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On the Estimation of Logistic Models with Banking Data Using Particle Swarm Optimization

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Abstract: This paper presents numerical works on estimating some logistic models using particle swarm optimization (PSO). The considered models are the Verhulst model, Pearl and Reed generalization model, von Bertalanffy model, Richards model, Gompertz model, hyper-Gompertz model, Blumberg model, Turner et al. model, and Tsoularis model. We employ data on commercial and rural banking assets in Indonesia due to their tendency to correspond with logistic growth. Most banking asset forecasting uses statistical methods concentrating solely on short-term data forecasting. In banking asset forecasting, deterministic models are seldom employed, despite their capacity to predict data behavior for an extended time. Consequently, this paper employs logistic model forecasting. To improve the speed of the algorithm execution, we use the Cauchy criterion as one of the stopping criteria. For choosing the best model out of the nine models, we analyze several considerations such as the mean absolute percentage error, the root mean squared error, and the value of the carrying capacity in determining which models can be unselected. Consequently, we obtain the best-fitted model for each commercial and rural bank. We evaluate the performance of PSO against another metaheuristic algorithm known as spiral optimization for benchmarking purposes. We assess the robustness of the algorithm employing the Taguchi method. Ultimately, we present a novel logistic model which is a generalization of the existence model. We evaluate its parameters and compare the result with the best-obtained model.

Keywords: banking data; logistic growth model; parameter estimation; particle swarm optimization; Taguchi method

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1. Introduction

The logistic growth model, or simply the logistic model, becomes an important part of multidisciplinary science that deals with modeling data that have a tendency to grow exponentially in the early phase, and in the next phase, it saturates and slows down until the growth stops. In mathematical biology, the logistic model is often used in modeling the growth of a population [1,2], the growth of some species size [3–6], and the cases of an endemic disease [7,8]. Meanwhile, in banking and finance, the model is used to model the growth of deposits and loans of a bank [9–12]. The benefit of using logistic model-to-model data is usually to know the prediction of the peak growth and when it occurs and to predict some number called carrying capacity.With this prediction, a policy can be made to deal with the control of the growth of the data.

The logistic model was first introduced by Pierre-François Verhulst in the middle of the nineteenth century [13]. Later, in 1911 and 1920, it was rediscovered by McKendrick and Pai and Pearl and Reed, respectively. McKendrick and Pai studied the rate of the multiplication of micro-organisms that must be proportional to the current number and the lack of foods [4]. Pearl and Reed derived the logistic function for representing the U.S. population growth [1]. Two years later, Pearl and Reed made a generalization by taking the growth rate parameter as a function of time [2].

Years later, for empirical purposes, many researchers modified the logistic model directly or indirectly from the previous researchers. For instance, von Bertalanffy studied the growth of length and weight of guppy fish (*Lebistes reticulatus*) [14] and the connections between metabolism and the growth rate of living organisms [15], Richards extended the von Bertalanffy model to become more flexible and tested the plant data [16], Blumberg proposed a modification in the Verhulst model for modeling the population or organism size [17], Turner et al. presented the generalization called generic growth model [18], and then Tsoularis generalized the generic growth model in [19]. It also needs to be noted that there is a Gompertz function [20] that belongs to the family of logistic functions. In [18], Turner et al. also discussed the hyper-Gompertz function.

In this paper, we use the logistic model to model banking assets growth. Many banking asset forecasting methods employ statistical methodologies that focus exclusively on short-term data predictions, such as [21–23]. Deterministic models are rarely utilized in banking asset forecasting, despite their ability to project data behavior over prolonged periods. Therefore, this work utilizes logistic model prediction. We use Indonesian banking data from the period January 2007–January 2020. The rationale for using the logistic model in analyzing the growth of banking assets is as follows. For a healthy bank, the amount of assets has a tendency to always go up exponentially, although, in some parts, it also fluctuates. However, for several reasons of the bank's internal and external limitations, there should be an upper threshold number for banking asset growth which can be seen as the carrying capacity in the logistic model. A bank is a financial institution with a main intermediary function, which is accepting deposits and providing loans. A bank can be classified as a commercial bank or a rural bank. The objects of this study are the data of assets of Indonesian commercial and rural banks. Commercial banks carry out their business activities conventionally or according to Sharia,in which there are also services in payment transmission services, while rural banks do not provide services in payment transmission. Commercial banks also have a wider range of financial products than rural banks, such as foreign exchange, insurance businesses, and equity participation. In terms of service areas, commercial banks can serve at the village, city, provincial, national, and international levels, and on the other hand, rural banks' service areas are limited to districts or cities. Therefore, the total assets of commercial banks are certainly far greater than rural banks; see Figure 1a,b. The monthly fluctuation of the assets can be seen in Figure 1c,d. The reason why we need to distinguish between commercial and rural bank data, without collecting them into a single data unit, is that we want to see whether the dynamics of growth in commercial banking assets (big banks) are different or the same as the dynamics of rural banking (small banks) asset growth. Whether the two data will fit into the same or different logistic models is our concern.

In the application of a logistic model to modeling data, the problem of estimating the model's parameters is unavoidable. In the logistic model, it can be said that the parameters can be divided into two groups, the first group is the parameters with a small estimated value, and the second group is the carrying capacity parameter which may have a very large value. The gap between the two groups causes parameter estimation problems in a search area that is in the form of a very long rectangle. Such a search area will complicate classical numerical methods; for example, Newton's method with its gradient and Hessian which requires guessing the initial value. The large value of carrying capacity will interfere with the performance of the Newton method. Meanwhile, a metaheuristic algorithm such as the spiral optimization (SpO) algorithm [24,25], which is very powerful for an optimization problem in a square search area as it has an iteration update in the form of spiral rotation, will also experience weaknesses in estimating the parameters of the logistic model because the update in each iteration will create search points that exit the search area.

