



Article

Compatible Taper and Volume Systems Based on Volume Ratio Models for Four Pine Species in Oaxaca Mexico

Albert Castillo-López ^{1,*}, Gerónimo Quiñonez-Barraza ², Ulises Diéguez-Aranda ³ and José Javier Corral-Rivas ⁴

¹ Doctorado Institucional en Ciencias Agropecuarias y Forestales, Universidad Juárez del Estado de Durango, Río Papaloapan, Valle del Sur, 34120 Durango, DGO, Mexico

² Campo Experimental Valle del Guadiana, Instituto Nacional de Investigaciones Forestales, Agrícolas y Pecuarias, Carretera Durango-Mezquital km 4.5, 34170 Durango, DGO, Mexico; quinonez.geronimo@inifap.gob.mx

³ Departamento de Ingeniería Agroforestal, Escuela Politécnica Superior de Lugo, Universidad de Santiago de Compostela, Campus Universitario, 27002 Lugo, Spain; ulises.dieguez@usc.es

⁴ Facultad de Ciencias Forestales, Universidad Juárez del Estado de Durango, Río Papaloapan, Valle del Sur, 34120 Durango, DGO, Mexico; jcorral@ujed.mx

* Correspondence: a_castillo@ujed.mx; Tel.: +52-951-218-0065

Abstract: Estimating tree volume components is an essential element in sustainable forest management. Compatible taper and merchantable outside-bark volume systems based on volume ratio models were globally fitted to four pine species (reduced models) with the aim to select the best reduced model and to fit it with dummy variables and additive effects using *Pinus pseudostrobus* Lindl as a base species. The study was carried out in the northern mountains of Oaxaca, Mexico. To fit the taper, merchantable volume, stem volume, branch volume, and total tree volume (stem volume and branch volume) equations, a taper dataset of 222, 230, 245, and 333 trees of *Pinus douglasiana* Martínez (*Pd*), *Pinus oaxacana* Mirov (*Po*), *Pinus patula* Schltd (*Pp*), and *Pinus pseudostrobus* Lindl (*Pps*), respectively, was used. In general, the compatible systems explained more than 97% in the observed variability for the four studied components: outside-bark diameter (d), merchantable outside-bark volume (V_m), stem outside-bark volume (V_s), and total tree volume (V_t). Alternatively, more than 52% of the observed variability for branch volume (V_b) was also explained. The developed compatible systems based on volume ratio models are a simple and consistent alternative for estimating the outside-bark diameter and variable outside-bark volume, as well as the components of commercial species for uneven-age and mixed-species forests in Oaxaca, Mexico.

Keywords: reduced and full compatible system; merchantable outside-bark volume; stem outside-bark volume; total tree volume; branch volume



Citation: Castillo-López, A.; Quiñonez-Barraza, G.; Diéguez-Aranda, U.; Corral-Rivas, J.J. Compatible Taper and Volume Systems Based on Volume Ratio Models for Four Pine Species in Oaxaca Mexico. *Forests* **2021**, *12*, 145. <https://doi.org/10.3390/f12020145>

Received: 9 December 2020

Accepted: 23 January 2021

Published: 27 January 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Sustainable forest management incorporates the use of statistical tools for estimating the total and merchantable volume of trees for the correct application of silvicultural treatments in a specific stand or group of stands, estimating the distribution of commercial products and the economic yield [1]. Forest species, site quality, stand density, and silvicultural treatments have a direct relationship with the tree profile [2–4] and these elements are used in the formulation of taper and volume systems [1,4,5].

In the last decades, numerous taper models have been proposed, starting with simple hyperbolic expressions, polynomial functions, segmented equations, or spline functions, and directly through volume ratio functions [6,7]. However, several studies have considered that the form of the stem does not correspond to a single equation of dendrometric type, but that the form differs to some extent with the tree species and the point in height along the stem [8]. Furthermore, the development during recent years of new flexible

approaches to derive compatible taper and volume systems to which data corresponding to different species and trees with variable stem forms can be accurately fitted (e.g., [1,9–11]). Therefore, this is a good justification for reviewing the appropriateness of compatible taper and volume equations systems derived from volume ratio functions which are based on relative tree height for different tree species. Ratio volume equations estimate the volume of a tree up to a certain diameter or height as a percentage of the total tree volume (stem volume and branch volume) [12–15]. These models consider the fitting of an equation that characterizes the tree form and are mathematical relationships between the diameters or sections of the stem at any point of the tree and the height at which the diameter is estimated [16].

However, up to date most of the research related to the estimation of tree volume has focused on developing compatible tree taper and stem volume models that describe the stem profile and estimate the total or partial stem volume, without considering the volume of branches [17,18]. The total tree volume models that include the branch component are required to provide base information for the sustainable use of the entire tree volume [17,19] because in Mexican forestry the total tree volume is the most important variable for forest management and planning. The suitable estimation of the branch volume is very important because these sections are increasingly used not only for bioenergy purposes but also for quantification of CO₂ sequestration in forest ecosystems [20,21]. The estimation of branch volume is also relevant for assessing the potential bioenergy purposes on the soil nutrient and carbon cycle [22].

The objective of this study was to fit three compatible taper and merchantable outside-bark volume equations as simultaneous systems for estimating the taper (d , cm), merchantable outside-bark volume (V_m , m³), stem outside-bark volume (V_s , m³), branch volume (V_b , m³), and total tree volume (V_t , m³) components for main pine species in Oaxaca, Mexico. The model fitting was tested for all combined species with a reduced equations system with the same set of global parameters for the four studied tree species and with a full equation system obtained by expanding in the best reduced system each global parameter by including an associated parameter and a dummy variable to differentiate the species and to assess whether separate models are necessary for the selected pine species.

2. Materials and Methods

2.1. Study Area

The study was carried out in mixed-species forests in the Regional Forest Management Unit (UMAFOR 2001), in the northeast of Oaxaca state in Mexico. The study area covers an area of 195,397 ha. The UMAFOR 2001 is located at geographic coordinates 17-09'56.93"—17-33'01.82" N and 96-16-42.75"—96-22'29" O (Figure 1). The elevation of the sampling areas ranges between 2031–3361 m. The annual mean temperature varies between 3–28 °C, with a mean annual rainfall of 3797 mm [23]. The predominant vegetation type corresponds to pine and oak species, mainly dominated by *Pinus oaxacana* Mirov, *Pinus douglasiana* Martínez, *Pinus patula* Schltdl, *Pinus pseudo-strobus* Lindl, *Pinus leiophylla* Schiede, *Pinus pringlei* Shaw, *Pinus rudis* Endl, *Pinus teocote* Schltdl, *Quercus crassifolia* Bonpl, *Quercus laurina* Bonpl, and *Quercus rugosa* Née.

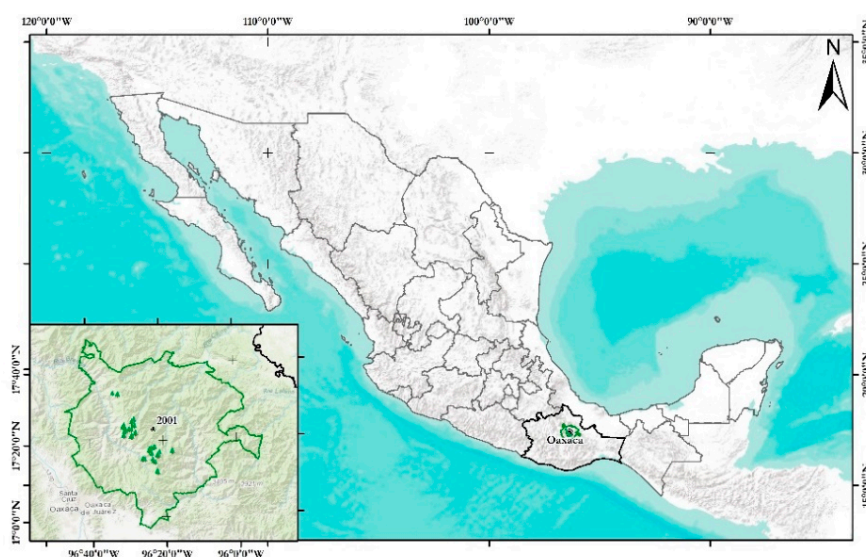


Figure 1. Location of the study area in Oaxaca, Mexico.

