```
In[1]:= Remove[a, b, c, A, B, C, Rx, Az, Bz, Cz, InvAz, u1, u2, Kcal,
        Qcal, Gcal, Pcal, Sigma, sigma, Gmatrix1, Gmatrix2, integrand1,
        integrand2, integrand1expim, integrand2expim, Fcal1, Fcal2, Fcal, Im]
```

    ... Remove: Symbol C is Protected and cannot be removed.

    ... Remove: Symbol Im is Protected and cannot be removed.

```
In[2]:= Remove[C]
```

    ... Remove: Symbol C is Protected and cannot be removed.

```
In[3]:= ClearAll["Global`*"]
```

```
In[4]:= Unprotect[C]
```
Out[4]= {C}

```
In[5]:= Unprotect[In]
```
Out[5]= {In}

```
In[6]:= Unprotect[Im]
```
Out[6]= {Im}

```
In[7]:= A1 = -3/2
```
Out[7]= $-\dfrac{3}{2}$

```
In[8]:= A2 = 7/10
```
Out[8]= $\dfrac{7}{10}$

```
In[9]:= B1 = -1
```
Out[9]= -1

```
In[10]:= B2 = 2/10
```
Out[10]= $\dfrac{1}{5}$

```
In[11]:= C1 = 1/2
```
Out[11]= $\dfrac{1}{2}$

```
In[12]:= Rx[z_] = {{1}}
```
Out[12]= {{1}}

```
In[13]:= I1 = IdentityMatrix[1]
```
Out[13]= {{1}}

```
In[14]:= Sigma = {{1}}
```
Out[14]= {{1}}

```
In[15]:= A[z_] = I1 + A1 * z + A2 * z^2
```
Out[15]= $\left\{\left\{1 - \dfrac{3z}{2} + \dfrac{7z^2}{10}\right\}\right\}$

In[16]:= **A[z]**

Out[16]= $\left\{\left\{1 - \dfrac{3\,z}{2} + \dfrac{7\,z^2}{10}\right\}\right\}$

In[17]:= **B[z_] = I1 + B1 * z + B2 * z^2**

Out[17]= $\left\{\left\{1 - z + \dfrac{z^2}{5}\right\}\right\}$

In[18]:= **C[z_] = I1 + C1 * z**

Out[18]= $\left\{\left\{1 + \dfrac{z}{2}\right\}\right\}$

In[19]:= **Solve[Det[A[z]] == 0]**

Out[19]= $\left\{\left\{z \to \dfrac{1}{14}\left(15 - \mathbb{i}\,\sqrt{55}\right)\right\}, \left\{z \to \dfrac{1}{14}\left(15 + \mathbb{i}\,\sqrt{55}\right)\right\}\right\}$

In[20]:= **Solve[Det[B[z]] == 0]**

Out[20]= $\left\{\left\{z \to \dfrac{1}{2}\left(5 - \sqrt{5}\right)\right\}, \left\{z \to \dfrac{1}{2}\left(5 + \sqrt{5}\right)\right\}\right\}$

In[21]:= **Solve[Det[C[z]] == 0]**

Out[21]= $\{\{z \to -2\}\}$

In[22]:= **p = 2**

Out[22]= 2

In[23]:= **q = 2**

Out[23]= 2

In[24]:= **r = 2**

Out[24]= 2

In[25]:= **n = 1**

Out[25]= 1

In[26]:= **m = 1**

Out[26]= 1

In[27]:= **ur[z_] := {{1}, {z}}**

In[28]:= **up[z_] := {{1}, {z}}**

In[29]:= **uq[z_] := {{1}, {z}}**

In[30]:= **Orm$n = ConstantArray[0, {r * m, n}]**

Out[30]= {{0}, {0}}

In[31]:= **Oqn$m = ConstantArray[0, {q * n, m}]**

Out[31]= {{0}, {0}}

In[32]:= **In = IdentityMatrix[n]**

Out[32]= $\{\{1\}\}$

In[33]:= **Imm = IdentityMatrix[m]**

Out[33]= $\{\{1\}\}$

In[34]:= **Gcal[z_] = Together[ArrayFlatten[{{KroneckerProduct[up[z], -Inverse[A[z]].B[z]]}, {Orm$n}, {KroneckerProduct[uq[z], In]}}]]**

