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Towards Sustainable Inventory Management: A Many-Objective Approach to Stock Optimization in Multi-Storage Supply Chains

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Abstract: Within the framework of sustainable supply chain management and logistics, this work tackles the complex challenge of optimizing inventory levels across varied storage facilities. It introduces a comprehensive many-objective optimization model designed to minimize holding costs, energy consumption, and shortage risk concurrently, thereby integrating sustainability considerations into inventory management. The model incorporates the distinct energy consumption profiles associated with various storage types and evaluates the influence of stock levels on energy usage. Through an examination of a 60-day production schedule, the dynamic relationship between inventory levels and operational objectives is investigated, revealing a well-defined set of optimal solutions that highlight the trade-off between energy savings and shortage risk. Employing a 30-day rolling forward analysis with daily optimization provides insights into the evolving nature of inventory optimization. Additionally, the model is extended to encompass a five-objective optimization by decomposing shortage risk, offering a nuanced comprehension of inventory risks. The outcomes of this research provide a range of optimal solutions, empowering supply chain managers to make informed decisions that strike a balance among cost, energy efficiency, and supply chain resilience.

Keywords: many-objective optimization; ideal stock optimization; sustainable supply chain; stock management; cost-effective logistics

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1. Introduction

Supply chain management is a topic of great relevance in today's industrial environment, and a widely researched topic in academia [1]. Resilience of the supply chain (SC) to external unpredictable factors is greatly sought after by companies, and the current data-heavy industrial scene allows the leverage of computational techniques in order to minimize disruptions [2]. Resilience to these SC disruptions has become of paramount importance as industries become more globalized and disruptions more frequent, stemming from natural disasters to geo-political conflicts [3] or global pandemics, such as the COVID-19 pandemic [4]. One of the final stages of the upstream supply chain is the storage of raw materials in companies' warehouses. This is a very complex problem on its own, as keeping, e.g., large amounts of stock can help companies reduce the risk of having a shortage of materials in unpredictable supply chain events, but it comes at a hefty cost, since holding stock has inherent costs—the longer and the greater the quantities of stock being held, the larger the costs.

At the same time, environmental conscience has risen significantly in the last few years. Increasingly, more companies start to take environmental aspects into considerations in their decision-making processes, diverging from purely profit-focused approaches. Nevertheless, it is very commonplace for sustainable solutions to also reduce costs—a solution that is focused on reducing energy consumption will invariably reduce energy costs. In the supply chain literature this sustainability trend has also been verified, with many publications on green (or sustainable) supply chains [5–10]. Indeed, according to Khan et al. [11], publications on sustainable supply chains have been steadily increasing, especially after 2013, where the yearly number of publications increased from 21 in 2013 to 66 in 2018. More recent publications on the subject show that this trend has continued, with Hmouda et al. [12] showing a continuing increase up until 2021 (note that the publication was submitted in 2022).

The concept of safety stock determination and optimization has been widely researched for decades [13], as establishing the minimum amount of stock necessary for companies to be resilient to most unpredictable phenomena is paramount. Different approaches towards the problem are found throughout the literature. Some researchers use statistical methods for the determination of the safety stock [14]; some tackle the safety stock placement of multi-stage supply chains using optimization and decomposition techniques [15], the combination of base stock and base backlog in a make-to-stock setting [16], and others deal with inventory control in conjunction with pricing and production rates [17,18].

A less researched and more complex problem is the optimization of the ideal stock levels. In contrast to determining safety stock, ideal stock optimization is not static in time and depends on many factors, some fixed, like warehouse capacity, and some highly changeable, like the production schedule and consequent stock requirements. While being a less researched topic than safety stock optimization, several publications are available on this topic. Daniel and Rajendran [19] deal with the problem of determining installation base-stock levels in a serial supply chain, using heuristics to expedite the convergence of the optimization algorithms used. The problem is formulated as a single-objective problem, focused on minimizing the total supply chain cost, comprised of the total holding cost and the total shortage cost; the problem is also solved as a bi-objective problem, considering as objectives the two costs separately. Haijema and Minner [20] analyse hybrid base-stock and constant order policies. The authors address the issue as a simulation-based optimization. Király et al. [21] use simulation to address the issue of inventory control of multi-echelon supply chains. The sustainability in supply chains is also considered as the distance travelled between nodes of the supply chains is also taken into account.

Multiobjective optimization is an optimization area focused on optimizing more than one objective. This means that when the problem has conflicting objectives, multiple non-dominated solutions can be found. This approach has been applied on a plethora of problems in industrial settings, namely, on a multi-effect desalination unit integrated with a fuel cell-based trigeneration system [22] or on data-driven soft sensors for a cleaner papermaking process [23]. Many-objective optimization is a subset of multiobjective optimization that regards optimization problems with three or more objectives. This allows more flexibility in modelling the problem but also comes at the cost—and with the opportunity—of substantially increasing the number of non-dominated solutions. This type of optimization has been applied in, e.g., semiconductor manufacturing [24] or pharmaceutical supply chains [25].

1.1. State-of-the-Art Review

Given the identified research gaps and the context of Industry 5.0, the research question for this study is the following: In a dynamic production environment with heterogeneous storage, how can data-driven inventory management strategies and decision-making frameworks be designed to simultaneously mitigate shortage risk, minimize holding costs, and promote sustainability? This research questions can be separated into three components of inventory management: sustainable inventory management; data-driven inventory management; multiobjective inventory management.

Recent advancements in inventory management have underscored the importance of incorporating components of sustainability and environment impact. Becerra et al. [26] provide a review on sustainable inventory management. The authors show that the majority of the articles analysed have their environmental focus on reducing, mostly resorting to approaches of either simulation or exact programming methods. Lv and Sun [27] propose a bi-objective robust optimization focused on carbon emissions and total system cost, by changing routing decisions. Vu and Ko [28] optimize a single-objective problem, which weighs different costs, including greenhouse gas emissions, and considers cold storage on a trans-shipment problem. Zhou et al. [29] minimize a single objective comprised of weighed factors that include CO_2 emissions and holding costs, by optimizing a set of binary routing variables and order quantities. Mishra et al. [30] present an optimal replenishment strategy, focused on a single-objective optimization with environmental emissions reduction and holding cost as components of their objective.

With the surge of big data, an increasing number of optimization strategies with a data-driven component have appeared. This trend has also been seen in inventory management. Beutel and Minner [31] focus their work on safety stock under causal demand forecast. A single objective is optimized which includes holding and shortage costs, by balancing inventory levels and satisfied demands. The aforementioned work by Lv and Sun [27] is focused on robust optimization, which captures the multi-period uncertain production demand.

Many publications on inventory management consider multiple objectives. To simplify the designation, as the definition is not consensual, in this work, an optimization with two objectives is classified as bi-objective optimization; three or more objectives are considered many-objective optimization—the main difference being that representation of the Pareto front is possible (with sufficient clarity) for only two objectives. The previously stated work by Lv and Sun [27] is an example of a bi-objective optimization approach, but there are additional instances. Sarwar et al. [32] present a bi-objective inventory control system, focused on minimizing the cost of inventory and carbon emissions. Tsai and Chen [33] present a many-objective approach to inventory optimization, considering three objectives, total inventory cost, average inventory level, frequency of inventory shortage, by changing the reorder point and order quantity.

Besides the publications described here, additional state-of-the-art research was considered. Table 1 shows the classification of all articles considered in terms of how many objectives there were, what the decision variables were, what the objectives were, what storage types were considered and whether or not the work featured any data-driven components. These publications are compared with the proposed approach in the following subsection.

