \frac{2599 a}{5} \right) \right)}{219\,024}, \right. \\
& \left. - \frac{3125 (-4 + 5 a) \left( -\frac{25\,100\,929}{25} + \frac{290\,473\,787 a}{125} - \frac{8\,571\,848 a^2}{5} + \frac{51\,431\,088 a^3}{125} \right)}{7\,884\,864 \left( -1 + \frac{4 a}{5} \right) (-5 + 3 a) (-5 + 4 a)}, \right. \\
& \left. 125 \left( -\frac{1600}{9} (-5 + 4 a) + \frac{675 (-5+3 a) (-4+5 a)}{13 (-3+5 a)} - \frac{1872 a (-1+a^2)}{\left( -1+\frac{4 a}{5} \right) (-5+3 a) (-5+4 a) (-3+5 a)} \right) \right. \\
& \left. \frac{3744}{}, \right. \\
& \left. - \frac{3125 \left( \frac{25\,541}{5} - \frac{108\,239 a}{25} \right)}{438\,048}, \frac{1875 (-5 + 3 a)}{2704}, \right. \\
& \left. 0, 0, -\frac{43\,925}{7488}, -\frac{375}{416}, 0, 0 \right\}, \\
& \left\{ \frac{375}{416} + \frac{15\,625 \left( \frac{2599}{5} + a \left( -1034 + \frac{2599 a}{5} \right) \right)}{219\,024}, \frac{25}{16} + \frac{625}{936} (37 + a (-70 + 37 a)), \right. \\
& \frac{25 (-4 + 5 a) \left( -\frac{1215 (3-5 a)}{13 (-5+3 a)} - \frac{5120 (-4+5 a)}{9 (-5+4 a)} \right)}{3744}, \frac{125 (-4 + 5 a) (-781 + 37 (35 - 12 a) a)}{1872 (-5 + 3 a) (-5 + 4 a)}, \\
& -\frac{3125 \left( 91 - \frac{481 a}{5} \right)}{24\,336}, \frac{125}{104} (-5 + 3 a), 0, 0, -\frac{375}{416}, -\frac{25}{16}, 0, 0 \Big\}, \\
& \left\{ -\frac{3125 (-4 + 5 a) \left( -\frac{25\,100\,929}{25} + \frac{290\,473\,787 a}{125} - \frac{8\,571\,848 a^2}{5} + \frac{51\,431\,088 a^3}{125} \right)}{7\,884\,864 \left( -1 + \frac{4 a}{5} \right) (-5 + 3 a) (-5 + 4 a)}, \right. \\
& \frac{25 (-4 + 5 a) \left( -\frac{1215 (3-5 a)}{13 (-5+3 a)} - \frac{5120 (-4+5 a)}{9 (-5+4 a)} \right)}{3744}, \\
& \frac{25 \left( -56\,946\,240 + \frac{360\,075\,456 a}{5} - \frac{413\,517\,312 a^2}{25} - \frac{462\,578\,688 a^3}{125} \right)}{31\,539\,456 \left( -1 + \frac{4 a}{5} \right) (-5 + 3 a) (-5 + 4 a) (-1 + a^2)} - \\
& \left( 25 \left( -1\,002\,843\,131 + \frac{12\,868\,251\,319 a}{5} - \frac{30\,487\,416\,186 a^2}{25} - \right. \right. \\
& \left. \left. \frac{227\,227\,141\,126 a^3}{125} + 1\,955\,925\,008 a^4 - 514\,310\,880 a^5 \right) \right) / \\
& \left( 31\,539\,456 \left( -1 + \frac{4 a}{5} \right) (-5 + 3 a) (-5 + 4 a) (-1 + a^2) \right), \\
& \frac{25 \left( 876\,096 a - \frac{6132\,672 a^2}{5} + \frac{10\,513\,152 a^3}{25} \right)}{1\,752\,192 \left( -1 + \frac{4 a}{5} \right) (-5 + 3 a) (-5 + 4 a) (-1 + a^2)} - \\
& \left( 25 \left( \frac{192}{25} a (-1\,310\,375 + a (2\,386\,315 + 2 a (-123\,131 + 125 a (-5827 + 2220 a))) \right) + \right. \\
& 25 (-649\,105 + a (754\,109 + 2 a (526\,835 + a (-877\,873 + 262\,500 a))) + \\
& \left. 4 (1\,572\,420 + \right.
\end{aligned}$$



$$\begin{aligned}
& -\frac{5a}{2\left(-1+\frac{4a}{5}\right)(-5+3a)}, \frac{5}{-5+3a}\}, \\
& \left\{-\frac{3125\left(\frac{25541}{5}-\frac{108239a}{25}\right)}{438048}, -\frac{3125\left(91-\frac{481a}{5}\right)}{24336}, \right. \\
& \left. -\frac{625(-4+5a)\left(-5625+\frac{4}{5}\left(-\frac{77639}{5}+\frac{324717a}{25}\right)\right)}{876096\left(-1+\frac{4a}{5}\right)(-5+3a)}, \right. \\
& \left. \frac{625\left(375+\frac{4}{5}\left(100-\frac{1443a}{5}\right)\right)(-4+5a)}{48672\left(-1+\frac{4a}{5}\right)(-5+3a)}, \right. \\
& \left. \frac{43925}{3744}, \frac{375}{208}, 0, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{\frac{1875(-5+3a)}{2704}, \frac{125}{104}(-5+3a), -\frac{3375(-4+5a)}{5408(-5+3a)}, \right. \\
& \left. -\frac{225(-4+5a)}{208(-5+3a)}, \frac{375}{208}, \frac{25}{8}, 0, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, \frac{25(-3428(-5+3a)+25(-785+384a)+\frac{48}{25}(-2240+2069a))}{4992\left(-1+\frac{4a}{5}\right)(-5+3a)}, \right. \\
& \left. \frac{75\left(25-\frac{452a}{5}\right)}{832\left(-1+\frac{4a}{5}\right)(-5+3a)}, 0, \right. \\
& \left. 0, \frac{43925}{2496}, \frac{1125}{416}, 0, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, \frac{6525}{832(-5+3a)}, \frac{435}{32(-5+3a)}, 0, 0, \frac{1125}{416}, \frac{75}{16}, 0, 0, 0, 0\right\}, \\
& \left\{-\frac{43925}{7488}, -\frac{375}{416}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416}, 0, 0\right\}, \\
& \left\{-\frac{375}{416}, -\frac{25}{16}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16}, 0, 0\right\}, \\
& \left\{0, 0, \frac{25\left(-\frac{1301}{5}+\frac{3228a}{25}\right)}{468\left(-1+\frac{4a}{5}\right)(-5+3a)}, \right. \\
& \left. -\frac{5a}{2\left(-1+\frac{4a}{5}\right)(-5+3a)}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416}\right\}, \\
& \left\{0, 0, \frac{75}{26(-5+3a)}, \frac{5}{-5+3a}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16}\right\}
\end{aligned}$$

In[198]:= Det[Gmatrix12a8]



$$\begin{aligned}
\text{Out}[198]= & \left( 170\,761\,236\,262\,381\,076\,812\,744\,140\,625 \right. \\
& \left( 1\,805\,664\,062\,500\,000\,000 - 38\,805\,820\,312\,500\,000\,000\,a + 397\,928\,313\,085\,937\,500\,000\,a^2 - \right. \\
& 2\,591\,120\,332\,988\,281\,250\,000\,a^3 + 12\,028\,506\,547\,090\,087\,890\,625\,a^4 - \\
& 42\,367\,680\,486\,603\,125\,000\,000\,a^5 + 117\,656\,218\,927\,675\,078\,125\,000\,a^6 - \\
& 264\,241\,171\,561\,359\,546\,875\,000\,a^7 + 488\,454\,001\,060\,526\,435\,156\,250\,a^8 - \\
& 752\,383\,793\,014\,272\,681\,875\,000\,a^9 + 974\,012\,895\,502\,928\,365\,312\,500\,a^{10} - \\
& 1\,065\,741\,716\,252\,512\,485\,725\,000\,a^{11} + 988\,794\,924\,121\,527\,389\,715\,625\,a^{12} - \\
& 778\,751\,359\,825\,439\,233\,757\,000\,a^{13} + 520\,118\,981\,809\,328\,888\,993\,500\,a^{14} - \\
& 293\,668\,074\,696\,522\,505\,106\,160\,a^{15} + 139\,396\,373\,681\,471\,574\,744\,964\,a^{16} - \\
& 55\,158\,413\,527\,356\,254\,736\,960\,a^{17} + 17\,973\,090\,232\,348\,988\,476\,800\,a^{18} - \\
& 4\,739\,039\,900\,035\,583\,616\,000\,a^{19} + 985\,830\,272\,779\,674\,240\,000\,a^{20} - \\
& 155\,715\,830\,535\,168\,000\,000\,a^{21} + 17\,549\,096\,767\,488\,000\,000\,a^{22} - \\
& \left. \left. 1\,256\,979\,824\,640\,000\,000\,a^{23} + 42\,998\,169\,600\,000\,000\,a^{24} \right) \right) / \\
& \left( 11\,062\,441\,448\,736\,885\,200\,388\,096 \, (-5 + 3a)^{10} \, (-5 + 4a)^{10} \, (-1 + a^2)^2 \right)
\end{aligned}$$

In[199]:= Simplify[Det[Gmatrix12a8]]

$$\begin{aligned}
\text{Out}[199]= & \left( 170\,761\,236\,262\,381\,076\,812\,744\,140\,625 \right. \\
& \left( 4 - 5a \right)^4 \left( 5375 - 22\,820a + 35\,778a^2 - 24\,640a^3 + 6400a^4 \right)^2 \Big/ \\
& \left( 11\,062\,441\,448\,736\,885\,200\,388\,096 \, (5 - 4a)^6 \, (5 - 3a)^2 \, (-1 + a^2)^2 \right)
\end{aligned}$$

In[200]:= Gmatrix128b = Gmatrix12 /. {a -> 8/10}

$$\begin{aligned}
\text{Out}[200]= & \left\{ \left\{ \frac{43\,925}{7488} + \frac{25 \left( -10\,413 + \frac{55\,341b}{5} - \frac{54\,756b^2}{25} \right)}{3744 \, (-5 + 3b) \, (-5 + 4b) \, (-1 + b^2)}, \right. \right. \\
& - \frac{625 \, (-35 + 37b)}{3744} + \frac{625 \left( 875 - 444b + \frac{4}{5} \left( -1706 + \frac{4}{5} (875 - 444b) + 840b \right) \right)}{1872 \, (-5 + 3b) \, (-5 + 4b)}, \\
& \frac{125 \, (-4 + 5b) \left( -820\,925 + \frac{4}{5} (462\,180 - 253\,008b) + 462\,180b \right)}{1\,752\,192 \, (-5 + 3b) \, (-5 + 4b)}, \\
& - \frac{1\,624\,375 \, (-4 + 5b)}{876\,096} + \frac{125 \left( \frac{576}{5 \left( -1 + \frac{4b}{5} \right)} - \frac{576}{-5 + 4b} \right)}{3744}, \\
& \frac{5 \left( 7285 - 3b \, (507 + 500b) + \frac{4}{5} \left( -1875 + b \, (-5785 + 3396b) \right) \right)}{416 \, (-5 + 3b) \, (-5 + 4b) \, (-1 + b^2)}, \\
& \frac{75}{16 \, (-5 + 3b)}, 0, 0, \frac{5 \, (-7285 + 3396b)}{832 \, (-5 + 3b) \, (-5 + 4b)}, -\frac{375}{416}, 0, 0 \Big\}, \\
& \left\{ - \frac{625 \, (-35 + 37b)}{3744} + \frac{625 \left( 875 - 444b + \frac{4}{5} \left( -1706 + \frac{4}{5} (875 - 444b) + 840b \right) \right)}{1872 \, (-5 + 3b) \, (-5 + 4b)}, \right. \\
& \frac{25}{8} + \frac{625 \, (37 + b \, (-70 + 37b))}{1872}, \\
& \frac{3125 \, (-4 + 5b) \left( -16\,835 + (28\,897 - 10\,500b)b + \frac{48}{5} (781 + 37b \, (-35 + 12b)) \right)}{876\,096 \, (-5 + 3b) \, (-5 + 4b)}, \\
& \frac{3125 \, (-4 + 5b) \left( -781 + \frac{4}{5} (420 - 444b) + 875b \right)}{438\,048}, - \frac{125 \left( 139 - 60b + \frac{4}{5} \left( -125 + 36b \right) \right)}{208 \, (-5 + 3b) \, (-5 + 4b)} \Big\},
\end{aligned}$$

$$\begin{aligned}
& -\frac{25}{8}, 0, 0, \frac{25}{416} \left( -15 + \frac{144}{5-4b} + \frac{208}{-5+3b} \right), \frac{125}{208} (-5+3b), 0, 0 \}, \\
& \left\{ \frac{125 (-4+5b) \left( -820925 + \frac{4}{5} (462180 - 253008b) + 462180b \right)}{1752192 (-5+3b) (-5+4b)}, \right. \\
& \left. \frac{3125 (-4+5b) \left( -16835 + (28897 - 10500b)b + \frac{48}{5} (781 + 37b(-35+12b)) \right)}{876096 (-5+3b) (-5+4b)}, \right. \\
& - \left( \left( 25 \left( \frac{24576}{5} b (-43925 - 15535b + 21084b^2) - \right. \right. \right. \\
& \quad \frac{512}{5} (-2635500 - 8213975b - 204025b^2 + 2956164b^3) + \\
& \quad 25 (-133077775 + 205853635b - 142252332b^2 + 37739520b^3) - \\
& \quad \frac{128}{125} (298090375 + 1945968250b - 1746928805b^2 + 336089892b^3) - \\
& \quad 4 (-1468518175 + 2342889320b - 1905168349b^2 + 583563300b^3) + \\
& \quad \left. \left. \frac{16}{25} (-4152214550 + 9094972995b - 9213408409b^2 + 3033829860b^3) \right) \right) \right) / \\
& \left( 31539456 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b) (-1+b^2) \right) \Big) - \\
& \left( 25 \left( 25 (-47709200 + 102369880b - 49400941b^2 - 20010655b^3 + 13177500b^4) + \right. \right. \\
& \quad \frac{16}{25} (-937070400 + 2724188560b - 2978197292b^2 + 1318277075b^3 + 25503125 \\
& \quad b^4 - 116512500b^5) + \frac{768}{125} (22464000 - 79158800b + 90688600b^2 - \\
& \quad 17951341b^3 - 30793375b^4 + 13177500b^5) - 4 (-292224400 + 642897160b - \\
& \quad \left. \left. 501750837b^2 + 287393120b^3 - 205349375b^4 + 65887500b^5) \right) \right) \Big) / \\
& \left( 31539456 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b) (-1+b^2) \right), \\
& \frac{625 \left( -\frac{6921369}{5} + \frac{10184616b}{5} - \frac{92950416b^2}{125} \right)}{15769728 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b)} - \\
& \left( 625 \left( -625 (4-5b)^2 (-875+444b) + \right. \right. \\
& \quad 20 (379600 + b (-1180440 + b (1661481 + 125b (-9019 + 2220b))) - \\
& \quad \frac{768}{125} (-187200 + b (-87920 + b (962744 + 125b (-7927 + 2220b))) + \\
& \quad \left. \left. \frac{16}{5} (-2150400 + b (3548560 + b (-539692 + 125b (-13561 + 5460b))) \right) \right) \right) / \\
& \left( 15769728 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b) \right), 0, 0, \\
& - \frac{3 \left( -11945 + 11141b - \frac{8772b^2}{5} \right)}{1664 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-1+b^2)},
\end{aligned}$$