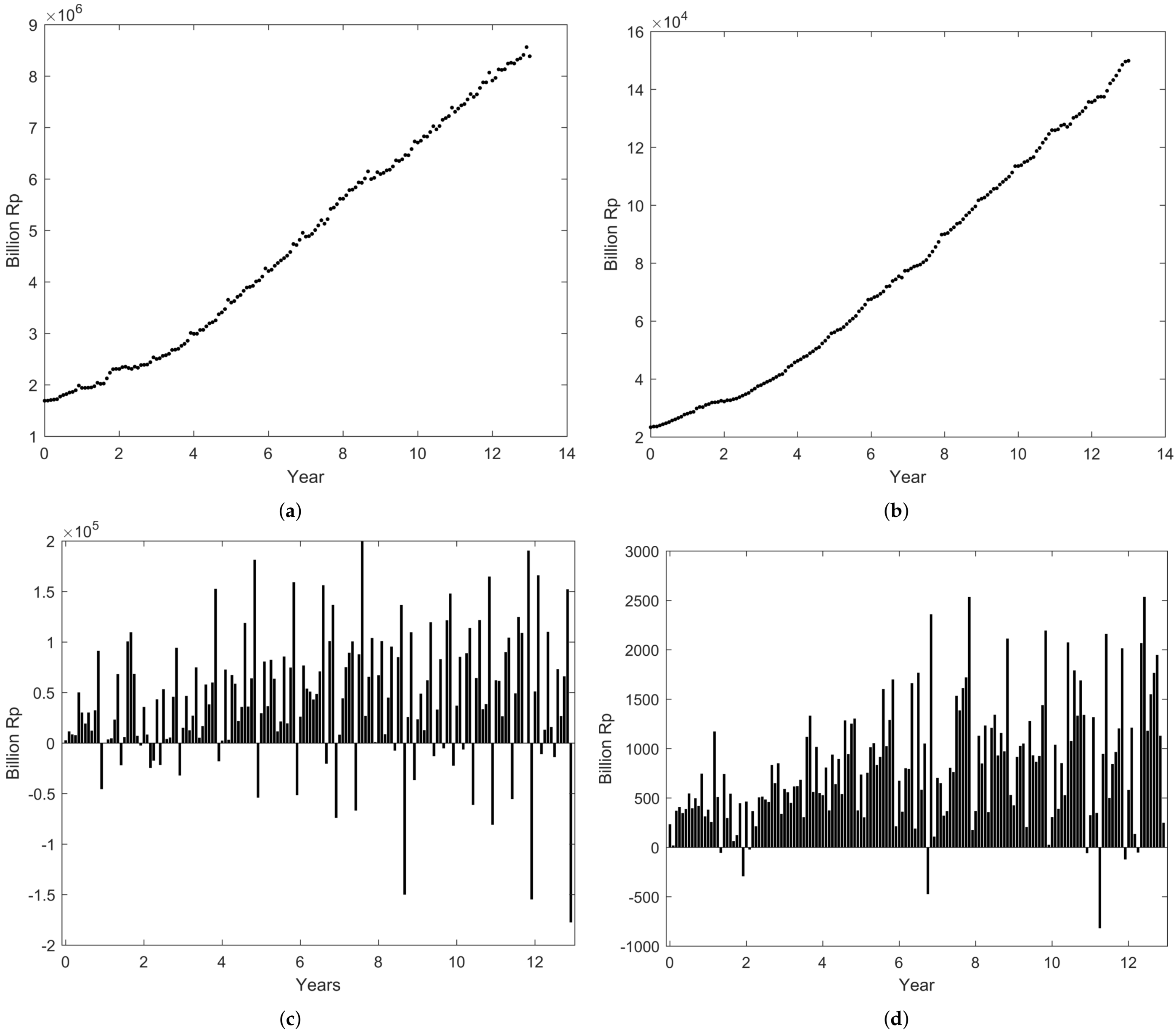


Figure 1. Total assets of (**a**) commercial banks and (**b**) rural banks in Indonesia in the period January 2007−January 2020. The monthly fluctuation of total assets of (**c**) commercial banks and (**d**) rural banks.

Particle swarm optimization (PSO), an optimizer based on the bird or fish-flocking behavior in search of optimal food resources, is one of the metaheuristic methods that is unaffected by the size of the search area because the updating of search points in the PSO method can adapt to the size of the search area. Thus, the algorithm will be very suitable for estimating the parameters of logistic models. To avoid discovering search points outside the search area, the PSO method might include a condition that ensures all search points remain within the search area. In this study, numerous logistic models' parameters are estimated using PSO. In order to reduce the time required to complete iterations, we employ the Cauchy criterion in addition to the maximum number of iterations as a stop condition. Using the Cauchy criterion, we can automatically halt the algorithm if we suspect that it has attained convergence. Although there have been so many variants of PSO in the literature, including recent developments in [26–29], this paper uses the standard PSO algorithm with a decreasing inertia weight [30,31]. To check the algorithm's robustness, we apply the Taguchi method [32,33] to determine the optimal parameters for PSO.

The organization of this paper is as follows. In Section 2, we present a review of nine logistic models used in this paper. Section 3 provides the PSO algorithm. Section 4 describes the implementation of the logistic models' parameter estimation as well as the strategy for the optimization problems. Section 5 presents the numerical result of the parameter estimation and determines the best logistic models for the commercial and rural bank data. In Section 6, we compare the result of PSO with the SpO algorithm as a benchmark. In Section 7, we check the robustness of PSO by tuning the parameters using the Taguchi method. In Section 8, we propose a new logistic model as the generalization of the existence model. We estimate this new model and compare the result with the best models obtained from Section 5. Section 9 discusses and concludes this study.

2. Logistic Models Review

2.1. Verhulst Model

The Verhulst model is given by

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry\Big(1 - \frac{y}{K}\Big), \tag{1}$$

where r is the growth rate parameter, and K is the carrying capacity of the environment.

The right-hand of (1) can be written as

$$ry - \frac{r}{K}y^2 = (r_1 - r_2)y - r_3 y^2.$$

In the study of the population of organisms, r_1 means the birth rate, r_2 means the death rate, and r_3 means the resultant between competition and cooperation rate among them [34].

By separating variables, the differential Equation (1) has solution

$$y(t) = \frac{K}{1 + \big(\frac{K - y(0)}{y(0)}\big)\exp(-rt)}. \tag{2}$$

The value of y that causes the right side of (1) to become maximum is called the inflection point,and the time when that happens is called inflection time. In the study of an epidemic, the inflection time can be viewed as the date when the daily cases reach maximum numbers. The inflection point of the logistic model can be found by solving the following equation for y,

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 0. \tag{3}$$

Let $y = y_{\text{inf}}$ be the solution of (3), i.e., the inflection point. Then, by solving (2) for t so that

$$y(t) = y_{\text{inf}}, \tag{4}$$

we will have the inflection time. Let us denote the inflection time with t_{inf}.

The Verhulst model has inflection point $y_{\text{inf}} = \frac{K}{2}$ and inflection time $t_{\text{inf}} = \frac{1}{r}\ln\big(\frac{K - y(0)}{y(0)}\big)$.

2.2. Pearl and Reed Generalization Model

By rearranging (1) into

$$\frac{K}{y(K - y)}\frac{\mathrm{d}y}{\mathrm{d}t} = r, \tag{5}$$

the left-hand side of (5) varies in time due to the changing of y. Assume that parameter K is constant (for discussion K is not conctant, $K = K(t)$, see [34,35]). Hence, parameter r must vary in time. This leads Pearl and Reed [2] generalizing (1) to be

$$\frac{\mathrm{d}y}{\mathrm{d}t} = r(t)y\Big(1 - \frac{y}{K}\Big). \tag{6}$$

Integrating (5) gives

$$y(t) = \frac{K}{1 + C\exp(-\int r(t)\mathrm{d}t)}, \tag{7}$$

where $C = \left(\frac{K-y(0)}{y(0)}\right)\exp(R(0))$, with $R(t) = \int r(t)\mathrm{d}t$.

For Pearl and Reed's generalization model, we need to find t_{inf} first by solving

$$\begin{aligned}
0 = \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} &= r'(t)y(t)\Big(1 - \frac{y(t)}{K}\Big) + r(t)\Big(1 - \frac{2}{K}y(t)\Big)y'(t) \\
&= r'(t)y(t)\Big(1 - \frac{y(t)}{K}\Big) - r(t)^2 y(t)\Big(1 - \frac{y(t)}{K}\Big)\Big(1 - \frac{2}{K}y(t)\Big),
\end{aligned}$$

or

$$0 = r'(t) + r(t)^2\Big(1 - \frac{2}{K}y(t)\Big).$$

Then, we calculate $y_{\text{inf}} = \left(\frac{r'(t_{\text{inf}})}{r(t_{\text{inf}})^2} + 1\right)\frac{K}{2}$. When $r(t)$ is constant, then we obtain $y_{\text{inf}} = \left(\frac{0}{r^2} + 1\right)\frac{K}{2} = \frac{K}{2}$ same as from the Verhulst model.

In their paper, Pearl and Reed take $r(t)$ as a polynomial, $r(t) = \sum_{n=1}^{N} n a_n t^{n-1}$. Then, (7) becomes

$$y(t) = \frac{K}{1 + \left(\frac{K-y(0)}{y(0)}\right)\exp(-\sum_{n=1}^{N} a_n t^n)}. \tag{8}$$

In the numerical works of this paper, we use $r(t) = a_1 + a_2 t + a_3 t^2$. It is the same $r(t)$ that is used in [2,8].

2.3. Von Bertalanffy Model

Ludwig von Bertalanffy derived his logistic model based on physiological reasons for modeling guppy fish growth and modified the Verhulst model so that it can accommodate the crude metabolic types of the fish [14,19]. The paper [5] uses the von Bertalanffy model to determine the length of Pacific bonito fish based on age and vice versa. The model is given below

$$\frac{\mathrm{d}y}{\mathrm{d}t} = r y^{\frac{2}{3}}\Big(1 - \Big[\frac{y}{K}\Big]^{\frac{1}{3}}\Big). \tag{9}$$

The number $\frac{2}{3}$ in $\mathrm{d}y/\mathrm{d}t = ry^{\frac{2}{3}} - (rK^{-\frac{1}{3}})y$ expresses the surface rule for a certain organism, i.e., the rate of the entire animal's metabolism is proportional to the $\frac{2}{3}$ power of its body weight [15].

The solution of (9) is

$$y(t) = \Big[K^{\frac{1}{3}} - (K^{\frac{1}{3}} - y(0)^{\frac{1}{3}})\exp\Big(-\frac{1}{3}rK^{-\frac{1}{3}}t\Big)\Big]^3, \tag{10}$$

with inflection point $y_{\text{inf}} = \frac{8}{27}K$ and inflection time $t_{\text{inf}} = \frac{3K^{1/3}}{r}\ln\left(\frac{3[K^{1/3} - y(0)^{1/3}]}{K^{1/3}}\right)$.