1.2. Proposed Approach

This article sets out to answer the previously introduced research question through the optimization of the ideal stock at the warehouse of a company, considered a multi-layered warehouse system with regular warehousing, storage tanks, and cold storage. It brings novelty as the under-researched topic of ideal stock is addressed through a many-objective optimization problem, considering the risk of shortage, holding costs, and sustainability, through energy consumption. This combination was not found in any published research. In contrast to safety stock-based strategies, the optimization of the ideal stock offers a more resilient alternative, with the capacity of being focused on the overarching goal of optimizing output, instead of purely offering a safety level. Furthermore, different storage locations are considered for the stocks to be optimized, each with their respective energy consumption. This strategy and the energy consumption modelling is also novel. It is modelled uniquely for each type of storage location, heavily influenced by the works conducted by Sabegh and Bingham [34], Zavvar Sabegh and Bingham [35], Wolisz et al. [36], Lewczuk et al. [37]. Finally, the shortage risk is modelled through a novel, dimensionless approach, comprised of three components—a risk of immediate shortage, shortage considering one standard deviation from the suppliers' lead time, and a general measure of capacity to fulfil material requirements.

The research is focused on many-objective optimization due to its role in the industry, where decision-makers are not substituted, but rather, their work is facilitated. This comes from the fact that the optimization does not output a single solution but rather a series of optimal solutions. Sustainability can also have a bigger role, as profit does not have to be the only objective any longer—while profit tends to be the biggest focus of companies, adding a sustainability objective in a problem may skew the decision-making process, if the results show that some solutions may have considerable improvements in sustainability at a small cost in profit. Bringing forward these scenarios and effectively improving decision-makers' visibility into possible decision strategies has extensive advantages. Not only are they cost-effective analyses, as cost is always an objective, but they also contribute to smart planning, energy efficiency, and overall environmentally friendly logistics systems. Furthermore, this approach contributes to energy management in manufacturing execution systems (MES), as stock management is a central aspect of it.

Table 1. Classification of inventory management state-of-the-art research regarding the type of optimization used, the decision variables considered, whether or not holding, shortage, and sustainability are considered objectives; whether or not cold storage, general warehousing, or tanks are addressed and modelled; and whether or not the approach has any data-driven component. Publications are ordered according to publication date (older publications first). Publications classified as single-objective approaches but with multiple objectives simply mean that those objectives are considered in the objective function, e.g., by adding holding and shortage costs.

Article	**Optimization Type**	**Decision Variables**	**Objective**			**Storage Type**			**Data-Driven**
			Holding	**Shortage**	**Sustainability**	**Cold Storage**	**Warehouse**	**Tanks**	
Daniel and Rajendran [19]	Bi	Installation base-stock levels	X	X			X		
Tsou [38]	Many	Order size and safety factor	X	X			X		
Liao et al. [39]	Many	Order quantities	X				X		
Beutel and Minner [31]	Single	Inventory levels, satisfied demands	X	X			X		X
Bouchery et al. [40]	Bi	Batch quantity and binary decision	X		X		X		
Tsai and Chen [33]	Many	Reorder point and order quantity	X	X			X		
Mishra et al. [30]	Single	Cycle time, selling price, preservation and environmental emission cost, ordering cost per cycle and per order	X		X		X		
Sarwar et al. [32]	Bi	Order quantity	X	X	X		X		
Singh et al. [41]	Single	Cycle length, credit period, production rate	X		X		X		
Sepehri et al. [42]	Single	Production run time, selling price, and two investment components	X		X		X		
Lv and Sun [27]	Bi	Binary routing decisions	X		X		X		X
Vu and Ko [28]	Single	Routing decisions			X	X			
Zhou et al. [29]	Single	Routing decisions and order quantities	X		X		X		
Proposed Approach	Many	Stored quantities	X	X	X	X	X	X	X

The results from Table 1 show that the proposed approach tackles a not very explored problem. Regarding the optimization type, six works considered a single objective, four considered two objectives, and three considered more than two (specifically, they all considered three objectives). The proposed approach is initially formulated as a three-objective problem but is relaxed into a five-objective optimization problem. The advantages of many-objective optimization problems have been addressed. Regarding the objectives considered, it can be seen that only a single publication simultaneously addresses holding, shortage, and sustainability. Unsurprisingly for inventory management problems, all publications consider the holding cost, and many (8 out of 13) consider a sustainability component—interestingly, only the older publication disregards sustainability. Only five publications consider shortage costs, with most not allowing for shortage. The least explored component in inventory management regards storage type. Indeed, most publications simply disregard this component—in these cases, articles were classified as regarding general warehouses. The proposed approach gives a considerable focus on the storage type and considers three different types, as the implication in terms of energy consumptions changes drastically. Finally, not many publications have data-driven considerations—however, with the advent of big data, it becomes easier and more necessary to include data as a driving aspect of optimization problems.

This work is also within the scope of the advent Industry 5.0, as the pillars of this new evolution of industrial technology are based on sustainability, human centricity, and resilience [43]. To this end, this research is greatly centred on the pillars of Industry 5.0: human-centric, as the solution outputted by the many-objective optimization model requires a human decision-maker with field expertise; sustainable, as one of the objectives is the reduction in energy consumption; resilient, as the risk of shortage is also minimized, and the optimization is data-driven, based on the production schedule of the company.

This study was inspired by the pharmaceutical industry. However, the methodology detailed ahead is perfectly applicable to any other industries with dynamic production schedules, as the balancing act of storing raw materials is a complex process that has to take into account many considerations. In this problem, a single warehouse, a single cold storage, and multiple tanks were considered; other industries may feature a different configuration of storage locations. Some may not require, e.g., cold storage, depending on the industry. These changes should be easily implemented as there is large flexibility to adapt to different companies.

2. Mathematical Formulation

As stated in the introduction, the objectives used for the optimization problem were the energy consumption, holding cost, and risk of shortage. These objectives were selected based on previous work on the topic (namely, the components of total holding cost and total shortage cost presented in [19]), and sustainability considerations required by the pillars of Industry 5.0. The specific objectives considered are shown below.

- $E \equiv$ energy consumption : total daily consumption of energy derived from acclimatizing the raw materials.
- $C_{Hold} \equiv$ holding cost: total daily cost of holding the stocks at the different storage locations.
- $R_{Sh} \equiv$ risk of shortage: measure of risk of not having sufficient raw materials for production, given the suppliers' lead time and the near-future raw material requirements.

While the holding costs are linearly dependant on the occupation degree of each storage location, the formulation of the energy consumption and risk of shortage are more complex and requires novel implementations with the specific conditions of the problem in mind. The main rationale for these changes stems from the opportunities provided by a many-objective formulation—as the objectives are independent of each other, they do not have to be in the same units, nor be weighed to evaluate their impact on a single solution. This justifies, for instance, why the risk of shortage is directly measured in lead times of the materials and their standard deviations.

2.1. Theoretical Background

To correctly formulate the objectives of the problem, a series of theorems supported by the existing literature can be used. Out of the three objectives, modelling the energy consumption is the most complex component. Each type of storage location has a different model for the energy consumption, as it varies in magnitude and dynamics. The energy consumed in the cold storage is the more nuanced component. Theorem 1 and its corresponding proof define the energy consumption dynamics of a refrigerator unit, with changing degrees of fullness and hysteresis bands.

Theorem 1. *The energy consumption of a refrigerator varies depending on its degree of fullness and the allowed hysteresis band.*

Proof of Theorem 1. According to Sabegh and Bingham [34] and Zavvar Sabegh and Bingham [35], a refrigerator filled with products has a higher specific heat capacity than a empty one. The authors present experimental data showing that an empty refrigerator increases its energy consumption with the increase in the hysteresis band, while one at 40% useful capacity reduces its energy consumption with the increase in the hysteresis band. □

Consider Theorems 2 and 3. These theorems regard the energy consumption in the warehouse.

