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Nonlinear Quantile Mixed-Effects Models for Prediction of the Maximum Crown Width of *Fagus sylvatica* L., *Pinus nigra* Arn. and *Pinus brutia* Ten.

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Abstract: In the current study, a novel approach combining quantile regression with nonlinear mixed-effects (*QR-NLME*) modeling was applied to predict the maximum crown width (cw_{max}) of three economically important forest species—the European beech (*Fagus sylvatica* L.), the black pine (*Pinus nigra* Arn.), and the Calabrian pine (*Pinus brutia* Ten.) at tree level. A power *QR-NLME* model was fitted first to a dataset including 1414 European beech trees obtained from 29 randomly distributed sample plots, 770 black pine trees from 25 sample plots, and 1880 Calabrian pine trees from 41 sample plots in Greece, to predict the cw_{max} at tree level. Additionally, a nonlinear mixed-effects model (*NLME*) was fitted to the same dataset to predict the average crown width at tree level for all species. In the second stage, the crown competition factor (*CCF*) was estimated based on the population average response of the cw_{max} predictions. The proposed approach presented sound results when compared with the outcomes of relevant models from other regions fitted to open-grown tree data, and therefore, it can be well implemented on clustered data structures, in cases of absence of open-grown tree data.

Keywords: stochastic approximation of the expectation-maximization algorithm; crown competition factor; stand density; forest management

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Citation: Raptis, D.I.; Kazana, V.; Kechagioglou, S.; Kazaklis, A.; Stamatiou, C.; Papadopoulou, D.; Tsitsoni, T. Nonlinear Quantile Mixed-Effects Models for Prediction of the Maximum Crown Width of *Fagus sylvatica* L., *Pinus nigra* Arn. and *Pinus brutia* Ten. *Forests* **2022**, *13*, 499. https://doi.org/10.3390/f13040499

Academic Editor: Karol Bronisz

Received: 14 February 2022
Accepted: 21 March 2022
Published: 23 March 2022

1. Introduction

Estimation of the basic crown dimensions at the tree or stand level is of high importance in forest management science [1]. This is mainly due to the fact that the tree crown is closely related to the total tree growth [2], the competition level [3], and the basic attributes at the stand level, which further affect the stand structure and composition, including the understory vegetation. Assuming that a tree's growing space (*GS*) is approximately circular-shaped [4], the prediction of the *GS* of the tree is based on the crown width dimensions. However, the trees growing under competition status are expected to develop shorter crown widths in comparison to those growing as open trees. In order to capture the tree crown's biological tendency, open-grown trees free of competition have been typically used in crown width modeling, leading to the establishment of the maximum crown width (cw_{max}) dimension. In this context, cw_{max} has been defined as the theoretical maximum crown diameter of an *i*th tree, with a specific DBH_i, growing in stand conditions, which would be observed had the specific tree been grown as an open-grown tree free of competition. According to Krajicek et al. [3], the mathematical expression of the cw_{max} may lead to the evaluation of the competition at stand level, through the use of the crown competition factor (*CCF*) concept.

CCF has been advocated as an objective competition measure in terms of evaluating the optimal stocking density and the consequent application of thinning strategies [5]. When trees within a stand can no longer achieve the maximum crown area without being limited by the neighboring trees, *CCF* values exceed 100%, and the competition level increases [6]. For *CCF* values higher than 100%, trees have less crown area available for maximum tree growth, but stand growth is still maintained at high levels due to the development of efficient foliage on the top of the remaining trees on-site [7]. In this sense, a *CCF* value equal to 100% implies that the tree at the stand has the maximum area available as if it would have been grown in an open-spaced area with no competition [8]. As a consequence, silvicultural treatments may affect significantly the *CCF* value levels within a stand. The removal of a small number of large-diameter trees reduces the competition level to the same degree as would the removal of many small-diameter trees [9].

From a practical perspective, the importance of the *CCF* determination is not limited only to the applied silvicultural treatments and, consequently, to the forest management planning. Due to its mathematical expression, it has been used as a significant regressor in forest growth modeling, in an effort to capture the competition status within a forest stand (e.g., [10–12]). In addition, based on the fact that the *CCF* is a distance-independent competition measure, there is no need for prior estimations of within-tree distances, and therefore, it can be easily estimated as long as the cw_{max} value is provided, which is clearly an advantage in modeling processes.

However, several problems have been reported with regard to the cw_{max} determination. For example, the cw_{max} value for a well-defined tree is not always available, whereas the cw_{max} modeling process is often a relatively complex task, particularly due to field difficulties related to the extensive traveling required in order to identify trees that meet the definition of an "open-grown tree" [13]. Furthermore, the definition of the typical "open-grown tree" is not quite clear, and this may introduce certain subjectivity [1]. Alternative modeling approaches for the prediction of the cw_{max} of certain forest species have been already attempted. Condés and Sterba [14] used a large dataset from the second Spanish National Forest Inventory to propose a modeling method for cw_{max} by defining "open-grown" trees as those whose residuals fell within both the upper 90th percentile of a general, double-logarithmic model developed to predict the crown width and the tree height based on the *DBH*, and the lower 10th percentile of the log of total height.

The majority of the applied methods for cw_{max} model development have used the *DBH* as the basic independent variable. In cases in which isolated open-grown trees have served as primary sampling units, the modeling process involved mainly the ordinary least squares (*OLS*) approaches usually presenting relatively reliable results. However, in most cases, the field data for forest models are obtained from sample plot installations of various sizes and shapes, and it is now well established that for multiple measurements from individual sampling units, the *OLS* method is inappropriate. This is because it leads to underestimates of the covariance matrix of the parameter estimates and the residual variance of the *OLS* equation [15,16]. While a number of methods have been evaluated as alternatives to solve this problem, the mixed-effects modeling approach is advocated as one of the most advanced tools for the analysis of clustered data [17]. Therefore, the correlation between measurements should be identified even in the case of using alternatives methods for analyses, such as quantile regression (*QR*).

