algorithms

Article

Correlation Analysis of Railway Track Alignment and Ballast Stiffness: Comparing Frequency-Based and Machine Learning Algorithms

Saeed Mohammadzadeh [1], Hamidreza Heydari [1], Mahdi Karimi [1] and Araliya Mosleh [2,*]

[1] School of Railway Engineering, Iran University of Science and Technology, Tehran 16846, Iran; mohammadz@iust.ac.ir (S.M.); h_heydari@iust.ac.ir (H.H.); m_karimi95@alumni.iust.ac.ir (M.K.)
[2] Faculty of Engineering, University of Porto, 4200-465 Porto, Portugal
* Correspondence: amosleh@fe.up.pt

Abstract: One of the primary challenges in the railway industry revolves around achieving a comprehensive and insightful understanding of track conditions. The geometric parameters and stiffness of railway tracks play a crucial role in condition monitoring as well as maintenance work. Hence, this study investigated the relationship between vertical ballast stiffness and the track longitudinal level. Initially, the ballast stiffness and track longitudinal level data were acquired through a series of experimental measurements conducted on a reference test track along the Tehran–Mashhad railway line, utilizing recording cars for geometric track and stiffness recordings. Subsequently, the correlation between the track longitudinal level and ballast stiffness was surveyed using both frequency-based techniques and machine learning (ML) algorithms. The power spectrum density (PSD) as a frequency-based technique was employed, alongside ML algorithms, including linear regression, decision trees, and random forests, for correlation mining analyses. The results showed a robust and statistically significant relationship between the vertical ballast stiffness and longitudinal levels of railway tracks. Specifically, the PSD data exhibited a considerable correlation, especially within the 1–4 rad/m wave number range. Furthermore, the data analyses conducted using ML methods indicated that the values of the root mean square error (RMSE) were about 0.05, 0.07, and 0.06 for the linear regression, decision tree, and random forest algorithms, respectively, demonstrating the adequate accuracy of ML-based approaches.

Keywords: machine learning; railway track maintenance; longitudinal level; data mining; ballast stiffness

Citation: Mohammadzadeh, S.; Heydari, H.; Karimi, M.; Mosleh, A. Correlation Analysis of Railway Track Alignment and Ballast Stiffness: Comparing Frequency-Based and Machine Learning Algorithms. *Algorithms* **2024**, *17*, 372. https://doi.org/10.3390/a17080372

Academic Editor: Ioannis Tsoulos

Received: 25 June 2024
Revised: 19 August 2024
Accepted: 19 August 2024
Published: 22 August 2024

1. Introduction

An improved and more astute comprehension of railway track conditions stands as a paramount concern within the industry, impacting operations, safety, passenger comfort, resource utilization, and beyond. Notably, a significant share of operational and maintenance costs, up to 60%, is attributed to ballast layer upkeep, necessitating the implementation of optimized strategies and resource allocation [1–3]. On the other hand, the most important factor ensuring the safe operation of rail vehicles lies in track geometry, which helps the efficiency and performance of railway tracks [4,5]. Investigating the correlation between the geometrical conditions of the track and the performance and quality of the components of ballasted tracks can lead us to a reasonable understanding of the condition of railway tracks. Railway managers pay particular attention to the geometrical conditions of tracks, recognizing the challenge inherent in assessing their operational status [6,7].

In modern railway maintenance practices such as track recording cars, mechanized machines play a pivotal role in recording, monitoring, and inspecting track geometrical conditions. While moving at the specified speed on the track, they can capture various parameters such as geometrical irregularities and track stiffness. Geometric parameters of railway tracks including longitudinal levels (LLs), horizontal alignments (ALs), track gauge (GAU), cross-level (XLV), and twist (TWI) play a crucial role in ensuring the safe operation of rail vehicles and improving the efficiency and performance of railway tracks. On the other hand, one of the other main factors that plays a key role in the maintenance of railway tracks is ballast stiffness. Even though track stiffness measurement is not commonly used in the inspection of railway tracks, there are a large amount of railway track specifications that are directly or indirectly related to track stiffness, which helps managers and railway engineers make maintenance decisions by discovering knowledge from these data [8–10]. So, the interpretation of track stiffness can help decision-makers and senior managers determine a strategy and plan for the maintenance and repair of railway tracks. In recent years, significant strides have been made in the development of continuous track measurement methods [11–18].

Track stiffness plays a crucial role in influencing the dynamic behavior of railway tracks and their interaction with wheels. In addition, the geometric quality of tracks is directly correlated with track maintenance, and both are directly related to track stiffness. A high stiffness of the track enhances its load-carrying capacity and reduces the displacement of the track. On the other hand, high track stiffness leads to an increase in dynamic forces, especially in the contact area between the wheel and the rail, leading to increased damage to track components, including sleepers and the ballast [19]. Changes in track stiffness along the route lead to changes in track/wheel interaction forces, resulting in varying settlements and geometric deterioration along the track. Numerous studies have been conducted on track stiffness in recent years, illuminating its pivotal role in railway infrastructure [20–23].

Generally, limited research has been performed to investigate the mutual effects of track stiffness, which represents the characteristics of the track substructure and track geometry, indicative of the track superstructure. Shi et al. conducted a critical review of studies and their future trends regarding track stiffness irregularity (TSI) and presented a new concept of critical values of track stiffness irregularity for the integrated management of track geometry [24]. Pozavak et al. delved into determining the optimal track stiffness concerning its effect on track geometric quality, considering it dependent on the individual stiffness of all superstructure and substructure elements, as well as their mutual compatibility [25]. In the Canadian railway network, Roghani et al. compared a dataset of track geometric failures which were related to track roughness and stiffness. This comparison, besides confirming the relationship between track stiffness and track failures, led to providing allowable values of roughness and stiffness, along with a risk diagram for the maintenance and repair of the railway track [26]. Berggren presented a novel method aimed at extracting new insights from collected data, sourced from geometric quality, dynamic stiffness, and ground-penetrating radar [27,28]. Nielsen et al. evaluated data collected by a track recording car from 1999 to 2016, focusing on investigating the geometric deterioration of the track within wavelengths of 1 to 25 m. They examined the correlation between changes in track stiffness and geometric irregularities to understand their impact on track maintenance and repair [29,30]. Grossoni et al. investigated the role of track vertical stiffness variations on the long-term track deterioration behavior through a series of computational experiments, leading to the presentation of a process to generate vertical stiffness representative of real-world track conditions [31].

Over the past few decades, numerous researchers have directed their attention towards condition monitoring and track maintenance [32–35]. One of the appropriative approaches that can be employed to achieve this aim is data mining, which is used in railway engineering in various subjects, as mentioned by Villarejo et al. [36], Bergquist and Söderholm [37], Famurewa et al. [38], Sauni et al. [39], and Thaduri et al. [40]. In the data mining approach, machine learning (ML) techniques are generally used to find data correlation. Several ML techniques (both supervised and unsupervised methods) are utilized to survey the maintenance of railway tracks. Sresakoolchai et al. tried to improve the efficiency of maintenance activities, especially railway component defects and track geometry, utilizing deep reinforcement learning integrated with digital twin techniques [41]. Gerum et al. used random forest (RF) and recurrent neural network (RNN) techniques to predict railway track maintenance work and found that the accuracy of the RNN and RF were about 80% and 77%, respectively [42]. In another study, Consilvio et al. used an unsupervised machine learning algorithm (K-means clustering) and a supervised learning method (support vector machine) to manage the railway earthwork and track circuit maintenance [43]. Falamarzi et al. used Pearson correlation analysis to predict the track gauge and realized that the prediction accuracy of the support vector machine (SVM) is higher in curved sections, while the artificial neural network (ANN) has better accuracy in straight sections [44]. Lee et al. predicted the track geometry degradation by the ANN and SVR techniques [45]. Martey et al. employed several unsupervised algorithms and supervised ML techniques (including cluster analysis, multiple linear regression, decision tree regression, random forest regression, and support vector regression) to analyze the influence of geocell installation on track geometry quality [46]. In order to predict the railway track geometry parameters, Sresakoolchai et al. developed deep ML models using 3D recurrent neural network-based techniques [47]. Popov et al. studied the potential of two unsupervised ML techniques (including autoencoder and K-means clustering) to localize track defects based on geometry measurements [48]. Mehrali et al. studied the use of data mining techniques in surveying railway track geometry parameters and track stiffness [49]. However, to the best of the authors' knowledge, the literature pertaining to the relationship between track stiffness and geometrical parameters has remained notably scarce. Therefore, there is a pressing need for further research and studies to formulate a coherent and systematic approach towards clarifying the connection between track stiffness and geometrical parameters.