Theorem 2. *The total energy consumption of a warehouse is calculated as the sum of the energy consumptions for the transportation of equipment, building heating and cooling, ventilation, lighting, IT networks, and other energy consumptions.*

Proof of Theorem 2. According to the practical case study by Lewczuk et al. [37], the total energy consumption is the combination of the stated components, regardless of the warehouse technology and level of automation. □

Theorem 3. *When subjected to heating at the same power output, an empty room increases its temperature faster than a furnished one but also cools down faster.*

Proof of Theorem 3. The emptier a warehouse is, the larger the ratio of air in it; the fuller a warehouse is, the more solid materials there are, and the ratio of air to contents decreases. The mechanisms of heat transfer to the air and to solid objects are different, and their capacity to retain and emit heat also differs. The thermal conductivity of air is smaller than most solid and liquid materials, meaning that it has a lower capacity to exchange heat from and to the environment. According to the experimental work by Wolisz et al. [36], an empty room subjected to the same heating power of a furnished one increases the temperature faster than a furnished one. The authors performed a test on an empty and furnished room, where the room was heated for 4 h (starting at 21 °C), and then allowed to cool for 4 more hours. The empty room increased to 23.3 °C and then reduced to 20.4 °C; the furnished room increased to 22.7 °C and then reduced to 20.7 °C. This experiment proves Theorem 3. □

Finally, a theorem regarding how the holding costs are modelled according to the storage degree of fullness can be seen in Theorem 4.

Theorem 4. *The holding costs of a warehouse are proportional to its degree of fullness.*

Proof of Theorem 4. As stated by Harrison et al. [44], *"holding costs are continuously incurred at a rate proportional to the storage level"*. □

To mathematically formulate each of the objectives, additional notation must be introduced and defined. Table 2 presents the variables required for this problem.

Table 2. Required variables for the problem formulation. All units are presented within square brackets. Variables without unit are dimensionless.

Variable	Description
General Variables	
X_i	Total stock of product i, $[IU]$ $[n_i \times 1]$
$A_{i,k}$	Binary matrix of association of product i to storage location k $[n_i \times n_k]$
S_k	Total quantities in storage location k, $[IU]$.
Lim_k	Limits for storage location k, $[IU]$.
H_k	Holding cost at storage k, per inventory unit of product, $[CU/IU]$.
n_i	Total number of products.
n_k	Total number of storage locations.
n_{kt}	Total number of tanks.
n_t	Total number of days considered for the future material requirements.

Table 2. *Cont.*

Variable	Description
Cold Storage Variables	
ET_{cold}	Correspondence of which storage location k is a cold storage $[n_k \times 1]$
S_{cold}	Total stock at the cold storage location, $[IU]$ $[1 \times 1]$
α_1	Individual freezer unit capacity, $[IU]$
α_2	Ratio of energy consumption increase in freezer units, with degree of fullness
α_3	Nominal daily energy consumption of each freezer unit, $[EU]$
α_4	Mid-point of the sigmoid energy consumption increase
α_5	Width of the sigmoid energy consumption increase
Warehouse Variables	
$ET_{warehouse}$	Correspondence of which storage location k is a warehouse $[n_k \times 1]$
$S_{warehouse}$	Total stock in the warehouse, $[IU]$ $[1 \times 1]$
$FC_{warehouse}$	Fixed energy consumption in the warehouse, $[EU]$ $[1 \times 1]$
β_1	Energy consumption per unit of material in the warehouse, consumed by transportation of stock, $[EU/IU]$ $[1 \times 1]$
β_2	Base energy consumed by HVAC in the warehouse, considering an empty warehouse, $[EU]$ $[1 \times 1]$
β_3	Linear decay rate of HVAC energy consumption in the warehouse, per unit of material, $[EU/IU]$ $[1 \times 1]$
Tank Variables	
ET_{tank}	Correspondence of which storage location k is a tank, with multiple tanks allowed $[n_k \times n_{kt}]$
S_{tank}	Total stock in each tank, $[IU]$ $[1 \times n_{kt}]$
γ	Energy consumption per tank, considering the tank at full capacity, $[EU]$ $[n_{kt} \times 1]$
Stock Shortage Calculation Variables	
L_i	Average lead time of purchasing product i, $[day]$.
σ_i	Standard deviation of the lead time of purchasing product i, $[day]$.
$N_{i,t}$	Requirements of material i for the t-th day after the problem, $[IU]$.
$B_{i,t}$	Stock of material i on day t after consumption has been subtracted and without stock replenishment. The values only contain the sign of the quantities, i.e., whether they are negative, null, or positive quantities.
ShD_i	Day when the base stock is exhausted. Takes the value $n_k + 1$ for materials whose base stock is never exhausted.
$Sh1_i$	Shortage value of type 1 for each material.
$Sh2_i$	Shortage value of type 2 for each material.
$Sh3_i$	Shortage value of type 3 for each material.
$SW1$	Weight of shortage type 1.
$SW2$	Weight of shortage type 2.
$SW3$	Weight of shortage type 3.

The decision variables for this problem are the total stocks of each product, i.e., X_i. These establish the ideal stocks for a given day, based on the near future conditions. Index i corresponds to the material, ranging from 1 to n_i; index k corresponds to the storage location, ranging from 1 to n_k; index t corresponds to the day, ranging from 1 to n_t. $n_{i,cold}$, $n_{i,warehouse}$, and $n_{i,tanks}$ correspond to the number of products in the cold storage, warehouse, and tanks, respectively, verifying the condition $n_{i,cold} + n_{i,warehouse} + n_{i,tanks} = n_i$.

2.2. Assumptions

A number of assumptions were considered for this problem's formulation.

- Three types of storage location were considered: warehouse, cold storage, and tanks. Each type could be in different locations and had different holding costs.
- The formulation considered a single warehouse and cold storage, but multiple tanks were allowed.
- Lead time for transportation of stock between storage locations was considered to be negligible.
- Holding cost was assumed to be linear with the amount of stock.
- While there was only one cold storage location, it contained multiple freezer units.
- For the calculation of the risk of shortage, a horizon of 60 days was considered.
- Filling a storage location more than its maximum capacity was not allowed.
- The problem was considered deterministic.

- All quantities were considered to be in generic units. Materials were in inventory units, $[IU]$, costs were in cost units, $[CU]$, and energy were in energy units, $[EU]$.
- The total energy consumption of a tank was linearly proportional to the rate of fullness of the tank.

Additionally, a few assumptions related to the proposed theorems can be presented.

- The energy consumption dynamics with the degree of fullness and allowed hysteresis band were assumed to be identical for industrial freezers as the ones proposed in [34,35] and presented in Theorem 1.
- The considered industrial freezers could automatically (or manually) adapt their hysteresis band according to the usage level, with a maximum allowed hysteresis of 2 °C—this means that according to Theorem 1, by changing the hysteresis band, the energy consumption could be kept the same with increasing occupation.
- The maximum allowed hysteresis of 2 °C allowed a constant energy consumption up to a utilization of 40%—after that, the energy consumption started to increase from nominal values. This increase stagnated after a certain capacity was reached.
- The increase in energy consumption was modelled as a sigmoid function, approximately constant at lower values, then increasing when the maximum hysteresis band was reached, and returning to constant values.
- The warehouse energy consumption components of transportation of equipment, building heating and cooling, and ventilation were considered to be dependent on the degree of fullness of the warehouse. The remaining components of lighting, IT networks, and other energy consumptions were considered to be fixed values.
- The energy consumption of the warehouse from the transportation of equipment was modelled linearly with the degree of fullness of the warehouse—more occupation required more movement of stock.
- Considering that a warehouse is required to be kept at a specific temperature, and ignoring the transient period required to increase the warehouse contents' inner temperature up to the environment temperature, the greater the ratio of fullness of the warehouse, the lower its non-transient energy consumption, as proved in Theorem 3. This means that the warehouse's energy consumption elements of heating, cooling, and ventilation were modelled as inversely proportional to its degree of fullness.