Based on the importance of the cw_{max} determination for forest management planning and silvicultural treatment purposes at the stand level, the aims of the current study were (i) the development of cw_{max} prediction models for three economically important species—the European beech, the black pine, and the Calabrian pine—by applying a novel quantile nonlinear mixed-effects modeling approach; (ii) the estimation of the corresponding *CCF* values at the species level; (iii) the comparison of the current study predictions with the outcomes of relevant models fitted to open-grown trees of other regions, in order to evaluate the results at species biological tendency level.

2. Materials and Methods

2.1. Data Sampling

The raw field data were obtained from 29 sample plots installed in the pure European beech forests of Rodopi Mountain-Range National Park (RMRNP), located in the Region of Eastern Macedonia and Thrace, in northern Greece, between 24°10′ E and 24°50′ E, 41°20′ N and 41°32′ N (Figure 1). The forests of this region account for 34% of the total growing stock of Greece's forest area [18]. The sample plots were randomly distributed using the "create random points" command of the ArcGis 10.2 (ESRI, Redlands, CA, USA) environment by using as base a well-defined shapefile of species expansion. Each circular-shaped sample plot covered a total area of 1000 m^2 (0.1 ha), and it was installed in the field using a fixed radius of 17.8 m at vertical projection. For all living tress the total height (*h*), the diameter at breast height (*DBH*), the crown base height (*cbh*), and the crown width (*cw*) were measured with a LaserAce 1000 Rangefinder (Trimble) instrument. The crown width was measured in two vertical directions using the Canopy Spread module of the same instrument. The total sample included 1414 individual European beech trees over a wide range of density and competition status.

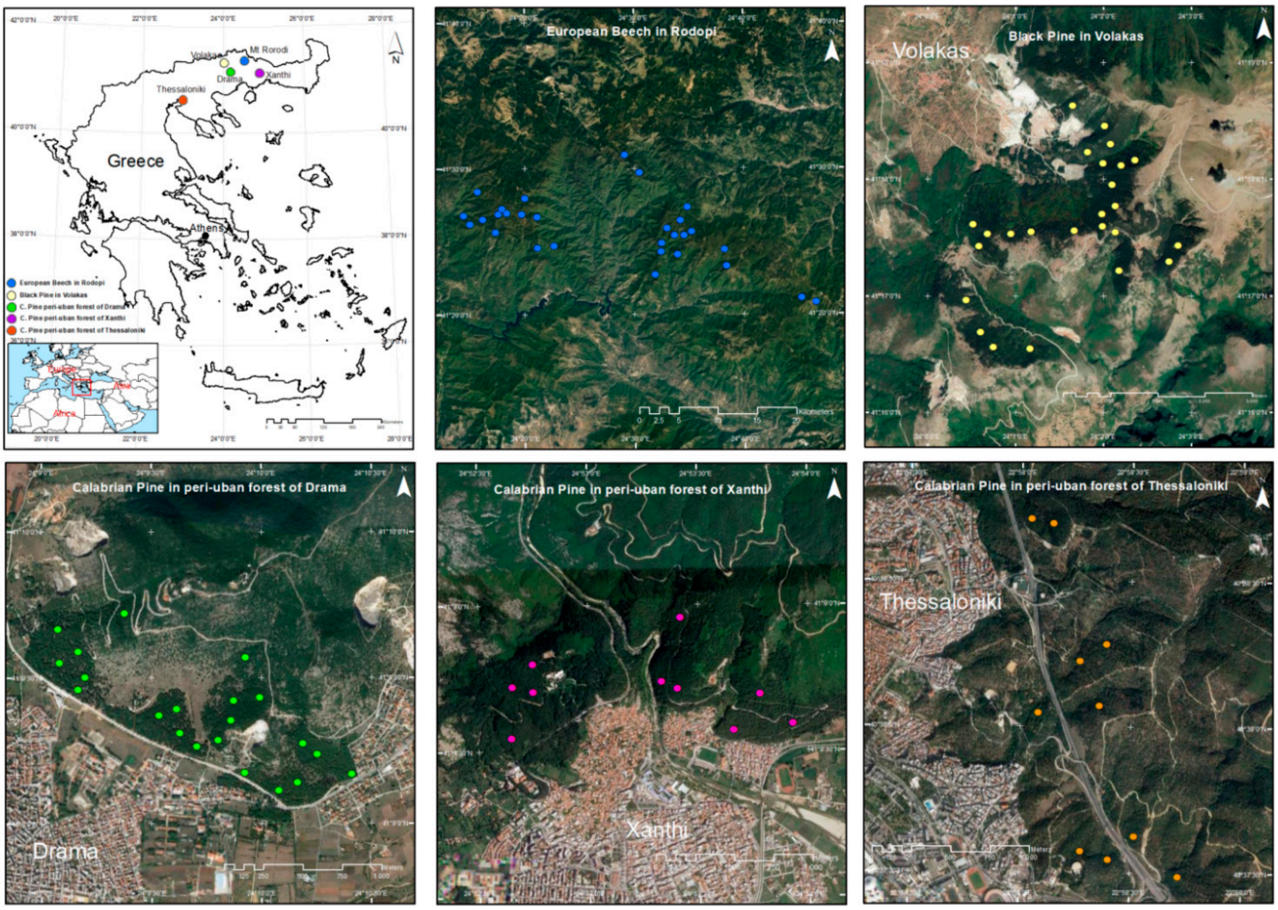


Figure 1. Distribution of the sample plots in the study areas.

The second dataset was composed of 770 trees obtained from 25 sample plots located in the pure black pine natural forest stands of the Volakas Drama area, in northern Greece, between 24°00′ E and 24°03′ E, 41°16′ N and 41°19′ N (Figure 1). For the field measurements, the same methodology as above was followed, but the installed sample plots were half the size, covering 500 m^2 (0.05 ha) each and thus corresponding to a 12.6 m fixed radius per plot at vertical projection.