In general, an innovative approach to the effective management of railway maintenance activities involves establishing a relationship between different track properties. This perspective allows engineers to rapidly assess track health conditions without extensive record-keeping. Based on the literature review conducted above, there is limited knowledge regarding the correlation between the geometric parameters and structural characteristics of railway tracks. Among the track properties, ballast stiffness and track geometry are two significant parameters that play a crucial role in maintenance, yet a comprehensive correlation study between them has not been conducted. Therefore, the current study aims to survey the relationship between vertical track stiffness and track geometry, with a specific focus on two crucial representatives: vertical ballast stiffness and the longitudinal level. To achieve this, track geometric data and vertical track deflection measurements were collected along the Tehran–Mashhad railway line in Iran. Various correlation analyses were then conducted using both frequency-based methods and machine learning techniques. First, the relationship between vertical rail deflection and longitudinal level was examined by calculating the power spectrum density (PSD) of the collected data. This analysis aims to investigate the correlation between the data obtained from the track recording car and stiffness recording car. By analyzing the PSD, this study seeks to identify patterns and relationships that can provide insights into the impact of vertical rail deflection on track longitudinal levels and ballast stiffness, contributing to improved track maintenance and performance evaluation. Next, this study evaluated and compared the effectiveness of frequency analysis with three machine learning algorithms—linear regression, decision tree, and random forest—in analyzing the relationship between vertical track stiffness and track geometry. These correlation analyses provide insights into how variations in ballast stiffness impact the vertical longitudinal level of the track, contributing to a deeper understanding of track performance and maintenance needs.

The insights mentioned above can reveal the potential of advanced analytical techniques in understanding and optimizing railway track maintenance strategies. By leveraging machine learning algorithms, railway maintenance teams can more accurately predict and address issues related to track conditions. The remainder of this paper is organized as follows: Section 2 presents the data acquisition process, detailing the methods and equipment used to gather the necessary data for track longitudinal level and vertical track deflection measurements along the Tehran–Mashhad railway line. Section 3 discusses the algorithms employed in correlation mining analyses, including both frequency-based techniques and machine learning algorithms such as linear regression, decision tree, and random forest algorithms. Section 4 provides a comprehensive analysis and discussion of the results, highlighting key findings and their implications. Finally, Section 5 concludes this paper, summarizing the main contributions and suggesting potential areas for future research.

2. Methodology of Data Acquisition

An overview of the site location of data collection and the acquisition techniques of geometrical parameters and track stiffness are discussed and described in this section.

2.1. Description of Field Measurement Site

The Tehran–Mashhad railway track is one of the important and high-traffic track railways of Iran's railway network. For a comprehensive study and investigation, a 15 km stretch of the Tehran–Mashhad railway line (from 230 to 245 km), which is located between the Semnan and Miandareh stations, was selected to conduct the field measurements. The specific location of the collected data is shown in Figure 1. The properties of the reference railway track, which were monitored and investigated in this study, are presented in Table 1.

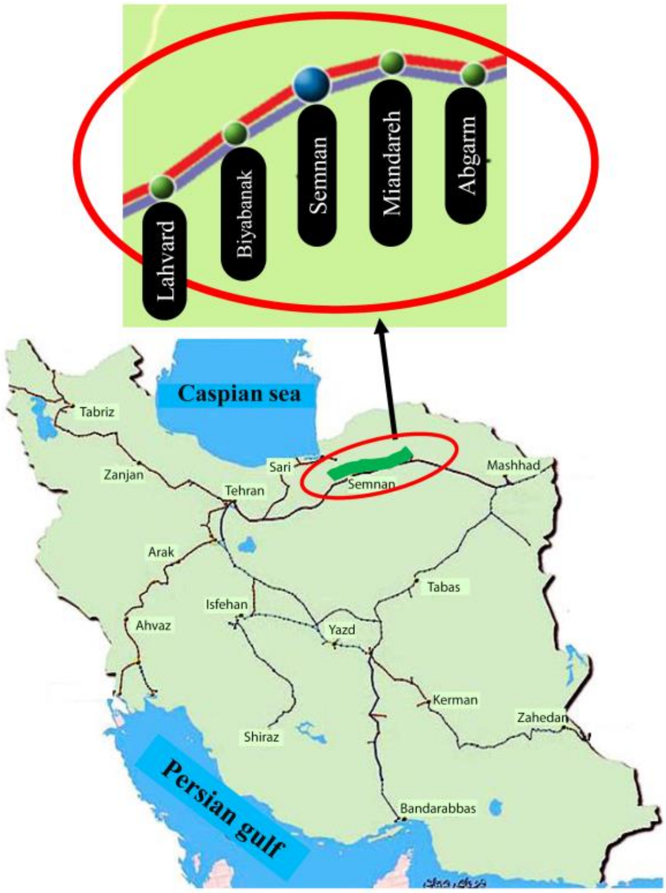


Figure 1. Data acquisition along Tehran–Mashhad railway line (between Semnan and Miandareh stations).

Table 1. The characteristics of the reference railway track.

Parameter	Description	Value	Unit
P	Axle load	22.5	Tons
K_p	Stiffness of fastening system	400	kN/mm
ls	Sleeper length	2.6	m
l_e	Effective length of sleeper	1.1	m
h_b	Ballast and sub-ballast depth	0.5	m
α	Ballast friction angle	35	degree
E_f	Subgrade elastic modulus	42.7	MPa

2.2. Data Acquisition of Track Longitudinal Level (LL)

Track geometric conditions are determined based on the spatial location and orientation of the rail within a three-dimensional framework. This framework is represented by a coordinate system comprising three axes: (1) the x-axis parallel to the direction of train movement, (2) the y-axis perpendicular to the rail horizontally, and (3) the z-axis extends perpendicular to the track vertically. According to the European standard EN 13848-1 [50], which evaluates track geometric quality, there are five geometric parameters used to assess the track: longitudinal levels (LLs), horizontal alignments (ALs), track gauge (GAU), cross-level (XLV), and twist (TWI). The longitudinal levels refer to the vertical deviations of the rail level from the reference track serving as a fundamental metric in gauging track geometry.

In this study, the data were collected by an EM120 track recording car, as depicted in Figure 2. This sophisticated equipment enables thorough and efficient data collection, facilitating comprehensive analysis and informed decision-making in track maintenance and management. In these machines, data acquisition components are mobilized along both the vertical and horizontal axes by pneumatic cylinders, enabling the measurement of geometric parameters. Parameters such as longitudinal levels (LLs) are determined by assessing the displacement of the middle point with respect to the two initial and final points [46]. The track geometric parameters are automatically measured and logged along the referenced railway line.

Figure 2. A view of the geometric track recording car (EM120).

2.3. Data Acquisition of Vertical Ballast Stiffness

In this study, an onboard technique was utilized to acquire the rail displacements and thus track stiffness. As illustrated in Figure 3, the data collection process involves attaching a solid beam to the bogie of the wagon, with a laser and camera set positioned at the ends of the beam's arms [22,23,51]. This rail track measurement system comprises three interconnected units: the sensor unit, the processing unit, and the central unit. These components are securely attached to the railcar body via a girder beam. The sensor unit collects critical data related to rail behavior, including a camera for visual information, lasers for precise measurements, and an incremental rotary encoder. The central unit, which includes a microcontroller, battery, and GPS receiver, manages data flow and communication. Cabling connects the central unit to the sensor and processing units. A 2D laser type is used because it can scan a line across the rail, emitting light at a 60-degree angle relative to the horizontal. The camera captures images of the rail surface and stores them in the processing unit. Camera specifications include high resolution (e.g., Full HD or 4K) and a wide-angle lens (e.g., 140 degrees) to cover the track area. The camera regulation with 720p full HD image captures clear video footage. The processing unit, housed in a laptop, calculates rail displacement using image processing techniques. Known laser light angles and distances allow for the accurate determination of the rail surface height relative to the sensor. This integrated system enables the continuous monitoring of rail conditions, contributing to safety and maintenance efforts.

Figure 3. Data acquisition of track stiffness: (**a**) recording car; (**b**) laser/camera system installed to bogie frame.

The described setup allows for precise measurements by capturing the deflection and deformation of the track, thereby providing accurate assessments of its stiffness. The installed sensors record the distance between the two lasers, with variations occurring as the sensors' distance from the rail surface fluctuates. These changes allow for the establishment of geometric relationships between the recorded parameters. Integrating these relationships makes it possible to calculate the distance between the lasers and the rail surface. This method ensures the precise tracking of the rail's surface profile, enabling accurate measurements essential for determining track stiffness and overall rail integrity. A schematic of geometric relationships that lead to computing track deflection is shown in Figure 4. As depicted in this figure, the height of the measuring device (h) from the rail surface can be geometrically extracted using the following equations [23,51]:

$$h = \begin{cases} tan\theta_1(l_1 + L_1) \\ tan\theta_2(l_2 + L_2) \end{cases} \tag{1}$$

where, in the above relationships, L_1 and L_2 represent the horizontal distances of the lasers from the camera, θ_1 and θ_2 are the angles between lasers and the horizon, l_1 and l_2 denote the horizontal distances between the center of the camera, the points where the lasers intersect with the rail, and h is the vertical distance between the camera/lasers and the rail surface.

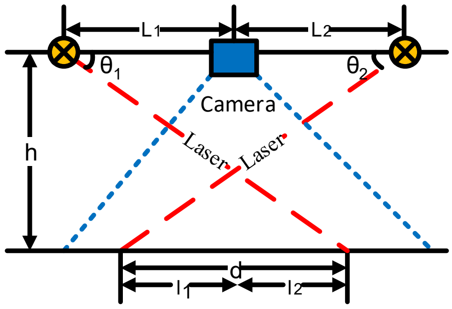


Figure 4. Geometric representation of track deflection measuring using laser/camera system.