Out[34]= $\left\{\left\{-\dfrac{2\left(5-5z+z^2\right)}{10-15z+7z^2}\right\}, \left\{-\dfrac{2z\left(5-5z+z^2\right)}{10-15z+7z^2}\right\}, \{0\}, \{0\}, \{1\}, \{z\}\right\}$

In[35]:= **MatrixForm[%]**

Out[35]//MatrixForm=

$$\begin{pmatrix} -\dfrac{2\left(5-5z+z^2\right)}{10-15z+7z^2} \\ -\dfrac{2z\left(5-5z+z^2\right)}{10-15z+7z^2} \\ 0 \\ 0 \\ 1 \\ z \end{pmatrix}$$

In[36]:= **Kcal[z_] = Together[ArrayFlatten[{{KroneckerProduct[up[z], -Inverse[A[z]].C[z]]}, {KroneckerProduct[ur[z], Imm]}, {Oqn$m}}]]**

Out[36]= $\left\{\left\{-\dfrac{5\left(2+z\right)}{10-15z+7z^2}\right\}, \left\{-\dfrac{5z\left(2+z\right)}{10-15z+7z^2}\right\}, \{1\}, \{z\}, \{0\}, \{0\}\right\}$

In[37]:= **MatrixForm[%]**

Out[37]//MatrixForm=

$$\begin{pmatrix} -\dfrac{5\left(2+z\right)}{10-15z+7z^2} \\ -\dfrac{5z\left(2+z\right)}{10-15z+7z^2} \\ 1 \\ z \\ 0 \\ 0 \end{pmatrix}$$

In[38]:= **sigma[z_] = Together$\left[$Transpose[Inverse[B[z]]].Inverse[Sigma].Inverse$\left[B\left[1/z\right]\right]\right]$**

Out[38]= $\left\{\left\{\dfrac{25z^2}{\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}\right\}\right\}$

In[39]:= **MatrixForm[%]**

Out[39]//MatrixForm=

$$\left(\dfrac{25z^2}{\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}\right)$$

In[40]:= `Pcal[z_] = Together[Gcal[z].Sigma.Transpose[Gcal[1/z]]]`

Out[40]= $\left\{\left\{\dfrac{4\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\ \dfrac{4\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{z\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\ 0,\right.\right.$

$\left.0,\ -\dfrac{2\left(5-5z+z^2\right)}{10-15z+7z^2},\ -\dfrac{2\left(5-5z+z^2\right)}{z\left(10-15z+7z^2\right)}\right\},\ \left\{\dfrac{4z\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\right.$

$\left.\dfrac{4\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\ 0,\ 0,\ -\dfrac{2z\left(5-5z+z^2\right)}{10-15z+7z^2},\ -\dfrac{2\left(5-5z+z^2\right)}{10-15z+7z^2}\right\},$

$\{0,0,0,0,0,0\},\ \{0,0,0,0,0,0\},\ \left\{-\dfrac{2\left(1-5z+5z^2\right)}{7-15z+10z^2},\ -\dfrac{2\left(1-5z+5z^2\right)}{z\left(7-15z+10z^2\right)},\ 0,\ 0,\ 1,\ \dfrac{1}{z}\right\},$

$\left\{-\dfrac{2z\left(1-5z+5z^2\right)}{7-15z+10z^2},\ -\dfrac{2\left(1-5z+5z^2\right)}{7-15z+10z^2},\ 0,\ 0,\ z,\ 1\right\}\right\}$

In[41]:= `MatrixForm[%]`

Out[41]//MatrixForm=

$$\begin{pmatrix} \dfrac{4\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & \dfrac{4\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{z\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & 0 & 0 & -\dfrac{2\left(5-5z+z^2\right)}{10-15z+7z^2} & -\dfrac{2\left(5-5z+z^2\right)}{z\left(10-15z+7z^2\right)} \\[6pt] \dfrac{4z\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & \dfrac{4\left(5-5z+z^2\right)\left(1-5z+5z^2\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & 0 & 0 & -\dfrac{2z\left(5-5z+z^2\right)}{10-15z+7z^2} & -\dfrac{2\left(5-5z+z^2\right)}{10-15z+7z^2} \\[6pt] 0 & 0 & 0 & 0 & 0 & 0 \\[6pt] 0 & 0 & 0 & 0 & 0 & 0 \\[6pt] -\dfrac{2\left(1-5z+5z^2\right)}{7-15z+10z^2} & -\dfrac{2\left(1-5z+5z^2\right)}{z\left(7-15z+10z^2\right)} & 0 & 0 & 1 & \dfrac{1}{z} \\[6pt] -\dfrac{2z\left(1-5z+5z^2\right)}{7-15z+10z^2} & -\dfrac{2\left(1-5z+5z^2\right)}{7-15z+10z^2} & 0 & 0 & z & 1 \end{pmatrix}$$