$$\begin{aligned}
& \frac{6525}{832(-5+3b)}, \\
& - \frac{5(-4+5b) \left( -5625 + b \left( -375 + \frac{12}{5} (5785 - 3396b) \right) b + 20b(-1165 + 624b) \right)}{21632 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b) (-1+b^2)}, \\
& - \frac{3375(-4+5b)}{10816(-5+3b)}, \\
& - \frac{-625 + 60(10-3b)b + \frac{12}{5}(100+b(-120+61b))}{52 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-1+b^2)}, \\
& \left. \frac{75}{26(-5+3b)} \right\}, \\
& \left\{ - \frac{1624375(-4+5b)}{876096} + \frac{125 \left( \frac{576}{5 \left( -1 + \frac{4b}{5} \right)} - \frac{576}{-5+4b} \right)}{3744}, \right. \\
& \left. \frac{3125(-4+5b) \left( -781 + \frac{4}{5}(420-444b) + 875b \right)}{438048}, \right. \\
& \left. \frac{625 \left( -\frac{6921369}{5} + \frac{10184616b}{5} - \frac{92950416b^2}{125} \right)}{15769728 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b)} - \right. \\
& \left. \left( 625 \left( -625(4-5b)^2(-875+444b) + \right. \right. \right. \\
& \quad 20(379600+b(-1180440+b(1661481+125b(-9019+2220b)))) - \\
& \quad \frac{768}{125}(-187200+b(-87920+b(962744+125b(-7927+2220b)))) + \\
& \quad \left. \left. \frac{16}{5}(-2150400+b(3548560+b(-539692+125b(-13561+5460b)))) \right) \right) \right) / \\
& \left( 15769728 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b) \right), \frac{15325}{2496} - \\
& \frac{1}{7884864} \times 625 \left( -364(-44+25b)(4+25b) + \frac{192}{25}(7312+4625b(-8+5b)) - \right. \\
& \quad \left. 25(22288+4625b(-8+5b)) \right), 0, 0, \frac{75 \left( 25 + \frac{4}{5}(-160+87b) \right)}{832 \left( -1 + \frac{4b}{5} \right) (-5+3b)}, \\
& - \frac{2175}{416}, - \frac{125(-4+5b) \left( 375 + b \left( 100 + \frac{12}{5}(-125+36b) \right) \right)}{10816 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b)}, \\
& \frac{1125(-4+5b)}{5408}, \\
& \frac{10}{13 \left( -1 + \frac{4b}{5} \right)}, \\
& - \frac{25}{13} \left. \right\}, \\
& \left\{ \frac{5(7285-3b(507+500b) + \frac{4}{5}(-1875+b(-5785+3396b)))}{416(-5+3b)(-5+4b)(-1+b^2)}, \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{125 \left( 139 - 60 b + \frac{4}{5} (-125 + 36 b) \right)}{208 (-5 + 3 b) (-5 + 4 b)}, \\
& 0, \\
& 0, \\
& \frac{445 - 117 b}{32 (-5 + 3 b) (-1 + b^2)}, \\
& \frac{75}{80 - 48 b}, \\
& 0, \\
& 0, \\
& 0, \\
& 0, \\
& 0, \\
& 0 \}, \left\{ \frac{75}{16 (-5 + 3 b)}, \right. \\
& - \frac{25}{8}, \\
& 0, 0, \\
& \frac{75}{80 - 48 b}, \\
& \frac{25}{8}, 0, \\
& 0, 0, 0, \\
& 0, 0 \}, \\
& \left\{ 0, 0, - \frac{3 \left( -11945 + 11141 b - \frac{8772 b^2}{5} \right)}{1664 \left( -1 + \frac{4b}{5} \right) (-5 + 3 b) (-1 + b^2)}, \right. \\
& \frac{75 \left( 25 + \frac{4}{5} (-160 + 87 b) \right)}{832 \left( -1 + \frac{4b}{5} \right) (-5 + 3 b)}, \\
& 0, 0, \\
& \frac{3 (445 - 117 b)}{64 (-5 + 3 b) (-1 + b^2)}, \\
& \frac{225}{160 - 96 b}, \\
& 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, \frac{6525}{832 (-5 + 3 b)}, - \frac{2175}{416}, 0, 0, \right. \\
& \frac{225}{160 - 96 b}, \frac{75}{16}, \\
& 0, 0, 0, 0 \}, \\
& \left\{ \frac{5 (-7285 + 3396 b)}{832 (-5 + 3 b) (-5 + 4 b)}, \right. \\
& \left. \frac{25}{416} \left( -15 + \frac{144}{5 - 4 b} + \frac{208}{-5 + 3 b} \right) \right\},
\end{aligned}$$

$$\begin{aligned}
& - \frac{5(-4+5b) \left( -5625 + b \left( -375 + \frac{12}{5} (5785 - 3396b) b + 20b(-1165 + 624b) \right) \right)}{21632 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b) (-1+b^2)}, \\
& - \frac{125(-4+5b) \left( 375 + b \left( 100 + \frac{12}{5} (-125 + 36b) \right) \right)}{10816 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-5+4b)}, \\
& 0, 0, 0, 0, \\
& \frac{445 - 117b}{64(-5+3b)(-1+b^2)}, \\
& \frac{75}{160-96b}, 0, 0 \}, \\
& \left\{ -\frac{375}{416}, \frac{125}{208}(-5+3b), -\frac{3375(-4+5b)}{10816(-5+3b)}, \right. \\
& \left. \frac{1125(-4+5b)}{5408}, 0, 0, 0, \right. \\
& 0, \frac{75}{160-96b}, \frac{25}{16}, 0, 0 \}, \\
& \left\{ 0, 0, -\frac{-625+60(10-3b)b + \frac{12}{5}(100+b(-120+61b))}{52 \left( -1 + \frac{4b}{5} \right) (-5+3b) (-1+b^2)}, \right. \\
& \left. \frac{10}{13 \left( -1 + \frac{4b}{5} \right)}, 0, 0, 0, 0, 0, 0, \right. \\
& \left. \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b} \right\}, \\
& \left\{ 0, 0, \frac{75}{26(-5+3b)}, -\frac{25}{13}, 0, 0, 0, 0, 0, \right. \\
& \left. 0, \frac{75}{160-96b}, \frac{25}{16} \right\} \}
\end{aligned}$$

In[201]:= Det[Gmatrix128b]

```
Out[201]= (95 367 431 640 625
(68 152 764 282 226 562 500 000 000 - 1 415 649 587 133 789 062 500 000 000 b +
13 946 499 109 417 358 398 437 500 000 b2 - 86 488 796 450 228 781 738 281 250 000 b3 +
377 564 213 292 550 903 131 103 515 625 b4 - 1 227 460 890 228 296 404 868 164 062 500
b5 + 3 058 183 089 483 222 534 684 082 031 250 b6 -
5 888 841 354 560 437 152 555 273 437 500 b7 + 8 621 908 667 636 090 111 193 740 234 375
b8 - 8 923 919 806 688 813 625 617 875 000 000 b9 +
4 563 387 525 895 334 682 598 985 937 500 b10 + 4 290 341 836 365 192 892 401 899 375 000
b11 - 13 980 028 049 849 982 102 663 838 265 625 b12 +
19 263 421 881 468 980 853 901 373 837 500 b13 -
17 110 541 117 155 555 841 755 793 528 750 b14 +
9 005 407 230 065 300 343 091 924 096 500 b15 +
379 394 346 932 385 645 735 863 338 025 b16 -
6 716 423 571 380 853 855 200 159 455 960 b17 +
8 536 510 928 836 434 446 209 048 610 944 b18 -
7 079 107 390 406 255 848 844 517 628 800 b19 +
4 491 861 872 105 454 640 158 928 348 512 b20 -
2 288 922 873 839 299 895 666 455 883 520 b21 +
953 248 591 731 522 608 253 060 620 544 b22 -
325 686 601 463 306 051 208 861 342 720 b23 + 90 832 437 826 806 752 330 953 017 600 b24 -
20 414 280 807 951 465 945 109 248 000 b25 + 3 615 858 505 472 912 602 928 640 000 b26 -
486 849 950 913 218 688 000 000 000 b27 + 46 897 856 003 122 560 000 000 000 b28 -
2 882 535 984 000 000 000 000 000 b29 + 85 030 560 000 000 000 000 000 b30)) /
(9 167 498 816 495 026 176 (-5 + 3 b)12 (-5 + 4 b)10 (-1 + b2)6)
```

```
In[202]:= Simplify[Det[Gmatrix128b]]
```

```
Out[202]= 
$$\frac{95\,367\,431\,640\,625\,(10\,567 - 25\,000\,b + 15\,625\,b^2)^2\,(500 - 1105\,b + 744\,b^2 - 180\,b^3)^4}{9\,167\,498\,816\,495\,026\,176\,(5 - 4\,b)^4\,(5 - 3\,b)^8\,(-1 + b^2)^4}$$

```

What follows shows that the method (specific to a square matrix C) described by Klein and Spreij (2010) gives the same results.

```
In[203]:= BkI2 = KroneckerProduct[B, I2]
```

```
Out[203]= {{-b, 0, 0, 0}, {0, -b, 0, 0}, {-1/2, 0, -3/5, 0}, {0, -1/2, 0, -3/5}}
```

```
In[204]:= MatrixForm[BkI2]
```

```
Out[204]//MatrixForm=
```

$$\begin{pmatrix} -b & 0 & 0 & 0 \\ 0 & -b & 0 & 0 \\ -\frac{1}{2} & 0 & -\frac{3}{5} & 0 \\ 0 & -\frac{1}{2} & 0 & -\frac{3}{5} \end{pmatrix}$$

```
In[205]:= I2kA = KroneckerProduct[I2, A]
```

```
Out[205]= {{-4/5, 0, 0, 0}, {-1/2, -a, 0, 0}, {0, 0, -4/5, 0}, {0, 0, -1/2, -a}}
```

In[206]:= **MatrixForm[I2kA]**

Out[206]//MatrixForm=

$$\begin{pmatrix} -\frac{4}{5} & 0 & 0 & 0 \\ -\frac{1}{2} & -a & 0 & 0 \\ 0 & 0 & -\frac{4}{5} & 0 \\ 0 & 0 & -\frac{1}{2} & -a \end{pmatrix}$$

In[207]:=

In[208]:= **MatrixForm[I4]**

Out[208]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[209]:= **mI4 = KroneckerProduct[-I2, I2]**

Out[209]=  $\{\{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, -1\}\}$

In[210]:= **MatrixForm[mI4]**

Out[210]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[211]:= **mBkI2 = KroneckerProduct[-B, I2]**

Out[211]=  $\{\{b, 0, 0, 0\}, \{0, b, 0, 0\}, \{\frac{1}{2}, 0, \frac{3}{5}, 0\}, \{0, \frac{1}{2}, 0, \frac{3}{5}\}\}$

In[212]:= **MatrixForm[mBkI2]**

Out[212]//MatrixForm=

$$\begin{pmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ \frac{1}{2} & 0 & \frac{3}{5} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{3}{5} \end{pmatrix}$$

In[213]:= **SmB = ArrayFlatten[{{mI4, mBkI2}}]**

Out[213]=  $\{\{-1, 0, 0, 0, b, 0, 0, 0\}, \{0, -1, 0, 0, 0, b, 0, 0\},$   
 $\{0, 0, -1, 0, \frac{1}{2}, 0, \frac{3}{5}, 0\}, \{0, 0, 0, -1, 0, \frac{1}{2}, 0, \frac{3}{5}\}\}$

In[214]:= **MatrixForm[SmB]**

Out[214]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & b & 0 & 0 \\ 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{3}{5} \end{pmatrix}$$

In[215]:= **SA = ArrayFlatten[{ {I4, I2kA} }]**

Out[215]=  $\left\{ \left\{ 1, 0, 0, 0, -\frac{4}{5}, 0, 0, 0 \right\}, \left\{ 0, 1, 0, 0, -\frac{1}{2}, -a, 0, 0 \right\}, \right.$   
 $\left. \left\{ 0, 0, 1, 0, 0, 0, -\frac{4}{5}, 0 \right\}, \left\{ 0, 0, 0, 1, 0, 0, -\frac{1}{2}, -a \right\} \right\}$

In[216]:= **MatrixForm[SA]**

Out[216]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -a & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & -a \end{pmatrix}$$

In[217]:= **SmBA = ArrayFlatten[{ {mI4, mBkI2}, {I4, I2kA} }]**

Out[217]=  $\left\{ \left\{ -1, 0, 0, 0, b, 0, 0, 0 \right\}, \left\{ 0, -1, 0, 0, 0, b, 0, 0 \right\}, \right.$   
 $\left\{ 0, 0, -1, 0, \frac{1}{2}, 0, \frac{3}{5}, 0 \right\}, \left\{ 0, 0, 0, -1, 0, \frac{1}{2}, 0, \frac{3}{5} \right\}, \left\{ 1, 0, 0, 0, -\frac{4}{5}, 0, 0, 0 \right\},$   
 $\left. \left\{ 0, 1, 0, 0, -\frac{1}{2}, -a, 0, 0 \right\}, \left\{ 0, 0, 1, 0, 0, 0, -\frac{4}{5}, 0 \right\}, \left\{ 0, 0, 0, 1, 0, 0, -\frac{1}{2}, -a \right\} \right\}$

In[218]:= **MatrixForm[SmBA]**

Out[218]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & b & 0 & 0 \\ 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{3}{5} \\ 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -a & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & -a \end{pmatrix}$$

In[219]:= **Det[SmBA]**

Out[219]= 
$$\frac{-960 a + 1600 a^2 + 960 b - 400 a b - 2000 a^2 b - 1200 b^2 + 2000 a b^2}{10000}$$

In[220]:= **MatrixRank[SmBA]**

Out[220]= 8

In[221]:= **mCkI2 = KroneckerProduct[-C, I2]**

Out[221]=  $\left\{ \{a, 0, 0, 0\}, \{0, a, 0, 0\}, \left\{ \frac{1}{2}, 0, \frac{7}{10}, 0 \right\}, \left\{ 0, \frac{1}{2}, 0, \frac{7}{10} \right\} \right\}$



In[222]:= **MatrixForm**[mCkI2]

Out[222]//MatrixForm=

$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ \frac{1}{2} & 0 & \frac{7}{10} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{7}{10} \end{pmatrix}$$

In[223]:= **SmCA** = **ArrayFlatten**[{{mI4, mCkI2}, {I4, I2kA}}]

Out[223]=  $\left\{ \{-1, 0, 0, 0, a, 0, 0, 0\}, \{0, -1, 0, 0, 0, a, 0, 0\}, \right.$   
 $\left\{0, 0, -1, 0, \frac{1}{2}, 0, \frac{7}{10}, 0\right\}, \left\{0, 0, 0, -1, 0, \frac{1}{2}, 0, \frac{7}{10}\right\}, \left\{1, 0, 0, 0, -\frac{4}{5}, 0, 0, 0\right\},$   
 $\left\{0, 1, 0, 0, -\frac{1}{2}, -a, 0, 0\right\}, \left\{0, 0, 1, 0, 0, 0, -\frac{4}{5}, 0\right\}, \left\{0, 0, 0, 1, 0, 0, -\frac{1}{2}, -a\right\} \}$

In[224]:= **MatrixForm**[SmCA]

Out[224]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & a & 0 & 0 \\ 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{7}{10} & 0 \\ 0 & 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{7}{10} \\ 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -a & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & -a \end{pmatrix}$$

In[225]:= **Det**[SmCA]

Out[225]= 0

In[226]:= **MatrixRank**[SmCA]

Out[226]= 7

In[227]:= **SmBOA** = **ArrayFlatten**[{{mI4, mBkI2}, {O4, O4}, {I4, I2kA}}]

Out[227]=  $\left\{ \{-1, 0, 0, 0, b, 0, 0, 0\}, \{0, -1, 0, 0, 0, b, 0, 0\}, \left\{0, 0, -1, 0, \frac{1}{2}, 0, \frac{3}{5}, 0\right\}, \right.$   
 $\left\{0, 0, 0, -1, 0, \frac{1}{2}, 0, \frac{3}{5}\right\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\},$   
 $\{0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0\}, \left\{1, 0, 0, 0, -\frac{4}{5}, 0, 0, 0\right\},$   
 $\left\{0, 1, 0, 0, -\frac{1}{2}, -a, 0, 0\right\}, \left\{0, 0, 1, 0, 0, 0, -\frac{4}{5}, 0\right\}, \left\{0, 0, 0, 1, 0, 0, -\frac{1}{2}, -a\right\} \}$

In[228]:= **MatrixForm**[SmBOA]

Out[228]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & b & 0 & 0 \\ 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{3}{5} & 0 \\ 0 & 0 & 0 & -1 & 0 & \frac{1}{2} & 0 & \frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & -a & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{4}{5} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & -a \end{pmatrix}$$

In[229]:= **u2**[z\_] := {{1}, {z}}

In[230]:= **MatrixForm**[u2[z]]

Out[230]//MatrixForm=

$$\begin{pmatrix} 1 \\ z \end{pmatrix}$$

In[231]:= **u2kI4**[z\_] = **KroneckerProduct**[u2[z], I4]

Out[231]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1},  
{z, 0, 0, 0}, {0, z, 0, 0}, {0, 0, z, 0}, {0, 0, 0, z}}

In[232]:= **u2kI4**[z]

Out[232]= {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1},  
{z, 0, 0, 0}, {0, z, 0, 0}, {0, 0, z, 0}, {0, 0, 0, z}}

In[233]:= **MatrixForm**[u2kI4[z]]

Out[233]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ z & 0 & 0 & 0 \\ 0 & z & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & z \end{pmatrix}$$

In[234]:=

In[235]:= **InvAzkI2**[z\_] = **KroneckerProduct**[**InvAz**[z], **I2**]

$$\text{Out[235]} = \left\{ \left\{ \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \right. \\ \left. \left\{ \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\}$$

In[236]:= **I2kInvAz**[z\_] = **KroneckerProduct**[**I2**, **InvAz**[z]]

$$\text{Out[236]} = \left\{ \left\{ \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \left\{ \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \left\{ 0, 0, \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\}$$

In[237]:= **LR**[z\_] =  $\begin{pmatrix} \text{InvAzkI2}[z] & \mathbf{04} & \mathbf{04} \\ \mathbf{04} & \mathbf{04} & \mathbf{04} \\ \mathbf{04} & \mathbf{04} & \text{I2kInvAz}[z] \end{pmatrix}$

$$\text{Out[237]} = \left\{ \left\{ \left\{ \left\{ \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \right. \right. \right. \\ \left. \left\{ \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \right. \\ \left. \left\{ 0, \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\}, \\ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\ \left\{ \left\{ \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \left\{ \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \right. \\ \left. \left\{ 0, 0, \frac{1 - a z}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \left\{ 0, 0, \frac{z}{2 \left( 1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4z}{5}}{1 - \frac{4z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\} \right\}$$

In[238]:= LR[z\_] = ArrayFlatten[LR[z]]

Out[238]= 
$$\left\{ \left\{ \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$

$$\left\{ 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}} \right\} \}$$

In[239]:= LR[z]

$$\begin{aligned}
 \text{Out[239]} = & \left\{ \left\{ \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right. \\
 & \left\{ 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
 & \left\{ \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
 & \left\{ 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \\
 & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
 & \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
 & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \\
 & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \\
 & \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \\
 & \left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\}
 \end{aligned}$$

In[240]:= **MatrixForm**[LR[z]]