2.2. Data

The database considered 3173, 3218, 3660, and 4769 outside-bark diameter measurements at different heights obtained from 222, 230, 245, and 333 trees of *Pinus douglasiana* Martínez (*Pd*), *Pinus oaxacana* Mirov (*Po*), *Pinus patula* Schltd (*Pp*), and *Pinus pseudostrabus* Lindl (*Pps*), respectively. All trees were randomly selected in the study area. The diameter, height, site index, and crown class condition were considered. Diameter at breast height (1.3 m above ground level) (*dbh*; cm) was measured to the nearest 0.1 cm in each tree. The trees were later felled leaving stumps of average height 0.15 m, and total tree height (*H*; m) was measured to the nearest 0.01 m. The trees were cut into bolts as follows. The first three bolts were of the constant length of 0.3 m and the third log was of variable length because the upper diameter coincided with *dbh*. These measurements were considered to describe the tree profile in the lower section of each tree in which taper changes so rapidly low on the bole. Subsequent bolts were 2.54 m long. All tree branches, defined as those with a diameter at base greater than or equal to 5 cm, were also sectioned at variable lengths, those lengths ranged from 1.0 to 10 m. The minimum and maximum perpendicular outside-bark diameters were measured in each cross-section (of trunk or branch) to the nearest 0.1 cm and then the arithmetically average was registered. The axes for these measurements depended on the stem form. The volumes of the stem bolts and branches were calculated with the Smalian formula and the top with the cone equation. Table 1 shows the descriptive statistics including number of trees and observations (*n*), mean, minimum and maximum values, and standard deviation (SD) for the main tree variables of the studied tree species. Figure 2 shows the relative volume versus relative height for the species under study.

Table 1. Descriptive statistics of the studied pine tree species.

Species	Variable	Trees	<i>n</i>	Minimum	Mean	Maximum	SD
<i>Pd</i>	<i>dbh</i> (cm)	222	3173	8.95	39.02	74.00	13.46
	<i>H</i> (m)			7.40	25.11	38.98	6.24
	<i>d</i> (cm)			0.00	26.16	84.60	16.16
	<i>h</i> (m)			0.08	11.27	38.98	9.48
	V_m (m ³)			0.0000	0.9943	5.9168	1.0681
	V_s (m ³)			0.0360	1.6618	5.9168	1.1787
	V_b (m ³)			0.0005	0.0230	0.3267	0.0382
	V_t (m ³)			0.0402	1.6848	6.1381	1.2044
<i>Po</i>	<i>dbh</i> (cm)	230	3218	8.55	32.31	70.50	11.92
	<i>H</i> (m)			7.84	25.69	40.61	6.04
	<i>d</i> (cm)			0.00	21.64	77.25	13.72
	<i>h</i> (m)			0.03	11.08	40.61	9.39
	V_m (m ³)			0.0000	0.6536	5.7910	0.8069
	V_s (m ³)			0.0363	1.1058	5.7910	0.9693
	V_b (m ³)			0.0005	0.0135	0.1441	0.0199
	V_t (m ³)			0.0405	1.1192	5.9058	0.9837
<i>Pp</i>	<i>dbh</i> (cm)	245	3660	9.60	41.54	80.50	15.23
	<i>H</i> (m)			7.10	29.94	49.28	7.62
	<i>d</i> (cm)			0.00	28.14	88.50	16.91
	<i>h</i> (m)			0.05	13.21	49.28	11.04
	V_m (m ³)			0.0000	1.3366	8.9478	1.5338
	V_s (m ³)			0.0242	2.2816	8.9478	1.8388
	V_b (m ³)			0.0003	0.0252	0.1759	0.0301
	V_t (m ³)			0.0284	2.3068	9.0721	1.8628
<i>Pps</i>	<i>dbh</i> (cm)	333	4769	8.30	39.60	81.15	14.63
	<i>H</i> (m)			9.36	28.18	45.55	6.86
	<i>d</i> (cm)			0.00	27.09	91.50	16.68
	<i>h</i> (m)			0.07	12.28	45.55	10.39
	V_m (m ³)			0.0000	1.1356	9.2150	1.3631
	V_s (m ³)			0.0425	1.9719	9.2150	1.6818
	V_b (m ³)			0.0004	0.0267	0.3876	0.0390
	V_t (m ³)			0.0467	1.9986	9.3117	1.7080

n = number of observations; SD = standard deviation; *dbh* = diameter at breast height (cm); *H* = total tree height (m); *d* = outside-bark diameter at height *h* (cm); *h* = upper-height measure from the ground to the top tree height (m); V_m = merchantable outside-bark volume (m³); V_s = stem outside-bark volume (m³); V_b = branch volume (m³); V_t = total tree stem volume (m³).

2.3. Total Tree Volume Equation

In the forest literature, there are different volume equations to predict tree volume, including the equation of Schumacher and Hall [24], which has been widely used for different commercial species [1,3,25,26], and therefore it was selected to estimate the stem outside-bark volume (V_s) in this study (Equation (1)). To predict the branch volume (V_b), the power model was used with a scaled intercept parameter on the exponential function (Equation (2)). Equation (2) is a modification of Equation (1), but the parameter of height variable was not significantly different to zero at 5% of significance level.

$$V_s = \alpha_0 dbh^{\alpha_1} H^{\alpha_2} \quad (1)$$

$$V_b = e^{\gamma_0} dbh^{\gamma_1} \quad (2)$$

where V_s = stem outside-bark volume (m³); *dbh* = diameter at breast height (cm); V_b = branch volume (m³); *H* = total tree height (m); α_i and γ_i = parameters to be estimated through regression analysis.

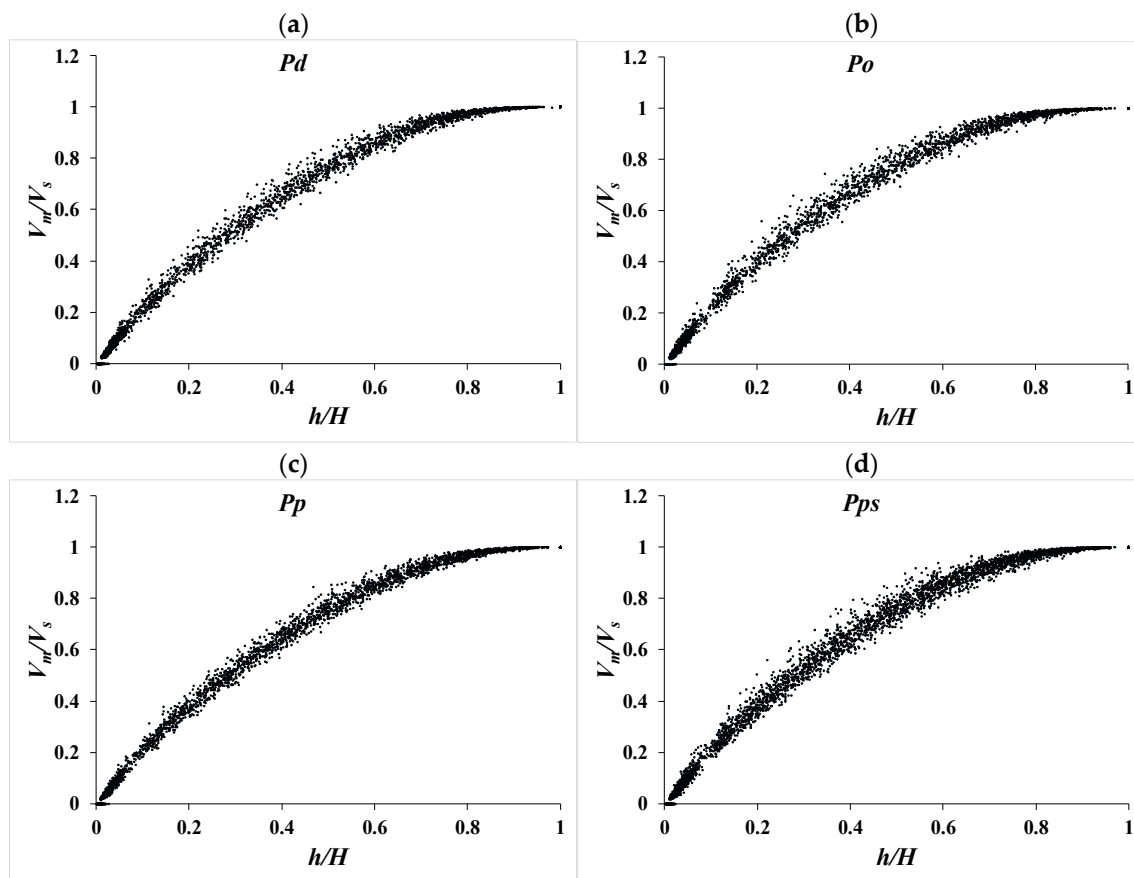


Figure 2. Relative volume (V_m/V_s) and relative height (h/H) for *Pinus douglasiana* (Pd; a); *Pinus oaxacana* (Po; b); *Pinus patula* (Pp; c); and *Pinus pseudostrobus* (Pps; d).