In[42]:= `Qcal[z_] = Together[Kcal[z].Rx[z].Transpose[Kcal[1/z]]]`

Out[42]= $\left\{\left\{\dfrac{25z\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\right.\right.$

$\left.\dfrac{25\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\ -\dfrac{5\left(2+z\right)}{10-15z+7z^2},\ -\dfrac{5\left(2+z\right)}{z\left(10-15z+7z^2\right)},\ 0,\ 0\right\},$

$\left\{\dfrac{25z^2\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\ \dfrac{25z\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)},\ -\dfrac{5z\left(2+z\right)}{10-15z+7z^2},\right.$

$\left.-\dfrac{5\left(2+z\right)}{10-15z+7z^2},\ 0,\ 0\right\},\ \left\{-\dfrac{5z\left(1+2z\right)}{7-15z+10z^2},\ -\dfrac{5\left(1+2z\right)}{7-15z+10z^2},\ 1,\ \dfrac{1}{z},\ 0,\ 0\right\},$

$\left\{-\dfrac{5z^2\left(1+2z\right)}{7-15z+10z^2},\ -\dfrac{5z\left(1+2z\right)}{7-15z+10z^2},\ z,\ 1,\ 0,\ 0\right\},\ \{0,0,0,0,0,0\},\ \{0,0,0,0,0,0\}\right\}$

In[43]:= `MatrixForm[%]`

Out[43]//MatrixForm=

$$\begin{pmatrix} \dfrac{25z\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & \dfrac{25\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & -\dfrac{5\left(2+z\right)}{10-15z+7z^2} & -\dfrac{5\left(2+z\right)}{z\left(10-15z+7z^2\right)} & 0 & 0 \\[6pt] \dfrac{25z^2\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & \dfrac{25z\left(2+z\right)\left(1+2z\right)}{\left(10-15z+7z^2\right)\left(7-15z+10z^2\right)} & -\dfrac{5z\left(2+z\right)}{10-15z+7z^2} & -\dfrac{5\left(2+z\right)}{10-15z+7z^2} & 0 & 0 \\[6pt] -\dfrac{5z\left(1+2z\right)}{7-15z+10z^2} & -\dfrac{5\left(1+2z\right)}{7-15z+10z^2} & 1 & \dfrac{1}{z} & 0 & 0 \\[6pt] -\dfrac{5z^2\left(1+2z\right)}{7-15z+10z^2} & -\dfrac{5z\left(1+2z\right)}{7-15z+10z^2} & z & 1 & 0 & 0 \\[6pt] 0 & 0 & 0 & 0 & 0 & 0 \\[6pt] 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[44]:= **integrand1[z_] = Simplify[Together[KroneckerProduct[Pcal[z], sigma[z]]]]**

Out[44]= $\left\{\left\{\dfrac{100\,z^2}{\left(10 - 15\,z + 7\,z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\ \dfrac{100\,z}{\left(10 - 15\,z + 7\,z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\right.\right.$

$\left. 0,\ 0,\ -\dfrac{50\,z^2}{\left(1 - 5\,z + 5\,z^2\right)\,\left(10 - 15\,z + 7\,z^2\right)},\ -\dfrac{50\,z}{\left(1 - 5\,z + 5\,z^2\right)\,\left(10 - 15\,z + 7\,z^2\right)}\right\},$

$\left\{\dfrac{100\,z^3}{\left(10 - 15\,z + 7\,z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\ \dfrac{100\,z^2}{\left(10 - 15\,z + 7\,z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\ 0,\ 0,\right.$

$\left. -\dfrac{50\,z^3}{\left(1 - 5\,z + 5\,z^2\right)\,\left(10 - 15\,z + 7\,z^2\right)},\ -\dfrac{50\,z^2}{\left(1 - 5\,z + 5\,z^2\right)\,\left(10 - 15\,z + 7\,z^2\right)}\right\},\ \{0, 0, 0, 0, 0, 0\},$