Out[240]//MatrixForm=

$$\begin{pmatrix} \frac{1-a z}{1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1-a z}{1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{z}{2\left(1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}\right)} & 0 & \frac{1-\frac{4 z}{5}}{1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{z}{2\left(1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}\right)} & 0 & \frac{1-\frac{4 z}{5}}{1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1-a z}{1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{z}{2\left(1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}\right)} & \frac{1-\frac{4 z}{5}}{1-\frac{4 z}{5}-a z+\frac{4 a z^2}{5}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[241]:= Phi[z\_] = LR[z].SmBOA.u2kI4[z]

$$\begin{aligned}
 \text{Out[241]} = & \left\{ \left\{ -\frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{bz(1-az)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, \theta, \theta, \theta \right\}, \right. \\
 & \left\{ \theta, -\frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{bz(1-az)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, \theta, \theta \right\}, \\
 & \left\{ -\frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( \frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + \frac{bz}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right. \\
 & \left. \theta, -\frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{3\left(1-\frac{4z}{5}\right)z}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, \theta \right\}, \\
 & \left\{ \theta, -\frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( \frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + \frac{bz}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right. \\
 & \left. \theta, -\frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{3\left(1-\frac{4z}{5}\right)z}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right\}, \{ \theta, \theta, \theta, \theta \}, \{ \theta, \theta, \theta, \theta \}, \\
 & \{ \theta, \theta, \theta, \theta \}, \{ \theta, \theta, \theta, \theta \}, \left\{ \frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{4z(1-az)}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, \theta, \theta, \theta \right\}, \\
 & \left\{ \frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( -\frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{2z}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right. \\
 & \left. \frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{a\left(1-\frac{4z}{5}\right)z}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, \theta, \theta \right\}, \\
 & \left\{ \theta, \theta, \frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{4z(1-az)}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, \theta \right\}, \\
 & \left\{ \theta, \theta, \frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( -\frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{2z}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right. \\
 & \left. \frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{a\left(1-\frac{4z}{5}\right)z}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} \right\} \}
 \end{aligned}$$

In[242]:= Together[Phi[z]]

$$\text{Out[242]} = \left\{ \left\{ -\frac{5(-1+bz)}{-5+4z}, 0, 0, 0 \right\}, \left\{ 0, -\frac{5(-1+bz)}{-5+4z}, 0, 0 \right\}, \right. \\ \left\{ \frac{(-4+5b)z^2}{2(-5+4z)(-1+az)}, 0, \frac{5-3z}{5(-1+az)}, 0 \right\}, \\ \left\{ 0, \frac{(-4+5b)z^2}{2(-5+4z)(-1+az)}, 0, \frac{5-3z}{5(-1+az)} \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \\ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\} \}$$

In[243]:= Theta[z\_] =

KroneckerProduct[I2, Transpose[Inverse[Bz[z]]].I2.Inverse[Bz[1/z]]]

$$\text{Out[243]} = \left\{ \left\{ \frac{1}{4 \left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)} + \frac{\left( 1 - \frac{3}{5z} \right) \left( 1 - \frac{3z}{5} \right)}{\left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)}, \right. \right. \\ \left. \frac{\left( 1 - \frac{b}{z} \right) z}{2 \left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)}, 0, 0 \right\}, \\ \left\{ \frac{1-bz}{2 \left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) z \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)}, \right. \\ \left. \frac{\left( 1 - \frac{b}{z} \right) (1-bz)}{\left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)}, 0, 0 \right\}, \\ \left\{ 0, 0, \frac{1}{4 \left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)} + \frac{\left( 1 - \frac{3}{5z} \right) \left( 1 - \frac{3z}{5} \right)}{\left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)}, \right. \\ \left. \frac{\left( 1 - \frac{b}{z} \right) z}{2 \left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)} \right\}, \left\{ 0, 0, \right. \\ \left. \frac{1-bz}{2 \left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) z \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)}, \frac{\left( 1 - \frac{b}{z} \right) (1-bz)}{\left( 1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z} \right) \left( 1 - \frac{3z}{5} - bz + \frac{3bz^2}{5} \right)} \right\} \}$$

In[244]:= Together[Theta[z]]

$$\text{Out[244]} = \left\{ \left\{ \frac{z(60-161z+60z^2)}{4(b-z)(-5+3z)(-3+5z)(-1+bz)}, \frac{25z^2}{2(-5+3z)(-3+5z)(-1+bz)}, 0, 0 \right\}, \right. \\ \left\{ -\frac{25z}{2(-b+z)(-5+3z)(-3+5z)}, -\frac{25z}{(-5+3z)(-3+5z)}, 0, 0 \right\}, \\ \left\{ 0, 0, \frac{z(60-161z+60z^2)}{4(b-z)(-5+3z)(-3+5z)(-1+bz)}, \frac{25z^2}{2(-5+3z)(-3+5z)(-1+bz)} \right\}, \\ \left\{ 0, 0, -\frac{25z}{2(-b+z)(-5+3z)(-3+5z)}, -\frac{25z}{(-5+3z)(-3+5z)} \right\} \}$$



In[245]:= MatrixForm[%]

Out[245]//MatrixForm=

$$\begin{pmatrix} \frac{z (60-161 z+60 z^2)}{4 (b-z) (-5+3 z) (-3+5 z) (-1+b z)} & \frac{25 z^2}{2 (-5+3 z) (-3+5 z) (-1+b z)} & 0 & 0 \\ -\frac{25 z}{2 (-b+z) (-5+3 z) (-3+5 z)} & -\frac{25 z}{(-5+3 z) (-3+5 z)} & 0 & 0 \\ 0 & 0 & \frac{z (60-161 z+60 z^2)}{4 (b-z) (-5+3 z) (-3+5 z) (-1+b z)} & \frac{25 z}{2 (-5+3 z) (-3+5 z)} \\ 0 & 0 & -\frac{25 z}{2 (-b+z) (-5+3 z) (-3+5 z)} & -\frac{25}{(-5+3 z)} \end{pmatrix}$$

In[246]:= AR[z\_] = Phi[z].Theta[z].Transpose[Phi[1/z]]

Out[246]=

{ ... 1 ... }

large output

show less

show more

show all

set size limit...

In[247]:= RatAR[z\_] = Together[AR[z]]

$$\begin{aligned} \text{Out[247]} = & \left\{ \left\{ -\frac{25 z (60 - 161 z + 60 z^2)}{4 (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \right. \right. \\ & -\frac{625 (b - z) z^2}{2 (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\ & -\frac{5 (-4 + 5 b) z (60 - 161 z + 60 z^2)}{8 (a - z) (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\ & -\frac{125 (-4 + 5 b) z^2}{4 (-a + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, 0, 0, 0, 0, \\ & \left. \frac{5 z (60 - 161 z + 60 z^2)}{4 (-b + z) (-5 + 3 z) (-5 + 4 z) (-3 + 5 z)}, -\frac{125 z^2}{2 (-5 + 3 z) (-5 + 4 z) (-3 + 5 z)}, 0, 0 \right\}, \\ & \left\{ -\frac{625 z (-1 + b z)}{2 (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \right. \\ & \frac{625 (b - z) z (-1 + b z)}{(-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\ & \frac{125 (-4 + 5 b) z (-1 + b z)}{4 (-a + z) (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\ & \frac{125 (-4 + 5 b) z (-1 + b z)}{2 (-a + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, 0, 0, 0, 0, \\ & \frac{125 z (-1 + b z)}{2 (-b + z) (-5 + 3 z) (-5 + 4 z) (-3 + 5 z)}, \frac{125 z (-1 + b z)}{(-5 + 3 z) (-5 + 4 z) (-3 + 5 z)}, 0, 0 \right\}, \\ & \left\{ \frac{5 (-4 + 5 b) z^3 (60 - 161 z + 60 z^2)}{8 (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \right. \\ & \frac{125 (-4 + 5 b) (b - z) z^4}{4 (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\ & \frac{z (60 - 161 z + 60 z^2) (1200 - 5180 z + 8376 z^2 - 1000 b z^2 + 625 b^2 z^2 - 5180 z^3 + 1200 z^4)}{400 (-a + z) (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\ & \left. \frac{z^2 (1200 - 5180 z + 8376 z^2 - 1000 b z^2 + 625 b^2 z^2 - 5180 z^3 + 1200 z^4)}{8 (-a + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \right. \end{aligned}$$

$$\begin{aligned}
& 0, 0, 0, 0, -\frac{(-4+5b)z^3(60-161z+60z^2)}{8(-b+z)(-5+3z)(-5+4z)(-3+5z)(-1+az)(-1+bz)}, \\
& \frac{25(-4+5b)z^4}{4(-5+3z)(-5+4z)(-3+5z)(-1+az)(-1+bz)}, \\
& \frac{z(60-161z+60z^2)}{20(-b+z)(-3+5z)(-1+az)(-1+bz)}, -\frac{5z^2}{2(-3+5z)(-1+az)(-1+bz)} \}, \\
& \left\{ \frac{125(-4+5b)z^3}{4(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \right. \\
& -\frac{125(-4+5b)(b-z)z^3}{2(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \\
& \frac{z(1200-5180z+8376z^2-1000bz^2+625b^2z^2-5180z^3+1200z^4)}{8(-a+z)(-b+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \\
& \left. -\frac{z(1200-5180z+8376z^2-1000bz^2+625b^2z^2-5180z^3+1200z^4)}{4(-a+z)(-5+3z)(-5+4z)(-4+5z)(-3+5z)(-1+az)}, \right. \\
& \left. -\frac{25(-4+5b)z^3}{4(-b+z)(-5+3z)(-5+4z)(-3+5z)(-1+az)}, \right. \\
& -\frac{5z}{2(-5+3z)(-5+4z)(-3+5z)(-1+az)}, -\frac{5z}{2(b-z)(-3+5z)(-1+az)}, \\
& \left. \frac{5z}{(-3+5z)(-1+az)} \right\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \left\{ \frac{5z(60-161z+60z^2)}{4(-5+3z)(-4+5z)(-3+5z)(-1+bz)}, \right. \\
& \frac{125(b-z)z^2}{2(-5+3z)(-4+5z)(-3+5z)(-1+bz)}, \\
& \frac{(-4+5b)z(60-161z+60z^2)}{8(a-z)(-b+z)(-5+3z)(-4+5z)(-3+5z)(-1+bz)}, \\
& \frac{25(-4+5b)z^2}{4(-a+z)(-5+3z)(-4+5z)(-3+5z)(-1+bz)}, 0, 0, 0, 0, \\
& \frac{25z^2}{z(60-161z+60z^2)}, 0, 0, 0 \}, \\
& -\frac{125z}{4(-b+z)(-5+3z)(-3+5z)(-1+bz)}, \frac{125z(-b+z)}{2(-5+3z)(-3+5z)(-1+bz)}, 0, 0 \}, \\
& \left\{ \frac{125z}{2(-5+3z)(-4+5z)(-3+5z)}, \frac{125z(-b+z)}{(-5+3z)(-4+5z)(-3+5z)}, \right. \\
& \frac{25(-4+5b)z}{4(a-z)(-b+z)(-5+3z)(-4+5z)(-3+5z)}, \\
& \frac{25(-4+5b)z}{25(-4+5b)z}, 0, 0, 0, 0, \\
& \frac{25z}{2(a-z)(-5+3z)(-4+5z)(-3+5z)}, \\
& -\frac{25z}{2(-b+z)(-5+3z)(-3+5z)}, -\frac{25z}{(-5+3z)(-3+5z)}, 0, 0 \}, \\
& \left\{ 0, 0, \frac{z(60-161z+60z^2)}{20(-a+z)(-b+z)(-5+3z)(-1+bz)}, \right. \\
& \frac{5z^2}{2(-a+z)(-5+3z)(-1+bz)}, 0, 0, 0, 0, 0, 0, \\
& \left. -\frac{z(60-161z+60z^2)}{4(-b+z)(-5+3z)(-3+5z)(-1+bz)}, \frac{25z^2}{2(-5+3z)(-3+5z)(-1+bz)} \right\},
\end{aligned}$$

$$\left\{ \begin{aligned} &0, 0, \frac{5z}{2(-a+z)(-b+z)(-5+3z)}, \frac{5z}{(-a+z)(-5+3z)}, 0, 0, 0, 0, \\ &0, 0, -\frac{25z}{2(-b+z)(-5+3z)(-3+5z)}, -\frac{25z}{(-5+3z)(-3+5z)} \end{aligned} \right\}$$

In[248]:= **InvBz[z\_] = Inverse[Bz[z]]**

$$\text{Out[248]} = \left\{ \left\{ \frac{1 - \frac{3z}{5}}{1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}}, 0 \right\}, \left\{ \frac{z}{2\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, \frac{1 - bz}{1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}} \right\} \right\}$$

In[249]:= **Tempor[z\_] = Transpose[InvBz[z]].I2.InvBz[1/z]**

$$\begin{aligned} \text{Out[249]} = & \left\{ \left\{ \frac{1}{4\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} + \frac{\left(1 - \frac{3}{5z}\right)\left(1 - \frac{3z}{5}\right)}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, \right. \\ & \left. \frac{\left(1 - \frac{b}{z}\right)z}{2\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} \right\}, \\ & \left\{ \frac{1 - bz}{2\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)z\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, \frac{\left(1 - \frac{b}{z}\right)(1 - bz)}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} \right\} \end{aligned}$$

In[250]:= **Psi[z\_] = KroneckerProduct[Rx, Tempor[z]]**

$$\begin{aligned} \text{Out[250]} = & \left\{ \left\{ 2 \left( \frac{1}{4\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} + \frac{\left(1 - \frac{3}{5z}\right)\left(1 - \frac{3z}{5}\right)}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} \right), \right. \\ & \left. \frac{\left(1 - \frac{b}{z}\right)z}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, 0, 0 \right\}, \\ & \left\{ \frac{1 - bz}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)z\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, \right. \\ & \left. \frac{2\left(1 - \frac{b}{z}\right)(1 - bz)}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, 0, 0 \right\}, \left\{ 0, 0, \right. \\ & 3 \left( \frac{1}{4\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} + \frac{\left(1 - \frac{3}{5z}\right)\left(1 - \frac{3z}{5}\right)}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} \right), \\ & \left. \frac{3\left(1 - \frac{b}{z}\right)z}{2\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} \right\}, \left\{ 0, 0, \right. \\ & \left. \frac{3(1 - bz)}{2\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)z\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)}, \frac{3\left(1 - \frac{b}{z}\right)(1 - bz)}{\left(1 + \frac{3b}{5z^2} - \frac{3}{5z} - \frac{b}{z}\right)\left(1 - \frac{3z}{5} - bz + \frac{3bz^2}{5}\right)} \right\} \end{aligned}$$

In[251]:= SmCOA = ArrayFlatten[{ {mI4, mCkI2}, {I4, I2kA}, {O4, O4} }]

Out[251]=  $\left\{ \left\{ -1, 0, 0, 0, a, 0, 0, 0 \right\}, \left\{ 0, -1, 0, 0, 0, a, 0, 0 \right\}, \left\{ 0, 0, -1, 0, \frac{1}{2}, 0, \frac{7}{10}, 0 \right\}, \right.$   
 $\left. \left\{ 0, 0, 0, -1, 0, \frac{1}{2}, 0, \frac{7}{10} \right\}, \left\{ 1, 0, 0, 0, -\frac{4}{5}, 0, 0, 0 \right\}, \left\{ 0, 1, 0, 0, -\frac{1}{2}, -a, 0, 0 \right\}, \right.$   
 $\left. \left\{ 0, 0, 1, 0, 0, 0, -\frac{4}{5}, 0 \right\}, \left\{ 0, 0, 0, 1, 0, 0, -\frac{1}{2}, -a \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$   
 $\left. \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0, 0, 0, 0, 0 \right\} \right\}$

In[252]:=  $WR[z\_]= \begin{pmatrix} \text{InvAzkI2}[z] & \text{O4} & \text{O4} \\ \text{O4} & \text{I2kInvAz}[z] & \text{O4} \\ \text{O4} & \text{O4} & \text{O4} \end{pmatrix}$

Out[252]=  $\left\{ \left\{ \left\{ \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \left\{ 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \right.$   
 $\left. \left\{ \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \right.$   
 $\left. \left\{ 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\},$   
 $\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\},$   
 $\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\},$   
 $\left\{ \left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\}, \right.$   
 $\left. \left\{ \left\{ \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0 \right\}, \left\{ \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0 \right\}, \right.$   
 $\left. \left\{ 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0 \right\}, \left\{ 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}} \right\} \right\},$   
 $\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\},$   
 $\left\{ \left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\}, \right.$   
 $\left. \left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\}, \right.$   
 $\left. \left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\} \right\}$

In[253]:= `WR[z_] = ArrayFlatten[WR[z]]`

Out[253]= 
$$\left\{ \left\{ \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\}, \right.$$

$$\left\{ 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, 0, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, \frac{1 - a z}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0, 0 \right\},$$

$$\left\{ 0, 0, 0, 0, 0, 0, \frac{z}{2 \left( 1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5} \right)}, \frac{1 - \frac{4 z}{5}}{1 - \frac{4 z}{5} - a z + \frac{4 a z^2}{5}}, 0, 0, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$$

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}$$

In[254]:= MyGamma[z\_] = WR[z].SmCOA.u2kI4[z]

Out[254]=  $\left\{ \left\{ -\frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{az(1-az)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, 0, 0, 0 \right\}, \right.$   
 $\left\{ 0, -\frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{az(1-az)}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, 0, 0 \right\},$   
 $\left\{ -\frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( \frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + \frac{az}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right.$   
 $\left. 0, -\frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{7\left(1-\frac{4z}{5}\right)z}{10\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, 0 \right\},$   
 $\left\{ 0, -\frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( \frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + \frac{az}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right.$   
 $\left. 0, -\frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} + \frac{7\left(1-\frac{4z}{5}\right)z}{10\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right\},$   
 $\left\{ \frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{4z(1-az)}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, 0, 0, 0 \right\},$   
 $\left\{ \frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( -\frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{2z}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right.$   
 $\left. \frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{a\left(1-\frac{4z}{5}\right)z}{1-\frac{4z}{5}-az+\frac{4az^2}{5}}, 0, 0 \right\},$   
 $\left\{ 0, 0, \frac{1-az}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{4z(1-az)}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)}, 0 \right\},$   
 $\left\{ 0, 0, \frac{z}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} + z \left( -\frac{1-\frac{4z}{5}}{2\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} - \frac{2z}{5\left(1-\frac{4z}{5}-az+\frac{4az^2}{5}\right)} \right), \right.$   
 $\left. \frac{1-\frac{4z}{5}}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} - \frac{a\left(1-\frac{4z}{5}\right)z}{1-\frac{4z}{5}-az+\frac{4az^2}{5}} \right\},$   
 $\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

In[255]:= BR[z\_] = MyGamma[z].Psi[z].Transpose[MyGamma[1/z]]

Out[255]= { ... 1 ... }

large output

show less

show more

show all

set size limit...