The horizontal distance of the laser beams (d) is equal to the summation of l_1 and l_2 ($d = l_1 + l_2$), so the vertical height of the measuring device can be derived from the below equation:

$$d = h(cot\theta_1 + cot\theta_2) - (L_1 + L_2) \tag{2}$$

By determining the distance of laser beams (d) through image processing, the height of the measuring device (h) can be calculated according to Equation (2). If the measuring system is rigidly fixed to the railcar frame and wheelset, it can be assumed that the distance between the measuring device and wheel/rail contact point (H) remains constant as the railcar moves at low speed. Therefore, the relative rail deflection (y_r) can be determined as follows:

$$y_r = H - h \tag{3}$$

In summary, measuring the laser beam distance (d) provides the relative displacement between the railcar and railway track (h), which can then be used to calculate the relative rail displacement by assuming rigidity in the wheel/side frame system.

Therefore, the recording of the described camera/laser system can be related to the relative track deflection, enabling the calculation of the track modulus using the Winkler model. In soft track conditions, the rail rises at the wheel/rail contact point, causing the laser tracks to converge. Conversely, in hard track conditions, the distance between the lasers increases. By identifying the vertical deflection of the rail, train axle load, the stiffness of the fastening system, the elastic modulus of the subgrade, the length of the sleeper, and the ballast layer depth, the stiffness of the ballast layer can be calculated using the following equations: [52–54]

$$\frac{1}{k_t} = \frac{1}{k_p} + \frac{1}{k_b} + \frac{1}{k_s} \tag{4}$$

$$k_s = l_s(l_e + 2h_b tg\alpha)E_f \tag{5}$$

$$E_f = c_s^2\rho \tag{6}$$

where, in Equation (4), k_t is track stiffness, k_p is the stiffness of the railpad of the fastening system, k_b is ballast stiffness, and k_s is subgrade stiffness. In Equations (5) and (6), k_s is subgrade stiffness, l_s is the sleeper length, and l_e is the effective length of the sleeper, typically considered to be 0.6 times the sleeper length; h_b is the height of the ballast, E_f is the subgrade elastic modulus, c_s is the shear wave velocity, α is the angle of stress distribution, and ρ is the density of the ballast material.

The laser-based rail track inspection system, as described, offers precise measurements by capturing rail deflection and deformation. However, like any technology, it has limitations and challenges, and the following delves into some of these aspects:

(1) Sensor Positioning and Stability: Maintaining consistent sensor positioning is critical. Vibrations, track irregularities, and mechanical wear can affect sensor alignment. If sensors deviate from their optimal position, measurement accuracy may suffer.
(2) Surface Reflectivity: Rail surfaces vary in reflectivity due to factors like corrosion, dirt, and surface texture. Laser sensors may struggle with highly reflective or uneven surfaces, affecting data quality.
(3) Environmental Conditions: Adverse weather (rain, snow, fog) can impact laser performance. Poor visibility affects accurate measurements, especially during inclement weather.
(4) Speed Dependency: The system's effectiveness depends on train speed (low speed is preferred). At high speeds, rapid data acquisition yields less and inequality measurements.
(5) Geometric Defects: Some defects (e.g., cracks, squats) have longitudinal characteristics and so affect obtained results. Traditional 2D imaging struggles to detect such defects accurately.
(6) Wheel/Side Frame Assumption: The system assumes the rigidity of the wheel/side frame system. Deviations from this assumption (e.g., wheel wear) may impact measurements.
(7) Camera Quality and Configuration: Camera resolution, lens quality, and field of view affect image clarity. Poor camera settings may compromise data accuracy.
(8) Maintenance and Calibration: Regular maintenance and calibration are essential. Neglecting these tasks can lead to measurement errors.

In summary, while laser-based rail inspection offers valuable insights, addressing these challenges is crucial for maintaining accurate and reliable measurements. The referenced studies [22,23,51] provide further insights into this topic.

3. Algorithms Used in Correlation Mining Analyses

3.1. Overview

A schematic flowchart depicting the step-by-step process followed in the current study is presented in Figure 5. As shown in this figure, the analysis study comprises three main phases: (a) field measurements using track recording, (b) employing frequency-based methods for correlation analyses, and (c) utilizing data mining analyses through machine learning algorithms.

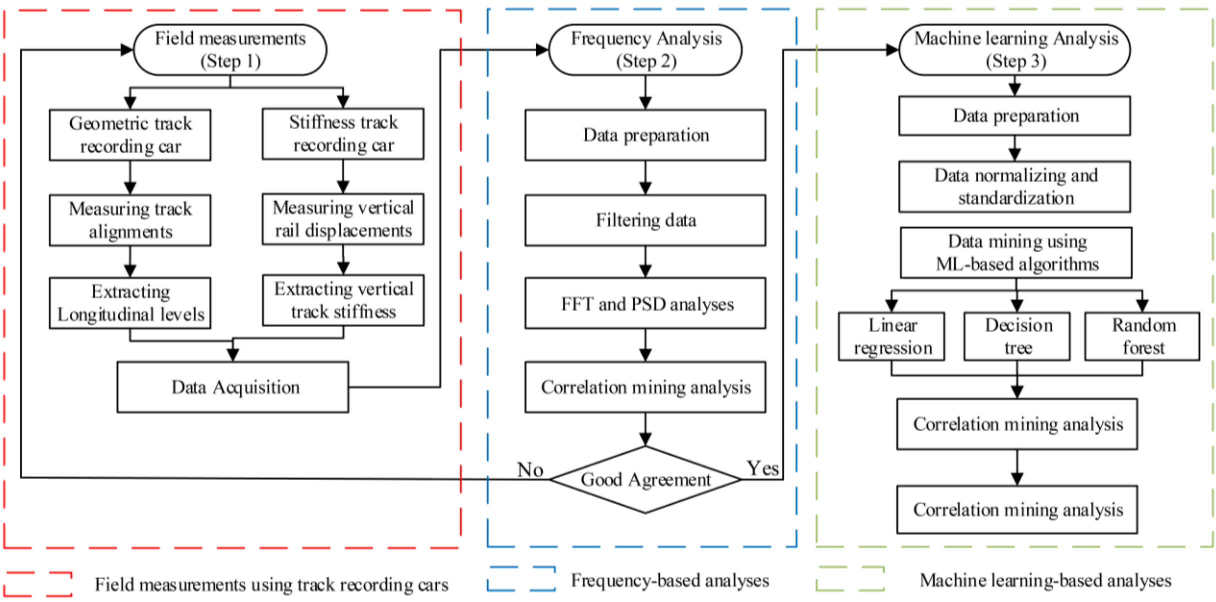


Figure 5. Process of correlation analyses using frequency-based and ML-based algorithms.

The process of the first block is explained in the previous section, while the subsequent steps are described in the following section.

3.2. Frequency-Based Technique

In the current survey, the power spectrum density (PSD) is used as a frequency-based technique to investigate the relationship between railway ballast stiffness and track alignment. The PSD of a signal determines how much of a signal's power is distributed across various frequencies [55]. The power spectrum of a signal can be calculated once for all signals, known as a periodogram, or by averaging different segments of a time signal from a periodogram to obtain the PSD. Ideally, a power spectrum requires an infinite sequence of continuous data; however, in practice, there are limitations in sampling frequency and data length, resulting in limited data. Consequently, it is crucial to determine how these constraints affect the PSD of a signal [56,57]. By examining the PSD, this study aims to identify significant patterns and relationships that can help understand how variations in ballast stiffness influence the track's longitudinal alignment.

3.3. Machine Learning-Based Algorithms

In this study, the relationship between vertical rail deflections and longitudinal levels was examined using predictive monitoring algorithms, such as linear regression, decision trees, and random forests. After data preprocessing through preparation and normalization, the accuracy measures were used to compare the performance of the mentioned ML-based algorithms in the mining of the correlation between rail displacements and both longitudinal levels of the left and right rails. A schematic of the ML-based methods used in the current study is illustrated in Figure 6. The three accuracy measures include the Mean Absolute Error (MAE), which measures the average magnitude of the errors, Mean Squared Error (MSE), which measures the average squared magnitude of the errors, and root mean square error (RMSE), which measures the square root of the average squared magnitude of the errors.

3.3.1. Linear Regression Method

In statistics, linear regression is a method used to model the relationship between a response (target) variable and one or more descriptive variables (features). This technique is frequently employed to uncover linear relationships between variables. It operates under the assumption that descriptive variables, which have values independent of each other, can effectively predict the response variable, whose value is neither dependent on these descriptive variables nor under the researcher's control. The primary objective of regression analysis is to identify and quantify the linear relationship between the target and features, facilitating predictions and insights based on the data.

The correlation coefficient is a statistical measure used to evaluate the relationship between variables, ranging in value from −1 to +1. A correlation coefficient close to either −1 or +1 indicates a strong relationship between variables. A coefficient near +1 signifies a direct relationship, meaning that as one variable increases, the other also increases. Conversely, a coefficient near −1 signifies an inverse relationship, where an increase in one variable corresponds to a decrease in the other. The form of the linear regression model is represented by the below equation:

$$Y = \beta_1 \cdot X + \beta_0 + \epsilon \quad (7)$$

where X and Y represent the independent and dependent variables, respectively. Also, the parameters of the linear model are ϵ: error, β_0: distance from the origin or intercept, and β_1: coefficient or line slope.

The line slope and distance from the origin in a linear regression model respectively indicate the sensitivity of the dependent variable to the independent variables and the value of the dependent variable when the independent variable equals zero [58–60]. These parameters play crucial roles in understanding the relationship between variables and predicting outcomes based on the model. The slope indicates the rate of change in the dependent variable for a unit change in the independent variable, providing insights into the strength and direction of the relationship. Meanwhile, the distance from the origin represents the intercept, which determines the value of the dependent variable when the independent variable is absent or equals zero, providing a reference point for the regression line.