Comparing the von Bertalanffy model with the Verhulst model, for assumption $y(t) > 1$ for all t, yields

$$ry(t)^{\frac{2}{3}}\Big(1 - \Big[\frac{y(t)}{K}\Big]^{\frac{1}{3}}\Big) < ry(t)\Big(1 - \frac{y(t)}{K}\Big) \text{ for all } t.$$

So, if we set the same parameters value r and K for both models, the von Bertalanffy model needs more time to reach its carrying capacity. Therefore, if we want to make the von Bertalanffy model close to the Verhulst model, we need to set the parameter r of the von Bertalanffy model bigger than the parameter r of the Verhulst model.

2.4. Richards Model

Inspired by the von Bertalanffy model, Richards modified the Verhulst model to be more flexible for the empirical use of fitting plant data [16]. The model is written below

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{r}{\alpha} y \Big(1 - \Big[\frac{y}{K}\Big]^{\alpha}\Big), \tag{11}$$

where $\alpha > 0$ is called the shape parameter. The Richards model is sometimes called the generalized logistic equation following Nelder in [36].

The solution of (11) is

$$y(t) = \frac{K}{\big[1 + \big(-1 + \big[\frac{K}{y(0)}\big]^{\alpha}\big) \exp(-rt)\big]^{1/\alpha}}, \tag{12}$$

with inflection point $y_{\mathrm{inf}} = \frac{K}{(\alpha+1)^{1/\alpha}}$ and inflection time $t_{\mathrm{inf}} = \frac{1}{r} \ln\big(\frac{[K/y(0)]^{\alpha} - 1}{\alpha}\big)$.

2.5. Gompertz Model

In 1825, Benjamin Gompertz sent a letter to Francis Baily about the nature of a designed function to describe the law of human mortality [20]. Later, the function was called the Gompertz function.

The following differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry \ln\Big(\frac{K}{y}\Big), \tag{13}$$

is referred to as the Gompertz model. The model can be viewed as a derivation of Richards differential Equation (11), in which α tends to zero.

The solution of (13) is

$$y(t) = K\Big(\frac{y(0)}{K}\Big)^{\exp(-rt)}, \tag{14}$$

and it is called the Gompertz function. The model has inflection point $y_{\mathrm{inf}} = \frac{K}{\exp(1)}$ and inflection time $t_{\mathrm{inf}} = \frac{1}{r} \ln\big(\ln\big(\frac{K}{y(0)}\big)\big)$.

2.6. Hyper-Gompertz Model

Turner et al. [18] studied a model which they called the hyper-Gompertz model, by putting γ as the power of $\ln(K/y)$ in (13), where $\gamma > 0$ and $\gamma \neq 1$. The hyper-Gompertz model is given by

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry\Big[\ln\Big(\frac{K}{y}\Big)\Big]^{\gamma}. \tag{15}$$

By manipulating (15) in the following way,

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\ln\Big[\frac{y}{K}\Big]\Big) = \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}t} = r\Big[\ln\Big(\frac{K}{y}\Big)\Big]^{\gamma} = (-1)^{\gamma} r\Big[\ln\Big(\frac{y}{K}\Big)\Big]^{\gamma},$$

the solution of (15) is

$$y(t) = K\exp\Big(\Big[(-1)^{\gamma}(-\gamma+1)rt + \big[\ln(\frac{y(0)}{K})\big]^{-\gamma+1}\Big]^{\frac{1}{-\gamma+1}}\Big). \tag{16}$$

It has the inflection point $y_{\mathrm{inf}} = \frac{K}{\exp(\gamma)}$ and inflection time $t_{\mathrm{inf}} = \frac{(-\gamma)^{-\gamma+1} - [\ln(y(0)/K)]^{-\gamma+1}}{(-1)^{\gamma}(-\gamma+1)r}$.

2.7. Blumberg Model

Another modification of the logistic model is proposed by Blumberg [17], written below

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry^{\beta}\left(1 - \frac{y}{K}\right)^{\gamma}. \tag{17}$$

Because of the difficulties of solving (17) analytically, the differential Equation (17) will be solved numerically using the Runge–Kutta method.

The inflection point of the Blumberg model is $y_{\text{inf}} = \frac{\beta K}{\beta+\gamma}$. Meanwhile, the inflection point t_{inf} is taken numerically by solving

$$\min_{i=0,\dots,T} |y(t_i) - y_{\text{inf}}|,$$

with $y(t_i)$ is the Runge–Kutta approximation, $t_i = i\Delta t$, $\Delta t > 0$.

2.8. Turner et al. Model

In a more general way than the Blumberg model, Turner et al. offered this differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry^{1+\alpha(-\gamma+1)}\left(1 - \left[\frac{y}{K}\right]^{\alpha}\right)^{\gamma}, \tag{18}$$

which they called a generic growth model [18]. The parameters α and γ are positive and yield $-1+\gamma < 1/\alpha$. $\gamma \neq 1$. Unlike (17), the model (18) has an analytical solution.

By rewriting (18) as follows

$$\frac{\mathrm{d}y}{\mathrm{d}t} = rK^{-\alpha\gamma}y^{\alpha+1}(K^{\alpha}y^{-\alpha} - 1)^{\gamma},$$

and using an integration substitution method for $K^{\alpha}y^{-\alpha} - 1$, we obtain

$$y(t) = K/\left(1 + \left[(\gamma-1)\alpha K^{\alpha(-\gamma+1)}rt + \left(\left[\frac{K}{y(0)}\right]^{\alpha} - 1\right)^{-\gamma+1}\right]^{1/(-\gamma+1)}\right)^{1/\alpha}. \tag{19}$$

The Turner et al. model has an inflection point $y_{\text{inf}} = \left[\frac{1+\alpha(-\gamma+1)}{1+\alpha}\right]^{1/\alpha}K$ and inflection time

$$t_{\text{inf}} = \frac{\left[\left(\frac{K}{y(0)}\right)^{\alpha} - 1\right]^{-\gamma+1} - \left[\frac{\alpha\gamma}{1+\alpha(-\gamma+1)}\right]^{-\gamma+1}}{r\alpha(-\gamma+1)K^{\alpha(-\gamma+1)}}$$

2.9. Tsoularis Model

Completing the work of Turnet et al., Tsoularis in [19] generalized the generic growth model to be like this

$$\frac{\mathrm{d}y}{\mathrm{d}t} = ry^{\beta}\left(1 - \left[\frac{y}{K}\right]^{\alpha}\right)^{\gamma}. \tag{20}$$

Note that this model is the generalization of all logistic models that have been mentioned before, of course, except the Pearl and Reed generalization. Like the Blumberg model, the Tsoularis model also can not be solved analytically, hence the model will be solved numerically using the Runge–Kutta method.

The inflection point of the Tsoularis model is $y_{\text{inf}} = \left(\frac{\beta}{\beta+\alpha\gamma}\right)^{1/\alpha}K$, where the inflection time will be solved as in the Blumberg model case.

3. Particle Swarm Optimization

As a part of the swarm intelligence family, particle swarm optimization (PSO) is a metaheuristic algorithm inspired by the way bird or fish schools find foods [37]. PSO was first introduced by Kennedy and Eberhart in 1995 [30]. The algorithm portrays the motions of a group of particles with some velocities and the best-remembered position for finding the best position of the group in a closed search space. The best-remembered position acts as the local optimum, and the best position of the group acts as the global optimum. Shi and Eberhart [31] acquainted an inertia weight parameter in the velocity calculation to obtain more stability between the seeking of local optimum and global optimum. Many papers have utilized PSO, its variants, or its hybrid with other algorithms for advanced optimization problems; for examples, see [38–47].

The PSO algorithm for a minimization problem $\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x})$ is written in Algorithm 1.