The parameters of V_s equation correspond to α_i ($i = 0, 1, 2$) were fully compatible with taper (d) and merchantable volume (V_m) equations. The total tree volume (V_t) was modeled as the sum of V_s and V_b components (Equation (3)).

$$V_t = \alpha_0 dbh^{\alpha_1} H^{\alpha_2} + e^{\gamma_0} dbh^{\gamma_1} \quad (3)$$

2.4. Compatible Taper and Volume Systems Derived from Ratio Functions Based on Relative Height

The ratio volume models that depend on the relative height were studied by Broad and Wake [7], Cao and Burkhart [27], Cao, Burkhart and Max [14], Parresol, et al. [28], Bullock and Burkhart [29], and most recently by Zhao and Kane [9], who used biomass and volume functions for the ratio of commercial height. Van Deusen et al. [30] and Reed and Green [15] derived taper models compatible with volume ratio functions based on total height. The function is $R(p) = \frac{V_m}{V_s}$, V_m (m^3) corresponds to the cumulative volume of the stem from the base to the height h (m) above the ground, where V_s (m^3) is the total stem volume and $p = \frac{h}{H}$ is the ratio between the merchantable (h) and total height (H). Zhao and Kane [9] and Lynch, Zhao, Harges, and McTague [10] proposed four conditions that ratio volume functions must meet are (I) $R = 0$, if $p = 0$, (II) $R = 1$, if $h = H$, (III) $\partial R / \partial h \geq 0$, for $0 \leq p \leq 1$ y (IV) $\partial^2 R / \partial^2 p \leq 0$, for $0 \leq p \leq 1$, also these properties were satisfied by Quiñonez-Barraza, Zhao, and De los Santos-Posadas [1] in compatible taper and volume equations for pine species in Durango, Mexico. The compatible taper and volume equations systems using in this study are shown in Table 2. In a preliminary analysis, these three systems performed better than others. These systems were arranged for d , and V_m components and were called CS1, CS2, and CS3 [9,10]. These systems are compatible with V_s , V_b , and V_t components in Equations (1)–(3).

Table 2. Compatible taper and merchantable volume systems based on volume ratio models fitted to studied species.

System	Equation
CS1	$V_m = \alpha_0 dbh^{\alpha_1} H^{\alpha_2} \frac{h}{H} \exp\left[\beta_0 \left(1 - \frac{h}{H}\right)\right] \varepsilon_{ij}; 0 < \beta_0 \leq 1, 0 \leq \frac{h}{H} \leq 1$ $d = \left[\frac{\alpha_0 dbh^{\alpha_1} H^{\alpha_2}}{Hk} \exp\left[\beta_0 \left(1 - \frac{h}{H}\right)\right] \left(1 - \beta_0 \frac{h}{H}\right) \right]^{0.5} + \varepsilon_{ij}$
CS2	$V_m = \alpha_0 dbh^{\alpha_1} H^{\alpha_2} \left[1 - \left(1 - \frac{h}{H}\right)^{\beta_0} \exp\left(-\beta_1 \frac{h}{H}\right) \right] \varepsilon_{ij}; \beta_0 \geq 1, \beta_1 > 0, 0 \leq \frac{h}{H} \leq 1$ $d = \left\{ \frac{\alpha_0 dbh^{\alpha_1} H^{\alpha_2}}{Hk} \exp\left(-\beta_1 \frac{h}{H}\right) \left(1 - \frac{h}{H}\right)^{\beta_0 - 1} \left[\beta_0 + \beta_1 \left(1 - \frac{h}{H}\right) \right] \right\}^{0.5} + \varepsilon_{ij}$
CS3	$V_m = \alpha_0 dbh^{\alpha_1} H^{\alpha_2} \left[1 - \left(1 - \frac{h}{H}\right)^{\beta_0} \right]^{(1 - \beta_1 \exp(-\exp(\beta_2 dbh^{\alpha_1} H^{\alpha_2})))} \varepsilon_{ij}; \beta_0 \geq 0, 0 \leq \frac{h}{H} \leq 1$ $d = \left[\frac{\alpha_0 dbh^{\alpha_1} H^{\alpha_2}}{Hk} \left(1 - \frac{h}{H}\right)^{\beta_0 - 1} \left(1 - \beta_1 \exp(-\exp(\beta_2 dbh^{\alpha_1} H^{\alpha_2}))\right) \left(1 - \left(1 - \frac{h}{H}\right)^{\alpha}\right)^{(-\beta_1 \exp(-\exp(\beta_2 dbh^{\alpha_1} H^{\alpha_2})))} \right]^{0.5} + \varepsilon_{ij}$

V_m = merchantable outside-bark volume from the base to the height h (m); dbh = diameter at breast height (cm); H = total tree height (m); h = upper height (m); d = upper diameter at height h ; $k = \pi/40,000$ for metric units; α_i, β_i = parameters to be estimated; ε_{ij} = error of j th measurement in the i th tree.

The used dummy variables for the full model were considered as additive effects to *Pinus pseudostrobus* (*Pps*) because this was the most abundant and representative tree species in the dataset. The dummy variables were expressed following the method studied by Quiñonez-Barraza et al. [31]; $I_j = \begin{cases} 1 & \text{if } sp = j \\ 0 & \text{otherwise} \end{cases}$, where I_j represents the dummy variable for species (sp); $j = 2$ for *Pd*, $j = 3$ for *Po*, and $j = 4$ for *Pp*. The global parameters were rewritten based on dummy variables, so that α_i and β_i could be represented as $\alpha_i = \alpha_{i1} + \alpha_{i2}I_2 + \alpha_{i3}I_3 + \alpha_{i4}I_4$ and $\beta_i = \beta_{i1} + \beta_{i2}I_2 + \beta_{i3}I_3 + \beta_{i4}I_4$. The full model with dummy variables only considered the significant parameters different from zero at significance level of 1% ($\alpha = 0.01$) in modeling process.

2.5. Model Fitting, Autocorrelation, Heteroscedasticity, and Equations Comparison

The components of the compatible taper and volume systems were simultaneously fitted using the iterative seemingly unrelated regression technique (ITSUR) [32] with MODEL procedure of the SAS/ETS® [33]. This technique generates an accurately estimated parameter when the error components are correlated in taper or merchantable volume equations for a specific system. The compatibility between the taper and volume equations allowed the estimated parameter to be the same for each CS and these guaranteed optimizations on fitting process by ITSUR technique [1,31,34,35]. To guarantee compatibility by degrees of freedom between the taper and the volume equations (V_s, V_b, V_t), we used the ratio of one over the number of sections of each tree (ni), which generated the weight variable ($w = 1/ni$) [11,31,36,37]. This procedure was incorporated into the SAS script as *resid.* (V_s, V_b, V_t) = *resid.* (V_s, V_b, V_t) \sqrt{w} for V_s, V_b and V_t , respectively. This process allowed to fit the stem volume, total tree volume, and branch volume equations with the same dataset of the taper outside-bark merchantable volume.

The main problems associated with taper and volume equations that do not satisfy the fundamental assumptions of regression analysis are autocorrelation and heteroscedasticity [38]. To correct the autocorrelation of the error term and obtain efficient model parameters [39–41], a second-order continuous autocorrelation structure was used (CAR2) [42–44].

$$e_{ij} = d_1 \rho_1^{h_{ij} - h_{ij-1}} e_{ij-1} + d_2 \rho_2^{h_{ij} - h_{ij-2}} e_{ij-2} + \varepsilon_{ij} \tag{4}$$

where e_{ij} is the j th ordinary residual on the i th individual tree; $d_1 = 1$ for $j > 1$; $d_2 = 1$ for $j > 2$; $d_1 = 0$ for $j = 1$; $d_2 = 0$ for $j \leq 2$; $h_{ij} - h_{ij-1}$ is the distance separating the j th from the j th-1 observations within each tree; $h_{ij} - h_{ij-2}$ is the distance separating the j th from the j th-2 observations within each tree; and ρ_1 and ρ_2 , are the first- and second-order autoregressive parameters.