The third dataset was composed of 1880 Calabrian pine trees obtained from a total of 41 randomly distributed sample plots in northern Greece, of which 21 and 10 plots were located, respectively, in the peri-urban forests of Drama (between 24°09′ E and 24°11′ E, 41°09′ N and 41°10′ N) and Xanthi (24°52′ E and 24°54′ E, 41°08′ N and 41°09′ N) cities in the Region of Eastern Macedonia and Thrace and 10 plots in the peri-urban forest of Thessaloniki city in the Region of Central Macedonia, extending between 22°58′ E and 22°59′ E, 40°37′ N and 40°39′ N (Figure 1). The randomly distributed sample plots were circular-shaped of 1000 m^2 (0.1 ha), each with a fixed radius of 17.8 m at vertical projection. Within these established sample plots, the same variables as those described previously for the other 2 datasets were measured at tree level, using similar equipment.

The European beech stands in the study area are managed for wood production. Thinnings in the form of selective cuttings are applied periodically between 20 and 30 years according to the approval by the Forest Service management plan. This silvicultural system, as part of the sustainable forest management concept, aims primarily at soil protection by ensuring continuous forest cover over time. The same system is applied in the natural black pine forests growing in the wider area. In the case of the Calabrian pine peri-urban forests, a different management system is followed aiming primarily for fire protection and recreation activity. This system is based on different combinations of undergrowth clearings, pruning of dead crown lower parts, and some thinnings at specific points (on both sides of roads, paths, ridges) in order to prevent fire from crowning.

2.2. Analysis

The analysis was partly based on the approach of Russell and Weiskittel [1], using a *QR* technique, which allows the estimation of the response variable, that is, the crown width, for any percentile of the data [19]. According to the same authors, large datasets consist of various crown widths corresponding to both, open- and forest-grown trees. In order to estimate the maximum potential crown width for a given species at a specific *DBH*, the 99th percentile can be fitted to represent the cw_{max} value that could be safely assumed similar to open-grown trees. However, in the current research, due to the clustered structure of the raw data and the inherent correlation between measurements, it was preferred to combine *QR* with nonlinear mixed-effects models (*NLMEs*), which have effectively been used for analyzing similar grouped data structures [20,21]. The proposed *QR-NLME* model is currently based on the implementation of the stochastic approximation of the expectation-maximization (*SAEM*) algorithm [22] and the asymmetric Laplace (AL) distribution, as it has been described by Galarza et al. [20] through the *qrNLMM* package in R (cran) open source statistical software. In addition, based on the sample size, the 90th percentile was finally fitted, which mostly corresponds to suggestions by Condés and Sterba [14].

Despite the fact that the linear model is the most commonly used for cw_{max} predictions [23], a nonlinear (power) model of the mathematical expression $y = ax^b$ was selected in the current study, because the specific form has presented increased accuracy for crown width modeling of European beech [24], black pine [25], and Calabrian pine trees [26], and it can be easily linearized through logarithmic transformation. Russell and Weiskittel [1] also used the same model form for cw_{max} modeling of various species both conifers and hardwoods. In addition, a simple mixed-effects model was separately fitted to the same datasets aiming at an average (mean) crown width response. A detailed description of the mixed-effects fitting approach is provided by Calama and Montero [17]. In both those modeling procedures, it was assumed that both model parameters were composed of a random part, as long as convergence was feasible, following the suggestions of Pinheiro and Bates [27].

The mixed-effects modeling approach provides predictions for the population average (marginal) and for the subject-specific response. For the latter, additional data are required in order to calibrate the mixed-effects model at the local level (sample plot or stand). However, in some cases, the required data to localize the random part are difficult to obtain [28,29], and the population average response can be used instead, by setting the random effects equal to zero. Since data for open-grown trees are rarely available, the fixed part of the mixed-effects model was considered as a reasonable solution due to the low inter-subject variability, after percentile separation. In addition, the utility of the mixed-modeling approach in the specific case aimed at the population average response in an effort to capture the biological cw_{max} tendency at the species level, and therefore, local calibration was not necessary. The final cw_{max} models were compared graphically with those that were proposed by Hasenauer [30] for European beech and black pine in Austria, and by Kara and Topaçoğlu [31] for Calabrian pine in Turkey.

For the estimation of the *CCF* value at the final stage, the fixed part of the cw_{max} model was used as a potential unbiased solution at the population level. The statistical analysis was performed in the *R* (cran) open source statistical software environment, along with the *qrNLMM*, *ggplot2*, *lmfor*, and *NLME* libraries.

Mathematical Expression

The general formulation of the mixed-effects models is [17,20]

$$y_i = f(\Phi_i, x_i) + e_i \tag{1}$$

and

$$\Phi_i = A_i \beta_p + B_i b_i \tag{2}$$

where y is the response variable of the ith subject, Φ_i is a parameter vector of dimension r, and f is the nonlinear function form. The e_i term represents the independent and normally distributed vector of random errors. The parameter vector can be defined through Equation (2), where β_p is the regression coefficient corresponding to the pth quantile, A_i and B_i are design matrices of dimensions $r \times d$ and $r \times q$, for the fixed and random effects specific to each plot, and b_i is a q-dimensional random effects vector associated with the ith subject. Hence, the pth quantile regression for the nonlinear mixed-effects model can be expressed as [20]

$$Q_p = (y_{ij} | x_{ij},\ \beta_p) = f(\Phi_i,\ x_{ij}) = f(A_i \beta_p + B_i b_i,\ \ x_{ij}) + e_{ij} \tag{3}$$

In addition, for the specific case, the mathematical form of the basic mixed-effects model was

$$cw = (\beta_1 + b_1) DBH^{(\beta_2 + b_2)} + e \tag{4}$$

where β_1, β_2 is the fixed part and b_1, b_2 is the random part of the model, cw is the crown width (m), and DBH is the diameter at breast height (cm).