2.3. Objectives Formulation

The calculation of the energy consumption was performed separately for each type of storage location, cold storage, warehouse, or tank. The general formulation is as shown in Equation (1).

$$E = E_{cold} + E_{warehouse} + E_{tank} \quad [EU] \tag{1}$$

2.3.1. Energy Consumption

The energy consumption of freezer units followed Theorem 1 and the assumptions previously identified. Figure 1 shows an example of the evolution of the energy consumption with the amount of stock. The example considered 3 individual freezer units, and the second freezer was only used after the first was completely full.

The energy consumption of the cold storage E_{cold} can then be formulated as shown in Equation (2). Note that the square brackets with no upper horizontal segments correspond to the floor operator, while the square brackets with no lower horizontal segments correspond to the ceiling operator.

$$E_{cold} = \left\lfloor \frac{\alpha_2}{1 + \exp\left(-\left(\frac{S_{cold}-1}{\alpha_1} - \left\lfloor \frac{S_{cold}-1}{\alpha_1} \right\rfloor\right) \cdot \frac{\alpha_5}{\alpha_4} + \alpha_5\right)} + \left\lceil \frac{S_{cold}}{\alpha_1} \right\rceil \right\rfloor \cdot \alpha_3 \quad [EU] \tag{2}$$

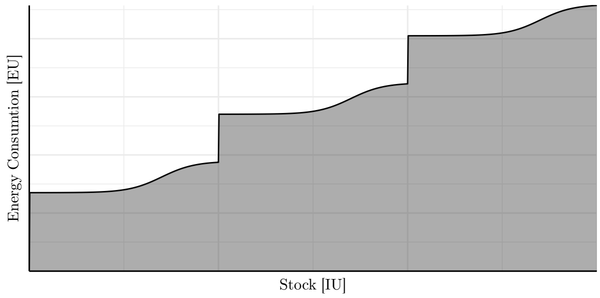

Figure 1. Example of energy consumption evolution with the stock.

Variable S_{cold}, the total stock in the cold storage, can be calculated as shown in Equation (3).

$$S_{cold} = \left(X^{[n_i \times 1]}\right)^T \cdot \left(A^{[n_i \times n_k]} \cdot ET_{cold}^{[n_k \times 1]}\right) \quad [IU] \tag{3}$$

Regarding the energy consumption of the warehouse, $E_{warehouse}$, Theorems 2 and 3 were considered, along with the assumptions provided. The formulation of the energy consumption of the warehouse is shown in expressions (4) and (5).

$$E_{warehouse} = FC_{warehouse} + E_{WH_{transport}} + E_{WH_{HVAC}} \quad [EU] \tag{4}$$

$$\begin{cases} E_{WH_{transport}} = S_{warehouse} \cdot \beta_1 \quad [EU] \\ E_{WH_{HVAC}} = \beta_2 - S_{warehouse} \cdot \beta_3 \quad [EU] \end{cases} \tag{5}$$

Variable $S_{warehouse}$, the total stock in the warehouse, can be calculated as shown in Equation (6).

$$S_{warehouse} = \left(X^{[n_i \times 1]}\right)^T \cdot \left(A^{[n_i \times n_k]} \cdot ET_{warehouse}^{[n_k \times 1]}\right) \quad [IU] \tag{6}$$

Tanks are generally simpler in terms of their energy consumption. For these reasons, the total energy consumption of the tanks is linearly proportional to the rate of fullness of the tank.

$$E_{tank} = \sum_{kt=1}^{n_{kt}} \frac{\gamma_{kt} \cdot S_{tank_{kt}}}{Lim_{kt}} \quad [EU] \tag{7}$$

The stock on each tank can be calculated as shown in expression (8). Contrarily to the stocks in the warehouse and cold storage, S_{tank} has a size of $[1 \times n_{kt}]$, that is, a stock for each tank.

$$S_{tank} = \left(X^{[n_i \times 1]}\right)^T \cdot \left(A^{[n_i \times n_k]} \cdot ET_{tank}^{[n_k \times n_{kt}]}\right) \quad [IU] \tag{8}$$

2.3.2. Holding Cost

The calculation of the holding costs required the holding costs per inventory unit for each storage location (H_k) and the conclusion from Theorem 4 showing that the holding costs are proportional to storage level. This calculation was performed as shown in expression (9). Note that the cost per unit of material in each storage location does not necessarily have to have a monetary value. If, e.g., storage location 1 has a cost per unit of 1 and storage location 2 has a cost of 0.9, this means that storage location 2 has 90% of the cost of storage location 1. Since this optimization was conducted in a many-objective way, the different objective values did not have to be comparable in absolute terms. For a correct analysis, the total stock in each storage location, S_k, was divided by the limit of each storage location, to convert the value into a percentage of fill of each storage location.

$$C_{Hold} = \sum_{k=1}^{n_k} \left(\frac{S_k \cdot H_k}{Lim_k}\right) \quad [CU] \tag{9}$$

The total stock in each storage location was calculated as shown in expression (10)

$$S_k = \sum_{i=1}^{n_i} (X_i \cdot A_{ik}) \quad [IU] \tag{10}$$

While it may seem that the holding costs and the energy consumption objectives may be similar and evolve in a similar way, it does not necessarily take place that way, e.g., a tank without substantial energy consumption per inventory unit of stock may have a very high holding cost, for business-related reasons, such as concurrency with other raw materials.

2.3.3. Risk of Shortage

One of the major advantages of many-objective optimization is that objectives do not have to match their units, as they are compared separately. This means that there is no effort to obtain a monetary cost for shortage, and the risk of shortage can be calculated directly based on the lead time from the suppliers and given the production schedule of how soon a given material is completely consumed.

The final objective evaluated the risk of the optimized stocks not satisfying the production requirements. This considered the daily material requirements for an horizon of 60 days ($N_{i,t}$), the average lead time of the suppliers of each material (L_i), and the standard deviation of the lead time of the materials (σ_i). The first step was the calculation of the evolution of the daily stocks $B_{i,t}$, based on the optimized stocks X_i and on the daily material requirements $N_{i,t}$. This is shown in expression (11).

$$B_{i,t} = \begin{cases} X_i - N_{i,t} & t = 1 \\ B_{i,t-1} - N_{i,t} & t > 1 \end{cases} \tag{11}$$

This expression simply establishes whether the stock of a material on a given day is positive or negative. The next stage was the calculation of the shortage day of each material ShD_i, as shown in expression (12). The function *sign* was used to collapse the value from $B_{i,t}$ into either 1 or -1. For consistency of the expression, if a value of $B_{i,t}$ was 0, the function *sign* returned the value 1, while the original *sign* function would return the value 0; this was to ensure that the expression below worked properly if such a case took place.

$$ShD_i = n_t - \left[\frac{n_t}{2} - \left(\sum_{t=1}^{n_t} \frac{sign(B_{i,t})}{2} \right) - 1 \right] \tag{12}$$

After obtaining on which days the stock of each material was exhausted, the calculation of the risk was performed. Three types of risk were considered:

- The first and gravest type occurred when the shortage day was closer to the day of the optimization than the materials' mean lead time. This means that in average conditions, the stock would not be able to be replenished and there would be material shortage.
- The second type of shortage took place when the shortage day was closer to the day of the optimization than the materials' average lead time with one standard deviation.
- The third type of shortage was inversely proportional to the shortage day of each material without restocking; the further away the day when a material ran out, the smaller the risk.