To ensure homoscedasticity in the volume equations, we used a power function of the residual variance ($\sigma_i^2 = (dbh^2H)^\varnothing$), the value of \varnothing was estimated by the method proposed by Harvey [45], using the errors of the fitted equation as dependent variable. The parameters were estimated with the arrangement $resid.(V_m, V_s, V_b, V_t) = resid.(V_m, V_s, V_b, V_t) / [(dbh^2H)^\varnothing]^{0.5}$ for V_s , V_t , and V_b , respectively [1,46]. The fixed \varnothing 's parameter were 1.9423, 1.2813, 1.3214, and 1.8507 for Pd , Po , Pp , and Pps , respectively.

The evaluation and selection of the most efficient compatible system based on volume ratio equations was performed with the known fit statistics as the adjusted coefficient of determination (R^2_{adj}), the root mean square error (RMSE), the Akaike's information criterion (AIC), and the average error (E). In addition, graphic analyses of the residuals were examined to identify point for which the systems provide especially poor or good predictions [47]. Fit statistics are defined by the following expressions:

$$R^2_{adj} = 1 - \left[\frac{n - 1 \sum_{i=1}^n (y_i - y_i)^2}{n - p - 1 \sum_{i=1}^n (y_i - \bar{y})^2} \right] \quad (5)$$

$$RMSE = \left[\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n - 1} \right]^{0.5} \quad (6)$$

$$AIC = n \ln \left[\sum_{i=1}^n \frac{(y_i - \hat{y}_i)^2}{n} \right] + 2p \quad (7)$$

$$E = \left| \sum_{i=1}^n \frac{(y_i - \hat{y}_i)}{n} \right| \quad (8)$$

where y_i , \hat{y}_i , and \bar{y} are the observed, predicted, and average values of the dependent variable (d , V_m , V_s , V_b , and V_t); n is the number of observations; p is the number of parameters for each equation.

3. Results

3.1. Reduced Compatible Systems

Table 3 shows the estimated parameters and the fitting statistics of three reduced compatible taper and volume systems (RCSs) derived from height-based ratio equations analyzed in this study for all combined species. All parameters were significantly different to zero at 1% of the significance level. The systems considered using of a second-order continuous-time autoregressive error structure (CAR2) to correct the autocorrelation of the errors and a power function for heteroscedasticity correction in volume equations (V_m , V_s , V_b , V_t). Overall, the three compatible systems accounted for 97% of the variance of d , V_m , V_s , and V_t , and 50% of the variance of V_b . The RMSE values ranged between 1.75 and 2.29 cm for diameter, from 0.17 to 0.19 m³ for merchantable outside-bark volume, from 0.17 to 0.24 for V_s , from 0.0221 to 0.0222 for V_b , and lately from 0.253 to 0.254 for corresponding V_t . The RCS3 offered the best fit statistics in R^2_{adj} , RMSE, and E in the components d , V_m and V_s and slightly better in the components V_b and V_t .

Figure 3 illustrates the trend of residuals in box and whisker of d and V_m components for each reduce compatible system (RCS1, RCS2, RCS3) by relative height classes (h/H ; %). The RCS3 presented better accuracy than RCS1 and RCS2 in the different relative height categories (i.e., both d and V_m residuals are closer to 0 in comparison to RCS1 and RCS2). Thus, the RCS3 was proposed as the most suitable for describing the stem profile and predicting volume component of the four pine studied species.

Table 3. Parameter estimates and fit statistics for the reduced compatible systems analyzed.

System	Parameter	Estimator	SE	t-Value	Pr > t	Component	R ² _{adj}	RMSE	AIC	E
RCS1	α_0	0.000065	0.000001	64.83	<0.00001	<i>d</i>	0.980	2.2940	24617.8	0.259
	α_1	1.972095	0.004204	469.06	<0.00001	<i>V_m</i>	0.977	0.1919	-48929	-0.052
	α_2	0.848143	0.005625	150.77	<0.00001	<i>V_s</i>	0.972	0.2472	-2871.4	-0.005
	β_0	0.981334	0.001591	616.85	<0.00001	<i>V_b</i>	0.509	0.0222	-7831.3	0.001
	γ_0	-15.434798	0.112485	-137.22	<0.00001	<i>V_t</i>	0.971	0.2539	-2814.3	-0.005
	γ_1	3.087980	0.027454	112.48	<0.00001					
	ρ_1	0.888375	0.003789	234.45	<0.00001					
	ρ_2	0.315585	0.007199	43.84	<0.00001					
RCS2	α_0	0.000065	0.000001	64.38	<0.00001	<i>d</i>	0.980	2.2735	24352.8	0.185
	α_1	1.971378	0.004209	468.43	<0.00001	<i>V_m</i>	0.978	0.1902	-49190	-0.048
	α_2	0.849937	0.005626	151.07	<0.00001	<i>V_s</i>	0.972	0.2472	-2871.4	-0.005
	β_0	1.659596	0.017481	94.94	<0.00001	<i>V_b</i>	0.509	0.0222	-7831.3	0.001
	β_1	1.039716	0.021714	47.88	<0.00001	<i>V_t</i>	0.971	0.2539	-2814.2	-0.004
	γ_0	-15.432212	0.112549	-137.12	<0.00001					
	γ_1	3.087266	0.027468	112.39	<0.00001					
	ρ_1	0.889772	0.003760	236.65	<0.00001					
RCS3	α_0	0.000061	0.000001	64.54	<0.00001	<i>d</i>	0.988	1.7566	16707.8	0.019
	α_1	1.975390	0.004096	482.27	<0.00001	<i>V_m</i>	0.981	0.1749	-51673.1	-0.021
	α_2	0.859596	0.005357	160.47	<0.00001	<i>V_s</i>	0.972	0.2474	-2869.6	0.005
	β_0	2.016540	0.005936	339.70	<0.00001	<i>V_b</i>	0.509	0.0221	-7831.2	0.001
	β_2	0.380476	0.003884	97.95	<0.00001	<i>V_t</i>	0.971	0.2541	-2812.2	0.005
	β_3	0.000002	0.000000	15.19	<0.00001					
	γ_0	-15.529946	0.113751	-136.53	<0.00001					
	γ_1	3.109574	0.027748	112.07	<0.00001					
ρ_1	0.919566	0.004373	210.30	<0.00001						
ρ_2	0.819601	0.004317	189.84	<0.00001						

SE = the standard error of the parameter; t-value = the value of the student's t distribution; Pr > |t| = the probability value associated with student's t-distribution; R²_{adj} = adjusted coefficient of determination; RMSE = root mean square error, AIC = Akaike's criterion information, E = absolute mean error. *Noted that gamma parameters represent the branch equation (Equation (2)).

3.2. Full Compatible System

Table 4 shows the estimated parameters and fitting statistics for the full compatible system (FCS3) based on RCS3 for the five components *d*, *V_m*, *V_s*, *V_b*, and *V_t*, after expanding each parameter by including an associated parameter and a dummy variable to differentiate among species. Results of dummy variables indicate that there were differences among the compatible taper and volume systems from different pine species (i.e., all the associated parameters were different from zero ($p < 0.0001$)). For the taper equation, the RMSE was 1.74 cm, while for components *V_m*, *V_s*, *V_t*, and *V_b*, RMSE values were 0.168, 0.234, 0.24119 and 0.02178 m³, respectively. R²_{adj} values were greater than 97.4% for *d*, *V_m*, *V_s*, and *V_t* equations, and of 52.8% for the *V_b* equation. The full model indicated that estimated α_i 's are different for all species and just for estimated β_0 's and β_2 's some parameters are the same. Thus, *Pps* and *Po* share the same β_0 parameter, while, *Pps*, *Po*, and *Pd* share the same β_2 parameter.