$\{0, 0, 0, 0, 0, 0\},\ \left\{-\dfrac{50\,z^2}{\left(5 - 5\,z + z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\ -\dfrac{50\,z}{\left(5 - 5\,z + z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\right.$

$\left. 0,\ 0,\ \dfrac{25\,z^2}{\left(5 - 5\,z + z^2\right)\,\left(1 - 5\,z + 5\,z^2\right)},\ \dfrac{25\,z}{\left(5 - 5\,z + z^2\right)\,\left(1 - 5\,z + 5\,z^2\right)}\right\},$

$\left\{-\dfrac{50\,z^3}{\left(5 - 5\,z + z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\ -\dfrac{50\,z^2}{\left(5 - 5\,z + z^2\right)\,\left(7 - 15\,z + 10\,z^2\right)},\ 0,\right.$

$\left.\left. 0,\ \dfrac{25\,z^3}{\left(5 - 5\,z + z^2\right)\,\left(1 - 5\,z + 5\,z^2\right)},\ \dfrac{25\,z^2}{\left(5 - 5\,z + z^2\right)\,\left(1 - 5\,z + 5\,z^2\right)}\right\}\right\}\right\}$

In[45]:= **integrand2[z_] = Simplify[Together[KroneckerProduct[Qcal[z], sigma[z]]]]**

Out[45]= $\left\{\left\{\dfrac{625\,z^3\,(2+z)\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},\right.\right.$

$\dfrac{625\,z^2\,(2+z)\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},$

$-\dfrac{125\,z^2\,(2+z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)},$

$\left.-\dfrac{125\,z\,(2+z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)},\ 0,\ 0\right\},$

$\left\{\dfrac{625\,z^4\,(2+z)\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},\right.$

$\dfrac{625\,z^3\,(2+z)\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},$

$-\dfrac{125\,z^3\,(2+z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)},$

$\left.-\dfrac{125\,z^2\,(2+z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(10-15\,z+7\,z^2\right)},\ 0,\ 0\right\},$

$\left\{-\dfrac{125\,z^3\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},\right.$

$-\dfrac{125\,z^2\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},\ \dfrac{25\,z^2}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)},$

$\left.\dfrac{25\,z}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)},\ 0,\ 0\right\},\ \left\{-\dfrac{125\,z^4\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},\right.$

$-\dfrac{125\,z^3\,(1+2\,z)}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)\,\left(7-15\,z+10\,z^2\right)},\ \dfrac{25\,z^3}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)},$

$\left.\dfrac{25\,z^2}{\left(5-5\,z+z^2\right)\,\left(1-5\,z+5\,z^2\right)},\ 0,\ 0\right\},\ \{0,0,0,0,0,0\},\ \{0,0,0,0,0,0\}\right\}$

In[46]:= **integrand1expim[f_] = $\left(1 \big/ \left(2\, \text{Pi}\right)\right)$ * integrand1[Exp[I f]]**

Out[46]= $\Big\{\Big\{ \dfrac{50\, e^{2\,i\,f}}{\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},$

$\dfrac{50\, e^{i\,f}}{\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},\ 0,\ 0,$

$-\dfrac{25\, e^{2\,i\,f}}{\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\pi},\ -\dfrac{25\, e^{i\,f}}{\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\pi}\Big\},$

$\Big\{ \dfrac{50\, e^{3\,i\,f}}{\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},\ \dfrac{50\, e^{2\,i\,f}}{\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},$

$0,\ 0,\ -\dfrac{25\, e^{3\,i\,f}}{\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\pi},$

$-\dfrac{25\, e^{2\,i\,f}}{\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\left(10 - 15\, e^{i\,f} + 7\, e^{2\,i\,f}\right)\pi}\Big\},\ \{0, 0, 0, 0, 0, 0\},\ \{0, 0, 0, 0, 0, 0\},$

$\Big\{ -\dfrac{25\, e^{2\,i\,f}}{\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},\ -\dfrac{25\, e^{i\,f}}{\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},$

$0,\ 0,\ \dfrac{25\, e^{2\,i\,f}}{2\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\pi},\ \dfrac{25\, e^{i\,f}}{2\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\pi}\Big\},$

$\Big\{ -\dfrac{25\, e^{3\,i\,f}}{\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},\ -\dfrac{25\, e^{2\,i\,f}}{\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(7 - 15\, e^{i\,f} + 10\, e^{2\,i\,f}\right)\pi},$