If a material had a shortage of the first type, the other two shortages were null; if the material had the second shortage type, the third type was null. This means that for each material, only the gravest type of shortage was considered. The calculation of the 3 types of shortage is shown in expressions (13)–(15).

$$Sh1_i = \max(L_i - ShD_i, 0) \tag{13}$$

$$Sh2_i = \begin{cases} 0, & Sh1_i > 0 \\ \max(L_i + \sigma_i - ShD_i, 0), & Sh1_i = 0 \end{cases} \tag{14}$$

$$Sh3_i = \begin{cases} 0, & Sh1_i > 0 \lor Sh2_i > 0 \lor ShD_i = n_t + 1 \\ \lceil -\max(-(L_i + \sigma_i - ShD_i), 0) + n_t \rceil, & otherwise \end{cases} \tag{15}$$

Finally, the shortage risk was calculated. This calculation weighed the types of risk differently. Generally speaking, the first type of risk should be weighted larger than the second, and the second larger than the third, to prioritize their reduction in the optimization. The calculation of the shortage risk was performed has shown in expression (16).

$$R_{Sh} = \sum_{i=1}^{n_i} (Sh1_i \cdot SW1 + Sh2_i \cdot SW2 + Sh3_i \cdot SW3) \tag{16}$$

2.4. Constraints

The only constraint applied to this problem, besides the non-negativity constraint of the decision variables, was the total stock per storage location not exceeding its capacity. This is the formulation shown in Equation (17).

$$S_k \leq Lim_k \quad [IU], \quad \forall_{k \in \{1, \cdots, n_k\}} \tag{17}$$

The complete formulation of the problem is as shown in expression (18).

$$\begin{array}{ll} \min\limits_{\mathbf{X}} & F_1 = E \quad [EU] \\ & F_2 = C_{Hold} \quad [CU] \\ & F_3 = R_{Sh} \\ s.t. & \mathbf{X} \geq 0 \\ & S_k \leq Lim_k \quad [IU], \quad \forall_{k \in \{1, \cdots, n_k\}} \end{array} \tag{18}$$

3. Optimization Approach

The formulated problem is a nonlinear, convex, constrained, many-objective problem. This is a very complex problem that requires the usage of metaheuristic optimization algorithms. Considering this, the algorithm used for the optimization was the non-dominated sorting genetic algorithm NSGA-III [45]. This algorithm is an evolutionary algorithm suited to many-objective problems. While the NSGA-II algorithm could also be applied to many-objective problems, some researchers have shown the advantages of using NSGA-III. Ishibuchi et al. [46] presented a plethora of benchmarks tested on both algorithms; NSGA-III outperformed NSGA-II on all problems except knapsack ones. Ciro et al. [47] showed that NSGA-III obtained better results on an open shop-scheduling problem with resource constraints. While there are many ways of dealing with constraints, the implementation used for this approach was a simple feasibility-first approach that did not compute the objective values for unfeasible solutions but rather assigned them the value of the worst objective value out of the entire population plus the constraint violation [48]. Furthermore, all variables and solutions were encoded as real numbers, and box constraints were in place on all variables to disable negative quantities and exploding quantities. To this end, the box constraints required decision variables to be greater or equal to 0 and smaller than the capacity of their storage location (e.g., a product in cold storage was allowed to have values as high as the capacity of the entire cold storage). The formulation described was implemented in Python, specifically using the Pymoo library [49].

Table 3 shows the values selected for each parameter used for the results' analysis. Refer back to Table 2 for a further description of the parameters.

Table 3. Values used for the aforementioned formulation. Check the parameters' descriptions in Table 2. The column value shows either the value of the parameter or a range of values. Parameters Lim_1 and H_1 regard cold storage; Lim_2 and H_2 regard the warehouse; $Lim_{3:n_k}$ and $H_{3:n_k}$ regard the tanks.

Parameter	Value
$n_{i,cold}$	50
$n_{i,warehouse}$	250
$n_{i,tanks}$	24
n_t	60
α_1	50,000 [IU]
α_2	40%
α_3	135 [EU]
α_4	0.7
α_5	8
$FC_{warehouse}$	100 [EU]
β_1	0.0004 [EU/IU]
β_2	250 [EU]
β_3	0.00003 [EU/IU]
γ	$[30, 90]$ [EU]
SW_1	100
SW_2	10
SW_3	0.1
Lim_1	2.5×10^6 [IU]
Lim_2	1.25×10^7 [IU]
$Lim_{3:n_k}$	$[50, 150]$ [IU]
H_1	1 [CU/IU]
H_2	0.5 [CU/IU]
$H_{3:n_k}$	$[0.3, 0.7]$ [CU/IU]
L_i	$[5, 20]$ [day]
σ_i	$[1, 20]$ [day]

Table 4 shows the parameters used for the three components that control the optimization algorithm: the determination of the reference directions; the NSGA-III algorithm's parameters themselves, and the parameters of the termination criterion.

Table 4. Parameters used for the determination of the reference directions, NSGA-III algorithm, and termination criterion. Duplicate individuals are eliminated.

Parameter	Value
Reference direction method	Das–Dennis
Reference direction dimensions	3
Reference direction number of partitions	12
Population size	200
Initial sampling	Random
Selection method	Tournament selection
Selection pressure	2
Crossover method	Simulated binary crossover
Crossover Eta	30
Number of offspring	2
Mutation method	Polynomial mutation
Mutation Eta	20
Termination tolerance	0.1
Generation window	30
Termination criterion calculation generation period	10
Maximum number of generations	2000

After obtaining the results from the optimization, if the number of solutions was sufficiently large that the advantages and trade-offs of each solution were not easily analysed, a filter was applied for each objective. The algorithm design is shown in Algorithm 1, which details in pseudocode the proposed optimization process, considering the NSGA-III optimization algorithm and the parameters found in Table 4.

Algorithm 1 Ideal stock many-objective optimization

procedure IDEAL STOCK PROBLEM(X, $Pars$)
    Calculate $S_{cold}\,(X)$ ▷ Equation (3)
    Calculate $S_{warehouse}\,(X)$ ▷ Equation (6)
    Calculate $S_{tank}\,(X)$ ▷ Equation (8)
    Calculate E_{cold}, $E_{warehouse}$, and E_{tank} ▷ Equations (2), (4) and (7)
    Calculate E ▷ Equation (1)
    Calculate $S\,(X)$ ▷ Equation (10)
    Calculate C_{hold} ▷ Equation (9)
    Calculate $B\,(X)$ ▷ Equation (11)
    Calculate ShD ▷ Equation (12)
    Calculate $Sh1$, $Sh2$, and $Sh3$ ▷ Equations (13)–(15)
    Calculate R_{Sh} ▷ Equation (16)
    Calculate constraint violation $S_k - Lim_k$ ▷ Equation (17)
Introduce the parameters of the formulation problem into the ideal stock problem function (Table 3)
Obtain the reference direction using the Das–Dennis method. Consider the number of dimensions of the problem and an adequate number of partitions
Define the optimization's termination criteria
while Termination criteria not met **do**
    Run the optimization
while Number of solutions is large **do**
    Apply a filter to objective i
    Store the non-dominant solutions that adhere to the filter

4. Results' Analysis

The optimization was run for the specified conditions. For consistency, this optimization was repeated 10 times. Regarding the algorithm analysis, the 10 optimizations took an average of 114.2 s, varying from 78.7 s to 128.9 s. As previously mentioned, duplicate individuals were eliminated, meaning that each of the 10 optimizations achieved a different final population; this varied from 19 to 30 members, with an average size of 26.7. While the optimization was run for a maximum of 2000 iterations, the implemented termination criterion caused the optimization to end beforehand. The optimization took between 1370 and 2000 iterations, with an average of 1874 iterations.

Figure 2 shows a parallel plot of the optimization solutions for the first test run. Additionally, Figure 3 shows the Pareto fronts between the three objectives. To reduce the clutter, only 3 out of the 10 optimization results are shown, randomly selected.

The first conclusion that can be drawn from the plots is the nicely shaped Pareto front between the energy consumption and the risk, which can also be seen by the inverse behaviour of the results seen in the parallel plot. This simply means that generally, a reduction in energy consumption (caused by a reduction in the stock kept) leads to a higher shortage risk. The slope of the Pareto front between these two objectives also indicates that a small relative increase in energy consumption leads to a substantial reduction in shortage risk.