Figure 4 shows the trend of residuals in box and whisker of *d* and *V_m* components by relative height (h/H) of the FCS3 system for the pine species studied. According to Figure 4, FCS3 described well the data for both outside-bark diameter along the stem and cumulative stem volume from the base to the height *h*.

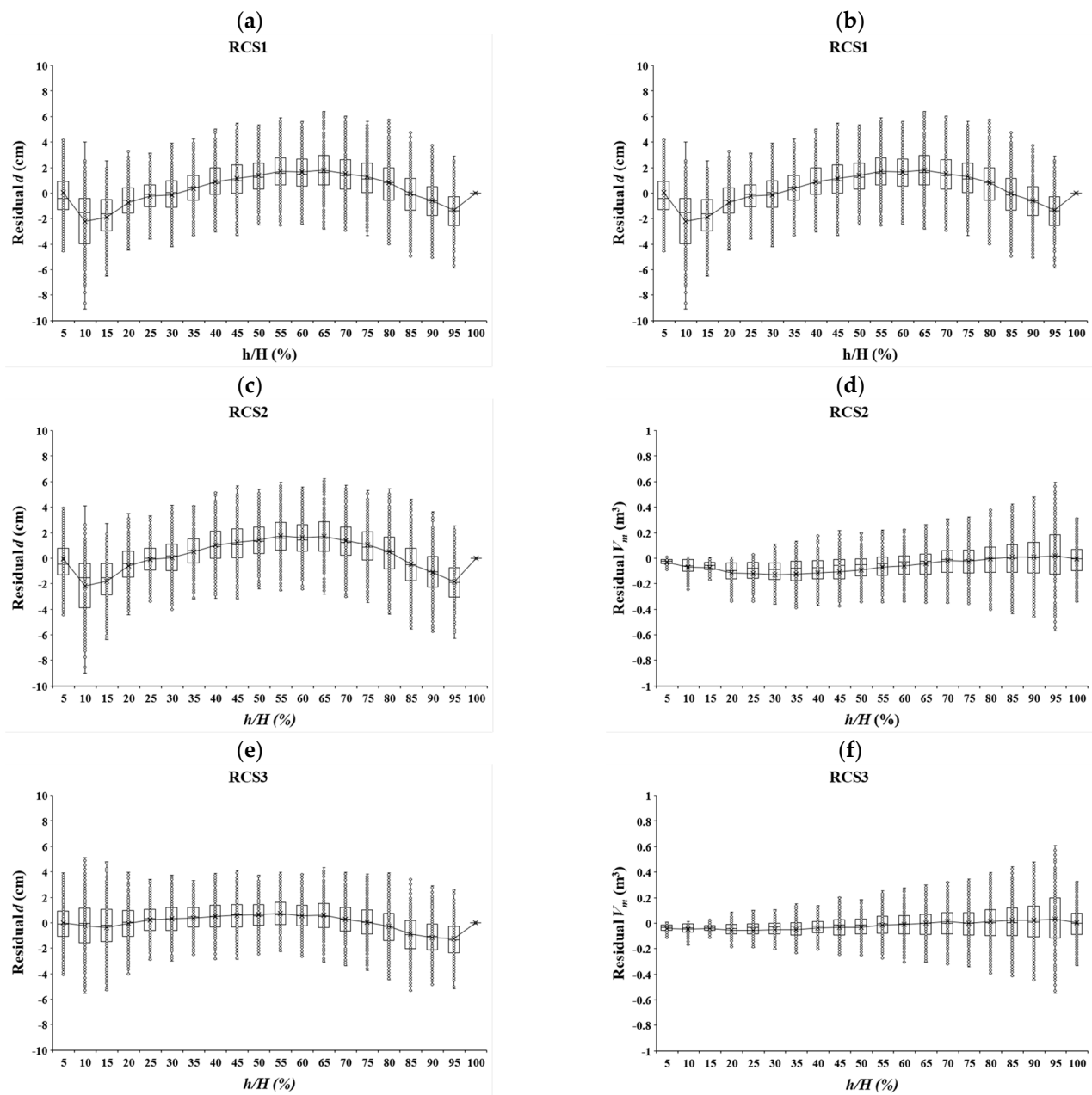


Figure 3. Distribution of residuals of d and V_m of the reduced compatible systems RCS1 (a,b), RCS2 (c,d), and RCS3 (e,f); “o” is the mean, “-” is the median, and “I” is the error bar representing the interquartile range (Q1–Q3).

Table 4. Estimated parameters and fit statistics of the simultaneous fitting of full compatible system (FCS3).

System	Parameter	Estimator	SE	t-Value	Pr > t
FCS3	$\alpha_0(Pps)$	0.000062	0.000002	36.30	<0.00001
	$\alpha_0(Pd)$	0.000045	0.000004	9.99	<0.00001
	$\alpha_0(Po)$	-0.000030	0.000002	-14.33	<0.00001
	$\alpha_0(Pp)$	0.000021	0.000003	7.56	<0.00001
	$\alpha_1(Pps)$	2.092634	0.006420	325.75	<0.00001
	$\alpha_1(Pd)$	-0.209980	0.011200	-18.82	<0.00001
	$\alpha_1(Po)$	-0.052660	0.012500	-4.20	<0.00001
	$\alpha_1(Pp)$	-0.273110	0.009830	-27.79	<0.00001
	$\alpha_2(Pps)$	0.732078	0.009500	77.05	<0.00001
	$\alpha_2(Pd)$	0.070796	0.016300	4.34	<0.00001
	$\alpha_2(Po)$	0.238758	0.017000	14.05	<0.00001
	$\alpha_2(Pp)$	0.217209	0.013600	15.97	<0.00001
	$\beta_0(Pps)$	2.007968	0.008380	239.74	<0.00001
	$\beta_0(Pd)$	0.136165	0.015500	8.80	<0.00001
	$\beta_0(Pp)$	-0.074190	0.013400	-5.53	<0.00001
	$\beta_1(Pps)$	0.406536	0.005880	69.15	<0.00001
	$\beta_1(Pd)$	-0.085270	0.008330	-10.24	<0.00001
	$\beta_1(Po)$	0.079601	0.008180	9.73	<0.00001
	$\beta_1(Pp)$	-0.075960	0.009430	-8.06	<0.00001
	$\beta_2(Pps)$	0.000003	0.000000	20.40	<0.00001
	$\beta_2(Pp)$	-0.000004	0.000000	-9.14	<0.00001
	$\rho_1(Pps)$	0.915668	0.004430	206.91	<0.00001
	$\rho_2(Pps)$	0.815828	0.004420	184.73	<0.00001
	$\gamma_0(Pps)$	-14.267400	0.163000	-87.54	<0.00001
	$\gamma_0(Pd)$	-7.146090	0.346600	-20.62	<0.00001
	$\gamma_0(Po)$	-1.522470	0.390500	-3.90	<0.00001
	$\gamma_0(Pp)$	-2.177400	0.296700	-7.34	<0.00001
	$\gamma_1(Pps)$	2.816073	0.039800	70.75	<0.00001
	$\gamma_1(Pd)$	1.768450	0.084100	21.02	<0.00001
	$\gamma_1(Po)$	0.364510	0.098400	3.70	0.0002
$\gamma_1(Pp)$	0.480006	0.071700	6.70	<0.00001	
Component		R^2_{adj}	RMSE	AIC	E
d		0.9885	1.74006	16428.178	0.039
V_m		0.9826	0.16808	-52848.07	-0.019
V_s		0.975	0.23406	-2983.556	0.007
V_b		0.5282	0.02178	-7871.256	0.001
V_t		0.9742	0.24119	-2919.868	0.008

SE = the standard error of the parameter, t-value = value of the student's t distribution, Pr > |t| = probability value associated with student's t -distribution; R^2_{adj} = adjusted coefficient of determination; RMSE = root mean square error, AIC = Akaike's criterion information, E = absolute mean error.