Assuming that the vertical projection of a tree crown is approximately circular shaped, the general maximum crown projection area (*cpa*) of the ith tree crown can be estimated from the following type:

$$cpa_i = \frac{\pi}{4} cw_{i\ max}^2 \ \ or \ \ cpa_i = \frac{\pi}{4}\left(\beta_1 DBH_i^{\beta_2}\right)^2 \tag{5}$$

where β_1, β_2 is the fixed part of the quantile nonlinear mixed-effects model (*QR-NLME*) when no auxiliary information is available, and cpa_i is the vertical crown projection area of ith tree (m^2).

The fixed part of the model may also be localized per plot through a limited sample of data, as one of the main attributes of mixed-effects models, which is not recommended in this specific case.

The *CCF* of a forest stand including n trees, growing on a total area A, is expressed as the quotient between the sum of the potential (maximum) crown projection areas cpa_1 + cpa_2 + … cpa_n of all living trees and the total area A, multiplied by 100 [6] as follows:

$$CCF = \frac{1}{A} \sum_{i=1}^{n} cpa_i 100 \tag{6}$$

where *CCF* is the crown competition factor (dimensionless); *A* is the total stand area (m^2), which may represent the plot area [9,32]; *n* is the total number of living trees.

3. Results

The descriptive statistics of the total sample (n = 4064) per species are presented in Table 1.

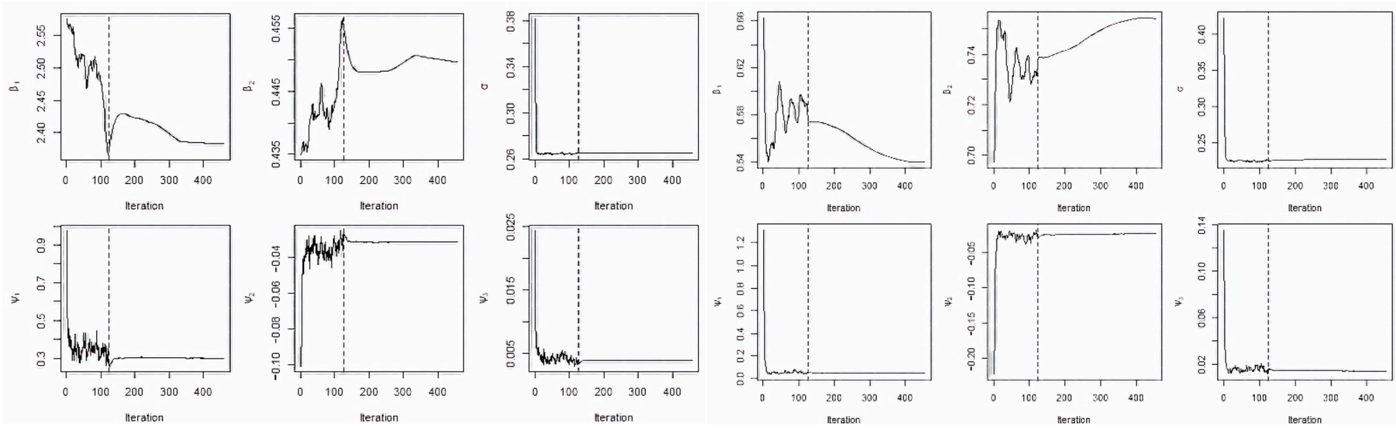
Table 1. Descriptive statistics of the total sample.

***Fagus Sylvatica* L. (*n* = 1414)**	**Min**	**Mean**	**Max**	**SD**
DBH (cm)	0.2	24.40	88.5	14.04
h (m)	1.5	18.27	34.4	6.85
cw (m)	0.9	7.60	19.6	2.91
Number of trees per ha	220	488	880	166
Basal area ($m^2 \cdot ha^{-1}$)	11.59	30.32	48.52	8.75
***Pinus nigra* Arn. (*n* = 770)**				
DBH (cm)	3.1	29.17	77.2	12.71
h (m)	1.9	16.45	24.8	4.76
cw (m)	0.3	4.78	15.0	2.27
Number of trees per ha	320	616	1060	165
Basal area ($m^2 \cdot ha^{-1}$)	26.87	48.94	82.50	12.85
***Pinus brutia* Ten. (*n* = 1880)**				
DBH (cm)	0.7	28.15	77.9	9.84
h (m)	1.7	14.44	21.7	4.12
cw (m)	0.4	6.42	15.0	2.31
Number of trees per ha	210	498	750	129
Basal area ($m^2 \cdot ha^{-1}$)	11.39	32.87	60.88	12.71

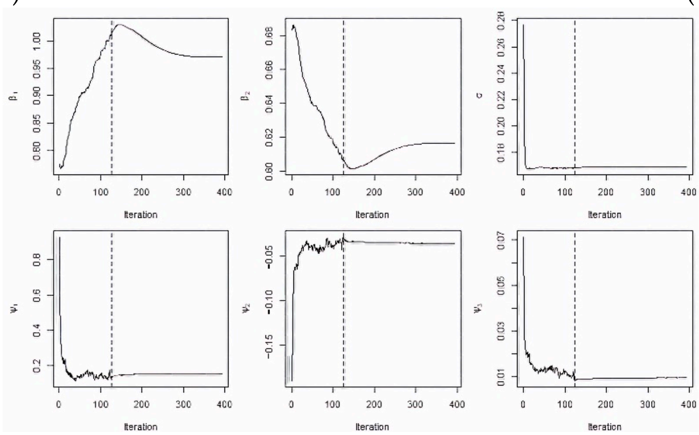
3.1. Convergence

The *QR-NLME* model successfully converged in all cases using the default values for maximum iterations (*MaxIter* = 500) and for the cut point (c = 0.25), which determines the percentage of initial iterations with no memory (Figure 2). It should be noted that convergence is not always ensured, and different starting values of parameter combinations should be tested instead.