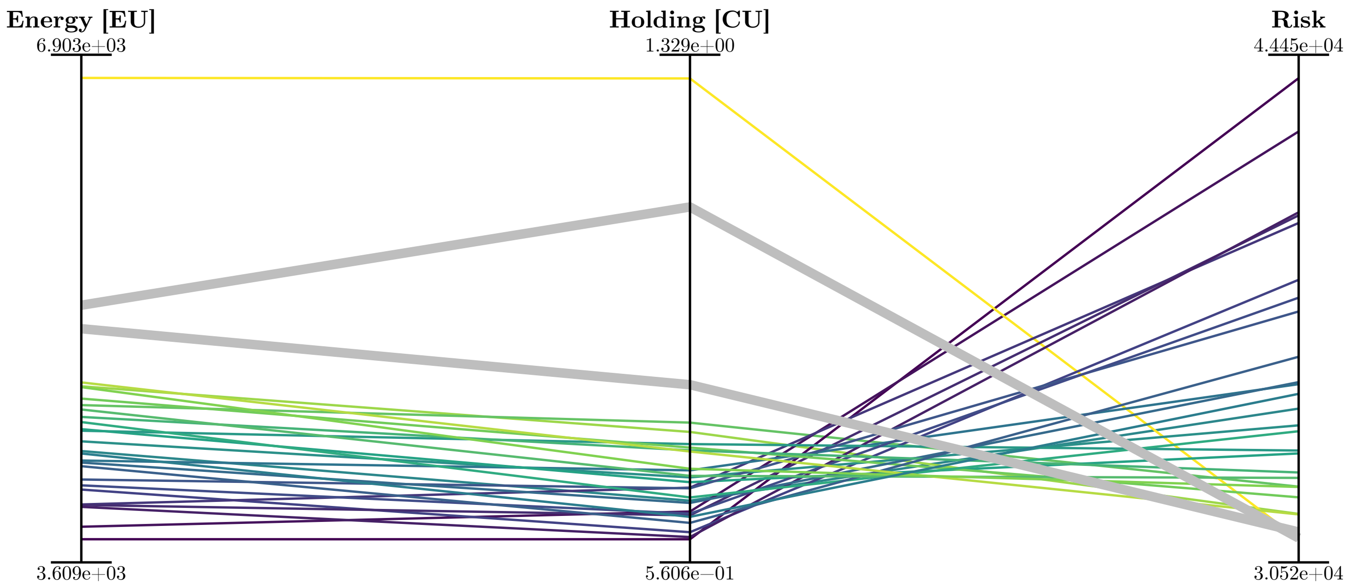

Figure 2. Parallel plot of the many-objective optimization solutions. Each line corresponds to an optimal solution. The colour scale supplied regards the first objective, the energy consumption, simply to aid in identifying the solutions across the remaining objectives.

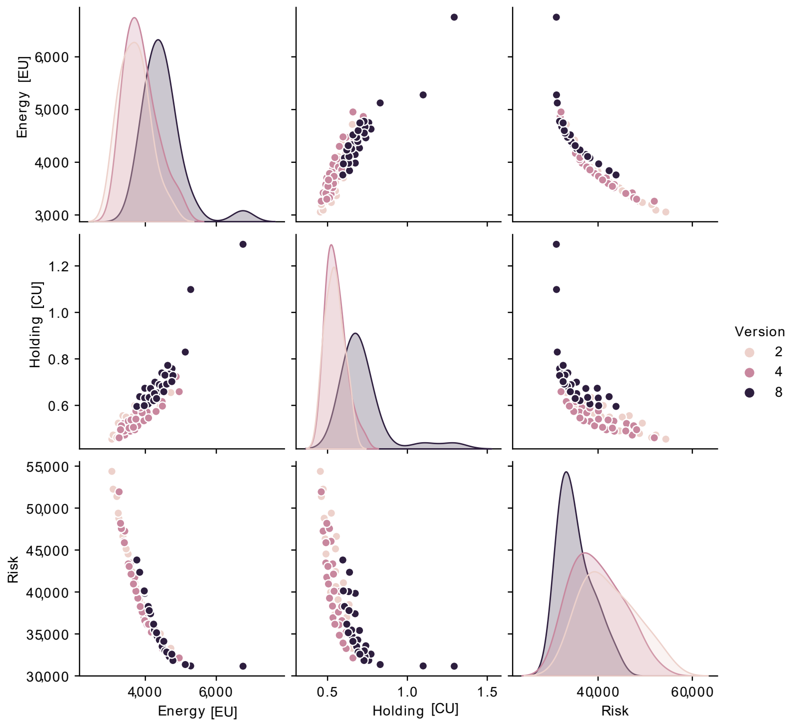

Figure 3. Pareto fronts between each objective for 3 distinct optimization results, out of the 10 repetitions.

The holding costs had a slightly different relation to the two other objectives. They had an approximate proportionality with the energy consumption. This is an expected behaviour, as both objectives were directly proportional to the amount of stock in the storage locations. However, the relation between the two was not always direct, since the holding costs per storage location were not necessarily proportional to the energy costs per storage location. This simply means that a storage location may have a large energy consumption but a small holding cost, as is the case with the cold storage in this setting. From Figure 2, the general rule is that the more similar the slope between the energy consumption and holding costs between two optimization solutions, the more proportional the stocks considered. Solutions with contrasting slopes tended to have non-proportional stocks of each type of storage location. An example of this can be seen in the two solutions shown in Figure 2 as the grey lines. For clarity, these solutions were indexed as solution 4 (the one with the largest holding cost) and solution 14.

Table 5 shows that the stock mix was indeed very different between the two solutions, even though these shared similar energy consumption and shortage risk values. The overarching conclusion that can be drawn from this comparison is that for a small reduction in the shortage risk, without substantially changing the level of energy consumption, the stocks would have to be increased, especially in the tanks, which tripled from scenario 14 to 4.

Table 5. Total inventory units at each storage location (tanks' inventory levels are aggregated) for each of the optimal solutions tested.

Storage Location	Solution Index 4	Solution Index 14
Cold	7.67×10^5	7.55×10^5
Tanks	6.93×10^1	2.19×10^1
Warehouse	7.37×10^6	7.03×10^6

4.1. Results' Evolution

As previously stated, the daily product requirements were collected for 90 days, while the optimization only considered 60. This means that the ideal stocks could be calculated for 30 days to observe the temporal evolution of the ideal stocks. The optimizations ran for each of the rolling forward periods and were run 10 times each. Figure 4 shows the 30 Pareto fronts of risk against energy. It is important to mention that for each day, only one version is shown: the one whose median values of the three objectives had the smallest Euclidean distance to the origin. Note that the Pareto curves shown are smoothed approximations to reduce the visual clutter and allow an easier analysis.