Algorithm 1 PSO Algorithm

procedure
  Initialization: $I \subset \mathbb{R}^n$ is the search space; m is the number of particles; $\mathbf{x}_i(0) \in I$ is the initial position of particle i, $i = 1, 2, \ldots, m$; $\mathbf{v}_i(0) \in \mathbb{R}^n$ is the initial velocity of particle i, $i = 1, 2, \ldots, m$; c_1 and c_2 are the weighted coefficients; w is the inertia weight parameter.
  Set $\mathbf{x}_i^{\text{best}}(0) = \mathbf{x}_i(0)$ as the initial best position of particle i.
  Set $\mathbf{x}^{\text{gbest}}(0) = \mathbf{x}^{\text{g}}$, where $f(\mathbf{x}^{\text{g}}) = \min_i f(\mathbf{x}_i(0))$, as the initial best position of the group.
  $k = 0$.
  if the stopping criteria is not reached **then**
    for $\forall i \in \{1, 2, \ldots, m\}$ **do**
      Update the particle velocity:

$$\begin{aligned}\mathbf{v}_i(k+1) = w \times \mathbf{v}_i(k) &+ (c_1 \times \text{random}(0,1) \\ &\times (\mathbf{x}_i^{\text{best}}(k) - \mathbf{x}_i(k))) \\ &+ (c_2 \times \text{random}(0,1) \\ &\times (\mathbf{x}^{\text{gbest}}(k) - \mathbf{x}_i(k))).\end{aligned}$$

      Update the particle position: $\mathbf{x}_i^{\text{new}} = \mathbf{x}_i(k) + \mathbf{v}_i(k+1)$
      Check whether the updated position is still in the search space or not by the following condition:
      if $\mathbf{x}_i^{\text{new}} \in I$ **then**
        Set $\mathbf{x}_i(k+1) = \mathbf{x}_i^{\text{new}}$
      else
        Set $\mathbf{x}_i(k+1) = \mathbf{x}_i(k)$
      Update the best position of each particle and the best position of the group:
      if $f(\mathbf{x}_i(k+1)) \leq f(\mathbf{x}_i^{\text{best}}(k))$ **then**
        $\mathbf{x}_i^{\text{best}}(k+1) = \mathbf{x}_i(k+1)$.
        if $f(\mathbf{x}_i(k+1)) \leq f(\mathbf{x}^{\text{gbest}}(k))$ **then**
          $\mathbf{x}^{\text{gbest}}(k+1) = \mathbf{x}_i(k+1)$.
        else
          $\mathbf{x}^{\text{gbest}}(k+1) = \mathbf{x}^{\text{gbest}}(k)$.
      else
        $\mathbf{x}_i^{\text{best}}(k+1) = \mathbf{x}_i^{\text{best}}(k)$.
        $\mathbf{x}^{\text{gbest}}(k+1) = \mathbf{x}^{\text{gbest}}(k)$.
    $k = k + 1$.
    $w = \rho \times w$, where ρ is the decreasing parameter of w, $0 < \rho < 1$.
  return $\mathbf{x}^{\text{gbest}}$ as the minimum point of $f(\mathbf{x})$.

It is common to set the following PSO parameters: $c_1 = 2$, $c_2 = 2$, $w = 1$, and ρ as a number which is very close to 1; for example, $\rho = 0.99$.

PSO is designed to make the global minimum of the objective function in the current iteration less than or equal to the global minimum of the objective function in previous iterations, that is $f(\mathbf{x}^{\text{gbest}}(k)) \leq f(\mathbf{x}^{\text{gbest}}(j))$, for all $j \leq k$. In the optimization for data fitting, the objective function is usually the error between the data and the model. Therefore, the objective function has values greater or equal to zero, which is bounded below. The number zero is one of the lower bounds. Now, suppose that $f \geq 0$. Thus, the sequence $u_k = f(\mathbf{x}^{\text{gbest}}(k))$ is a decreasing sequence and bounded below. By the Monotone Convergence Theorem, u_k is convergent. And by the Cauchy convergence criterion, a sequence of real numbers is convergent if and only if it is a Cauchy sequence. The Cauchy sequence is defined as follows: a sequence u_k of real numbers is said to be a Cauchy sequence if for every $\epsilon > 0$ there exists a natural number $H(\epsilon)$ such that for all natural numbers $k, j \geq H(\epsilon)$, the terms x_k, x_j satisfy $|x_k - x_j| < \epsilon$ [48].

We adopt the Cauchy convergence criterion as one of the stopping criteria of the PSO. In this paper, we use two stopping criteria: the maximum iteration number ($k_{\max}$) and the Cauchy stopping criterion. We will stop the iteration if $k = k_{\max}$ or the Cauchy stopping criterion is reached. The Cauchy stopping criterion is explained as follows: set a natural number N, if $|f(\mathbf{x}^{\text{gbest}}(k)) - f(\mathbf{x}^{\text{gbest}}(k-j))| < \epsilon$ for some $j \geq 1$ and $\epsilon > 0$, and for all k where $j \leq k \leq j + N$, then the current iteration must be stopped. So, if the iteration of the Cauchy stopping criterion is reached, then we must stop there and do not need to continue until maximum iteration. If this happens, much time can be saved. And if the Cauchy criterion is too difficult to make happen, although we have reached the maximum iteration, we will stop the iteration there rather than waiting too long for the sake of the Cauchy criterion being reached.

4. Implementation

The test sample (banks) consists of Indonesian commercial and rural banks. The number of commercial and rural banks in Indonesia has decreased over the years (see Figure 2, the data are taken from the Indonesia Central Bureau of Statistics's website https://www.bps.go.id (accessed on 23 October 2024)) due to various factors like mergers, consolidations, and regulatory changes. This decrease is largely due to government efforts to stabilize the banking sector and encourage mergers among smaller banks to create stronger institutions. Despite that, the total assets seem to increase each year; see Figure 1a,b. The data are monthly total assets in the period January 2007–January 2020 in terms of a billion rupiah (Rp) taken from Indonesia Financial Service Authority's website (https://www.ojk.go.id accessed on 1 March 2020), as shown in Figure 1. Let $y_d(t_i)$ be the data of banking assets at time t_i, where $t_i = i\Delta t$, $i = 0, 1, \ldots, 156$. $\Delta t = 1/12$ is the time increment, t_0 is January 2007, and t_{156} is January 2020. We use the data from January 2007 until December 2018 as the training data, that is the data that are implemented to the logistic models for gaining the parameter value of the models. The data from January 2019 until January 2020 are used as the validation data of the models' prediction in the future assets value.

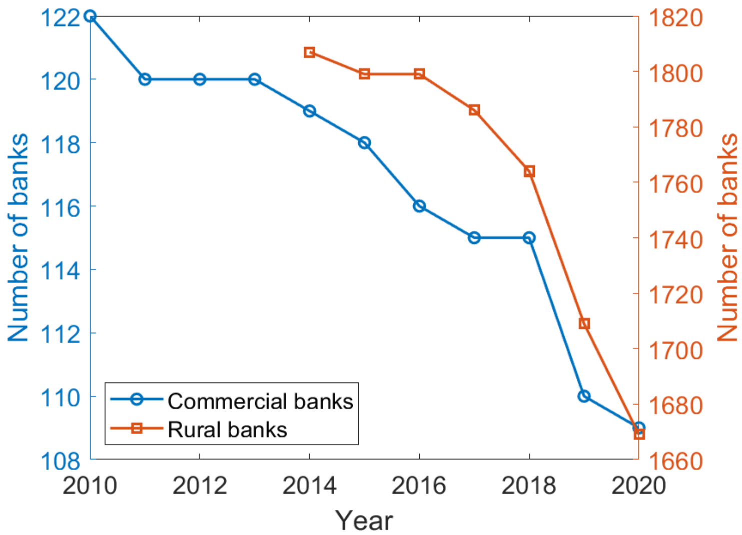


Figure 2. The number of commercial and rural banks in Indonesia over the years.

Parameters of logistic models can be divided into two categories. One consists of a growth rate parameter and shape parameters which can be estimated in a short interval of search space; for example, [0,1], [0,5], etc. The other one is a carrying capacity that may require a very long interval of search space; for example, $[0, 10^3]$, $[10^4, 10^8]$, etc. Instead of picking an arbitrary interval for estimating the carrying capacity, we can pick an interval based on a rough estimated value of the carrying capacity.