$0,\ 0,\ \dfrac{25\, e^{3\,i\,f}}{2\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\pi},\ \dfrac{25\, e^{2\,i\,f}}{2\left(5 - 5\, e^{i\,f} + e^{2\,i\,f}\right)\left(1 - 5\, e^{i\,f} + 5\, e^{2\,i\,f}\right)\pi}\Big\}\Big\}$

In[47]:= **integrand2expim[f_] = $\left(1 \big/ \left(2\,\text{Pi}\right)\right)$ * integrand2[Exp[I f]]**

$$\left\{\left\{\frac{625\,e^{3\,i\,f}\,\left(2+e^{i\,f}\right)\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},\right.\right.$$

$$\frac{625\,e^{2\,i\,f}\,\left(2+e^{i\,f}\right)\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},$$

$$-\frac{125\,e^{2\,i\,f}\,\left(2+e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\pi},$$

$$\left.-\frac{125\,e^{i\,f}\,\left(2+e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\pi},\,0,\,0\right\},$$

$$\left\{\frac{625\,e^{4\,i\,f}\,\left(2+e^{i\,f}\right)\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},\right.$$

$$\frac{625\,e^{3\,i\,f}\,\left(2+e^{i\,f}\right)\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},$$

$$-\frac{125\,e^{3\,i\,f}\,\left(2+e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\pi},$$

$$\left.-\frac{125\,e^{2\,i\,f}\,\left(2+e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(10-15\,e^{i\,f}+7\,e^{2\,i\,f}\right)\,\pi},\,0,\,0\right\},$$

$$\left\{-\frac{125\,e^{3\,i\,f}\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},\right.$$

$$-\frac{125\,e^{2\,i\,f}\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},$$

$$\left.\frac{25\,e^{2\,i\,f}}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\pi},\,\frac{25\,e^{i\,f}}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\pi},\,0,\,0\right\},$$

$$\left\{-\frac{125\,e^{4\,i\,f}\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},\right.$$

$$-\frac{125\,e^{3\,i\,f}\,\left(1+2\,e^{i\,f}\right)}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\left(7-15\,e^{i\,f}+10\,e^{2\,i\,f}\right)\,\pi},$$

$$\left.\frac{25\,e^{3\,i\,f}}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\pi},\,\frac{25\,e^{2\,i\,f}}{2\,\left(5-5\,e^{i\,f}+e^{2\,i\,f}\right)\,\left(1-5\,e^{i\,f}+5\,e^{2\,i\,f}\right)\,\pi},\,0,\,0\right\},$$

$$\left.\{0,\,0,\,0,\,0,\,0,\,0\},\,\{0,\,0,\,0,\,0,\,0,\,0\}\right\}$$

In[48]:= **Fcal1 = Integrate[integrand1expim[f], {f, 0, 2 Pi}]**

Out[48]= $\left\{\left\{\frac{425}{48},\,\frac{125}{16},\,0,\,0,\,-\frac{2150}{449},\,-\frac{2000}{449}\right\},\,\left\{\frac{125}{16},\,\frac{425}{48},\,0,\,0,\,-\frac{1750}{449},\,-\frac{2150}{449}\right\},\,\{0,\,0,\,0,\,0,\,0,\,0\},\right.$

$\left.\{0,\,0,\,0,\,0,\,0,\,0\},\,\left\{-\frac{2150}{449},\,-\frac{1750}{449},\,0,\,0,\,\frac{75}{22},\,\frac{125}{44}\right\},\,\left\{-\frac{2000}{449},\,-\frac{2150}{449},\,0,\,0,\,\frac{125}{44},\,\frac{75}{22}\right\}\right\}$

In[49]:= `Fcal2 = Integrate[integrand2expim[f], {f, 0, 2 Pi}]`

Out[49]= $\left\{\left\{\dfrac{205\,071\,875}{948\,288}, \dfrac{64\,779\,375}{316\,096}, -\dfrac{376\,875}{19\,756}, -\dfrac{113\,750}{4939}, 0, 0\right\},\right.$