In[256]:= RatBR[z\_] = Together[BR[z]]

$$\begin{aligned}
 \text{Out[256]} = & \left\{ \left\{ \frac{25 (a - z) z (-1 + a z) (60 - 161 z + 60 z^2)}{2 (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \right. \right. \\
 & \frac{625 z^2 (-a + z) (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
 & - \frac{5 (-4 + 5 a) z (-1 + a z) (60 - 161 z + 60 z^2)}{4 (a - z) (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
 & - \frac{125 (-4 + 5 a) z^2 (-1 + a z)}{2 (-a + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + b z)}, \\
 & - \frac{5 z (-1 + a z) (60 - 161 z + 60 z^2)}{2 (-b + z) (-5 + 3 z) (-5 + 4 z) (-3 + 5 z) (-1 + b z)}, \\
 & - \frac{125 z^2 (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-3 + 5 z) (-1 + b z)}, 0, 0, 0, 0, 0, 0 \Big\}, \\
 & \left\{ - \frac{625 z (-a + z) (-1 + a z)}{(-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \right. \\
 & - \frac{1250 z (-a + z) (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\
 & - \frac{125 (-4 + 5 a) z (-1 + a z)}{2 (-a + z) (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\
 & - \frac{125 (-4 + 5 a) z (-1 + a z)}{(-a + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z)}, \\
 & - \frac{250 z (-1 + a z)}{(-b + z) (-5 + 3 z) (-5 + 4 z) (-3 + 5 z)}, \\
 & - \frac{250 z (-1 + a z)}{(-5 + 3 z) (-5 + 4 z) (-3 + 5 z)}, 0, 0, 0, 0, 0, 0 \Big\}, \\
 & \left\{ - \frac{5 (-4 + 5 a) (a - z) z^3 (60 - 161 z + 60 z^2)}{4 (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \right. \\
 & - \frac{125 (-4 + 5 a) z^4 (-a + z)}{2 (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\
 & - \left( \left( z (60 - 161 z + 60 z^2) (4200 - 17550 z + 27527 z^2 - 2000 a z^2 + \right. \right. \\
 & \left. \left. 1250 a^2 z^2 - 17550 z^3 + 4200 z^4) \right) / (400 (-a + z) (-b + z) \right. \\
 & \left. (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)) \right), \\
 & \frac{z^2 (4200 - 17550 z + 27527 z^2 - 2000 a z^2 + 1250 a^2 z^2 - 17550 z^3 + 4200 z^4)}{8 (-a + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\
 & - \frac{(-4 + 5 a) z^3 (60 - 161 z + 60 z^2)}{4 (-b + z) (-5 + 3 z) (-5 + 4 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\
 & - \frac{25 (-4 + 5 a) z^4}{2 (-5 + 3 z) (-5 + 4 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\
 & - \frac{3 z (-10 + 7 z) (60 - 161 z + 60 z^2)}{40 (-b + z) (-5 + 3 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, \\
 & - \frac{15 z^2 (-10 + 7 z)}{4 (-5 + 3 z) (-3 + 5 z) (-1 + a z) (-1 + b z)}, 0, 0, 0, 0 \Big\}, \\
 & \left\{ \frac{125 (-4 + 5 a) z^3 (-a + z)}{2 (-b + z) (-5 + 3 z) (-5 + 4 z) (-4 + 5 z) (-3 + 5 z) (-1 + a z)}, \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{125 (-4 + 5a) z^3 (-a + z)}{(-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \\
& \frac{z (4200 - 17550z + 27527z^2 - 2000az^2 + 1250a^2z^2 - 17550z^3 + 4200z^4)}{8 (-a + z) (-b + z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \\
& \frac{z (4200 - 17550z + 27527z^2 - 2000az^2 + 1250a^2z^2 - 17550z^3 + 4200z^4)}{4 (-a + z) (-5 + 3z) (-5 + 4z) (-4 + 5z) (-3 + 5z) (-1 + az)}, \\
& \frac{25 (-4 + 5a) z^3}{2 (-b + z) (-5 + 3z) (-5 + 4z) (-3 + 5z) (-1 + az)}, \\
& \frac{25 (-4 + 5a) z^3}{(-5 + 3z) (-5 + 4z) (-3 + 5z) (-1 + az)}, \\
& \frac{15z (-10 + 7z)}{4 (-b + z) (-5 + 3z) (-3 + 5z) (-1 + az)}, \\
& \frac{15z (-10 + 7z)}{2 (-5 + 3z) (-3 + 5z) (-1 + az)}, \{0, 0, 0, 0\}, \\
& \left\{ -\frac{5(a - z)z(60 - 161z + 60z^2)}{2 (-b + z) (-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)}, \right. \\
& \quad \frac{125z^2(-a + z)}{(-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)}, \\
& \quad \frac{(-4 + 5a)z(60 - 161z + 60z^2)}{4(a - z) (-b + z) (-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)}, \\
& \quad \frac{25(-4 + 5a)z^2}{2(-a + z) (-5 + 3z) (-4 + 5z) (-3 + 5z) (-1 + bz)}, \\
& \quad \frac{z(60 - 161z + 60z^2)}{2(-b + z) (-5 + 3z) (-3 + 5z) (-1 + bz)}, \\
& \quad \frac{25z^2}{(-5 + 3z) (-3 + 5z) (-1 + bz)}, \{0, 0, 0, 0, 0, 0\}, \\
& \left\{ -\frac{125(a - z)z}{(-b + z) (-5 + 3z) (-4 + 5z) (-3 + 5z)}, -\frac{250(a - z)z}{(-5 + 3z) (-4 + 5z) (-3 + 5z)}, \right. \\
& \quad \frac{25(-4 + 5a)z}{2(a - z) (-b + z) (-5 + 3z) (-4 + 5z) (-3 + 5z)}, \\
& \quad \frac{25z}{(a - z) (-5 + 3z) (-4 + 5z) (-3 + 5z)}, -\frac{25z}{(-b + z) (-5 + 3z) (-3 + 5z)}, \\
& \quad \frac{50z}{(-5 + 3z) (-3 + 5z)}, \{0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, \frac{3z(-7 + 10z)(60 - 161z + 60z^2)}{40(-a + z) (-b + z) (-5 + 3z) (-3 + 5z) (-1 + bz)}, \right. \\
& \quad \frac{15z^2(-7 + 10z)}{4(-a + z) (-5 + 3z) (-3 + 5z) (-1 + bz)}, 0, \\
& \quad 0, -\frac{3z(60 - 161z + 60z^2)}{4(-b + z) (-5 + 3z) (-3 + 5z) (-1 + bz)}, \\
& \quad \frac{75z^2}{2(-5 + 3z) (-3 + 5z) (-1 + bz)}, \{0, 0, 0, 0\}, \\
& \left. \left\{ 0, 0, \frac{15z(-7 + 10z)}{4(-a + z) (-b + z) (-5 + 3z) (-3 + 5z)}, \frac{15z(-7 + 10z)}{2(-a + z) (-5 + 3z) (-3 + 5z)}, \right. \right.
\end{aligned}$$



$$\begin{aligned} &0, 0, -\frac{75 z}{2(-b+z)(-5+3z)(-3+5z)}, -\frac{75 z}{(-5+3z)(-3+5z)}, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\ &\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \end{aligned}$$

In[257]:= RatARexpim[t\_] = RatAR[Exp[I t]]

$$\begin{aligned} \text{Out[257]} = &\left\{ -\frac{25 e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{4(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \right. \\ &-\frac{625 e^{2 i t} (b - e^{i t})}{2(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \\ &-\frac{5(-4+5 b) e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8(a - e^{i t})(-b + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \\ &-\frac{125(-4+5 b) e^{2 i t}}{4(-a + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \\ &0, 0, 0, 0, \frac{5 e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{4(-b + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-3+5 e^{i t})}, \\ &-\frac{125 e^{2 i t}}{2(-5+3 e^{i t})(-5+4 e^{i t})(-3+5 e^{i t})}, 0, 0\}, \\ &\left\{ -\frac{625 e^{i t} (-1 + b e^{i t})}{2(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \right. \\ &\frac{625 e^{i t} (b - e^{i t})(-1 + b e^{i t})}{(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \\ &\frac{125(-4+5 b) e^{i t} (-1 + b e^{i t})}{4(-a + e^{i t})(-b + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \\ &\frac{125(-4+5 b) e^{i t} (-1 + b e^{i t})}{2(-a + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})}, \\ &0, 0, 0, 0, \frac{125 e^{i t} (-1 + b e^{i t})}{2(-b + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-3+5 e^{i t})}, \\ &\frac{125 e^{i t} (-1 + b e^{i t})}{(-5+3 e^{i t})(-5+4 e^{i t})(-3+5 e^{i t})}, 0, 0\}, \\ &\left\{ \frac{5(-4+5 b) e^{3 i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})(-1+a e^{i t})(-1+b e^{i t})}, \right. \\ &\frac{125(-4+5 b) e^{4 i t} (b - e^{i t})}{4(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})(-1+a e^{i t})(-1+b e^{i t})}, \\ &-\left( (e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t}) (1200 - 5180 e^{i t} + 8376 e^{2 i t} - 1000 b e^{2 i t} + \right. \\ &\quad 625 b^2 e^{2 i t} - 5180 e^{3 i t} + 1200 e^{4 i t})) / (400(-a + e^{i t})(-b + e^{i t})(-5+3 e^{i t}) \\ &\quad (-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t})(-1+a e^{i t})(-1+b e^{i t}))), (e^{2 i t} \\ &\quad (1200 - 5180 e^{i t} + 8376 e^{2 i t} - 1000 b e^{2 i t} + 625 b^2 e^{2 i t} - 5180 e^{3 i t} + 1200 e^{4 i t})) / \\ &\quad (8(-a + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-4+5 e^{i t})(-3+5 e^{i t}) \\ &\quad (-1+a e^{i t})(-1+b e^{i t}))), 0, 0, 0, 0, \\ &\quad \frac{(-4+5 b) e^{3 i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{8(-b + e^{i t})(-5+3 e^{i t})(-5+4 e^{i t})(-3+5 e^{i t})(-1+a e^{i t})(-1+b e^{i t})}, \\ &\quad \frac{25(-4+5 b) e^{4 i t}}{4(-5+3 e^{i t})(-5+4 e^{i t})(-3+5 e^{i t})(-1+a e^{i t})(-1+b e^{i t})}, \end{aligned}$$

$$\begin{aligned}
& \frac{e^{it} (60 - 161 e^{it} + 60 e^{2it})}{20 (-b + e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it}) (-1 + b e^{it})}, \\
& - \frac{5 e^{2it}}{2 (-3 + 5 e^{it}) (-1 + a e^{it}) (-1 + b e^{it})} \Big\}, \\
& \left\{ \frac{125 (-4 + 5 b) e^{3it}}{4 (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \right. \\
& - \frac{125 (-4 + 5 b) e^{3it} (b - e^{it})}{2 (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \\
& - \left( (e^{it} (1200 - 5180 e^{it} + 8376 e^{2it} - 1000 b e^{2it} + \right. \\
& \quad 625 b^2 e^{2it} - 5180 e^{3it} + 1200 e^{4it})) / (8 (-a + e^{it}) (-b + e^{it}) \\
& \quad (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})) \Big), \\
& \left. \frac{e^{it} (1200 - 5180 e^{it} + 8376 e^{2it} - 1000 b e^{2it} + 625 b^2 e^{2it} - 5180 e^{3it} + 1200 e^{4it})}{4 (-a + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \right. \\
& \quad \frac{0, 0, 0, 0, - \frac{25 (-4 + 5 b) e^{3it}}{4 (-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \\
& \quad - \frac{25 (-4 + 5 b) e^{3it}}{2 (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \\
& \quad - \frac{5 e^{it}}{2 (b - e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it})}, \left. \frac{5 e^{it}}{(-3 + 5 e^{it}) (-1 + a e^{it})} \right\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{5 e^{it} (60 - 161 e^{it} + 60 e^{2it})}{4 (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \right. \\
& \quad \frac{125 e^{2it} (b - e^{it})}{2 (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \\
& \quad \frac{(-4 + 5 b) e^{it} (60 - 161 e^{it} + 60 e^{2it})}{8 (a - e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \\
& \quad \frac{25 (-4 + 5 b) e^{2it}}{4 (-a + e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \\
& \quad \frac{e^{it} (60 - 161 e^{it} + 60 e^{2it})}{0, 0, 0, 0, - \frac{25 e^{2it}}{4 (-b + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \\
& \quad \frac{25 e^{2it}}{2 (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, 0, 0 \Big\}, \\
& \left\{ \frac{125 e^{it}}{2 (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \frac{125 e^{it} (-b + e^{it})}{(-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \right. \\
& \quad \frac{25 (-4 + 5 b) e^{it}}{4 (a - e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \\
& \quad \frac{25 (-4 + 5 b) e^{it}}{2 (a - e^{it}) (-5 + 3 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, 0, 0, 0, 0, \\
& \quad \frac{25 e^{it}}{2 (-b + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it})}, - \frac{25 e^{it}}{(-5 + 3 e^{it}) (-3 + 5 e^{it})}, 0, 0 \Big\},
\end{aligned}$$

$$\left\{0, 0, \frac{e^{it} (60 - 161 e^{it} + 60 e^{2it})}{20 (-a + e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-1 + b e^{it})}, \right. \\ \left. - \frac{5 e^{2it}}{2 (-a + e^{it}) (-5 + 3 e^{it}) (-1 + b e^{it})}, 0, 0, 0, 0, 0, \right. \\ \left. 0, - \frac{e^{it} (60 - 161 e^{it} + 60 e^{2it})}{4 (-b + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \right. \\ \left. \frac{25 e^{2it}}{2 (-5 + 3 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})} \right\}, \\ \left\{0, 0, \frac{5 e^{it}}{2 (-a + e^{it}) (-b + e^{it}) (-5 + 3 e^{it})}, \frac{5 e^{it}}{(-a + e^{it}) (-5 + 3 e^{it})}, 0, 0, 0, \right. \\ \left. 0, 0, 0, - \frac{25 e^{it}}{2 (-b + e^{it}) (-5 + 3 e^{it}) (-3 + 5 e^{it})}, - \frac{25 e^{it}}{(-5 + 3 e^{it}) (-3 + 5 e^{it})} \right\}$$