Pearl and Reed [1] give a formula to determine the rough estimation of the carrying capacity of the Verhulst logistic function in (2). They use three data points that have the same distance range $h > 0$. Those points are named (s_1, x_1), (s_2, x_2), and (s_3, x_3), where $s_2 - s_1 = s_3 - s_2 = h$, and $x_1 = y_d(s_1)$, $x_2 = y_d(s_2)$, $x_3 = y_d(s_3)$. By substituting those points into (2) and doing some eliminations, we obtain

$$K = \frac{2x_1x_2x_3 - (x_1 + x_3)x_2^2}{x_1x_3 - x_2^2}. \tag{21}$$

We choose s_2 as the time around the mid-time of banking assets data, $s_2 = \frac{71}{12}, \frac{72}{12}$ and some $h = \frac{71}{12}, \frac{69}{12}, \frac{67}{12}$. From those s_2 and h, we calculate $s_1 = s_2 - h$ and $s_3 = s_2 + h$. Thus, we obtain $s_1 = 0, \frac{2}{12}, \frac{4}{12}$ and $s_2 = \frac{142}{12}, \frac{140}{12}, \frac{138}{12}$ for the case $s_2 = \frac{71}{12}$, and $s_1 = \frac{1}{12}, \frac{3}{12}, \frac{5}{12}$ and $s_2 = \frac{143}{12}, \frac{141}{12}, \frac{139}{12}$ for the case $s_2 = \frac{72}{12}$. Having those s_j and the associated $x_j = y_d(s_j)$, $j = 1, 2, 3$, from (21), we obtain rough values for the carrying capacities of Indonesian commercial and rural banking assets that are presented in Table 1. From Table 1, we can clearly see why the Pearl and Reed formula is called a rough calculation of the carrying capacity of the logistic model, since different s_1, s_2, and s_3 will produce different carrying capacity values.

Table 1. Rough values of the carrying capacity (in billion Rp) of commercial and rural banks in Indonesia.

s_1	s_2	s_3	Rough K for Com. Bank	Rough K for Rural Bank
0	71/12	142/12	12,431,437	206,126
2/12	71/12	140/12	12,022,655	199,172
4/12	71/12	138/12	11,342,372	200,580
1/12	72/12	143/12	14,254,951	215,716
3/12	72/12	141/12	13,313,955	204,998
5/12	72/12	139/12	12,619,803	204,283

Perhaps the best-estimated value (using an optimization method) of the carrying capacities of the logistic models is smaller or greater than the rough values. Experience shows that their value is not very much different [8,10]. In this paper, for all logistics models, the carrying capacity will be estimated using PSO at an interval, say $[a, b]$. In the case of commercial banks, $a =$ 8,068,346 (the last data of the training data of commercial banks) and $b =$ 142,549,510 (the maximum rough carrying capacity value of commercial banks from Table 1 times 10). And for the case of rural banks, $a =$ 135,693 (the last data of the training data of rural banks) and $b =$ 2,157,160 (the maximum rough carrying capacity value of rural banks from Table 1 times 10). Meanwhile, to estimate the parameters of logistic models in Section 2 such as $a_1, a_2, a_3, r, \alpha, \beta, \gamma$, we use the intervals presented in Table 2. The parameter r of the von Bertalanffy model is estimated at interval $[0, 50]$, because the parameter r requires a larger value to match the behavior of the Verhulst model.

Table 2. Some intervals as the search space for estimating the parameter of logistic models.

Parameter	The Search Space
a_1, a_2, a_3 for Pearl–Reed generalization model	$[0, 1]$
r for Verhulst, Richards, Gompertz, hyper-Gompertz, Blumberg, Turner et al., and Tsoularis models	$[0, 1]$
r for von Bertalanffy model	$[0, 50]$
α for Richards, Turner et al., and Tsoularis models	$[0, 5]$
β for Blumberg and Tsoularis models	$[0, 3]$
γ for hyper-Gompertz model	$[0.75, 3]$
γ for Blumberg, Turner et al., and Tsoularis models	$[1, 5]$

We have set the search space for the parameter estimation. Now, we set some parameters in PSO algorithm: $m = 1000$, $c_1 = 2$, $c_2 = 2$, $w = 1$, $\rho = 0.99$. Let $\mathbf{x}(k)$ be the vector of the logistic model's parameters at k-th iteration. The objective function of the parameter estimation being used here is the Mean Absolute Percentage Error (MAPE), which is calculated as follows

$$f(\mathbf{x}(k)) = \frac{100\%}{144} \sum_{i=0}^{143} \left| \frac{y(t_i, \mathbf{x}(k)) - y_d(t_i)}{y_d(t_i)} \right|,$$

where $y_d(t_i)$ is the training data of banking assets at time $t_i = i/12$ and $y(t_i, \mathbf{x}(k))$ is the value of assets based on the logistic model's solution, $i = 0, 1, \ldots, 143$.

We set the stopping criteria for the algorithm, required the iteration passes the 20-th iteration, as follows

$$k = k_{\max} \text{ or } |f(\mathbf{x}(k)) - f(\mathbf{x}(k-20))| < \epsilon,$$

where $k_{\max} = 150$ and $\epsilon = 0.001\%$.

We run the PSO twenty times, and choose the obtained parameters that have the minimum MAPE. The obtained model is used to validate the validation data by calculating

$$\text{MAPE validation} = \frac{100\%}{13} \sum_{i=144}^{156} \left| \frac{y(t_i, \mathbf{x}^{\text{gbest}}) - y_d(t_i)}{y_d(t_i)} \right|.$$

Apart from MAPE, as an additional consideration to find out how close the logistic model is to the data, we also calculate the Root Mean Square Error (RMSE) for the training data and the validation data below

$$\text{RMSE training} = \sqrt{\frac{\sum_{i=0}^{143} (y(t_i, \mathbf{x}^{\text{gbest}}) - y_d(t_i))^2}{144}},$$

$$\text{RMSE validation} = \sqrt{\frac{\sum_{i=144}^{156} (y(t_i, \mathbf{x}^{\text{gbest}}) - y_d(t_i))^2}{13}}.$$

5. Numerical Results

The results of the parameter estimation are presented in Table 3 for commercial banks and in Table 4 for rural banks. The letters P-R, V, vB, R, G, hG, B, Tu, Ts refer to the Pearl and Reed generalization model (8), Verhulst model (2), von Bertalanffy model (10), Richards model (12), Gompertz model (14), hyper-Gompertz model (16), Blumberg model (17), Turner et al. model (19), and Tsoularis model (20), respectively.

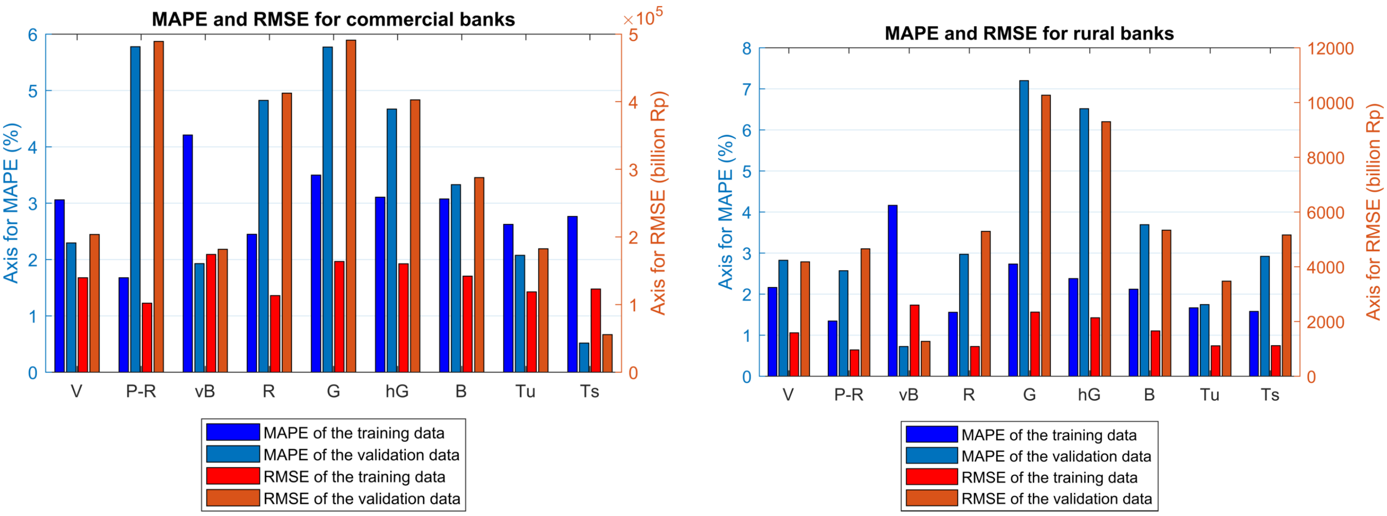
In general, for the selection of the best model, the first thing to review is the smallest error between the data and the model. Usually, the model with the smallest error will be chosen, and the model with the biggest error is not selected. To provide a better visualization of the errors in Tables 3 and 4, we present the chart of MAPE and RMSE in Figure 3. First, we analyze the MAPE of the training data, because it is used as the objective function of the estimation parameters. From the figure, the Pearl–Reed generalization model has the smallest MAPE, and it is followed by the Richards model. Meanwhile, the von Bertalanffy model has the biggest MAPE, followed by the Gompertz model and the hyper-Gompertz model. Based on the information on the MAPE values of the training and validation data, the first models to be out from consideration are the von Bertalanffy model, the Gompertz model, and the hyper-Gompertz model.

