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Editorial

"Multi-Objective and Multi-Level Optimization: Algorithms and Applications": Foreword by the Guest Editor

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Decision making in real-world applications frequently calls for taking into account multiple goals to come up with viable solutions. Multi-objective optimization is frequently the result of the association of (conflicting) objectives defined by either a single decision maker or a group of decision makers. In this case, optimal solutions do not have the same image values as in single-objective optimization but are rather non-dominated, equivalent, and enable the definition of the Pareto front. We enter the realm of game theory when objectives belong to several non-cooperative decision makers; furthermore, when goals and/or decision makers are hierarchically arranged, this necessitates dealing with nested optimization problems and, consequently, multi-level optimization.

All these problems are computationally difficult to solve, and their resolution typically involves reformulating them into several single-objective problems or one single-objective problem, introducing additional constraints. Moreover, to limit the computational burden, before their resolution, it is worthwhile to reduce the number of objectives to a very limited (significative) number by applying proper methodologies.

This Special Issue, "Multi-Objective and Multi-Level Optimization: Algorithms and Applications", presents original articles focused on multi-objective and multi-level optimization. Five papers have been collected which present innovative applications and theory development, as reported in the following.

Attainment surfaces, which were first described by Fonseca and Fleming more than 25 years ago, give multi-objective optimization researchers a way to precisely visualize the region dominated by a Pareto Optimal Front. Many studies link the non-dominated solutions with curves to depict approximated Pareto Optimal Fronts (POFs). It is incorrect to utilize a curve to unite these non-dominated solutions, according to Fonseca and Fleming's reasoning. Intermediate solutions between any two non-dominated solutions identified using curves suggest that these solutions may exist; however, there is no assurance they do. Non-dominated solutions, according to Fonseca and Fleming, can be used to create an envelope that divides dominated and non-dominated regions rather than a curve. The envelope formed by the non-dominated solutions is referred to as an attainment surface. Attainment surfaces have not been widely applied. The first article [1] of this Special Issue investigates some of the drawbacks that might have prevented this performance measure from being adopted, wherein the quantitative Knowles and Corne measure is analyzed based on attainment surfaces. The analysis demonstrates that the attainment surface's shape influences the outcomes of the Knowles and Corne technique. An improved Knowles and Corne method for bi-objective POF comparisons is proposed. In addition, assuming m objective functions, the porcupine measure, an m-dimensional attainment-surface-based quantitative metric, is suggested for comparing the effectiveness of multi-objective optimization techniques. A computationally and empirically improved version of the porcupine measure is devised by the authors.

To identify pores and fractures (both natural and induced, even at the micro-level) in the wells of a hydrocarbon reservoir, the authors of the second paper [2] of this Special Issue propose a novel meta-heuristic image analysis method using multi-objective optimization.

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The algorithm is called "Pixel-wise *k*-Immediate Neighbors", and offers better identification accuracy in the presence of grayscale sample rock images. In the oil and gas business, porous and fracture imaging is being employed widely to forecast the amount of petroleum present under suitable trap circumstances. These characteristics have many uses in the transportation of contaminants, the storage of radioactive waste in bedrock and the storage of carbon dioxide. In the recent literature, there are a few techniques presented that are able to automatically detect pores and cracks from images. Support Vector Machines (SVMs) are used as a classification tool by many researchers, while deep learning systems are used by a small number of them. SVMs are supervised learning models with associated learning algorithms that analyze data for classification and regression analysis; deep (supervised, semi-supervised or unsupervised) learning is based on artificial neural networks with representation learning (the term "deep" refers to the use of multiple layers in the network). The reported accuracy of the latter, however, is unsatisfactory due to several reasons. Despite its limitations, as discussed in [2], the multi-objective method suggested by the authors outperformed the most popular deep learning system approaches in terms of classification accuracy.