In[258]:= RatBRexpim[t\_] = RatBR[Exp[I t]]

$$\text{Out[258]} = \left\{ \left\{ \frac{25 e^{it} (a - e^{it}) (-1 + a e^{it}) (60 - 161 e^{it} + 60 e^{2it})}{2 (-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \right. \right. \\ \frac{625 e^{2it} (-a + e^{it}) (-1 + a e^{it})}{(-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, \\ \frac{5 (-4 + 5 a) e^{it} (-1 + a e^{it}) (60 - 161 e^{it} + 60 e^{2it})}{4 (a - e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})} \\ \left. - \frac{125 (-4 + 5 a) e^{2it} (-1 + a e^{it})}{125 (-4 + 5 a) e^{2it} (-1 + a e^{it})}, \right. \\ \left. - \frac{2 (-a + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{5 e^{it} (-1 + a e^{it}) (60 - 161 e^{it} + 60 e^{2it})}, \right. \\ \left. \frac{2 (-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}{125 e^{2it} (-1 + a e^{it})}, \right. \\ \left. - \frac{625 e^{it} (-a + e^{it}) (-1 + a e^{it})}{(-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it}) (-1 + b e^{it})}, 0, 0, 0, 0, 0, 0 \right\}, \\ \left\{ - \frac{625 e^{it} (-a + e^{it}) (-1 + a e^{it})}{(-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \right. \\ \frac{1250 e^{it} (-a + e^{it}) (-1 + a e^{it})}{(-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \\ \frac{125 (-4 + 5 a) e^{it} (-1 + a e^{it})}{2 (-a + e^{it}) (-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \\ \frac{125 (-4 + 5 a) e^{it} (-1 + a e^{it})}{(-a + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it})}, \\ \frac{125 e^{it} (-1 + a e^{it})}{(-b + e^{it}) (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it})}, \\ \frac{250 e^{it} (-1 + a e^{it})}{(-5 + 3 e^{it}) (-5 + 4 e^{it}) (-3 + 5 e^{it})}, 0, 0, 0, 0, 0, 0 \right\}, \\ \left\{ - \left( \left( 5 (-4 + 5 a) e^{3it} (a - e^{it}) (60 - 161 e^{it} + 60 e^{2it}) \right) / \left( 4 (-b + e^{it}) (-5 + 3 e^{it}) \right. \right. \right. \\ \left. \left. \left. (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it}) (-1 + b e^{it}) \right) \right), \right. \\ \frac{125 (-4 + 5 a) e^{4it} (-a + e^{it})}{2 (-5 + 3 e^{it}) (-5 + 4 e^{it}) (-4 + 5 e^{it}) (-3 + 5 e^{it}) (-1 + a e^{it}) (-1 + b e^{it})}, \\ \left. - \left( \left( e^{it} (60 - 161 e^{it} + 60 e^{2it}) (4200 - 17550 e^{it} + 27527 e^{2it} - 2000 a e^{2it} + \right. \right. \right. \\ \left. \left. \left. 1250 a^2 e^{2it} - 17550 e^{3it} + 4200 e^{4it}) \right) / \left( 400 (-a + e^{it}) (-b + e^{it}) \right) \right) \right\}$$

$$\begin{aligned}
& \left( (-5+3e^{it}) (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it}) (-1+be^{it}) \right), \\
& \left( e^{2it} (4200 - 17550e^{it} + 27527e^{2it} - 2000ae^{2it} + 1250a^2e^{2it} - \right. \\
& \quad \left. 17550e^{3it} + 4200e^{4it}) \right) / \left( 8 (-a+e^{it}) (-5+3e^{it}) \right. \\
& \quad \left. (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it}) (-1+be^{it}) \right), \\
& \quad \left( -4+5a \right) e^{3it} (60 - 161e^{it} + 60e^{2it}) \\
& - \frac{4 (-b+e^{it}) (-5+3e^{it}) (-5+4e^{it}) (-3+5e^{it}) (-1+ae^{it}) (-1+be^{it})}{25 (-4+5a) e^{4it}}, \\
& \frac{2 (-5+3e^{it}) (-5+4e^{it}) (-3+5e^{it}) (-1+ae^{it}) (-1+be^{it})}{3e^{it} (-10+7e^{it}) (60 - 161e^{it} + 60e^{2it})}, \\
& \frac{40 (-b+e^{it}) (-5+3e^{it}) (-3+5e^{it}) (-1+ae^{it}) (-1+be^{it})}{15e^{2it} (-10+7e^{it})}, \\
& - \frac{4 (-5+3e^{it}) (-3+5e^{it}) (-1+ae^{it}) (-1+be^{it})}{125 (-4+5a) e^{3it} (-a+e^{it})}, 0, 0, 0, 0 \}, \\
& \left\{ \frac{2 (-b+e^{it}) (-5+3e^{it}) (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it})}{125 (-4+5a) e^{3it} (-a+e^{it})}, \right. \\
& \quad \left. (-5+3e^{it}) (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it}) \right), \\
& - \left( \left( e^{it} (4200 - 17550e^{it} + 27527e^{2it} - 2000ae^{2it} + \right. \right. \\
& \quad \left. \left. 1250a^2e^{2it} - 17550e^{3it} + 4200e^{4it}) \right) / \left( 8 (-a+e^{it}) (-b+e^{it}) \right. \right. \\
& \quad \left. \left. (-5+3e^{it}) (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it}) \right) \right), \\
& - \left( \left( e^{it} (4200 - 17550e^{it} + 27527e^{2it} - 2000ae^{2it} + 1250a^2e^{2it} - \right. \right. \\
& \quad \left. \left. 17550e^{3it} + 4200e^{4it}) \right) / \right. \\
& \quad \left. \left( 4 (-a+e^{it}) (-5+3e^{it}) (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it}) \right) \right), \\
& \quad \left. \left( 4 (-a+e^{it}) (-5+3e^{it}) (-5+4e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+ae^{it}) \right) \right), \\
& \quad \left. 25 (-4+5a) e^{3it} \right), \\
& - \frac{2 (-b+e^{it}) (-5+3e^{it}) (-5+4e^{it}) (-3+5e^{it}) (-1+ae^{it})}{25 (-4+5a) e^{3it}}, \\
& - \frac{(-5+3e^{it}) (-5+4e^{it}) (-3+5e^{it}) (-1+ae^{it})}{15e^{it} (-10+7e^{it})}, \\
& \frac{4 (-b+e^{it}) (-5+3e^{it}) (-3+5e^{it}) (-1+ae^{it})}{15e^{it} (-10+7e^{it})}, \\
& \frac{2 (-5+3e^{it}) (-3+5e^{it}) (-1+ae^{it})}{5e^{it} (a - e^{it}) (60 - 161e^{it} + 60e^{2it})}, 0, 0, 0, 0 \}, \\
& \left\{ - \frac{2 (-b+e^{it}) (-5+3e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+be^{it})}{125e^{2it} (-a+e^{it})}, \right. \\
& - \frac{(-5+3e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+be^{it})}{(-4+5a) e^{it} (60 - 161e^{it} + 60e^{2it})}, \\
& \frac{4 (a - e^{it}) (-b+e^{it}) (-5+3e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+be^{it})}{25 (-4+5a) e^{2it}}, \\
& \frac{2 (-a+e^{it}) (-5+3e^{it}) (-4+5e^{it}) (-3+5e^{it}) (-1+be^{it})}{e^{it} (60 - 161e^{it} + 60e^{2it})}, \\
& - \frac{2 (-b+e^{it}) (-5+3e^{it}) (-3+5e^{it}) (-1+be^{it})}{25e^{2it}}, \\
& \left. \frac{(-5+3e^{it}) (-3+5e^{it}) (-1+be^{it})}{(-5+3e^{it}) (-3+5e^{it}) (-1+be^{it})}, 0, 0, 0, 0, 0, 0 \right\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ -\frac{125 e^{i t} (a - e^{i t})}{(-b + e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \right. \\
& -\frac{250 e^{i t} (a - e^{i t})}{(-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& \frac{25 (-4 + 5 a) e^{i t}}{2 (a - e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& \frac{25 (-4 + 5 a) e^{i t}}{(a - e^{i t}) (-5 + 3 e^{i t}) (-4 + 5 e^{i t}) (-3 + 5 e^{i t})}, \\
& -\frac{25 e^{i t}}{(-b + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, \\
& -\frac{50 e^{i t}}{(-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, 0, 0, 0, 0, 0, 0 \Big\}, \\
& \left\{ 0, 0, \frac{3 e^{i t} (-7 + 10 e^{i t}) (60 - 161 e^{i t} + 60 e^{2 i t})}{40 (-a + e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \right. \\
& -\frac{15 e^{2 i t} (-7 + 10 e^{i t})}{4 (-a + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, 0, \\
& 0, -\frac{3 e^{i t} (60 - 161 e^{i t} + 60 e^{2 i t})}{4 (-b + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, \\
& \frac{75 e^{2 i t}}{2 (-5 + 3 e^{i t}) (-3 + 5 e^{i t}) (-1 + b e^{i t})}, 0, 0, 0, 0 \Big\}, \\
& \left\{ 0, 0, \frac{15 e^{i t} (-7 + 10 e^{i t})}{4 (-a + e^{i t}) (-b + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, \right. \\
& \frac{15 e^{i t} (-7 + 10 e^{i t})}{2 (-a + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, 0, 0, \\
& -\frac{75 e^{i t}}{2 (-b + e^{i t}) (-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, -\frac{75 e^{i t}}{(-5 + 3 e^{i t}) (-3 + 5 e^{i t})}, 0, 0, 0, 0 \Big\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \Big\}
\end{aligned}$$

In[259]:= Assuming[a > -1 && a < 1 && b > -1 && b < 1,

GmatrixAR = Integrate[(1/(2 Pi)) RatARexpim[t], {t, 0, 2 Pi}]]

$$\begin{aligned}
\text{Out[259]} = & \left\{ \left\{ \frac{43925}{7488}, -\frac{625 (-35 + 37 b)}{3744}, \right. \right. \\
& \frac{5 (-4 + 5 b) (-820925 + a (462180 - 253008 b) + 462180 b)}{14976 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)}, \\
& \frac{125 (-875 + 444 a) (-4 + 5 b)}{7488 (-5 + 3 a) (-5 + 4 a)}, 0, 0, 0, 0, \frac{5 (-7285 + 3396 b)}{832 (-5 + 3 b) (-5 + 4 b)}, -\frac{375}{416}, 0, 0 \Big\}, \\
& \left\{ -\frac{625 (-35 + 37 b)}{3744}, \frac{625 (37 + b (-70 + 37 b))}{1872}, \right. \\
& \frac{125 (-4 + 5 b) (-16835 + (28897 - 10500 b) b + 12 a (781 + 37 b (-35 + 12 b)))}{7488 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)}, \\
& \frac{125 (-4 + 5 b) (-781 + a (420 - 444 b) + 875 b)}{3744 (-5 + 3 a) (-5 + 4 a)}, 0, 0, 0,
\end{aligned}$$

$$\begin{aligned}
& 0, \frac{25}{416} \left( -15 + \frac{144}{5-4b} + \frac{208}{-5+3b} \right), \frac{125}{208} (-5+3b), 0, 0 \}, \\
& \left\{ \frac{5(-4+5b)(-820925+a(462180-253008b)+462180b)}{14976(-5+3a)(-5+4a)(-5+3b)(-5+4b)}, \right. \\
& \frac{125(-4+5b)(-16835+(28897-10500b)b+12a(781+37b(-35+12b)))}{7488(-5+3a)(-5+4a)(-5+3b)(-5+4b)}, \\
& (25(-47709200+102369880b-49400941b^2-20010655b^3+13177500b^4) + \\
& a^2(-937070400+2724188560b-2978197292b^2+1318277075b^3+25503125b^4 - \\
& 116512500b^5) + 12a^3(22464000-79158800b+90688600b^2-17951341b^3 - \\
& 30793375b^4+13177500b^5) - 5a(-292224400+642897160b - \\
& 501750837b^2+287393120b^3-205349375b^4+65887500b^5)) / \\
& (748800(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)), \\
& (-625(4-5b)^2(-875+444b) + \\
& 25a(379600+b(-1180440+b(1661481+125b(-9019+2220b)))) - \\
& 12a^3(-187200+b(-87920+b(962744+125b(-7927+2220b)))) + \\
& 5a^2(-2150400+b(3548560+b(-539692+125b(-13561+5460b)))) / \\
& (14976(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)), 0, 0, 0, \\
& (-4+5b)(5625+b(4875-5625a+20(1165-624b)b+3ab(-5785+3396b))) \\
& 0, - \frac{1664(-5+3a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)}{675(-4+5b)}, \\
& \frac{832(-5+3a)(-5+3b)}{-625+60(10-3b)b+3a(100+b(-120+61b))}, \\
& \frac{20(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}{15}, \\
& - \frac{2(-5+3a)(-5+3b)}{15} \}, \\
& \left\{ \frac{125(-875+444a)(-4+5b)}{7488(-5+3a)(-5+4a)}, \frac{125(-4+5b)(-781+a(420-444b)+875b)}{3744(-5+3a)(-5+4a)}, \right. \\
& (-625(4-5b)^2(-875+444b) + \\
& 25a(379600+b(-1180440+b(1661481+125b(-9019+2220b)))) - \\
& 12a^3(-187200+b(-87920+b(962744+125b(-7927+2220b)))) + \\
& 5a^2(-2150400+b(3548560+b(-539692+125b(-13561+5460b)))) / \\
& (14976(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)), \\
& (-455a(-44+25b)(4+25b)+12a^2(7312+4625b(-8+5b)) - \\
& 25(22288+4625b(-8+5b))) / (7488(-5+3a)(-5+4a)(-1+a^2)), \\
& 0, 0, 0, 0, \frac{25(-4+5b)(375+b(100+3a(-125+36b)))}{832(-5+3a)(-5+3b)(-5+4b)(-1+ab)}, \\
& - \frac{225(-4+5b)}{416(-5+3a)}, - \frac{5a}{2(-5+3a)(-1+ab)}, \frac{5}{-5+3a} \}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ \frac{5(-7285+3396b)}{832(-5+3b)(-5+4b)}, \frac{25}{416} \left( -15 + \frac{144}{5-4b} + \frac{208}{-5+3b} \right), \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{(-4+5b)(-5625+b(-4875+5625a+3a(5785-3396b)b+20b(-1165+624b)))}{1664(-5+3a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)}, \\
& \frac{25(-4+5b)(375+b(100+3a(-125+36b)))}{832(-5+3a)(-5+3b)(-5+4b)(-1+ab)}, 0, \\
& \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b}, 0, 0\}, \\
& \left\{ -\frac{375}{416}, \frac{125}{208}(-5+3b), \frac{675(-4+5b)}{832(-5+3a)(-5+3b)}, \right. \\
& \left. -\frac{225(-4+5b)}{416(-5+3a)}, 0, 0, 0, 0, \frac{75}{160-96b}, \frac{25}{16}, 0, 0\right\}, \\
& \left\{ 0, 0, \frac{-625+60(10-3b)b+3a(100+b(-120+61b))}{20(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}, \right. \\
& \left. -\frac{5a}{2(-5+3a)(-1+ab)}, 0, 0, 0, 0, 0, 0, \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b}\right\}, \\
& \left\{ 0, 0, -\frac{15}{2(-5+3a)(-5+3b)}, \frac{5}{-5+3a}, 0, 0, 0, 0, 0, 0, \frac{75}{160-96b}, \frac{25}{16}\right\}
\end{aligned}$$

In[260]:= Assuming[a > -1 && a < 1 && b > -1 && b < 1,

GmatrixBR = Integrate[(1/(2 Pi)) RatBRexpim[t], {t, 0, 2 Pi}]]

$$\begin{aligned}
\text{Out[260]} = & \left\{ \left\{ \frac{25(-5(8785+a(-15406+8785a))-13(1195+a(-3514+1195a))b+}{12(1757+a(-3830+1757a))b^2} \right\} / (3744(-5+3b)(-5+4b)(-1+b^2)), \right. \\
& \frac{625(875-444b+a(-1706+a(875-444b)+840b))}{1872(-5+3b)(-5+4b)}, \\
& (5(-4+5a)(-820925+12a^3b(43925+(15535-21084b)b)+5b(-100139+ \\
& 12b(4165+1872b))+a^2(12+35b)(-43925+b(-15535+21084b))+ \\
& a(1425055+b(1666042-b(237245+551292b)))) / \\
& (7488(-5+3a)(-5+4a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)), \\
& \frac{125 \left( -\frac{135(-5+3a)(-4+5a)}{(-3+5a)(-5+3b)} + \frac{320(-5+4a)}{-5+4b} - \frac{1872a(-1+a^2)}{(-5+3a)(-5+4a)(-3+5a)(-1+ab)} \right)}{3744}, \\
& \frac{5(7285-3b(507+500b)+a(-1875+b(-5785+3396b)))}{416(-5+3b)(-5+4b)(-1+b^2)}, \\
& \frac{375(-5+3a)}{208(-5+3b)}, \\
& 0, 0, 0, 0, 0, 0\}, \\
& \left\{ -\frac{625(-875+444b+a(1706-840b+a(-875+444b)))}{1872(-5+3b)(-5+4b)}, \right. \\
& \frac{625}{936}(37+a(-70+37a)), \\
& \frac{25(-4+5a) \left( \frac{243(3-5a)}{(-5+3a)(-5+3b)} + \frac{1024(-4+5a)}{(-5+4a)(-5+4b)} \right)}{3744}, \\
& \frac{125(-4+5a)(-781+37(35-12a)a)}{1872(-5+3a)(-5+4a)}, \\
& \left. -\frac{125(139-60b+a(-125+36b))}{208(-5+3b)(-5+4b)}, \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{125}{104} (-5+3a), 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ \left( 5(-4+5a) \left( -820925 + 12a^3b(43925 + (15535 - 21084b)b) + 5b(-100139 + \right. \right. \right. \\
& \quad \left. \left. \left. 12b(4165 + 1872b) \right) + a^2(12 + 35b)(-43925 + b(-15535 + 21084b)) + \right. \right. \\
& \quad \left. \left. \left. a(1425055 + b(1666042 - b(237245 + 551292b))) \right) \right) \right\} / \\
& \quad (7488(-5+3a)(-5+4a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)), \\
& \frac{25(-4+5a) \left( \frac{243(3-5a)}{(-5+3a)(-5+3b)} + \frac{1024(-4+5a)}{(-5+4a)(-5+4b)} \right)}{3744}, \\
& - \left( (-15000a^5b(-43925 - 15535b + 21084b^2) + \right. \\
& \quad a^2(4152214550 - 9094972995b + 9213408409b^2 - 3033829860b^3) + \\
& \quad 25(133077775 - 205853635b + 142252332b^2 - 37739520b^3) + \\
& \quad 250a^4(-2635500 - 8213975b - 204025b^2 + 2956164b^3) + \\
& \quad 5a(-1468518175 + 2342889320b - 1905168349b^2 + 583563300b^3) + \\
& \quad \left. a^3(596180750 + 3891936500b - 3493857610b^2 + 672179784b^3) \right) / \\
& \quad (748800(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)), \\
& - \left( (25(-649105 + a(754109 + 2a(526835 + a(-877873 + 262500a)))) + 5(1572420 + \right. \\
& \quad a(4089089 + 5a(-2697125 + 2a(554737 - 3125a(-173 + 84a))))))b + \\
& \quad 12a(-1310375 + a(2386315 + 2a(-123131 + 125a(-5827 + 2220a))))b^2 \Big) / \\
& \quad (14976(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)), \\
& \quad (-4+5a)(5625 + b(4875 - 5625a + 20(1165 - 624b)b + 3ab(-5785 + 3396b))) \Big) \\
& \quad - \frac{832(-5+3a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)}{675(-4+5a)}, \\
& \frac{416(-5+3a)(-5+3b)}{3(25(-785 + 384a) - 4285(-5+3a)b + 3(-2240 + 2069a)b^2)}, \\
& \frac{640(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}{1305}, \\
& - \frac{64(-5+3a)(-5+3b)}{0, 0, 0, 0 \}, \\
& \left\{ \frac{125 \left( -\frac{135(-5+3a)(-4+5a)}{(-3+5a)(-5+3b)} + \frac{320(-5+4a)}{-5+4b} - \frac{1872a(-1+a^2)}{(-5+3a)(-5+4a)(-3+5a)(-1+ab)} \right)}{3744}, \right. \\
& \frac{125(-4+5a)(-781 + 37(35 - 12a)a)}{1872(-5+3a)(-5+4a)}, \\
& - \left( (25(-649105 + a(754109 + 2a(526835 + a(-877873 + 262500a)))) + 5(1572420 + \right. \\
& \quad a(4089089 + 5a(-2697125 + 2a(554737 - 3125a(-173 + 84a))))))b + \\
& \quad 12a(-1310375 + a(2386315 + 2a(-123131 + 125a(-5827 + 2220a))))b^2 \Big) / \\
& \quad (14976(-5+3a)(-5+4a)(-1+a^2)(-5+3b)(-5+4b)(-1+ab)), \\
& \quad -1310375 + a(2386315 + 2a(-123131 + 125a(-5827 + 2220a))) \Big) \\
& \quad \frac{7488(-5+3a)(-5+4a)(-1+a^2)}{25(-4+5a)(375 + b(100 + 3a(-125 + 36b)))}, \\
& \frac{416(-5+3a)(-5+3b)(-5+4b)(-1+ab)}{225(-4+5a)}, \\
& - \frac{208(-5+3a)}{208(-5+3a)},
\end{aligned}$$