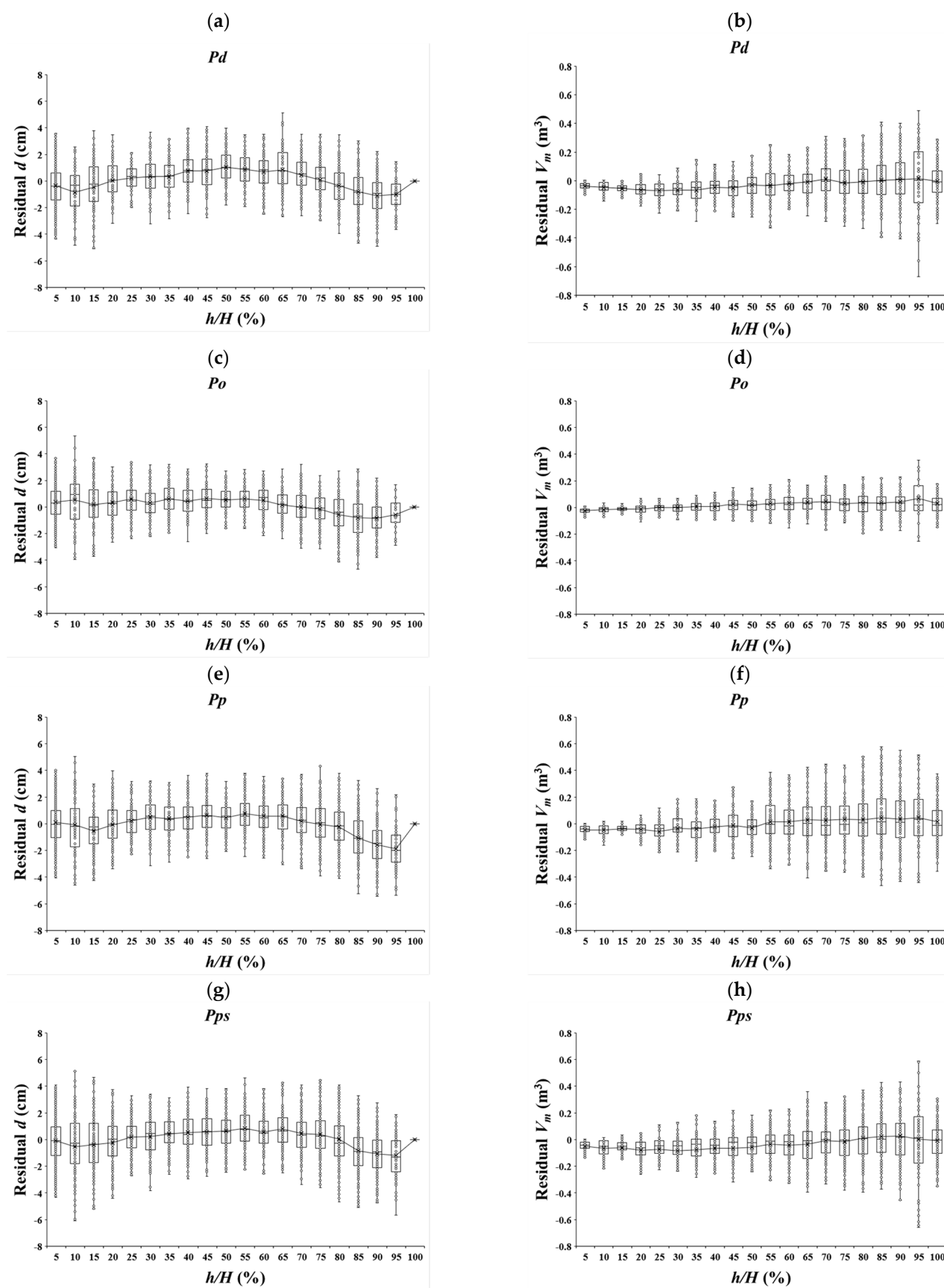


Figure 4. Distribution of the residuals of outside-bark d and V_m of FCS3 for Pd (a,b), Po (c,d), Pp (e,f), and Pps (g,h); “o” is the mean, “-” the median of the residuals, and “I” the error bar representing the interquartile range (Q1–Q3).

4. Discussion

4.1. Reduced Compatible Systems (RCS)

Three reduced compatible systems based on volume ratio equations evaluated in this study (RCS1, RCS2, and RCS3) allowed accuracy modeling of d , V_m , V_s , and V_t ($R^2_{adj} > 0.97$) for all combined data of Pd , Po , Pp , and Pps , although they were less accurate in estimating V_b ($R^2_{adj} = 0.50$) in terms of the fit statistics (Table 3). However, RCS3 performed slightly better than the other systems on fitting the data (i.e., it shows higher values of R^2_{adj} , lower values of RMSE, and E for the components d , V_m , and V_s and slightly higher for the components V_b and V_t). This compatible system based on ratio volume models was one of the top five for pine species in Durango and Michoacán Mexico [1], and [36], respectively. Moreover, this compatible system was reported as the best for slash pine in Georgia and north Florida, USA [11]. These findings supported the selection of this RCS and then to fit it as a full compatible taper and merchantable outside-bark volume system.

The use of a second-order continuous autocorrelation structure (CAR2) performed similarly for correcting the residual dependency in d for the three evaluated reduced equations systems. This result agrees with previous reports that have shown that a CAR2 allowed correction of residual dependency in compatible taper and volume systems developed for temperate and tropical tree species in Mexico (e.g., Vargas-Larreta, et al. [48]).

The distribution of residuals of studied compatible systems (Figure 3) showed for d , there are not important overestimations or underestimation in all height classes, while for V_m small overestimations were observed in the large classes (25 and 55%). However, accuracy was better for RCS3 in all relative height classes compared to RCS1 and RCS2, exhibiting only small overestimations in the height classes of 25–35%. Thus, the reduced compatible system based on volume ratio RCS3 was proposed as the most suitable to be tested as a full model with dummy variables to assess whether separate models are necessary for describing the stem profile and predicting stem volume of the four pine species analyzed in this study. Similar approaches have been used to assess if there are no differences in predicting taper and merchantable volume for different species [31,38,49]. Some differences were showed for loblolly pine in two different plantations of the southeastern USA and Oklahoma USA for the same compatible system [10], also for some pine species in Durango Mexico [1].

4.2. Full Compatible System (FCS)

Results of the simultaneous fitting of FCS3 as full model with dummy variables and additive effects to differentiate among the studied pine species indicated significant differences for all of them and highly accurate predictions of d , V_m , V_s , V_t components, but as for the reduced system, predictions were less accurate in the V_b equation. Consequently, stem taper differs among the four species analyzed. This result is consistent with other studies regarding the development of compatible taper and volume systems in different pine species that also reported differences in tree taper [31,38,49]. In these cases, the compatible segmented-stem system reported by Fang et al. [50] was used for accounting differences between stem profiles for different species of Durango Mexico. Recently Quiñonez-Barraza, Zhao, and De los Santos-Posadas [1] reported that the compatible taper and merchantable volume systems based on ratio volume models perform better than the segmented-stem system for most studied species. Consequently, the FCS3 in this study performed well for the four computed species.

The accuracy for predicting of volume branch component of FCS3 was lower ($R^2_{adj} = 0.50$) in comparison with the other components as commonly reported in the literature [37,51]. According to Ver Planck and MacFarlane [18], tree branches are difficult to model due to the variability of sizes and profile form present in the tree crowns, a situation that explains the relatively low values of fit statistics obtained in predicting this component. However, the estimation of branch volume by FCS3 may help to improve forest inventories by reducing error in whole-tree biomass, volume, or carbon estimation of these ecosystems [52].

The use of a CAR2 autocorrelation structure also corrected the residual dependency in d for FCS3. The distribution of residuals of FCS3 (Figure 4) showed for d there are no important overestimations or underestimation in all height classes and studied species, while for V_m small overestimations were observed in the height categories of 25–55% in *Pps*, and small underestimations in the height categories of 60–75% in *Pp*. Overestimations are probably caused by the fact that the observed “true” volumes of sectioned trees, calculated using Smalian’s formula, are slightly overestimated [53] and the overlapping bolts method of Bailey [54] should be better than the Smalian method but the percentage gain in fit statistics could be low. A future study could consider the overlapping bolts method for merchantable volume estimations.

The resulting developed full model is integrated by equations that are based on volume ratio functions and they generated consistent results and showed accurate estimations of V_m for upper-diameters and -heights and without implementing numerical methods or integration and this is an important element in a parsimonious completely compatible taper, total and merchantable volume [55], also in this case for branch volume and total tree volume. Moreover, its use allowed accurate estimates of taper, merchantable outside-bark volume, stem outside-bark volume, branch volume, and total tree outside-bark volume for *Pd*, *Po*, *Pp*, and *Pps*.