Based on the variance of the parameters at plot level (Figure 3 and Table 2), it was assumed that both parameters of the average (mean) mixed-effects nonlinear model contain random parts, and the model was not simplified further. In addition, for the same model, a power form variance function was used in order to stabilize the error's variance, since it presented the best fitting ability during the preliminary research stage. The variance function was of the form $Var(e) = \sigma^2 DBH^{2k}$, where σ^2 is the residual variance of the estimated model, and $\boldsymbol{k}$ is the variance function coefficient. The average model converged for the three species and the residual plots revealed no heteroscedastic trends across the total range of the fitted values (Figure 4), after weighting. The value of the k coefficient is also presented in Table 2.

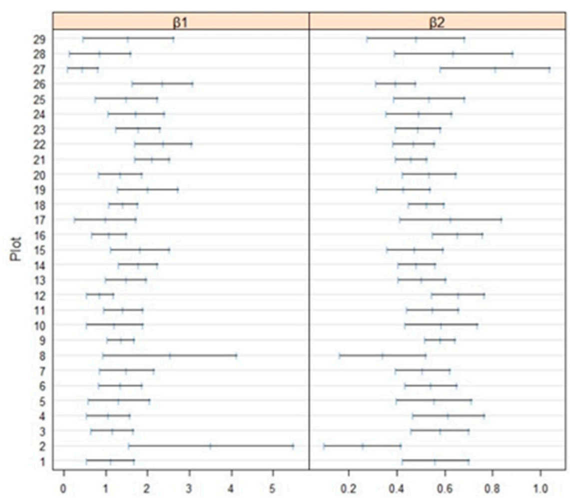


(**a**) (**b**)



(**c**)

Figure 2. Graphical summary of successful convergence for the fixed-effect parameters of the cw_{max} model, variance components of the random effects, and nuisance parameters, extracted from the *qrNLMM* package for European beech (**a**), black pine (**b**), and Calabrian pine (**c**) data. The vertical dashed lines delimit the beginning of the almost sure convergence, as defined by the cut-point parameter (c = 0.25).



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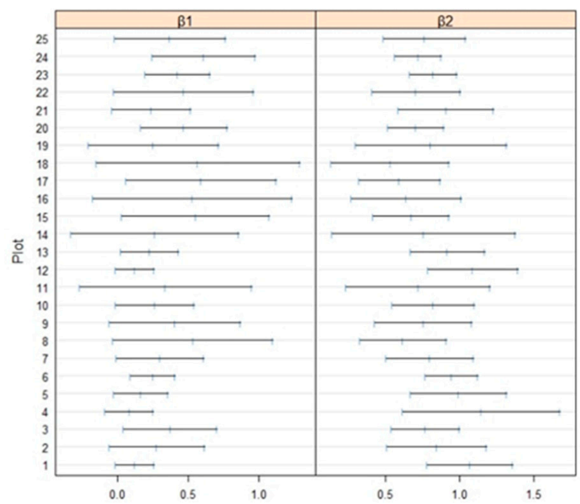

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Figure 3. *Cont.*

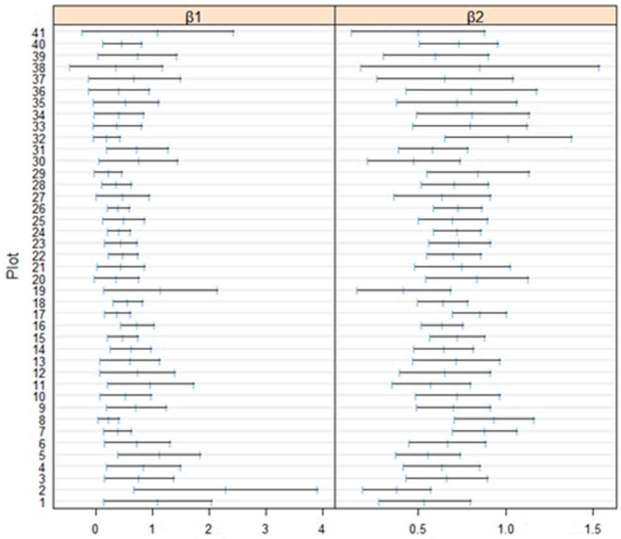

Figure 3. The 95% confidence intervals on the average power model parameters per sample plot for European beech (**a**), black pine (**b**), and Calabrian pine (**c**) data, based on the *nlsList* fit (*NLME* package). A considerable variability among plot parameter estimates for all species can be observed, revealing the need for introducing random effects to account for the between-plot variation.

Table 2. Model parameters and fitting statistics for all species.

Model Form $\beta_1 DBH^{\beta_2}$			
	Fagus sylvatica	*Pinus nigra*	*Pinus brutia*
Maximum Crown Width Model			
β_1 (SE)	2.38356 (0.0692)	0.53990 (0.0377)	0.96984 (0.0423)
β_2 (SE)	0.44960 (0.0083)	0.75397 (0.0231)	0.61605 (0.0091)
$var(b_1)$	0.29972	0.04670	0.15569
$var(b_2)$	0.00392	0.01397	0.09540
$\mathrm{cov}(b_1,b_2)$	−0.03149	−0.02432	−0.03528
σ^2	0.07032	0.05151	0.02862
AIC	6017.14	3069.26	6375.88
Log likelihood	−3002.57	−1528.63	−3181.94
Percentile	90	90	90
Average (Mean) Crown Width Model			
β_1 (SE)	1.49139 (0.0810)	0.31579 (0.0275)	0.54825 (0.0356)
β_2 (SE)	0.52723 (0.0148)	0.81158 (0.0245)	0.71199 (0.0174)
$var(b_1)$	0.11863	0.00489	0.02402
$var(b_2)$	0.00338	0.00293	0.00413
$\mathrm{cov}(b_1,b_2)$	−0.01821	−0.00265	−0.00799
σ^2	0.59047	0.06751	0.04278
k	0.21597	0.47467	0.47917
AIC	5201.23	2561.58	5474.78
Log likelihood	−2593.61	−1273.79	−2730.39
Fitting index	0.74459	0.68786	0.74843
Root mean square error	1.4702	1.2649	1.0194
Bias	0.0000	−0.0068	−0.0033