$$\begin{aligned}
& - \frac{15 (25 + a (-160 + 87 b))}{64 (-5 + 3 a) (-5 + 3 b) (-1 + a b)}, \\
& \frac{435}{32 (-5 + 3 a)}, 0, 0, 0, 0\}, \\
& \left\{ \frac{5 (7285 - 3 b (507 + 500 b) + a (-1875 + b (-5785 + 3396 b)))}{416 (-5 + 3 b) (-5 + 4 b) (-1 + b^2)}, \right. \\
& - \frac{125 (139 - 60 b + a (-125 + 36 b))}{208 (-5 + 3 b) (-5 + 4 b)}, \\
& \frac{(-4 + 5 a) (-5625 + b (-4875 + 5625 a + 3 a (5785 - 3396 b) b + 20 b (-1165 + 624 b)))}{832 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b) (-1 + b^2)}, \\
& \frac{25 (-4 + 5 a) (375 + b (100 + 3 a (-125 + 36 b)))}{416 (-5 + 3 a) (-5 + 3 b) (-5 + 4 b) (-1 + a b)}, \\
& \frac{445 - 117 b}{32 (-5 + 3 b) (-1 + b^2)}, \\
& \frac{75}{80 - 48 b}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ -\frac{375 (-5 + 3 a)}{208 (-5 + 3 b)}, \frac{125}{104} (-5 + 3 a), \frac{675 (-4 + 5 a)}{416 (-5 + 3 a) (-5 + 3 b)}, \right. \\
& -\frac{225 (-4 + 5 a)}{208 (-5 + 3 a)}, \frac{75}{80 - 48 b}, \\
& \frac{25}{8}, 0, 0, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, \frac{3 (25 (-785 + 384 a) - 4285 (-5 + 3 a) b + 3 (-2240 + 2069 a) b^2)}{640 (-5 + 3 a) (-5 + 3 b) (-1 + a b) (-1 + b^2)}, \right. \\
& - \frac{15 (25 + a (-160 + 87 b))}{64 (-5 + 3 a) (-5 + 3 b) (-1 + a b)}, 0, 0, \\
& - \frac{3 (-445 + 117 b)}{64 (-5 + 3 b) (-1 + b^2)}, \frac{225}{160 - 96 b}, 0, 0, 0, 0\}, \\
& \left\{ 0, 0, -\frac{1305}{64 (-5 + 3 a) (-5 + 3 b)}, \frac{435}{32 (-5 + 3 a)}, \right. \\
& 0, 0, \frac{225}{160 - 96 b}, \frac{75}{16}, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
& \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \}
\end{aligned}$$

In[261]:= Gmatrix = GmatrixAR + GmatrixBR

$$\begin{aligned}
\text{Out[261]} = & \left\{ \frac{43925}{7488} + (25 (-5 (8785 + a (-15406 + 8785 a)) - 13 (1195 + a (-3514 + 1195 a)) b + \right. \\
& \left. 12 (1757 + a (-3830 + 1757 a)) b^2) \right) / (3744 (-5 + 3 b) (-5 + 4 b) (-1 + b^2)), \\
& - \frac{625 (-35 + 37 b)}{3744} + \frac{625 (875 - 444 b + a (-1706 + a (875 - 444 b) + 840 b))}{1872 (-5 + 3 b) (-5 + 4 b)}, \\
& \frac{5 (-4 + 5 b) (-820925 + a (462180 - 253008 b) + 462180 b)}{14976 (-5 + 3 a) (-5 + 4 a) (-5 + 3 b) (-5 + 4 b)} + \\
& (5 (-4 + 5 a) (-820925 + 12 a^3 b (43925 + (15535 - 21084 b) b) + 5 b (-100139 +
\end{aligned}$$

$$\begin{aligned}
& 12b(4165 + 1872b) + a^2(12 + 35b)(-43925 + b(-15535 + 21084b)) + \\
& a(1425055 + b(1666042 - b(237245 + 551292b))) \Big) / \\
& (7488(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)(-1 + ab)(-1 + b^2)), \\
& \frac{125(-875 + 444a)(-4 + 5b)}{7488(-5 + 3a)(-5 + 4a)} + \\
& \frac{125 \left( -\frac{135(-5+3a)(-4+5a)}{(-3+5a)(-5+3b)} + \frac{320(-5+4a)}{-5+4b} - \frac{1872a(-1+a^2)}{(-5+3a)(-5+4a)(-3+5a)(-1+ab)} \right)}{3744}, \\
& \frac{5(7285 - 3b(507 + 500b) + a(-1875 + b(-5785 + 3396b)))}{416(-5 + 3b)(-5 + 4b)(-1 + b^2)}, \\
& -\frac{375(-5 + 3a)}{208(-5 + 3b)}, \\
& 0, \\
& 0, \frac{5(-7285 + 3396b)}{832(-5 + 3b)(-5 + 4b)}, \\
& -\frac{375}{416}, 0, 0 \Big\}, \\
& \left\{ -\frac{625(-35 + 37b)}{3744} - \frac{625(-875 + 444b + a(1706 - 840b + a(-875 + 444b)))}{1872(-5 + 3b)(-5 + 4b)}, \right. \\
& \frac{625}{936}(37 + a(-70 + 37a)) + \frac{625(37 + b(-70 + 37b))}{1872}, \\
& \frac{25(-4 + 5a) \left( \frac{243(3-5a)}{(-5+3a)(-5+3b)} + \frac{1024(-4+5a)}{(-5+4a)(-5+4b)} \right)}{3744} + \\
& \frac{125(-4 + 5b)(-16835 + (28897 - 10500b)b + 12a(781 + 37b(-35 + 12b)))}{7488(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)}, \\
& \frac{125(-4 + 5a)(-781 + 37(35 - 12a)a)}{1872(-5 + 3a)(-5 + 4a)} + \\
& \frac{125(-4 + 5b)(-781 + a(420 - 444b) + 875b)}{3744(-5 + 3a)(-5 + 4a)}, \\
& -\frac{125(139 - 60b + a(-125 + 36b))}{208(-5 + 3b)(-5 + 4b)}, \frac{125}{104}(-5 + 3a), 0, 0, \\
& \frac{25}{416} \left( -15 + \frac{144}{5 - 4b} + \frac{208}{-5 + 3b} \right), \frac{125}{208}(-5 + 3b), 0, 0 \Big\}, \\
& \left\{ \frac{5(-4 + 5b)(-820925 + a(462180 - 253008b) + 462180b)}{14976(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)} + \right. \\
& (5(-4 + 5a)(-820925 + 12a^3b(43925 + (15535 - 21084b)b) + 5b(-100139 + \\
& 12b(4165 + 1872b)) + a^2(12 + 35b)(-43925 + b(-15535 + 21084b)) + \\
& a(1425055 + b(1666042 - b(237245 + 551292b)))) \Big) / \\
& (7488(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)(-1 + ab)(-1 + b^2)), \\
& \frac{25(-4 + 5a) \left( \frac{243(3-5a)}{(-5+3a)(-5+3b)} + \frac{1024(-4+5a)}{(-5+4a)(-5+4b)} \right)}{3744} + \\
& \frac{125(-4 + 5b)(-16835 + (28897 - 10500b)b + 12a(781 + 37b(-35 + 12b)))}{7488(-5 + 3a)(-5 + 4a)(-5 + 3b)(-5 + 4b)}, \\
& - \left( (-15000a^5b(-43925 - 15535b + 21084b^2) + \right. \\
& \left. a^2(4152214550 - 9094972995b + 9213408409b^2 - 3033829860b^3) + \right.
\end{aligned}$$

$$\begin{aligned}
& 25 (133\,077\,775 - 205\,853\,635\,b + 142\,252\,332\,b^2 - 37\,739\,520\,b^3) + \\
& 250\,a^4 (-2\,635\,500 - 8\,213\,975\,b - 204\,025\,b^2 + 2\,956\,164\,b^3) + \\
& 5\,a (-1\,468\,518\,175 + 2\,342\,889\,320\,b - 1\,905\,168\,349\,b^2 + 583\,563\,300\,b^3) + \\
& a^3 (596\,180\,750 + 3\,891\,936\,500\,b - 3\,493\,857\,610\,b^2 + 672\,179\,784\,b^3) \Big/ (748\,800 \\
& (-5 + 3\,a) (-5 + 4\,a) (-1 + a^2) (-5 + 3\,b) (-5 + 4\,b) (-1 + a\,b) (-1 + b^2) \Big) + \\
& (25 (-47\,709\,200 + 102\,369\,880\,b - 49\,400\,941\,b^2 - 20\,010\,655\,b^3 + 13\,177\,500\,b^4) + \\
& a^2 (-937\,070\,400 + 2\,724\,188\,560\,b - 2\,978\,197\,292\,b^2 + 1\,318\,277\,075\,b^3 + 25\,503\,125\,b^4 - \\
& 116\,512\,500\,b^5) + 12\,a^3 (22\,464\,000 - 79\,158\,800\,b + 90\,688\,600\,b^2 - 17\,951\,341\,b^3 - \\
& 30\,793\,375\,b^4 + 13\,177\,500\,b^5) - 5\,a (-292\,224\,400 + 642\,897\,160\,b - \\
& 501\,750\,837\,b^2 + 287\,393\,120\,b^3 - 205\,349\,375\,b^4 + 65\,887\,500\,b^5) \Big/ \\
& (748\,800 (-5 + 3\,a) (-5 + 4\,a) (-1 + a^2) (-5 + 3\,b) (-5 + 4\,b) (-1 + a\,b) (-1 + b^2) \Big), \\
& - \Big( (25 (-649\,105 + a (754\,109 + 2\,a (526\,835 + a (-877\,873 + 262\,500\,a) \Big) \Big) + 5 (1\,572\,420 + \\
& a (4\,089\,089 + 5\,a (-2\,697\,125 + 2\,a (554\,737 - 3125\,a (-173 + 84\,a) \Big) \Big) \Big) \Big) \Big) b + \\
& 12\,a (-1\,310\,375 + a (2\,386\,315 + 2\,a (-123\,131 + 125\,a (-5827 + 2220\,a) \Big) \Big) \Big) \Big) b^2 \Big) \Big/ \\
& (14\,976 (-5 + 3\,a) (-5 + 4\,a) (-1 + a^2) (-5 + 3\,b) (-5 + 4\,b) (-1 + a\,b) \Big) + \\
& (-625 (4 - 5\,b)^2 (-875 + 444\,b) + \\
& 25\,a (379\,600 + b (-1\,180\,440 + b (1\,661\,481 + 125\,b (-9019 + 2220\,b) \Big) \Big) \Big) - \\
& 12\,a^3 (-187\,200 + b (-87\,920 + b (962\,744 + 125\,b (-7927 + 2220\,b) \Big) \Big) \Big) + \\
& 5\,a^2 (-2\,150\,400 + b (3\,548\,560 + b (-539\,692 + 125\,b (-13\,561 + 5460\,b) \Big) \Big) \Big) \Big) \Big/ \\
& (14\,976 (-5 + 3\,a) (-5 + 4\,a) (-1 + a^2) (-5 + 3\,b) (-5 + 4\,b) (-1 + a\,b) \Big), \\
& (-4 + 5\,a) (5625 + b (4875 - 5625\,a + 20 (1165 - 624\,b) b + 3\,a\,b (-5785 + 3396\,b) \Big) \Big) \\
& - \frac{832 (-5 + 3\,a) (-5 + 3\,b) (-5 + 4\,b) (-1 + a\,b) (-1 + b^2)}{675 (-4 + 5\,a)}, \\
& \frac{416 (-5 + 3\,a) (-5 + 3\,b)}{3 (25 (-785 + 384\,a) - 4285 (-5 + 3\,a) b + 3 (-2240 + 2069\,a) b^2)}, \\
& \frac{640 (-5 + 3\,a) (-5 + 3\,b) (-1 + a\,b) (-1 + b^2)}{1305}, \\
& \frac{64 (-5 + 3\,a) (-5 + 3\,b)}{(-4 + 5\,b) (5625 + b (4875 - 5625\,a + 20 (1165 - 624\,b) b + 3\,a\,b (-5785 + 3396\,b) \Big) \Big)}, \\
& \frac{1664 (-5 + 3\,a) (-5 + 3\,b) (-5 + 4\,b) (-1 + a\,b) (-1 + b^2)}{675 (-4 + 5\,b)}, \\
& \frac{832 (-5 + 3\,a) (-5 + 3\,b)}{-625 + 60 (10 - 3\,b) b + 3\,a (100 + b (-120 + 61\,b) \Big)}, \\
& \frac{20 (-5 + 3\,a) (-5 + 3\,b) (-1 + a\,b) (-1 + b^2)}{15}, \\
& - \frac{125 (-875 + 444\,a) (-4 + 5\,b)}{2 (-5 + 3\,a) (-5 + 3\,b)} \Big\}, \\
& \Big\{ \frac{125 (-875 + 444\,a) (-4 + 5\,b)}{7488 (-5 + 3\,a) (-5 + 4\,a)} + \\
& \frac{125 \Big( -\frac{135 (-5 + 3\,a) (-4 + 5\,a)}{(-3 + 5\,a) (-5 + 3\,b)} + \frac{320 (-5 + 4\,a)}{-5 + 4\,b} - \frac{1872\,a (-1 + a^2)}{(-5 + 3\,a) (-5 + 4\,a) (-3 + 5\,a) (-1 + a\,b)} \Big)}{3744}, \\
& \frac{125 (-4 + 5\,a) (-781 + 37 (35 - 12\,a) a)}{1872 (-5 + 3\,a) (-5 + 4\,a)} + \\
& \frac{125 (-4 + 5\,b) (-781 + a (420 - 444\,b) + 875\,b)}{3744 (-5 + 3\,a) (-5 + 4\,a)},
\end{aligned}$$