5. Conclusions

In the present study, accurate completely compatible taper outside-bark volume systems based on ratio volume functions were developed and tested. These compatible systems included predicting for taper, merchantable outside-bark volume, stem outside-bark volume, branch volume, and total tree volume for *Pd*, *Po*, *Pp*, and *Pps*. The statistical and graphical analysis showed that the compatible taper system RCS3 performed better than RCS1 and RCS2 systems, and thus RCS3 was recommended to be tested as a full system (FCS3) with dummy variables to assess whether separate models are necessary for describing the stem profile of the four pine analyzed species in this study. The simultaneous fitting of FCS3 indicated that the stem taper differs among the four species analyzed and, therefore, separate models are needed; therefore, full model is suitable for the prediction of studied components. The resulting compatible taper and outside-bark volume equations are simple reliable tools for direct estimation of timber stocks and for classification of merchantable products in mixed-forests and uneven-aged forests in the northeast of Oaxaca state, Mexico. These completely compatible taper and volume models could be used to predict merchantable outside-bark volume to different upper-diameter and -heights and total tree volume for studied species and are a good biometric tool for informed decision-making in forest management and planning.

Author Contributions: Data analysis, model fitting, manuscript writing, A.C.-L., G.Q.-B., U.D.-A., and J.J.C.-R.; manuscript review and statistical support, A.C.-L., G.Q.-B., U.D.-A., and J.J.C.-R. All authors have read and agreed to the published version of the manuscript.

Funding: This study was partially financed by the CONAFOR—CONACYT funding for the project called Biometric system for the planning of sustainable forest management of ecosystems with timber potential in Mexico.

Data Availability Statement: Data sharing is not applicable to this article.

Acknowledgments: The main author would like to thank the Mexican Council of Science and Technology (CONACYT) for sponsoring his doctoral studies at Universidad Juárez del Estado de Durango.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Quiñonez-Barraza, G.; Zhao, D.; De los Santos-Posadas, H.M. Compatible Taper and Stem Volume Equations for Five Pine Species in Mixed-Species Forests in Mexico. *For. Sci.* **2019**, *65*, 602–613. [[CrossRef](#)]
2. Hocker, H.W. *Introducción a La Biología Forestal*; AGT: Mexico DF, Mexico, 1984; 446p.

3. Hernández Pérez, D.; de los Santos Posadas, H.M.; Ángeles Pérez, G.; Valdez Lazalde, J.R.; Haller, V.; Hugo, V. Taper and Commercial volume system for *Pinus patula* Schltdl. et Cham. in Zacualtipán, Hidalgo State. *Rev. Mex. De Cienc. For.* **2013**, *4*, 34–45.
4. Uranga-Valencia, L.P.; De los Santos-Posadas, H.M.; Valdez-Lazalde, J.R.; López-Upton, J.; Navarro-Garza, H. Total volume and taper for *Pinus patula* Schiede ex Schltdl. et Cham. at three forest conditions. *Agrociencia* **2015**, *49*, 787–801.
5. Ramírez-Martínez, A.; Santiago-García, W.; Quiñonez-Barraza, G.; Ruiz-Aquino, F.; Antúnez, P. Taper and total volume model for *Pinus ayacahuite* Ehren. *Madera Y Bosques* **2018**, *24*, 24. [[CrossRef](#)]
6. Burkhart, H.E.; Tomé, M. *Modeling Forest Trees and Stands*; Springer Science & Business Media: New York, NY, USA, 2012; 497p.
7. Broad, L.R.; Wake, G.C. Derivative Based Methods for Constructing Volume-Ratio and Taper Equations. *For. Sci.* **1995**, *41*, 157–167. [[CrossRef](#)]
8. Herrero, C.; Lizarralde, I.; Bravo, F. Ecuaciones de perfil de fuste para *Pinus sylvestris* y *Quercus pyrenaica* en los páramos ácidos de Castilla y León. In Proceedings of the IV Congreso Forestal Español, Zaragoza, Spain, 1 June 2005.
9. Zhao, D.; Kane, M. New Variable-Top Merchantable Volume and Weight Equations Derived Directly from Cumulative Relative Profiles for Loblolly Pine. *For. Sci.* **2017**, *63*, 261–269. [[CrossRef](#)]
10. Lynch, T.B.; Zhao, D.; Harges, W.; McTague, J.P. Deriving compatible taper functions from volume ratio equations based on upper-stem height. *Can. J. For. Res.* **2017**, *47*, 1424–1431. [[CrossRef](#)]
11. Zhao, D.; Lynch, T.B.; Westfall, J.; Coulston, J.; Kane, M.; Adams, D.E. Compatibility, Development, and Estimation of Taper and Volume Equation Systems. *For. Sci.* **2018**, *65*, 1–13. [[CrossRef](#)]
12. Burkhart, H.E. Cubic-foot volume of loblolly pine to any merchantable top limit. *South. J. Appl. For.* **1977**, *1*, 7–9. [[CrossRef](#)]
13. Clutter, J.L.J.F.S. Development of taper functions from variable-top merchantable volume equations. *For. Sci.* **1980**, *26*, 117–120. [[CrossRef](#)]
14. Cao, Q.V.; Burkhart, H.E.; Max, T.A.J.F.S. Evaluation of two methods for cubic-volume prediction of loblolly pine to any merchantable limit. *For. Sci.* **1980**, *26*, 71–80. [[CrossRef](#)]
15. Reed, D.D.; Green, E.J.J.F.S. Compatible stem taper and volume ratio equations. *For. Sci.* **1984**, *30*, 977–990. [[CrossRef](#)]
16. Dorado, F.C.; Gonzalez, J.A.J.F.S. Construcción de una tarifa de cubicación con clasificación de productos para *Pinus radiata* D. Don en Galicia basada en una función de perfil del tronco. *For. Syst.* **2000**, *9*, 253–268.
17. MacFarlane, D.W. Predicting branch to bole volume scaling relationships from varying centroids of tree bole volume. *Can. J. For. Res.* **2010**, *40*, 2278–2289. [[CrossRef](#)]
18. Ver Planck, N.R.; MacFarlane, D.W. Modelling vertical allocation of tree stem and branch volume for hardwoods. *For. Int. J. For. Res.* **2014**, *87*, 459–469. [[CrossRef](#)]
19. Zakrzewski, W.T. Estimating Wood Volume of the Stem and Branches of Sugar Maple (*Acer saccharum* Marsh.) Using a Stem Profile Model with Implicit Height. *For. Sci.* **2011**, *57*, 117–133. [[CrossRef](#)]
20. Gasol, C.M.; Martínez, S.; Rigola, M.; Rieradevall, J.; Anton, A.; Carrasco, J.; Ciria, P.; Gabarrell, X. Feasibility assessment of poplar bioenergy systems in the Southern Europe. *Renew. Sustain. Energy Rev.* **2008**, *13*, 801–812. [[CrossRef](#)]
21. Tavankar, F.; Eynollahi, Y. Amount and characteristics of logging residues in selection cutting stand in the Northern forests of Iran. *Int. J. Biosci.* **2013**, *3*, 35–42.
22. Merino, A.; Balboa, M.; Soalleiro, R.R.; Gonzalez, J.A. Nutrient exports under different harvesting regimes in fast-growing forest plantations in southern Europe. *For. Ecol. Manag.* **2005**, *207*, 325–339. [[CrossRef](#)]
23. García, E.J.C. *Clasificación climática según Köppen modificada por Enriqueta García*; Universidad Nacional Autónoma de México: Mexico City, México, 2004; 97p.
24. Schumacher, F.X.; Hall, F.D.S. Logarithmic expression of timber-tree volume. *J. Agric. Res.* **1933**, *47*, 719–734.
25. Sun, Y.; Liang, X.; Liang, Z.; Welham, C.; Li, W. Deriving Merchantable Volume in Poplar through a Localized Tapering Function from Non-Destructive Terrestrial Laser Scanning. *Forests* **2016**, *7*, 87. [[CrossRef](#)]
26. Ramos-Uvilla, J.A.; García-Magaña, J.J.; Hernández-Ramos, J.; García-Cuevas, X.; Velarde-Ramírez, J.C.; Muñoz-Flores, H.J.; García Espinoza, G.G. Ecuaciones y tablas de volumen para dos especies de *Pinus* de la Sierra Purhépecha, Michoacán. *Rev. Mex. De Cienc. For.* **2014**, *5*, 92–109.
27. Cao, Q.v.; Burkhart, H.E. Cubic-foot volume of loblolly pine to any height limit. *South. J. Appl. For.* **1980**, *4*, 166–168. [[CrossRef](#)]
28. Parresol, B.R.; Hotvedt, J.E.; Cao, Q.V. A volume and taper prediction system for bald cypress. *Can. J. For. Res.* **1987**, *17*, 250–259. [[CrossRef](#)]
29. Bullock, B.P.; Burkhart, H.E. Equations for Predicting Green Weight of Loblolly Pine Trees in the South. *South. J. Appl. For.* **2003**, *27*, 153–159. [[CrossRef](#)]
30. Van Deusen, P.C.; Matney, T.G.; Sullivan, A.D. A Compatible System for Predicting the Volume and Diameter of Sweetgum Trees to any Height. *South. J. Appl. For.* **1982**, *6*, 159–163. [[CrossRef](#)]
31. Quiñonez-Barraza, G.; De los Santos-Posadas, H.M.; Álvarez-González, J.G.; Velázquez-Martínez, A. Compatible taper and merchantable volume system for major pine species in Durango, Mexico. *Agrociencia* **2014**, *48*, 553–567.
32. Zellner, A. An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias. *J. Am. Stat. Assoc.* **1962**, *57*, 348–368. [[CrossRef](#)]
33. SAS Institute. *SAS/GRAPH 9.1 Reference*; SAS Institute: Cary, NC, USA, 2004.