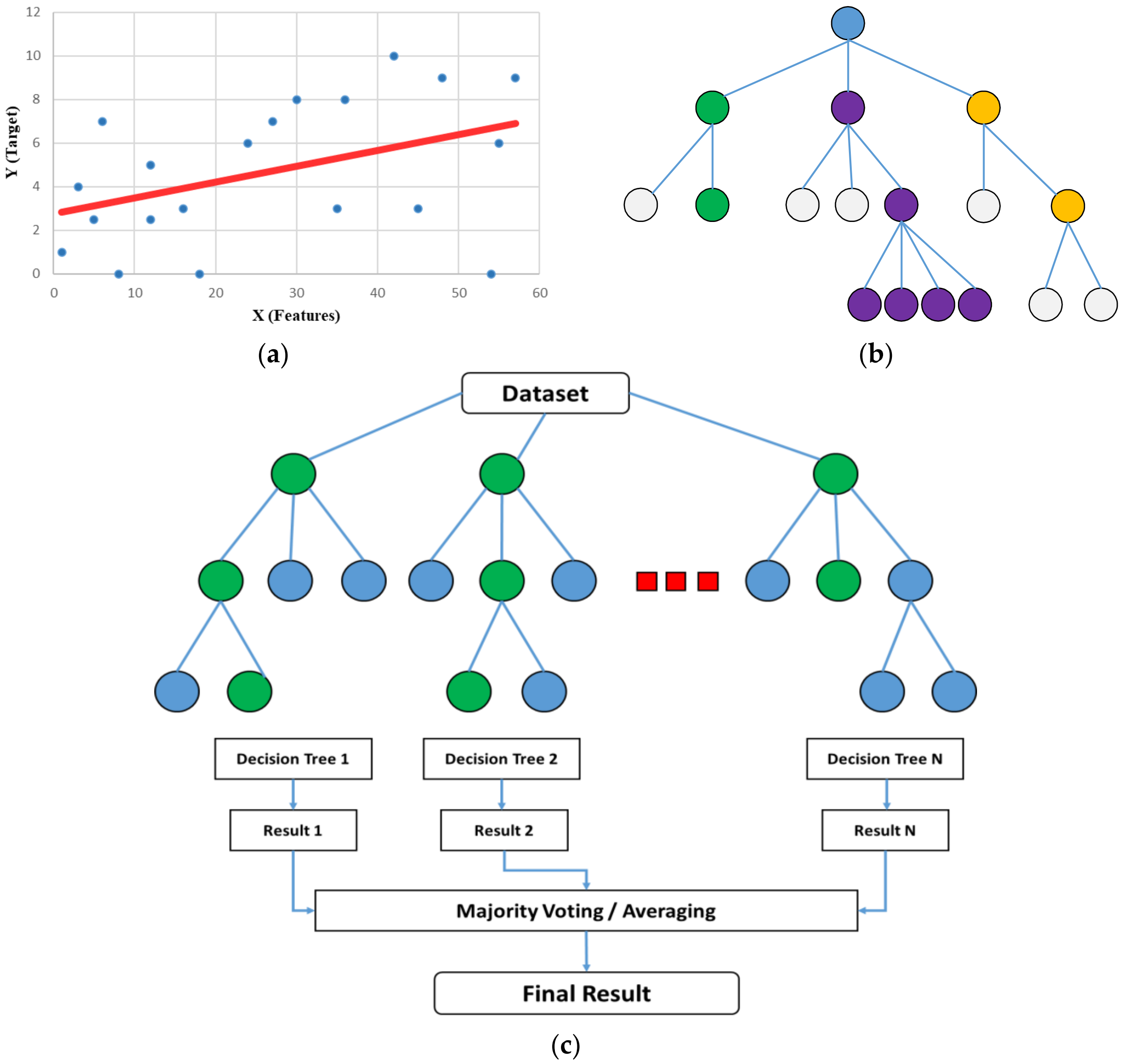


Figure 6. Machine learning-based algorithms used in current study: (**a**) linear regression algorithm, (**b**) decision tree algorithm, and (**c**) random forest algorithm.

As shown in Figure 6a, linear regression models the relationship between the outcome variable and explanatory variables using a linear function. In the current case, the features include the longitudinal levels of railway tracks, which came from the track geometry recording car. The vertical rail deflection as well as ballast stiffness are targeted as dependent variables. Linear regression estimates the coefficients (slope and intercept) of a linear equation that best fits the data. The model aims to minimize the difference between predicted and actual vertical ballast stiffness. The input data should be organized as a matrix, where each row corresponds to a measurement and columns represent the features. The algorithm learns the optimal coefficients by minimizing the sum of squared errors between predicted and actual deflections.

3.3.2. Decision Tree Method

The decision tree is a popular machine learning algorithm used for solving classification and regression problems due to its easy understanding and interpretation. This algorithm constructs a tree structure from the training data, representing a series of decisions to predict the target variable. It recursively divides the data into subsets based on features that provide the most information for making a final decision. The structure of a decision includes root nodes, branches, and leaf nodes. Each internal node represents a test on a feature, each branch represents the outcome of that test, and each leaf node holds a class label. The highest node in the tree is the root node. This technique is user-friendly, requiring no knowledge, and its learning and classification steps are straightforward and quick to execute [58–60].

So, a decision tree is a hierarchical model that makes decisions based on a series of questions or conditions. Thus, similar to linear regression, the same features (longitudinal levels) as input data and same target (vertical rail deflection and ballast stiffness) as outcome data were used in the current analyses. Decision trees split the data based on feature values to create a tree-like structure, as depicted in Figure 6b. Each internal node represents a test on a feature, and each branch represents the outcome of that test. The leaf nodes (terminal nodes) contain the predicted deflection values. Input data should be organized in a tabular format, with rows representing measurements and columns corresponding to features.

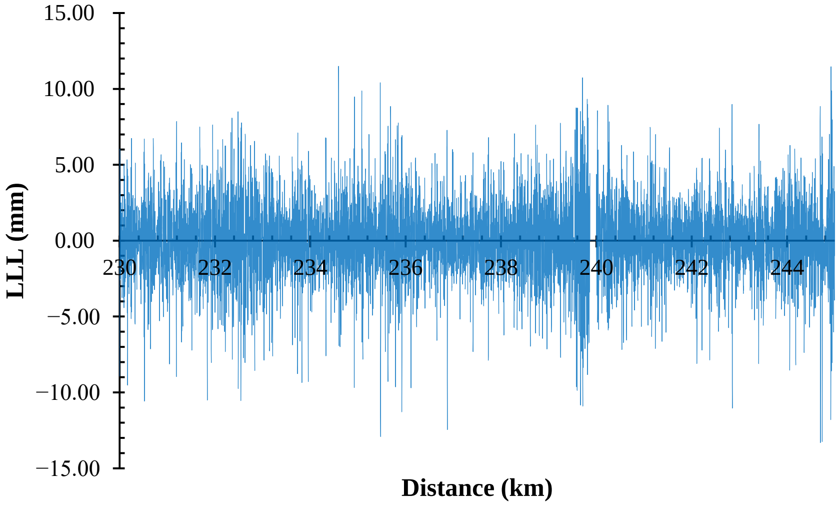
3.3.3. Random Forest Algorithm

Random forest is a supervised learning algorithm that randomly constructs the forest algorithm and includes a group of decision trees. As a type of ensemble learning algorithm, it combines the predictions of multiple decision trees to improve accuracy and robustness. Each decision tree in the random forest is trained on a random subset of training data and a random subset of features. The final output of the random forest is determined by aggregating the predictions of all the individual decision trees. This approach allows for the algorithm's accuracy and significantly reduces the risk of overfitting, where the model performs well on training data but poorly on test data [58–60].

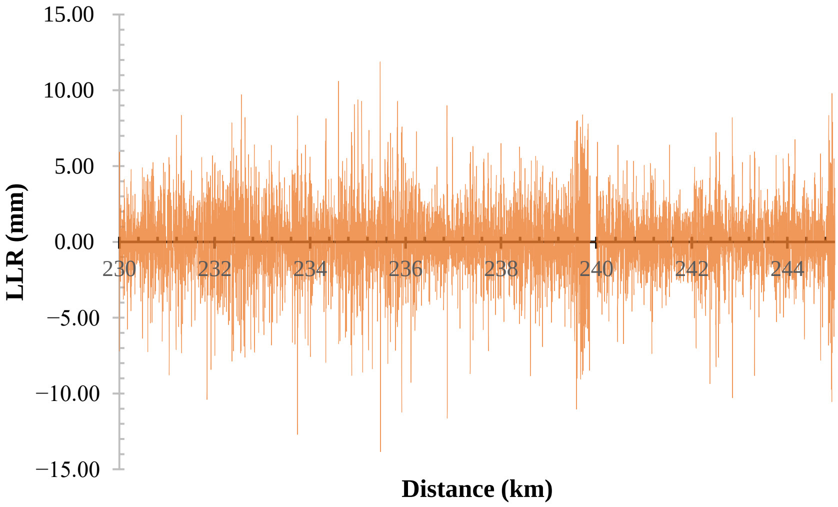
As illustrated in Figure 6c, random forest is an ensemble method that combines multiple decision trees to improve prediction accuracy. The same features and targets, as mentioned in previous algorithms, were utilized as independent and dependent variables, respectively. Random forest introduces additional randomness during tree growth by considering only a random subset of features at each split. The input data format remains the same as for decision trees. Random forest aggregates predictions from individual trees to make a more accurate overall prediction.