$\left\{\dfrac{64\,779\,375}{316\,096}, \dfrac{205\,071\,875}{948\,288}, -\dfrac{285\,875}{19\,756}, -\dfrac{376\,875}{19\,756}, 0, 0\right\}, \left\{-\dfrac{376\,875}{19\,756}, -\dfrac{285\,875}{19\,756}, \dfrac{75}{22}, \dfrac{125}{44}, 0, 0\right\},$

$\left.\left\{-\dfrac{113\,750}{4939}, -\dfrac{376\,875}{19\,756}, \dfrac{125}{44}, \dfrac{75}{22}, 0, 0\right\}, \{0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0\}\right\}$

In[50]:= `Fcal = Fcal1 + Fcal2`

Out[50]= $\left\{\left\{\dfrac{213\,468\,175}{948\,288}, \dfrac{67\,248\,875}{316\,096}, -\dfrac{376\,875}{19\,756}, -\dfrac{113\,750}{4939}, -\dfrac{2150}{449}, -\dfrac{2000}{449}\right\},\right.$

$\left\{\dfrac{67\,248\,875}{316\,096}, \dfrac{213\,468\,175}{948\,288}, -\dfrac{285\,875}{19\,756}, -\dfrac{376\,875}{19\,756}, -\dfrac{1750}{449}, -\dfrac{2150}{449}\right\},$

$\left\{-\dfrac{376\,875}{19\,756}, -\dfrac{285\,875}{19\,756}, \dfrac{75}{22}, \dfrac{125}{44}, 0, 0\right\}, \left\{-\dfrac{113\,750}{4939}, -\dfrac{376\,875}{19\,756}, \dfrac{125}{44}, \dfrac{75}{22}, 0, 0\right\},$

$\left.\left\{-\dfrac{2150}{449}, -\dfrac{1750}{449}, 0, 0, \dfrac{75}{22}, \dfrac{125}{44}\right\}, \left\{-\dfrac{2000}{449}, -\dfrac{2150}{449}, 0, 0, \dfrac{125}{44}, \dfrac{75}{22}\right\}\right\}$

In[51]:= `MatrixForm[%]`

Out[51]//MatrixForm=

$$\begin{pmatrix} \dfrac{213\,468\,175}{948\,288} & \dfrac{67\,248\,875}{316\,096} & -\dfrac{376\,875}{19\,756} & -\dfrac{113\,750}{4939} & -\dfrac{2150}{449} & -\dfrac{2000}{449} \\ \dfrac{67\,248\,875}{316\,096} & \dfrac{213\,468\,175}{948\,288} & -\dfrac{285\,875}{19\,756} & -\dfrac{376\,875}{19\,756} & -\dfrac{1750}{449} & -\dfrac{2150}{449} \\ -\dfrac{376\,875}{19\,756} & -\dfrac{285\,875}{19\,756} & \dfrac{75}{22} & \dfrac{125}{44} & 0 & 0 \\ -\dfrac{113\,750}{4939} & -\dfrac{376\,875}{19\,756} & \dfrac{125}{44} & \dfrac{75}{22} & 0 & 0 \\ -\dfrac{2150}{449} & -\dfrac{1750}{449} & 0 & 0 & \dfrac{75}{22} & \dfrac{125}{44} \\ -\dfrac{2000}{449} & -\dfrac{2150}{449} & 0 & 0 & \dfrac{125}{44} & \dfrac{75}{22} \end{pmatrix}$$

In[52]:= `InvFcal = Inverse[Fcal]`

Out[52]= $\left\{\left\{\dfrac{20\,376\,759}{157\,562\,500}, -\dfrac{14\,408\,937}{157\,562\,500}, \dfrac{3\,493\,389}{31\,512\,500}, \dfrac{84\,951}{315\,125}, \dfrac{5\,585\,286}{39\,390\,625}, -\dfrac{3\,058\,008}{39\,390\,625}\right\},\right.$

$\left\{-\dfrac{14\,408\,937}{157\,562\,500}, \dfrac{11\,625\,591}{157\,562\,500}, -\dfrac{2\,863\,827}{31\,512\,500}, -\dfrac{40\,713}{315\,125}, -\dfrac{3\,981\,498}{39\,390\,625}, \dfrac{2\,693\,544}{39\,390\,625}\right\},$

$\left\{\dfrac{3\,493\,389}{31\,512\,500}, -\dfrac{2\,863\,827}{31\,512\,500}, \dfrac{6\,757\,119}{6\,302\,500}, -\dfrac{41\,159}{63\,025}, \dfrac{966\,306}{7\,878\,125}, -\dfrac{669\,768}{7\,878\,125}\right\},$