Table 3. Result of PSO in estimating parameters of logistic models using commercial banking assets data in Indonesia.

	a_1	a_2	a_3		K	Stop	t_{inf}	y_{inf}	MAPE t.	MAPE v.	RMSE t.	RMSE v.
P-R	0.1375	0.0156	4.28×10^{-10}		8,228,341	62	7.33	5,114,283	1.68%	5.78%	101,902	489,191
	r	α	β	γ								
V	0.1844				15,071,474	25	11.22	7,535,737	3.06%	2.29%	139,536	203,508
vB	29.3160				142,549,510	25	44.89	42,236,895	4.21%	1.93%	174,153	181,636
R	0.6557	4.2911			8,255,810	42	8.16	5,599,424	2.45%	4.82%	113,273	412,614
G	0.0431				86,377,000	27	31.76	31,776,323	3.50%	5.77%	163,688	491,075
hG	0.0432			1	85,794,137	115	31.70	31,561,899	3.11%	4.67%	160,263	402,819
B	0.0718		1.0683	4.7342	48,868,588	66	13.17	8,997,285	3.07%	3.33%	142,053	287,811
Tu	0.2014	2.6711		1.0062	946,0970	150	8.54	5,778,078	2.62%	2.07%	118,677	182,535
Ts	0.5401	2.9998	0.9158	3.0812	13,585,176	150	14.08	6,091,104	2.77%	0.52%	122,988	55,416

Table 4. Result of PSO in estimating parameters of logistic models using rural banking assets data in Indonesia.

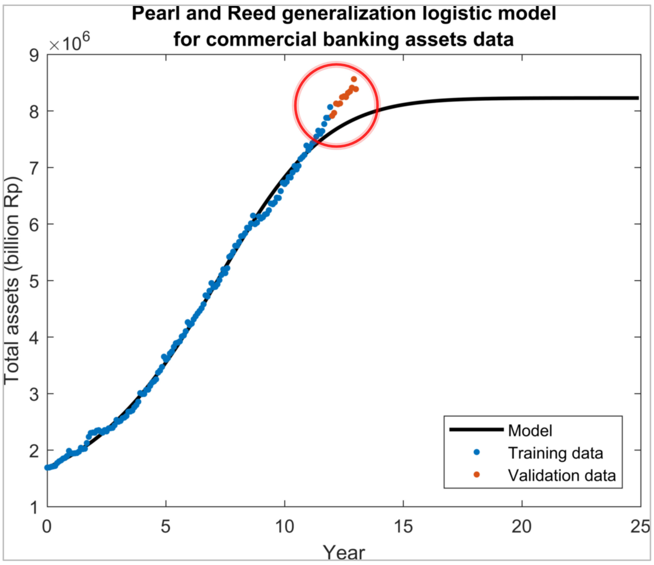
	a_1	a_2	a_3		K	Stop	t_{inf}	y_{inf}	MAPE t.	MAPE v.	RMSE t.	RMSE v.
P-R	0.1838	0.0080	4.00×10^{-9}		162,036	57	8.42	94,498	1.34%	2.57%	961	4655
	r	α	β	γ								
V	0.2052				259,816	24	11.28	129,908	2.16%	2.82%	1587	4180
vB	8.3632				2,157,160	26	38.58	612,388	4.16%	0.72%	2597	1274
R	0.5296	3.0127			153,790	74	8.63	96,968	1.57%	2.97%	1086	5291
G	0.0443				1,919,658	24	33.49	706,203	2.73%	7.20%	2339	10,268
hG	0.0447			1	1,872,938	146	33.09	689,016	2.38%	6.51%	2131	9299
B	0.0878		1.0875	4.2360	712,615	64	12.33	145,580	2.12%	3.69%	1656	5337
Tu	0.2165	2.4256		1.0072	166,654	150	8.86	99,583	1.66%	1.74%	1111	3475
Ts	0.2477	4.3027	0.9657	4.6135	195,562	150	13.75	95,839	1.58%	2.92%	1117	5159



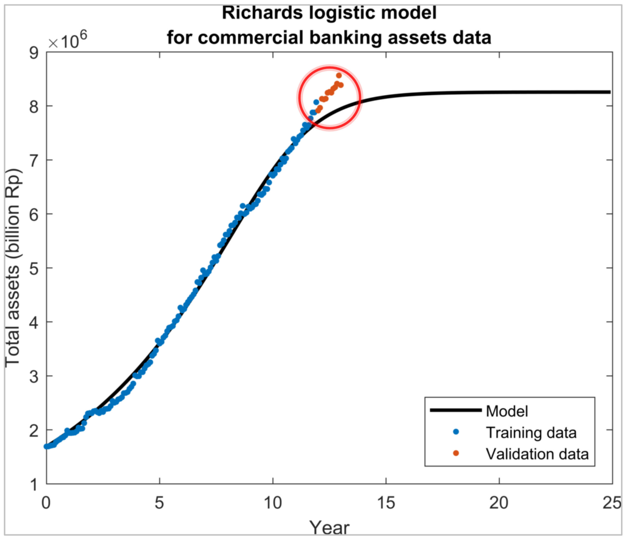
(**a**) (**b**)

Figure 3. MAPE and RMSE of the obtained models for (**a**) commercial banks and (**b**) rural banks.

From the perspective of the MAPE of the validation data, for the commercial banking case, the Pearl–Reed generalization model has the biggest MAPE followed by the Gompertz model, the Richards model, and the hyper-Gompertz model. For the rural banking case, the Gompertz model has the biggest validation MAPE, followed by the hyper-Gompertz model. The Pearl–Reed generalization model and the Richards model have small MAPE of the training data but their MAPE of the validation data is big. Let us look at their plots versus the data in Figure 4. From the figure, those models are very good at approaching the training data, but they fail to predict the validation data. The prediction strays too far from the data; see the parts marked by a red circle. Also, as we can see from the figure, the prediction failure is caused by the values of their carrying capacity being less than the highest of the validation data ($y_d(t_{155}) = 8{,}562{,}974$) for the commercial banking case, and are just a little bit higher than the highest of the validation data ($y_d(t_{156}) = 149{,}872$) for the rural banking case.



(**a**)



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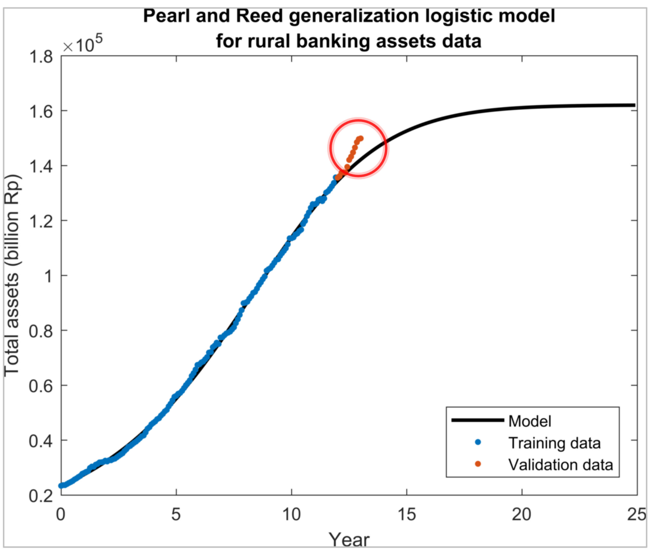
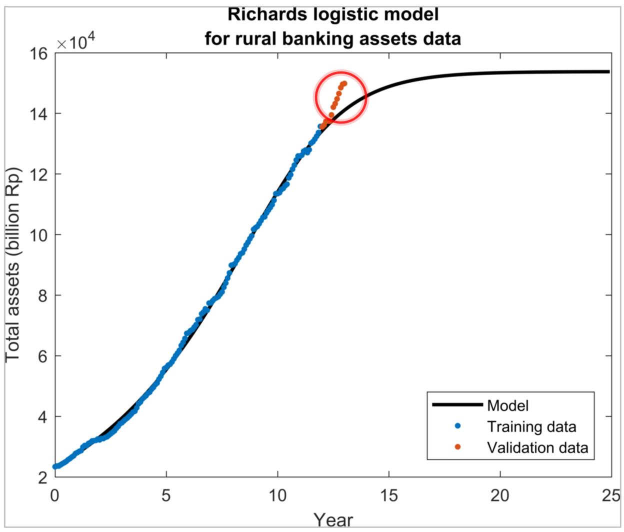


(**c**)



(**d**)

Figure 4. Plot of the Pearl–Reed generalization model versus the data of (**a**) commercial banks and (**c**) rural banks and the Richards model versus the data of (**b**) commercial banks and (**d**) rural banks.