4. Results and Discussion

In this section, the results of this survey examining the relationship between ballast stiffness and the track longitudinal level are presented, utilizing frequency-based and ML-based techniques. Figure 7 shows the data distribution of longitudinal levels (for both left and right rails) recorded by the recording car (presented in Figure 3) of the geometry track in the designated study location. Moreover, Figure 8 illustrates the change in the vertical deflection of the rail along the Tehran–Mashhad railway line, specifically between kilometers 230 and 245, providing a clear representation of these differences in track conditions.



(**a**)



(**d**)

Figure 7. Longitudinal levels collected from experimental measurements: (**a**) left rail (LLL), (**b**) right rail (LLR).

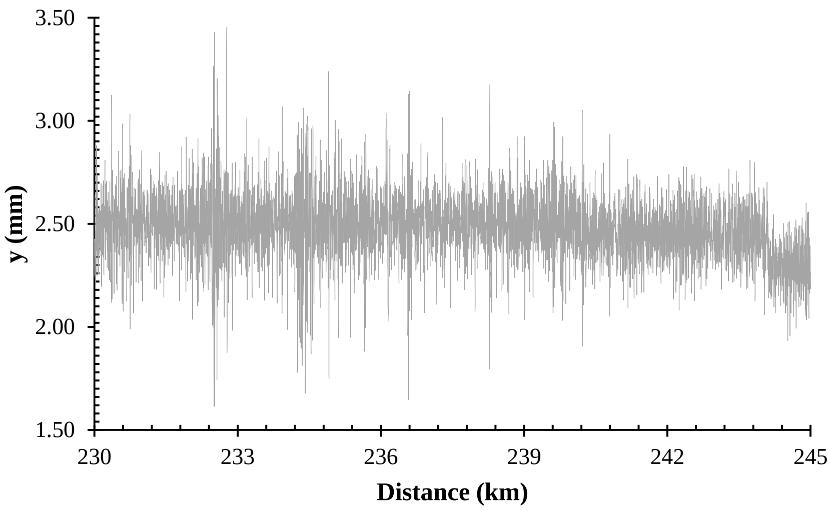


Figure 8. Variation in vertical rail deflection along reference railway line.

The characteristics of the reference railway track are presented in Table 1. Based on the mentioned equations presented in Section 2.3 and railway track properties, the ballast stiffness can be calculated based on Equations (4)–(6). The variation in ballast stiffness values which were calculated for the current case study are shown in Figure 9.

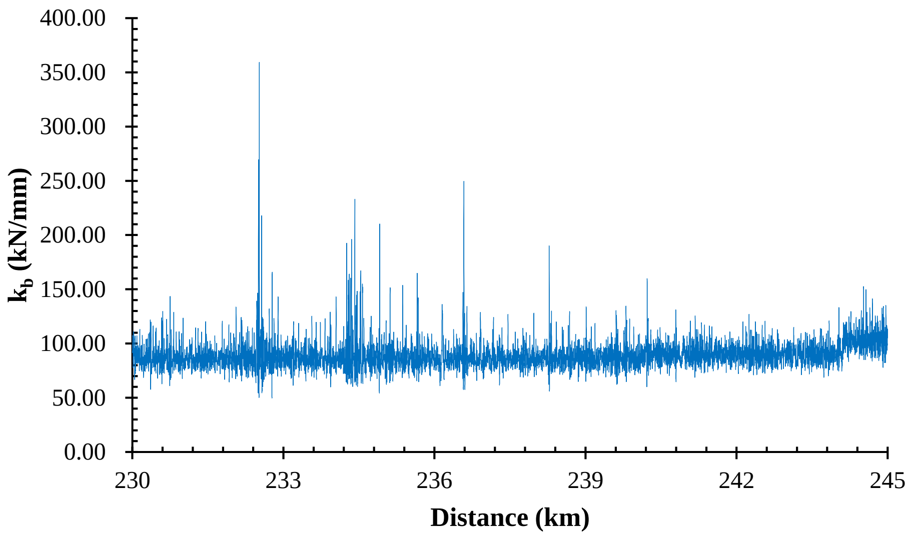


Figure 9. The calculated vertical ballast stiffness along the reference railway line.

4.1. Correlation Analyses through Frequency-Based Technique

The PSD of vertical rail deflection (*y-PSD*) and longitudinal levels are compared together in Figure 10. As shown in this figure, the vertical rail deflection and the longitudinal levels of the track are closely correlated, with their trends aligning in the frequency domain. The Pearson correlation matrix of the vertical rail displacements and longitudinal levels (for both left and right rails), which were extracted by PSD analyses, are presented in Table 2. Based on the correlation results presented in this table, the correlation between the vertical rail deflection and longitudinal level is 0.668 for the left rail and 0.669 for the right rail. These values indicate a relatively meaningful relationship between the PSD of the data for both rails. These obtained results are synchronized with the outcomes of Mehrali et al. [49]. Their concluded results show that the Pearson correlation between vertical rail deflections and longitudinal levels are 0.655 and 0.749 for the left and right rails, respectively.

Additionally, there is a considerable correlation in the power spectrum density, particularly within the 1–4 rad/m wave number range corresponding to wavelengths of 1.5 and 3 m. This meaningful correlation within specific wave number ranges highlights significant patterns in how vertical rail deflection interacts with longitudinal track alignment. These findings are in agreement with the study of Nielsen et al. [29,30] which investigated the contributions of longitudinal track unevenness and track stiffness variations. In this study, track unevenness and stiffness were measured in the Furet railway track of Sweden, and their contribution was evaluated using the PSD function. Their obtained results showed that there is very good agreement between track unevenness and stiffness for wave numbers up to 5 rad/m.

Another remark found from the correlation analysis is that the left and right rails are not completely aligned together. Rail misalignments can indicate vertical and horizontal track irregularities but can also be a consequence of a designed cant of railway tracks. So, considering the alignment or misalignment between rails is crucial for several reasons: safety, comfort and passenger experience, infrastructure deterioration, maintenance efficiency, etc. In Table 2, the Pearson correlation between the left and right rail deflections extracted is about 0.90. This shows that the left and right have not completely aligned together, and this could be due to the twist and superelevation of track or rail irregularities. In straight parts of the track without curves, these misalignments can be described as consequences of track settlement and/or short-wave rail surface irregularities.

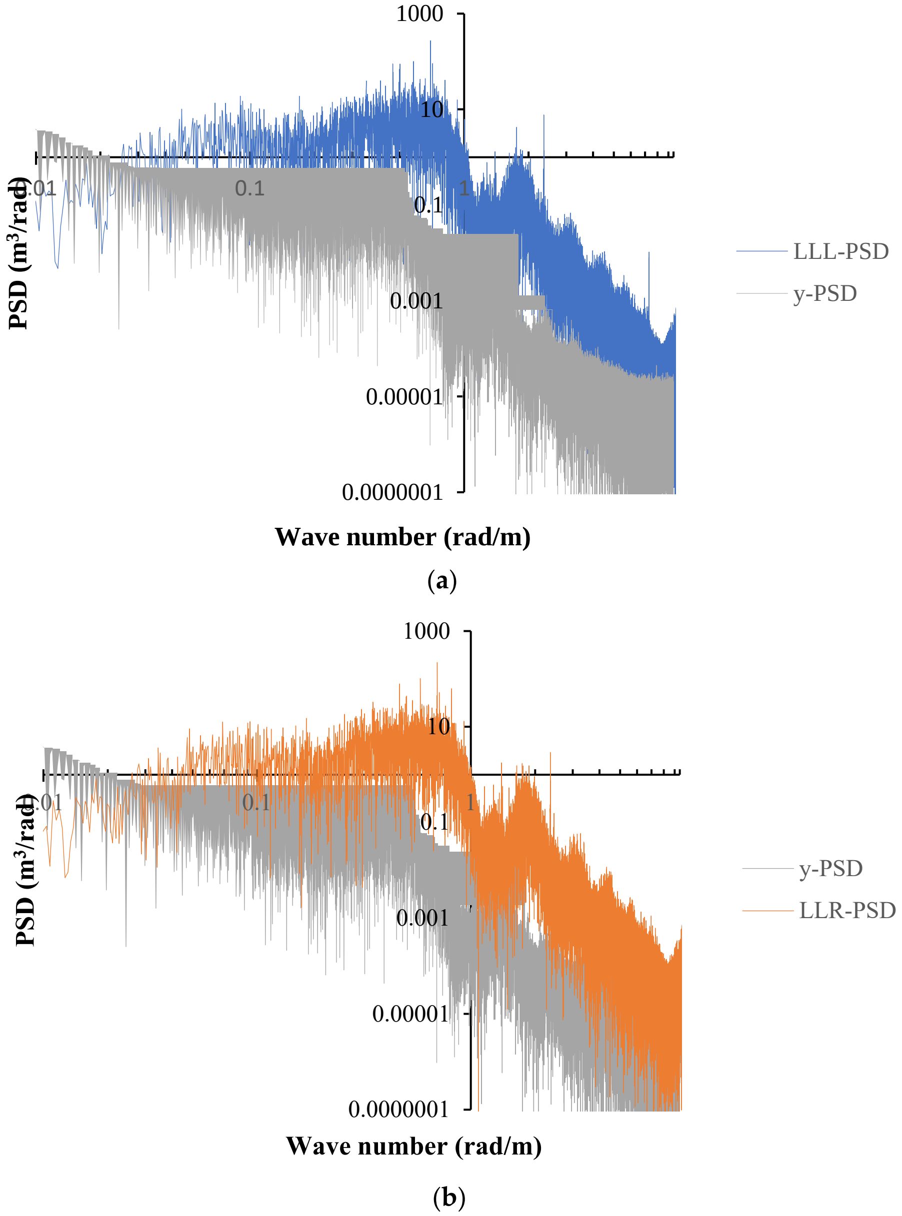
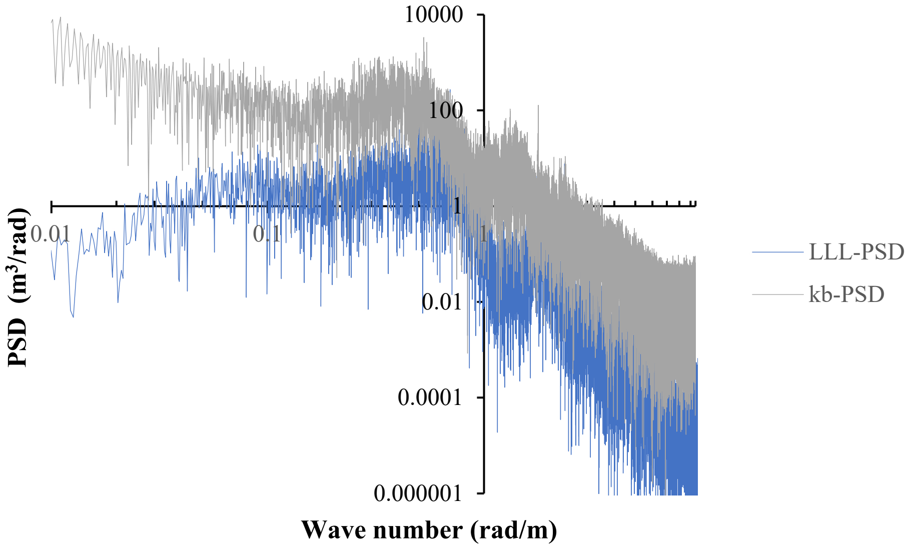