$$\begin{aligned}
& - \left( \left( 25 (-649105 + a (754109 + 2a (526835 + a (-877873 + 262500a))) \right) + 5 (1572420 + \right. \\
& \quad a (4089089 + 5a (-2697125 + 2a (554737 - 3125a (-173 + 84a))) \right) \right) b + \\
& \quad 12a (-1310375 + a (2386315 + 2a (-123131 + 125a (-5827 + 2220a))) \right) b^2 \Big/ \\
& \quad \left( 14976 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab) \right) \Big) + \\
& \quad \left( -625 (4 - 5b)^2 (-875 + 444b) + \right. \\
& \quad 25a (379600 + b (-1180440 + b (1661481 + 125b (-9019 + 2220b))) \right) - \\
& \quad 12a^3 (-187200 + b (-87920 + b (962744 + 125b (-7927 + 2220b))) \right) + \\
& \quad 5a^2 (-2150400 + b (3548560 + b (-539692 + 125b (-13561 + 5460b))) \right) \Big/ \\
& \quad \left( 14976 (-5 + 3a) (-5 + 4a) (-1 + a^2) (-5 + 3b) (-5 + 4b) (-1 + ab) \right), \\
& -1310375 + a (2386315 + 2a (-123131 + 125a (-5827 + 2220a))) \Big) \\
& \quad \frac{7488 (-5 + 3a) (-5 + 4a) (-1 + a^2)}{+} \\
& \quad \left( -455a (-44 + 25b) (4 + 25b) + 12a^2 (7312 + 4625b (-8 + 5b)) - \right. \\
& \quad \left. 25 (22288 + 4625b (-8 + 5b)) \right) \Big/ \left( 7488 (-5 + 3a) (-5 + 4a) (-1 + a^2) \right), \\
& 25 (-4 + 5a) (375 + b (100 + 3a (-125 + 36b))) \\
& \quad \frac{416 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab)}{225 (-4 + 5a)}, \\
& - \frac{208 (-5 + 3a)}{15 (25 + a (-160 + 87b))}, \\
& - \frac{64 (-5 + 3a) (-5 + 3b) (-1 + ab)}{435}, \\
& \frac{32 (-5 + 3a)}{25 (-4 + 5b) (375 + b (100 + 3a (-125 + 36b)))}, \\
& \quad \frac{832 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab)}{225 (-4 + 5b)}, \\
& - \frac{416 (-5 + 3a)}{5a}, \\
& - \frac{2 (-5 + 3a) (-1 + ab)}{5}, \\
& \left. \frac{-5 + 3a}{-5 + 3a} \right\}, \\
& \left\{ \frac{5 (7285 - 3b (507 + 500b) + a (-1875 + b (-5785 + 3396b)))}{416 (-5 + 3b) (-5 + 4b) (-1 + b^2)}, \right. \\
& \quad \frac{125 (139 - 60b + a (-125 + 36b))}{208 (-5 + 3b) (-5 + 4b)}, \\
& \quad \frac{(-4 + 5a) (-5625 + b (-4875 + 5625a + 3a (5785 - 3396b) b + 20b (-1165 + 624b)))}{832 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab) (-1 + b^2)}, \\
& \quad \frac{25 (-4 + 5a) (375 + b (100 + 3a (-125 + 36b)))}{416 (-5 + 3a) (-5 + 3b) (-5 + 4b) (-1 + ab)}, \\
& \quad \frac{445 - 117b}{32 (-5 + 3b) (-1 + b^2)}, \\
& \quad \frac{80 - 48b}{75}, \\
& \quad 0, 0, 0, \\
& \quad \left. 0, 0, 0 \right\}, \\
& \left\{ -\frac{375 (-5 + 3a)}{208 (-5 + 3b)}, \frac{125}{104} (-5 + 3a), \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{675(-4+5a)}{416(-5+3a)(-5+3b)}, \\
& -\frac{225(-4+5a)}{208(-5+3a)}, \\
& \frac{75}{80-48b}, \frac{25}{8}, 0, \\
& 0, 0, 0, 0, 0\}, \\
& \left\{0, 0, \frac{3(25(-785+384a)-4285(-5+3a)b+3(-2240+2069a)b^2)}{640(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}, \right. \\
& -\frac{15(25+a(-160+87b))}{64(-5+3a)(-5+3b)(-1+ab)}, \\
& 0, 0, -\frac{3(-445+117b)}{64(-5+3b)(-1+b^2)}, \\
& \left. \frac{225}{160-96b}, 0, 0, 0, 0\right\}, \\
& \left\{0, 0, -\frac{1305}{64(-5+3a)(-5+3b)}, \frac{435}{32(-5+3a)}, \right. \\
& 0, 0, \frac{225}{160-96b}, \\
& \left. \frac{75}{16}, 0, 0, 0, 0\right\}, \\
& \left\{\frac{5(-7285+3396b)}{832(-5+3b)(-5+4b)}, \frac{25}{416}\left(-15+\frac{144}{5-4b}+\frac{208}{-5+3b}\right), \right. \\
& \frac{(-4+5b)(-5625+b(-4875+5625a+3a(5785-3396b)b+20b(-1165+624b)))}{1664(-5+3a)(-5+3b)(-5+4b)(-1+ab)(-1+b^2)}, \\
& \frac{25(-4+5b)(375+b(100+3a(-125+36b)))}{832(-5+3a)(-5+3b)(-5+4b)(-1+ab)}, \\
& \left. 0, 0, 0, 0, \frac{445-117b}{64(-5+3b)(-1+b^2)}, \right. \\
& \frac{75}{160-96b}, 0, 0\}, \\
& \left\{-\frac{375}{416}, \frac{125}{208}(-5+3b), \frac{675(-4+5b)}{832(-5+3a)(-5+3b)}, \right. \\
& -\frac{225(-4+5b)}{416(-5+3a)}, 0, 0, 0, \\
& 0, \frac{75}{160-96b}, \frac{25}{16}, 0, 0\}, \\
& \left\{0, 0, \frac{-625+60(10-3b)b+3a(100+b(-120+61b))}{20(-5+3a)(-5+3b)(-1+ab)(-1+b^2)}, \right. \\
& \frac{5a}{2(-5+3a)(-1+ab)}, \\
& 0, 0, 0, 0, 0, 0, \\
& \left. \frac{445-117b}{64(-5+3b)(-1+b^2)}, \frac{75}{160-96b}\right\}, \\
& \left\{0, 0, -\frac{15}{2(-5+3a)(-5+3b)}, \frac{5}{-5+3a}, 0, 0, \right.
\end{aligned}$$

$$0, 0, 0, 0, \frac{75}{160 - 96b}, \frac{25}{16}\}}\}$$

In[262]:= **Gmatrix88 = Gmatrix /. {a → 8 / 10, b → 8 / 10}**

$$\begin{aligned} \text{Out[262]} = & \left\{ \left\{ \frac{43925}{2496}, \frac{1125}{416}, 0, 0, -\frac{43925}{3744}, -\frac{375}{208}, 0, 0, -\frac{43925}{7488}, -\frac{375}{416}, 0, 0 \right\}, \right. \\ & \left\{ \frac{1125}{416}, \frac{75}{16}, 0, 0, -\frac{375}{208}, -\frac{25}{8}, 0, 0, -\frac{375}{416}, -\frac{25}{16}, 0, 0 \right\}, \\ & \left\{ 0, 0, \frac{1414633075}{31539456}, \frac{14914625}{1752192}, 0, 0, -\frac{12984425}{584064}, -\frac{32625}{10816}, 0, 0, -\frac{490325}{54756}, -\frac{375}{338} \right\}, \\ & \left\{ 0, 0, \frac{14914625}{1752192}, \frac{66775}{7488}, 0, 0, -\frac{875}{192}, -\frac{2175}{416}, 0, 0, -\frac{250}{117}, -\frac{25}{13} \right\}, \\ & \left\{ -\frac{43925}{3744}, -\frac{375}{208}, 0, 0, \frac{43925}{3744}, \frac{375}{208}, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ -\frac{375}{208}, -\frac{25}{8}, 0, 0, \frac{375}{208}, \frac{25}{8}, 0, 0, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, -\frac{12984425}{584064}, -\frac{875}{192}, 0, 0, \frac{43925}{2496}, \frac{1125}{416}, 0, 0, 0, 0 \right\}, \\ & \left\{ 0, 0, -\frac{32625}{10816}, -\frac{2175}{416}, 0, 0, \frac{1125}{416}, \frac{75}{16}, 0, 0, 0, 0 \right\}, \\ & \left\{ -\frac{43925}{7488}, -\frac{375}{416}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416}, 0, 0 \right\}, \\ & \left\{ -\frac{375}{416}, -\frac{25}{16}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16}, 0, 0 \right\}, \\ & \left\{ 0, 0, -\frac{490325}{54756}, -\frac{250}{117}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416} \right\}, \\ & \left. \left\{ 0, 0, -\frac{375}{338}, -\frac{25}{13}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16} \right\} \right\} \end{aligned}$$

In[263]:= **Det[Gmatrix88]**

Out[263]= 0

In[264]:= **GmatrixA8 = Gmatrix /. {b → 8 / 10}**

$$\begin{aligned} \text{Out[264]} = & \left\{ \left\{ \frac{43925}{7488} - \frac{1}{3942432} \times \right. \right. \\ & 15625 \left( -\frac{52}{5} (1195 + a(-3514 + 1195a)) + \frac{192}{25} (1757 + a(-3830 + 1757a)) - \right. \\ & \left. \left. 5(8785 + a(-15406 + 8785a)) \right), \frac{375}{416} + \frac{15625 \left( \frac{2599}{5} + a(-1034 + \frac{2599a}{5}) \right)}{219024}, \right. \\ & \left. \frac{3125(-4 + 5a) \left( -\frac{25100929}{25} + \frac{290473787a}{125} - \frac{8571848a^2}{5} + \frac{51431088a^3}{125} \right)}{7884864 \left( -1 + \frac{4a}{5} \right) (-5 + 3a) (-5 + 4a)}, \right. \\ & \left. \frac{125 \left( -\frac{1600}{9} (-5 + 4a) + \frac{675(-5+3a)(-4+5a)}{13(-3+5a)} - \frac{1872a(-1+a^2)}{\left( -1 + \frac{4a}{5} \right) (-5+3a) (-5+4a) (-3+5a)} \right)}{3744}, \right. \\ & \left. -\frac{3125 \left( \frac{25541}{5} - \frac{108239a}{25} \right)}{438048}, \frac{1875(-5 + 3a)}{2704} \right\} \end{aligned}$$

$$\begin{aligned}
& 0, 0, -\frac{43925}{7488}, -\frac{375}{416}, 0, 0\}, \\
& \left\{ \frac{375}{416} - \frac{15625 \left( -\frac{2599}{5} + \left( 1034 - \frac{2599a}{5} \right) a \right)}{219024}, \frac{25}{16} + \frac{625}{936} (37 + a (-70 + 37a)), \right. \\
& \frac{25 (-4 + 5a) \left( -\frac{1215 (3-5a)}{13 (-5+3a)} - \frac{5120 (-4+5a)}{9 (-5+4a)} \right)}{3744}, \frac{125 (-4 + 5a) (-781 + 37 (35 - 12a) a)}{1872 (-5 + 3a) (-5 + 4a)}, \\
& \left. -\frac{3125 \left( 91 - \frac{481a}{5} \right)}{24336}, \frac{125}{104} (-5 + 3a), 0, 0, -\frac{375}{416}, -\frac{25}{16}, 0, 0 \right\}, \\
& \left\{ -\frac{3125 (-4 + 5a) \left( -\frac{25100929}{25} + \frac{290473787a}{125} - \frac{8571848a^2}{5} + \frac{51431088a^3}{125} \right)}{7884864 \left( -1 + \frac{4a}{5} \right) (-5 + 3a) (-5 + 4a)}, \right. \\
& \frac{25 (-4 + 5a) \left( -\frac{1215 (3-5a)}{13 (-5+3a)} - \frac{5120 (-4+5a)}{9 (-5+4a)} \right)}{3744}, \\
& -\frac{25 \left( -56946240 + \frac{360075456a}{5} - \frac{413517312a^2}{25} - \frac{462578688a^3}{125} \right)}{31539456 \left( -1 + \frac{4a}{5} \right) (-5 + 3a) (-5 + 4a) (-1 + a^2)} + \\
& \left( 25 \left( 1002843131 - \frac{12868251319a}{5} + \frac{30487416186a^2}{25} + \right. \right. \\
& \left. \left. \frac{227227141126a^3}{125} - 1955925008a^4 + 514310880a^5 \right) \right) / \\
& \left( 31539456 \left( -1 + \frac{4a}{5} \right) (-5 + 3a) (-5 + 4a) (-1 + a^2) \right), \\
& \frac{25 \left( 876096a - \frac{6132672a^2}{5} + \frac{10513152a^3}{25} \right)}{1752192 \left( -1 + \frac{4a}{5} \right) (-5 + 3a) (-5 + 4a) (-1 + a^2)} - \\
& \left( 25 \left( \frac{192}{25} a (-1310375 + a (2386315 + 2a (-123131 + 125a (-5827 + 2220a))) \right) + \right. \\
& 25 (-649105 + a (754109 + 2a (526835 + a (-877873 + 262500a))) + \\
& 4 (1572420 + \\
& \left. \left. a (4089089 + 5a (-2697125 + 2a (554737 - 3125a (-173 + 84a)))) \right) \right) / \\
& \left( 1752192 \left( -1 + \frac{4a}{5} \right) (-5 + 3a) (-5 + 4a) (-1 + a^2) \right), \\
& \frac{625 \left( 5625 + \frac{4}{5} \left( \frac{77639}{5} - \frac{324717a}{25} \right) \right) (-4 + 5a)}{876096 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, -\frac{3375 (-4 + 5a)}{5408 (-5 + 3a)}, \\
& \frac{25 (-3428 (-5 + 3a) + 25 (-785 + 384a) + \frac{48}{25} (-2240 + 2069a))}{4992 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, \\
& \frac{6525}{832 (-5 + 3a)}, 0, 0, \\
& \frac{25 \left( -\frac{1301}{5} + \frac{3228a}{25} \right)}{468 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, \frac{75}{26 (-5 + 3a)} \},
\end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{125 \left( -\frac{1600}{9} (-5+4a) + \frac{675 (-5+3a) (-4+5a)}{13 (-3+5a)} - \frac{1872a (-1+a^2)}{\left(-1+\frac{4a}{5}\right) (-5+3a) (-5+4a) (-3+5a)} \right)}{3744}, \right. \\
& \frac{125 (-4+5a) (-781+37(35-12a)a)}{1872 (-5+3a) (-5+4a)}, \\
& \frac{25 \left( 876096a - \frac{6132672a^2}{5} + \frac{10513152a^3}{25} \right)}{1752192 \left(-1+\frac{4a}{5}\right) (-5+3a) (-5+4a) (-1+a^2)} - \\
& \left( 25 \left( \frac{192}{25} a (-1310375 + a(2386315 + 2a(-123131 + 125a(-5827 + 2220a))) \right) \right) + \\
& 25 (-649105 + a(754109 + 2a(526835 + a(-877873 + 262500a))) + \\
& 4(1572420 + \\
& a(4089089 + 5a(-2697125 + 2a(554737 - 3125a(-173 + 84a)))) \right) \Big) \Big) \Big) / \\
& \left( 1752192 \left(-1+\frac{4a}{5}\right) (-5+3a) (-5+4a) (-1+a^2) \right), \\
& \frac{-187200 + 262080a - 89856a^2}{7488 (-5+3a) (-5+4a) (-1+a^2)} + \\
& \frac{-1310375 + a(2386315 + 2a(-123131 + 125a(-5827 + 2220a)))}{7488 (-5+3a) (-5+4a) (-1+a^2)}, \\
& \frac{625 \left( 375 + \frac{4}{5} \left( 100 - \frac{1443a}{5} \right) \right) (-4+5a)}{48672 \left(-1+\frac{4a}{5}\right) (-5+3a)}, - \frac{225 (-4+5a)}{208 (-5+3a)}, \\
& \frac{75 \left( 25 - \frac{452a}{5} \right)}{832 \left(-1+\frac{4a}{5}\right) (-5+3a)}, \\
& \frac{435}{32 (-5+3a)}, 0, 0, \\
& - \frac{5a}{2 \left(-1+\frac{4a}{5}\right) (-5+3a)}, \frac{5}{-5+3a} \Big\}, \\
& \left\{ - \frac{3125 \left( \frac{25541}{5} - \frac{108239a}{25} \right)}{438048}, - \frac{3125 \left( 91 - \frac{481a}{5} \right)}{24336}, \right. \\
& - \frac{625 (-4+5a) \left( -5625 + \frac{4}{5} \left( -\frac{77639}{5} + \frac{324717a}{25} \right) \right)}{876096 \left(-1+\frac{4a}{5}\right) (-5+3a)}, \\
& \frac{625 \left( 375 + \frac{4}{5} \left( 100 - \frac{1443a}{5} \right) \right) (-4+5a)}{48672 \left(-1+\frac{4a}{5}\right) (-5+3a)}, \\
& \frac{43925}{3744}, \frac{375}{208}, 0, 0, 0, 0, 0, 0 \Big\}, \\
& \left\{ \frac{1875 (-5+3a)}{2704}, \frac{125}{104} (-5+3a), - \frac{3375 (-4+5a)}{5408 (-5+3a)}, \right. \\
& - \frac{225 (-4+5a)}{208 (-5+3a)}, \frac{375}{208}, \frac{25}{8}, 0, 0, 0, 0, 0, 0 \Big\},
\end{aligned}$$



$$\begin{aligned}
& \left\{ 0, 0, \frac{25 \left( -3428 (-5 + 3a) + 25 (-785 + 384a) + \frac{48}{25} (-2240 + 2069a) \right)}{4992 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, \right. \\
& \quad \frac{75 \left( 25 - \frac{452a}{5} \right)}{832 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, 0, \\
& \quad 0, \frac{43925}{2496}, \frac{1125}{416}, 0, 0, 0, 0, 0 \left. \right\}, \\
& \left\{ 0, 0, \frac{6525}{832 (-5 + 3a)}, \frac{435}{32 (-5 + 3a)}, 0, 0, \frac{1125}{416}, \frac{75}{16}, 0, 0, 0, 0 \right\}, \\
& \left\{ -\frac{43925}{7488}, -\frac{375}{416}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416}, 0, 0 \right\}, \\
& \left\{ -\frac{375}{416}, -\frac{25}{16}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16}, 0, 0 \right\}, \\
& \left\{ 0, 0, \frac{25 \left( -\frac{1301}{5} + \frac{3228a}{25} \right)}{468 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, \right. \\
& \quad -\frac{5a}{2 \left( -1 + \frac{4a}{5} \right) (-5 + 3a)}, 0, 0, 0, 0, 0, 0, \frac{43925}{7488}, \frac{375}{416} \left. \right\}, \\
& \left\{ 0, 0, \frac{75}{26 (-5 + 3a)}, \frac{5}{-5 + 3a}, 0, 0, 0, 0, 0, 0, \frac{375}{416}, \frac{25}{16} \right\} \left. \right\}
\end{aligned}$$

In[265]:= Det[GmatrixA8]