34. Borders, B.E.; Patterson, W.D. Projecting Stand Tables: A Comparison of the Weibull Diameter Distribution Method, a Percentile-Based Projection Method, and a Basal Area Growth Projection Method. *For. Sci.* **1990**, *36*, 413–424. [[CrossRef](#)]
35. Santiago-García, W.; Pérez-López, E.; Quiñonez-Barraza, G.; Rodríguez-Ortiz, G.; Santiago-García, E.; Ruiz-Aquino, F.; Tamarit-Urias, J. A Dynamic System of Growth and Yield Equations for *Pinus patula*. *Forests* **2017**, *8*, 465. [[CrossRef](#)]
36. García-Espinoza, G.; Aguirre-Calderón, O.; Quiñonez-Barraza, G.; Alanís-Rodríguez, E.; De los Santos-Posadas, H.; García-Magaña, J. Taper and Volume Systems Based on Ratio Equations for *Pinus pseudostrobus* Lindl. in Mexico. *Forests* **2018**, *9*, 344. [[CrossRef](#)]
37. Quiñonez-Barraza, G.; Zhao, D.; de los Santos-Posadas, H.M.; Santiago-García, W.; Tamarit-Urias, J.C.; Nájera-Luna, J.A. Compatible taper, volume, green weight, biomass and carbon concentrations system for *Quercus sideroxylla* Bonpl. *Rev. Chapingo Ser. Hortic.* **2018**, *XXV*, 49–69. [[CrossRef](#)]
38. Corral-Rivas, J.J.; Diéguez-Aranda, U.; Corral Rivas, S.; Castedo Dorado, F. A merchantable volume system for major pine species in El Salto, Durango (Mexico). *For. Ecol. Manag.* **2007**, *238*, 118–129. [[CrossRef](#)]
39. Parresol, B.R.; Vissage, J.S. *White Pine Site Index for the Southern Forest Survey*; U.S. Department of Agriculture, Forest Service, Southern Research Station: Asheville, NC, USA, 1998. [[CrossRef](#)]
40. Parresol, B.R. Modeling Multiplicative Error Variance: An Example Predicting Tree Diameter from Stump Dimensions in Baldcypress. *For. Sci.* **1993**, *39*, 670–679. [[CrossRef](#)]
41. Castillo-López, A.; Santiago-García, W.; Vargas-Larreta, B.; Quiñonez-Barraza, G.; Solis-Moreno, R.; Corral Rivas, J.J. Dynamic site index models for four pine species in Oaxaca. *Rev. Mex. De Cienc. For.* **2018**, *9*, 9. [[CrossRef](#)]
42. Zimmerman, D.L.; Núñez-Antón, V.; Gregoire, T.G.; Schabenberger, O.; Hart, J.D.; Kenward, M.G.; Molenberghs, G.; Verbeke, G.; Pourahmadi, M.; Vieu, P.; et al. Parametric modelling of growth curve data: An overview. *Test* **2001**, *10*, 1–73. [[CrossRef](#)]
43. Nord-Larsen, T.; Cao, Q.V. A diameter distribution model for even-aged beech in Denmark. *For. Ecol. Manag.* **2006**, *231*, 218–225. [[CrossRef](#)]
44. Crecente-Campo, F.; Rojo Alboreca, A.; Diéguez-Aranda, U.J.A.F.S. A merchantable volume system for *Pinus sylvestris* L. in the major mountain ranges of Spain. *Ann. Sci.* **2009**, *66*, 808. [[CrossRef](#)]
45. Harvey, A.C. Estimating Regression Models with Multiplicative Heteroscedasticity. *Econometrica* **1976**, *44*, 461–465. [[CrossRef](#)]
46. Tang, X.; Pérez-Cruzado, C.; Fehrmann, L.; Álvarez-González, J.G.; Lu, Y.; Kleinn, C. Development of a compatible taper function and stand-level merchantable volume model for Chinese fir plantations. *PLoS ONE* **2016**, *11*, e0147610. [[CrossRef](#)] [[PubMed](#)]
47. Goelz, J.; Burk, T. Development of a well-behaved site index equation: Jack pine in north central Ontario. *Can. J. For. Res.* **1992**, *22*, 776–784. [[CrossRef](#)]
48. Vargas-Larreta, B.; Corral-Rivas, J.J.; Aguirre-Calderón, O.A.; López-Martínez, J.O.; De los Santos-Posadas, H.M.; Zamudio-Sánchez, F.J.; Treviño-Garza, E.J.; Martínez-Salvador, M.; Aguirre-Calderón, C.G. SiBiFor: Sistema Biométrico Forestal para el manejo de los bosques de México. *Rev. Chapingo Ser. Hortic.* **2017**, *XXIII*, 437–455. [[CrossRef](#)]
49. López-Martínez, J.O.; Vargas-Larreta, B.; Aguirre-Calderón, O.A.; Aguirre-Calderón, C.G.; Macario-Mendoza, P.A.; Martínez-Salvador, M.; Álvarez-González, J.G. Compatible taper-volume systems for major tropical species in Mexico. *For. Int. J. For. Res.* **2019**, *93*, 56–74. [[CrossRef](#)]
50. Fang, Z.; Borders, B.E.; Bailey, R.L. Compatible Volume-Taper Models for Loblolly and Slash Pine Based on a System with Segmented-Stem Form Factors. *For. Sci.* **2000**, *46*, 1–12. [[CrossRef](#)]
51. Simental-Cano, B.; López-Sánchez, C.A.; Wehenkel, C.; Vargas-Larreta, B.; Álvarez-González, J.G.; Corral-Rivas, J.J. Species-specific and regional volume models for 12 forest species in Durango, Mexico. *Rev. Chapingo. Ser. Cienc. For. Y Del Ambiente* **2017**, *23*, 155–171. [[CrossRef](#)]
52. Valenzuela, C.; Acuña, E.; Ortega, A.; Quiñonez-Barraza, G.; Corral-Rivas, J.; Cancino, J. Variable-top stem biomass equations at tree-level generated by a simultaneous density-integral system for second growth forests of roble, raulí, and coigüe in Chile. *J. For. Res.* **2019**, *30*, 993–1010. [[CrossRef](#)]
53. Kozak, A. A variable-exponent taper equation. *Can. J. For. Res.* **1988**, *18*, 1363–1368. [[CrossRef](#)]
54. Bailey, R.L. Upper stem volumes from stem analysis data: An overlapping bolts method. *Can. J. For. Res.* **1995**, *25*, 170–173. [[CrossRef](#)]
55. Jordan, L.; Berenhaut, K.; Souter, R.; Daniels, R.F. Parsimonious and Completely Compatible Taper, Total, and Merchantable Volume Models. *For. Sci.* **2005**, *51*, 578–584. [[CrossRef](#)]