$\left\{\dfrac{84\,951}{315\,125}, -\dfrac{40\,713}{315\,125}, -\dfrac{41\,159}{63\,025}, \dfrac{122\,004}{63\,025}, \dfrac{91\,416}{315\,125}, -\dfrac{22\,368}{315\,125}\right\},$

$\left\{\dfrac{5\,585\,286}{39\,390\,625}, -\dfrac{3\,981\,498}{39\,390\,625}, \dfrac{966\,306}{7\,878\,125}, \dfrac{91\,416}{315\,125}, \dfrac{43\,941\,576}{39\,390\,625}, -\dfrac{34\,912\,628}{39\,390\,625}\right\},$

$\left.\left\{-\dfrac{3\,058\,008}{39\,390\,625}, \dfrac{2\,693\,544}{39\,390\,625}, -\dfrac{669\,768}{7\,878\,125}, -\dfrac{22\,368}{315\,125}, -\dfrac{34\,912\,628}{39\,390\,625}, \dfrac{40\,436\,184}{39\,390\,625}\right\}\right\}$

In[53]:= `Det[Fcal]`

Out[53]= $\dfrac{9\,616\,851\,806\,640\,625}{899\,250\,130\,944}$

In[54]:= `N[Det[Fcal], 6]`

Out[54]= 10\,694.3

In[55]:= **N[Sqrt[Diagonal[InvFcal]] / Sqrt[500], 6]**

Out[55]= {0.0160826, 0.0121478, 0.0463062, 0.0622222, 0.0472342, 0.0453110}

In[56]:= **Fcal1 = NIntegrate[integrand1expim[f], {f, 0, 2 Pi}]**

⋯ NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

⋯ NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

⋯ NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

⋯ General: Further output of NIntegrate::izero will be suppressed during this calculation.

Out[56]= $\{\{8.85417, 7.8125, 0., 0., -4.78842 - 1.73472 \times 10^{-16}\, i, -4.45434 + 2.77556 \times 10^{-17}\, i\},$
$\{7.8125, 8.85417, 0., 0., -3.89755 - 3.97293 \times 10^{-13}\, i, -4.78842 - 1.73472 \times 10^{-16}\, i\},$
$\{0., 0., 0., 0., 0., 0.\}, \{0., 0., 0., 0., 0., 0.\},$
$\{-4.78842 + 2.77556 \times 10^{-16}\, i, -3.89755 + 2.77556 \times 10^{-17}\, i, 0., 0., 3.40909, 2.84091\},$
$\{-4.45434 - 7.63278 \times 10^{-16}\, i, -4.78842 + 2.77556 \times 10^{-16}\, i, 0., 0., 2.84091, 3.40909\}\}$

In[57]:= **Fcal2 = NIntegrate[integrand2expim[f], {f, 0, 2 Pi}]**

⋯ NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

⋯ NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

⋯ NIntegrate: Integral and error estimates are 0 on all integration subregions. Try increasing the value of the MinRecursion option. If value of integral may be 0, specify a finite value for the AccuracyGoal option.

⋯ General: Further output of NIntegrate::izero will be suppressed during this calculation.

Out[57]= $\{\{216.255, 204.936, -19.0765 - 1.36666 \times 10^{-11}\, i, -23.031 - 3.05311 \times 10^{-16}\, i, 0., 0.\},$
$\{204.938, 216.255, -14.4703 - 2.94556 \times 10^{-15}\, i, -19.0765 - 1.36666 \times 10^{-11}\, i, 0., 0.\},$
$\{-19.0765 - 5.16964 \times 10^{-12}\, i, -14.4703 + 2.22045 \times 10^{-15}\, i, 3.40909, 2.84091, 0., 0.\},$
$\{-23.0311 + 0.0000709438\, i, -19.0765 - 5.16964 \times 10^{-12}\, i, 2.84091, 3.40909, 0., 0.\},$
$\{0., 0., 0., 0., 0., 0.\}, \{0., 0., 0., 0., 0., 0.\}\}$