Out[265]= (170 761 236 262 381 076 812 744 140 625  
(1 805 664 062 500 000 000 - 38 805 820 312 500 000 000 a + 397 928 313 085 937 500 000 a<sup>2</sup> -  
2 591 120 332 988 281 250 000 a<sup>3</sup> + 12 028 506 547 090 087 890 625 a<sup>4</sup> -  
42 367 680 486 603 125 000 000 a<sup>5</sup> + 117 656 218 927 675 078 125 000 a<sup>6</sup> -  
264 241 171 561 359 546 875 000 a<sup>7</sup> + 488 454 001 060 526 435 156 250 a<sup>8</sup> -  
752 383 793 014 272 681 875 000 a<sup>9</sup> + 974 012 895 502 928 365 312 500 a<sup>10</sup> -  
1 065 741 716 252 512 485 725 000 a<sup>11</sup> + 988 794 924 121 527 389 715 625 a<sup>12</sup> -  
778 751 359 825 439 233 757 000 a<sup>13</sup> + 520 118 981 809 328 888 993 500 a<sup>14</sup> -  
293 668 074 696 522 505 106 160 a<sup>15</sup> + 139 396 373 681 471 574 744 964 a<sup>16</sup> -  
55 158 413 527 356 254 736 960 a<sup>17</sup> + 17 973 090 232 348 988 476 800 a<sup>18</sup> -  
4 739 039 900 035 583 616 000 a<sup>19</sup> + 985 830 272 779 674 240 000 a<sup>20</sup> -  
155 715 830 535 168 000 000 a<sup>21</sup> + 17 549 096 767 488 000 000 a<sup>22</sup> -  
1 256 979 824 640 000 000 a<sup>23</sup> + 42 998 169 600 000 000 a<sup>24</sup>) ) /  
(11 062 441 448 736 885 200 388 096 (-5 + 3 a)<sup>10</sup> (-5 + 4 a)<sup>10</sup> (-1 + a<sup>2</sup>)<sup>2</sup>)

In[266]:= Simplify[Det[GmatrixA8]]

Out[266]= (170 761 236 262 381 076 812 744 140 625  
(4 - 5 a)<sup>4</sup> (5375 - 22 820 a + 35 778 a<sup>2</sup> - 24 640 a<sup>3</sup> + 6400 a<sup>4</sup>)<sup>2</sup>) /  
(11 062 441 448 736 885 200 388 096 (5 - 4 a)<sup>6</sup> (5 - 3 a)<sup>2</sup> (-1 + a<sup>2</sup>)<sup>2</sup>)

In[267]:= fact1 = (4 - 5 a) /. {a -> 8 / 10}

Out[267]= 0

In[268]:= fact = (5375 - 22 820 a + 35 778 a<sup>2</sup> - 24 640 a<sup>3</sup> + 6400 a<sup>4</sup>) /. {a → 8 / 10}

Out[268]=  $\frac{567}{25}$

In[269]:= Solve[5375 - 22 820 a + 35 778 a<sup>2</sup> - 24 640 a<sup>3</sup> + 6400 a<sup>4</sup> == 0] // N

Out[269]= {{a → 0.72542 - 0.0796576 i}, {a → 0.72542 + 0.0796576 i},  
{a → 1.19958 - 0.371407 i}, {a → 1.19958 + 0.371407 i}}

In[270]:= Gmatrix8B = Gmatrix /. {a → 8 / 10}

Out[270]= 
$$\left\{ \left\{ \frac{43925}{7488} + \frac{25 \left( -10413 + \frac{55341b}{5} - \frac{54756b^2}{25} \right)}{3744(-5+3b)(-5+4b)(-1+b^2)}, \right. \right.$$

$$- \frac{625(-35+37b)}{3744} + \frac{625 \left( 875 - 444b + \frac{4}{5} \left( -1706 + \frac{4}{5} (875 - 444b) + 840b \right) \right)}{1872(-5+3b)(-5+4b)},$$

$$\frac{125(-4+5b) \left( -820925 + \frac{4}{5} (462180 - 253008b) + 462180b \right)}{1752192(-5+3b)(-5+4b)},$$

$$- \frac{1624375(-4+5b)}{876096} + \frac{125 \left( \frac{576}{5 \left( -1 + \frac{4b}{5} \right)} - \frac{576}{-5+4b} \right)}{3744},$$

$$\frac{5 \left( 7285 - 3b(507 + 500b) + \frac{4}{5} (-1875 + b(-5785 + 3396b)) \right)}{416(-5+3b)(-5+4b)(-1+b^2)},$$

$$\frac{75}{16(-5+3b)}, 0, 0, \frac{5(-7285 + 3396b)}{832(-5+3b)(-5+4b)}, -\frac{375}{416}, 0, 0 \Big\},$$

$$\left\{ -\frac{625(-35+37b)}{3744} - \frac{625 \left( -875 + 444b + \frac{4}{5} (1706 - 840b + \frac{4}{5} (-875 + 444b)) \right)}{1872(-5+3b)(-5+4b)}, \right.$$

$$\frac{25}{8} + \frac{625(37+b(-70+37b))}{1872},$$

$$\frac{3125(-4+5b) \left( -16835 + (28897 - 10500b)b + \frac{48}{5} (781 + 37b(-35+12b)) \right)}{876096(-5+3b)(-5+4b)},$$

$$\frac{3125(-4+5b) \left( -781 + \frac{4}{5} (420 - 444b) + 875b \right)}{438048}, -\frac{125 \left( 139 - 60b + \frac{4}{5} (-125 + 36b) \right)}{208(-5+3b)(-5+4b)},$$

$$-\frac{25}{8}, 0, 0, \frac{25}{416} \left( -15 + \frac{144}{5-4b} + \frac{208}{-5+3b} \right), \frac{125}{208}(-5+3b), 0, 0 \Big\},$$

$$\left\{ \frac{125(-4+5b) \left( -820925 + \frac{4}{5} (462180 - 253008b) + 462180b \right)}{1752192(-5+3b)(-5+4b)}, \right.$$

$$\frac{3125(-4+5b) \left( -16835 + (28897 - 10500b)b + \frac{48}{5} (781 + 37b(-35+12b)) \right)}{876096(-5+3b)(-5+4b)},$$

$$\left( 25 \left( -\frac{24576}{5}b(-43925 - 15535b + 21084b^2) + \right. \right.$$

$$\frac{16}{25} (4152214550 - 9094972995b + 9213408409b^2 - 3033829860b^3) +$$

$$\left. 25 (133077775 - 205853635b + 142252332b^2 - 37739520b^3) + \right.$$

$$\begin{aligned}
& \frac{512}{5} \left( -2635500 - 8213975b - 204025b^2 + 2956164b^3 \right) + \\
& 4 \left( -1468518175 + 2342889320b - 1905168349b^2 + 583563300b^3 \right) + \\
& \frac{64}{125} \left( 596180750 + 3891936500b - 3493857610b^2 + 672179784b^3 \right) \Big) \Big) / \\
& \left( 31539456 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b) (-1 + b^2) \right) - \\
& \left( 25 \left( 25 \left( -47709200 + 102369880b - 49400941b^2 - 20010655b^3 + 13177500b^4 \right) + \right. \right. \\
& \quad \frac{16}{25} \left( -937070400 + 2724188560b - 2978197292b^2 + 1318277075b^3 + 25503125 \right. \\
& \quad \left. \left. b^4 - 116512500b^5 \right) + \frac{768}{125} \left( 22464000 - 79158800b + 90688600b^2 - \right. \right. \\
& \quad \left. \left. 17951341b^3 - 30793375b^4 + 13177500b^5 \right) - 4 \left( -292224400 + 642897160b - \right. \right. \\
& \quad \left. \left. 501750837b^2 + 287393120b^3 - 205349375b^4 + 65887500b^5 \right) \right) \Big) \Big) / \\
& \left( 31539456 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b) (-1 + b^2) \right), \\
& \frac{625 \left( -\frac{6921369}{5} + \frac{10184616b}{5} - \frac{92950416b^2}{125} \right)}{15769728 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b)} - \\
& \left( 625 \left( -625(4 - 5b)^2(-875 + 444b) + \right. \right. \\
& \quad 20(379600 + b(-1180440 + b(1661481 + 125b(-9019 + 2220b)))) - \\
& \quad \frac{768}{125}(-187200 + b(-87920 + b(962744 + 125b(-7927 + 2220b)))) + \\
& \quad \left. \frac{16}{5}(-2150400 + b(3548560 + b(-539692 + 125b(-13561 + 5460b)))) \right) \Big) \Big) / \\
& \left( 15769728 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b) \right), 0, 0, \\
& \frac{3 \left( -11945 + 11141b - \frac{8772b^2}{5} \right)}{1664 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-1 + b^2)}, \\
& \frac{6525}{832(-5 + 3b)}, \\
& \frac{5(-4 + 5b) \left( 5625 + b \left( 375 + 20(1165 - 624b)b + \frac{12}{5}b(-5785 + 3396b) \right) \right)}{21632 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b) (-1 + b^2)}, \\
& - \frac{3375(-4 + 5b)}{10816(-5 + 3b)}, \\
& - \frac{-625 + 60(10 - 3b)b + \frac{12}{5}(100 + b(-120 + 61b))}{52 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-1 + b^2)}, \\
& \frac{75}{26(-5 + 3b)} \Big\},
\end{aligned}$$

$$\begin{aligned}
& \left\{ -\frac{1\,624\,375\,(-4+5b)}{876\,096} + \frac{125\left(\frac{576}{5\left(-1+\frac{4b}{5}\right)} - \frac{576}{-5+4b}\right)}{3744}, \right. \\
& \frac{3125\,(-4+5b)\left(-781+\frac{4}{5}(420-444b)+875b\right)}{438\,048}, \\
& \frac{625\left(-\frac{6\,921\,369}{5} + \frac{10\,184\,616b}{5} - \frac{92\,950\,416b^2}{125}\right)}{15\,769\,728\left(-1+\frac{4b}{5}\right)(-5+3b)(-5+4b)} - \\
& \left( 625\left(-625(4-5b)^2(-875+444b) + \right. \right. \\
& \quad 20(379\,600+b(-1\,180\,440+b(1\,661\,481+125b(-9019+2220b)))) - \\
& \quad \frac{768}{125}(-187\,200+b(-87\,920+b(962\,744+125b(-7927+2220b)))) + \\
& \quad \left. \left. \frac{16}{5}(-2\,150\,400+b(3\,548\,560+b(-539\,692+125b(-13\,561+5460b))))\right) \right) \Bigg/ \\
& \left( 15\,769\,728\left(-1+\frac{4b}{5}\right)(-5+3b)(-5+4b) \right), \frac{15\,325}{2496} - \\
& \frac{1}{7\,884\,864} \times 625\left(-364(-44+25b)(4+25b) + \frac{192}{25}(7312+4625b(-8+5b)) - \right. \\
& \quad \left. 25(22\,288+4625b(-8+5b))\right), 0, 0, \frac{75\left(25+\frac{4}{5}(-160+87b)\right)}{832\left(-1+\frac{4b}{5}\right)(-5+3b)}, \\
& -\frac{2175}{416}, -\frac{125(-4+5b)\left(375+b\left(100+\frac{12}{5}(-125+36b)\right)\right)}{10\,816\left(-1+\frac{4b}{5}\right)(-5+3b)(-5+4b)}, \\
& \frac{1125(-4+5b)}{5408}, \\
& \frac{10}{13\left(-1+\frac{4b}{5}\right)}, \\
& -\frac{25}{13}\Bigg\}, \\
& \left\{ \frac{5\left(7285-3b(507+500b)+\frac{4}{5}(-1875+b(-5785+3396b))\right)}{416(-5+3b)(-5+4b)(-1+b^2)}, \right. \\
& -\frac{125\left(139-60b+\frac{4}{5}(-125+36b)\right)}{208(-5+3b)(-5+4b)}, \\
& 0, 0, \\
& \frac{445-117b}{32(-5+3b)(-1+b^2)}, \\
& \frac{75}{80-48b}, 0, \\
& 0, 0, 0, 0, 0\Bigg\}, \\
& \left\{ \frac{75}{16(-5+3b)}, -\frac{25}{8}, 0, 0, \frac{75}{80-48b}, \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{25}{8}, 0, 0, 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, -\frac{3 \left( -11945 + 11141 b - \frac{8772 b^2}{5} \right)}{1664 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-1 + b^2)}, \right. \\
& \frac{75 \left( 25 + \frac{4}{5} (-160 + 87b) \right)}{832 \left( -1 + \frac{4b}{5} \right) (-5 + 3b)}, 0, \\
& 0, -\frac{3 (-445 + 117b)}{64 (-5 + 3b) (-1 + b^2)}, \\
& \frac{225}{160 - 96b}, 0, 0, 0, 0 \}, \\
& \left\{ 0, 0, \frac{6525}{832 (-5 + 3b)}, -\frac{2175}{416}, 0, 0, \right. \\
& \frac{225}{160 - 96b}, \frac{75}{16}, 0, 0, 0, 0 \}, \\
& \left\{ \frac{5 (-7285 + 3396b)}{832 (-5 + 3b) (-5 + 4b)}, \frac{25}{416} \left( -15 + \frac{144}{5 - 4b} + \frac{208}{-5 + 3b} \right), \right. \\
& \frac{5 (-4 + 5b) \left( -5625 + b \left( -375 + \frac{12}{5} (5785 - 3396b) b + 20b (-1165 + 624b) \right) \right)}{21632 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b) (-1 + b^2)}, \\
& -\frac{125 (-4 + 5b) \left( 375 + b \left( 100 + \frac{12}{5} (-125 + 36b) \right) \right)}{10816 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-5 + 4b)}, \\
& 0, 0, 0, 0, \frac{445 - 117b}{64 (-5 + 3b) (-1 + b^2)}, \\
& \frac{75}{160 - 96b}, 0, 0 \}, \\
& \left\{ -\frac{375}{416}, \frac{125}{208} (-5 + 3b), -\frac{3375 (-4 + 5b)}{10816 (-5 + 3b)}, \right. \\
& \frac{1125 (-4 + 5b)}{5408}, 0, 0, 0, \\
& 0, \frac{75}{160 - 96b}, \frac{25}{16}, 0, 0 \}, \\
& \left\{ 0, 0, -\frac{-625 + 60 (10 - 3b) b + \frac{12}{5} (100 + b (-120 + 61b))}{52 \left( -1 + \frac{4b}{5} \right) (-5 + 3b) (-1 + b^2)}, \right. \\
& \frac{10}{13 \left( -1 + \frac{4b}{5} \right)}, 0, 0, 0, 0, 0, 0, \\
& \frac{445 - 117b}{64 (-5 + 3b) (-1 + b^2)}, \frac{75}{160 - 96b} \}, \\
& \left\{ 0, 0, \frac{75}{26 (-5 + 3b)}, -\frac{25}{13}, 0, 0, 0, 0, 0, 0, \frac{75}{160 - 96b}, \frac{25}{16} \right\} \}
\end{aligned}$$

In[271]:= Det[Gmatrix8B]

```
Out[271]= (95 367 431 640 625
(68 152 764 282 226 562 500 000 000 - 1 415 649 587 133 789 062 500 000 000 b +
13 946 499 109 417 358 398 437 500 000 b2 - 86 488 796 450 228 781 738 281 250 000 b3 +
377 564 213 292 550 903 131 103 515 625 b4 - 1 227 460 890 228 296 404 868 164 062 500
b5 + 3 058 183 089 483 222 534 684 082 031 250 b6 -
5 888 841 354 560 437 152 555 273 437 500 b7 + 8 621 908 667 636 090 111 193 740 234 375
b8 - 8 923 919 806 688 813 625 617 875 000 000 b9 +
4 563 387 525 895 334 682 598 985 937 500 b10 + 4 290 341 836 365 192 892 401 899 375 000
b11 - 13 980 028 049 849 982 102 663 838 265 625 b12 +
19 263 421 881 468 980 853 901 373 837 500 b13 -
17 110 541 117 155 555 841 755 793 528 750 b14 +
9 005 407 230 065 300 343 091 924 096 500 b15 +
379 394 346 932 385 645 735 863 338 025 b16 -
6 716 423 571 380 853 855 200 159 455 960 b17 +
8 536 510 928 836 434 446 209 048 610 944 b18 -
7 079 107 390 406 255 848 844 517 628 800 b19 +
4 491 861 872 105 454 640 158 928 348 512 b20 -
2 288 922 873 839 299 895 666 455 883 520 b21 +
953 248 591 731 522 608 253 060 620 544 b22 -
325 686 601 463 306 051 208 861 342 720 b23 + 90 832 437 826 806 752 330 953 017 600 b24 -
20 414 280 807 951 465 945 109 248 000 b25 + 3 615 858 505 472 912 602 928 640 000 b26 -
486 849 950 913 218 688 000 000 000 b27 + 46 897 856 003 122 560 000 000 000 b28 -
2 882 535 984 000 000 000 000 000 b29 + 85 030 560 000 000 000 000 000 b30) ) /
(9 167 498 816 495 026 176 (-5 + 3 b)12 (-5 + 4 b)10 (-1 + b2)6)
```

```
In[272]:= Simplify[%]
```

```
Out[272]= 
$$\frac{95\,367\,431\,640\,625\,(10\,567 - 25\,000\,b + 15\,625\,b^2)^2\,(500 - 1105\,b + 744\,b^2 - 180\,b^3)^4}{9\,167\,498\,816\,495\,026\,176\,(5 - 4\,b)^4\,(5 - 3\,b)^8\,(-1 + b^2)^4}$$

```

```
In[273]:= fact1 = (10 567 - 25 000 b + 15 625 b2) /. {b -> 8 / 10}
```

```
Out[273]= 567
```

```
In[274]:= fact2 = (500 - 1105 b + 744 b2 - 180 b3) /. {b -> 8 / 10}
```

```
Out[274]= 0
```

```
In[275]:= Solve[10 567 - 25 000 b + 15 625 b2 == 0]
```

```
Out[275]= {{b ->  $\frac{1}{125} (100 - 9 \sqrt{7})$ }, {b ->  $\frac{1}{125} (100 + 9 \sqrt{7})$ }}
```

```
In[276]:= Gmatrix1288 - Gmatrix88
```

```
Out[276]= {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

In[277]:= **Det [%]**

Out[277]= 0

In[278]:= **Simplify[Together[Gmatrix12a8 - GmatrixA8]]**

Out[278]= {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}

In[279]:= **Simplify[Together[Gmatrix128b - Gmatrix8B]]**

Out[279]= {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}

In[280]:= **Simplify[Together[Gmatrix12 - Gmatrix]]**

Out[280]= {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
 {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}