Figure 10. The PSD of vertical rail deflections (y) and longitudinal levels: (**a**) LLL, (**b**) LLR.

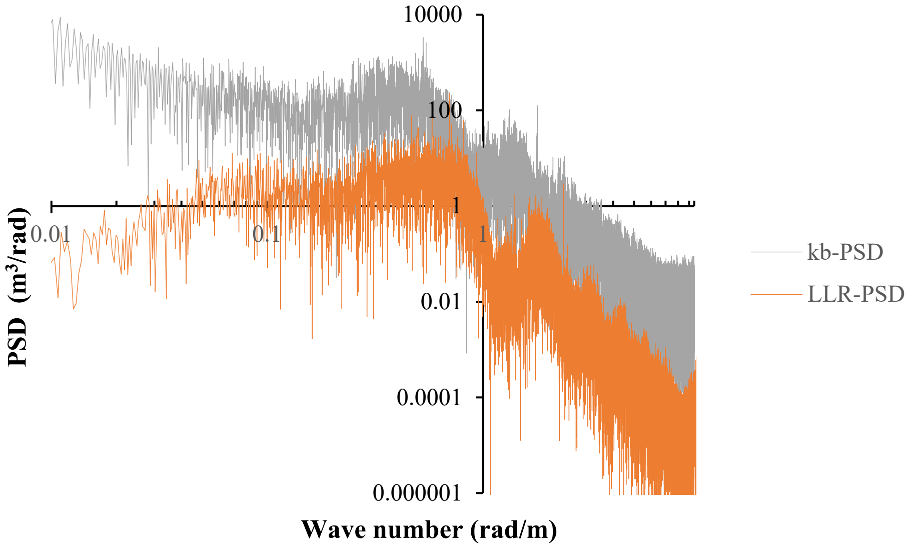
Table 2. Pearson correlation matrix of vertical rail deflections (y) and longitudinal levels (LLL, LLR) extracted by PSD analyses.

Parameters	LLL-PSD	LLR-PSD	y-PSD
LLL-PSD	1	0.902	0.668
LLR-PSD	0.902	1	0.669
y-PSD	0.668	0.669	1

The relationship between vertical ballast stiffness and the longitudinal level was also analyzed by calculating the PSD of the data, as illustrated in Figure 11. This figure seeks to uncover the correlation between vertical ballast stiffness and the longitudinal levels of the track.



(**a**)



(**b**)

Figure 11. The PSD of vertical ballast stiffness (kb) and longitudinal levels: (**a**) LLL, (**b**) LLR.

In Table 3, the Pearson correlation matrix of the vertical ballast stiffness (k_b) and track longitudinal levels (for both left and right rails) is extracted using frequency analyses. Based on the correlation results shown in this table, the correlation between vertical ballast stiffness and the longitudinal level is 0.672 for the left rail and 0.674 for the right rail. These values indicate a remarkable relationship between the PSD of these data for both rails. Also, there is a notable correlation in the power spectrum density, particularly within the wavelengths of 1.5 and 3 m (corresponding to 1–4 rad/m wave number range). This correlation in specific wave number ranges highlights significant patterns in the interaction between vertical ballast stiffness and longitudinal track alignment. This finding makes the relationship intelligible between ballast stiffness and longitudinal levels from the infrastructure managing perspective. This can provide a practical framework for assessing track health that is outlined based on the correlation between ballast stiffness and track longitudinal levels. Thus, the structural condition of railway superstructures and their related maintenance requirements are evoked by extracting the ballast stiffness from longitudinal levels. This insightful framework can promote maintenance decision-making in the planning of railway track management.

Table 3. Pearson correlation matrix of vertical ballast stiffness (k_b) and longitudinal levels (LLL, LLR) extracted by PSD analyses.

Parameters	LLL-PSD	LLR-PSD	y-PSD
LLL-PSD	1	0.902	0.672
LLR-PSD	0.902	1	0.674
k_b-PSD	0.672	0.674	1

4.2. Relationship Surveying through Machine Learning Algorithms

The accuracy measures (MAE, MSE, and RMSE) obtained from three predictive machine learning (ML)-based models are presented in Table 4. This table compares the MAE, MSE, and RMSE values across different machine learning algorithms. The results indicate that for the left rail, the RMSE values are approximately 0.05 (linear regression), 0.07 (decision tree), and 0.06 (random forest). Similarly, for the right rail, the RMSE values are approximately 0.05 (linear regression), 0.06 (decision tree), and 0.06 (random forest). Notably, the linear regression model exhibits the lowest accuracy measures, followed by the random forest and decision tree models.

Table 4. Model accuracy measures of relationship between vertical rail deflections (y) and longitudinal levels (LLL, LLR) mined by machine learning algorithms.

ML Algorithms	y-LLL			y-LLR		
	MAE	MSE	RMSE	MAE	MSE	RMSE
Linear regression	0.038	0.003	0.054	0.038	0.003	0.054
Decision tree	0.050	0.005	0.071	0.041	0.003	0.058
Random forest	0.046	0.004	0.064	0.041	0.003	0.058

Table 5 shows the model accuracy measures of the relationship between vertical ballast stiffness (k_b) and longitudinal levels mined by ML algorithms.

Table 5. Model accuracy measures of relationship between vertical ballast stiffness (k_b) and longitudinal levels (LLL, LLR) mined by machine learning algorithms.

ML Algorithms	k_b-LLL			k_b-LLR		
	MAE	MSE	RMSE	MAE	MSE	RMSE
Linear regression	0.027	0.001	0.038	0.027	0.001	0.038
Decision tree	0.036	0.003	0.050	0.030	0.002	0.042
Random forest	0.033	0.002	0.045	0.030	0.002	0.041

Based on Table 5, the accuracy measures for both the left and right rails exhibit remarkable similarity. These values suggest an appropriate relationship between vertical ballast stiffness and longitudinal levels. The root mean square error (RMSE) of the linear regression model is approximately 0.04, which stands as the lowest RMSE among the various machine learning algorithms investigated. Additionally, the RMSE values for the decision tree and random forest models fall within the range of 0.04 to 0.05. Consequently, the linear regression, random forest, and decision tree models yield the best accuracy results, respectively. The slight discrepancy between the decision tree and random forest can be attributed to the fact that the random forest model generalizes the decision tree approach. Mehrali et al. [49] highlighted a relation between rail deflection and track alignments using the decision tree method. They clarified that rail deflections and track alignments in the range of 16.3 to 8.1 mm have a close relationship when utilizing this ML technique.