In[58]:= **Fcal = Fcal1 + Fcal2**

Out[58]= $\{\{225.109, 212.748, -19.0765 - 1.36666 \times 10^{-11}\, i, -23.031 - 3.05311 \times 10^{-16}\, i,$
$\quad -4.78842 - 1.73472 \times 10^{-16}\, i, -4.45434 + 2.77556 \times 10^{-17}\, i\},$
$\{212.75, 225.109, -14.4703 - 2.94556 \times 10^{-15}\, i, -19.0765 - 1.36666 \times 10^{-11}\, i,$
$\quad -3.89755 - 3.97293 \times 10^{-13}\, i, -4.78842 - 1.73472 \times 10^{-16}\, i\},$
$\{-19.0765 - 5.16964 \times 10^{-12}\, i, -14.4703 + 2.22045 \times 10^{-15}\, i, 3.40909, 2.84091, 0., 0.\},$
$\{-23.0311 + 0.0000709438\, i, -19.0765 - 5.16964 \times 10^{-12}\, i, 2.84091, 3.40909, 0., 0.\},$
$\{-4.78842 + 2.77556 \times 10^{-16}\, i, -3.89755 + 2.77556 \times 10^{-17}\, i, 0., 0., 3.40909, 2.84091\},$
$\{-4.45434 - 7.63278 \times 10^{-16}\, i, -4.78842 + 2.77556 \times 10^{-16}\, i, 0., 0., 2.84091, 3.40909\}\}$

In[59]:= **InvFcal = Inverse[Fcal]**

Out[59]= $\{\{0.129351 - 2.47432 \times 10^{-6} \, i, -0.0914674 + 1.74966 \times 10^{-6} \, i, 0.11088 - 2.12099 \times 10^{-6} \, i,$
$0.269633 - 5.15775 \times 10^{-6} \, i, 0.141821 - 2.71286 \times 10^{-6} \, i, -0.0776485 + 1.48532 \times 10^{-6} \, i\},$
$\{-0.0914688 + 1.18597 \times 10^{-6} \, i, 0.073798 - 8.38629 \times 10^{-7} \, i, -0.0908961 + 1.01661 \times 10^{-6} \, i,$
$-0.129238 + 2.47216 \times 10^{-6} \, i, -0.101099 + 1.3003 \times 10^{-6} \, i, 0.0683922 - 7.11928 \times 10^{-7} \, i\},$
$\{0.110871 + 5.99262 \times 10^{-6} \, i, -0.0908885 - 4.23754 \times 10^{-6} \, i, 1.07214 + 5.13688 \times 10^{-6} \, i,$
$-0.65303 + 0.0000124917 \, i, 0.122671 + 6.57033 \times 10^{-6} \, i, -0.0850241 - 3.59733 \times 10^{-6} \, i\},$
$\{0.269636 - 0.0000177653 \, i, -0.129237 + 0.0000125623 \, i, -0.653008 - 0.0000152284 \, i,$
$1.93592 - 0.0000370318 \, i, 0.290158 - 0.0000194779 \, i, -0.071016 + 0.0000106644 \, i\},$
$\{0.141821 - 2.66264 \times 10^{-6} \, i, -0.101098 + 1.88282 \times 10^{-6} \, i, 0.122681 - 2.28241 \times 10^{-6} \, i,$
$0.290154 - 5.55029 \times 10^{-6} \, i, 1.11556 - 2.91932 \times 10^{-6} \, i, -0.886335 + 1.59836 \times 10^{-6} \, i\},$
$\{-0.0776505 + 6.51708 \times 10^{-7} \, i, 0.0683928 - 4.6084 \times 10^{-7} \, i, -0.0850313 + 5.58642 \times 10^{-7} \, i,$
$-0.0710181 + 1.35849 \times 10^{-6} \, i, -0.886337 + 7.14535 \times 10^{-7} \, i, 1.02655 - 3.91216 \times 10^{-7} \, i\}\}$

In[60]:= **Det[Fcal]**

Out[60]= $10\,692.2 + 0.204528 \, i$

In[61]:= **N$\left[$Sqrt[Diagonal[InvFcal]] / Sqrt[500], 6$\right]$**

Out[61]= $\{0.0160842 - 1.53836 \times 10^{-7} \, i, 0.0121489 - 6.90291 \times 10^{-8} \, i, 0.0463065 + 1.10932 \times 10^{-7} \, i,$
$0.0622241 - 5.95136 \times 10^{-7} \, i, 0.0472348 - 6.18045 \times 10^{-8} \, i, 0.0453112 - 8.63397 \times 10^{-9} \, i\}$