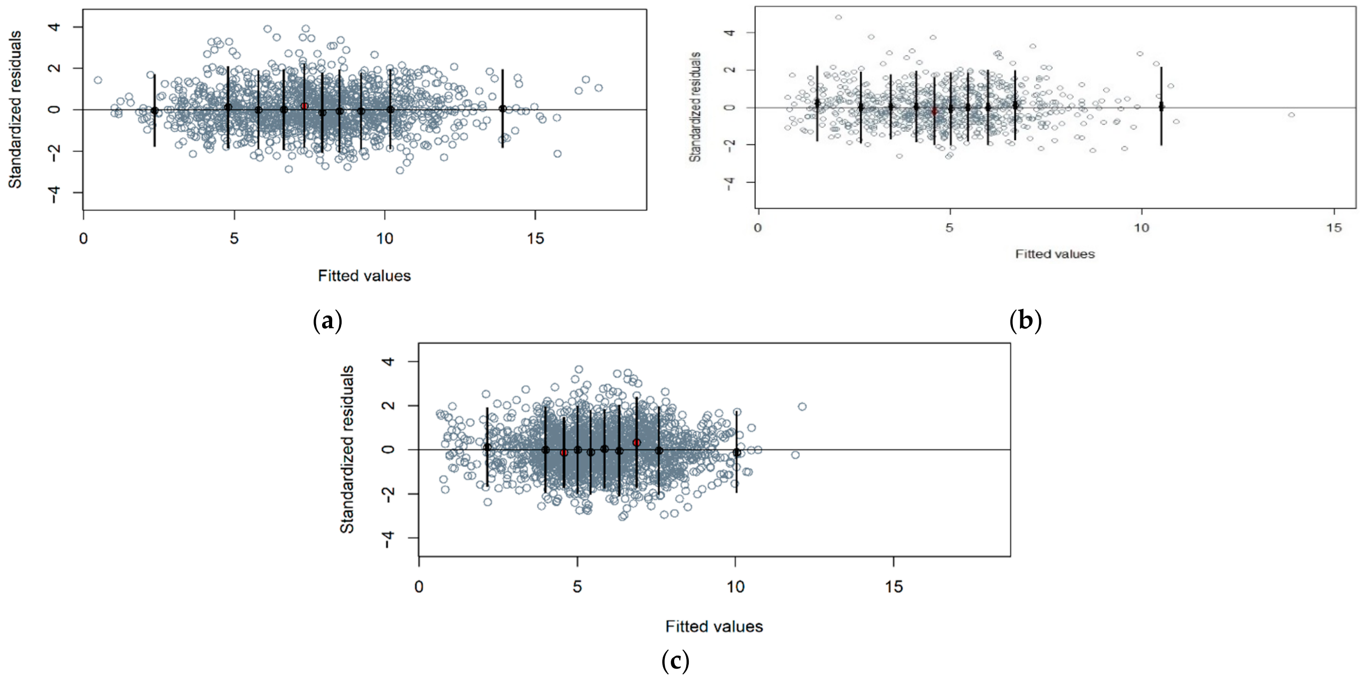

Figure 4. Standardized residual plot of the weighted average mixed-effect model, for European beech (**a**), black pine (**b**), and Calabrian pine (**c**) data. The large dots represent the mean values of the residuals according to 10 classes. The thick and thin vertical lines represent the 95% confidence interval of class mean and of one observation (mean ± SD), respectively. The red color indicates the lines that do not cross the baseline at $y = 0$.

3.2. Fitting Procedure

The results of the fitting procedure of both models for all species are presented in the following Table 2.

All model parameters were significantly different from zero according to the approximate t-criterion ($p < 0.001$ for all species). The Fitting index from Table 2, which is similar to R^2, lead to the conclusion that the mixed-effects model with the *DBH* as the sole independent variable managed to explain the 74.5%, 68.8%, and 74.8% of the crown width variance in European beech, black pine, and Calabrian pine, respectively, which is a very important finding in terms of practical use in field conditions. Both, the error and the prediction bias, ranged to low levels (Table 2).

3.3. Graphical Representation

Figure 5 depicts the graphical performance of the developed cw_{max} models separately for each of the three forest species. For comparison reasons, the regression (dashed/dotted) lines of the equivalent suggested cw_{max} models for the same species were added within the same graphs. It should be noted, however, that the selected models were developed using data from open-grown trees in Austria and Turkey, in contrast with the current research, in which the tree data of various densities and competition status were clustered at the sample plot level. The population responses (fixed effects) of the average (mean) models are graphically presented in dashed-line style. As it can be observed, a nonlinear relationship between the crown width and the *DBH* is revealed, especially for the European beech tree data, and to a lesser extent for the Calabrian pine tree data. The basic model of power form presented increased flexibility and managed to perform well against almost linear distributed data, as it can be concluded from the right (b) section of Figure 5, which corresponds to the black pine tree data.