In the third paper [3] of this Special Issue, a code called MolecGeom is provided. It is based on algorithms for stepwise distortions of bond angles and dihedral angles of polyatomic molecules. The energy for each molecule shape is used to construct Potential Energy Surfaces (PESs). To compute PESs, the Born–Oppenheimer approximation is used, which assumes that the atomic nuclei within a molecule are stationary with respect to the motion of its electrons. Applications involving PESs focus on how nuclear mobility affects molecular characteristics. This involves calculating reaction rates, forecasting vibrational spectra and identifying equilibrium geometries corresponding to stationary and saddle point energies. This multi-objective study focuses on the creation of a new technique for computing PESs and the investigation of the components of molecular geometry in terms of incremental change that provides thorough sampling while maintaining the accuracy of PES points. MolecGeom is used to create geometric data when calculating PESs for theoretical simulations of the vibrational–rotational spectra of water and formaldehyde.

The fourth paper [4] of this Special Issue proposes a multi-objective mathematical model and a Pareto analysis to determine the coefficients of an equation describing the thermal conductivity of refrigerants with a low global warming potential. The Non-dominated Sorting Genetic Algorithm II (NSGAII) was utilized for the first time to define a thermophysical characteristic like thermal conductivity. NSGA-II is a modified variant of NSGA that addresses a variety of constrained optimization problems. In contrast to the NSGA, which has a complexity of mn^3 (where m is the number of objectives and n is the population size), the NSGA-II is faster (complexity mn^2), has a better sorting algorithm, includes elitism to prevent an already-found Pareto optimal solution from being deleted, and uses an explicit diversity preservation mechanism. Standard GA (select, crossover, and mutation) is combined with non-dominated sorting in NSGA-II. Additionally, a fitness parameter called "Crowding Distance" is included to account for the density of solutions surrounding a given solution. The algorithm in [4] was used to enhance existing equations. In particular, the NSGAII method was used to optimize two different objective functions, i.e., maximizing the coefficient of determination and minimizing the average absolute relative deviation. After the optimization phase, ten selection approaches drawn from the literature were compared to select the best solution on the Pareto frontier.

Recent studies on energy consumption have shown how commercial food outlets are responsible for a non-negligible percentage of the overall energy consumption worldwide, highlighting the potential and importance of huge energy savings within commercial refrigeration systems. This topic is strictly related to that addressed in the fifth article [5] of this Special Issue, wherein, to create a process that is easier to operate, a supermarket refrigeration system's integrated design is adopted. The number and the discrete power of each compressor to be installed are the design variables of a multi-objective optimization problem proposed by the authors. In the latter, the control performance is assessed using six indices. Since it is unknown if design and performance are functionally dependent, the optimal configuration is determined experimentally. The article [5] has two goals, that is, to demonstrate the benefit of considering integrated design and adapting the Surface Response Method (SRM) to optimize problems without experimental variability, such as that used in the computational campaign proposed by the authors. A D-optimal design is used to carry out the experiments in the SRM framework. The SRM explores the relationships between several explanatory variables and one or more response variables. The main aim of the SRM is to use a sequence of designed experiments to obtain an optimal response. In the design of experiments, optimal designs are a class of experimental designs that are optimal with respect to a statistical criterion. The optimality of a design depends on the statistical model and is assessed with respect to a statistical criterion, which is related to the variance matrix of the estimator. When the statistical model has several parameters, the mean of the parameter estimator is a vector, and its variance is a matrix. The inverse matrix of the variance matrix is called the "information matrix". D optimality seeks to maximize the determinant of the information matrix. The problem in [5] is expressed as a mixture design with constraints and a synergistic cubic model. A desirability function is used to finally reduce the multi-objective problem to a single-objective one.

As can be inferred by the presentation of the above articles, there is an interesting variety of applications and methodologies used to cope with the topics of this Special Issue. We hope that the international scientific community will find the approaches presented and the applications considered in this collection of papers interesting and constructive. We also hope they will inspire further contributions on these topics, which certainly deserve further investigation.

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