The failure of the Pearl–Reed generalization model and the Richard model in producing an estimated carrying capacity which is far enough above the last validation data gives us insight into the carrying capacity value of the other models. Figure 5 presents the chart of the carrying capacity values of all models. From the figure, in addition to the two models mentioned, Turner et al. also produce the carrying capacity value which is close to the highest of the validation data. For the reasons of the digression of the models from the data and the closeness between the carrying capacity values with the highest validation data, we take out the Pearl and Reed model, the Richards model, and the Turner et al. model from the consideration as the best model.

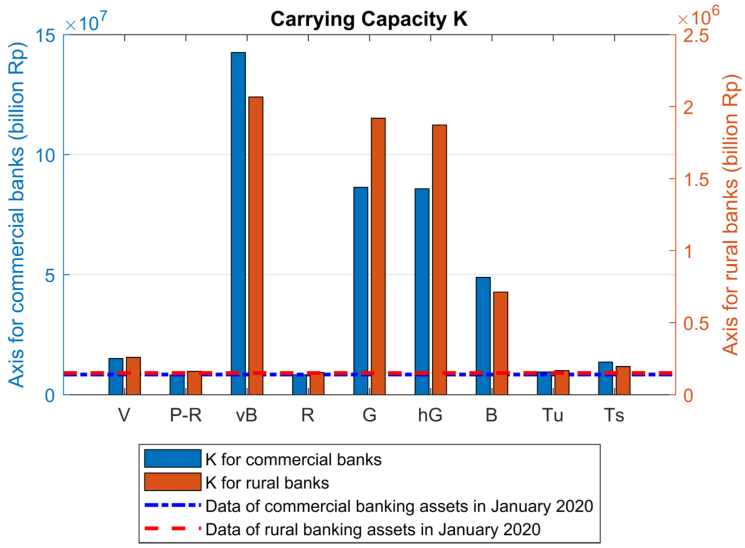


Figure 5. The carrying capacity of the obtained models.

Now, there are three models that need to be discussed: the Verhulst logistic model, the Blumberg model, and the Tsoularis model. From Figure 1, for the commercial banking case, the Tsoularis model has the smallest MAPE and RMSE of the training and validation data than the other two models. Therefore, we conclude that the Tsoularis model is the best logistic model to fit the Indonesia commercial banking data. Meanwhile, for the rural banking case, despite having the smallest MAPE and RMSE for the training data, the Tsoularis model strays far from the validation data as shown by the biggest value of MAPE and RMSE of the validation data. Now, for the rural banking case, we compare the Verhulst model and the Blumberg model. In Table 4, the Verhulst model has a smaller MAPE of the training data and also a smaller RMSE of the training and validation data than the Blumberg model, although the Blumberg model has a smaller MAPE of the training data. So, in the case of fitting the Indonesia rural banking data, we conclude that the Verhulst model is the best model.

6. Benchmarking with Spiral Optimization Algorithm

In this section, we compare the PSO performance with another metaheuristic algorithm as a benchmark. In this case, we employ the spiral optimization (SpO) which is a metaheuristic algorithm first introduced in [24,25]. It is inspired by the spiral phenomenon in nature. SPO describes a set of points that rotate around the global optimal point as the direction of rotation. The SpO has recently been used in many applications, such as [49–52]. The SpO algorithm has an updating scheme as follows:

$$\mathbf{x}_i(k+1) = S_n(\delta,\theta)\mathbf{x}_i(k) - (S_n(\delta,\theta) - I_n)\mathbf{x}^{\mathrm{gbest}}(k),$$

where $S_n(\delta,\theta) = \delta\Pi_{i=1}^{n-1}\left(\Pi_{j=1}^{i} R_{n-i,n+1-j}^{(n)}(\theta)\right)$, and $R_{i,j}^{(n)}(\theta)$ is $n \times n$ identity matrix with the ii and jj entries are $\cos\theta$, the ji entry is $\sin\theta$, and the ij entry is $-\sin\theta$.

The parameters of the method, θ and δ, represent the rotation angle, where $0 < \theta \leq \pi$, and the scaling radius of rotation, where $0 < \delta < 1$, respectively. In the majority of the literature, the ranges $\pi/4 \leq \theta \leq \pi/2$ and $0.9 \leq \delta \leq 0.99$ are employed.

We run the SpO algorithm to estimate the parameters of the Tsoularis model for the commercial bank case and the Verhulst model for the rural bank case. The comparison with the PSO's result is presented in Table 5. In both cases, the PSO result is superior because of the smaller MAPE training and validation compared to the SpO result.

Table 5. Result of SpO compared to PSO.

Algorithm	Tsoularis Model for Commercial Banks Data							Verhulst Model for Rural Banks Data			
	r	α	β	γ	K	MAPE t.	MAPE v.	r	K	MAPE t.	MAPE v.
SpO	0.8027	2.1869	0.8894	3.4229	20,757,461	3.04%	2.98%	0.2043	264,499	2.17%	3.24%
PSO	0.5401	2.9998	0.9158	3.0812	13,585,176	2.77%	0.52%	0.2052	259,816	2.16%	2.82%

7. Tuning PSO Parameters

A critical aspect of employing an algorithm for optimization problems is its robustness, typically described as the algorithm's capacity to function well amidst variations in its parameters. One approach to address the robustness issue is the Taguchi method [32,33]. It is a signal-to-noise (SN) ratio which can be defined as follows [53]:

$$SN = -10 \log_{10}(\text{MAPE})^2. \tag{22}$$

A higher value of the SN ratio means the parameter settings of the algorithm are more robust. Several studies have employed the Taguchi method in their optimization problems, especially when using PSO [26–28,53,54].

In this section, we apply a robustness check of the PSO's parameter settings using the Taguchi method. To this end, we consider five levels of robustness check for setting the parameters listed in Table 6. We execute the PSO algorithm with all parameter settings in the table to estimate the Tsoularis model's parameters for the commercial bank case and the Verhulst model's parameters for the rural bank case. At the end, we calculate each SN ratio based on the MAPE obtained from the parameter estimation. The result is illustrated in Figure 6. We must observe which parameter's value gives the highest SN ratio. It is found that the optimal setting of the PSO's parameters is identified as $m = 50$, $c_1 = 0.5$, $c_2 = 1.5$, $w = 3$, and $k_{max} = 50$.

Table 6. The levels of the PSO's parameters setting.

PSO's Parameter	Level 1	Level 2	Level 3	Level 4	Level 5
Number of particles (m)	10	50	100	200	300
Weighted coefficient (c_1)	0.5	1	1.5	2	2.5
Weighted coefficient (c_2)	0.5	1	1.5	2	2.5
Inertia weight (w)	1	2	3	4	5
Maximum iteration (k_{max})	25	50	75	100	150

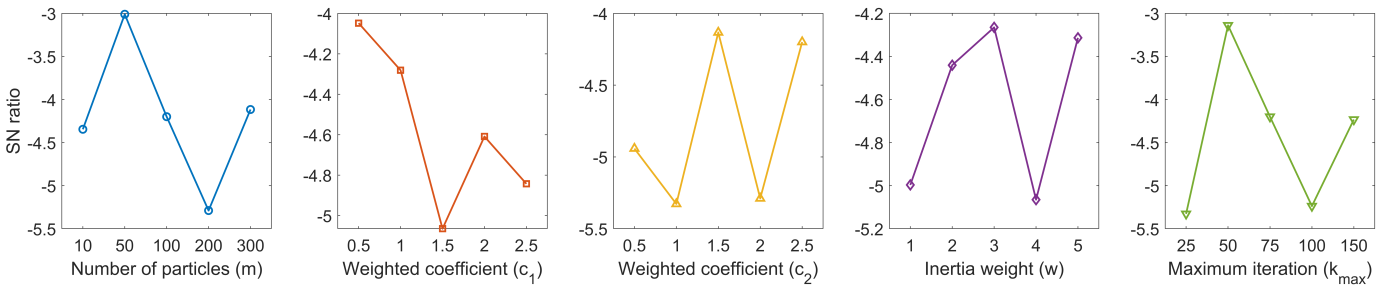


Figure 6. The SN ratio for PSO's parameters.