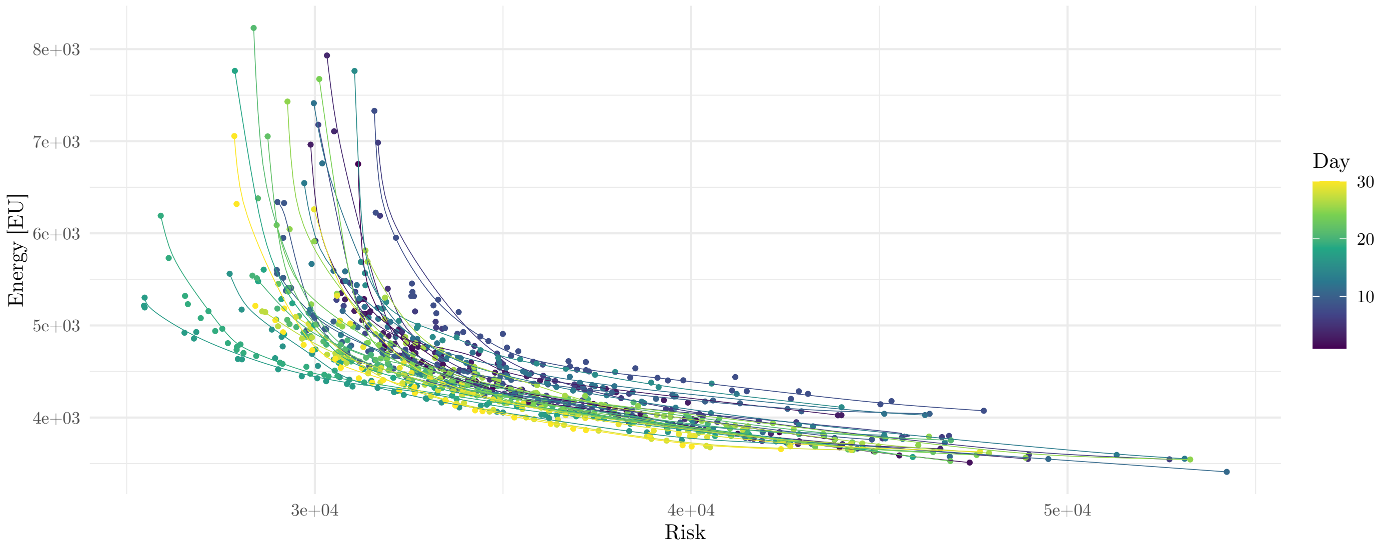

Figure 4. Approximated Pareto fronts for the optimizations with the lowest median objectives of each day of the rolling-forward schedule. The Pareto front's lines are smoothed for easier comprehension.

The figure shows that the Pareto fronts at the initial days tended to be slightly worst than the average (in regards to their distance to the origin), worsening their values until the 10th day. The Pareto fronts then moved closer to the origin, with the closest results around the 20th day. Finally, the fronts degraded slightly, ending on the 30th day slightly closer to the origin than on the first day. The distance to the origin broadly indicated the material requirements—Pareto fronts closer to the origin tended to regard production schedules with larger material requirements.

While there were some variations along the month, the stocks did not change extensively. This was a desirable behaviour, as the changes in the stock requirements from one day to the next were not very substantial—there were only changes in one day out of the 60 days considered, since when t changed to $t+1$, the material requirements changed from $N_{i,t:t+60}$ to $N_{i,t+1:t+61}$. Nevertheless, the figure shows that the Pareto fronts were further from the origin in the initial days, then improved until around $t = 20$ and ended at a reasonable distance. A smaller distance of one front to the origin meant that both objectives had better values than the one farther from the origin—this means that the problem's conditions changed sufficiently between those two scenarios, and the stock requirements in the front farther from the original were larger than the ones from the closer front.

4.2. Shortage Risk Segregation

The final analysis focused on separating the shortage risk formulation into its three components and using them independently when running the optimization. The formulation of this problem became as shown in expression (19). In terms of algorithm, the only change required in Algorithm 1 was to disregard row 12.

$$
\begin{array}{ll}
\min\limits_{\mathbf{X}} & F_1 = E \quad [EU] \\
 & F_2 = C_{Hold} \quad [CU] \\
 & F_3 = \sum Sh1_i \\
 & F_4 = \sum Sh2_i \\
 & F_5 = \sum Sh3_i \\
s.t. & \mathbf{X} \geq 0 \\
 & S_k \leq Lim_k \quad [IU], \quad \forall_{k \in \{1,\cdots,n_k\}}
\end{array}
\tag{19}
$$

The only alterations to the parameters presented in Table 4 were an increase in the number of reference direction dimensions from three to five and an increase in the population size from 200 to 2000. The optimization was then run 10 times to allow for a better representation of the solution space. As for the algorithm analysis, the average execution time was 453.4 s (358.5 s–524.4 s), the average final population size was 453.5 (410–514), and the average total number of iterations was 329 (270–390). The parallel plot of the complete solution space (including the unique solutions from the 10 optimization runs) is shown in Figure 5.

The parallel plot is a very complex display of information, showing an aggregation of many varied optimal solutions, each prioritizing a different set of objectives. Since the colour coding simply scales the energy consumption of the solution it regards, so as to allow for better tracking of solution along the other objectives, it can be seen that the first component of the risk is the one which is most closely related to the energy consumption, in an inverse proportion. This means that the higher the energy consumption, the lower the risk of type 1—since the larger the total stock the higher the energy consumption.

A many-objective optimization such as the one presented can be extremely useful for decision-makers to make their decision in a more informed way. A decision could determine that only solutions with a type 1 risk smaller than 400 would be acceptable. The result to this requirement would be the solutions shown in Figure 6.

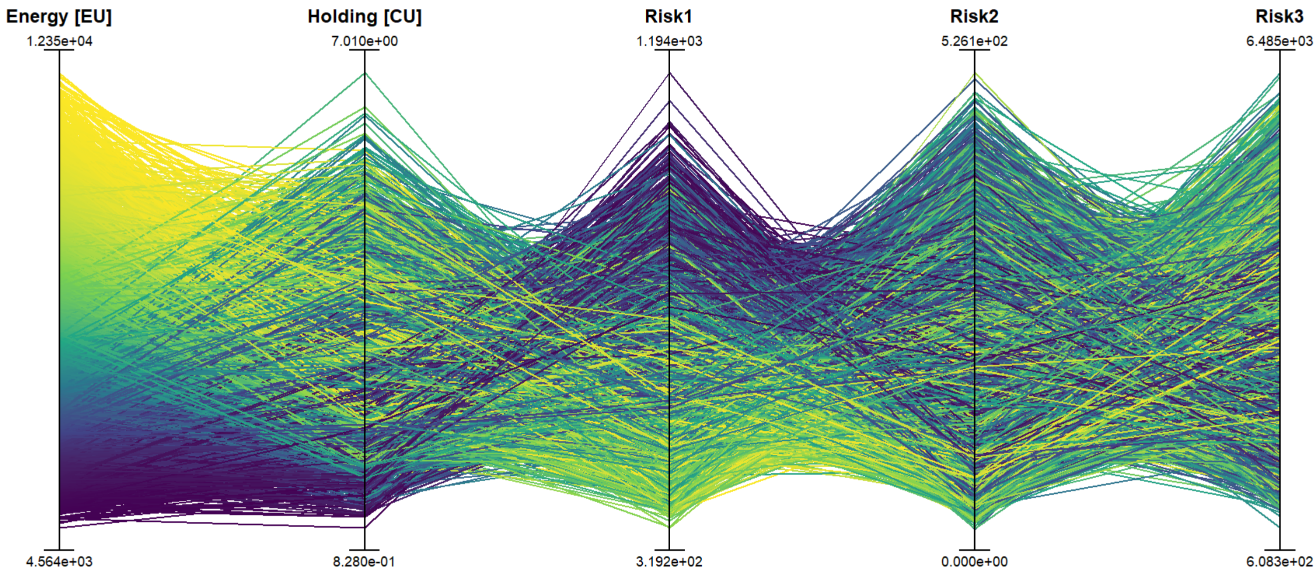

Figure 5. Parallel plot of the separated risk optimization for the complete solution space.