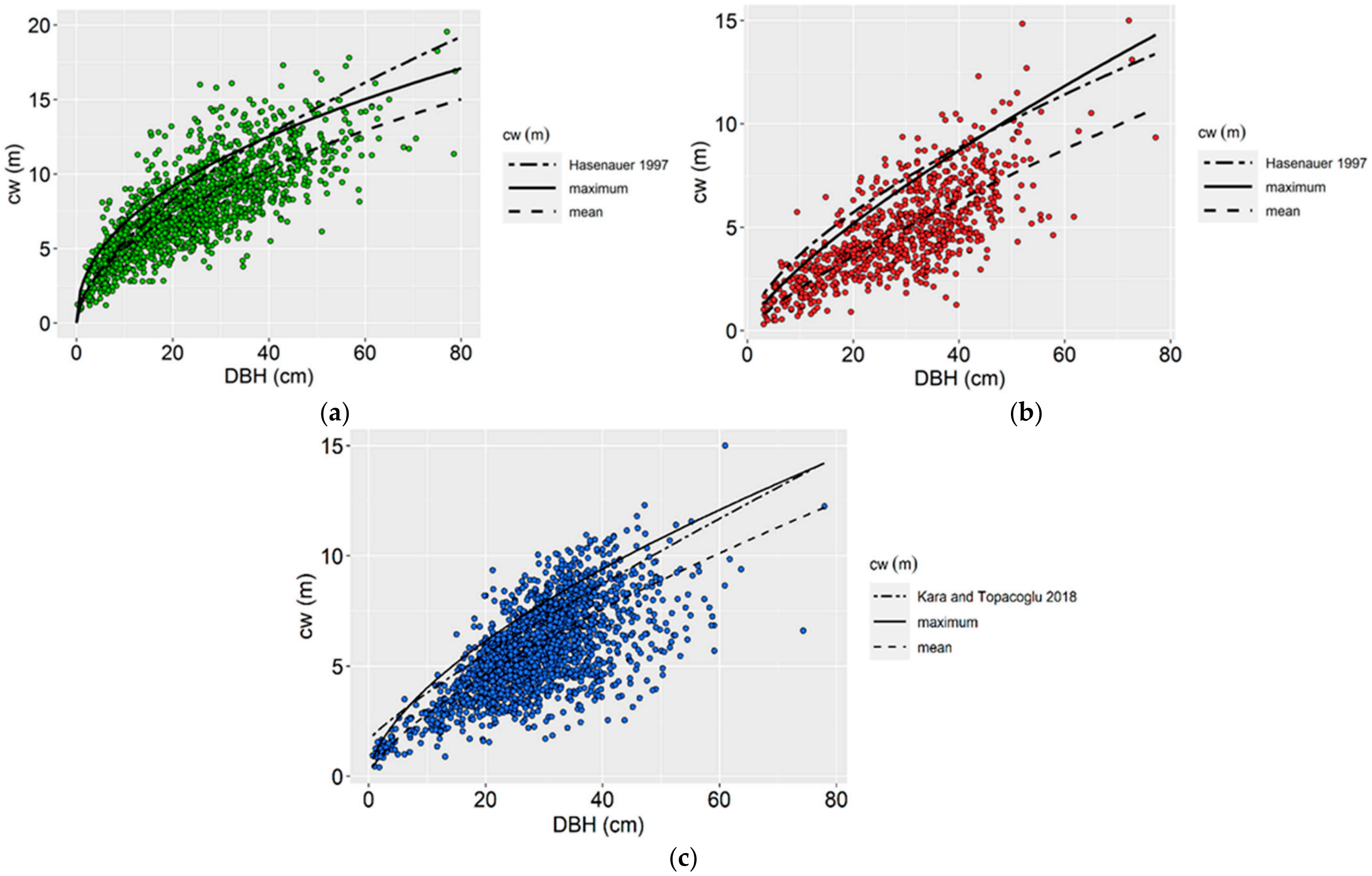

Figure 5. Comparative graphs of the developed models for cw_{max} prediction (solid line), along with a general model for the mean crown width (dashed line), for European beech (**a**), black pine (**b**), and Calabrian pine (**c**) tree data. The cw_{max} models by Hasenauer [30] for European beech and black pine in Austria, and by Kara and Topaçoğlu [31] for Calabrian pine in Turkey, are also graphically presented.

3.4. The Crown Competition Factor (CCF) Estimation

Using the population-average cw_{max} model and Equations (5) and (6), *CCF* was estimated for each species at the sample plot level. The results are presented in the following Table 3.

Table 3. The estimated *CCF* values per sample plot, based on the population average approach for cw_{max} estimation.

Species	Crown Competition Factor (CCF)			
	Min.	**Mean**	**Max.**	**SD**
Fagus sylvatica	218.3	378.5	598.6	89.9
Pinus nigra	138.8	245.2	388.8	58.9
Pinus brutia	86.5	214.2	376.4	65.9

The European beech presented the highest levels of competition in the specific area, where it reaches its optimal growth. For the pine species, the competition levels were relatively close, with black pine forest stands presenting slightly higher *CCF* values than those of Calabrian pine.

4. Discussion

In order to deal with the problem of the correlation of measurements from the same subjects, Sun et al. [21] introduced the quantile regression for the linear mixed-effects model into crown profile modeling, with encouraging results. Furthermore, the quantile regression provides the opportunity of estimating the conditional distribution of dependent variables while assessing the effects of regressors at various quantiles [33]. The same authors compared the quantile regression method and the mixed-effects modeling technique for height prediction of two forest species in Turkey. Cao and Wang [34] also compared the two methods to calibrate a taper equation for loblolly pine (*Pinus taeda* L.) species in Louisiana. To the best of our knowledge, this is the first time that these novel methods are linked together with a nonlinear model form in order to predict the cw_{max} dimension at tree level, accounting for clustered data structures from sample plots. This approach is also recommended in the case of hierarchical measurements over time, which is common in the case of permanent sample plots installation. From the modeling perspective, there are no similar approaches on cw_{max} prediction, and the results must be related to relevant studies of different modeling approaches.