5. Conclusions

The present paper focused on investigating the relationship between the longitudinal level and vertical ballast stiffness, using data mining techniques to analyze the collected data. This understanding is essential for various aspects in railway management including operational efficiency, safety, passenger comfort, and optimal resource allocation. So, this study was initially conducted by calculating ballast stiffness and longitudinal levels, using track recording cars, along a section of the Tehran–Mashhad railway line. The primary data were processed, and the correlation analyses between longitudinal levels and vertical ballast stiffness were conducted using frequency analysis techniques and machine learning algorithms such as linear regression, decision trees, and random forests. The following are the summarized results obtained from investigating the relationship between the longitudinal level and vertical ballast stiffness:

1. The frequency analysis showed that the power spectrum density of the vertical longitudinal level and ballast stiffness have approximately a 0.67 correlation. These values indicate a rather remarkable relationship between the PSD of these data. Additionally, it revealed a notable correlation in PSD data, particularly in the wavelength of 1–4 rad/m.
2. The correlation analyses, using machine learning methods, demonstrated an appropriate relationship between the rail longitudinal levels and vertical rail deflection. The concluded results revealed that the RMSE of the longitudinal levels and vertical rail deflections data ranged around 0.05 to 0.07. According to the outcomes, the linear regression, random forest, and decision trees achieved better accuracy, respectively.
3. Furthermore, the data mining analyses, used by ML-based algorithms, concluded that the longitudinal level of the rail and vertical ballast stiffness have a considerable relationship. The RMSE values of the linear regression, decision tree, and random forest algorithms were approximately 0.04, 0.045, and 0.05, respectively, indicating that the linear regression method yielded more accurate results.

Given that a significant portion of the maintenance costs are attributed to maintaining track stiffness, it becomes imperative to deploy strategies to optimize resource allocation. As recording geometric parameters is faster and more straightforward than assessing track stiffness, it is increasingly interesting to analyze the relationship between track stiffness and track longitudinal levels. In conclusion, this investigation has revealed a significant relationship between the track longitudinal levels, vertical rail deflection, and ballast stiffness of railway tracks. The findings demonstrate that machine learning-based algorithms exhibit a strong capability to mine and analyze this correlation effectively. This insight underscores the potential of advanced analytical techniques in understanding and optimizing railway track maintenance strategies. By leveraging machine learning algorithms, railway operators and maintenance teams can better predict and address issues related to track conditions, ultimately enhancing the safety, efficiency, and longevity of railway infrastructure. Moreover, to improve accuracy, future research can explore more advanced machine learning and deep learning techniques.

These research findings establish a fundamental link between ballast stiffness and geometric parameters within the context of infrastructure management. A pragmatic framework for evaluating track health conditions is proposed. By extracting ballast stiffness data from longitudinal measurements, critical insights are gained into the structural integrity of railway superstructures and their associated maintenance requirements. This innovative framework holds significant promise for enhancing decision-making in railway track management planning.

Author Contributions: Conceptualization, S.M. and H.H.; methodology, H.H.; software, H.H and M.K.; validation, S.M., H.H. and M.K.; formal analysis, H.H.; investigation, H.H. and M.K.; resources, S.M. and H.H; data curation, M.K.; writing—original draft preparation, H.H and M.K..; writing—review and editing, A.M.; visualization, A.M., M.K. and H.H.; supervision, A.M; project administration, S.M.; funding acquisition, S.M. and H.H All authors have read and agreed to the published version of the manuscript.

Funding: This work was financially supported by Base Funding-UIDB/04708/2020 and Programmatic Funding-UIDP/04708/2020 of the CONSTRUCT—Instituto de Estruturas e Construções, funded by national funds through the FCT/ MCTES (PIDDAC). The last author acknowledges Grant no. 2021.04272.CEECIND from the Stimulus of Scientific Employment, Individual Support (CEECIND)—4rd Edition provided by "FCT—Fundação para a Ciência".

Data Availability Statement: Data sharing is not applicable.

Acknowledgments: This work was financially supported by Base Funding-UIDB/04708/2020 and Programmatic Funding-UIDP/04708/2020 of the CONSTRUCT—Instituto de Estruturas e Construções, funded by national funds through the FCT/ MCTES (PIDDAC). The last author acknowledges Grant no. 2021.04272.CEECIND from the Stimulus of Scientific Employment, Individual Support (CEECIND)—4rd Edition provided by "FCT—Fundação para a Ciência".

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

Symbol	Description	Unit
LLL	Longitudinal level of left rail	mm
LLR	Longitudinal level of right rail	mm
AL	Horizontal alignment of railway track	mm
GAU	Track gauge of railway track	mm
XLV	Superelevation/cross-level of railway track	mm
TWI	Twist of railway track	%
Y	Vertical rail deflection	mm
P	Axle load	tons
K_p	Stiffness of fastening system	kN/mm
ls	Sleeper length	m
l_e	Effective length of sleeper	m
h_b	Ballast and sub-ballast depth	m
α	Ballast friction angle	degree
E_f	Subgrade elastic modulus	MPa

References

1. Esveld, C. *Modern Railway Track*; MRT Production: Zaltbommel, The Netherlands, 2014.
2. Sysyn, M.; Gerber, U.; Liu, J.; Fischer, S. Studying the Relation of the Residual Stresses in the Ballast Layer to the Elastic Wave Propagation. *Transp. J. Infrastruct. Geotechnol.* **2023**, *10*, 962–987. [CrossRef]
3. Davies, B.; Andrews, J. The impact of summer heatwaves on railway track geometry maintenance. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2021**, *235*, 1158–1171. [CrossRef]
4. Mosleh, A.; Montenegro, P.; Alves Costa, P.; Calçada, R. An approach for wheel flat detection of railway train wheels using envelope spectrum analysis. *Struct. Infrastruct. Eng.* **2020**, *17*, 1710–1729. [CrossRef]
5. Traquinho, N.; Vale, C.; Ribeiro, D.; Meixedo, A.; Montenegro, P.; Mosleh, A.; Calçada, R. Damage Identification for Railway Tracks Using Onboard Monitoring Systems in In-Service Vehicles and Data Science. *Machines* **2023**, *11*, 981. [CrossRef]
6. Mosleh, A.; Costa, P.A.; Calçada, R. A new strategy to estimate static loads for the dynamic weighing in motion of railway vehicles. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2020**, *234*, 183–200. [CrossRef]
7. Real, J.I.; Gómez, L.; Montalbán, L.; Real, T. Study of the influence of geometrical and mechanical parameters on ballasted railway tracks design. *J. Mech. Sci. Technol.* **2012**, *26*, 2837–2844. [CrossRef]
8. Ngo, T.; Indraratna, B. Use of Geogrid for Improved Performance of Ballasted Tracks: Experimental and DEM Approaches. *Int. J. Geosynth. Ground Eng.* **2024**, *10*, 33. [CrossRef]
9. Zhao, W.; Qiang, W.; Yang, F.; Jing, G.; Guo, Y. Data-driven ballast layer degradation identification and maintenance decision based on track geometry irregularities. *Int. J. Rail Transp.* **2023**, *12*, 581–603. [CrossRef]