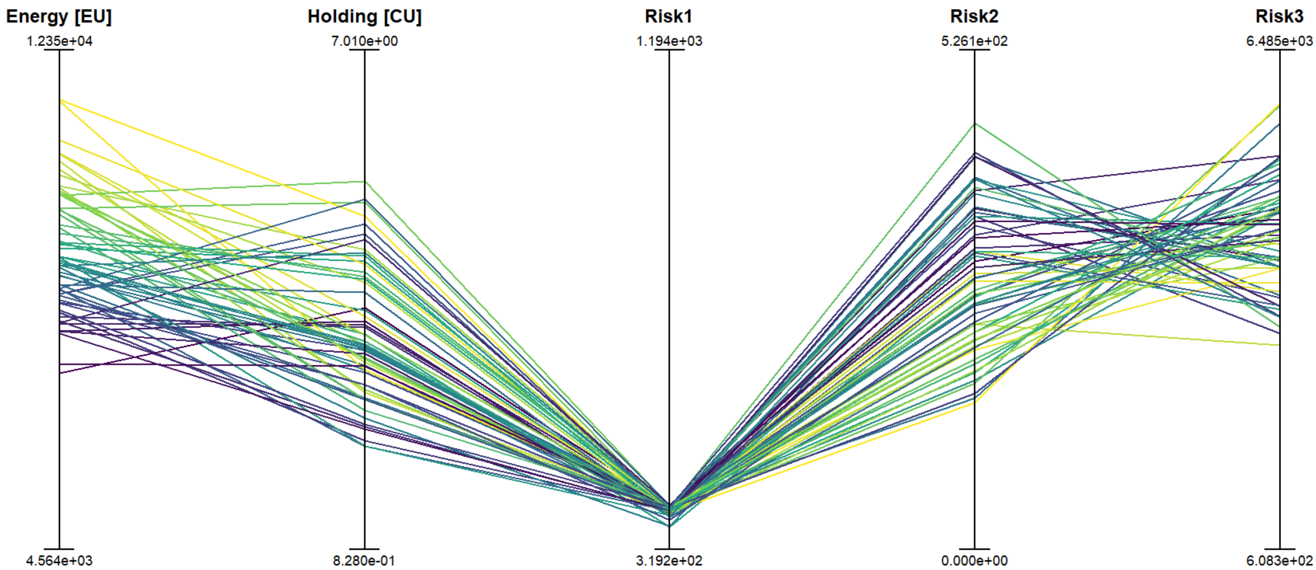

Figure 6. Parallel plot of the separated risk optimization for the complete solution space, only considering solutions with a risk of type 1 smaller than 400.

After obtaining this subset of parallel plots, the decision-maker could specify that only solutions with a holding cost higher than 4.8 are acceptable. The results would be as shown in Figure 7.

As can be seen, sequentially combining different requirements for the objectives allows the decision-makers to arrive at a restricted set of solutions that best fit the company's requirements.

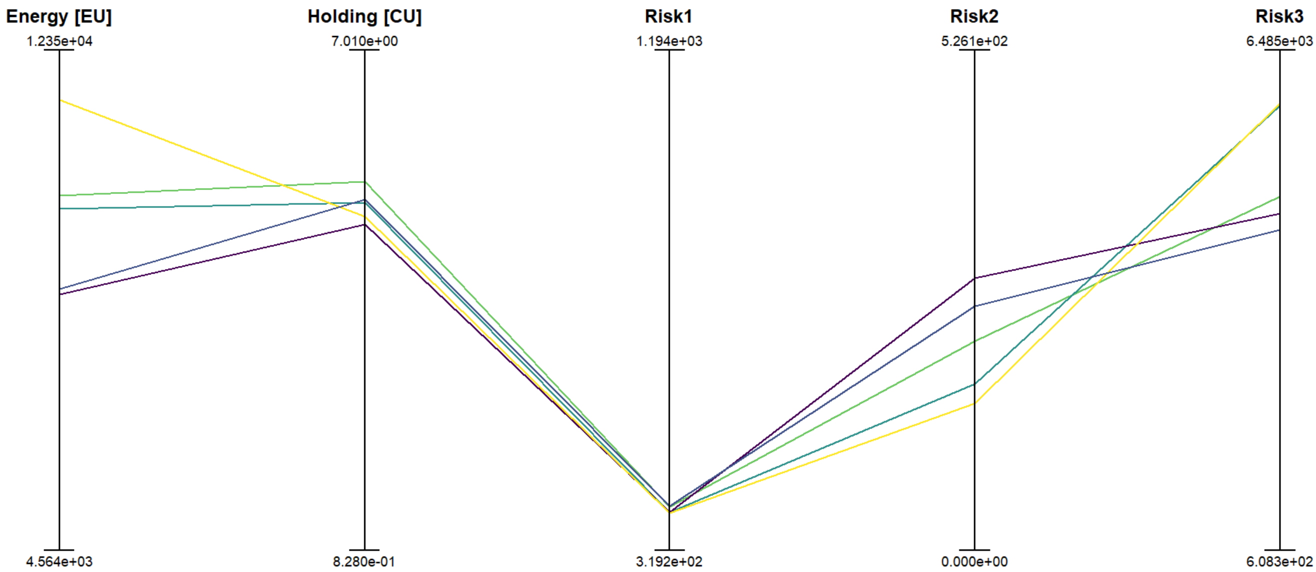

Figure 7. Parallel plot of the separated risk optimization for the complete solution space, only considering solutions with a risk of type 1 smaller than 400 and a holding cost larger than 4.8.

5. Conclusions

This research tackled a very important issue—stock management—using a methodology that complemented the decision-making process of stakeholders, rather than substituting it, with greater focus on the energy consumption of the solutions, in line with current environmental awareness.

The study revealed a diverse array of promising solutions, effectively balancing energy consumption, holding costs, and shortage risk. This many-objective optimization approach highlighted an inherent trade-off: minimizing one objective inevitably led to increases in others. This places the decision-maker in a pivotal role, tasked with selecting the solution that best aligns with the company's broader strategic goals. These findings also echo the concept of Pareto optimality, a state where it is impossible to improve one objective without compromising another. In a business context, this often translates into making strategic choices regarding resource allocation—for instance, optimizing energy consumption might necessitate higher holding costs. While this study's unique modelling and objectives preclude a direct comparison with the existing literature, it underscores the importance of Pareto efficient solutions in navigating complex, multi-faceted challenges like energy management and inventory control. The absence of a one-size-fits-all answer reinforces the need for bespoke strategies that reflect individual company priorities and risk tolerance.

The inclusion of energy consumption in the decision-making process can at first glance worsen the results—more energy efficient solutions tend to have a larger risk of shortage. However, brining awareness into this dimension can be useful in several ways. First of all, in scenarios were an improvement in energy consumption is achieved at a small cost to the other objectives, it provides a justification for decision-makers to select the said solution. Secondly, while choosing low-energy-consumption strategies may lead to larger risk of shortage, and consequently larger costs to the company, a focus on sustainability can provide marketing opportunities, as some industries benefit greatly from a stronger adherence to sustainability principles. According to Unal and Tascioglu [50], sustainability initiatives "*help companies establish a strong relationship with consumers, [. . .] which in turn creates a higher level of purchase intent and reduced sensitivity to price premiums*".

Logistics systems are pivotal components in companies and are often overlooked when it comes to the application of optimization and artificial intelligence algorithms, often applied solely to the shop-floor operations. However, the application of energy-efficient and environmentally friendly planning strategies in logistics systems, such as the present optimization of the ideal stock at storage locations, can improve visibility into the objectives considered and provide better results than fixed stock strategies, while being cost-effective analyses. Additionally, manufacturing execution systems (MES) often overlook the dimension of energy management; the proposed approach tackles both issues: a central issue of MES—stock optimization—with energy consumption considerations.

While this study demonstrates the effectiveness of many-objective optimization in balancing competing objectives in warehouse stock management, the model's focus on a single warehouse with specific storage types limits its generalizability to more complex supply chain configurations. Furthermore, the abundance of Pareto optimal solutions generated by the many-objective optimization model poses a challenge for decision-makers in selecting the most suitable option, as evidenced in Figure 5. Future research should prioritize extending the model to encompass multiple warehouses and diverse storage configurations, while also developing decision support tools or frameworks to facilitate the interpretation and selection of optimal solutions within a vast solution space. Additionally, strategies to reduce the number of quasi-redundant solutions and to prioritize those aligned with specific company strategies could significantly enhance the practicality and decision-making efficiency of the proposed approach. Finally, additional objectives may be added to the problem, taking into consideration, e.g., the actual costs of delivering products after their due date, or buying raw materials from low-lead-time suppliers at a premium.

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