8. New Logistic Model

In the section on logistic model reviews (Section 2), it is known what all logistic models have in common, e.g., that the growth rate r is a constant, except that only the Pearl and Reed generalization model has a time-varying-growth rate as $r(t) = a_1 + a_2 t + a_3 t^2$.Now, we propose a new logistic model by generalizing the Tsoularis model with time-varying-growth rate $r(t) = a_1 + a_2 t + a_3 t^2$ as follows:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = (a_1 + a_2 t + a_3 t^2) y^{\beta} \left(1 - \left[\frac{y}{K}\right]^{\alpha}\right)^{\gamma}. \tag{23}$$

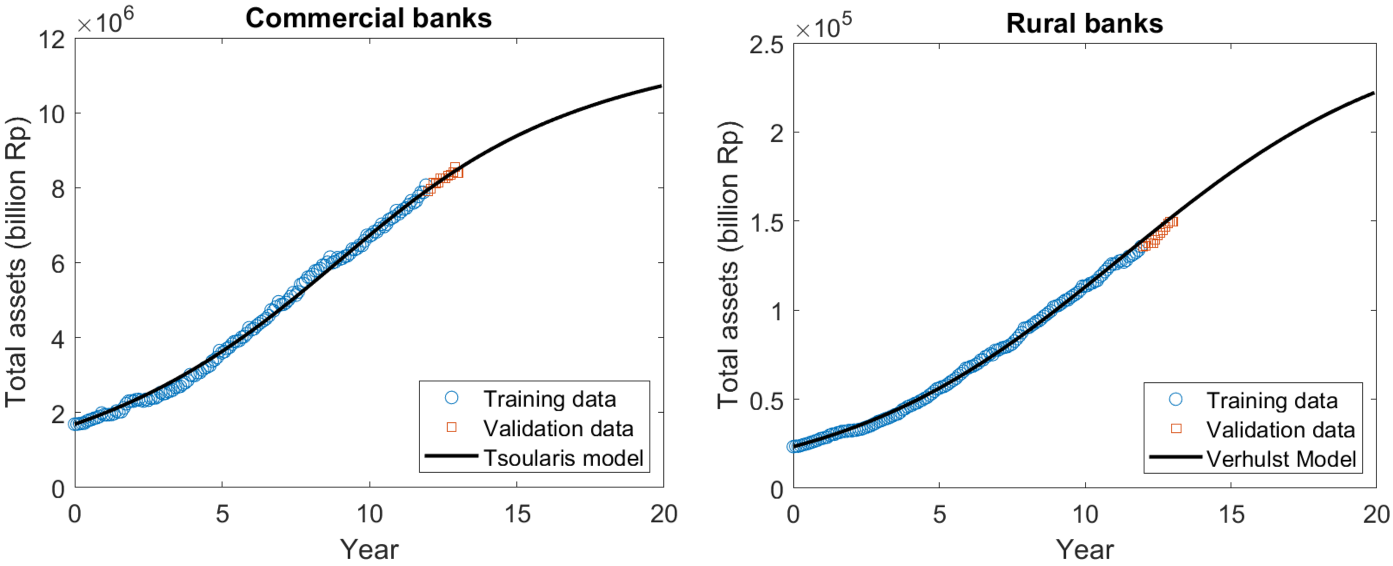
This model cannot be solved analytically. Consequently, numerical methods are required to estimate the parameters of the model. In this instance, we employ the Runge–Kutta method. We compare it with the outcomes from the Tsoularis model for commercial banks' data and the Verhulst model for rural banks' data. We have estimated all seven parameters of the new model (23), but we have that the result is superior compared to the benchmark models. The problem may arise from the new model having an increased number of parameters, and the nonautonomous nonlinear differential equation complicates precise numerical solutions. Therefore, we made a strategy, as follows. We use the obtained value of the Tsoularis model's parameters α, β, γ, and K from Tables 3 and 4, and we only estimate the parameters a_1, a_2 and a_3. The parameter estimations utilize the optimal PSO parameters derived from the Taguchi method in the preceding section. The outcome is displayed in Table 7. When compared to the results of the Tsoularis and Verhulst models, the newly proposed logistic model produces less MAPE training but higher MAPE validation. The new model demonstrates improved accuracy in capturing model fitting; however, it is unable to predict subsequent data for validation purposes.

Table 7. Result of PSO in estimating parameters of the new proposed logistic model (23).

Case Data	New Model	Tsoularis Model	Verhulst Model
Commercial banks	$a_1 = 0.3622$ $a_2 = 0.0932$ $a_3 = -0.0084$ $\alpha = 2.9998$ $\beta = 0.9158$ $\gamma = 3.0812$ $K = 13,585,176$ MAPE t. = 1.68% MAPE v. = 5.29%	$r = 0.5401$ $\alpha = 2.9998$ $\beta = 0.9158$ $\gamma = 3.0812$ $K = 13,585,176$ MAPE t. = 2.77% MAPE v. = 0.52%	None
Rural banks	$a_1 = 0.2114$ $a_2 = 0.0203$ $a_3 = -0.0019$ $\alpha = 4.3027$ $\beta = 0.9657$ $\gamma = 4.6135$ $K = 195,562$ MAPE t. = 1.58% MAPE v. = 6.37%	None	$r = 0.2052$ $K = 259,811$ MAPE t. = 2.16% MAPE v. = 2.82%

9. Discussion and Conclusions

The use of various logistic growth models to estimate the assets of commercial and rural banks in Indonesia provides important insights into the dynamics of banking growth. Each model brings its own assumptions about growth rates, carrying capacities, and the non-linear behavior of financial assets over time. In this study, nine logistic models, including the Verhulst, Gompertz, and Tsoularis models, were applied to model the asset data of commercial and rural banks. The findings revealed that the Tsoularis model, which is known for capturing complex growth dynamics, provided the best fit for commercial banks, whereas the Verhulst model was more appropriate for rural banks. The model fitting is illustrated in Figure 7. This variation underscores the need to tailor growth models to specific segments within the banking industry, reflecting their unique growth patterns and constraints.



(a) (b)

Figure 7. The result of data fitting and prediction of Indonesian (**a**) commercial and (**b**) rural banking data.

The results suggest that commercial banks, with larger asset bases and broader market exposure, experience growth that is more influenced by intricate factors, as captured by the Tsoularis model. This model's ability to account for variations in growth rates and environmental limits might explain its superior fit. Conversely, the Verhulst model, which assumes a simpler logistic growth dynamic, proved effective for rural banks, where growth might be more linear and constrained by local economic conditions. These outcomes indicate that asset growth in rural banks is less volatile and tends to stabilize at lower carrying capacities compared to commercial banks, which experience more complex and potentially exponential growth before stabilizing.

The PSO demonstrated higher performance with lower MAPE in both training and validation compared to the SpO algorithm used as a benchmark. The robustness of the PSO algorithm was verified in this study through parameter tuning using the Taguchi method. This study also attempted to introduce a new logistic model that incorporated a more complex structure with an increased number of parameters. This new model was found to be superior (for data fitting) but inferior (for data prediction) to the established Tsoularis and Verhulst models. The complexity introduced by the nonautonomous nonlinear differential equations in the new model posed significant challenges for numerical precision when all the parameters are estimated simultaneously. This underscores the potential trade-off between model complexity and practical applicability, suggesting that simpler models may sometimes yield more reliable outcomes in real-world scenarios where numerical accuracy and convergence are critical.

Despite these findings, several research gaps remain. Notably, while this study focused on modeling growth dynamics using traditional logistic models, future work could incorporate external macroeconomic factors such as inflation rates, policy shifts, and technological advancements that influence banking growth. Additionally, testing alternative optimization techniques beyond particle swarm optimization could reveal different insights into model performance. Lastly, exploring how these models perform in periods of economic downturn or rapid expansion would offer a more comprehensive understanding of their robustness and applicability across varying economic conditions.

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and I.G.; visualization, M.F.A.; supervision, K.A.S., N.S. and I.G.; project administration, M.F.A.; funding acquisition, M.F.A. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

The following abbreviations are used in this manuscript:

PSO	Particle Swarm Optimization
MAPE	Mean Absolute Percentage Error
RMSE	Root Mean Square Error

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