Hasenauer [30] developed cw_{max} models for European beech and black pine in Austria, while Kara and Topaçoğlu [31] suggested similar models for the Calabrian pine in Turkey. The results of the current study present some notable similarities with the outcomes of the reported models, as can be seen in Figure 5. The differences appearing between the models' predictions were expected since the data from the previous studies represented samples from completely different regions in Austria and Turkey. Similar findings were reported by Russell and Weiskittel [1], who compared their predictions with relevant models from other regions in the US. In addition, the open conditions for open-grown trees could have affected the crown width expression because those types of trees are usually exposed to climatic factors (wind, snow, etc.), compared with stand-grown trees, which are certainly affected by the competition status for light. The largest differences were observed on the European beech cw_{max} curves, mainly for small-diameter trees. The comparison between the Calabrian pine models leads to the conclusion that the corresponding curves had almost identical shapes, with the current model to provide slightly increased cw_{max} predictions, over a wide range of diameter classes. For the black pine, the cw_{max} curves are relatively similar, and the differences are very small, which might have a genetic explanation [35]. Hence, the proposed method seems to produce reliable cw_{max} predictions for the two conifer species, while additional research is needed in the case of European beech, for which data from the same region must be compared.

For the development of the average crown width model, the mixed-effects approach presented sound results, explaining the largest part of crown variation according to the corresponding fitting index (Table 2). The increased flexibility of the simple mixed-effects model has also been reported by other researchers [24,36], and it is based on the fact that the random part of the model was able to capture the effect of other factors rather than *DBH*, which were not included as additional regressors into the fitting model form. From that perspective, the random effects can be correlated with variables at a tree or stand level [37], leading to the development of generalized models that may, in turn, be expanded to mixed-effects and explain the functional mechanisms of crown allometry. The inherent simplicity of the proposed average crown width models is rather an advantage for practical use purposes since the *DBH* is the most common tree variable measured in forest inventories and management plans. Transferability on potential model applications should be based on the restrictions dictated by the model conditions, particularly those referred to the specific stand densities, as they are described in Table 1.

In order to select the 90% quantile for the *QR* analyses, different percentage values of quantiles were tested. Since *CCF* is assumed independent from site index [9], the value of 400 has been assumed as a reasonable upper bound, representing a stable condition between growth and mortality [9,14], although significantly higher values have been reported only as maximum values. In this context, Arney [9] reported a maximum *CCF* value of 701 for Douglas-fir (*Pseudotsuga menziesii*) naturally regenerated forests. The 99% *QR* led to unreasonable mean *CCF* values, especially for European beech species, which is further endorsed by the fact that these forests are subject to various management regimes. In addition, the 99% quantile could have been affected by outliers, common in large datasets, since no smoothing method has been applied. Similar findings were derived using 95% as quantile input. The 90% quantile presented reasonable results, which were further verified for all species according to Figure 5.

The analysis showed increased competition status for the European beech trees, while the corresponding values for the pine species were found to be at lower levels (Table 3). Since the *CCF* is supposed to be independent of the stand structure [13], those discrepancies may be attributed to the differences in the estimated cw_{max} for the studied species, provided that the stem density alone led to opposite conclusions (see the number of trees per ha in Table 1). From Table 2, it is inferred that, for example, a tree with 40 cm *DBH* is expected to develop a 12.52 m cw_{max} if it is a beech open-grown tree, 8.71 m if it is a black pine tree, and 9.41 m if it is a Calabrian pine tree. Hence, this may potentially lead to increased *CCF* values for the European beech plots, as it is depicted in Table 3. On the other hand, the difference in the competition status between black pine and Calabrian pine was expected to be larger, due to the differences between the management regimes, which are typically applied for the two species. Black pine forests are usually managed for wood production [38], while the Calabrian pine peri-urban forests are mainly managed for protection and recreation [39]. However, the adoption of selective cuttings as the primary silvicultural treatment in black pine forests has ensured the continuity of the forest cover, as well as the regeneration of this type of forest in a sustainable way.

Critical issues during the *CCF* application are the threshold values that have been proposed in order to schedule silvicultural treatments to regulate the stand competition in the frame of sustainable forest management. Arney [9] proposed the threshold value of 200 for the crown recession of coastal Douglas fir and 300 for suppression mortality. Castaldi et al. [40] reported increased values of *CCF* for the same species in Italy, reaching 619 in some forest stands. Since there are no similar values for the species of the current study, the *CCF* measure must be correlated with mortality and productivity aspects per species in a featured research study.

The application of the SAEM algorithm on a relatively large dataset was a complex process; thus, some convergence problems emerged. Indeed, for the Calabrian pine dataset, the model converged after 393 iterations within a total processing time of 36.7 min. For the European beech dataset, the model converged after 458 iterations and 30.3 min, and for the black pine after 454 iterations and 19.1 min of processing time. In addition, the model failed to converge in two cases, and the procedure had to start from the beginning.

5. Conclusions

The nonlinear quantile mixed-effects approach developed reliable cw_{max} models for two conifer species in northern Greece—the black pine (*Pinus nigra* Arn.) and the Calabrian pine (*Pinus brutia* Ten.). The proposed method can be safely applied in cases of the absence of open-grown tree data, while at the same time, it rectifies the problem of correlated observations from clustered data structures. In the case of the European beech (*Fagus sylvatica* L.) species, the proposed method should be applied with caution, and additional research is required.

Author Contributions: Conceptualization, D.I.R.; methodology, D.I.R. and V.K.; software, D.I.R.; formal analysis, D.I.R., V.K. and A.K.; investigation, D.I.R.; resources, D.I.R. and V.K.; data curation, D.I.R., S.K., C.S. and D.P.; writing—original draft preparation, D.I.R. and V.K.; writing—review and editing, A.K. and T.T.; project administration, V.K.; funding acquisition, V.K. and A.K. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partly funded by the European Regional Fund and the Eastern Macedonia and Thrace Region of Greece NSRF (ESPA) 2007-2013, MIS 430008.

Data Availability Statement: The data are available on reasonable request.

Acknowledgments: The authors would like to thank the three anonymous reviewers of the manuscript, who provided insightful comments that significantly improved our paper.

Conflicts of Interest: The authors declare no conflict of interest.

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