10. Vale, C.; Ribeiro, I.M.; Calcada, R. Integer programming to optimize tamping in railway tracks as preventive maintenance. *J. Transp. Eng.* **2012**, *138*, 123–131. [CrossRef]
11. Zhu, M.; Cheng, X.; Miao, L.; Sun, X.; Wang, S. Advanced Stochastic Modeling of Railway Track Irregularities. *Adv. Mech. Eng.* **2013**, *5*, 401637. [CrossRef]
12. Liu, R.K.X.P.; Sun, Z.Z.; Zou, C.; Sun, Q.-X. Establishment of track quality index standard recommendations for Beijing metro. *Discret. Dyn. Nat. Soc.* **2015**, *2015*, 473830. [CrossRef]
13. Khouy, I.A.; Schunnesson, H.; Juntti, U.; Nissen, A.; Larsson-Kråik, P.O. Evaluation of track geometry maintenance for a heavy haul railroad in Sweden; A case study. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2014**, *228*, 496–503. [CrossRef]
14. Chen, Q.; Niu, X.; Zuo, L.; Zhang, T.; Xiao, F.; Liu, Y.; Liu, J. A Railway Track Geometry Measuring Trolley System Based on Aided INS. *Sensors* **2018**, *18*, 538. [CrossRef] [PubMed]
15. Circelli, M.; Kaviani, N.; Licciardello, R.; Ricci, S.; Rizzetto, L.; Shahidzadeh Arabani, S.; Shi, D. Track geometry monitoring by an onboard computer-vision-based sensor system. *Transp. Res. Procedia* **2023**, *69*, 257–264. [CrossRef]
16. Soleimanmeigouni, I.; Ahmadi, A.; Nissen, A.; Xiao, X. Prediction of railway track geometry defects; a case study. *Struct. Infrastruct. Eng.* **2019**, *16*, 987–1001. [CrossRef]
17. Do, N.T.; Gül, M.; Nafari, S.F. Continuous evaluation of track modulus from a moving railcar using ANN-based techniques. *Vibration* **2020**, *3*, 149–161. [CrossRef]
18. Li, Q.; Bai, Z.; Chen, B.; Xin, H.; Cheng, Y.; Zhang, Q. High-speed railway track comprehensive measurement system based on GNSS/INS multi-sensor. In Proceedings of the China Satellite Navigation Conference (CSNC) 2020 Proceedings, Chengdu, China, 22–25 November 2020; Springer: Berlin/Heidelberg, Germany, 2020; Volume 1, pp. 3–14. [CrossRef]
19. Heydari, H.; Zakeri, J.A.; Taghipour, A. Investigating the wheel–rail dynamic response of transition system of concrete slab and ballasted tracks through train–track interaction model. *Innov. Infrastruct. Solut.* **2024**, *9*, 240. [CrossRef]
20. Li, M.X.D.; Berggren, E.G. A Study of the Effect of Global Track Stiffness and Its Variations on Track Performance: Simulation and Measurement. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2010**, *224*, 375–382. [CrossRef]
21. Pita, A.L.; Teixeira, P.F.; Robuste, F. High speed and track deterioration: The role of vertical stiffness of the track. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2004**, *218*, 31–40. [CrossRef]
22. Heydari, H.; Zakeri, J.A.; Esmaeili, M.; Varandas, J. Field Study of Using Additional Rails and Approach Slab as Transition Zone from Slab Track to Ballasted Track. *J. Rail Rapid Transit* **2017**, *232*, 970–978. [CrossRef]
23. Heydari, H. Evaluating the Dynamic Behavior of Railway-Bridge Transition Zone: Numerical and Field Measurements. *Can. J. Civ. Eng.* **2023**, *51*, 399–408. [CrossRef]
24. Can, S.; Yu, Z.; Lei, X.; Zhang, X.; Yunlong, G. A critical review on the vertical stiffness irregularity of railway ballasted track. *Constr. Build. Mater.* **2023**, *400*, 132715. [CrossRef]
25. Puzavac, L.; Popović, Z.; Lazarević, L. Influence of Track Stiffness on Track Behaviour under Vertical Load. *Traffic Transp.* **2012**, *24*, 405–412. [CrossRef]
26. Roghani, A.; Macciotta, R.; Hendry, M. Combining Track Quality and Performance Measures to Assess Track Maintenance Requirements. In Proceedings of the 2015 Joint Rail Conference, San Jose, CA, USA, 23–26 March 2015. [CrossRef]
27. Berggren, E.G. Efficient Track Maintenance; Methodology for Combined Analysis of Condition Data. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2010**, *224*, 353–360. [CrossRef]
28. Berggren, E.G.; Nissen, A.; Paulsson, B.S. Track deflection and stiffness measurements from a track recording car. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2014**, *228*, 570–580. [CrossRef]
29. Nielsen, J.C.; Berggren, E.G.; Hammar, A.; Jansson, F.; Bolmsvik, R. Degradation of railway track geometry—Correlation between track stiffness gradient and differential settlement. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2020**, *234*, 108–119. [CrossRef]
30. Germonpré, M.; Nielsen, J.C.O.; Degrande, G.; Lombaert, G. Contributions of longitudinal track unevenness and track stiffness variation to railway induced vibration. *J. Sound Vib.* **2018**, *437*, 292–307. [CrossRef]
31. Grossoni, I.; Andrade, A.R.; Bezin, Y.; Neves, S. The role of track stiffness and its spatial variability on long-term track quality deterioration. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2019**, *233*, 16–32. [CrossRef]
32. Tsunashima, H.; Ono, H.; Takata, T.; Ogata, S. Development and Operation of Track Condition Monitoring System Using In-Service Train. *Appl. Sci.* **2023**, *13*, 3835. [CrossRef]
33. Mosleh, A.; Montenegro, P.A.; Costa, P.A.; Calçada, R. Railway Vehicle Wheel Flat Detection with Multiple Records Using Spectral Kurtosis Analysis. *Appl. Sci.* **2021**, *11*, 4002. [CrossRef]
34. Tsunashima, H.; Honda, R.; Matsumoto, A. Track Condition Monitoring Based on In-Service Train Vibration Data Using Smartphones. In *New Research on Railway Engineering and Transport*; IntechOpen: London, UK, 2023. [CrossRef]
35. Malekjafarian, A.; OBrien, E.; Quirke, P.; Bowe, C. Railway Track Monitoring Using Train Measurements: An Experimental Case Study. *Appl. Sci.* **2019**, *9*, 4859. [CrossRef]
36. Villarejo, R.; Johansson, C.A.; Galar, D.; Sandborn, P.; Kumar, U. Context-driven decisions for railway maintenance. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2016**, *230*, 1469–1483. [CrossRef]
37. Bergquist, B.; Söderholm, P. Data analysis for condition-based railway infrastructure maintenance. *Qual. Reliab. Eng. Int.* **2015**, *31*, 773–781. [CrossRef]
38. Famurewa, S.M.; Zhang, L.; Asplund, M. Maintenance analytics for railway infrastructure decision support. *J. Qual. Maint. Eng.* **2017**, *23*, 310–325. [CrossRef]

39. Sauni, M.; Luomala, H.; Kolisoja, P.; Turunen, E. Investigating root causes of railway track geometry deterioration: A data mining approach. *Front. Built Environ.* **2020**, *6*, 122. [CrossRef]
40. Thaduri, A.; Galar, D.; Kumar, U. Railway assets: A potential domain for big data analytics. *Procedia Comput. Sci.* **2014**, *53*, 457–467. [CrossRef]
41. Sresakoolchai, J.; Kaewunruen, S. Railway infrastructure maintenance efficiency improvement using deep reinforcement learning integrated with digital twin based on track geometry and component defects. *Sci. Rep.* **2023**, *13*, 2439. [CrossRef]
42. Lopes Gerum, P.C.; Altay, A.; Baykal-Gürsoy, M. Data-driven predictive maintenance scheduling policies for railways. *Transp. Res Part C Emerg. Technol* **2019**, *107*, 137–154. [CrossRef]
43. Falamarzi, A.; Moridpour, S.; Nazem, M.; Cheraghi, S. Prediction of tram track gauge deviation using artificial neural network and support vector regression. *Aust. J. Civ. Eng.* **2019**, *17*, 63–71. [CrossRef]
44. Bergmeir, C.; Sáinz, G.; Martínez Bertrand, C.; Benítez, J.M. A Study on the Use of Machine Learning Methods for Incidence Prediction in High-Speed Train Tracks. In *Recent Trends in Applied Artificial Intelligence, Lecture Notes in Computer Science*; Springer: Berlin/Heidelberg, Germany, 2013; Volume 7906, pp. 674–683.
45. Lee, J.S.; Hwang, S.H.; Choi, I.Y.; Kim, I.K. Prediction of track deterioration using maintenance data and machine learning schemes. *J. Transp. Eng. A Syst.* **2018**, *144*. [CrossRef]
46. Martey, E.N.; Ahmed, L.; Attoh-Okine, N. Track geometry big data analysis: A machine learning approach. In Proceedings of the 2017 IEEE International Conference on Big Data, Boston, MA, USA, 11–14 December 2017; pp. 3800–3809. [CrossRef]
47. Sresakoolchai, J.; Kaewunruen, S. Track Geometry Prediction Using Three-Dimensional Recurrent Neural Network-Based Models Cross-Functionally Co-Simulated with BIM. *Sensors* **2022**, *23*, 391. [CrossRef]
48. Popov, K.; De Bold, R.; Chai, H.K.; Forde, M.C.; Ho, C.L.; Hyslip, J.P.; Kashani, H.F.; Kelly, R.; Hsu, S.S.; Rippin, M. Data-driven track geometry fault localisation using unsupervised machine learning. *Constr. Build. Mater.* **2023**, *377*, 131141. [CrossRef]
49. Mehrali, M.; Esmaeili, M.; Mohammadzadeh, S. Application of data mining techniques for the investigation of track geometry and stiffness variation. *Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit* **2020**, *234*, 439–453. [CrossRef]
50. ISO. *Railway Applications—Track—Track Geometry Quality—Part 1, Characterization of Track geometry*; International Organization for Standardization: Geneva, Switzerland, 2019.
51. McVey, B.; Farritor, S.; Norman, C.; Wood, N.; Arnold, R.; Feteh, M.; El-Sibaie, M. Track Modulus Measurement from a Moving Railcar. In Proceedings of the AREMA Annual Conference, Conference of Railway Engineering, Nashville, TN, USA, 19–22 September 2004; Engineering Technics Press: London, UK.
52. Zhai, W.M.; Wang, K.Y.; Lin, J.H. Modelling and experiment of railway ballast vibrations. *J. Sound Vib.* **2004**, *270*, 673–683. [CrossRef]
53. Heydari, H.; Khanie, N.; Naseri, R. Formulation and evaluation of the ballast shear interlocking coefficient based on analytical, experimental, and numerical analyses. *Constr. Build. Mater.* **2023**, *406*, 133457. [CrossRef]
54. Heydari, H.; Naseri, R.; Khanie, N. Investigating the effect of ballast contamination in vertical and shear interlocking stiffness: Experimental and numerical study. *Constr. Build. Mater.* **2024**, *428*, 136337. [CrossRef]
55. Brook, D.; Wynne, R.J. *Signal Processing: Principles and Applications*; Hodder Arnold: London, UK, 1988.
56. Stoica, P. *Spectral Analysis of Signals*, 1st ed.; Prentice Hall: Saddle River, NJ, USA, 2005.
57. Rao, K.; Swamy, M. Spectral Analysis of Signals. In *Digital Signal Processing*; Springer: Singapore, 2018. [CrossRef]
58. Mitchell, T.M. Machine learning and data mining. *Commun. ACM* **1999**, *42*, 30–36. [CrossRef]
59. Awad, M.; Khanna, R. *Machine Learning, In Efficient Learning Machines*; Apress: Berkeley, CA, USA, 2015. [CrossRef]
60. Zhou, Z.H. *Machine Learning*; Springer: Berlin/Heidelberg, Germany, 2021. [CrossRef]